Resonant Electromagnetic Responses with the Lorentz Integral Transform

- Motivation
- LIT method
- Low-energy continuum observables with LIT method

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  S-Factor S_{12} (Coulomb potential !)

  Isoscalar 0^+ resonance of ^4He

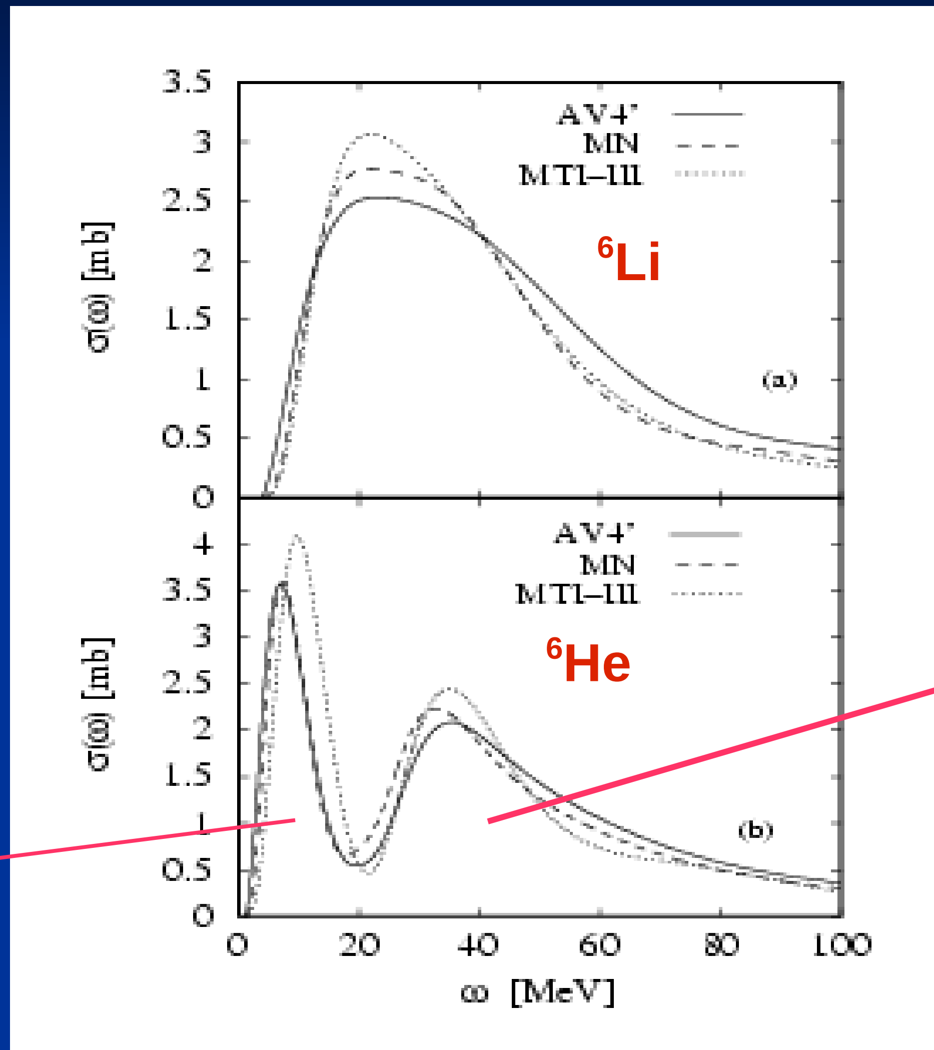
Motivation

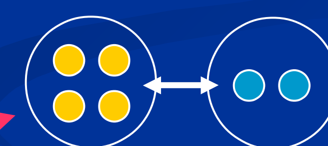
6-Body total photodisintegration

S.Bacca et al.
PRL 89 (2002) 052502,
PRC 69 (2004), 057001

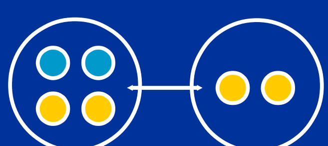
EIHH

Semirealistic NN potentials



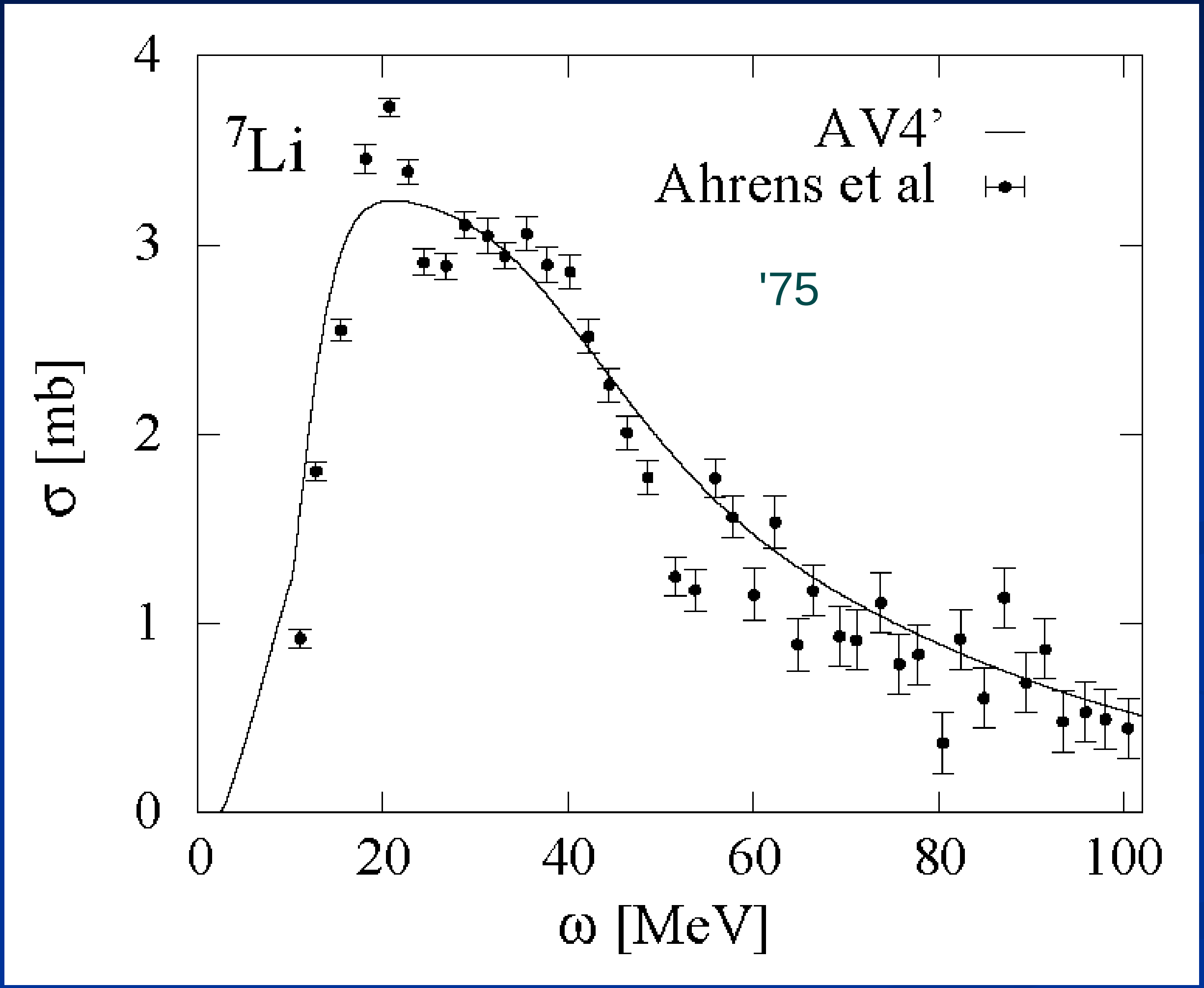


classical GT mode



soft mode

7-Body total photodisintegration

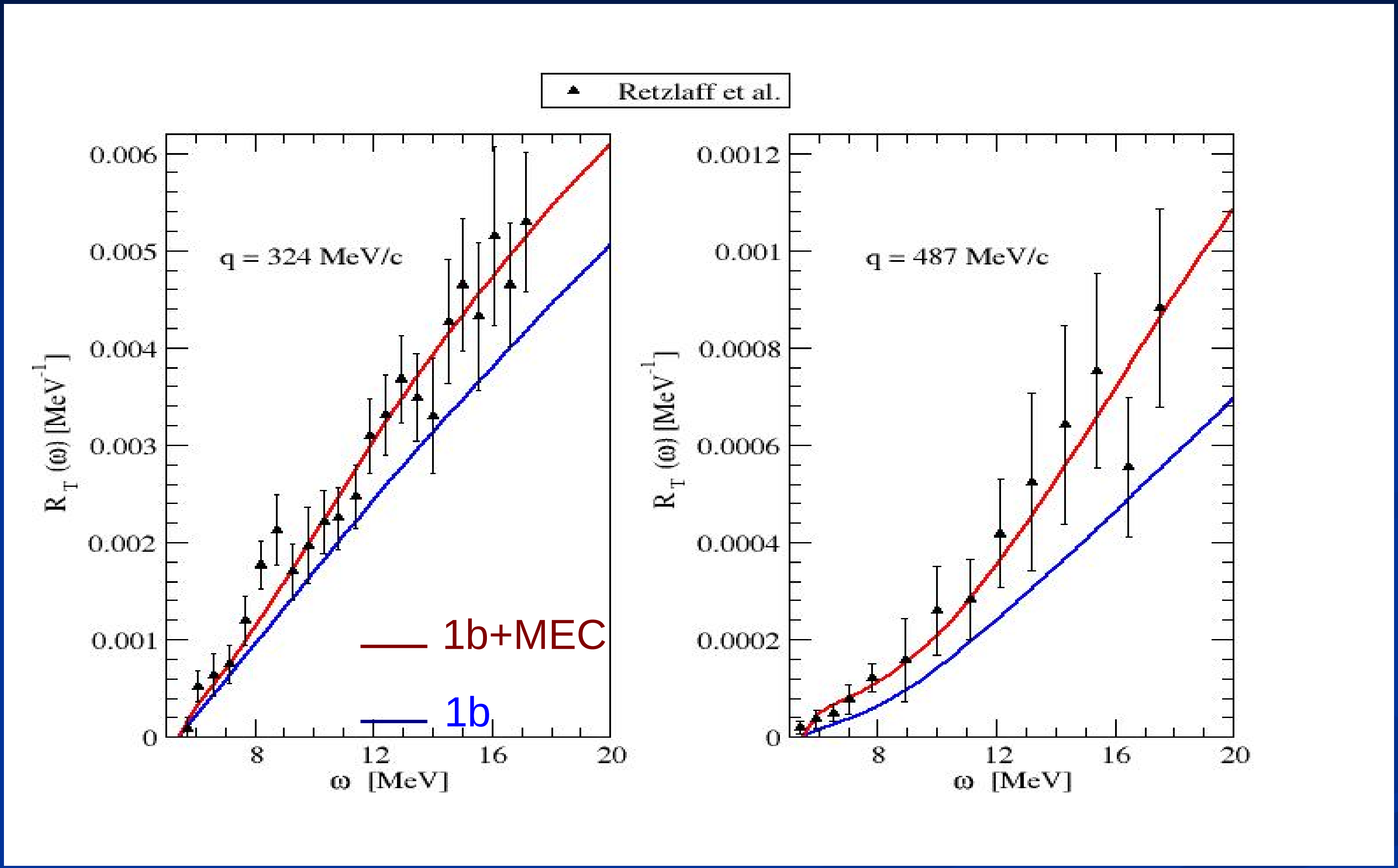


S.Bacca et al.
PLB 603(2004) 159

EIHH

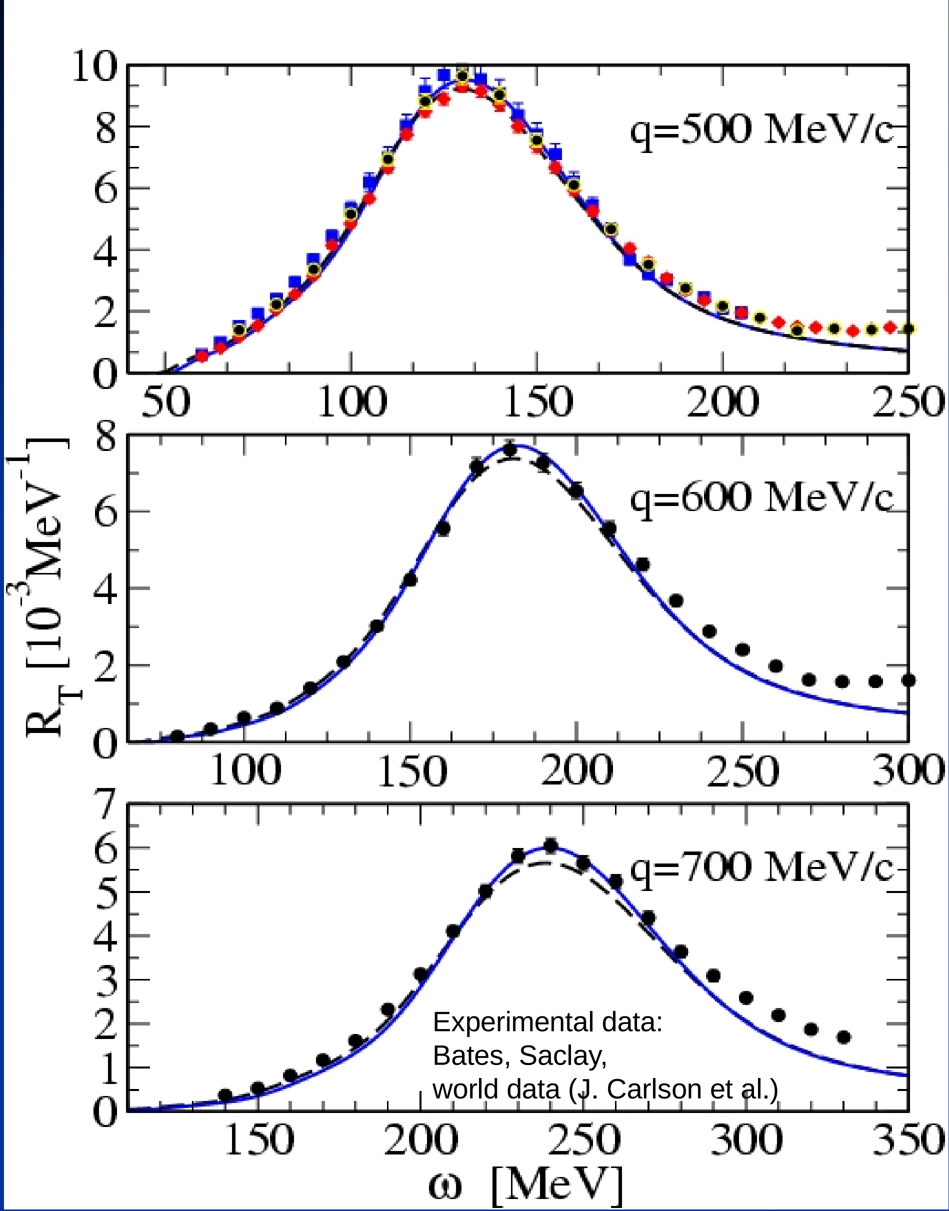
Semirealistic NN potential

Inelastic transverse ^{3}He (e,e') response function $R_T(q,\omega)$



V. Efros et al., FBS 47, 157 (2010)

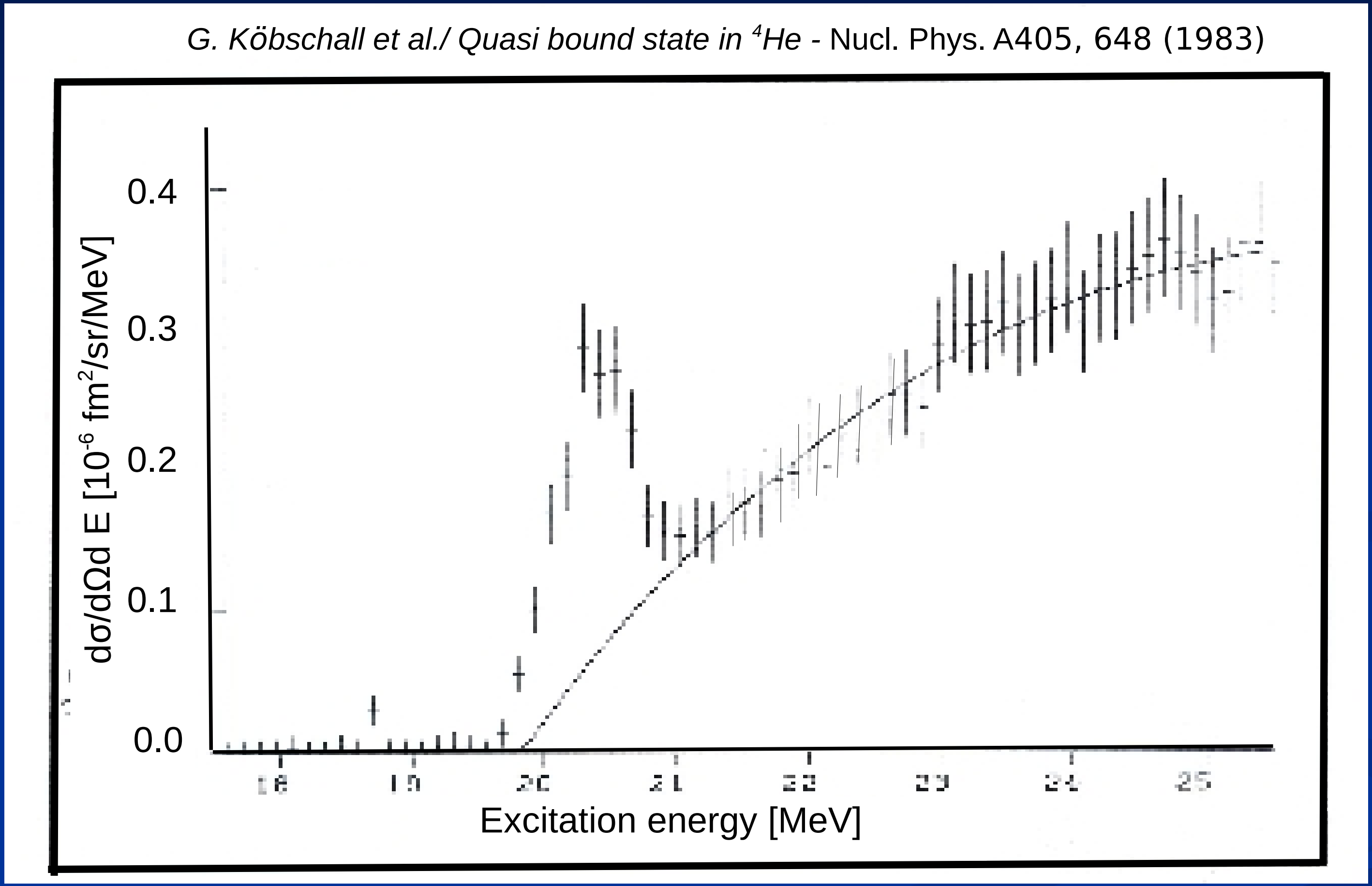
Inelastic transverse ^{3}He (e,e') response function $R_T(q,\omega)$



L. Yuan et al., PLB 706, 90 (2011)

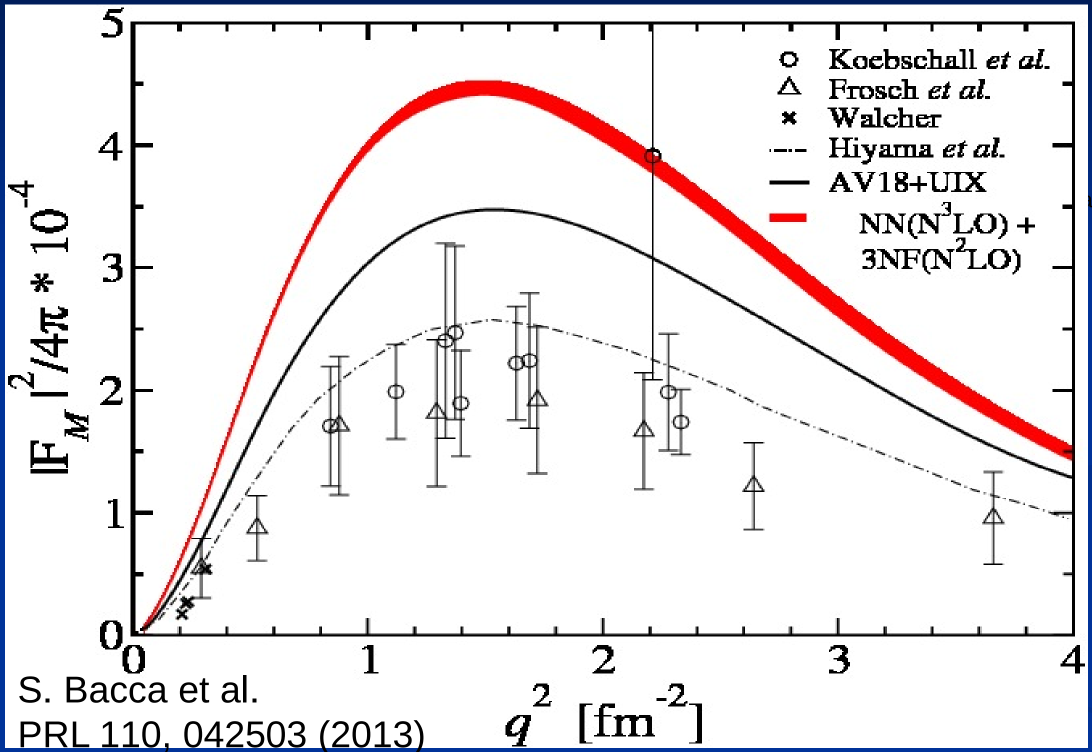
Resonances

0^+ Resonance in the ^{4}He compound system



Resonance at E_R = -8.2 MeV, i.e. above the ^{3}H-p threshold. Strong evidence in electron scattering off ^{4}He, Γ = 270±50 keV

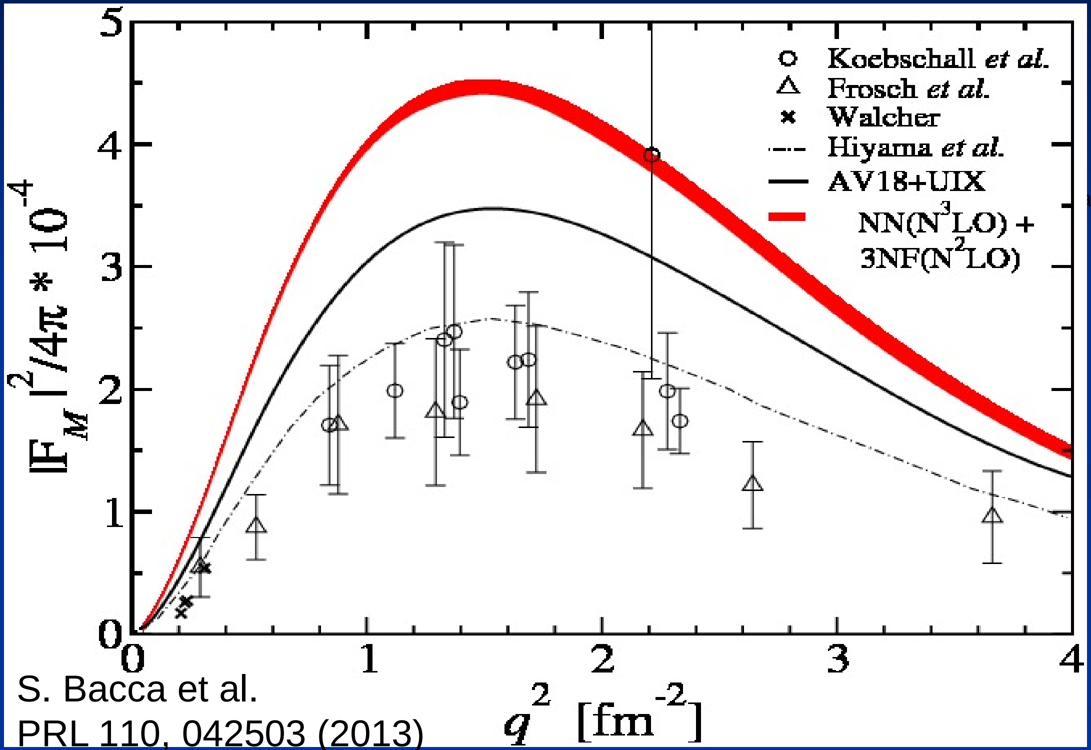
Comparison to experimental results



LIT/EIHH Calculation for AV18+UIX and Idaho-N3LO+N2LO

dash-dotted: AV8' + central 3NF (Hiyama et al.)

Comparison to experimental results



S. Bacca et al.
PRL 110, 042503 (2013)

Observable is strongly dependent on potential model

Breathing Mode? (S. Bacca et al., PRC 91, 024303 (2015))

- Resonance strength determined with LIT method
- But resonance width undetermined
- Why ?

LIT Method

LIT method

The LIT of a function $R(E)$ is defined as follows

$$\Rightarrow \quad L(\sigma) = \int dE\, \mathcal{L}(E,\sigma)\, R(E)\,,$$

where the kernel $\mathcal{L}$ is a Lorentzian,

$$\Rightarrow \quad \mathcal{L}(E,\sigma) = \frac{1}{(E-\sigma_R)^2 + \sigma_I^2}$$

For inclusive reactions the LIT $L(\sigma)$ is calculated by solving an equation of the form

$$\boxed{(H-\sigma)\,\tilde{\Psi} = S\,,}$$

where H is the Hamiltonian of the system under consideration and S is an asymptotically vanishing source term related to the operator inducing the specific reaction.

The solution $\tilde{\Psi}$ is localized and the LIT is given by

$$\boxed{L(\sigma) = \langle \tilde{\Psi} | \tilde{\Psi} \rangle\,.}$$

Alternative way:

$$L(\sigma) = -\frac{1}{\sigma_I} Im(\langle S | \frac{1}{\sigma_R + i\sigma_I - H} | S \rangle)\,.$$

The source term S for inclusive reactions has the form

$$|S\rangle = \theta|0\rangle \, ,$$

where the operator θ induces a specific electroweak reaction.

The corresponding response function is given by

$$R(E_f) = \int dE_f |\langle f|\theta|0\rangle|^2 \delta(E_f - E_0 - \omega)$$

Ingredients of the solution of the LIT equation via an expansion on a basis of dimension N:

N $\boxed{\text{eigenstates } \phi_n}$ with $\boxed{\text{eigenenergies } E_n}$

LIT states

and strength

$$S_n = |\langle \phi_n|\theta|0\rangle|^2$$

strength to a LIT state

leading to the following LIT

$$L(\sigma) = \sum_{i=1}^{N} \frac{S_n}{(\sigma_R - E_n)^2 + \sigma_I^2}$$

main point of the LIT :

Schrödinger-like equation with a source

$$\left(H - E_0 - \omega_0 + i\Gamma\right)\tilde{\Psi} = S$$

The $\tilde{\Psi}$ solution is unique and has ***bound state like*** asymptotic behavior

→ one can apply ***bound state methods***

Hypersperical Harmonics (HH)

Expansion of ground-state wave function and LIT states on HH basis

HH: (A-1) Jacobi coordinates $\longrightarrow$ 1 hyperradius + 3A-4 hyperangles

Acceleration of Convergence

with two-body coorelations: CHH
or with effective interaction (Lee-Suzuki approach): EIHH

Inversion of the LIT

- LIT is calculated for a fixed σ_I in many σ_R points

- Express the searched response function formally on a basis set with M basis basis functions $f_m(E)$ and open coefficients c_m with correct threshold behaviour for the $f_m(E)$ (e.g., $f_m = f_{thr}(E) \exp(-\alpha E/m)$)

- Make a LIT transform of the basis functions and determine coefficents c_m by a fit to the calculated LIT

- Increase M up to the point that a sufficient convergence is obtained (uncontrolled oscillations should not be present)

A regularization method is needed for the inversion

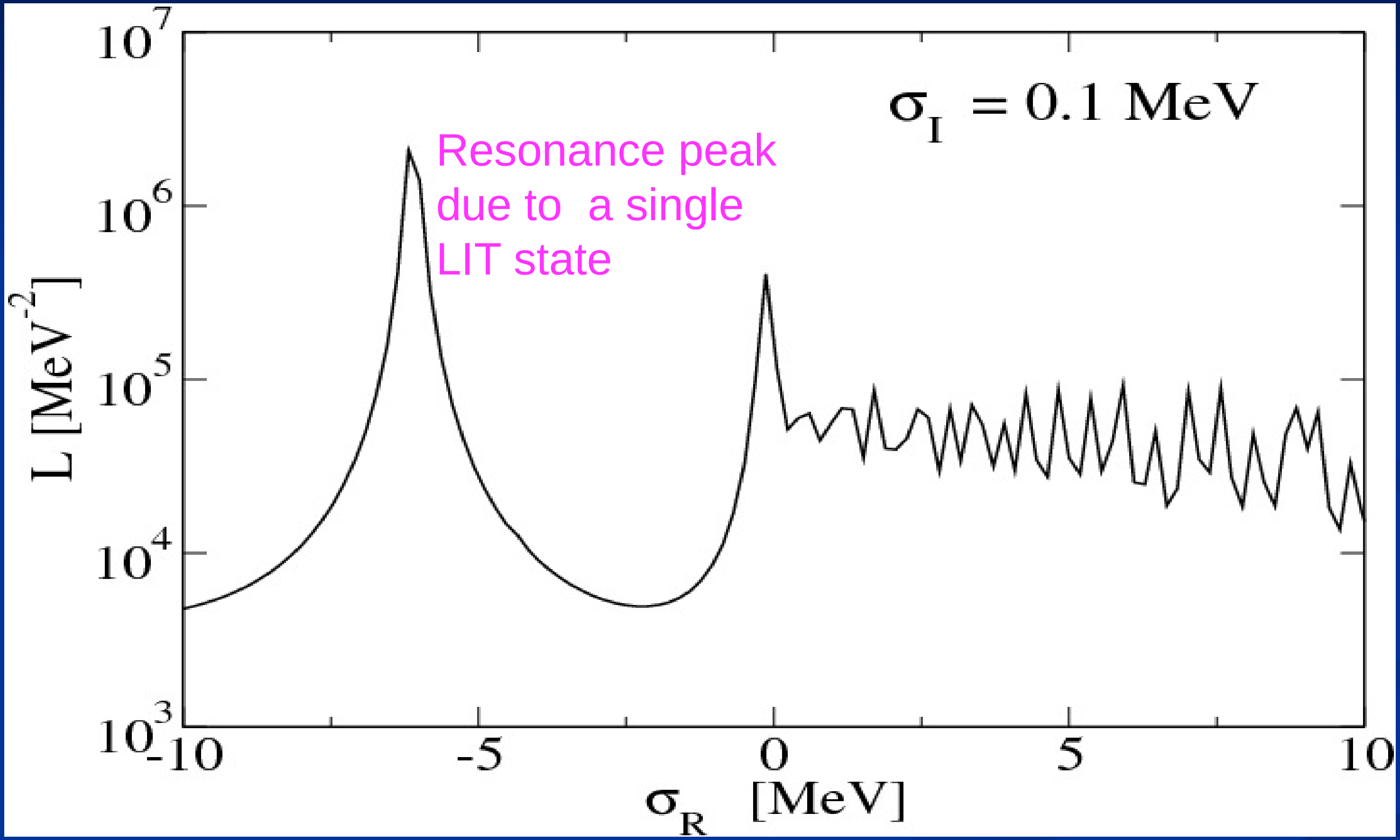
Why were we unable to determine the width of the ^{4}He isoscalar monopole resonance?

