Shape Coexistence and Competing Symmetries in Nuclei

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Shape Coexistence in Nuclei

Exp:	prolate-oblate	Kr, Se, Hg neutron-deficient isotopes
	spherical-prolate-oblate	^{186}Pb
	spherical-prolate	Sr neutron-rich isotopes ^{96}Zr, ^{78}Ni

- Shell model approach

  - Multiparticle-multihole intruder excitations across shell gaps
  - Drastic truncation of large SM spaces

- Mean-field approach (EDF)

  - Coexisting shapes associated with different minima of an energy surface
  - Beyond MF methods: restoration of broken symmetries

- Symmetry-based approach

  - Dynamical symmetries $\leftrightarrow$ phases
  - Geometry: coherent (intrinsic) states

Dynamical Symmetry

$$\begin{array}{ccccccc} G_{\rm dyn} & \supset & G & \supset & \cdots & \supset & G_{\rm sym} \\ \downarrow & & \downarrow & & & & \downarrow \\ [N] & & \langle\Sigma\rangle & & & & \Lambda \end{array}$$

$$\hat{H} = \sum_G a_G \, \hat{C}_G$$

- *Solvability of the **complete** spectrum* $E = E_{[N]\langle\Sigma\rangle\ldots\Lambda}$
- *Quantum numbers for **all** eigenstates* $|[N]\langle\Sigma\rangle\Lambda\rangle$

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- IBM: s (L=0) , d (L=2) bosons, N conserved *(Arima, