Learning about the structure of light nuclei from charge radii and form factors

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Workshop on
“Probing exotic structure of short-lived nuclei by electron scattering”
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Charge Radii and Electron Scattering

Our History

Charge Radii along isotope chains

- We did calculations for charge radii in Lithium, Beryllium, **Neon** and Magnesium isotopes in collaboration with measurements using collinear laser spectroscopy (COLLAPS)
- In this light nuclei charge radii reflect clustering (Lithium and Beryllium isotopes, ^{19}Ne and ^{20}Ne), neutron halos (^{11}Li, ^{11}Be), proton halos (^{17}Ne), deformation (Magnesium isotopes)

Elastic and inelastic electron scattering on ^{12}C

- In collaboration with experimental work at TU Darmstadt
- Inelastic electron scattering tests the Hoyle state wave function, proving its dilute nature
- Careful analysis for the extraction of the monopole transition matrix element that determines the pair decay width of the Hoyle state

Our Aim:

Solve the
nuclear many-body problem for exotic nuclei
with (realistic) NN interaction

Many-Body Method

Fermonic Molecular Dynamics
(FMD)

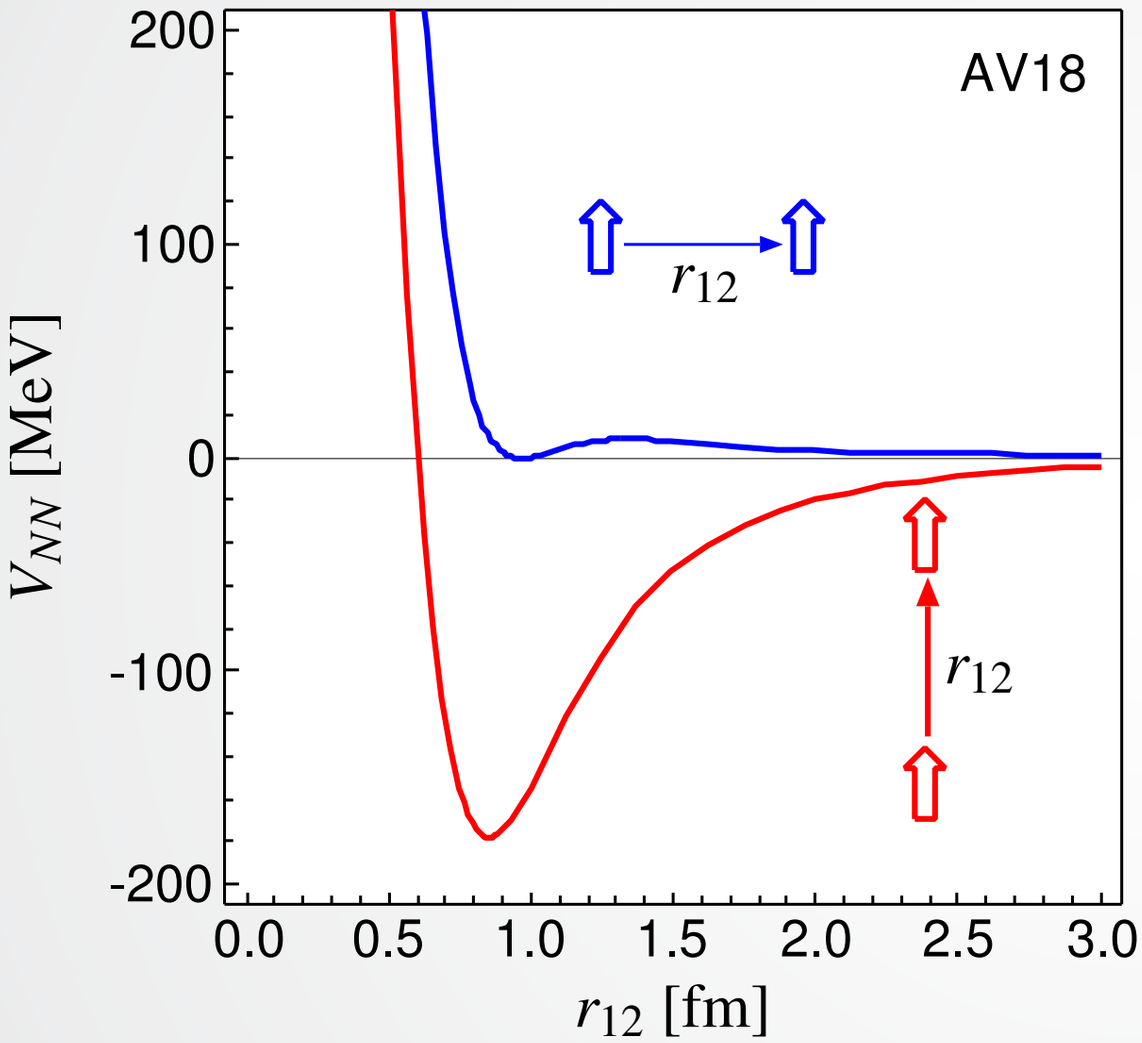
⟷

Realistic Effective Interaction

Unitary Correlation Operator Method
(UCOM)

Nucleon-Nucleon Interaction

S=1, T=0



- **repulsive core**: nucleons can not get closer than ≈ 0.5 fm → **central correlations**
- strong dependence on the orientation of the spins due to the **tensor force** (mainly from π-exchange) → **tensor correlations**
- the nuclear force will induce strong short-range correlations in the nuclear wave function

$$\hat{S}_{12} = 3(\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}}_{12})(\boldsymbol{\sigma}_2 \cdot \hat{\mathbf{r}}_{12}) - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$

Unitary Correlation Operator Method

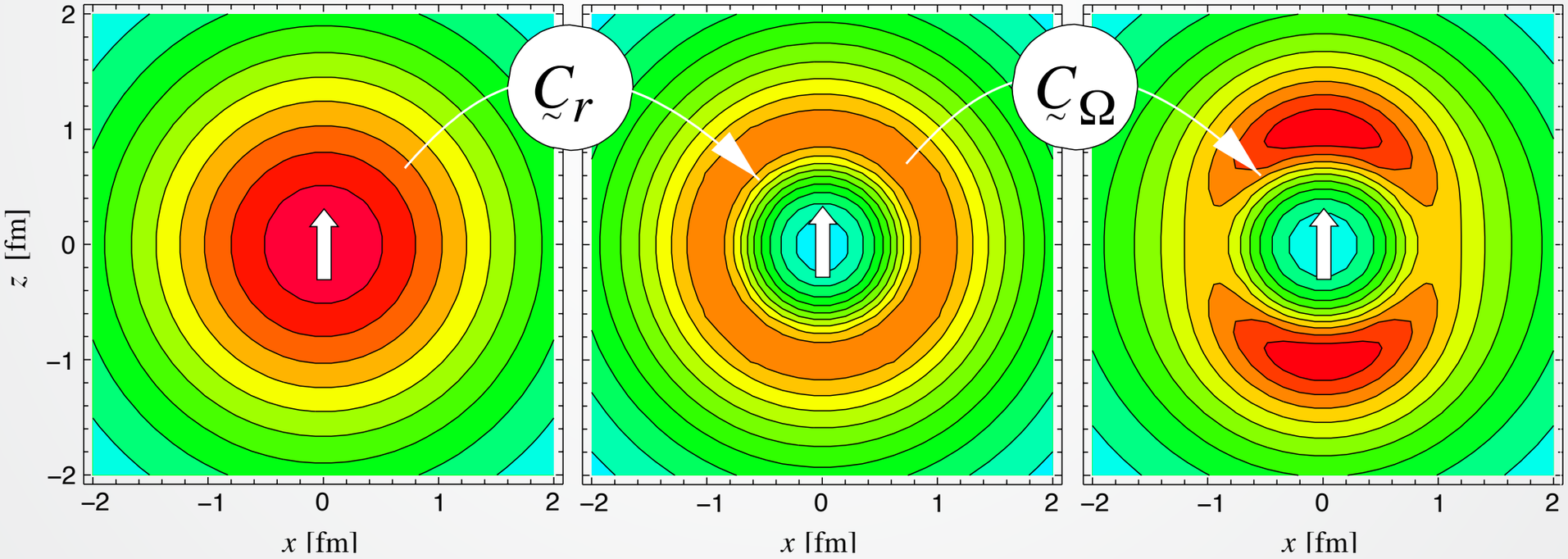
Correlation Operator

$$\hat{C} = \hat{C}_\Omega \hat{C}_r$$

Correlated Hamiltonian

$$\hat{C}^\dagger(\hat{T} + \hat{V})\hat{C} = \hat{T} + \hat{V}_{\mathrm{UCOM}} + \ldots$$

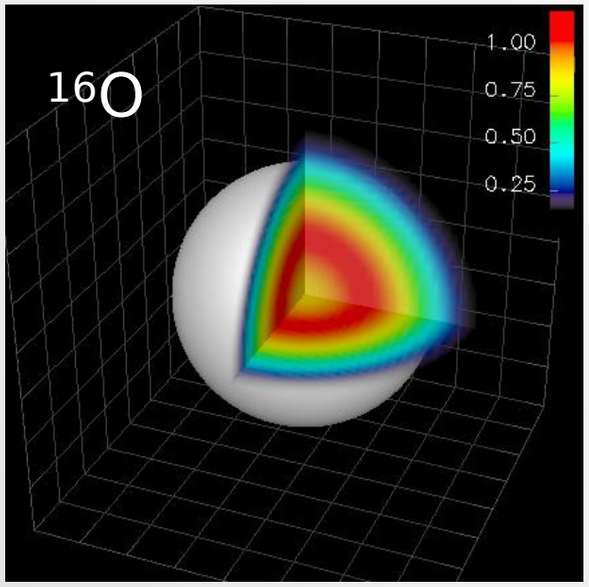
Central correlator shifts nucleons apart,
Tensor correlator aligns nucleons with spin

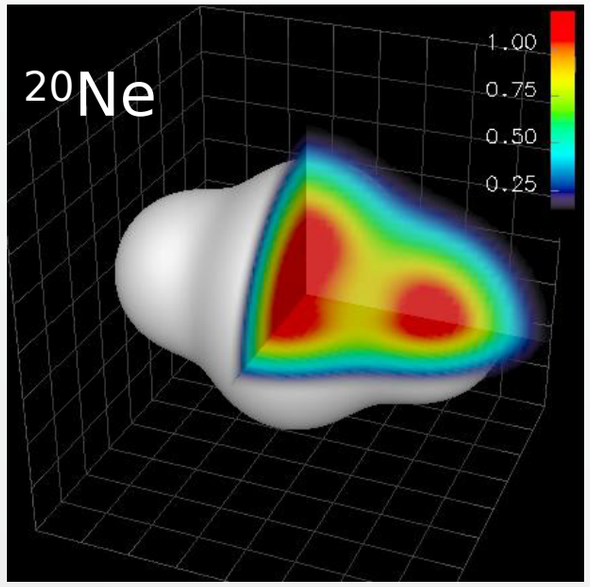


- add phenomenological correction term with momentum dependence and additional spin-orbit strength that is fitted to reproduce binding energies and radii of doubly-magic nuclei

Fermionic Molecular Dynamics

Nuclear structure calculations with a Gaussian wave-packet basis





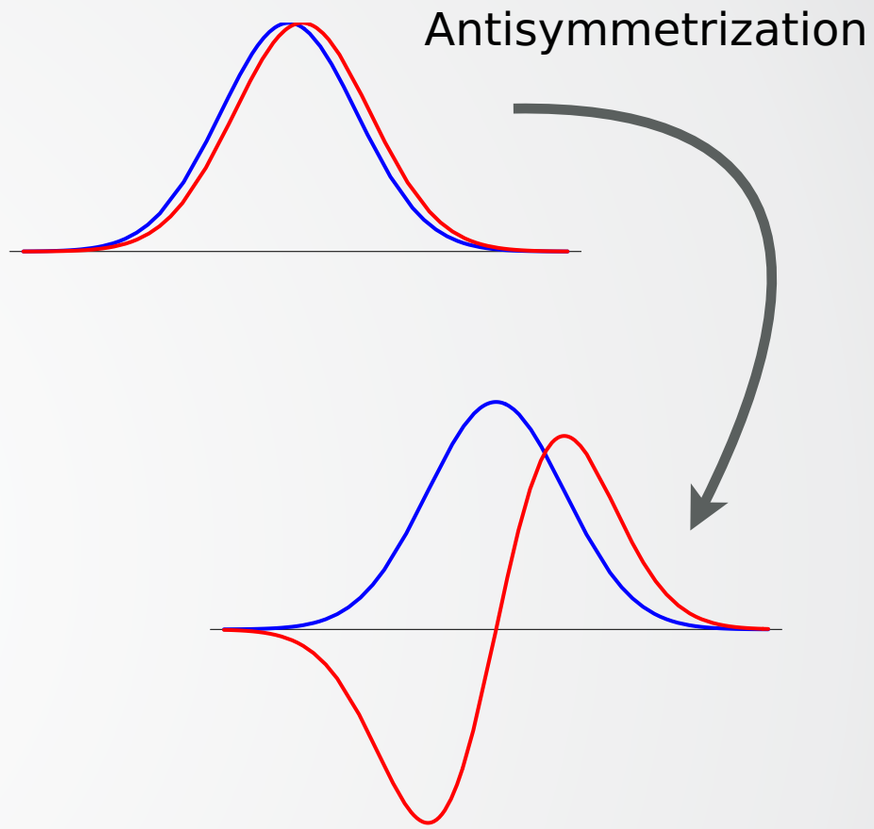
Fermionic Molecular Dynamics

Fermionic

Intrinsic many-body states

$$|Q\rangle = \hat{\mathcal{A}}\{|q_1\rangle \otimes \cdots \otimes |q_A\rangle\}$$

are antisymmetrized A-body states



Molecular

Single-particle states

$$\langle \mathbf{x}|q\rangle = \sum_i c_i \exp\left\{-\frac{(\mathbf{x}-\mathbf{b}_i)^2}{2a_i}\right\} \otimes |\chi_i^{\uparrow}, \chi_i^{\downarrow}\rangle \otimes |\xi\rangle$$

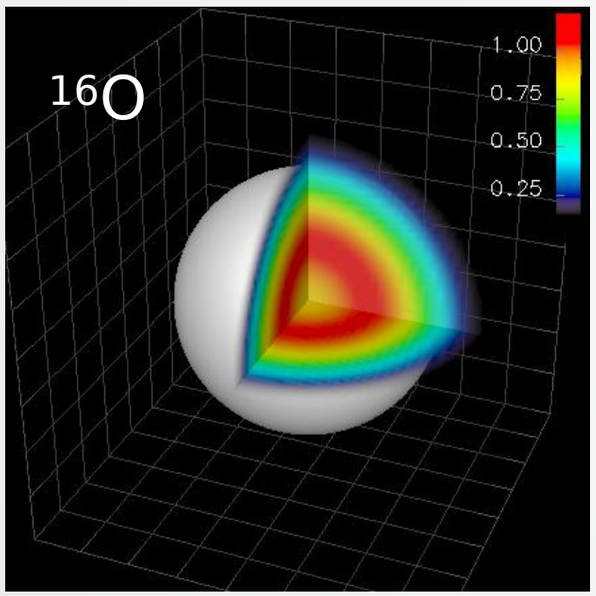
- Gaussian wave-packets in phase-space (complex parameter $\mathbf{b}_i$ encodes mean position and mean momentum), spin is free, isospin is fixed
- width a_i is an independent variational parameter for each wave packet
- use one or two wave packets for each single particle state

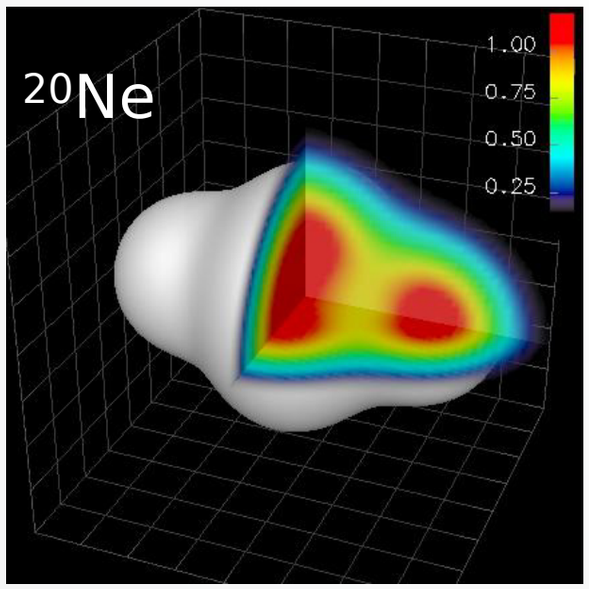
FMD basis contains
harmonic oscillator shell model
and **Brink-type cluster**
configurations as limiting cases

Projection after Variation

Variation and Projection

- minimize the energy of the intrinsic state
- intrinsic state may break symmetries of Hamiltonian
- restore inversion, translational and rotational symmetry by **projection on parity**, **angular (and linear) momentum**





Generator coordinates

- use generator coordinates (radii, quadrupole or octupole deformation, strength of spin-orbit force) to create additional basis states

Variation

$$\min_{\{q_\nu\}} \frac{\langle Q|\hat{H}-\hat{T}_{\mathrm{cm}}|Q\rangle}{\langle Q|Q\rangle}$$

Projection

$$\hat{P}^{\pi} = \frac{1}{2}(1+\pi\hat{\Pi})$$

$$\hat{P}^{J}{}_{MK} = \frac{2J+1}{8\pi^2}\int \mathrm{d}^3\Omega\, D^{J}{}_{MK}{}^{\star}(\Omega)\,\hat{R}(\Omega)$$

$$\hat{P}^{\mathbf{P}} = \frac{1}{(2\pi)^3}\int \mathrm{d}^3X\ \exp\{-i(\hat{\mathbf{P}}-\mathbf{P})\cdot\mathbf{X}\}$$

Variation after Projection

Variation after Projection

- Correlation energies can be quite large for well deformed and/or clustered states
- For light nuclei it is possible to perform real variation after projection
- Can be combined with generator coordinate method

Variation

$$\min_{\{q_\nu\}} \frac{\langle Q|\hat{H}-\hat{T}_{\mathrm{cm}}|Q\rangle}{\langle Q|Q\rangle}$$

Variation after Projection

$$\min_{\{q_\nu, c^{\alpha}{}_{K}\}} \frac{\sum_{KK'} c^{\alpha}{}_{K}{}^{\star}\langle Q|(\hat{H}-\hat{T}_{\mathrm{cm}})\hat{P}^{\pi}\hat{P}^{J}{}_{KK'}|Q\rangle c^{\alpha}{}_{K'}}{\sum_{KK'} c^{\alpha}{}_{K}{}^{\star}\langle Q|\hat{P}^{\pi}\hat{P}^{J}{}_{KK'}|Q\rangle c^{\alpha}{}_{K'}}$$

Multiconfiguration Mixing

- Set of N intrinsic states optimized for different spins and parities and for different values of generator coordinates are used as basis states
- Diagonalize in set of projected basis states

(Intrinsic) Basis States

$$\left\{\left|Q^{(a)}\right\rangle, a=1,\ldots,N\right\}$$

Generalized Eigenvalue Problem

$$\sum_{K'b}\underbrace{\langle Q^{(a)}|\hat{H}\hat{P}^{\pi}\hat{P}^{J}{}_{KK'}\hat{P}^{\mathbf{P}=0}|Q^{(b)}\rangle}_{\text{Hamiltonian kernel}} c^{\alpha}{}_{K'b} = E^{J^{\pi}\alpha}\sum_{K'b}\underbrace{\langle Q^{(a)}|\hat{P}^{\pi}\hat{P}^{J}{}_{KK'}\hat{P}^{\mathbf{P}=0}|Q^{(b)}\rangle}_{\text{norm kernel}} c^{\alpha}{}_{K'b}$$

Calculating Densities and Form Factors

- The proton/neutron form factors are given as the Fourier transforms of the corresponding densities

$$F_{p/n}(\mathbf{q}) = \int d^3x\, \rho_{p/n}(\mathbf{x})\, e^{i\mathbf{q}\cdot\mathbf{x}}$$

- Densities have to be calculated with respect to the total center-of-mass

$$\rho_{p/n}(\mathbf{x}) = \sum_{i=1}^{A} \left\langle Q^{(a)} \middle| \hat{P}_i^{p/n}\, \delta^3(\hat{\mathbf{x}}_i - \hat{\mathbf{X}} - \mathbf{x}) \middle| Q^{(b)} \right\rangle$$

