

Solution to long-standing puzzles on nuclear charge radii via $3N$ interaction

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Contents :

- I. Introduction
- II. Long-standing puzzles on nuclear charge radii
in $Z = \text{magic}$ nuclei
- III. Mean-field approaches with semi-realistic interaction
- IV. $3N$ LS interaction & isotope shifts in $Z = \text{magic}$ nuclei
- V. Summary

Collaborators:

for this study

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H.N. & T. Inakura, PRC 91, 021302(R)
H.N., PRC 92, 044307

for preceding studies

K. Sugiura (*Chiba U., Japan*)

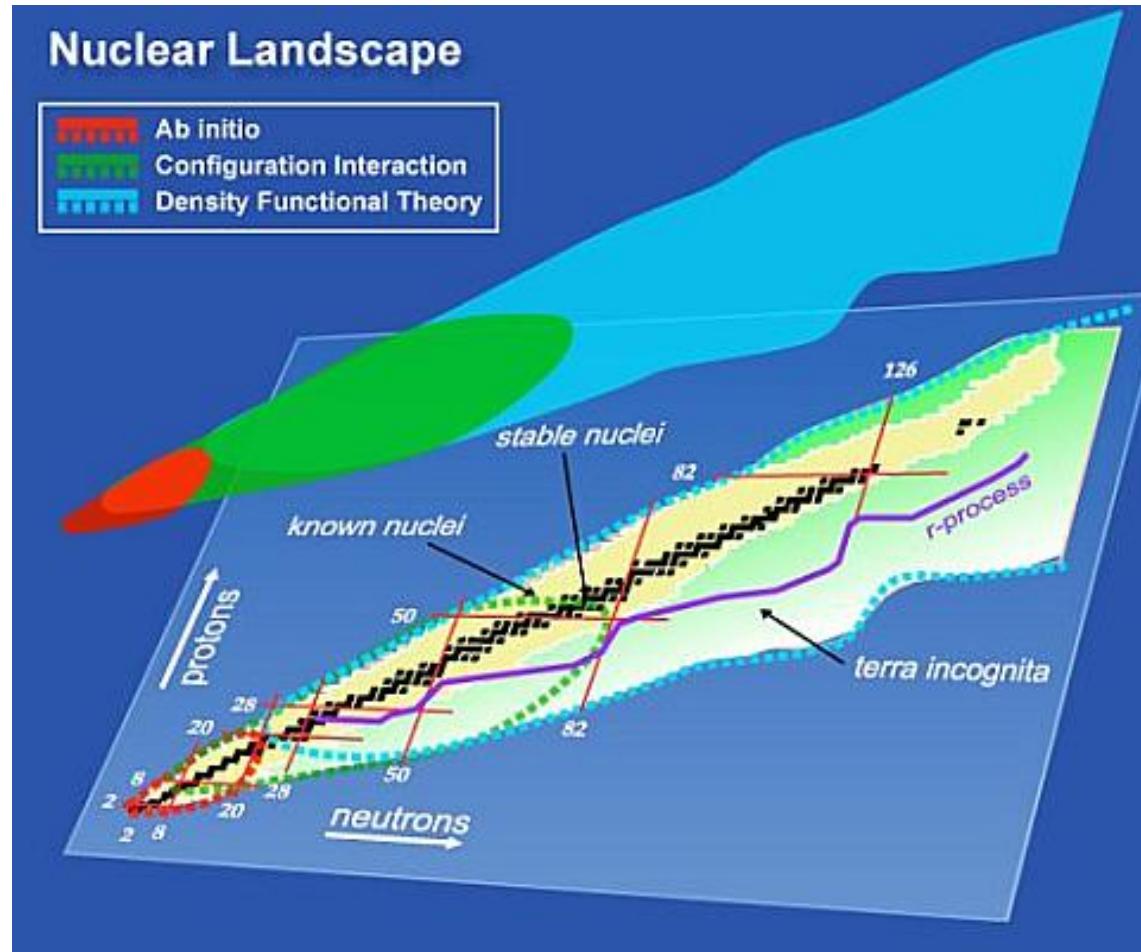
J. Margueron (*IPNL, France*)

special thanks to

M. Kohno (*RCNP, Japan*)

I. Introduction

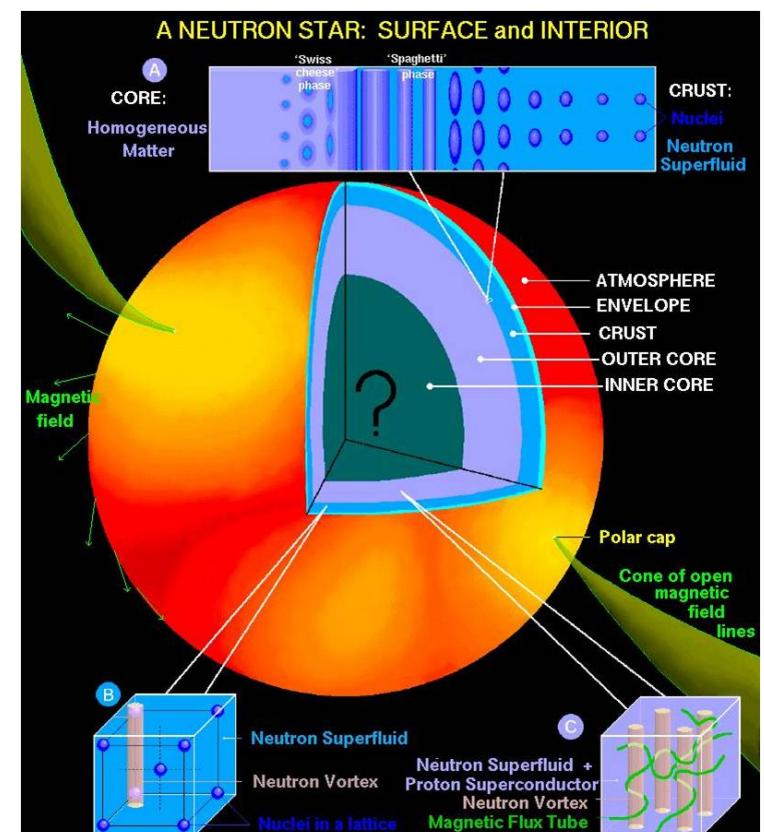
Comprehensive study of nuclear structure



(SciDAC UNDEF project: '07 – '12)

→ SCMF (or EDF) approach
... (effective) nucleonic int. ?

n-star — an extreme
of nuclear system

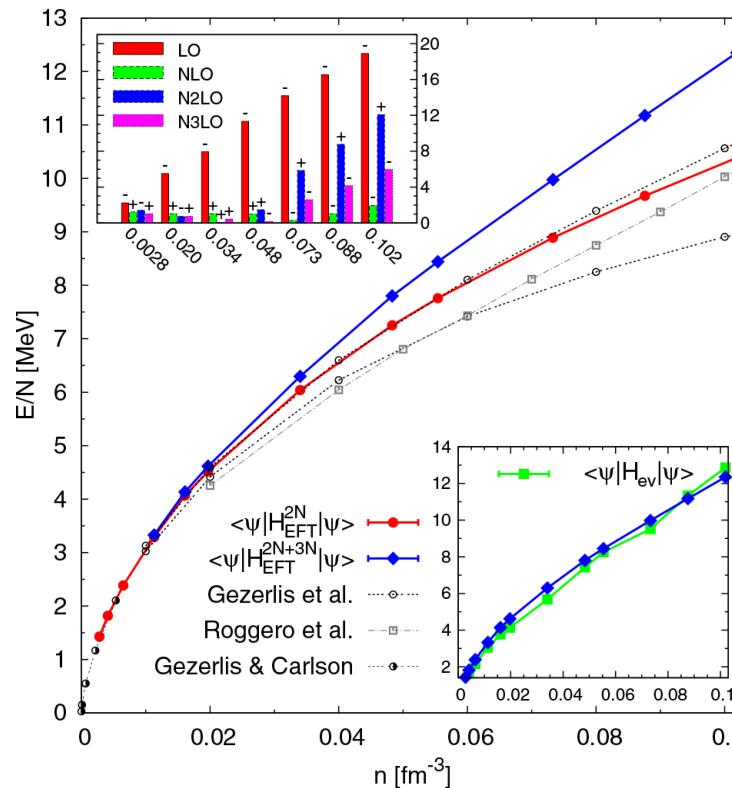


(by D. P. Page)

Nucleus — quantum system comprised of nucleons

$$H = K + V_{2N} + V_{3N} + \dots \quad |V_{2N}| \gg |V_{3N}|, \text{ but}$$

$$\begin{cases} V_{2N} \propto \rho^2 \text{ or } A^2 \\ V_{3N} \propto \rho^3 \text{ or } A^3 \end{cases} \rightarrow \text{more important for high } \rho \text{ (& large } A \text{ ?)}$$



AFQMC results
on pure n -matter
with χ EFT int.

G. Wlazłowski *et al.*, PRL 113, 182503

★ Effects of $3N$ int. ?

so far, argued in $\left\{ \begin{array}{l} \text{B.E. \& reactions in few-}N \text{ systems} \\ \text{B.E. \& some excited states} \\ \qquad \qquad \qquad \text{up to medium-mass nuclei} \\ \text{properties of (homogenous) infinite matter} \\ \qquad \qquad \qquad \text{but not yet comprehended sufficiently} \end{array} \right.$

- how significant ? (how strong ?)
- effects on other quantities (than energies) ?
- effects on medium-mass to heavy nuclei ?

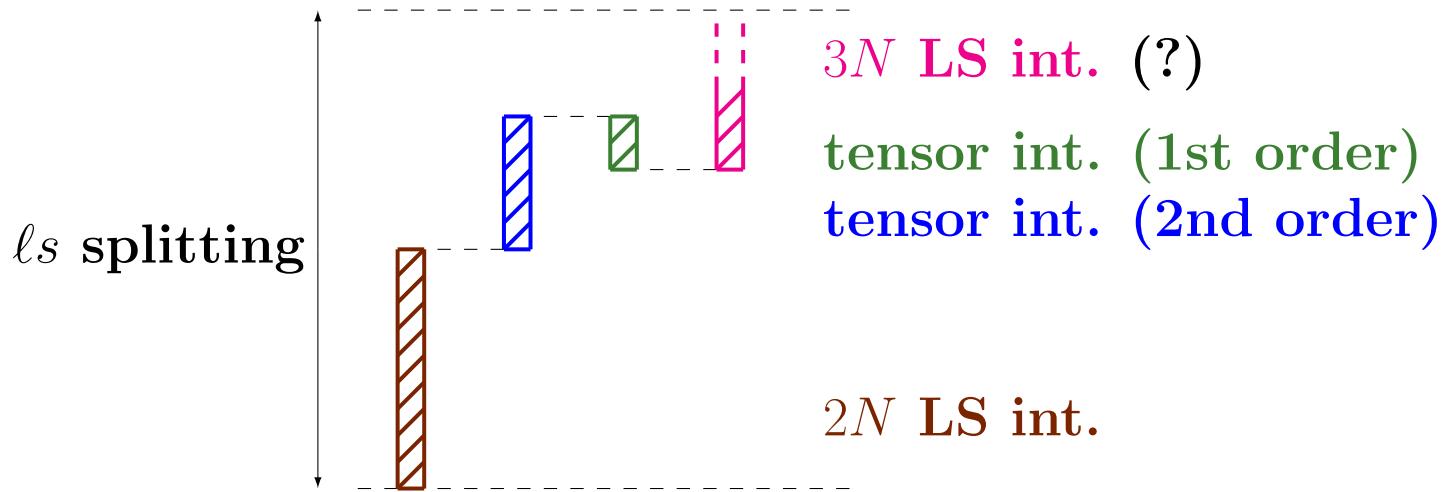
★ ℓs splitting in s.p. levels

\leftrightarrow shell structure \leftrightarrow magic number (in $Z, N > 20$)

— fundamental concept for nuclear structure
astrophysical importance

e.g. $\left\{ \begin{array}{l} \text{waiting point in } s\text{- \& } r\text{-processes} \\ \text{constraint on EoS} \quad \leftarrow \text{subtracting shell effects} \end{array} \right.$

origin? (\rightarrow correct prediction of shell structure)



χ EFT \rightarrow 3N LS int. (\rightarrow ρ -dep. LS int.)

N. Kaiser, PRC 68, 054001; M. Kohno, PRC 86, 061301(R)

strength? (\leftrightarrow convergence of χ EFT) exp. evidence?

II. Long-standing puzzles on nuclear charge radii in $Z =$ magic nuclei

Nuclear charge radius $\langle r^2 \rangle_c(A) [\approx \langle r^2 \rangle_p(A) + \langle r^2 \rangle_c(p)] \leftarrow (e, e) \text{ etc.}$

- model-indep. data
- (lowest-order) quantity reflecting proton dist. in nucleus
 - nuclear structure information additional to energy

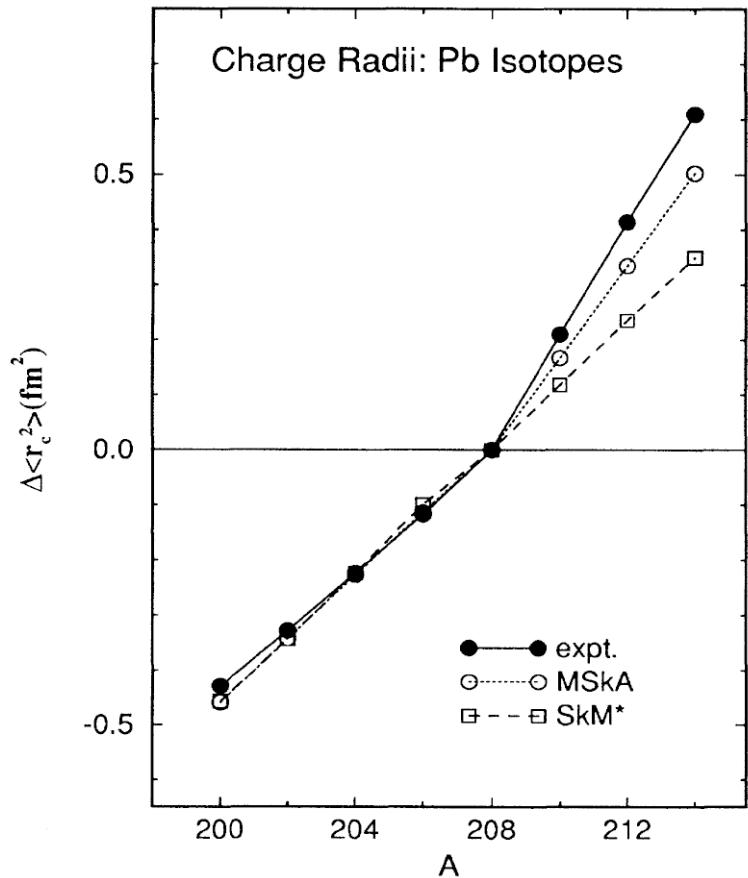
↪ Isotope shift ··· atomic X-ray freq. difference among isotopes

$$\propto \Delta \langle r^2 \rangle_c [= \Delta \langle r^2 \rangle_p] \text{ for heavy nuclei}$$

- exp. data — high-precision, incl. unstable nuclei
- good indicator to structure change *e.g.* spherical to deformed
- $Z =$ magic nuclei ··· (quite likely) spherical \rightarrow not so interesting?
 - it is !

★ Isotope shifts in Pb nuclei

$$\Delta \langle r^2 \rangle_p(^A\text{Pb}) := \langle r^2 \rangle_p(^A\text{Pb}) - \langle r^2 \rangle_p(^{208}\text{Pb})$$



exp. \Rightarrow **kink at $N = 126$!**

- not reproduced by Skyrme EDF (up to '95)
- reproduced by RMF due to isospin content of LS int. (not direct rel. effect)
- leading to extension of Skyrme EDF

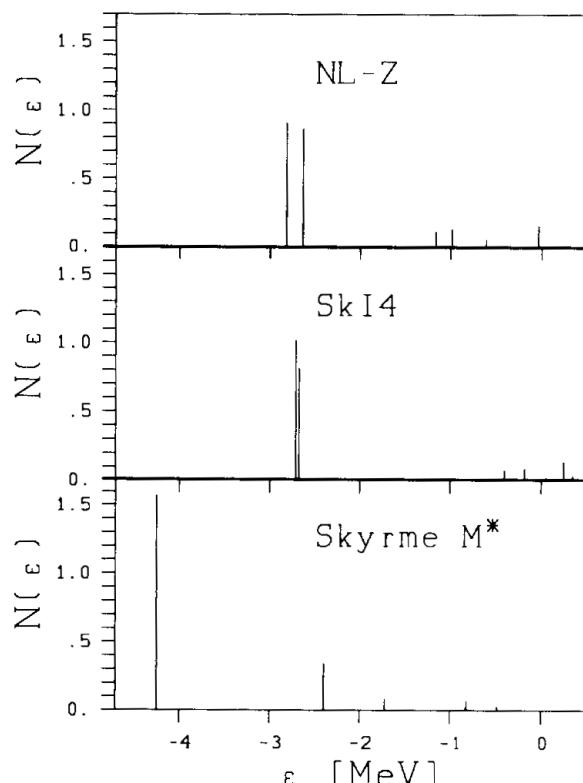
... but problem remains !

M.M. Sharma *et al.*, PRL 74, 3744

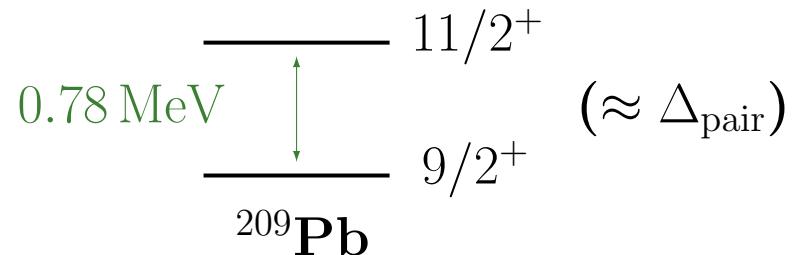
kink in $\Delta\langle r^2 \rangle_p(^A\text{Pb})$ at $N = 126$ ← $n0i_{11/2}$ occupation
 \uparrow
 p - n attraction

$\left\{ \begin{array}{l} \text{larger } \langle r^2 \rangle \\ \text{than neighboring orbits} \\ N < 126 \text{ — unocc.} \\ N > 126 \text{ — sizable occ. prob.} \\ (\because \text{pairing}) \end{array} \right.$

However, $\varepsilon_n(0i_{11/2}) \approx \varepsilon_n(1g_{9/2})$ required! (\leftrightarrow equal occ. prob.)



on the contrary ···

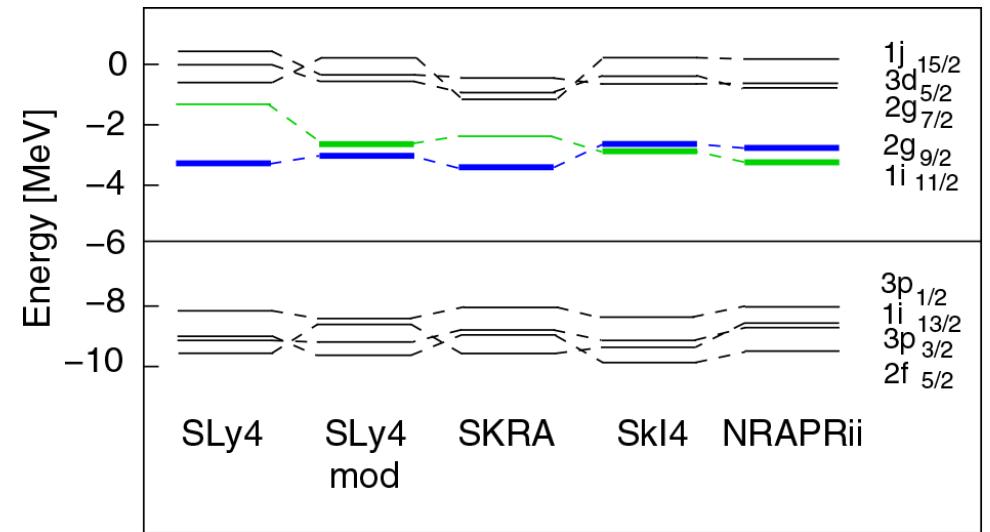
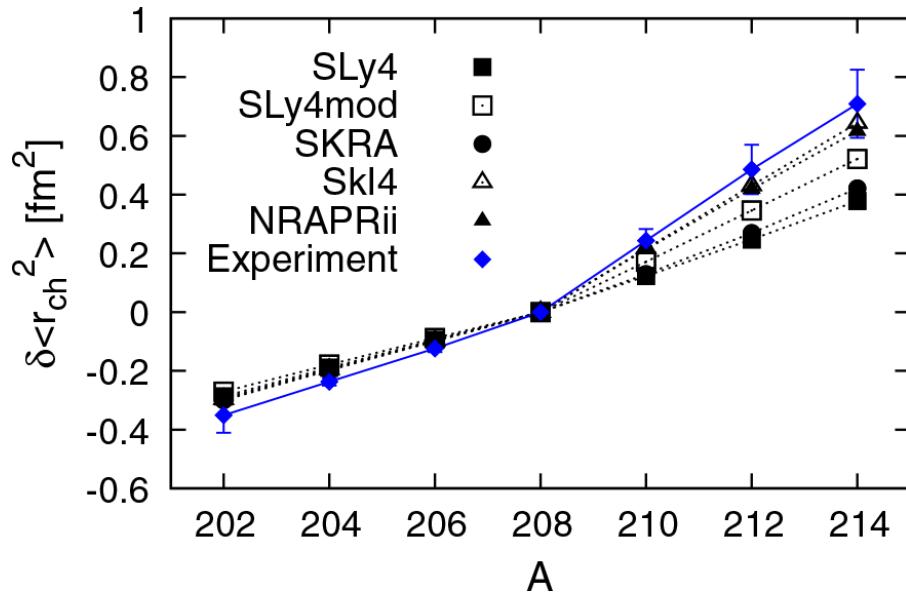


$\varepsilon_n(0i_{11/2}) - \varepsilon_n(1g_{9/2})$
 \leftrightarrow isospin content of LS int.

P.-G. Reinhard & H. Flocard,

NPA 584, 467

Further MF studies



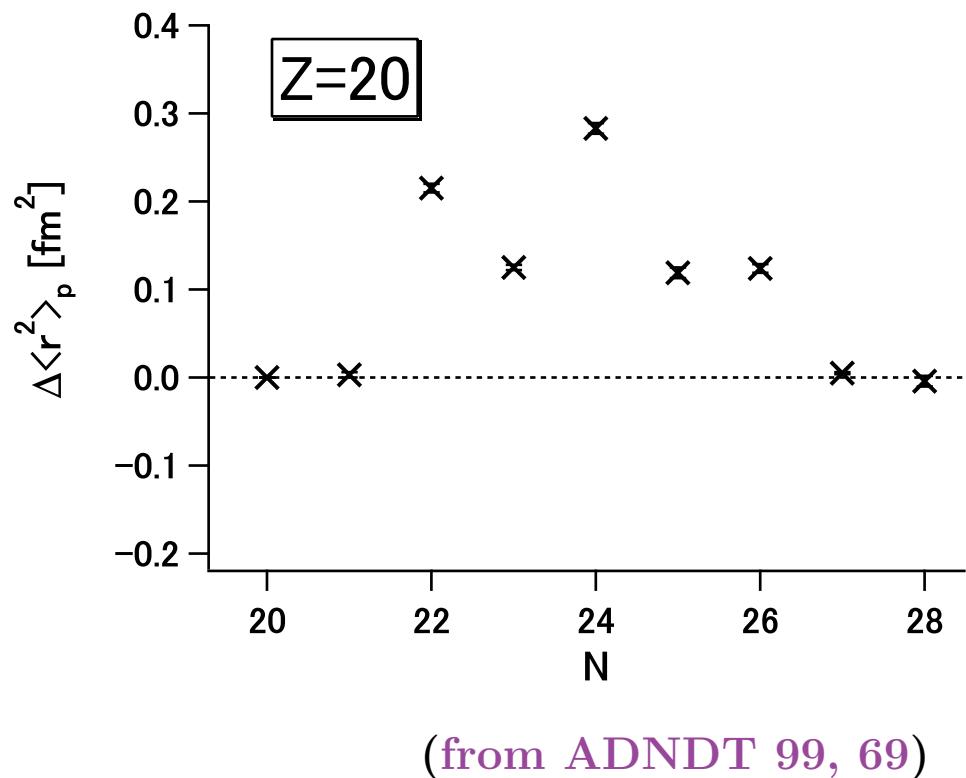
P.M. Goddard *et al.*, PRL 110, 032503

— kink reproduced only when $n1g_{9/2}$ & $n0i_{11/2}$ are degenerate
(or even inverted) !?

cf. def. & correlations beyond MF — influence on kink unlikely

M. Bender *et al.*, PRC 73, 034322

★ Charge radii of ^{40}Ca & ^{48}Ca



^{40}Ca & ^{48}Ca — doubly magic
→ MF cal. good (?)

exp. $\Rightarrow \langle r^2 \rangle_p(^{40}\text{Ca}) \approx \langle r^2 \rangle_p(^{48}\text{Ca})$!

- $\langle r^2 \rangle_p(^{40}\text{Ca}) < \langle r^2 \rangle_p(^{48}\text{Ca})$
in most MF cal.

- exception — RMF
G.A. Lalazissis *et al.*, ADNDT 71, 1
... physics ?

cf. $^{42-46}\text{Ca}$... beyond-MF effects ?

e.g. excitation across $Z = 20$ shell gap

E. Caurier *et al.*, PLB 522, 240

$(\nabla\rho)$ -dep. pairing ? S.A. Fayans, JETP Lett. 68, 169

III. Mean-field approaches with semi-realistic interaction

$\hat{v}_{\text{M3Y}} \approx G\text{-matrix}$

↓

$\hat{v}_{\text{M3Y-P}_n}$

H.N., PRC 68, 014316

central	... ρ -dep. introduced (\leftrightarrow saturation, incl. $3N$ effects)
LS	... modification (\leftrightarrow ℓs splitting, shown later) finite-range $\rightarrow T = 0 \& 1$ channels
tensor	... unchanged ($\hat{v}_{\text{M3Y-P}_n}^{(\text{TN})} = \hat{v}_{\text{M3Y}}^{(\text{TN})}$) \rightarrow realistic

‘M3Y-P6’ → reasonable shell structure (e.g. magic #)

H.N., PRC 87, 014336 ; H.N. & K. Sugiura, PTEP 2014, 033D02

→ yardstick

★ Incorporating $3N$ LS int.

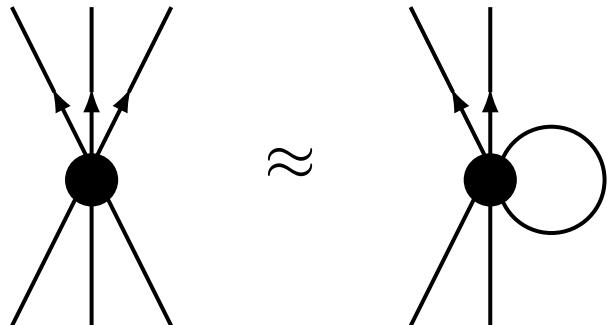
ℓs splitting — more or less fitted in MF cal. (\rightarrow shell structure)

… should not be changed seriously

(Do we really need $3N$ LS int. in MF approaches,
even if it significantly contributes to ℓs splitting ?)

$3N$ LS int. $\leftrightarrow \rho$ -dep. LS int. ($\hat{v}^{(LS\rho)}$)

M. Kohno, PRC 86, 061301(R)



$$v^{(LS\rho)} = 2i D[\rho(\mathbf{R}_{ij})] \mathbf{p}_{ij} \times \delta(\mathbf{r}_{ij}) \mathbf{p}_{ij} \cdot (\mathbf{s}_i + \mathbf{s}_j);$$

$$D[\rho(\mathbf{r})] = -w_1 \frac{\rho(\mathbf{r})}{1 + d_1 \rho(\mathbf{r})} \left(\approx -w_1 \rho(\mathbf{r}) \right)$$

$$\propto D[\rho] \cdot \hat{v}_{\text{Sky}}^{(LS)} \approx \rho \cdot \hat{v}_{\text{Sky}}^{(LS)}$$

↓

$$\left. \begin{array}{l} \text{M3Y-P6} \quad \leftarrow \hat{v}_{\text{M3Y}}^{(LS)} \times 2.2 \\ vs. \\ \text{M3Y-P6a} \quad \leftarrow \hat{v}_{\text{M3Y}}^{(LS)} + \hat{v}^{(LS\rho)} \end{array} \right\}$$

$$\Delta\varepsilon_{\text{M3Y-P6}}^{\ell s} \approx \Delta\varepsilon_{\text{M3Y-P6a}}^{\ell s} \rightarrow w_1 (> 0)$$

$$(\Delta\varepsilon^{\ell s}(n0i) \text{ at } {}^{208}\text{Pb})$$

→ influence on energies not large

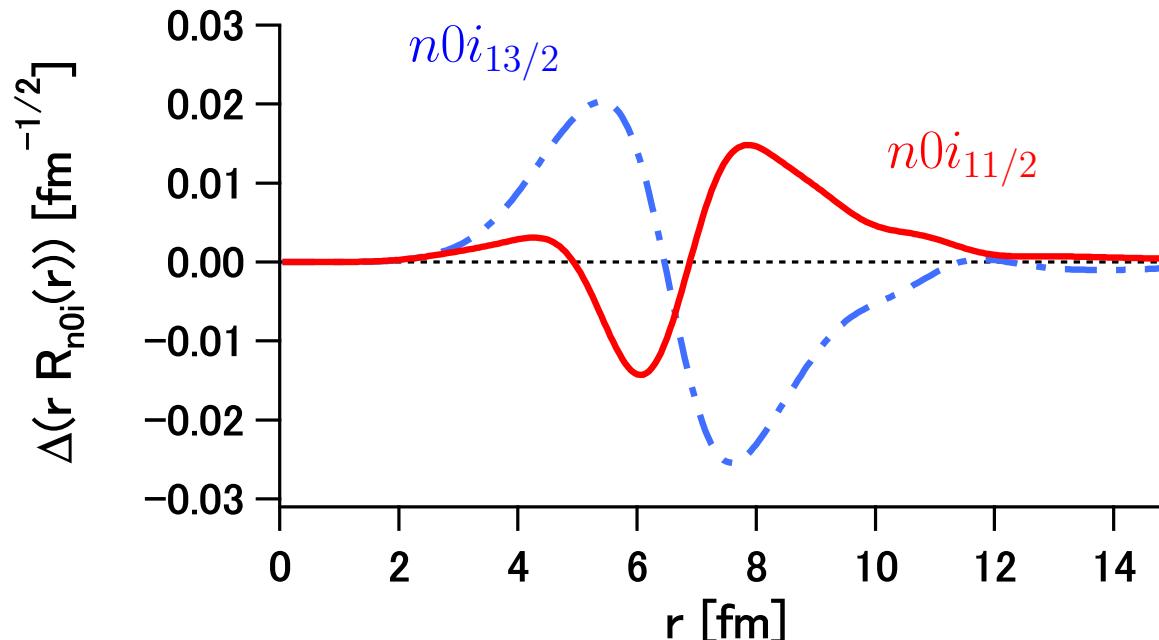
★ Influence on s.p. wave functions:

presence of $D[\rho]$ → $\begin{cases} \text{stronger LS for interior (higher } \rho\text{)} \\ \text{weaker LS for extetior (lower } \rho\text{)} \end{cases}$

→ $\begin{cases} j = \ell + 1/2 \text{ orbitals shrink} \\ j = \ell - 1/2 \text{ orbitals extends} \end{cases}$

e.g. larger $\langle r^2 \rangle$ for $n0i_{11/2}$

$$\Delta[r R_j(r)] := [r R_j(r)]_{\text{M3Y-P6a}} - [r R_j(r)]_{\text{M3Y-P6}} \quad @^{208}\text{Pb}$$

