Electron scattering as a probe of surface properties, proton shell structure of unstable nuclei and of the symmetry energy

Xavier Roca-Maza
Università degli Studi di Milano and INFN, Milano section

Probing exotic structure of short-lived nuclei by electron scattering
ECT*, Trento, July 16th-20th 2018

Motivation

- **In-medium nuclear (effective) interaction is not well understood for extreme values of isospin asymmetry, that is, far from the stability valley**
- **Experimental studies of elastic and inelastic electron scattering by unstable nuclei:**
  - determine e-m charge distribution **model independently**
  - access to isovector nuclear excitations
  - better understanding of nuclei under **extreme conditions**
- **Theoretical studies of elastic and inelastic electron scattering by unstable nuclei:**
  - Physical process **well understood**.
  - **Exact calculations available for the elastic channel** once the exact electromagnetic charge distribution is known
  - **Accurate calculations available for the bulk propetries of the nuclear response up to several tens of MeV** based on EDFs.
  - **Guidance for future experiments**

Table of contents

Exact solution: Dirac partial-wave (also known as DWBA) calculation of elastic scattering of electrons by nuclei. X. Roca-Maza *et al.* Phys. Rev. C **78**, 044332 (2008) & F. Salvat *et al.* Comp. Phys. Comm. **165** 157-190 (2005)

For studies on the nuclear charge distribution:

- $\mathbf{E_{beam}} \sim 2\pi \frac{\hbar c}{\lambda_{\mathrm{Nucl.size}}}$ where $\lambda_{\mathrm{nucl.size}} \sim 2\langle r^2\rangle^{1/2} \sim 2 - 10$ fm $\Rightarrow$ **100 − 600 MeV**.
- **Relativistic treatment** is needed $m_e c^2/E_{\mathrm{beam}} \lesssim 0.005$.
- At these energies, effect of **screening by the orbiting atomic electrons** is limited to scattering angles **smaller than 1 degree** (we will not calculate them here).
- The interaction potential is $V_{\mathrm{nucl.elec.}}$ calculated from ρ_{ch} (parametrized, model, ...)

$$V_{\mathrm{nucl.elec.}} = 4\pi Z_0 e^2 \left\{ \frac{1}{r}\int_0^r \rho_{\mathrm{ch}}(u) u^2 du + \int_r^\infty \rho_{\mathrm{ch}}(u) u du \right\}$$

- spherical symmetry assumed

Differential cross section for unpolarized electrons:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 + |g(\theta)|^2$$

where f and g are the **direct and spin-flip amplitudes** determined from the solution of the **Dirac equation for a central potential** $V_{\text{nucl.elec.}}(r)$ (asymptotically as a plane wave + outgoing spherical wave)

f and g admit a **partial-wave expansion** in terms of,

- the **phase-shifts** that represents the large r behavior of **Dirac spherical waves** $\Psi(\mathbf{r})$ function of the spherical spinors and two radial functions.
- Legendre and associated **Legendre polynomials**.

Sherman function: degree polarization from initially unpolarized beam scattered at angle θ

$$S(\theta) \equiv i\frac{f(\theta)g^*(\theta) - f^*(\theta)g(\theta)}{|f(\theta)|^2 + |g(\theta)|^2}$$

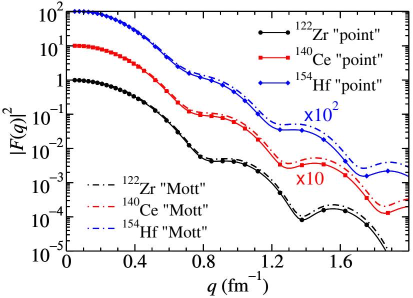
It may help in setting up detectors

Form factor

$$|F_{DWBA}(q)|^2 \equiv \frac{d\sigma/d\Omega}{d\sigma_{point}/d\Omega}$$

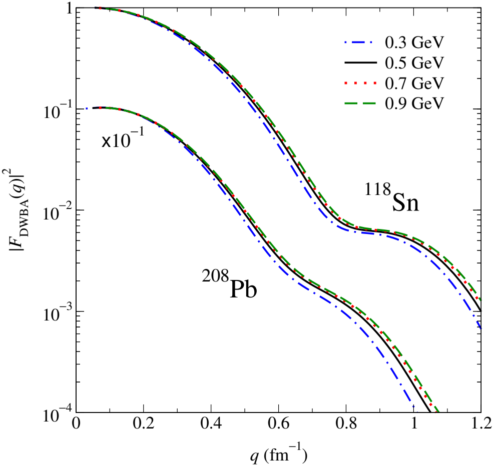
where $d\sigma_{point}/d\Omega$ is the DWBA solution for a point nucleus and $c\hbar q = 2E\sin(\theta/2)$.

- This definition, as compared to $\dfrac{d\sigma/d\Omega}{d\sigma_{Mott}/d\Omega}$, disentagles better the finite size effects of the nucleus.
- Nevertheless, it is found that the choice is not critical for our study in the low momentum transfer regime.