- Using the Fourier representation of the delta-function

$$\delta^3(\hat{\mathbf{x}}_i - \hat{\mathbf{X}} - \mathbf{x}) = \frac{1}{(2\pi)^3} \int d^3k\, e^{i\mathbf{k}\cdot(\hat{\mathbf{x}}_i - \hat{\mathbf{X}} - \mathbf{x})}$$

the form factor can be calculated using boosted Slater determinants

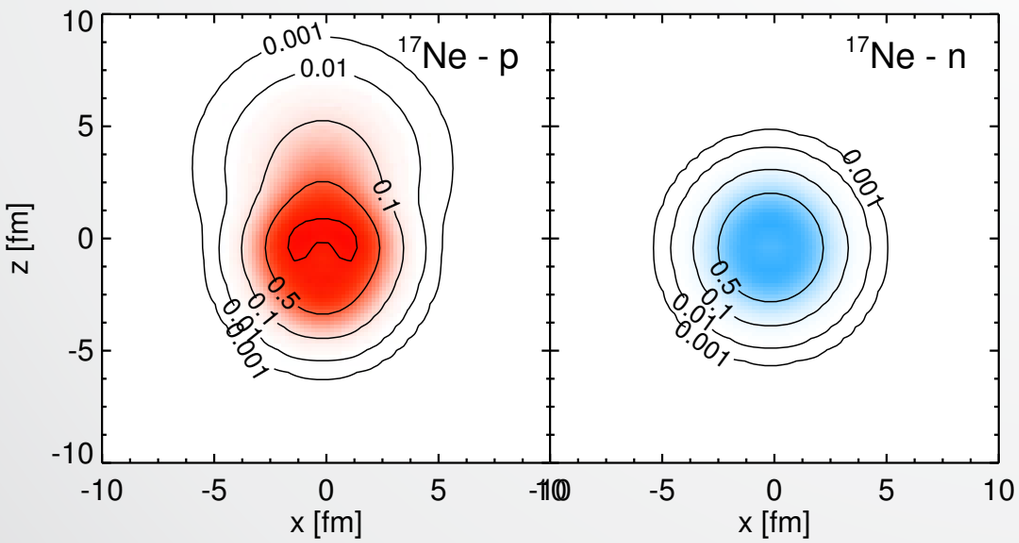
$$F_{p/n}(\mathbf{q}) = \sum_{i=1}^{A} \left\langle Q^{(a)} \middle| \hat{P}_i^{p/n}\, e^{i\mathbf{q}\cdot(\hat{\mathbf{x}}_i - \hat{\mathbf{X}})} \middle| Q^{(b)} \right\rangle = \sum_{i=1}^{A} \left\langle Q^{(a)} \middle| \hat{P}_i^{p/n}\, e^{i\mathbf{q}\cdot\hat{\mathbf{x}}_i} \middle| Q^{(b)}(\mathbf{v} = -\frac{1}{M}\mathbf{q}) \right\rangle$$

- Multiply with proton and neutron form factors to get the charge form factor

$$F_{\text{ch}}(\mathbf{q}) = F_p(\mathbf{q})\, F_{\text{proton}}(\mathbf{q}) + \frac{N}{Z} F_n(\mathbf{q})\, F_{\text{neutron}}(\mathbf{q})$$

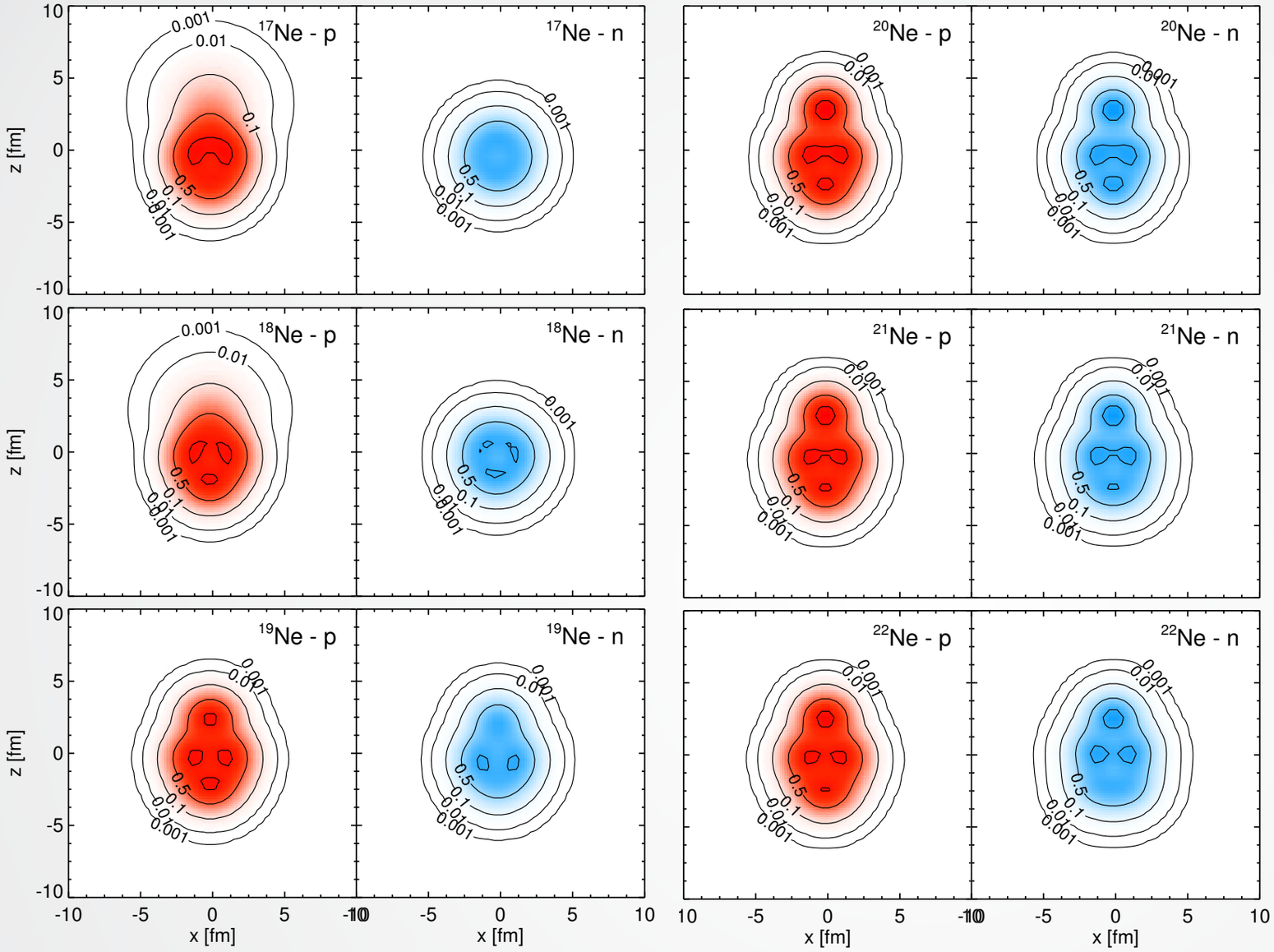
Neon Isotopes

Charge Radii as Signatures of Halos and Clustering in Ground States



Geithner, Neff *et al.*, Phys. Rev. Lett. **101**, 252502 (2008)

Neon Isotopes: Calculation



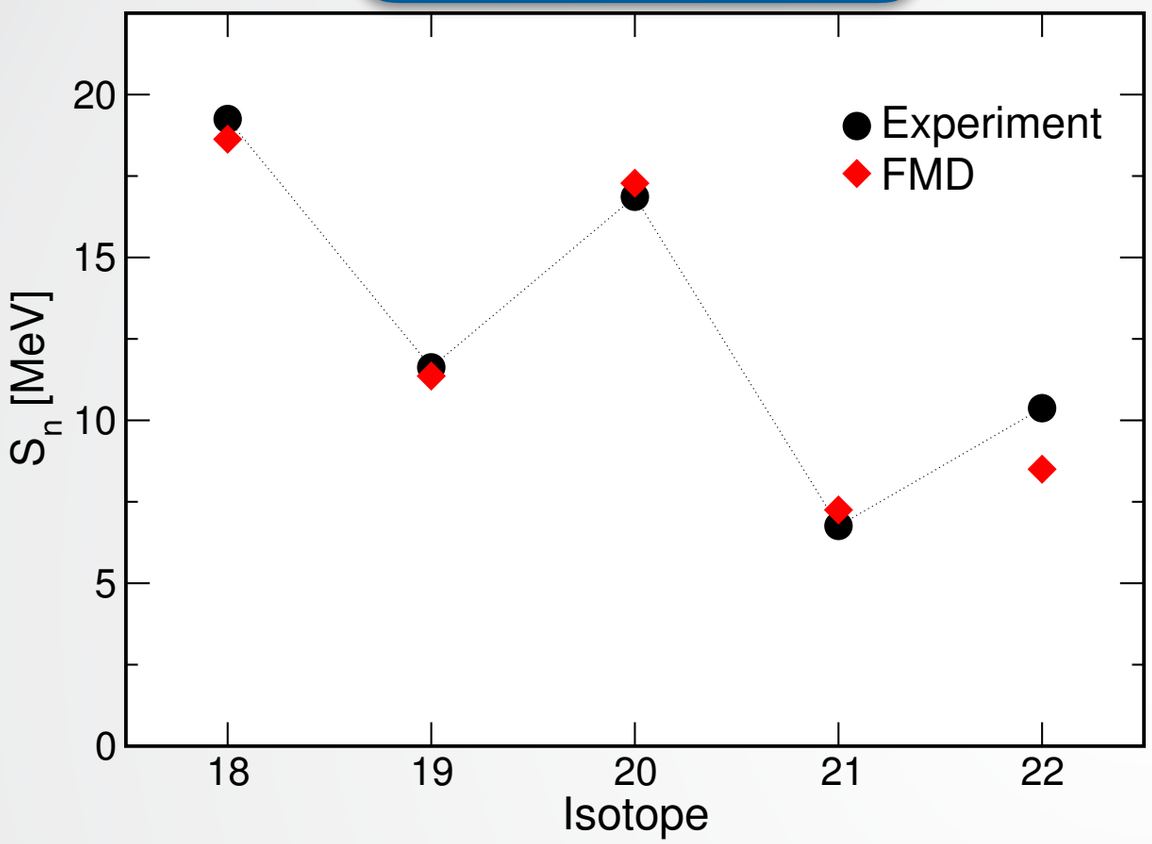
proton/neutron densities of dominant intrinsic FMD basis states

- UCOM(var) + phenomenological correction for saturation and spin-orbit
- Variation after parity projection on positive and negative parity
- Create basis states by cranking strength of spin-orbit force
- ^{15,16}O-“s^2” and ^{15,16}O-“d^2” minima in ^{17,18}Ne
- add explicit cluster configurations:

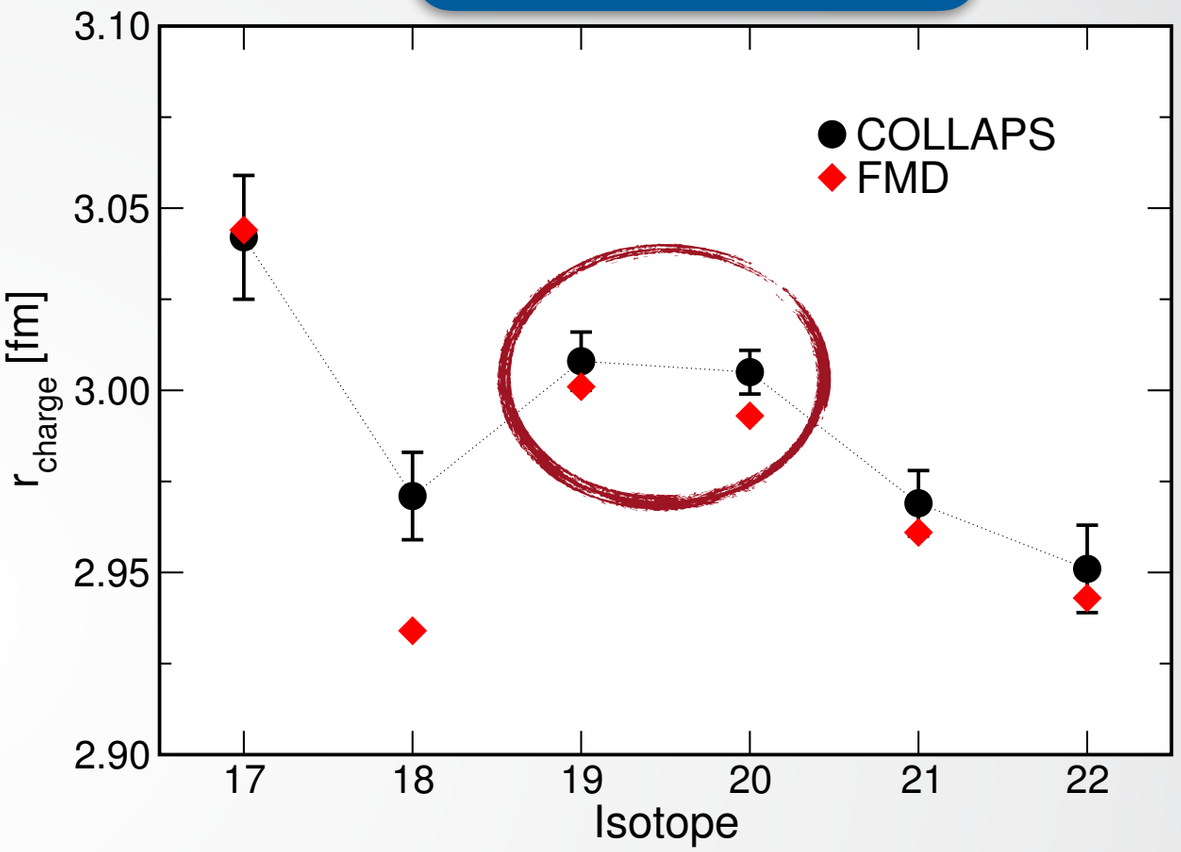
  ^{17}Ne: ^{14}O-^{3}He
  ^{18}Ne: ^{14}O-^{4}He
  ^{19}Ne: ^{16}O-^{3}He and ^{15}O-^{4}He
  ^{20}Ne: ^{16}O-^{4}He
  ^{21}Ne: “^{17}O”-^{4}He
  ^{22}Ne: “^{18}O”-^{4}He

Neon Isotopes: Energies and Radii

Separation Energies



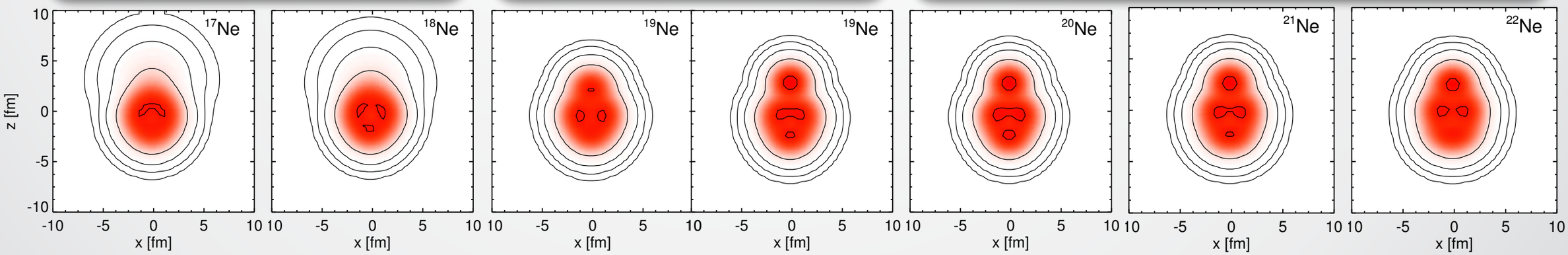
Charge Radii



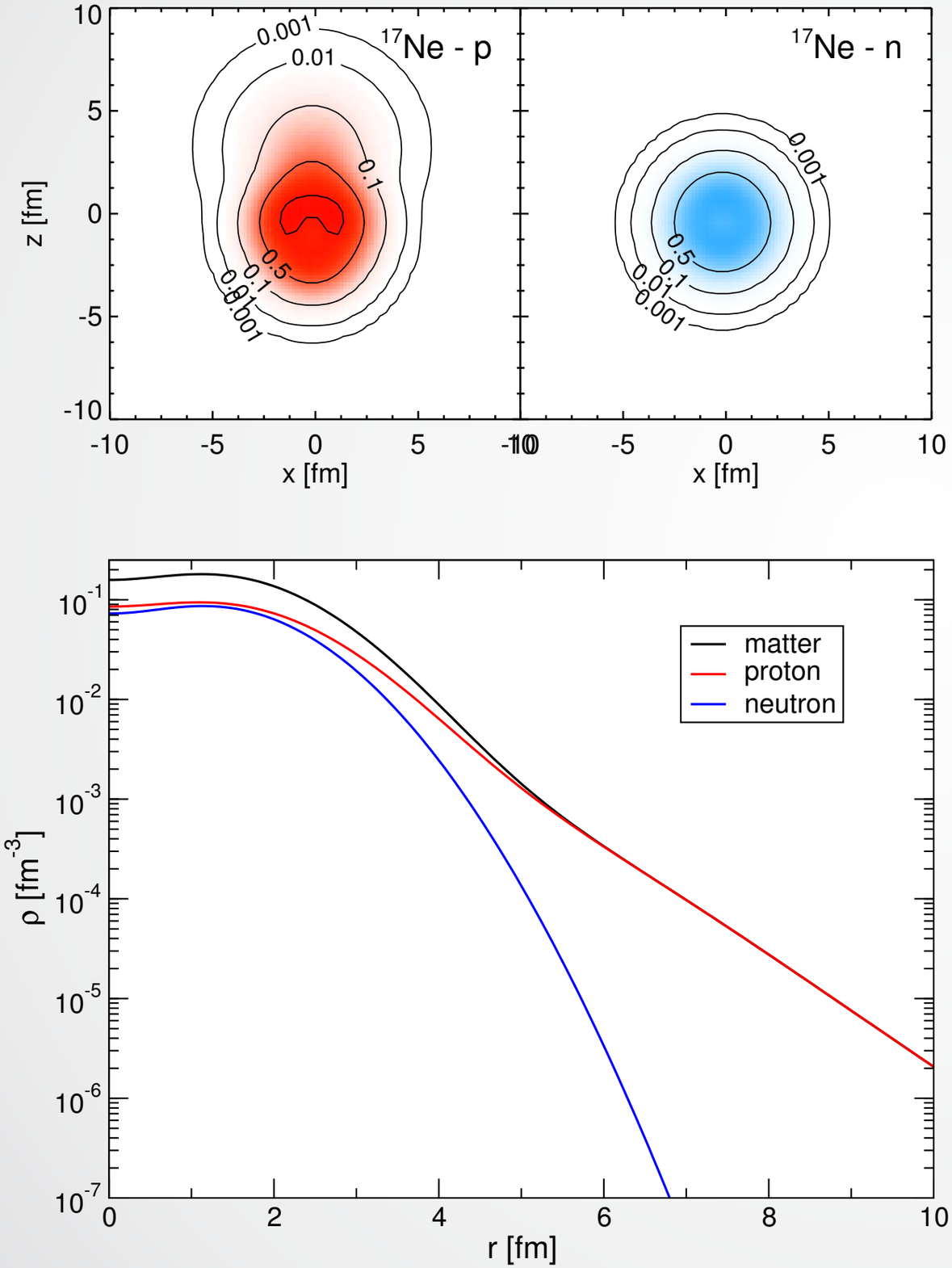
^{17,18}Ne: s^2/d^2 admixture

^{19}Ne: α, ^{3}He clustering

^{20,21,22}Ne: α clustering



^{17}Ne: two-proton halo ?



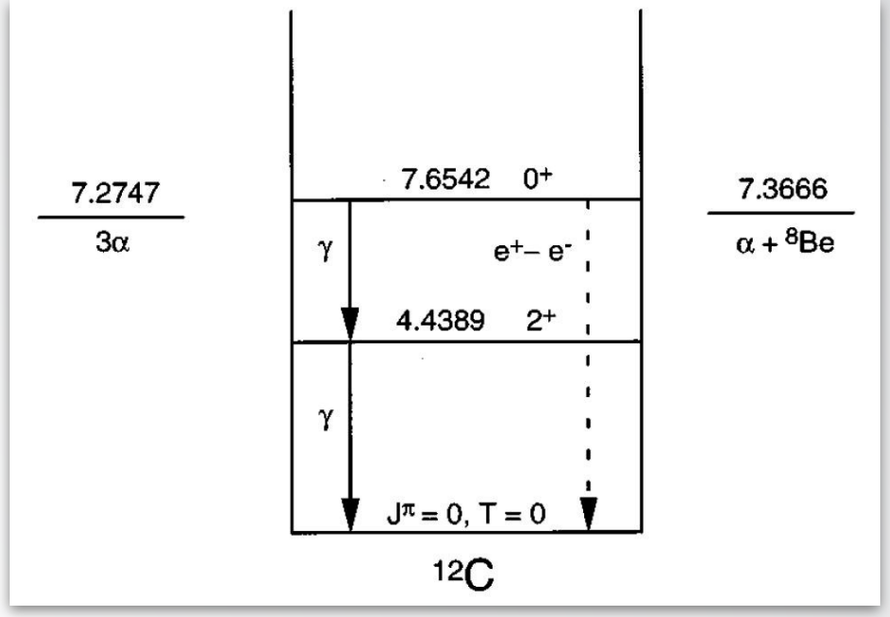
	FMD	Experiment
r_{ch} [fm]	3.04	3.042(21)
r_{mat} [fm]	2.75	2.75(7)
$B(E2;\ 1/2^- \rightarrow 3/2^-)$ [e^2 fm^4]	76.7	66^{+18}_{-25}
$B(E2;\ 1/2^- \rightarrow 5/2^-)$ [e^2 fm^4]	119.8	124(18)
s^2 occupancy	42 %	
d^2 occupancy	55 %	

A. Ozawa *et al.*, Nuc. Phys. **A693**, 32 (2001)
M.J.Chromik *et al.*.Phys. Rev. C**66**, 024313 (2002)

- proton skin $r_p - r_n = 0.45$ fm
- 40% probability to find a proton at distance r > 5 fm
- similar results are obtained in a three-body model

