

# Nuclear dipole photoabsorption cross section and polarizability in the Self-Consistent Green's Function approach

Probing exotic structure of short-lived nuclei  
by electron scattering

ECT\* (16-20 July 2018)

Collaborators:

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Vittorio Somà (CEA)

Petr Navrátil (TRIUMF)

Francesco Raimondi

(University of Surrey)



# Outline

- Self-consistent Green's function (SCGF) method
  - Non-perturbative treatment
  - Three-nucleon interactions
- Dipole Response Function and Polarisability in medium mass nuclei
  - $^{14}\text{O}$ ,  $^{16}\text{O}$ ,  $^{22}\text{O}$  and  $^{24}\text{O}$
  - $^{36}\text{Ca}$ ,  $^{40}\text{Ca}$ ,  $^{48}\text{Ca}$  and  $^{54}\text{Ca}$
  - $^{68}\text{Ni}$

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# Electromagnetic response in SCGF

OBSERVABLES

$$\begin{aligned} \sigma_\gamma(E) &= 4\pi^2 \alpha E R(E) && \text{PHOTOABSORPTION CROSS SECTION} \\ \alpha_D &= 2\alpha \int dE \frac{R(E)}{E} && \text{ELECTRIC DIPOLE POLARIZABILITY} \end{aligned}$$

Response  $R(E)$  depends on excited states of the nuclear system, when “probed” with dipole operator  $\hat{D}$

$$R(E) = \sum_{\nu} |\langle \psi_{\nu}^A | \hat{D} | \psi_0^A \rangle|^2 \delta_{E_{\nu}, E}$$



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$$R(E) = \sum_\nu \left| \langle \psi_\nu^A | \hat{D} | \psi_0^A \rangle \right|^2 \delta_{E_\nu, E}$$

$$\sum_{ab} \langle a | \hat{D} | b \rangle \langle \psi_\nu^A | c_a^\dagger c_b | \psi_0^A \rangle$$

s.p. matrix element of the dipole one-body operator

Nuclear structure component:  
Transition density matrix

# Polarization propagator and Bethe-Salpeter equation

$$\Pi_{\gamma\delta,\alpha\beta}(\omega) = \sum_n \frac{\langle \Psi_0^A | a_\delta^\dagger a_\gamma | \Psi_n^A \rangle \langle \Psi_n^A | a_\alpha^\dagger a_\beta | \Psi_0^A \rangle}{\hbar\omega - (E_n^A - E_0^A) + i\eta}$$

Two-body Propagator

$$\epsilon_n^\pi \equiv E_n^A - E_0^A$$

Energies of the excited states  
of the A-nucleon system

Equation for the polarization propagator

$$\Pi_{\gamma\delta,\alpha\beta}(\omega) = \Pi_{\gamma\delta,\alpha\beta}^f(\omega) + \Pi_{\gamma\delta,\mu\rho}^f(\omega) K_{\mu\sigma,\rho\nu}^{(p-h)}(\omega) \Pi_{\nu\sigma,\alpha\beta}(\omega)$$

Free polarization  
Propagator

p-h kernel

# Approximated solution of the Bethe-Salpeter equation

$$\Pi_{\gamma\delta,\alpha\beta}(\omega) = \Pi_{\gamma\delta,\alpha\beta}^f(\omega) + \Pi_{\gamma\delta,\mu\rho}^f(\omega)K_{\mu\sigma,\rho\nu}^{(p-h)}(\omega)\Pi_{\nu\sigma,\alpha\beta}(\omega)$$

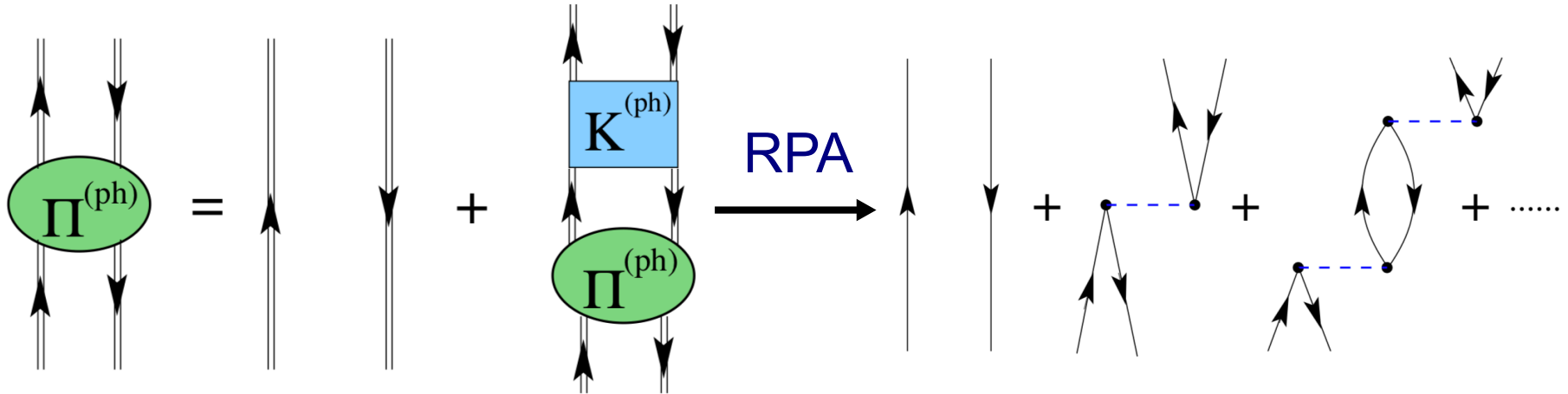


Fig. from PRC 68, 014311 (2003)

Extension of the RPA: 1) Fully-dressed (correlated) single-particle propagator in the RPA diagrams

C. Barbieri, W. Dickhoff PRC 68, 014311 (2003)

2) Reduction of the number of poles of the dressed propagator

C. Barbieri, M. Hjorth-Jensen PRC 79, 064313 (2009)

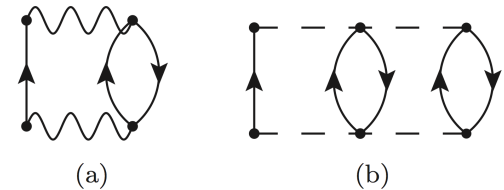
# Self-energy and Dyson equation

$$G_{\alpha\beta}(\omega) = G_{\alpha\beta}^{(0)}(\omega) + \sum_{\gamma\delta} G_{\alpha\gamma}^{(0)}(\omega) \Sigma_{\gamma\delta}^*(\omega) G_{\delta\beta}(\omega)$$

**Self-energy: effective potential affecting the s.p. propagation in the nuclear medium**

- Post-Hartree-Fock method based on self-consistency
- Based on realistic 2N and 3N forces
- Expansion of self-energy in Feynman diagrams
- Non-perturbative resummation of the correlations.