Mott DCS: $\dfrac{d\sigma_{Mott}}{d\Omega} = \left(\dfrac{Ze^2}{2E}\right)^2 \dfrac{\cos^2\theta}{\sin^4\theta}$; for small angles diverges as θ^{-4}

Energy Dependence in the e-m Form Factor

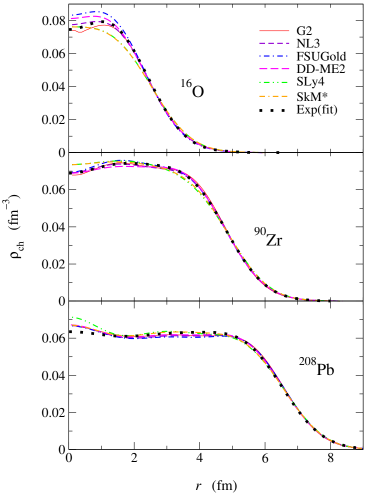


Test: The form factor in DWBA is almost energy-independent in the region we are interested in (low q-regime).

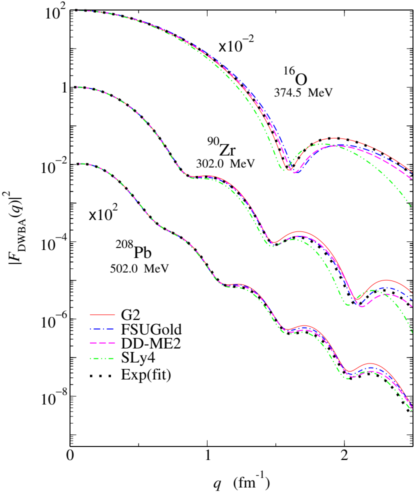
$F_{DWBA}(q)$ **is a good quantity for the study of the electromagnetic structure of the nucleus**

Experiment versus Theory in stable nuclei

Nuclear Model (NM) provides:

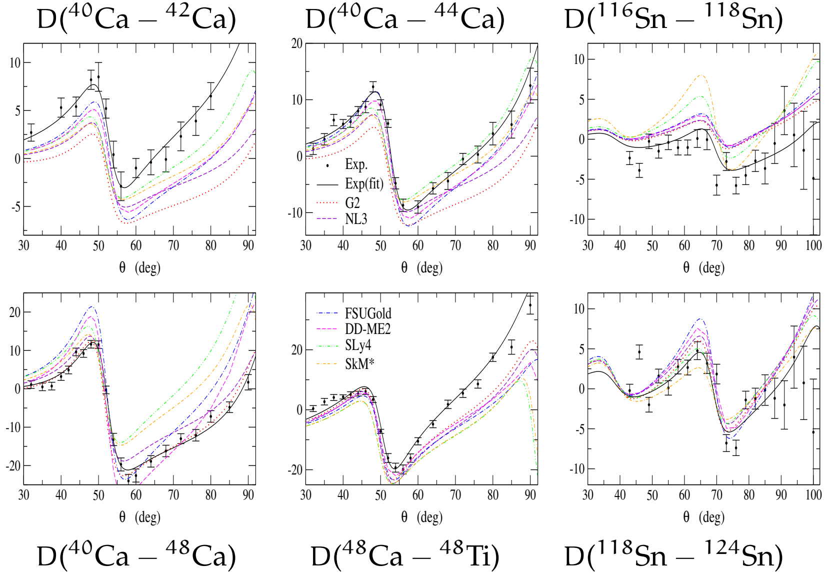


NM+DWBA provides:



... and a more demanding test:

$$D(A - B) \equiv (A - B)/(A + B)$$



How reliable are modern Energy Density Functionals?

In $F(q)$ and $d\sigma(\theta)$:

- Exist a quasi-model-independent q-regime: **Up to $1 - 1.5$ fm^{-1} for the studied models and scattering processes.**
- Exist discrepancies between models in the **high q-regime** **(remember differences in ρ_{ch} at low r)**
- If we define a **distance**, namely d_w, between experimental and theorical results:

$$d_w^2 = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{d\sigma_i^{exp} - d\sigma_i^{theo}}{\delta\sigma_i^{exp}} \right)^2$$

we observe that the criteria does not decide between models

Ability test for the models

Nucleus	E_e MeV	Exp. fit	DD-ME2	G2	NL3	FSUGold	SLy4	SKM*	"Best" model
		d_w^2							
^{16}O	374.5	**11.1**[b]	88.7	13.1	38.6	206.	191.	194.	**G2**
^{40}Ca	250.0	**7.18**[b]	3.15	16.2	13.9	0.84	24.4	24.3	**FSUGold**
	500.0	**3.48**[b]	1.49	42.9	19.7	5.79	40.0	39.0	**DD-ME2**
^{48}Ca	250.0	**6.66**[b]	4.85	9.74	7.14	4.08	14.9	13.6	**FSUGold**
	500.0	**3.19**[b]	1.11	17.0	3.53	2.57	21.84	18.5	**DD-ME2**
^{90}Zr	209.6	**0.78**[b]	0.87	2.21	1.36	0.65	6.53	5.36	**FSUGold**
	302.0	**0.86**[b]	0.91	9.92	3.27	0.67	9.35	7.19	**FSUGold**
^{118}Sn	225.0	**5.43**[a]	18.4	34.8	25.5	31.8	2.75	4.20	**SLy4**
^{208}Pb	248.2	**30.6**[b]	44.4	154.	74.8	89.5	89.2	61.0	**DD-ME2**
	502.0	**21.2**[b]	14.1	186.	50.5	61.1	95.9	76.5	**DD-ME2**
D(^{40}Ca – ^{42}Ca)	250.0	**0.56**[c]	9.1	28.3	16.0	11.1	9.1	12.9	**DD-ME2/SLy4**
D(^{40}Ca – ^{44}Ca)	250.0	**1.14**[c]	4.5	29.6	12.2	3.88	7.08	9.13	**FSUGold**
D(^{40}Ca – ^{48}Ca)	250.0	**1.06**[c]	16.4	4.89	7.74	38.5	94.1	49.3	**G2**
D(^{48}Ca – ^{48}Ti)	250.0	**2.49**[c]	18.0	19.6	31.0	37.8	71.8	64.9	**DD-ME2**
D(^{116}Sn – ^{118}Sn)	225.0	**2.05**[a]	8.05	7.80	9.00	10.1	13.2	18.5	**G2**
D(^{118}Sn – ^{124}Sn)	225.0	**4.03**[a]	5.35	6.98	7.50	9.22	7.05	7.18	**DD-ME2**

[a] A.S. Litvinenko et al., Nucl.Phys. **A182**, 265 (1972). [b] B. Dreher, J. Friedrich, K. Merle, H. Rothhaas and G. Luhrs, Nucl.Phys.**A235**, 219 (1974). [c] R.F. Frosch et al. Phys. Rev. **174** (1968) 1380.

Conclusions

- Theory is well understood and calculations are feasible: **exact solution of the scattering process** once the nuclear e-m charge distribution has been provided.
- **disagreement** with the experiment **due exclusively** to the **nuclear model**
- The defined **Form Factor**
  - include **all finite size effects**
  - is **nearly energy-independent** at low momentum transfer
- d_w^2 does not decide between the different models

How can theory help in the experimental analysis? Could we find simple and general trends for $F(q)$ along isotopic/isotonic chains?

Proposed methodology:

1.- use **NM + DWBA** results within the low q-region **as a guide**

2.- predict $F_{DWBA}(q)$ along isotopic and/or isotonic chains, **from the neutron-rich to proton-rich side**

3.- parameterize the charge form factor within the **Helm model** $\Rightarrow$ insights on the evolution of the surface and bulk contibutions to the charge distribution as the isospin number increases

- **Advantages**: Helm model has two parameters with clear physical meaning and analytic expression of $F(q)$
- **Disadvantages**: Helm model assumes PWBA, it is (in principle) needed to introduce a $q_{effective}$ that corrects the e-m effects on the phase of the incident electron wave functions (Coulomb distortions).

4.- Here, as a **benchmark NM**, we will use **G2** because it takes into account explicitly the electromagnetic structure of the nucleon self-consistently within the fitting procedure

Helm Model: **2** parameters

- Helm Charge Form Factor: R_0 & σ

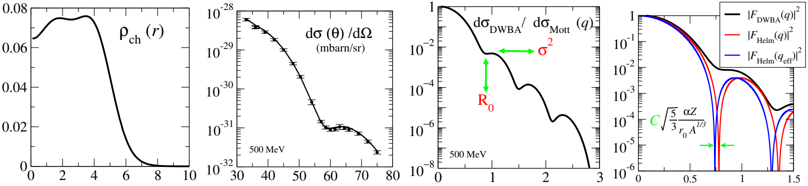
$$F_H(q) = \int e^{i\vec{q}\vec{r}} \rho_H(\vec{r}) d\vec{r} = \frac{3}{\mathbf{R_0} q} j_1(q\mathbf{R_0}) e^{-\sigma^2 q^2/2}$$

  where σ measures the surface fall-off of the density distribution and R_0 measures its bulk extension.
- How we determine the parameters:
  - R_0**:** one requires that the first zero of F_H occurs at the same q of F_{PWBA} (fourier transform of the self-consistent density). Therefore, it coincides with the sharp radius.
  - σ**:** is chosen to reproduce the height of the second maximum of $|F_{PWBA}|$

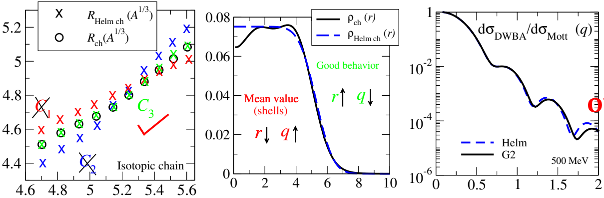
q_{eff}: 1 parameters

- **Effect** of the nuclear electrostatic **potential** on the **momentum transfer** (classical estimation):
$$\langle \hbar c q_{\text{eff}} \rangle \approx \langle \hbar c q - V \rangle = \int (\hbar c q - V(r))\, \rho_{ch}(r) d\mathbf{r}$$
considering a constant charge distribution in a spherical nucleus,
$$q_{eff} = q\left(1 + \mathbf{C}\frac{Z\alpha}{qR_{ch}}\right) \xrightarrow{m_e \ll q_e} q\left(1 + \mathbf{C}\frac{Z\hbar c\alpha}{ER_{ch}}\right)$$
where $R_{ch} \approx \sqrt{3/5} r_0 A^{1/3}$ and C=6/5
- One can always **reproduce** approximately the neglected **Coulomb phase shifts fitting** C in order to **reproduce** the **measured minima in** $F(q)$.
- **Isotopic chains:** correction produce a **change in the momentum transfer** of the **same order** than the change **produced** by adding/subtracting two more **neutrons**.
- **Isotonic chains:** correction can be **neglected** for our purposes here.

Testing Method Scheme:



+ DWBA → dσ/dΩ → $F_{DWBA}(q)$ → Helm Model + q_{eff}(**C**) →



C | best R_{ch}

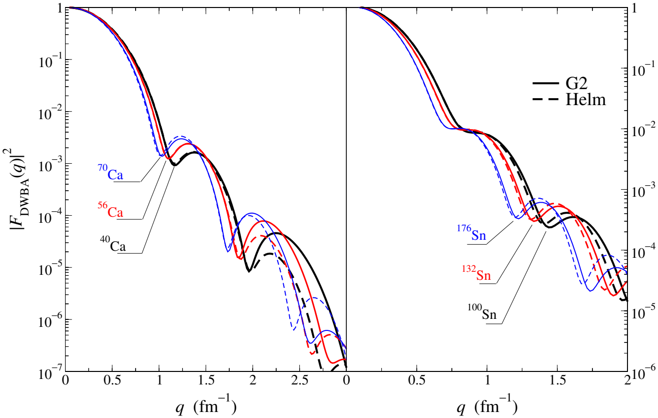
+ DWBA → **cross-check OK...**

Good method within the overlapped region

Results & Correlations: Z=50 and Z=20 isotopic chains

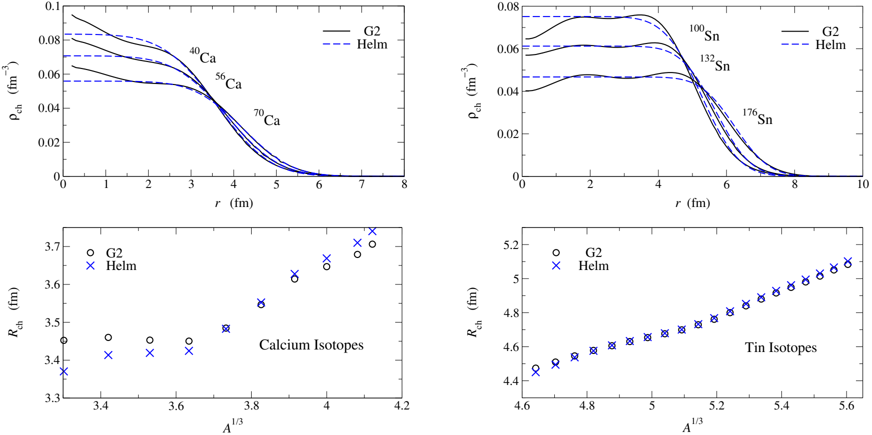
Charge Form Factor

F_{DWBA} increases and shifts towards smaller q as the neutron number increases

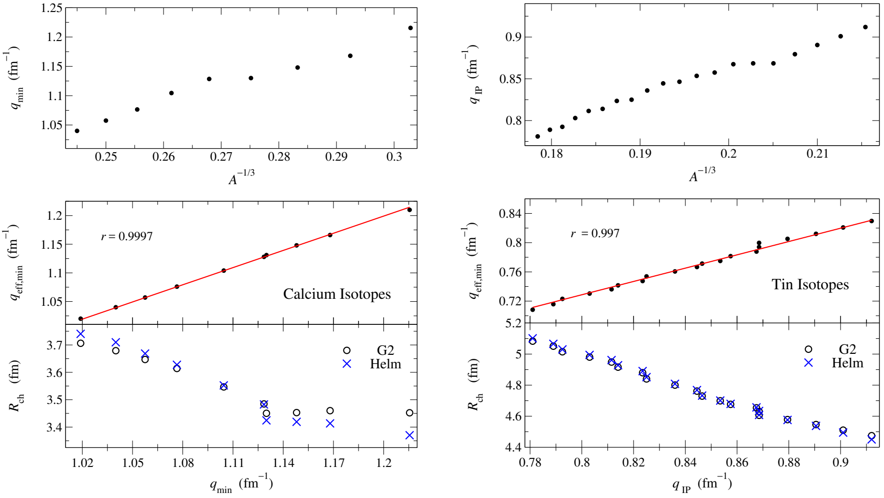


Methodology accurate for low-momentum transfer

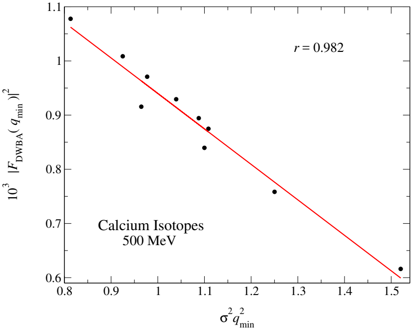
Helm and self-consistent charge densities and charge radii

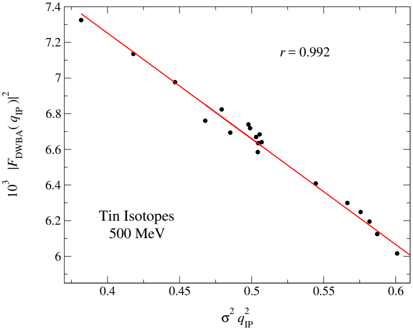


Correlations: evolution of first minimum or inflection point



Correlations: the smaller the bulk part of the nuclear charge distribution and the compact the surface, the smaller the form factor



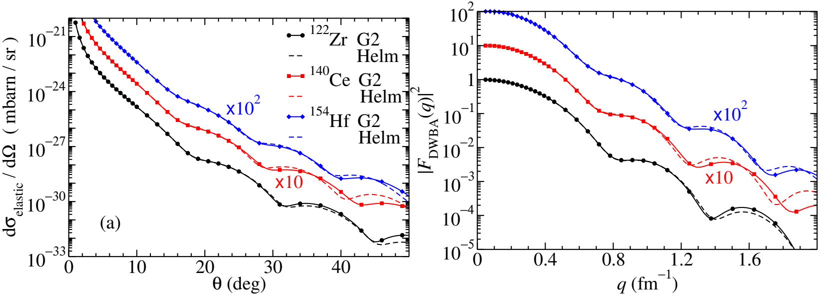


Therefore, if two or more isotopes have been measured ...

- linear correlations would provide, for an **unknown nucleus** of the chain, a **hint** on the value expected for the **square of the experimental electric charge form factor** at its first minimum
- if the value of the squared modulus of the **form factor is determined experimentally at its first minimum**, the **charge density in the Helm model can be sketched** from similar correlations

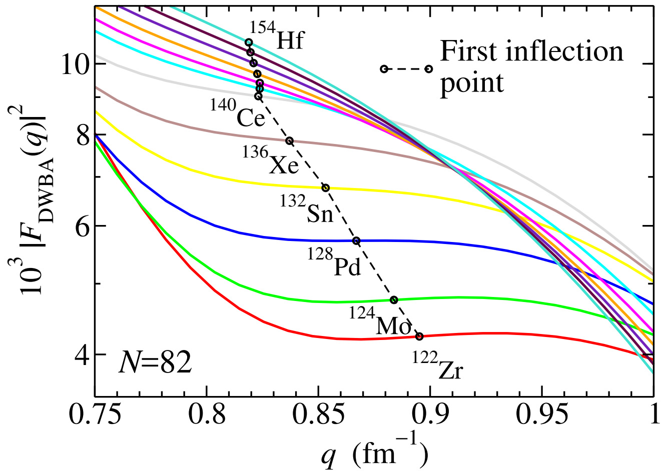
Results & Correlations: N=82, N=50 and N=14 isotonic chains

Differential cross sections and form factors

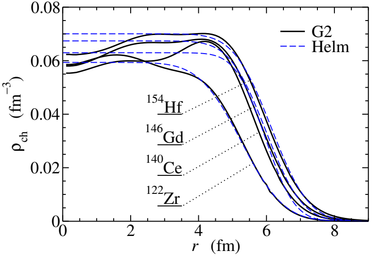


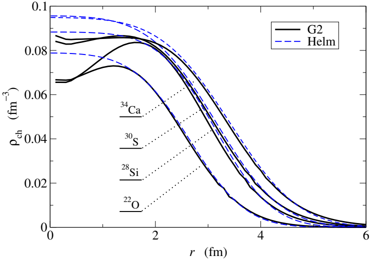
Methodology accurate for low-momentum transfer

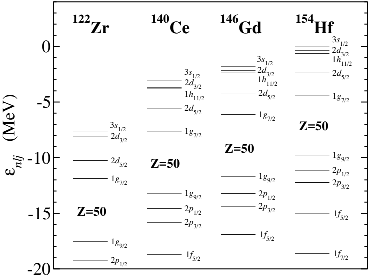
Charge form factors F_{DWBA} increases and shifts towards smaller q as the neutron number increases

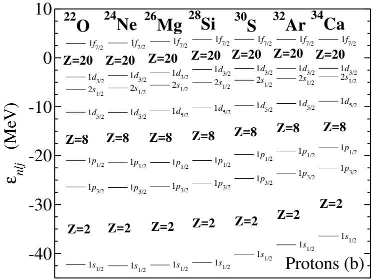


Charge densities and proton single particle levels

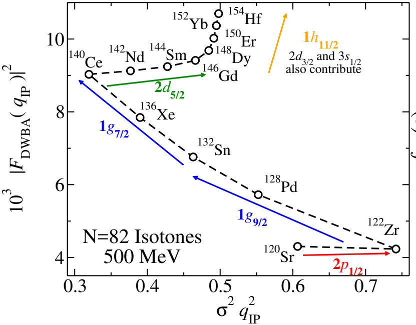


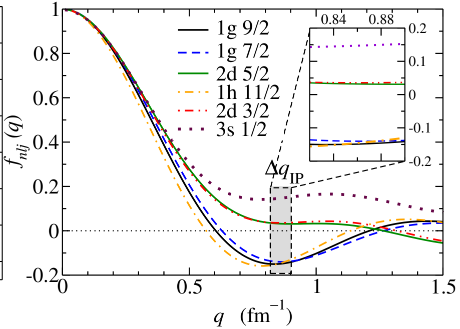




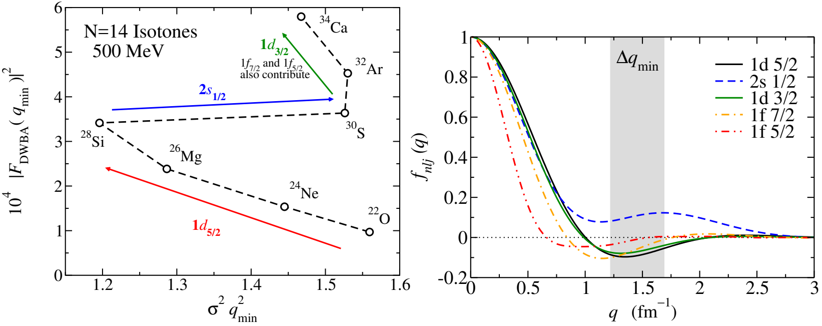


The increasing rate of the form factor basically depends on the proton level which is being filled



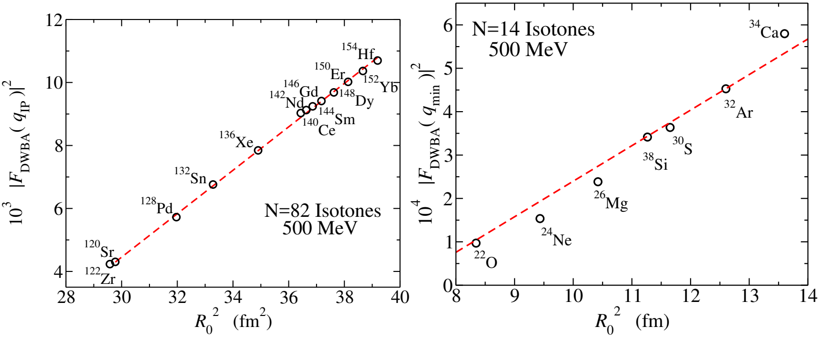


Also in lighter isotonic chains...



The larger the number of protons, the larger the formfactor

...this was clear, less clear was that it is almost linear along isotonic chains



Conclusions: isotopic chains

- The described analysis is **potentially useful** for future electron-nucleus elastic **scattering experiments**,
  - the **linear correlations** shown would provide, for an unknown nucleus of a chain, a hint on the value expected for $|F_{exp}(q_{min})|^2$ and for the DCS.
- The **exact analysis of the Coulomb phase shifts** applied to a exotic nuclei and compared with future measurments could, potentially, elucidate some aspects related with the **isospin asymmetry** of the nuclear force.

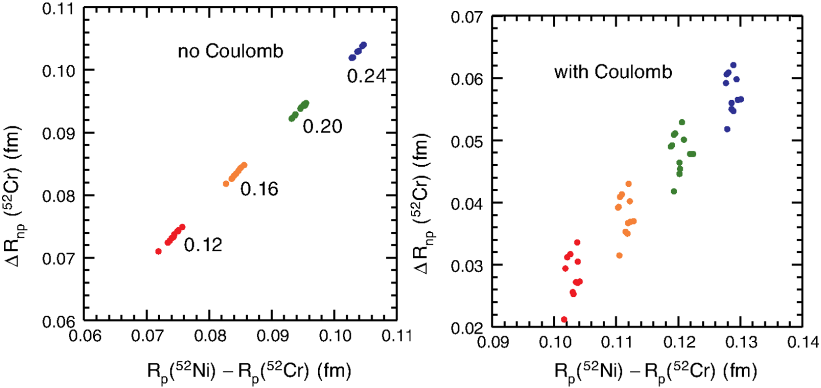
Conclusions: isotonic chains

- Rate of change of the **electric charge form factor** is **extremely sensitive** on the **proton level which is being filled**
  - levels with large n and small l contribute with opposite sign with respect to levels without radial nodes and large angular momenta.
  - plotting $|F(q)|^2$ against $\sigma^2 q_{min}^2$ magnifies such effects
- Therefore, **electron scattering** in isotonic chains can be a useful tool to **probe the proton shell structure of exotic nuclei**: filling order and occupancy of the different valence proton orbitals.

Differences in the proton radii of mirror nuclei

If isospin symmetry conserved (ISC) in nuclei

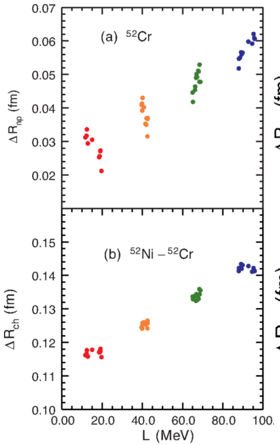
- $r_n(N,Z) = r_p(Z,N)$
- $\Delta r_{np}(N,Z) \equiv r_n(N,Z) - r_p(N,Z) = r_p(Z,N) - r_p(N,Z)$

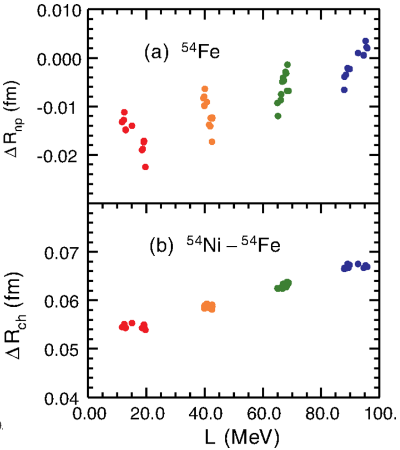


B. A. Brown, Phys. Rev. Lett. 119 122502 (2017)

Differences in the charge radii in mirror nuclei

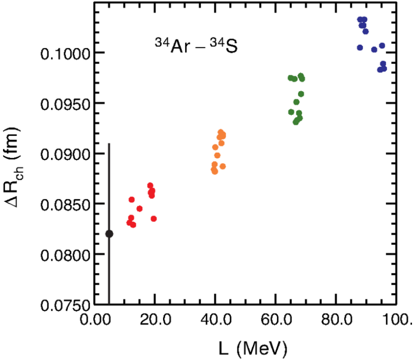
- $r_{ch} \approx \left(r_p^2 + R_P^2 + \frac{N}{Z} R_N^2 + \text{spin} - \text{orbit} \right)^{1/2}$
- If ISC $\Delta r_{np}(N, Z) \approx r_{ch}(Z, N) - r_{ch}(N, Z)$
- $\Delta r_{np}(N, Z)$ **in heavy nuclei is correlated with** L





Differences in the charge radii in mirror nuclei

- **Why correlation remains when isospin symmetry is broken by Coulomb?**
- **Why correlation improves for ΔR_{ch} as compared to that for ΔR_{np}?**
- **Other EDFs agree** (see *Phys. Rev. C 93 014314 (2018)*)



B. A. Brown, Phys. Rev. Lett. 119 122502 (2017)

Dipole polarizability: definition

From a macroscopic perspective

The electric **polarizability** measures the tendency of the nuclear **charge distribution to be distorted**

$$\alpha_D \sim \frac{\text{electric dipole moment}}{\text{external electric field}}$$

From a microscopic perspective

The electric **polarizability** is proportional to the **inverse energy weighted sum rule (IEWSR)** of the electric dipole response in nuclei

$$\alpha_D = \frac{8\pi}{9} e^2 \sum \frac{B(E1)}{E}$$

or

$$\alpha_D = \frac{\hbar c}{2\pi^2} \int \frac{\sigma_{\text{ph. abs.}}(E)}{E^2} dE$$

Dipole polarizability: macroscopic approach

The **dielectric theorem** establishes that the m_{-1} moment can be computed from the **expectation value of the Hamiltonian in the constrained ground state** $\mathcal{H}' = \mathcal{H} + \lambda\mathcal{D}$.