  L. Grigorenko et al., Phys. Rev. C 71, 051604 (2005)

- translational invariant point densities



Electron scattering on ^{12}C

FMD and Cluster Model Calculations

M. Chernykh, H. Feldmeier, T. Neff, P. von Neumann-Cosel, A. Richter, Phys. Rev. Lett. **98** (2007) 032501

M. Chernykh, H. Feldmeier, T. Neff, P. von Neumann-Cosel, A. Richter, Phys. Rev. Lett. **105** (2010) 022501

^{12}C: Microscopic α-Cluster Model

Basis States

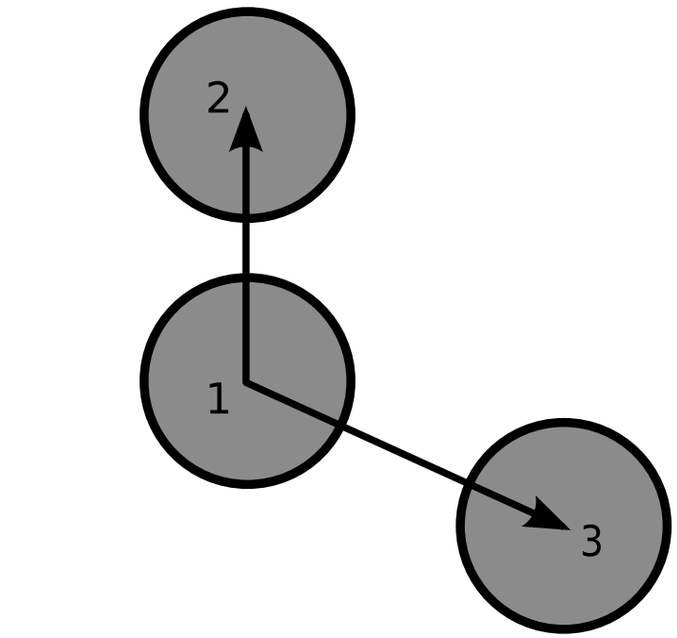
- ^{12}C is described as a system of three α-particles
- α-particles are given by HO $(0s)^4$ wave functions
- wave function is fully antisymmetrized

Effective Volkov NN-interaction

- parameters adjusted to give reasonable α binding energy and radius, $\alpha - \alpha$ scattering data, adjusted to reproduce ^{12}C ground state energy

'BEC' wave functions

- interpretation of the Hoyle state as a Bose-Einstein Condensate of α-particles using THSR wave function (Funaki, Tohsaki, Horiuchi, Schuck, Röpke)
- same interaction and α-cluster parameters used



$$R_{12} = (2, 4, \ldots, 10)\,\mathrm{fm}$$
$$R_{13} = (2, 4, \ldots, 10)\,\mathrm{fm}$$
$$\cos(\vartheta) = (1.0, 0.8, \ldots, -1.0)$$

165 configurations

$$\left|\Psi^{3\alpha}_{JMK\pi}(\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3)\right\rangle = \hat{P}^{\pi}\hat{P}^{J}_{MK}\hat{\mathcal{A}}\left\{\left|\Psi_{\alpha}(\mathbf{R}_1)\right\rangle \otimes \left|\Psi_{\alpha}(\mathbf{R}_2)\right\rangle \otimes \left|\Psi_{\alpha}(\mathbf{R}_3)\right\rangle\right\}$$

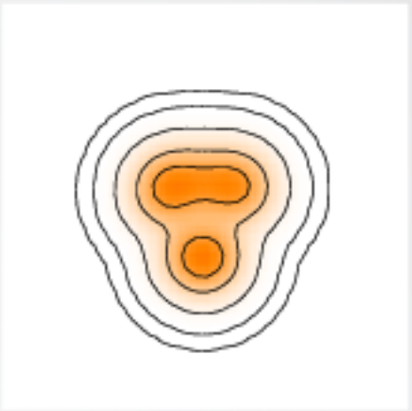
Kamimura, Nuc. Phys. **A351** (1981) 456

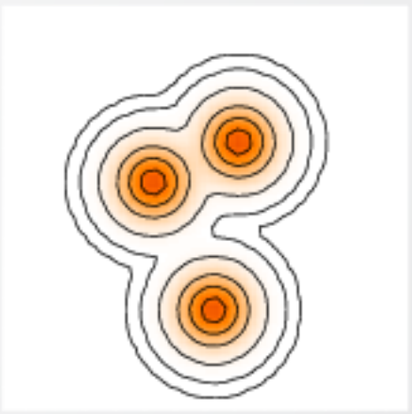
Funaki et al., Phys. Rev. C 67 (2003) 051306(R)

^{12}C: Fermionic Molecular Dynamics

Basis States

- 20 FMD states obtained in Variation after Projection on 0^+ and 2^+ with constraints on the radius
- 42 FMD states obtained in Variation after Projection on parity with constraints on radius and quadrupole deformation
- 165 explicit α-cluster configurations

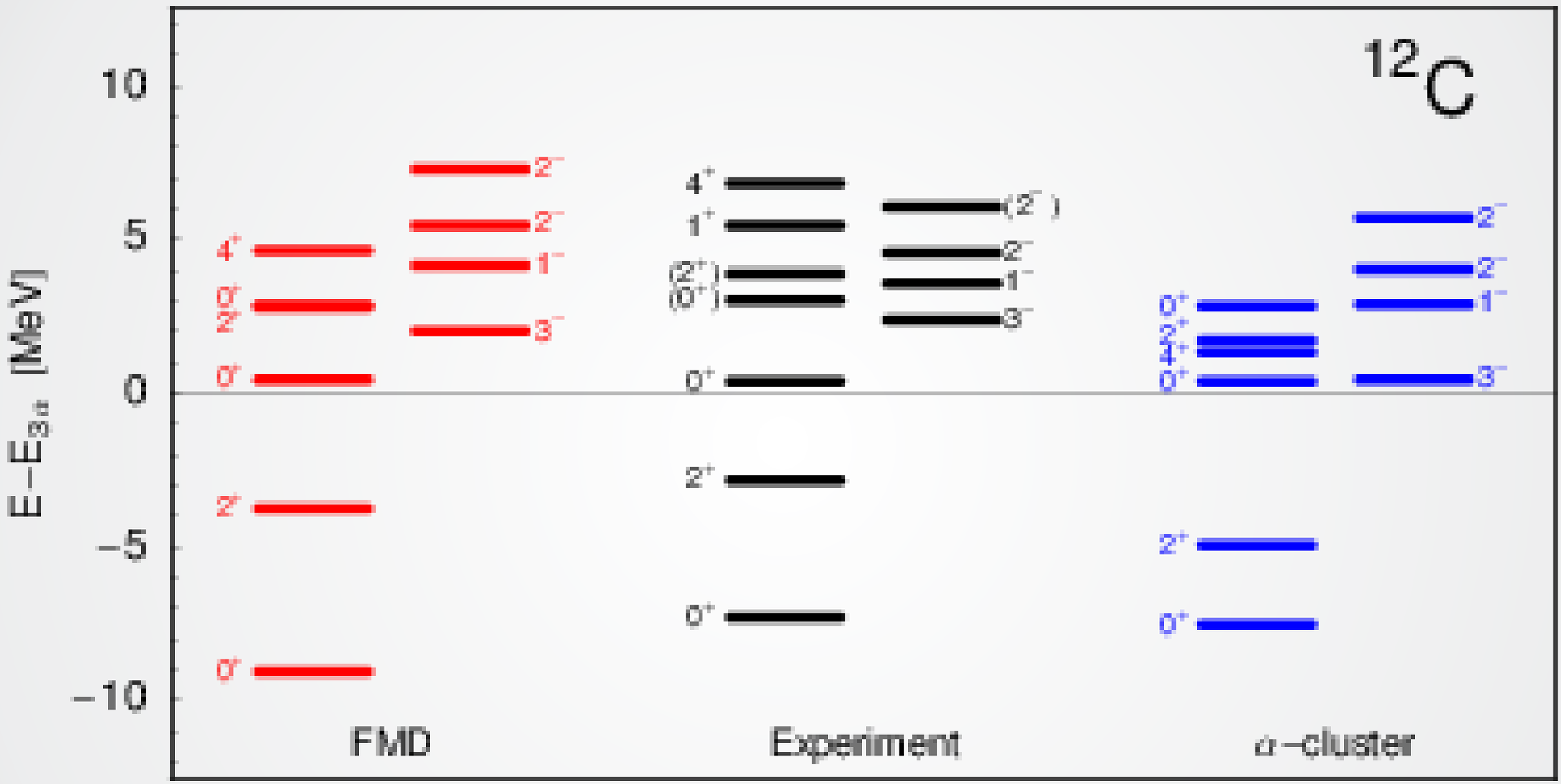




Effective UCOM NN-interaction

- UCOM interaction with phenomenological two-body correction term (momentum-dependent central and spin-orbit) fitted to doubly-magic nuclei
- not tuned for α-α scattering or ^{12}C properties

^{12}C: Spectrum



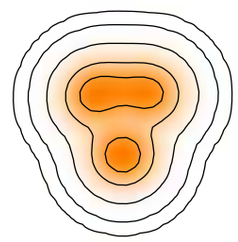
- FMD and cluster model get Hoyle state close to the 3-alpha threshold

^{12}C: Observables

	Exp[1]	Exp[2]	FMD	α-cluster	'BEC'[3]
$E(0_1^+)$	-92.16		-92.64	-89.56	-89.52
$E^*(2_1^+)$	4.44		5.31	2.56	2.81
$E(3\alpha)$	-84.89		-83.59	-82.05	-82.05
$E(0_2^+) - E(3\alpha)$	0.38		0.43	0.38	0.26
$E(0_3^+) - E(3\alpha)$	(3.0)	2.7(3)	2.84	2.81	
$E(2_2^+) - E(3\alpha)$	(3.89)	2.6(3)	2.77	1.70	
$r_{\text{charge}}(0_1^+)$	2.47(2)		2.53	2.54	
$r(0_1^+)$			2.39	2.40	2.40
$r(0_2^+)$			3.38	3.71	3.83
$r(0_3^+)$			4.62	4.75	
$r(2_1^+)$			2.50	2.37	2.38
$r(2_2^+)$			4.43	4.02	
$M(E0, 0_1^+ \to 0_2^+)$	5.4(2)		6.53	6.52	6.45
$B(E2, 2_1^+ \to 0_1^+)$	7.6(4)		8.69	9.16	
$B(E2, 2_1^+ \to 0_2^+)$	2.6(4)		3.83	0.84	
$B(E2, 2_2^+ \to 0_1^+)$	?		0.46	1.99	

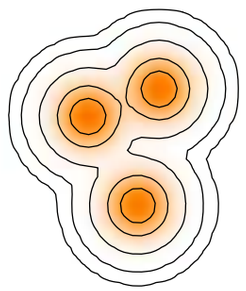
[1] Ajzenberg-Selove, Nuc. Phys. **A506**, 1 (1990)
[2] Itoh et al., Nuc. Phys. **A738**, 268 (2004)
[3] Funaki et al., Phys. Rev. C **67**, 051306(R) (2003)

^{12}C: Configurations

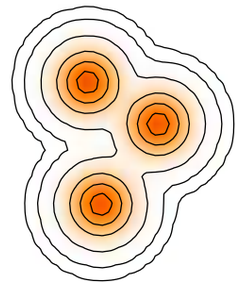


$\left|\left\langle \cdot \middle| 0_1^+ \right\rangle\right| = 0.94$

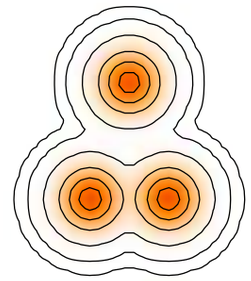
$\left|\left\langle \cdot \middle| 2_1^+ \right\rangle\right| = 0.93$