Second Order diagrams

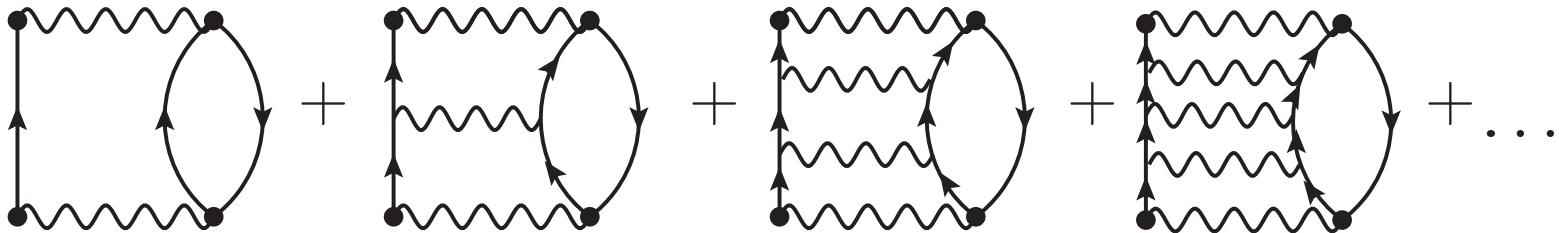


## Algebraic Diagrammatic Construction (ADC(n))

J. Schirmer and collaborators:  
Phys. Rev. A26, 2395 (1982)  
Phys. Rev. A28, 1237 (1983)

# Dyson ADC(n)

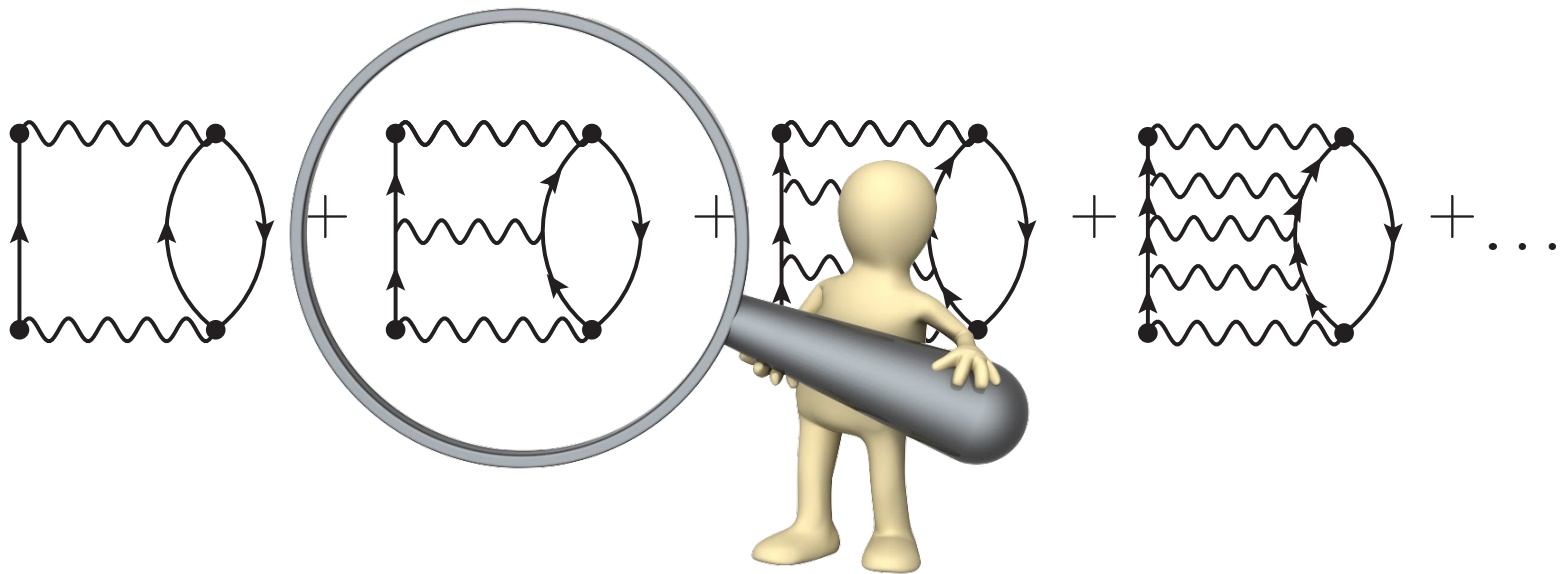
Self-energy expansion is treated **NON-perturbatively**:  
Entire classes of self-energy diagrams (ladder and ring) are summed  
at infinite order by means of a geometric series





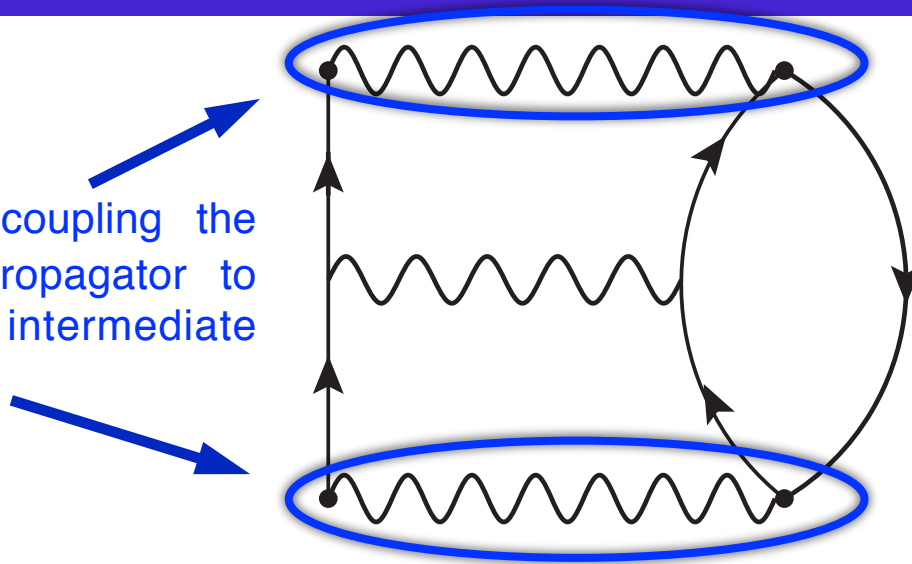
# Dyson ADC(3)

**Self-energy expansion is treated NON-perturbatively:**  
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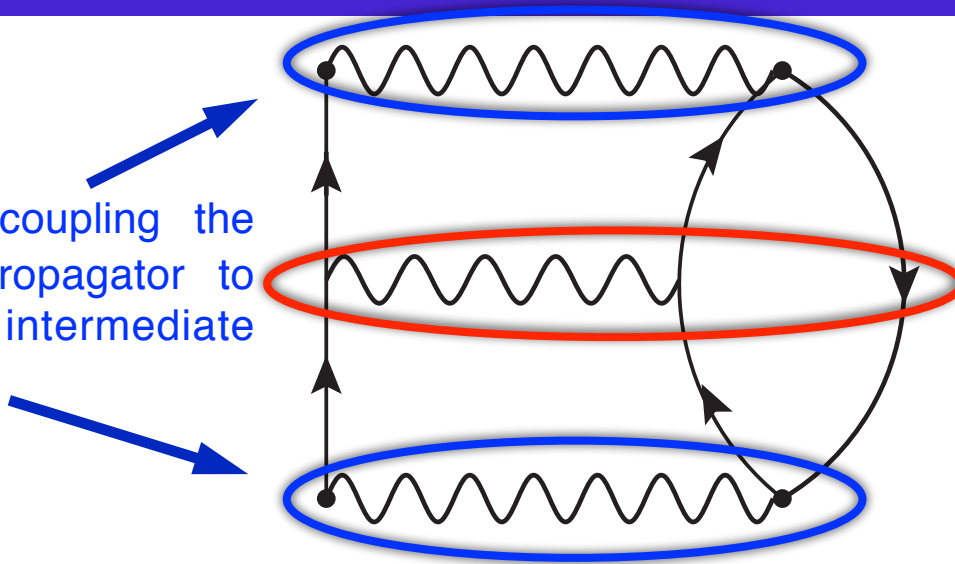
# Dyson ADC(3)

*M*: matrices coupling the single-particle propagator to more complex intermediate configurations



# Dyson ADC(3)

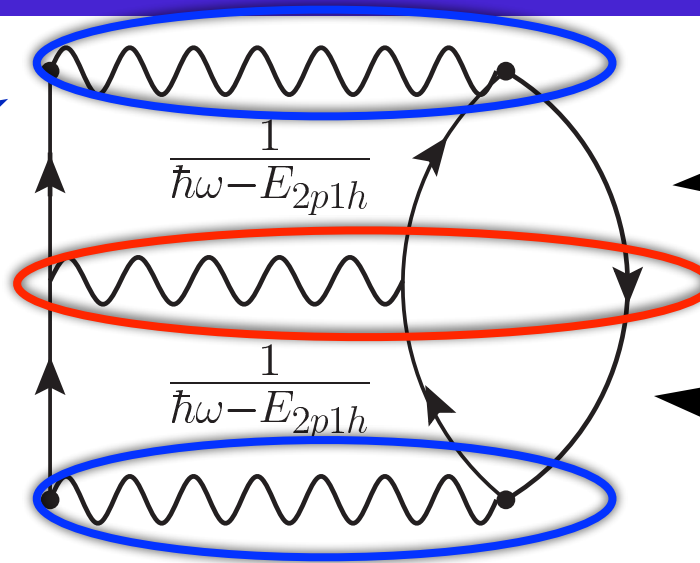
$M$ : matrices coupling the single-particle propagator to more complex intermediate configurations



$C$ : interaction matrix linked only to internal fermion lines

# Dyson ADC(3)

$M$ : matrices coupling the single-particle propagator to more complex intermediate configurations



Propagator  
(intermediate state configurations)

$C$ : interaction matrix linked only to internal fermion lines

Propagator  
(intermediate state configurations)

The set of ladder diagrams is a geometric series

$$\begin{array}{c}
 \begin{array}{ccccccc}
 \begin{array}{c} \text{Diagram 1} \\ \mathcal{M}^\dagger \frac{1}{\hbar\omega - E_{2p1h}} \mathcal{M} \end{array} & + & \begin{array}{c} \text{Diagram 2} \\ \mathcal{M}^\dagger \frac{1}{\hbar\omega - E_{2p1h}} C \frac{1}{\hbar\omega - E_{2p1h}} \mathcal{M} \end{array} & + & \begin{array}{c} \text{Diagram 3} \\ \mathcal{M}^\dagger \frac{1}{\hbar\omega - E_{2p1h}} C \frac{1}{\hbar\omega - E_{2p1h}} C \frac{1}{\hbar\omega - E_{2p1h}} \mathcal{M} \end{array} & + & \dots
 \end{array}
 \end{array}$$