Adopting the Droplet Model ($m_{-1} \propto \alpha_D$):

$$m_{-1} \approx \frac{A\langle r^2\rangle^{1/2}}{48J}\left(1+\frac{15}{4}\frac{J}{Q}A^{-1/3}\right)$$

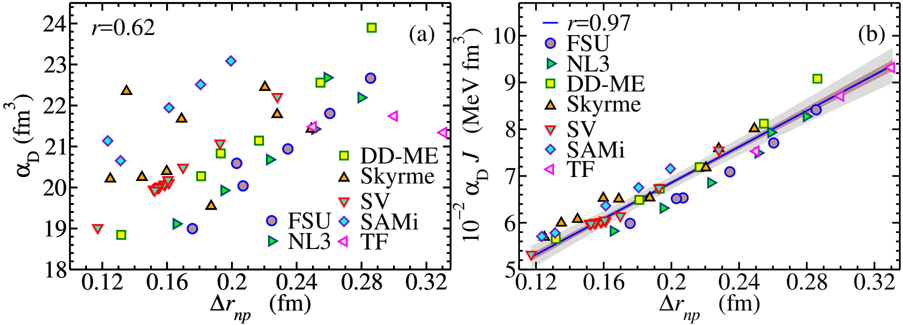
within the same model, connection with the neutron skin thickness:

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1+\frac{5}{2}\frac{\Delta r_{np}+\sqrt{\frac{3}{5}}\frac{e^2 Z}{70J}-\Delta r_{np}^{\mathrm{surface}}}{\langle r^2\rangle^{1/2}(I-I_C)}\right]$$

Is this correlation appearing also in EDFs?

Isovector Giant Dipole Resonance in ^{208}Pb:

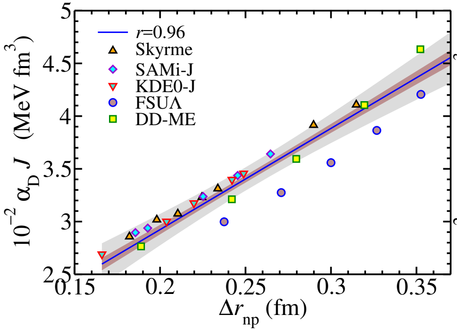
Dipole polarizability: microscopic results HF+RPA

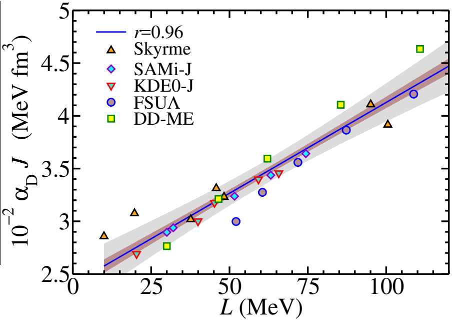


X. Roca-Maza, *et al.*, Phys. Rev. C 88, 024316 (2013).

$\alpha_D J$ is linearly correlated with Δr_{np} and no α_D alone within EDFs

Dipole polarizability in ^{132}Sn



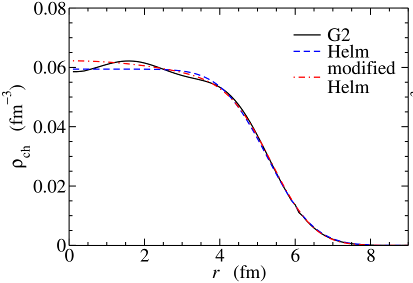


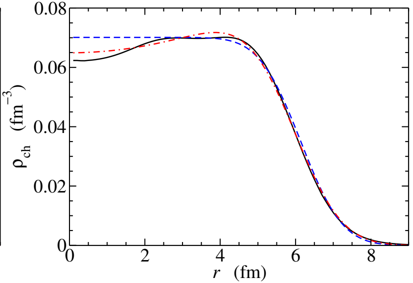
Thank you for your attention!

Extra Material

What happens if we use a more realistic ansatz for the parametrized density?

If we use a modified Helm density (3 parameters): able to reproduce the central "depression" or "bump" typically present in charge distribtions.

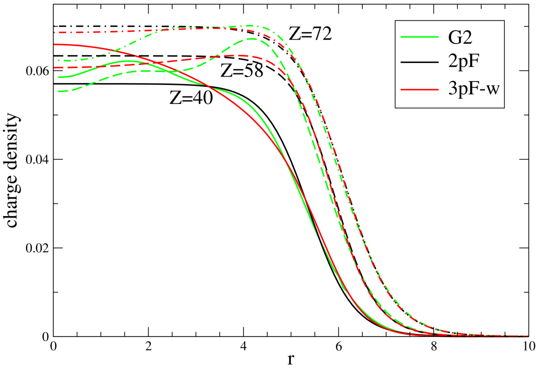




Just two examples of the N = 82 isotonic chain.

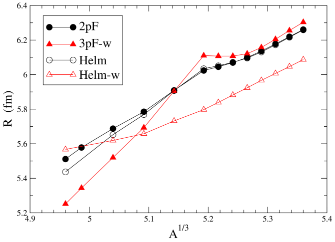
What happens if we use a more realistic ansatz for the parametrized density?

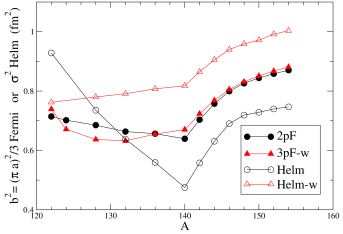
If we use a 2pF or 3pF density distributions.



How much affects the “new” parameterized density our previous analysis?

One finds different quantitative results but **similar behaviours** ⇒ our analysis still valid when using other density distributions





How much the form factor looks like?