To answer this let us check our very first LIT calculation from 1997:

^{4}He(e,e') inelastic longitudinal response function

with a central NN potential

Unpublished result from a CHH calculation with the TN potential (V. Efros, WL, G. Orlandini, PRL 78,432 (1997))



To study the problem better let us consider first instead of a four-body reaction a simpler three-body reaction:

$$^{3}\mathrm{He} + \gamma \longrightarrow \mathrm{d} + \mathrm{p}$$

at low energies

LIT calculation with central MTI/III NN potential in unretarded dipole approximation

Aim: Increase low-energy density of LIT states

To study the problem better let us consider first instead of a four-body reaction a **simpler three-body reaction**:

$$^{3}\mathrm{He} + \gamma \longrightarrow d + p$$

at low energies

LIT calculation with central MTI/III NN potential in unretarded dipole approximation

How: Increase number of basis states, both, hyperradial and hyperspherical

To study the problem better let us consider first instead of a four-body reaction a **simpler three-body reaction**:

$$^{3}\mathrm{He} + \gamma \longrightarrow d + p$$

at low energies

LIT calculation with central MTI/III NN potential in unretarded dipole approximation

Also note: hyperradial basis states consist in an expansion on Laguerre polynomials times a spatial cutoff $\exp(-\rho/b)$

Increase of b shifts spectrum to lower energies

To study the problem better let us consider first instead of a four-body reaction a simpler three-body reaction:

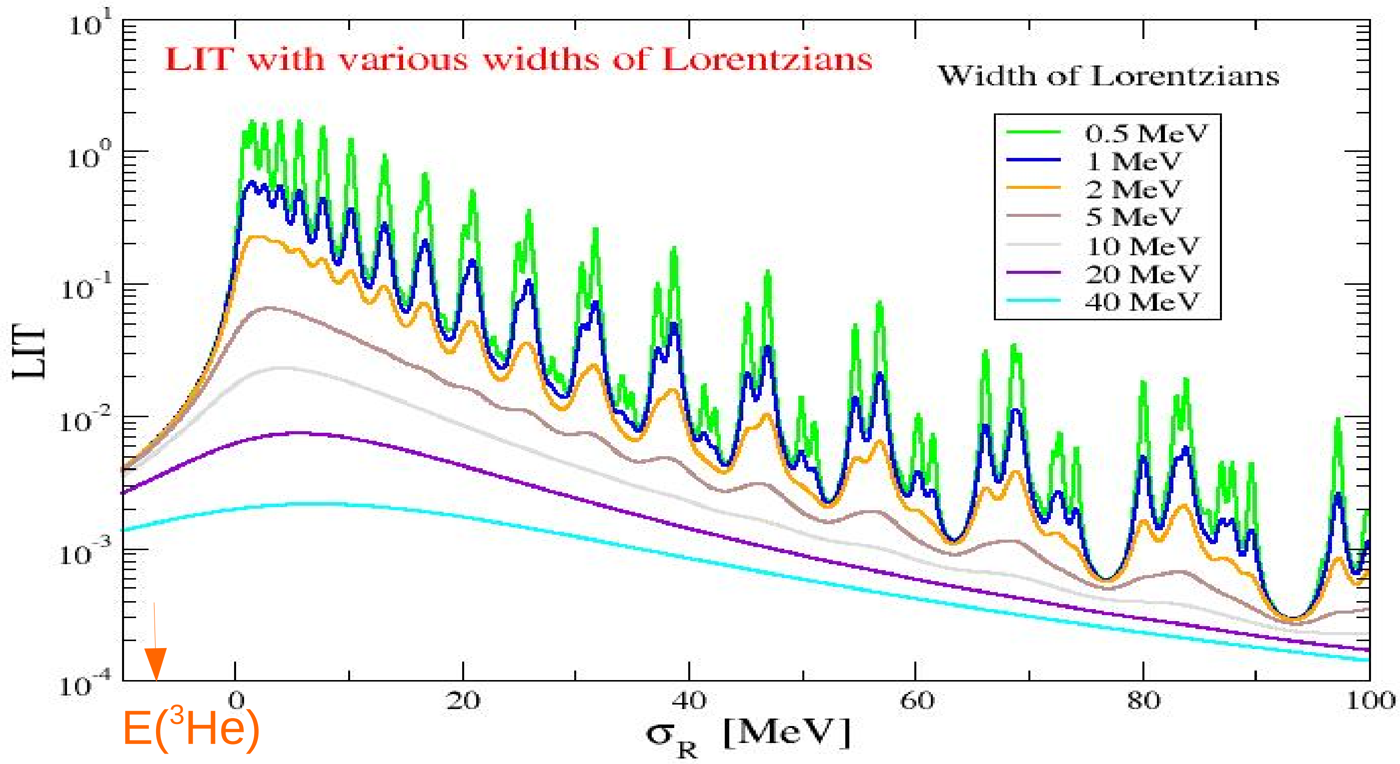
$$^{3}\mathrm{He} + \gamma \longrightarrow d + p$$

at low energies

LIT calculation with central MTI/III NN potential in unretarded dipole approximation

Next slide: LIT with 30 hyperspherical and 31 hyperradial basis functions ⇒ **930 basis states** with b = 0.6 fm

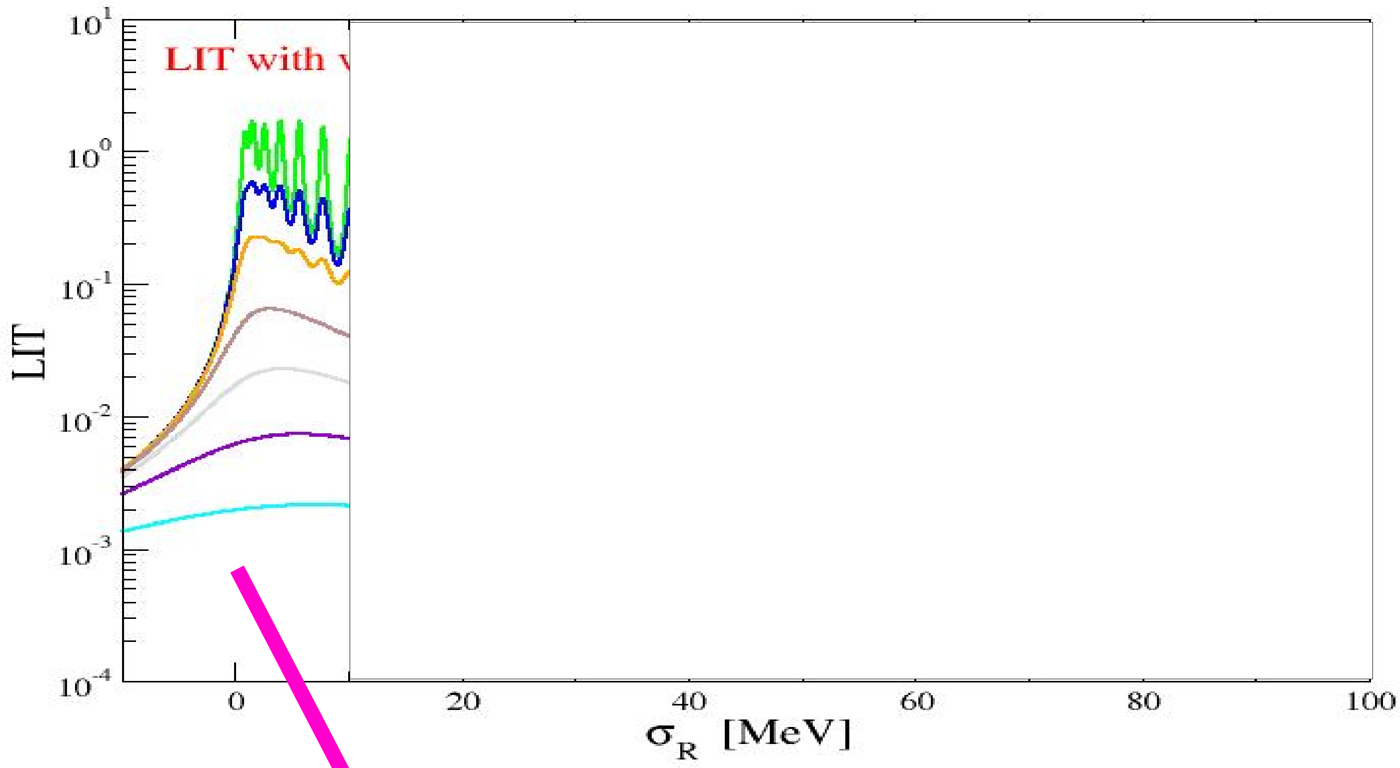
LIT with various widths of Lorentzians



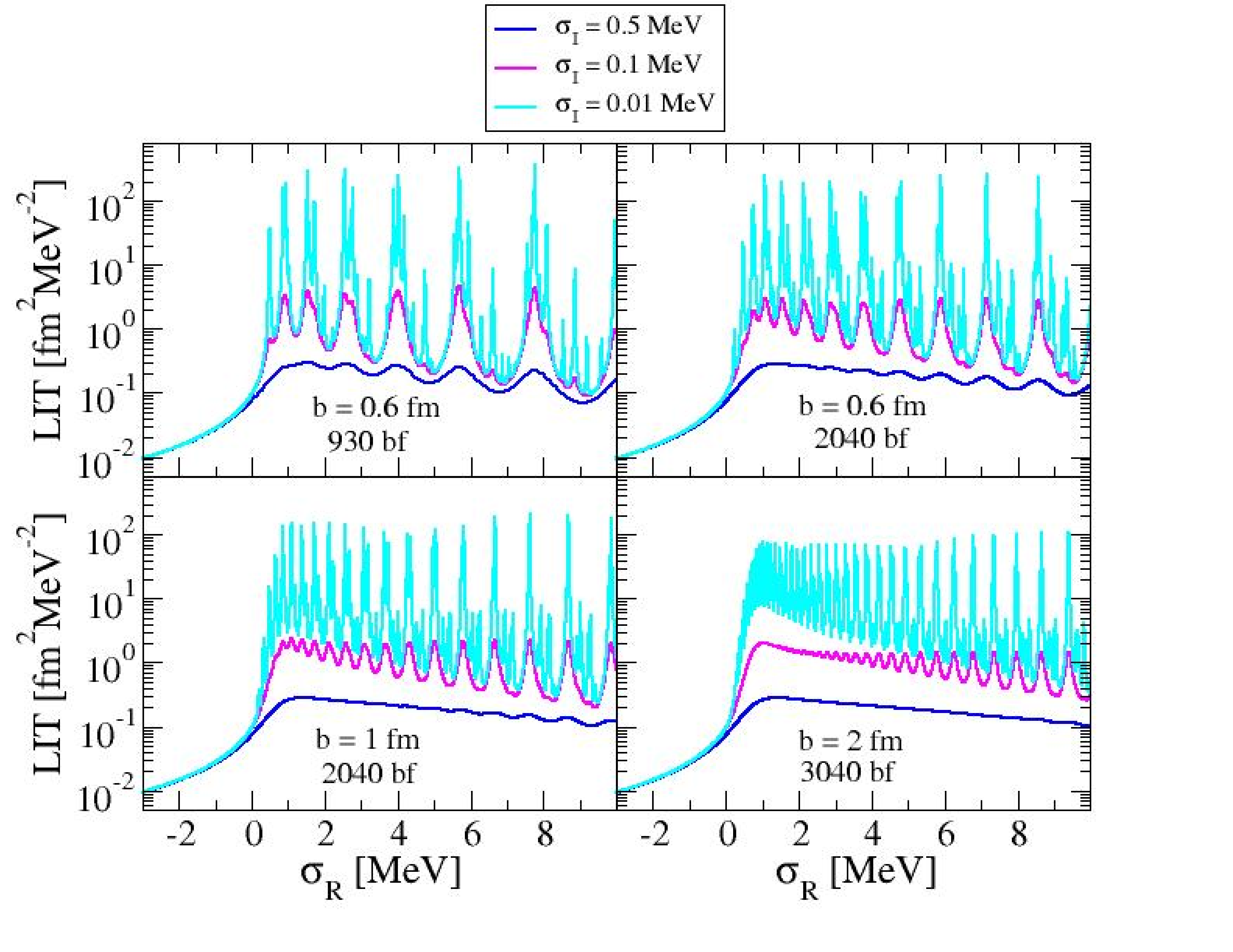
30 hyperspherical and 31 hyperradial basis functions

⇒ 930 basis states

b = 0.6 fm



Increase LIT state density and zOOM in


σ_I = 0.5 MeV
σ_I = 0.1 MeV
σ_I = 0.01 MeV
LIT [fm^2 MeV^-2]
b = 0.6 fm
930 bf
b = 0.6 fm
2040 bf
b = 1 fm
2040 bf
b = 2 fm
3040 bf
σ_R [MeV]

Observation

The LIT is a method with a controlled resolution

But not a single LIT state below three-body breakup threshold
In present LIT calculation! Similar problem as in the previous four-body case

Solution: use instead of the HH basis a somewhat modified basis

New A-body basis

Note one of the (A-1) Jacobi vectors can be written in the following form:

$$\boldsymbol{\eta} = \mathbf{r}_A - \mathbf{R}_{cm}(1,2,\ldots,A\text{-}1)$$

This is the coordinate one would use for the scattering of a nucleon with a (A-1)-nucleon system. In other words the relevant coordinate for a two-body breakup. Therefore

A-body HH basis $\longrightarrow$ (A-1)-body HH basis times expansion on $\boldsymbol{\eta}$

radial part: Laguerre polynomials

angular part: $Y_{LM}(\theta_\eta,\phi_\eta)$

Four-body system: HH for 3 particles plus 4-th particle coordinate $\boldsymbol{\eta}$

New A-body basis

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A-body HH basis $\longrightarrow$ (A-1)-body HH basis times expansion on $\boldsymbol{\eta}$

radial part: Laguerre polynomials

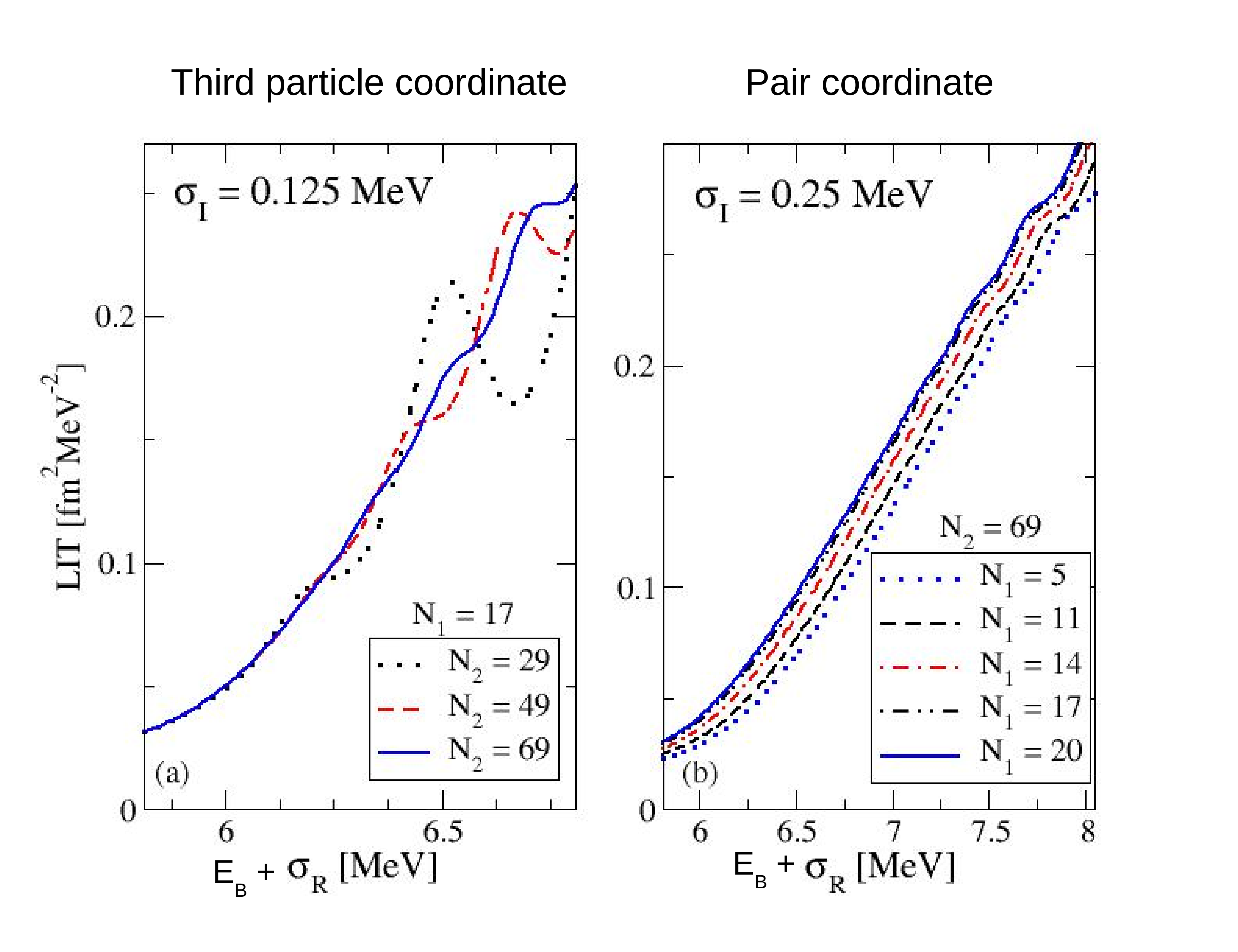
angular part: $Y_{LM}(\theta_\eta,\phi_\eta)$