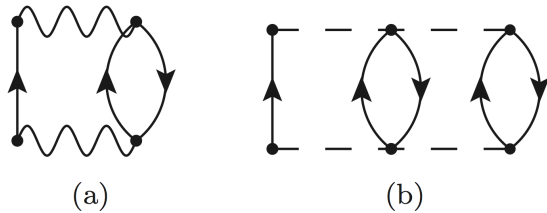
Sum

$$\mathcal{M}^\dagger \frac{1}{\hbar\omega - E_{2p1h} - C} \mathcal{M}$$

# Interaction-irreducible Self-Energy with NN and 3NFs

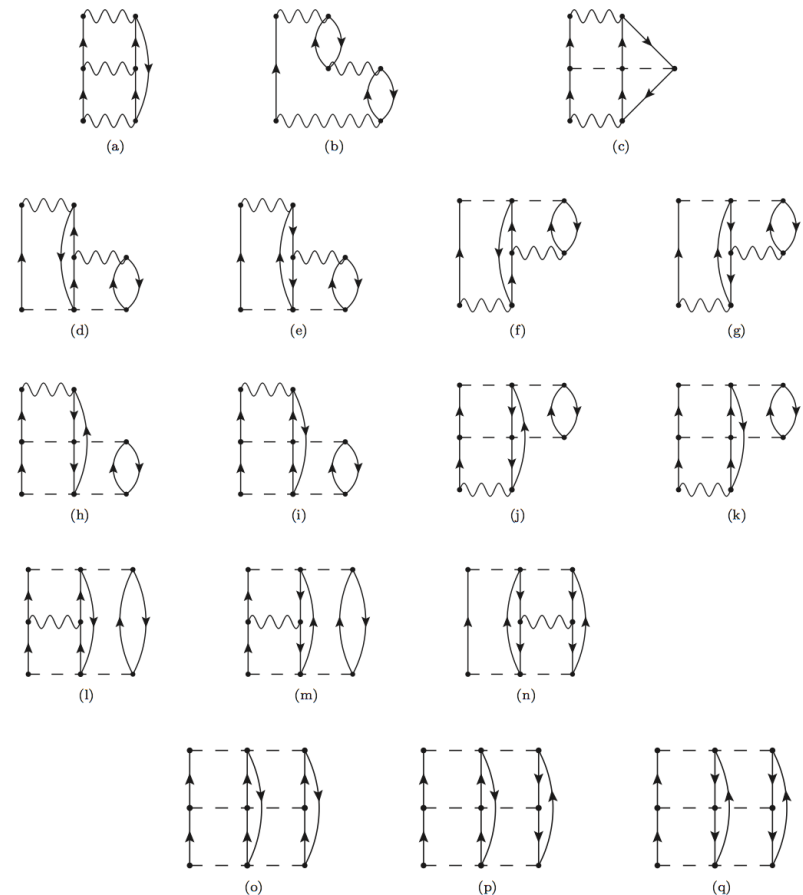
## ADC(2) ( $\neq$ 2<sup>nd</sup> order)

Second-order diagrams  
with NN and 3N forces



## ADC(3) ( $\neq$ 3<sup>rd</sup> order)

Third-order diagrams with NN and 3N forces



Complete set of ADC(3)  
working equations in  
FR, C. Barbieri  
Phys Rev C **97**, 054308 (2018)

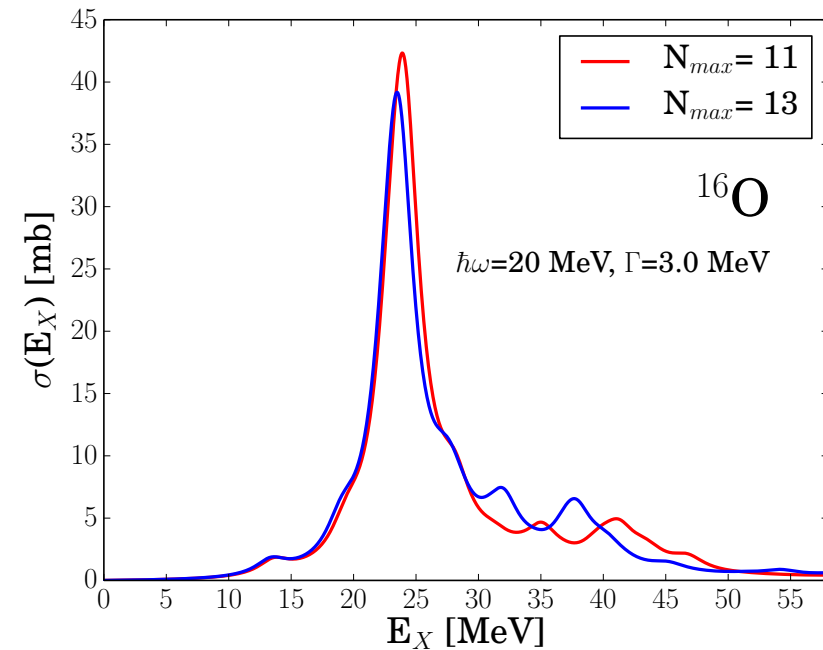


# Outline

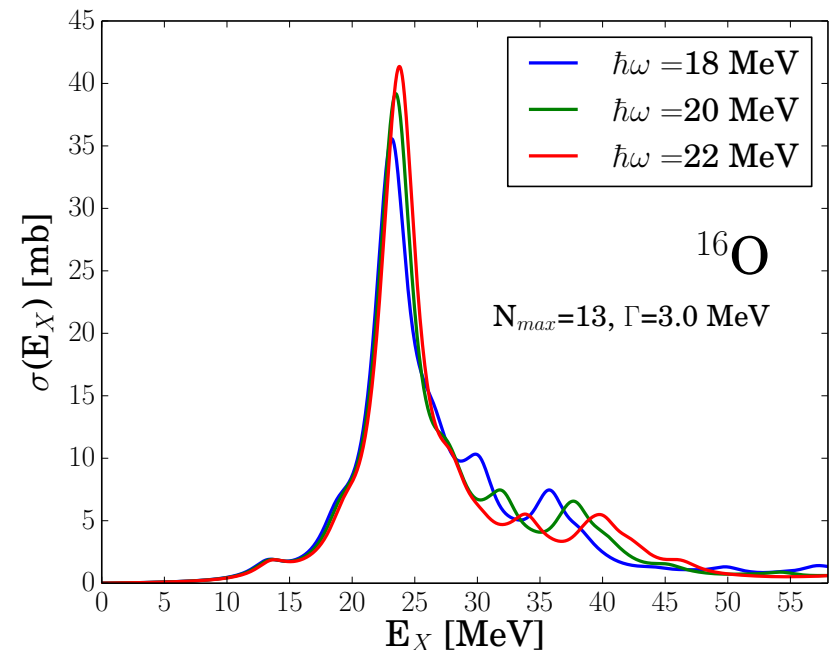
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# Features of the calculation

- NN and 3N nuclear interaction  $\text{NNLO}_{\text{sat}}$  (Phys. Rev. C 91, 051301(R))
- Electric dipole operator  $E1$   $\hat{Q}_{1m}^{T=1} = \frac{N}{N+Z} \sum_{p=1}^Z r_p Y_{1m} - \frac{Z}{N+Z} \sum_{n=1}^N r_n Y_{1m}$
- Single-particle harmonic oscillator basis ( $N_{\text{max}}, \hbar\omega$ )

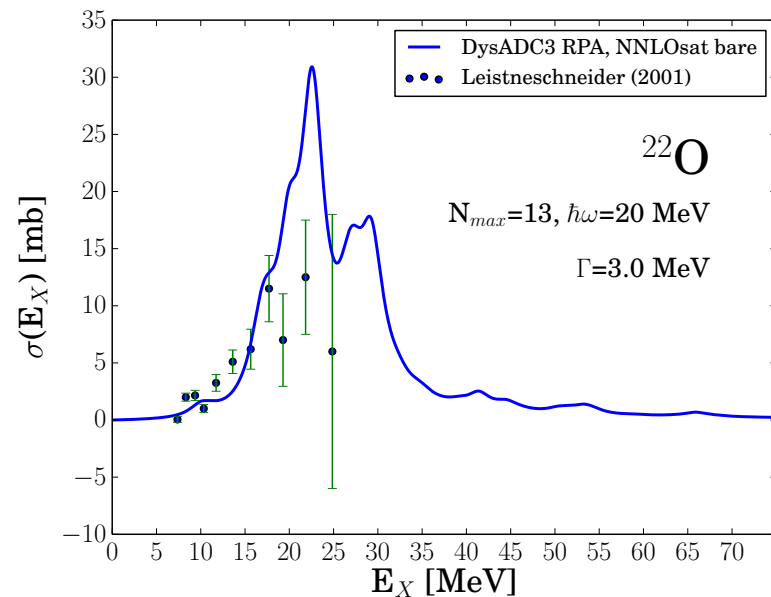
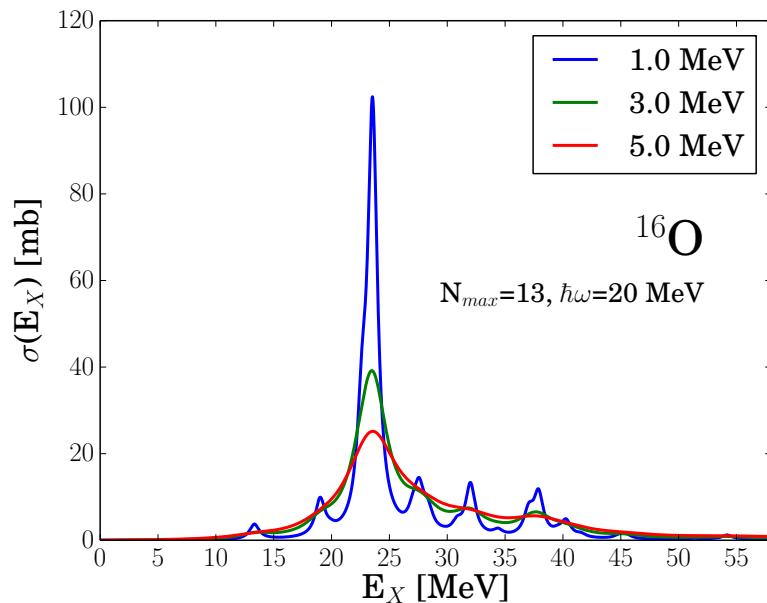
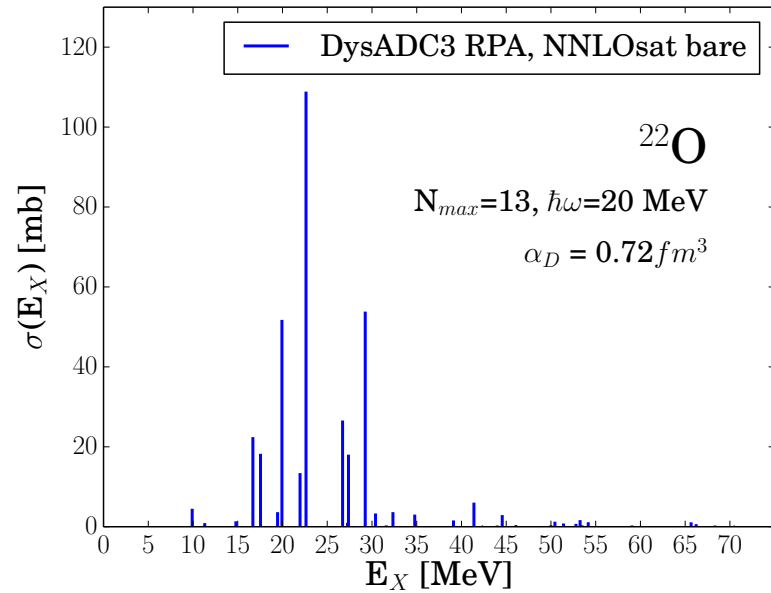
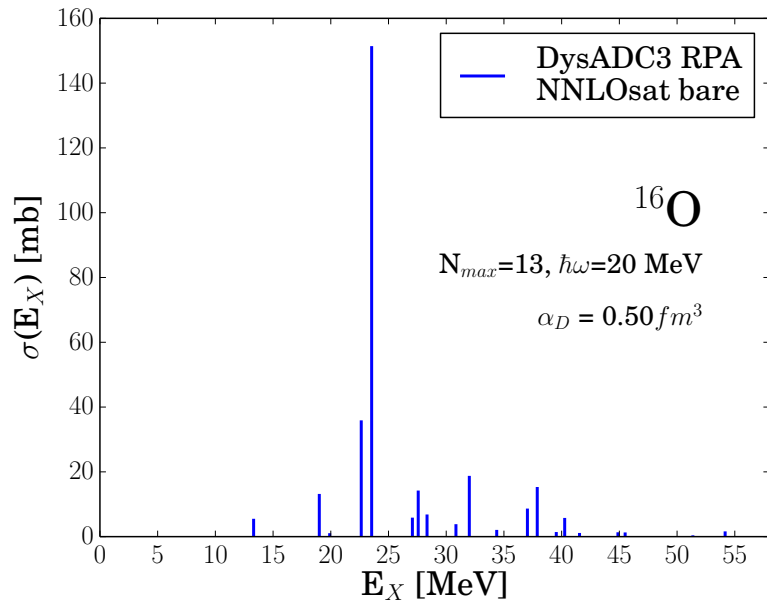