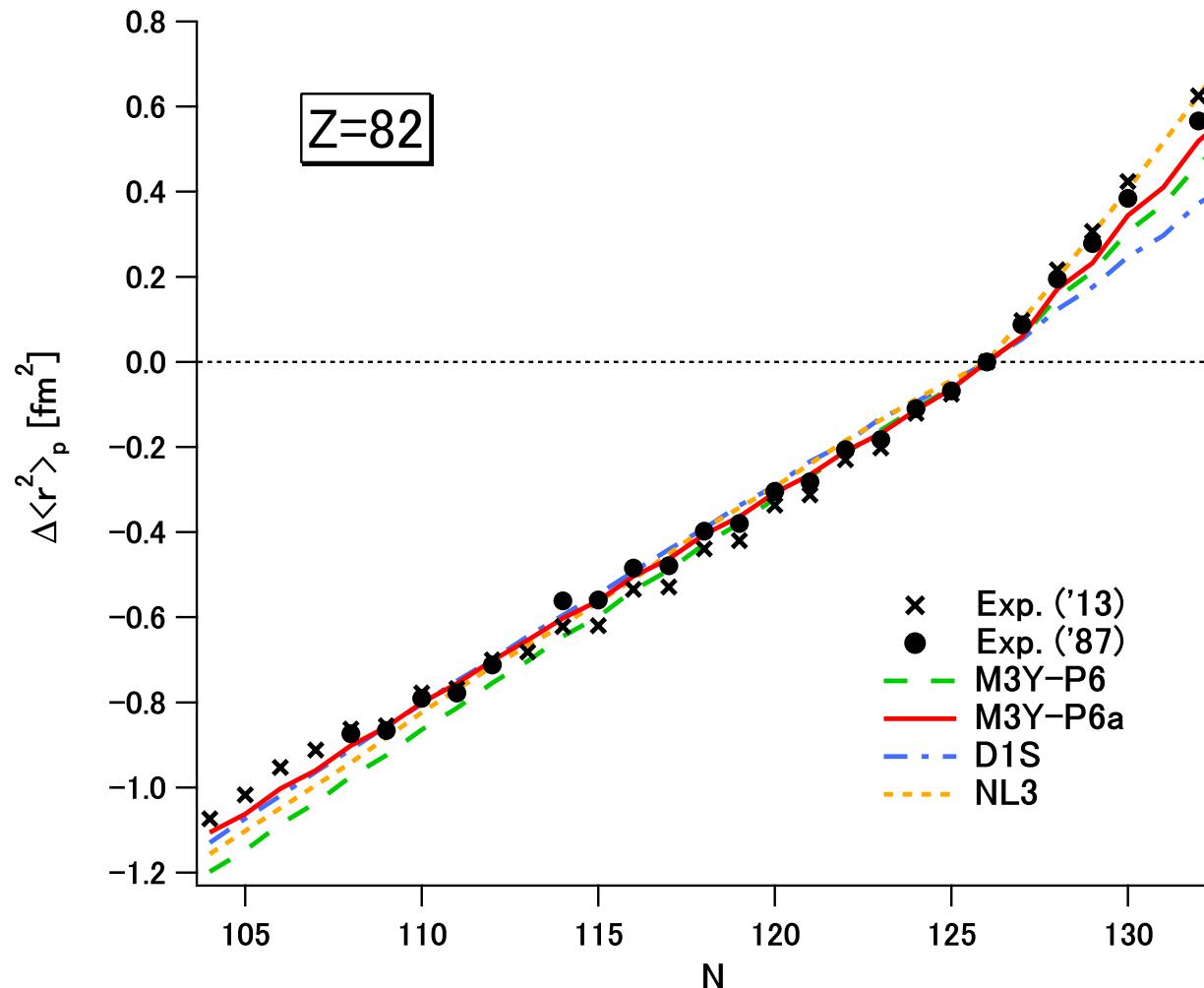


IV. $3N$ LS interaction & isotope shifts in $Z = \text{magic}$ nuclei

Spherical HFB cal. \rightarrow isotope shifts in $Z = \text{magic}$ nuclei

M3Y-P6 *vs.* M3Y-P6a \rightarrow effects of $3N$ LS int.

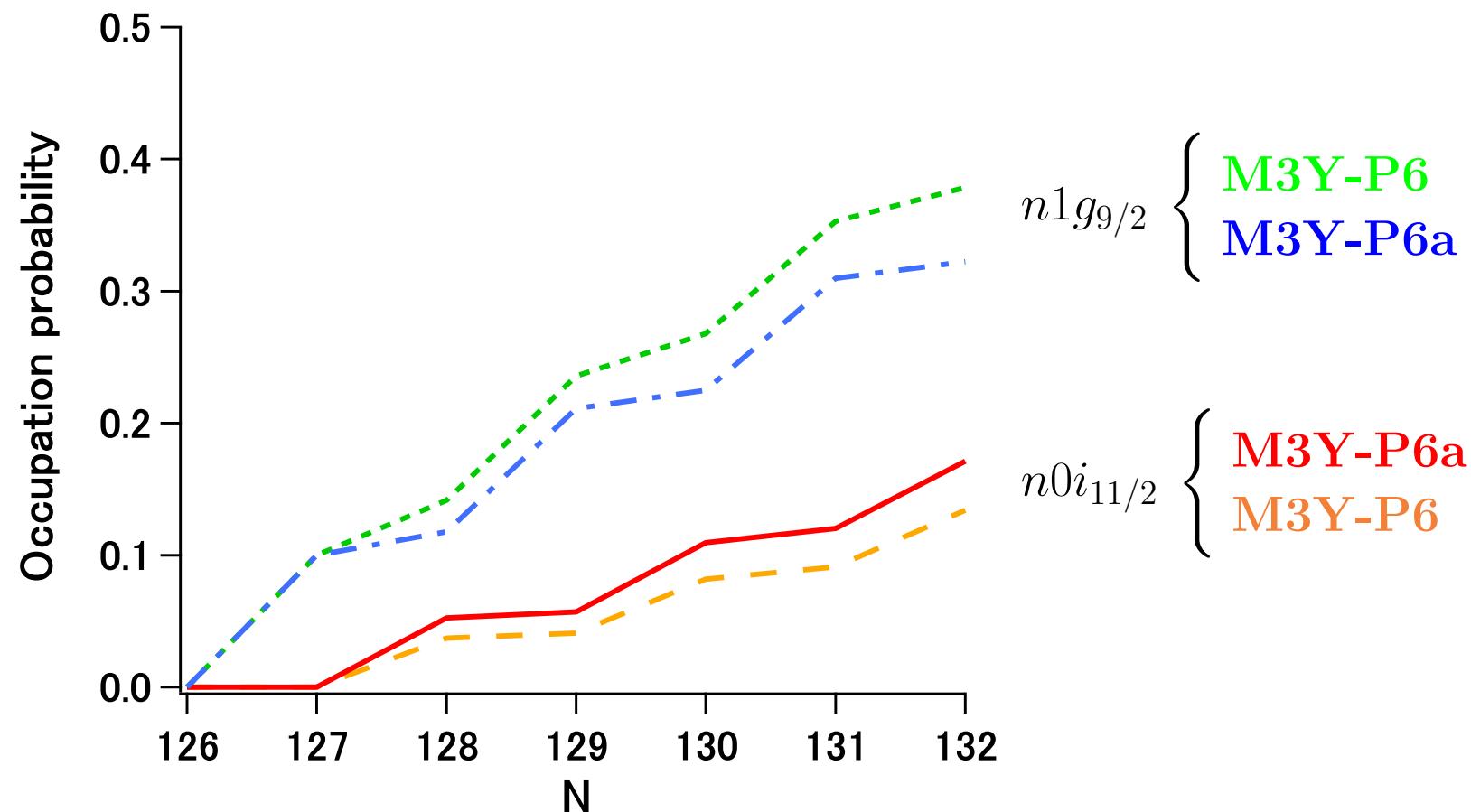
★ Isotope shifts of Pb nuclei $\Delta \langle r^2 \rangle_p(^A\text{Pb}) := \langle r^2 \rangle_p(^A\text{Pb}) - \langle r^2 \rangle_p(^{208}\text{Pb})$



S.p. energies & occ. prob.

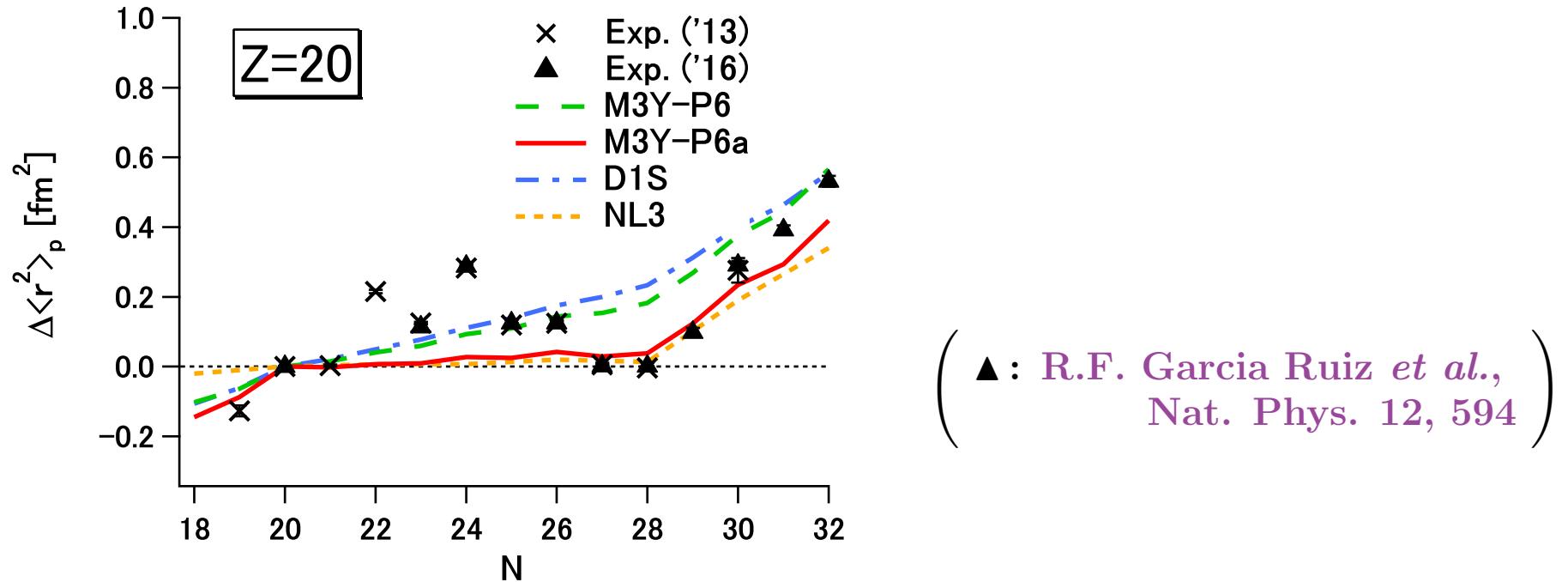
$$\varepsilon_n(0i_{11/2}) - \varepsilon_n(1g_{9/2}): \quad \left\{ \begin{array}{ll} \text{exp. @ } ^{209}\text{Pb} & \rightarrow 0.78 \text{ MeV} \\ \text{M3Y-P6a} & \rightarrow 0.72 \text{ MeV} \end{array} \right.$$