Three-body system: pair coordinate for two particles plus 3rd particle coordinate $\boldsymbol{\eta}$

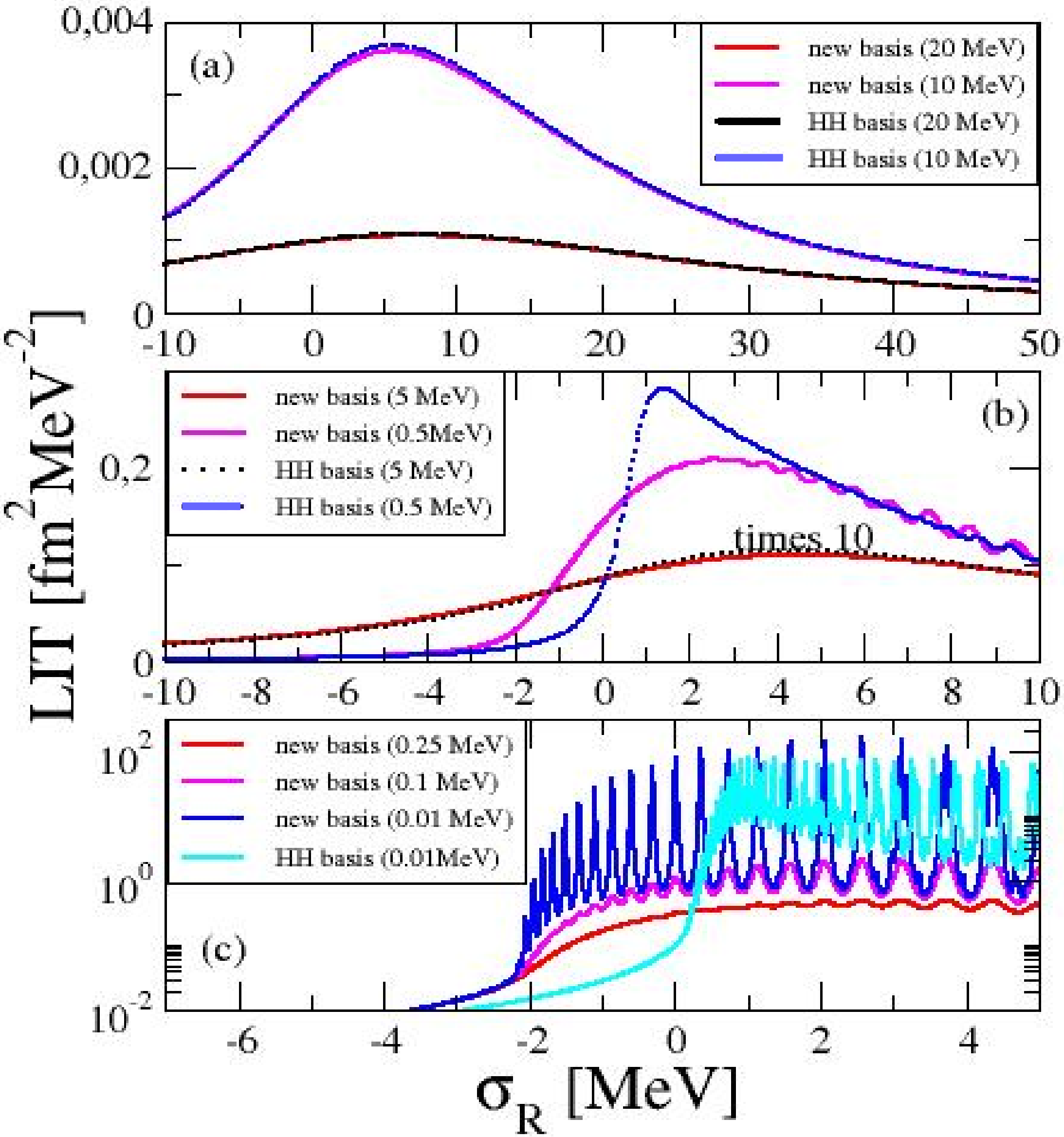
First three-body case

$$^3He + \gamma \longrightarrow d + p$$

With convergence for expansions in pair and third particle coordinate


Third particle coordinate
Pair coordinate
σ_I = 0.125 MeV
σ_I = 0.25 MeV
LIT [fm^2MeV^-2]
0.2
0.1
0
6
6.5
7
7.5
8
N_1 = 17
N_2 = 29
N_2 = 49
N_2 = 69
(a)
(b)
N_2 = 69
N_1 = 5
N_1 = 11
N_1 = 14
N_1 = 17
N_1 = 20
E_B + σ_R [MeV]
E_B + σ_R [MeV]