$$\delta(\alpha_D, N_{\text{max}}) = 2\%$$

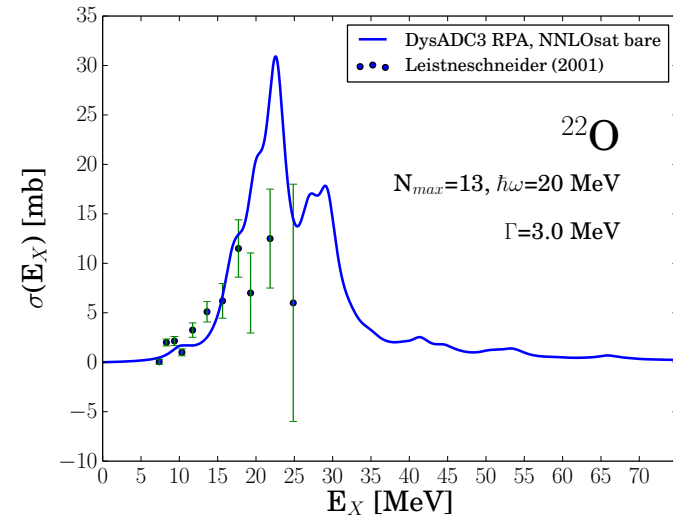
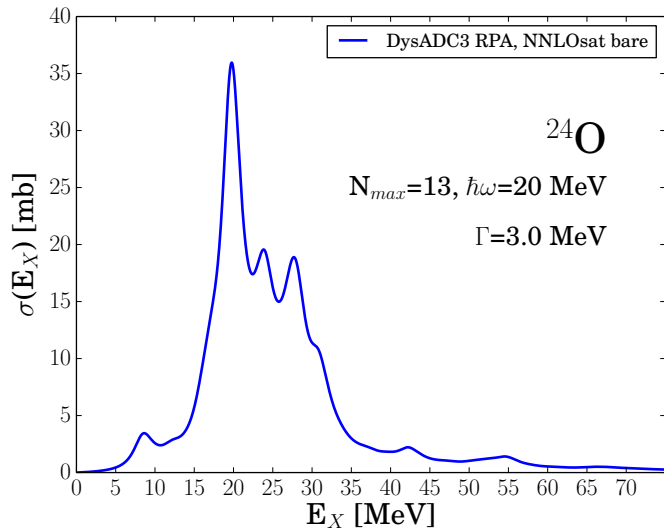
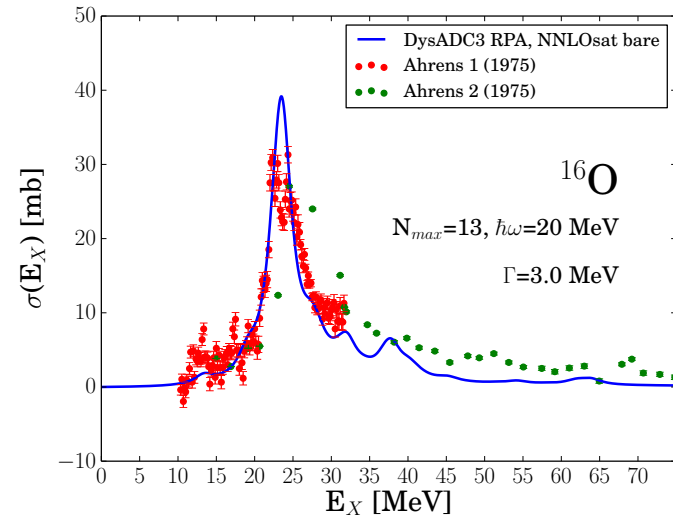
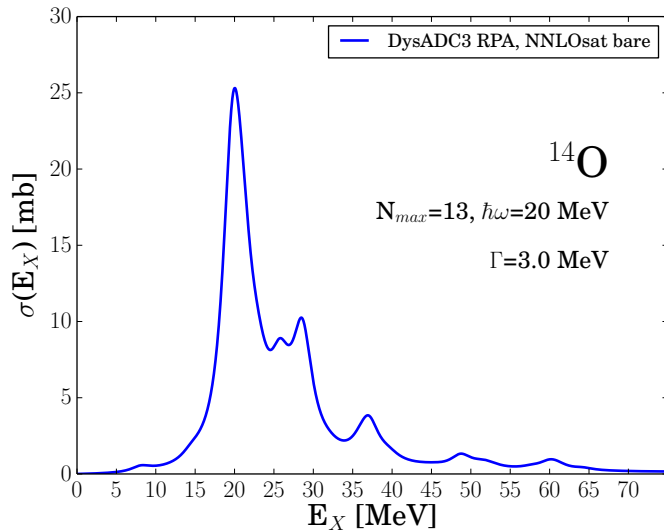


$$\delta(\alpha_D, \hbar\omega) = 1.5\%$$

# Discrete vs convoluted photoabsorption $\sigma$



# Results for Oxygen isotopes



$$\sigma_\gamma(E) = 4\pi^2\alpha E R(E)$$

- GDR position of  $^{16}\text{O}$  reproduced
- Hint of a soft dipole mode on the neutron-rich isotope

# Polarizability in $^{16}\text{O}$ and $^{22}\text{O}$

$$\alpha_D = 2\alpha \int dE \frac{R(E)}{E}$$

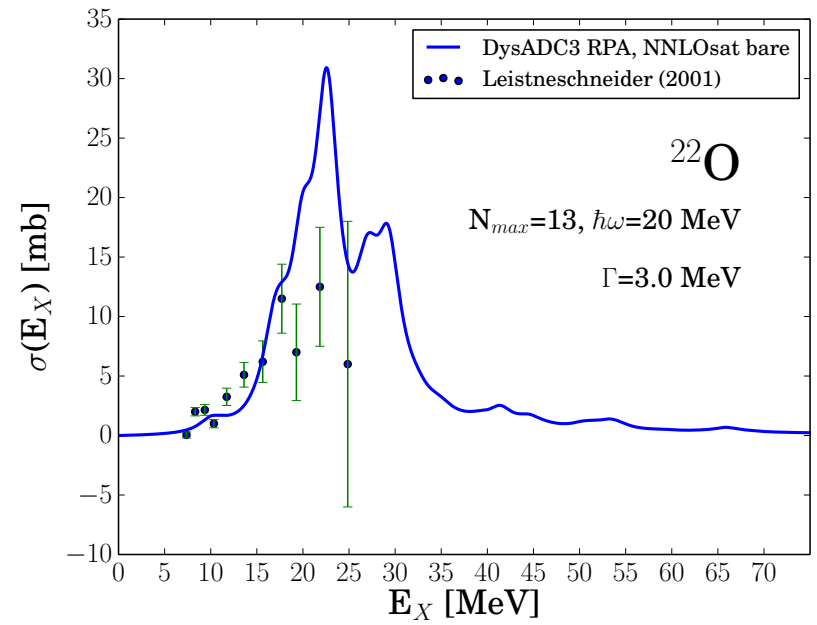
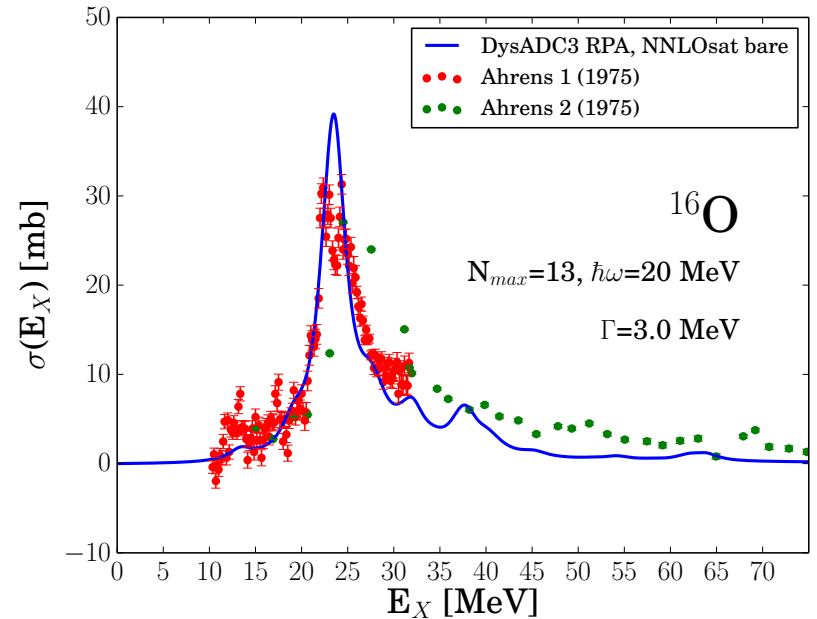
$\alpha_D$ ( $\text{fm}^3$ )	<i>Exp</i>	<i>SCGF</i>	<i>CC-LIT</i>
$^{16}\text{O}$	<b>0.585(9)</b>	<b>0.50</b>	<b>0.57(1) *</b> <b>0.528 **</b>
$^{22}\text{O}$	<b>0.43(4)</b>	<b>0.72</b>	<b>0.86(4) *</b> <b>na **</b>

$\delta(\alpha_D, \text{th-exp}) \approx 15\%$

Coupled-Cluster + LIT

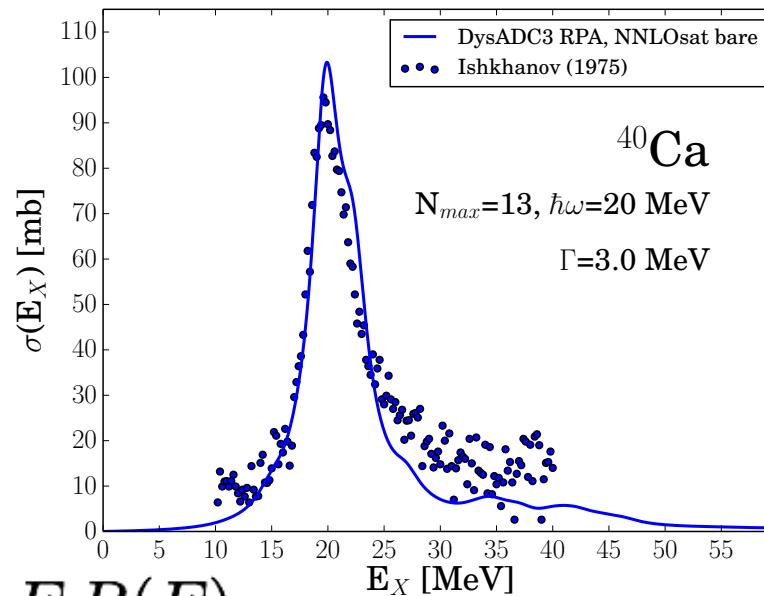
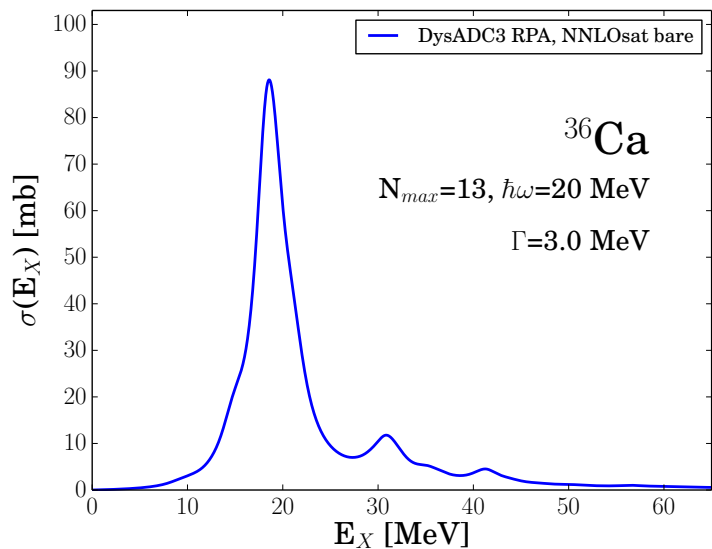
M. Miorelli et al arXiv:1804.01718:

\* Doubles/Doubles.      \*\* Triples/Triples

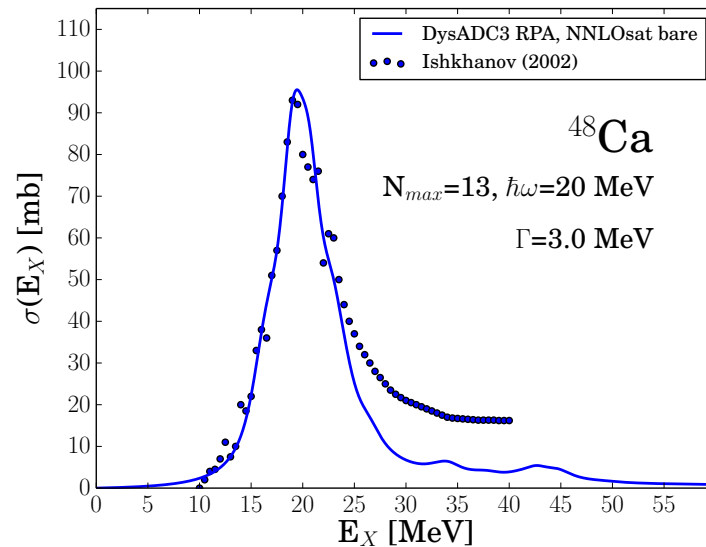
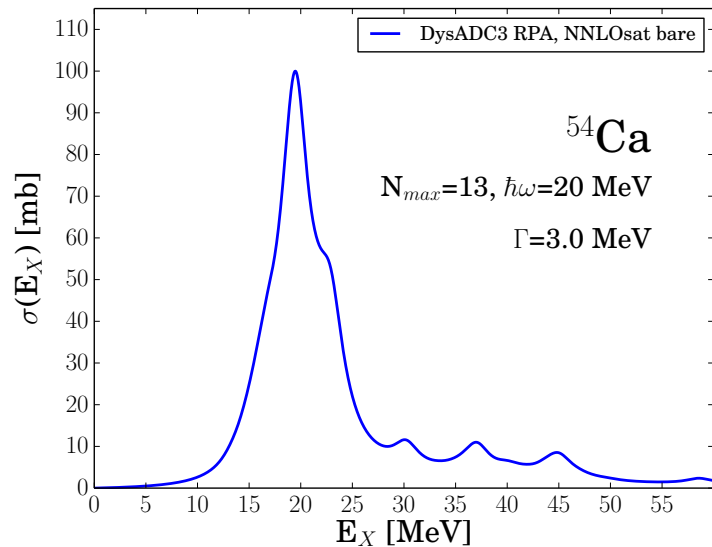




# Results for Calcium isotopes



$$\sigma_\gamma(E) = 4\pi^2 \alpha E R(E)$$



# Polarizability in $^{40}\text{Ca}$ and $^{48}\text{Ca}$

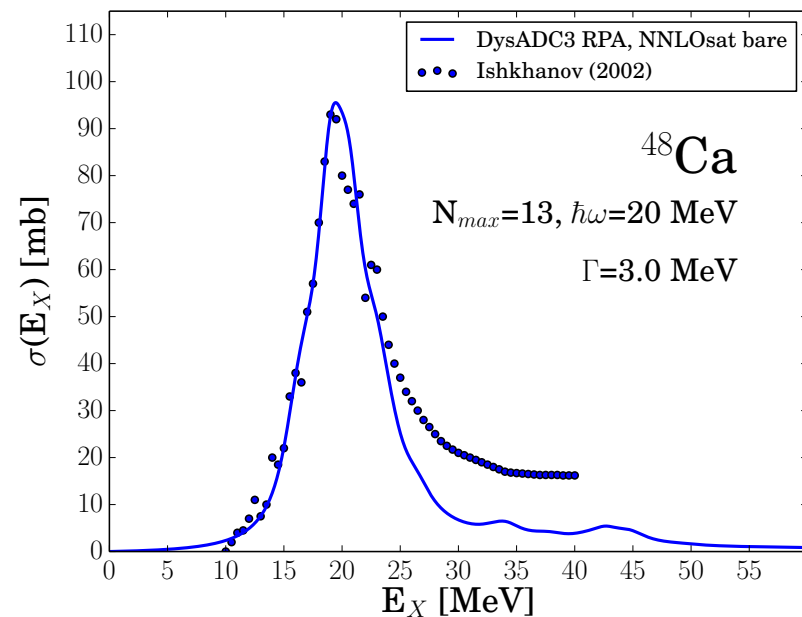
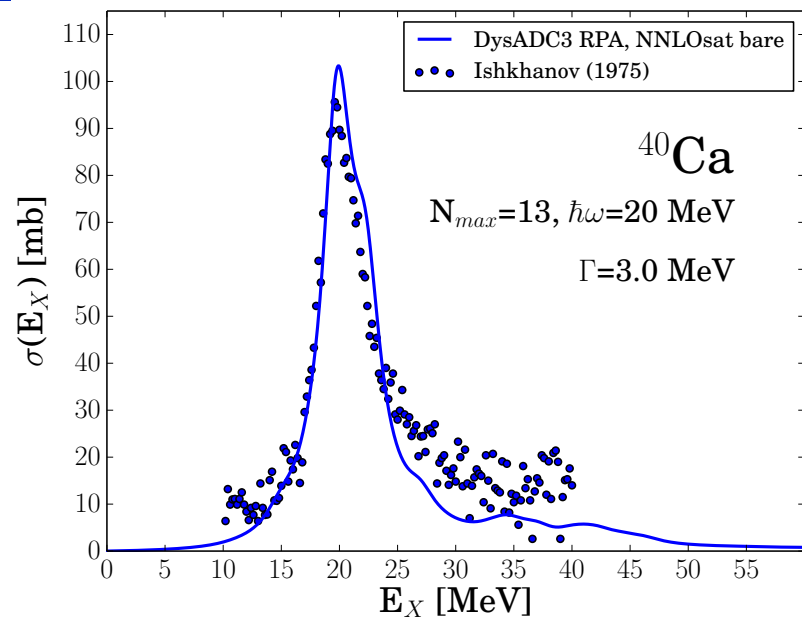
$$\alpha_D = 2\alpha \int dE \frac{R(E)}{E}$$

$\alpha_D$ (fm <sup>3</sup> )	Exp	SCGF	CC-LIT
$^{40}\text{Ca}$	2.23(3)	1.79	1.87(3) * na **
$^{48}\text{Ca}$	2.07(22) (Birkhan et al)	2.06	2.45 * 2.25(8)**

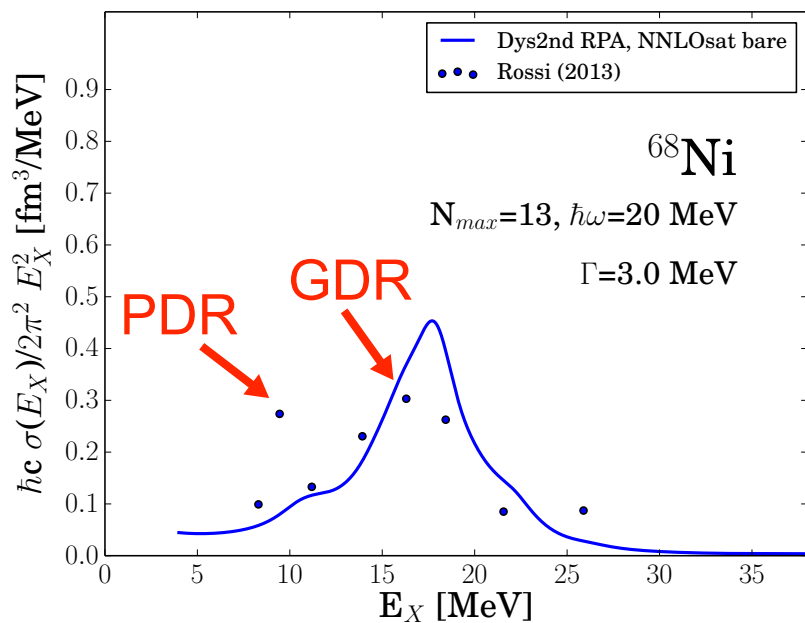
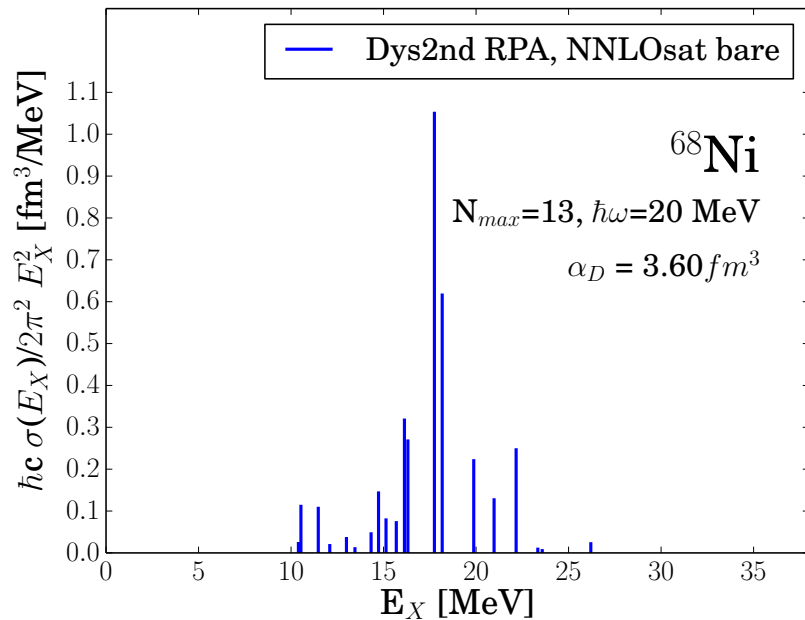
Coupled-Cluster + LIT

M. Miorelli et al arXiv:1804.01718:

\* Doubles/Doubles.      \*\* Triples/Triples



# Results for $^{68}\text{Ni}$



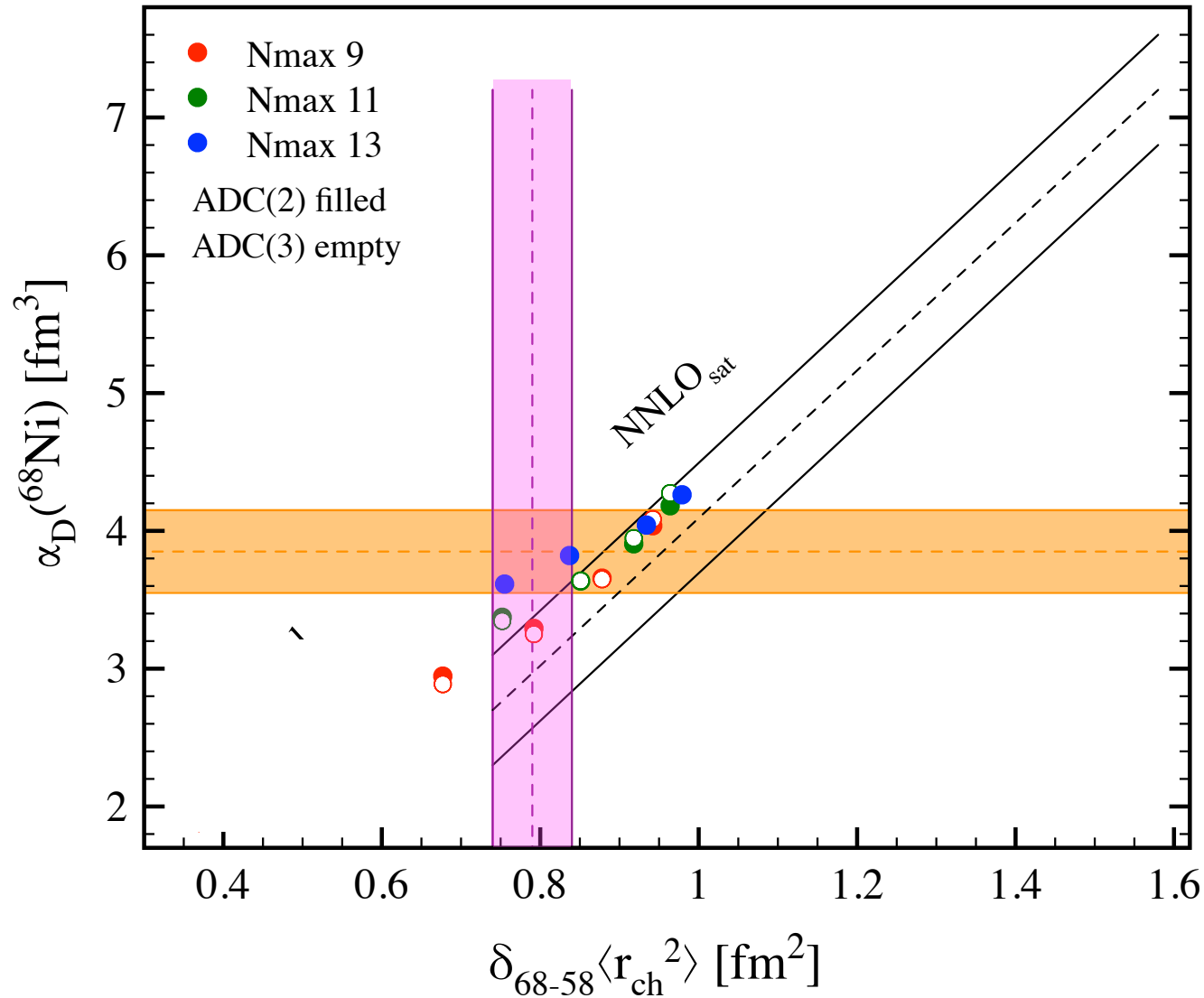
Comparison with experimental  
Coulomb excitation  
(Rossi *et al* PRL, 111, 242503 (2013))