occ. prob.



⇒ kink at $N = 126$ reproduced without $n1g_{9/2}$ - $n0i_{11/2}$ degeneracy!

★ Isotope shifts of Ca nuclei $\Delta \langle r^2 \rangle_p(^A\text{Ca}) := \langle r^2 \rangle_p(^A\text{Ca}) - \langle r^2 \rangle_p(^{40}\text{Ca})$

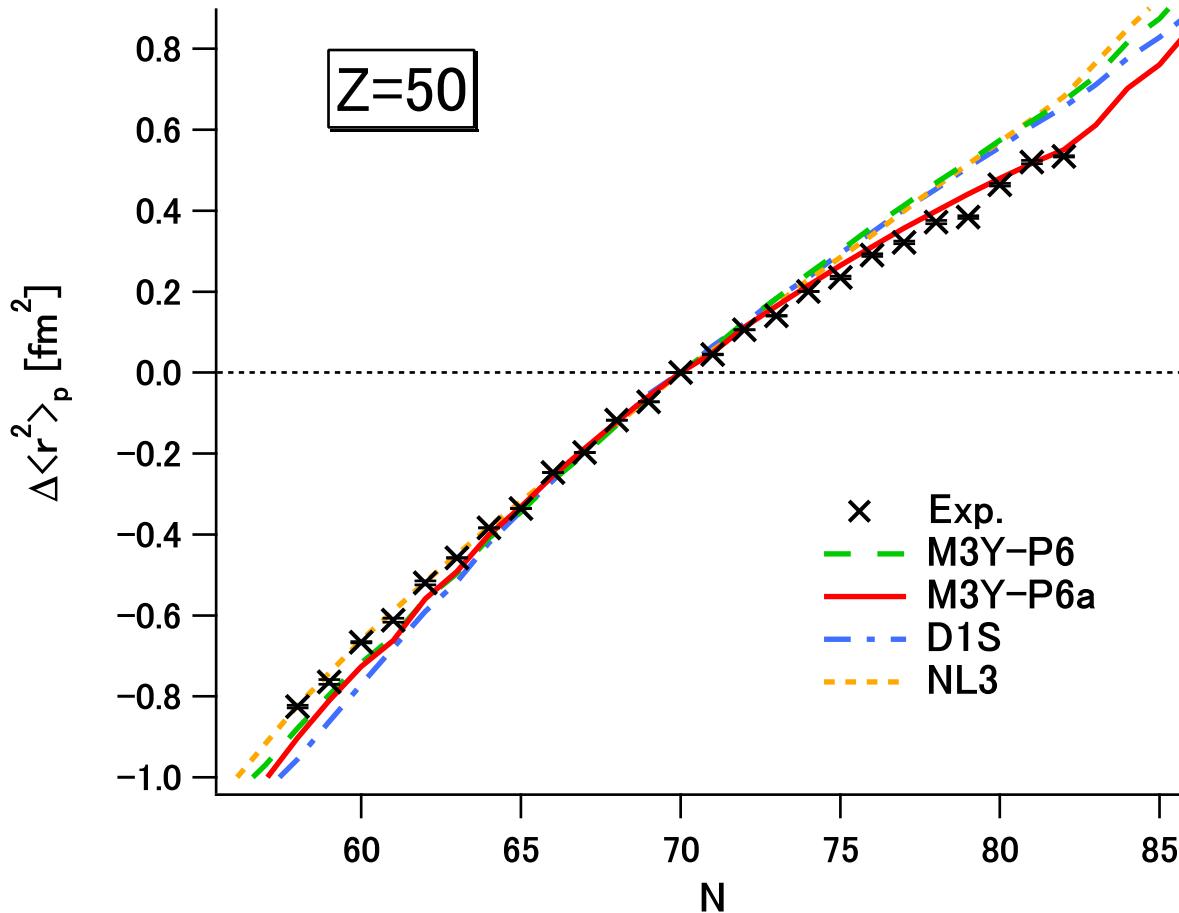


$\langle r^2 \rangle_p(^{40}\text{Ca}) \approx \langle r^2 \rangle_p(^{48}\text{Ca})$ reproduced !

\therefore) ρ -dep. LS $\rightarrow n0f_{7/2}$ shifts inward (RMF ?)

cf. χ EFT + CC / SRG also reproduce $\langle r^2 \rangle_p(^{40}\text{Ca}) \approx \langle r^2 \rangle_p(^{48}\text{Ca})$

★ Isotope shifts of Sn nuclei $\Delta \langle r^2 \rangle_p(^A\text{Sn}) := \langle r^2 \rangle_p(^A\text{Sn}) - \langle r^2 \rangle_p(^{120}\text{Sn})$



kink predicted at $N = 82$!

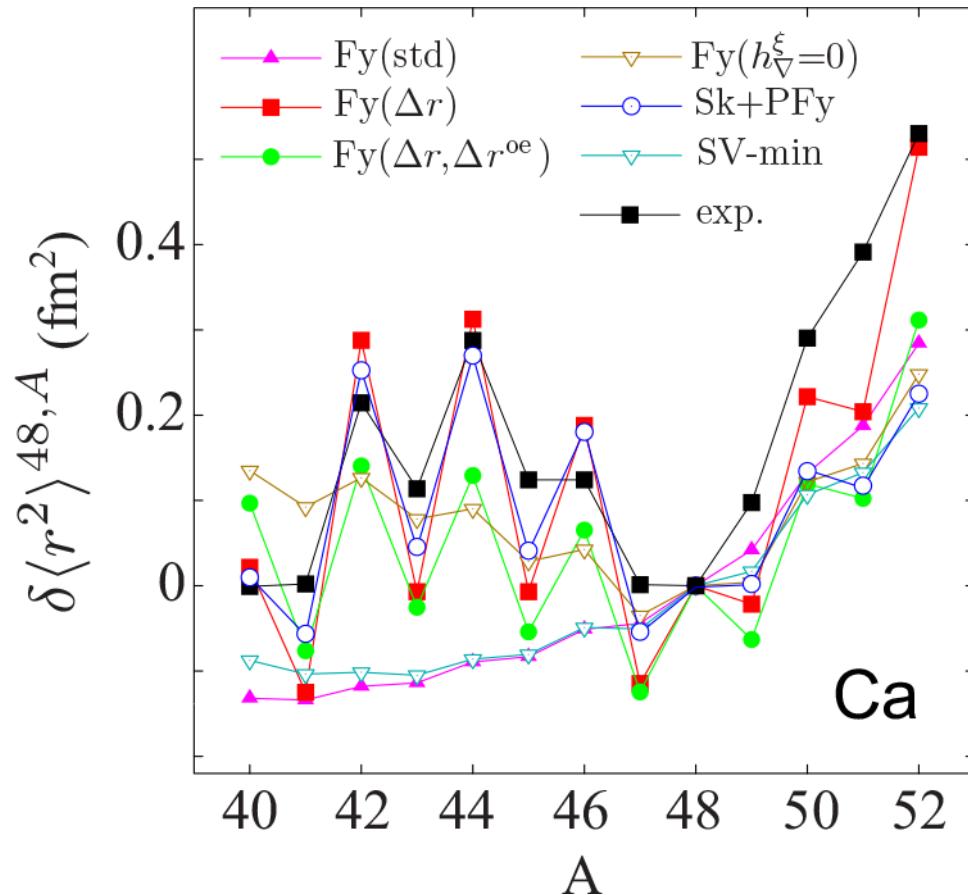
\therefore) ρ -dep. LS \rightarrow $\begin{cases} n0h_{11/2} \text{ shifts inward} & \rightarrow \text{slope in } 70 \lesssim N \leq 82 \\ n0h_{9/2} \text{ shifts outward} & \rightarrow \text{slope in } N > 82 \end{cases}$

— in sharp contrast to results w/o ρ -dep. LS (incl. RMF)
 \Rightarrow data? \rightarrow seems confirmed! (Garcia Ruiz' talk)

★ Back to ^{40}Ca - ^{48}Ca — differential charge distribution

- Fayan's EDF

P.-G. Reinhard & W. Nazarewicz, PRC 95, 064328



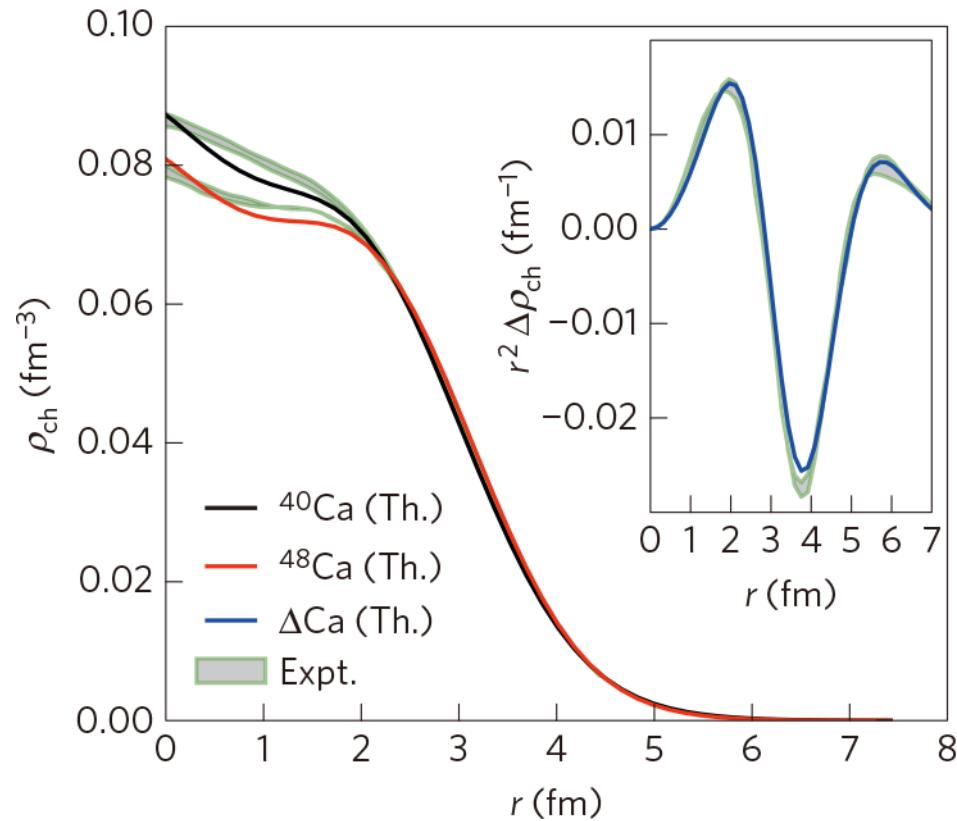
$^{42,44,46}\text{Ca}$ \leftarrow $[(\nabla\rho)^2 \cdot \kappa_\tau^2]$ term
 ^{48}Ca \leftarrow extensive $(\nabla\rho)^2$ -dep.
(— adjustable)

- other nuclei ?
- more precise information ?
 → to distinguish mechanism

⇒ differential charge density

- Result from χ EFT int.

G. Hagen *et al.*, Nat. Phys. 12, 186



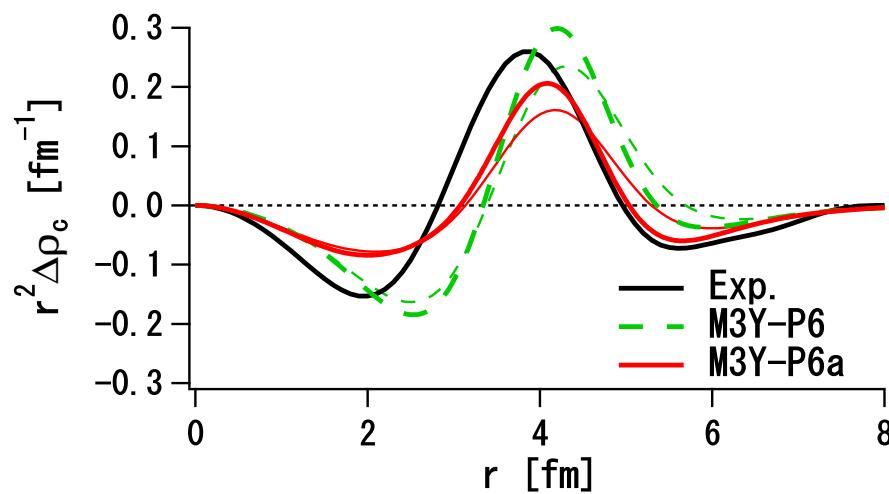
good agreement

via $2N + 3N$ int. (NNLO_{sat})

- favoring $3N$ solution ?
- mechanism ?

- M3Y-P6 *vs.* M3Y-P6a

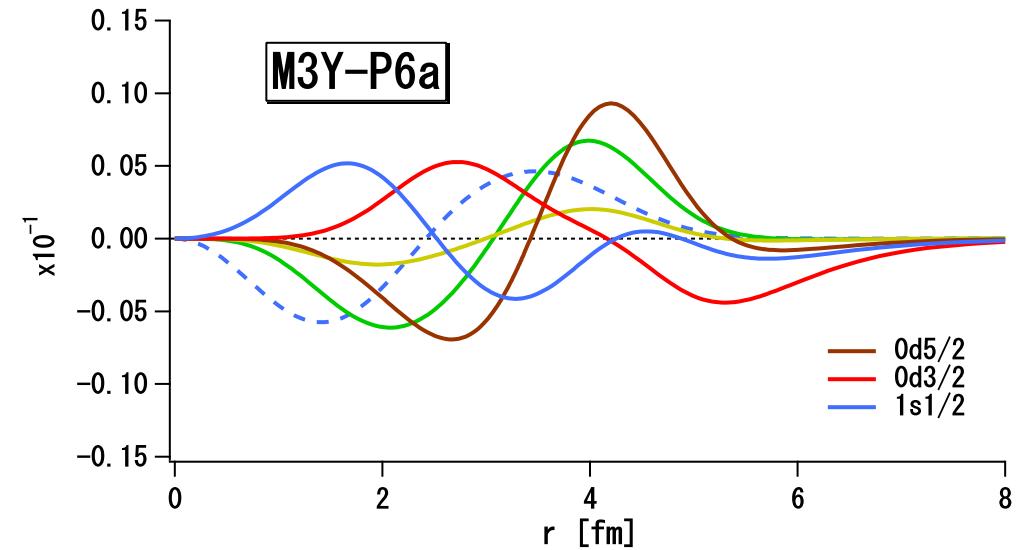
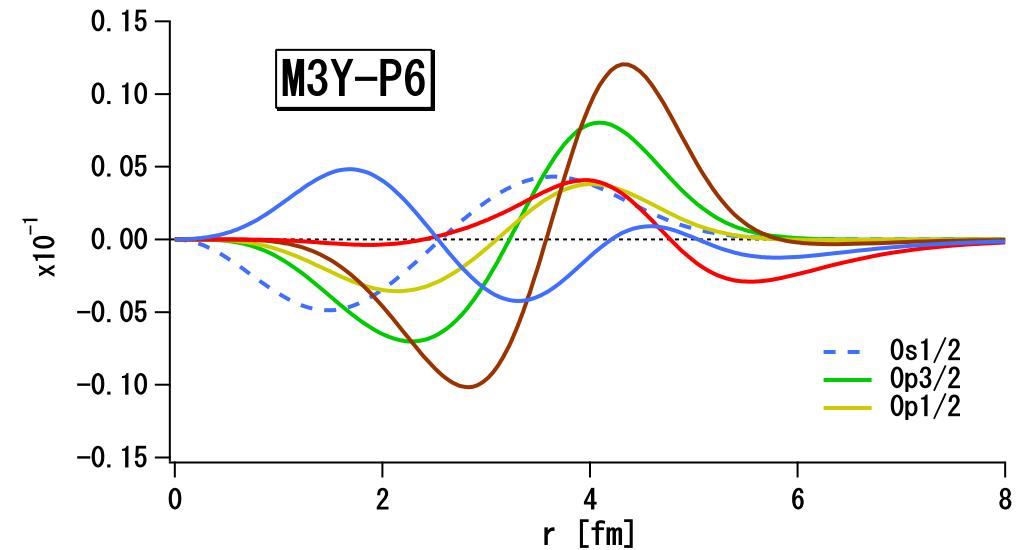
$$\Delta\rho_c := \rho_c(^{48}\text{Ca}) - \rho_c(^{40}\text{Ca})$$