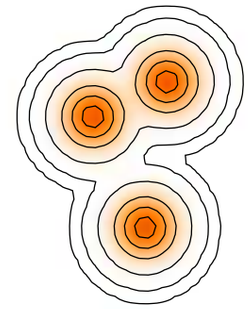
$\left|\left\langle \cdot \middle| 0_2^+ \right\rangle\right| = 0.72$



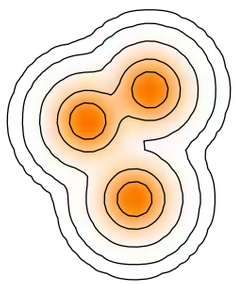
$\left|\left\langle \cdot \middle| 0_2^+ \right\rangle\right| = 0.71$



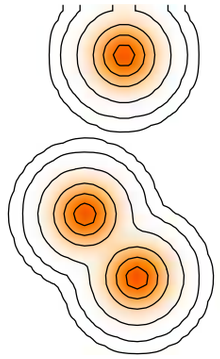
$\left|\left\langle \cdot \middle| 0_2^+ \right\rangle\right| = 0.61$



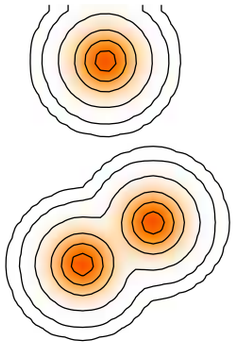
$\left|\left\langle \cdot \middle| 0_2^+ \right\rangle\right| = 0.61$



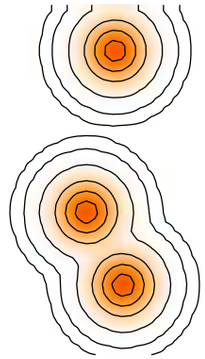
$\left|\left\langle \cdot \middle| 3_1^- \right\rangle\right| = 0.83$



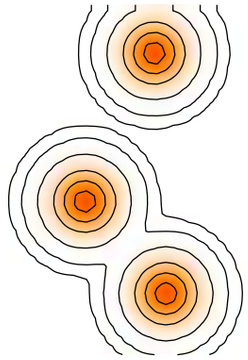
$\left|\left\langle \cdot \middle| 0_3^+ \right\rangle\right| = 0.50$



$\left|\left\langle \cdot \middle| 0_3^+ \right\rangle\right| = 0.49$



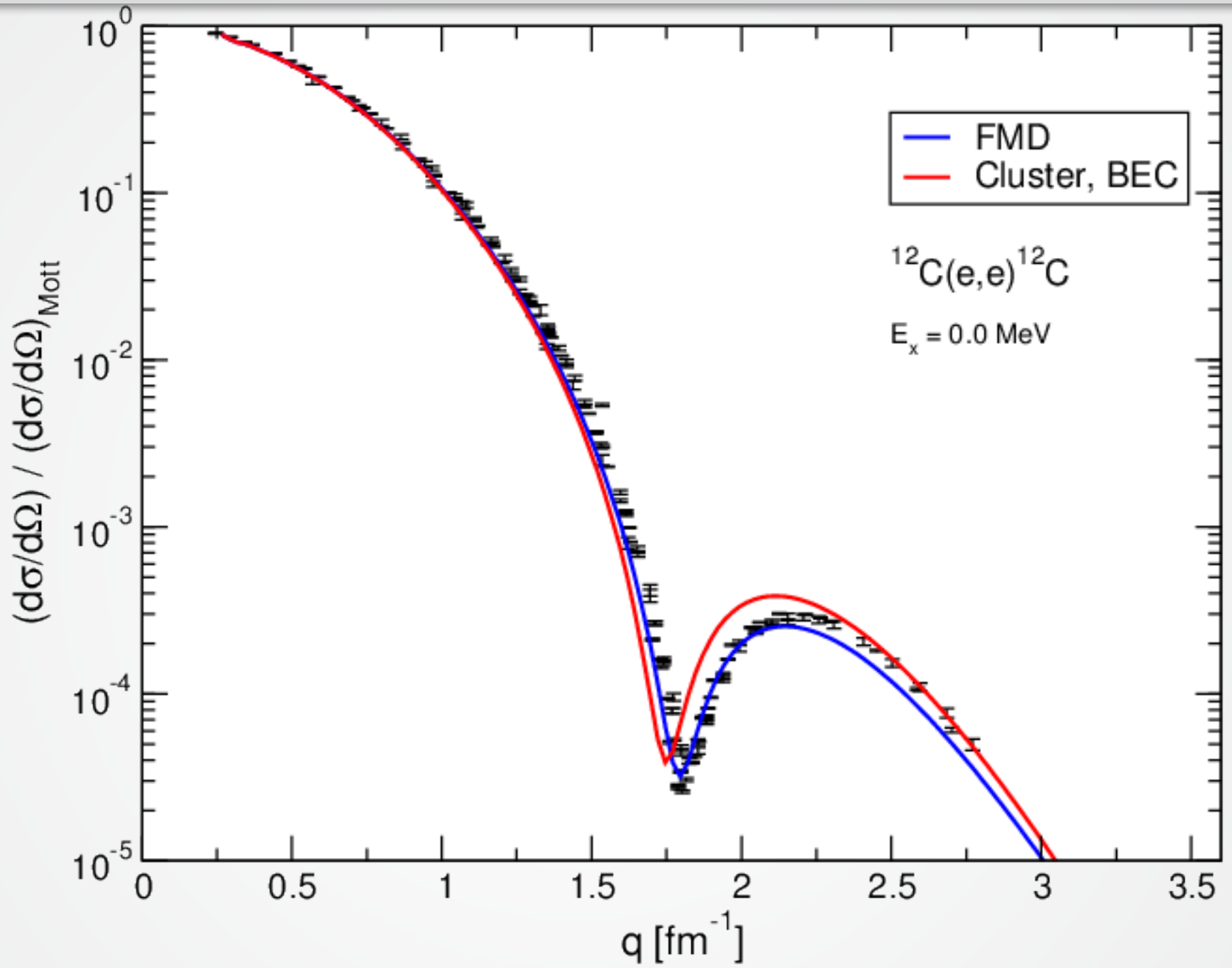
$\left|\left\langle \cdot \middle| 0_3^+ \right\rangle\right| = 0.44$



$\left|\left\langle \cdot \middle| 0_3^+ \right\rangle\right| = 0.41$

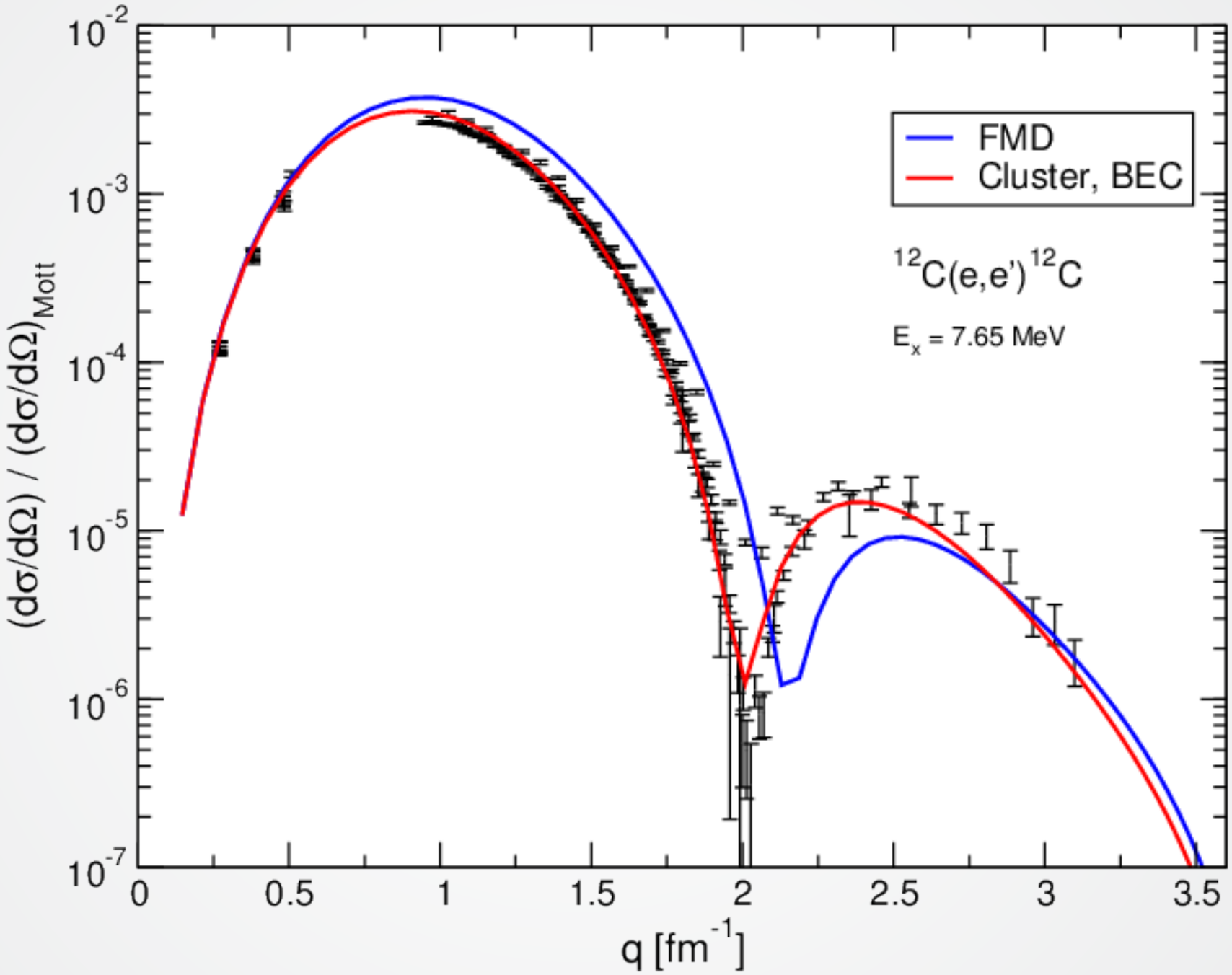
- Hoyle state dominated by triangular alpha-configurations, third 0^+ state by open triangles
- Pronounced clustering also in negative parity states

^{12}C: Elastic Cross Section



- Using DWBA to calculate the theoretical cross sections
- FMD doing a little bit better in reproducing the cross section

^{12}C: Inelastic Cross Section



- Inelastic cross section rather well described, confirming the dilute nature of the Hoyle state

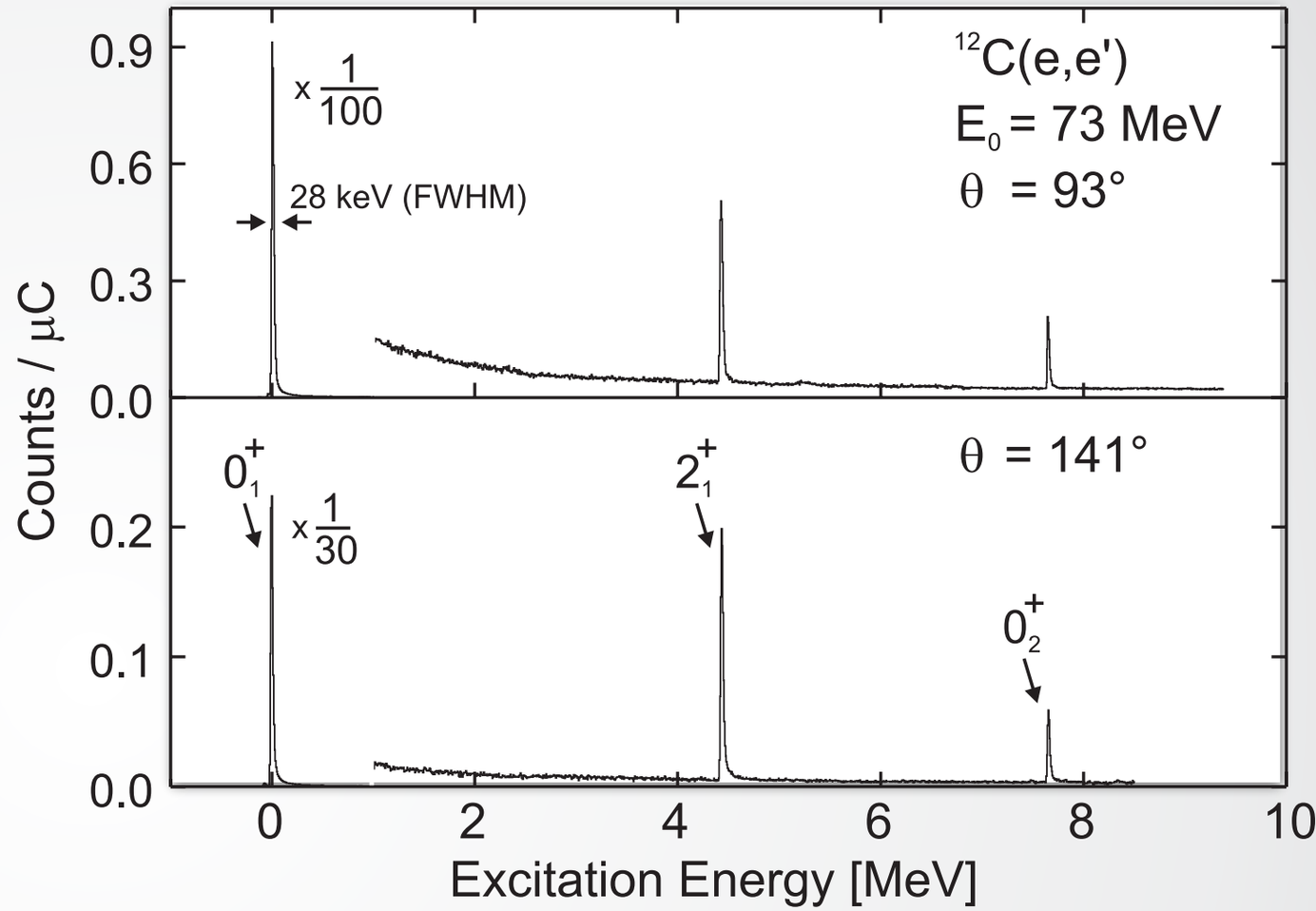
^{12}C: Inelastic Formfactor revisited

Motivation

- Astrophysics requires a precision value for the monopole matrix element
- Literature values differed by up to 10%

Our Approach

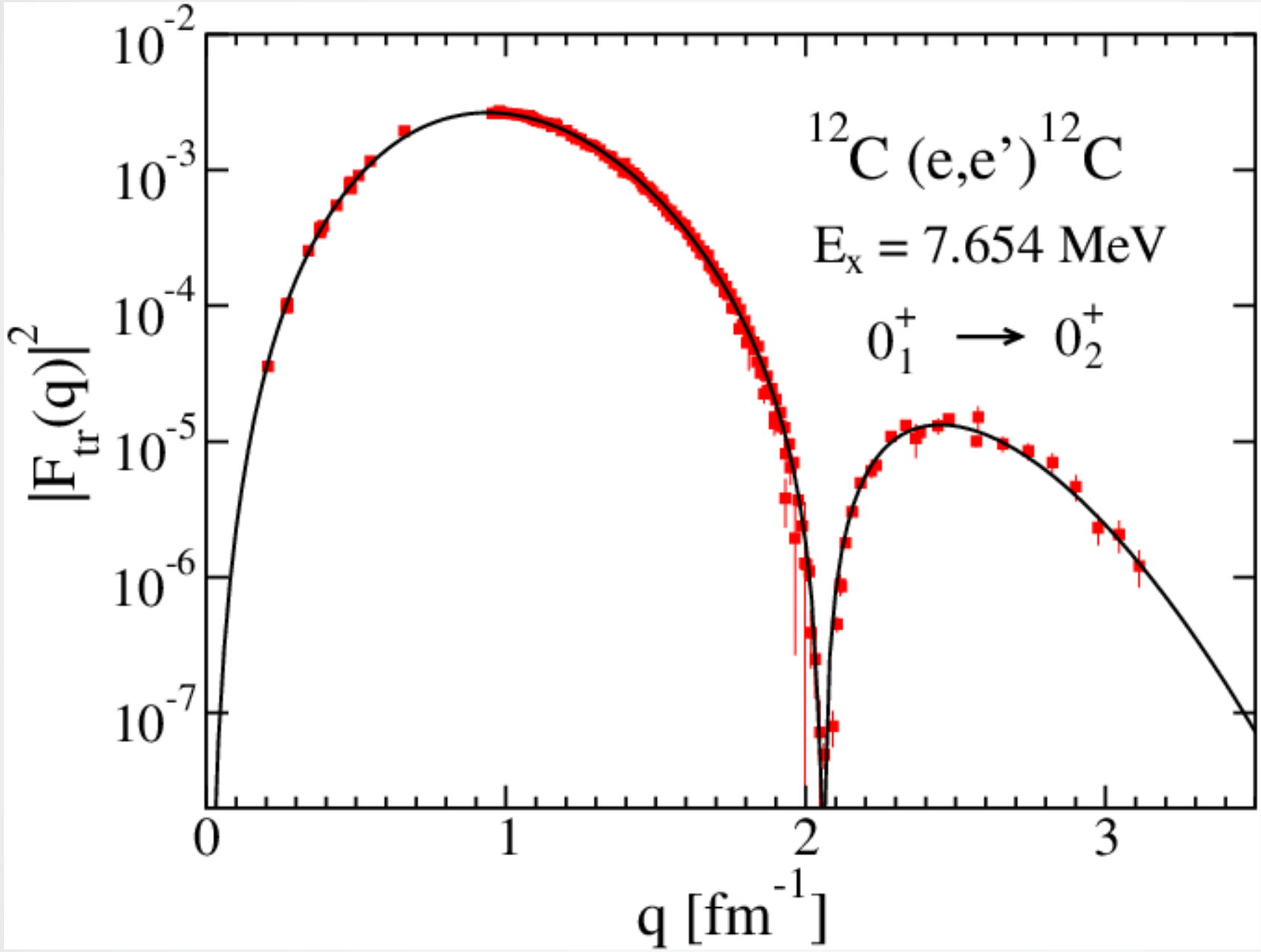
- Add new electron scattering data from Darmstadt at low momentum transfer
- Use a model independent approach to extract form factor/transition density
- Self-consistent calculation in DWBA
- Compare extraction from low-momentum data only with global form factor fit



$$F_{\mathrm{tr}}(q) = \frac{4\pi}{Z} \int_0^\infty \rho_{\mathrm{tr}}(r)\, j_0(qr)\, r^2\, dr$$

$$\left|F_{\mathrm{tr}}^{\mathrm{exp}}(q_i)\right|^2 = \frac{4\pi}{Z^2} B(C0, q_i, E_{0i})\, \frac{B^{\mathrm{PWBA}}(q_i)}{B^{\mathrm{DWBA}}(q_i, E_{0i})}$$

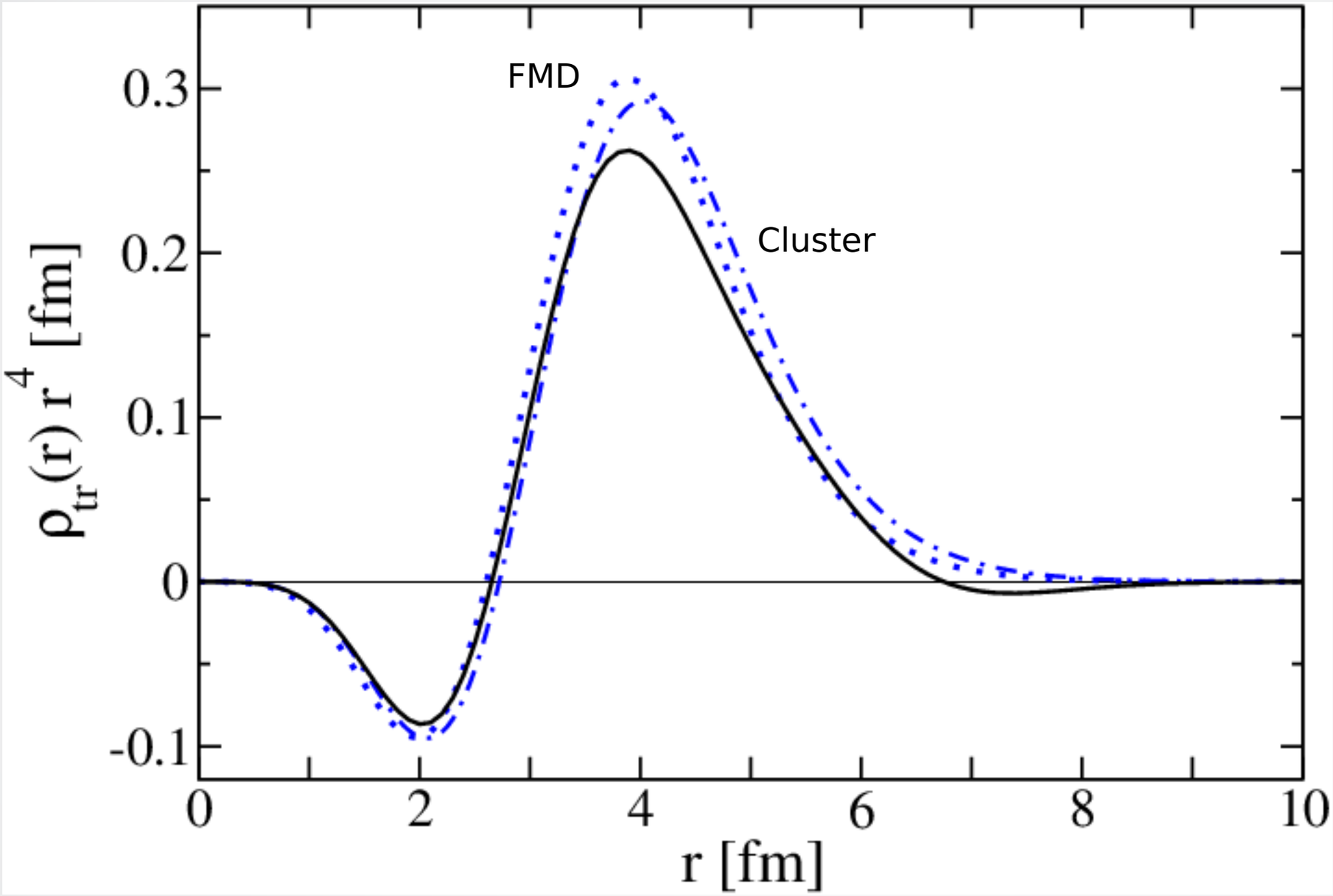
12C: Reanalyzed Monopole Form Factor



- Model independent ansatz:
  best fit with 6 parameters (b, c_1 … c_5)

$$F_{\mathrm{tr}}(q) = \frac{1}{Z}\, \mathrm{e}^{-\frac{1}{2}(b\,q)^2} \sum_{n=1}^{n_{\max}} c_n\, (b\,q)^{2n}$$