	<i>Exp</i>	<i>SCGF</i>
<i>Pigmy</i> (MeV)	<b>9.55(17)</b>	<b>10.53</b> <b>11.47</b>
<i>Giant</i> (MeV)	<b>17.1(2)</b>	<b>17.75</b>
$\alpha_D$ ( $\text{fm}^3$ )	<b>3.88(31)</b>	<b>3.60</b>

$$\delta(\alpha_D, \text{th-exp}) \approx 7\%$$

# $\alpha_D$ -isotopic shift correlation line for $^{68}\text{Ni}$

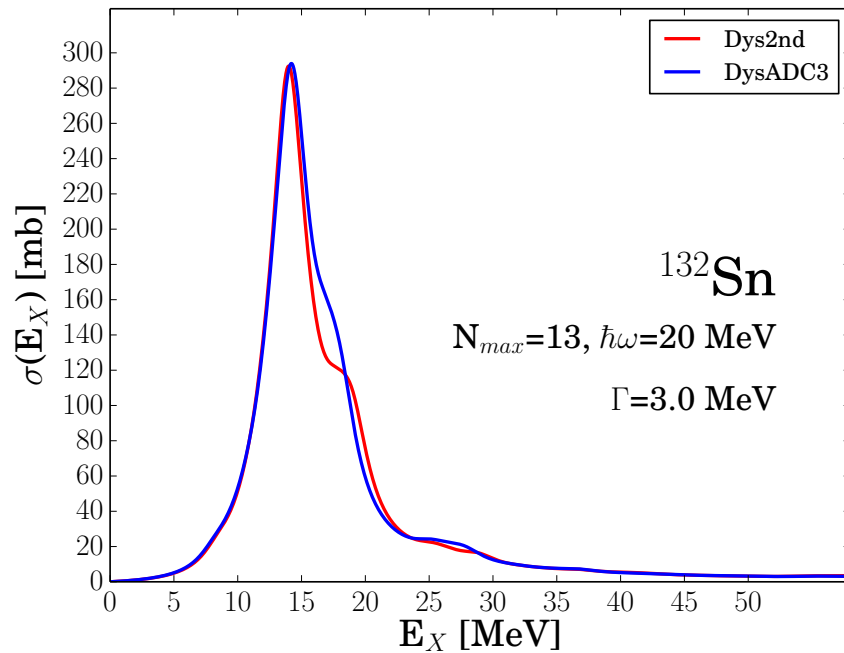
COLLAPS  
Collaboration (R. Garcia Ruiz talk's on Monday)



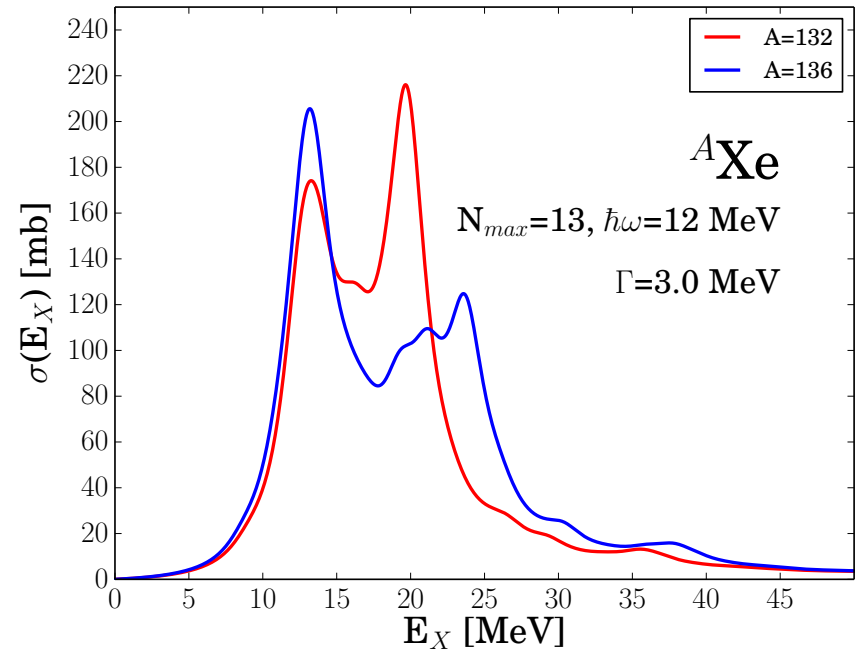
Rossi et al  
PRL, 111, 242503 (2013)

# Going to heavier nuclei: Sn and Xe

## ADC(2) vs ADC(3) many-body truncation



## Test case for SCRIT (K. Tsukada's talk on Tuesday)



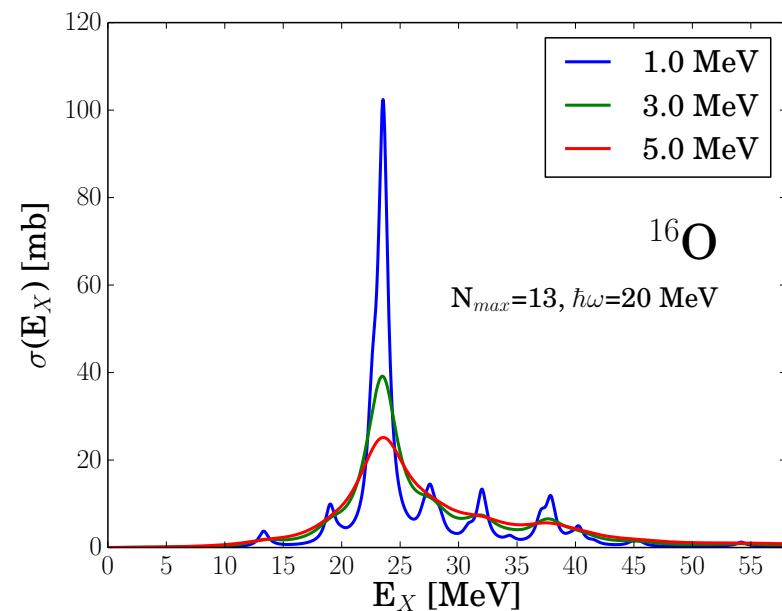
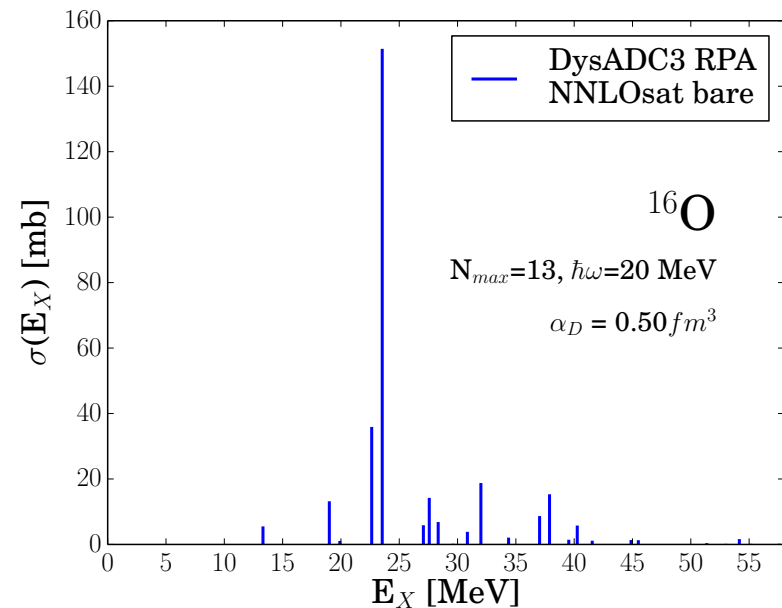


# Conclusion and perspectives

- Dipole response and polarisability calculated from first principles
- Continuum to be included
- Correlations: going beyond 1<sup>st</sup> order RPA approximations