LIT results with HH and new basis

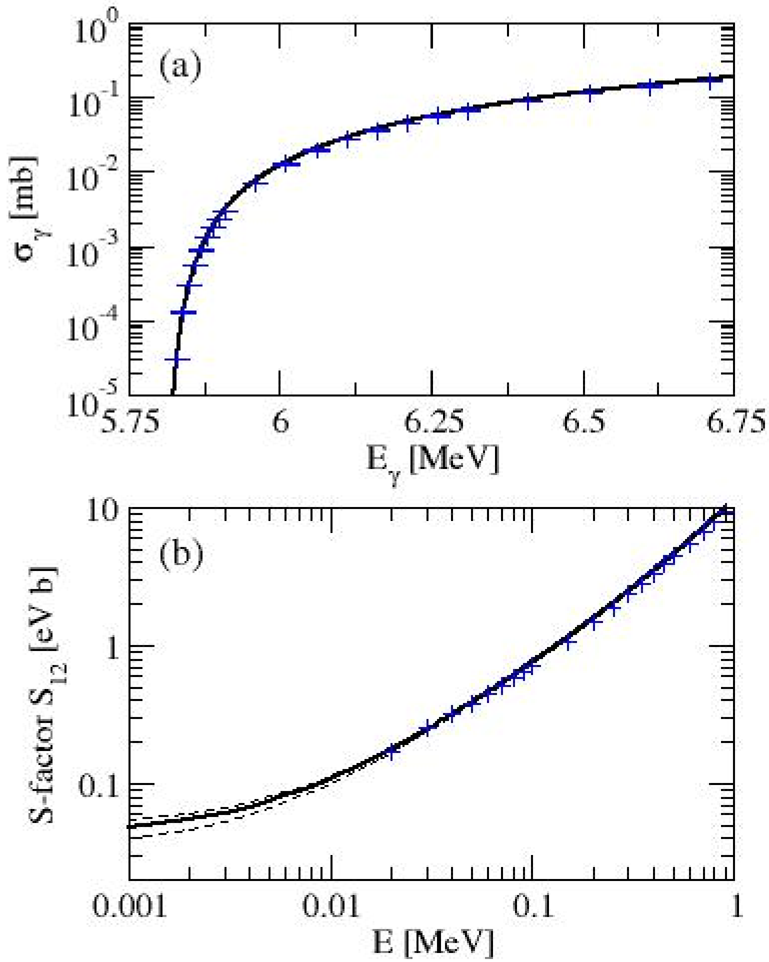


Inversions

Implement correct threshold behaviour for $^3He + \gamma \longrightarrow d + p$

Due to Coulomb potential: usual Gamow factor

Comparison with explicit calculation of continuum state



Cross section
$^3He + \gamma \longrightarrow d + p$

LIT: full curves
cont. wf: +

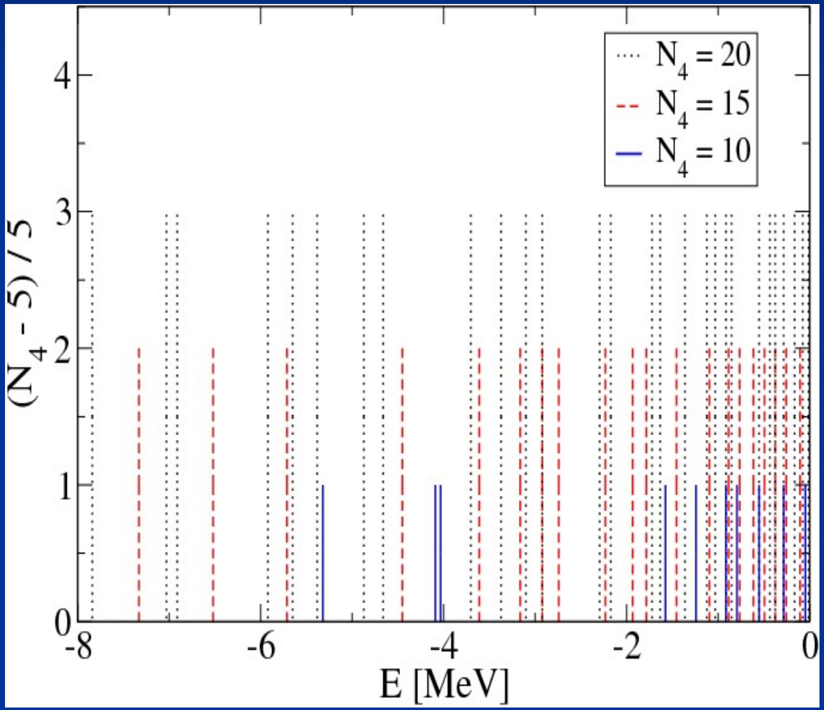
S-factor
$d + p \longrightarrow {}^3He + \gamma$

Error due to
inversion: dashed
(Standard deviation from inversions with 11-18 basis functions) S. Deflorian, V. Efros, WL, FBS 58:3 (2017)

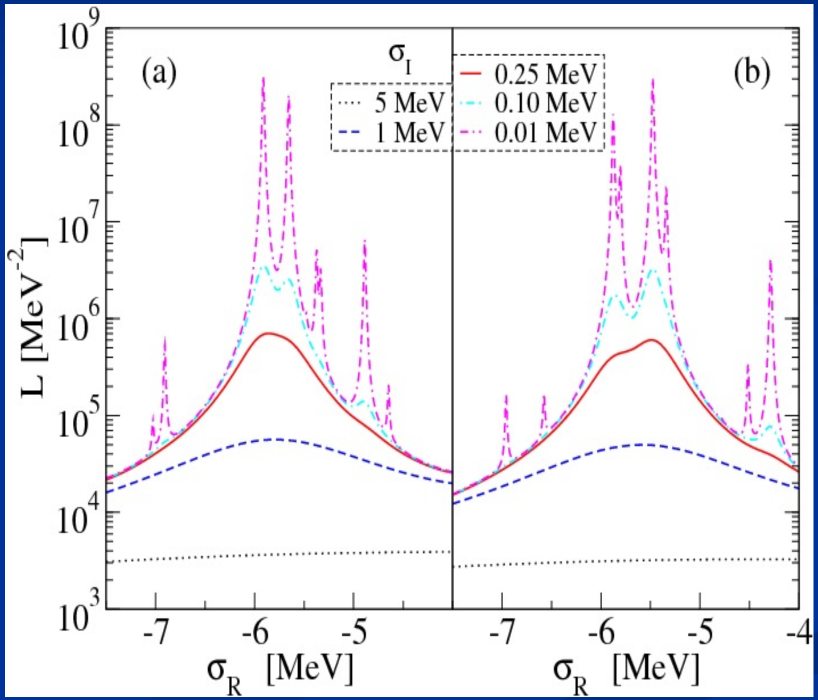
Back to the ^{4}He resonance

Results with new basis

Number of basis functions in 4-th particle coordinate

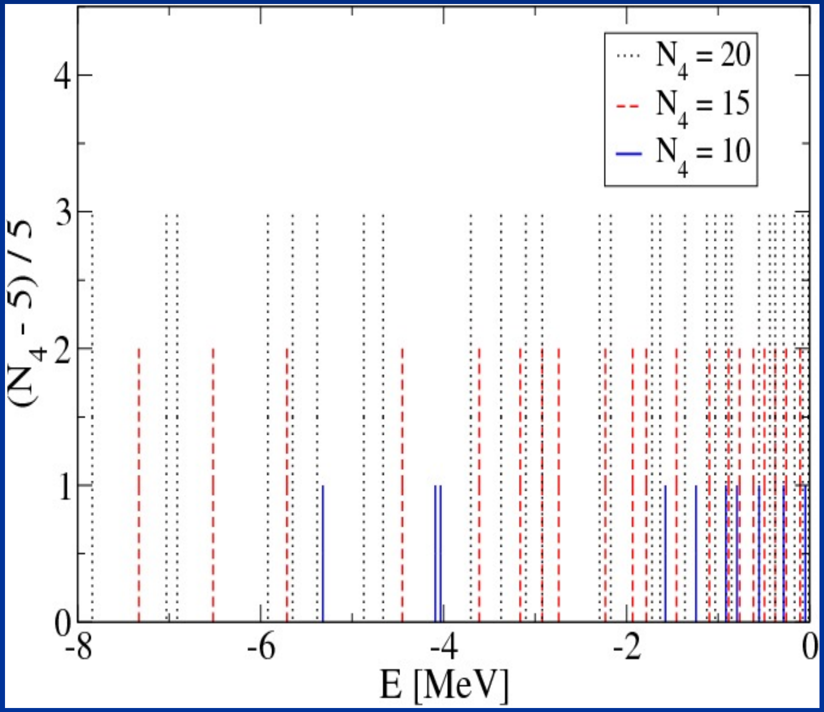


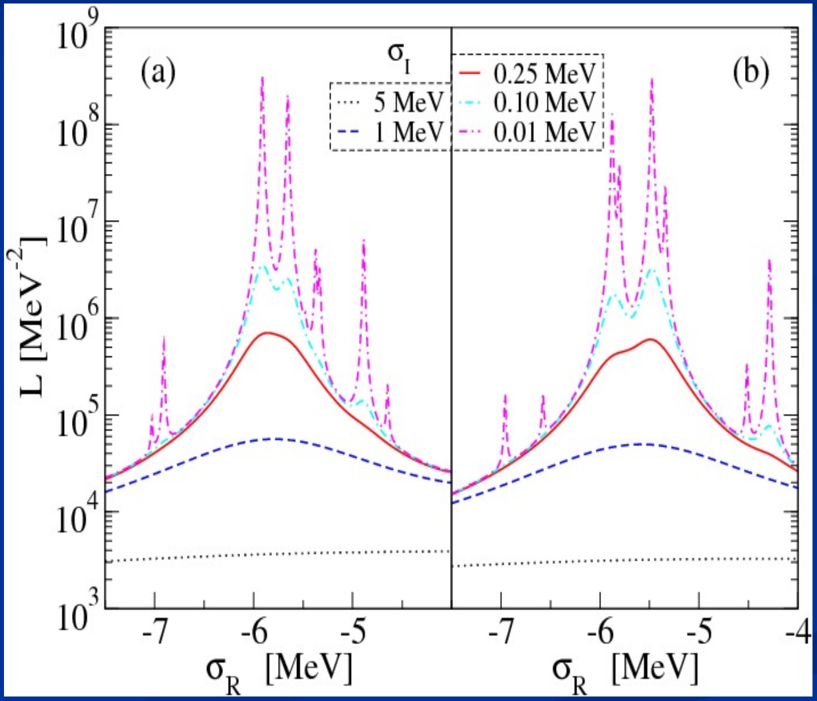
$N_4 = 20$ $N_4 = 21$



LIT

Results with new basis





Inversion: Γ = 180(70) keV

WL, PRC 91, 054001 (2015)

Summary and Outlook

- LIT method: ab initio method for reaction cross sections
- Extension to calculations for A>4 with realistic nuclear interaction possible
    - HH but not beyond A =10
    - other ab initio methods (coupled cluster)
    - Halo EFT