dip at $r \approx 5.5$ fm

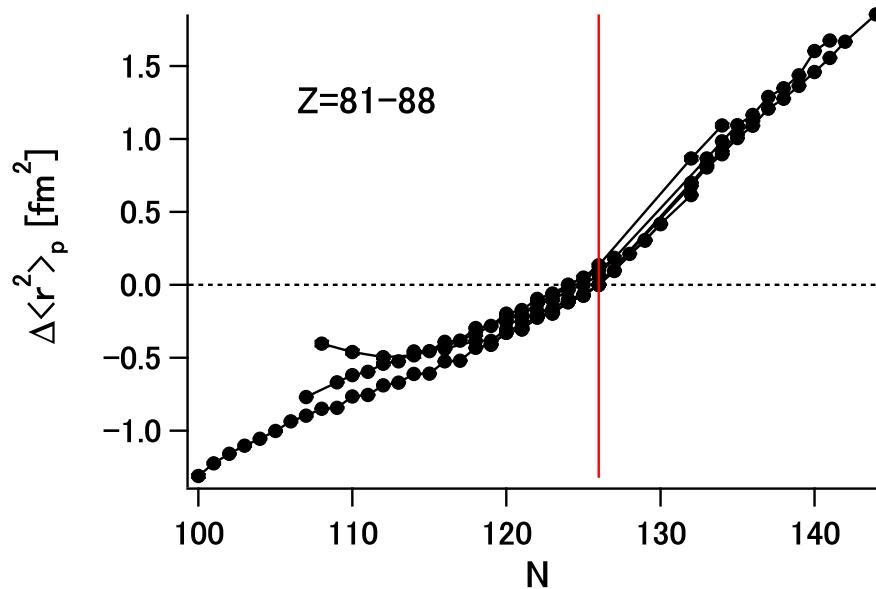
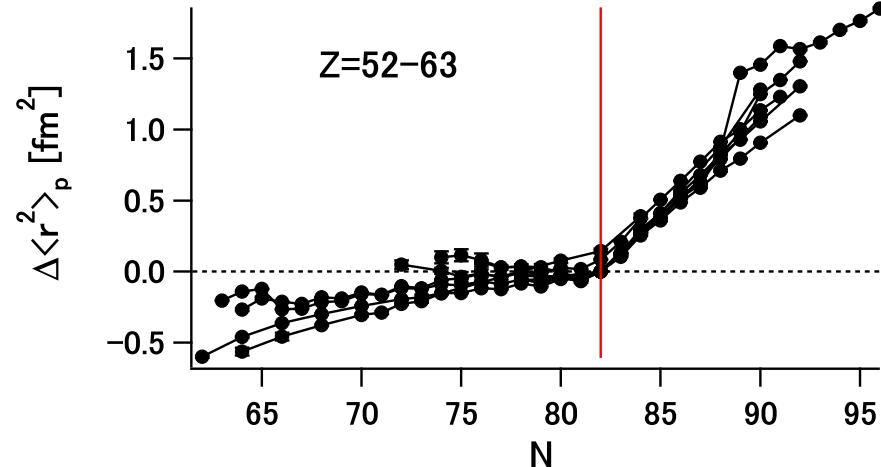
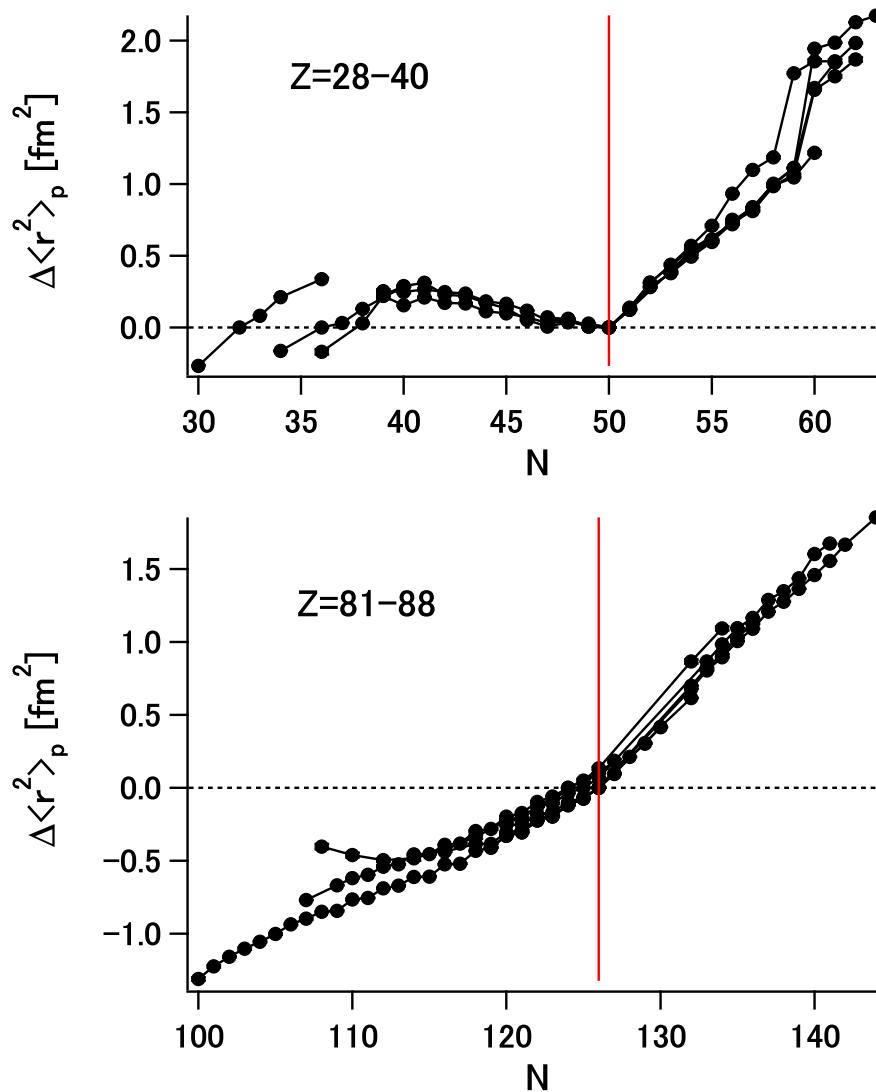
$$\rightarrow \langle r^2 \rangle_p(^{40}\text{Ca}) \approx \langle r^2 \rangle_p(^{48}\text{Ca})$$

contribution of s.p. orbitals :



shrinkage of $n0f_{7/2} \rightarrow$ shrinkage of $p0d_{3/2} \rightarrow$ dip at $r \approx 5.5$ fm !

★ ρ -dep. LS \rightarrow kink generally expected wherever N is jj -closed magic
 . . . consistent with exp. data



(from ADNDT 99, 69)

Note: deformation will be responsible for a certain part

V. Summary

1. We have investigated effects of $3N$ LS int. on isotope shifts of nuclei.
← sph. HFB with semi-realistic int. M3Y-P6 & its variant M3Y-P6a
2. With M3Y-P6a which contains ρ -dep. LS channel,
 - isotope shifts of the Pb nuclei are described fairly well without fictitious $n1g_{9/2}$ - $n0i_{11/2}$ degeneracy,
 - almost equal charge radii between ^{40}Ca and ^{48}Ca are reproduced,
 - isotope shifts of the Sn nuclei are in agreement with available data, and a kink is predicted at $N = 82$. → seems confirmed !
3. Results may be regarded as evidence for $3N$ LS interaction
based on χ EFT, indep. of ℓs splitting.
 - qualitative evidence for $3N$ interaction ! (?)
(cf. RMF, Fayen's EDF ?)
4. Exp. data on $\rho_c(r)$ may confirm physics ! (e.g. ^{40}Ca vs. ^{48}Ca)