# Backup slide

# Discrete spectrum convolution

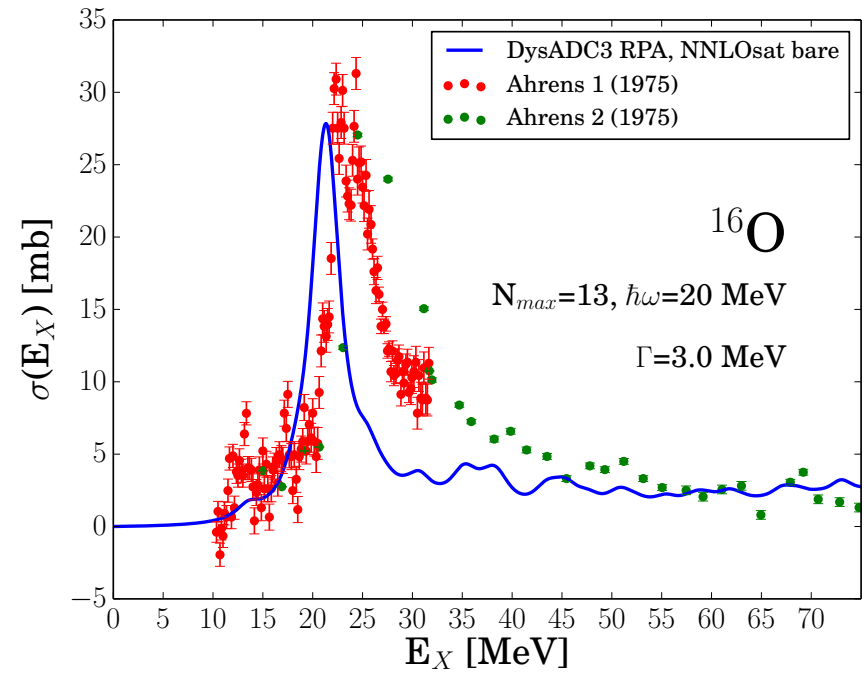
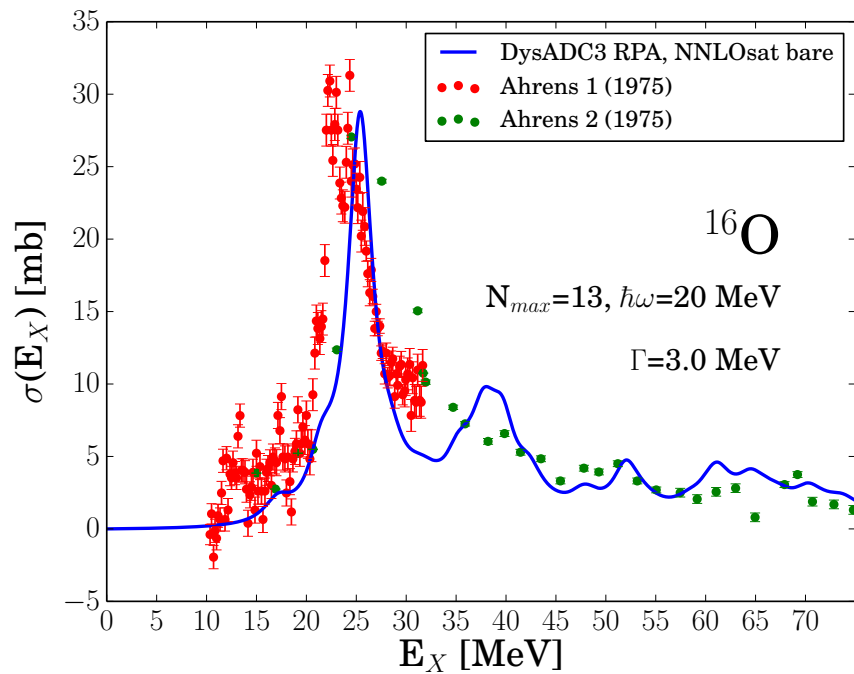


No treatment of the continuum

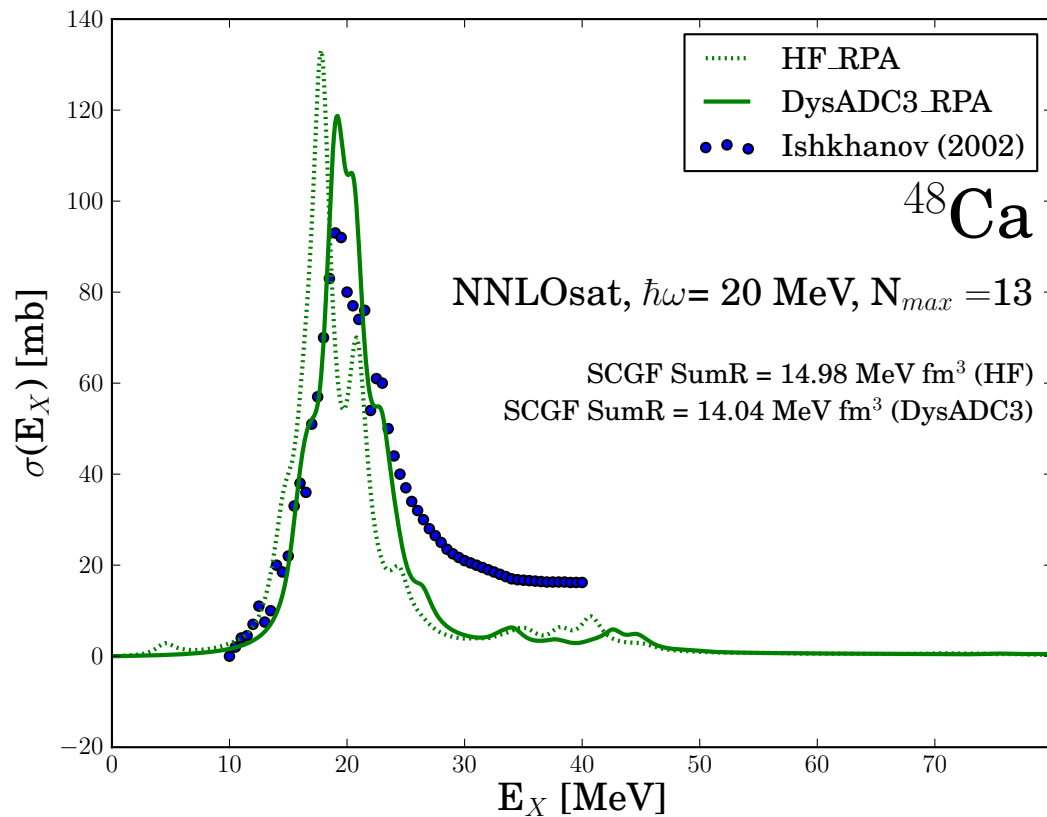
$$R_\Gamma(E) = \sum_n (\langle \Psi_n^A | \hat{Q}_{1m}^{T=1} | \Psi_0^A \rangle)^2 \frac{\Gamma/2\pi}{(E_n^A - E)^2 + \Gamma^2/4}$$

$\Gamma$  width of the Lorentzian

# Different reductions of the dressed propagator



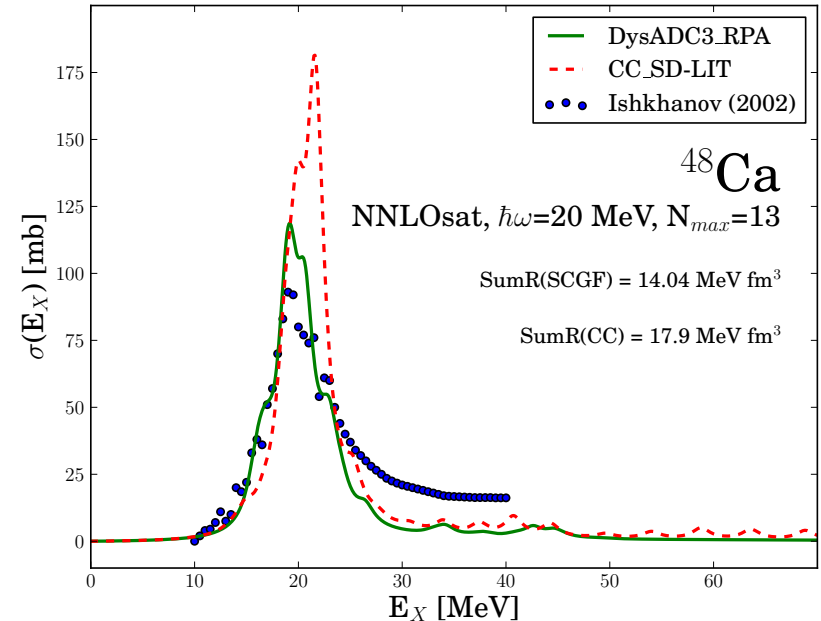
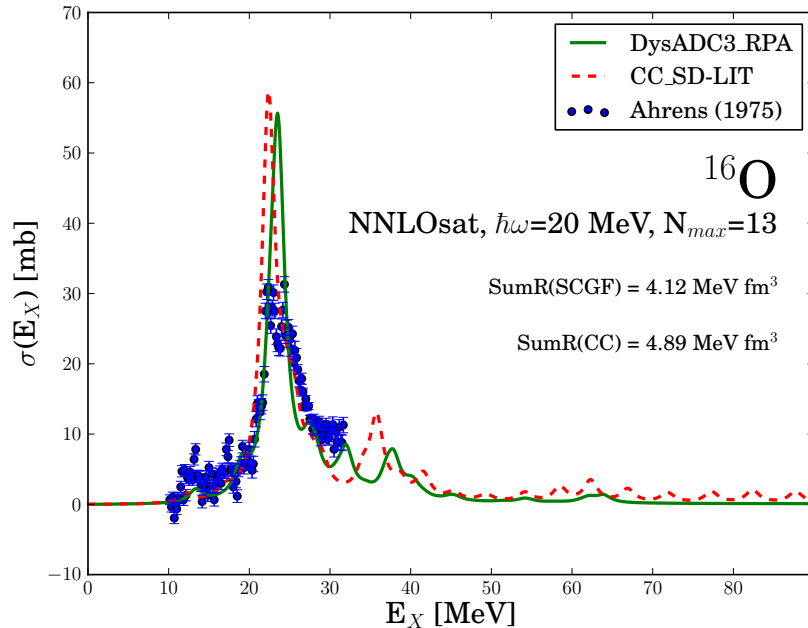
# Role of the correlations included in the reference state



Role of correlations beyond Hartree-Fock expected to be important for other observables

# Comparison with CC-LIT (Couple Cluster- Lorentz Integral Transform method)

In collaboration with [M. Miorelli](#) and [S. Bacca](#) (TRIUMF, University of Mainz)



- CC-Singles-Doubles (analogous to 2<sup>nd</sup> RPA)
- LIT reduces a continuum state problem to a bound-state-like problem

Different treatment of the correlations:

**SCGF**

Reference state correlated  
RPA (first-order two-body correlator)

**CC-SD-LIT**

HF Reference state  
Singles-Doubles