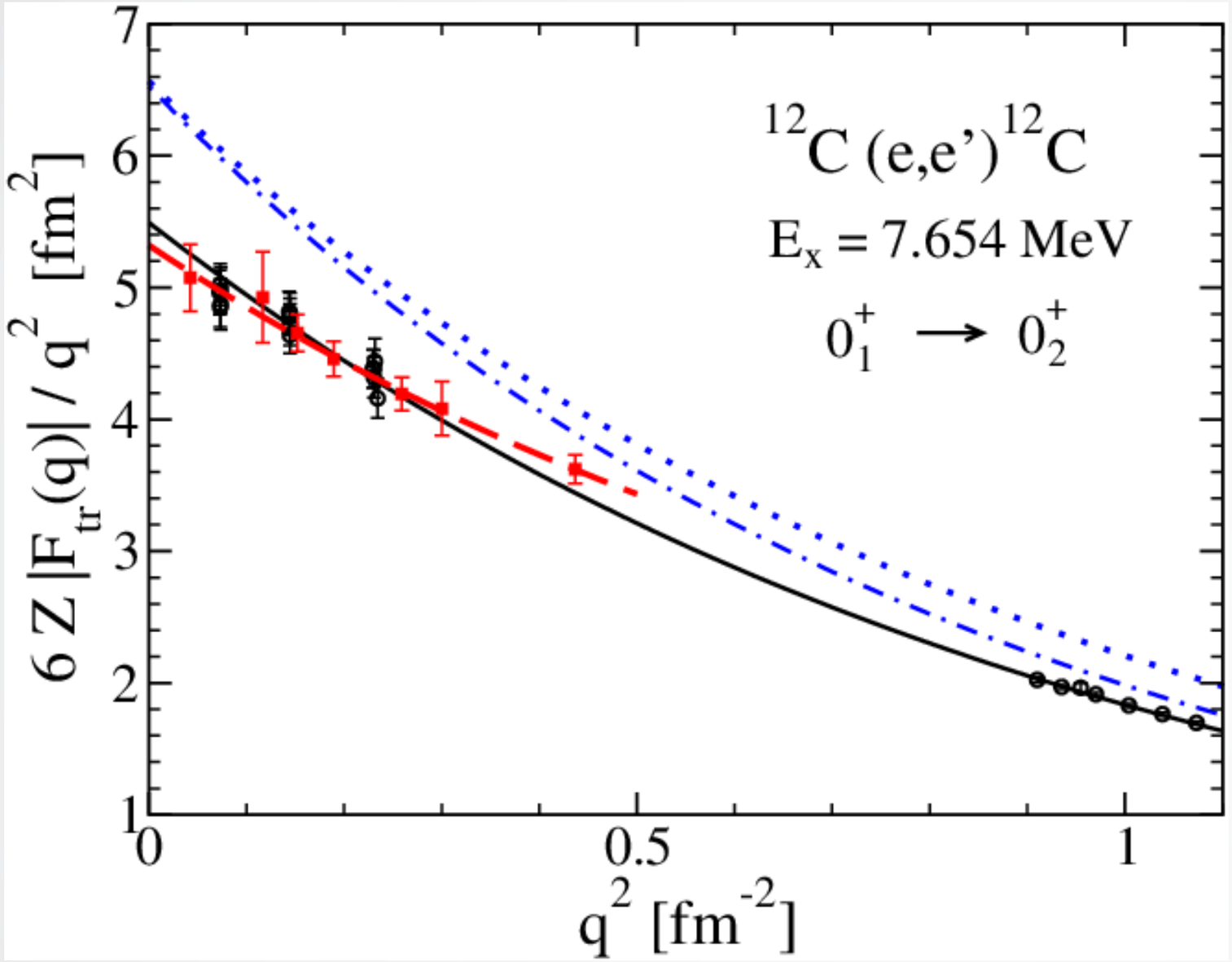
^{12}C: Transition Densities



- Models agree reasonably well with extracted transition density, deviations at large distances

$$\rho_{\mathrm{tr}}(r) = \frac{1}{b^3}\, \mathrm{e}^{-\frac{1}{2}(\frac{r}{b})^2} \sum_{n=0}^{n_{\max}} d_n \left(\frac{r}{b}\right)^{2n}$$

Monopole Matrix Element



Extract monopole matrix element from low-momentum data

$$-6Z\frac{F_{\mathrm{tr}}(q)}{q^2} = \langle r^2 \rangle_{\mathrm{tr}} - \frac{q^2}{20}\langle r^4 \rangle_{\mathrm{tr}} + \dots$$

$$M(E0) = (5.29 \pm 0.14)\,\mathrm{fm}^2$$

Extract monopole matrix element from global fit

$$M(E0) = (5.47 \pm 0.09)\,\mathrm{fm}^2$$

- Due to experimental error bars the fit to low-momentum data is less precise than the extraction from the global fit

Summary and Conclusions

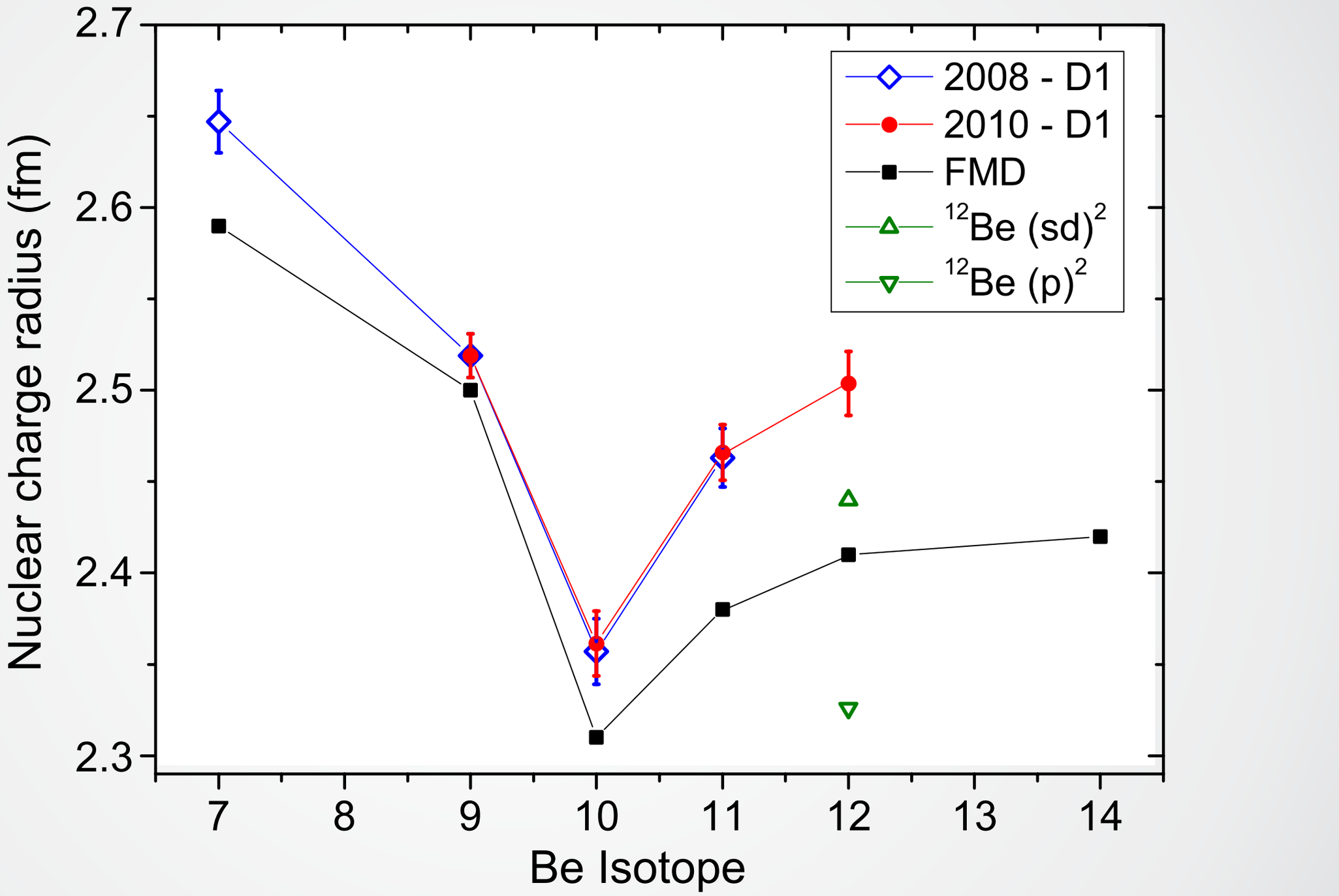
Charge Radii

- The charge radius is sensitive to exotic structures like proton- and neutron-halos and clustering
- FMD with its Gaussian wave-packet basis has the flexibility to describe such exotic nuclei
- The evolution of the charge radii along isotopes chains is especially illuminating and is favorable when comparing theory and experiment
- Important to treat proton- and neutron form factors/radii and center-of-mass properly

Electron scattering and Formfactors

- Elastic and inelastic electron scattering are good tools to investigate the wave functions of ground and excited states
- Model independent fits to form factors/densities are preferable especially for exotic cases
- DWBA analysis should be done carefully
- Data at low momentum transfer are sufficient to determine the charge radius (or monopole matrix element) in principle but global fit is more stable
- Improved calculations for ^{12}C with a ^{8}Be-^{4}He continuum have been done, but we have not yet calculated form factors

Beryllium Isotopes



Krieger *et al.*, Phys. Rev. Lett. **108**, 142501 (2012)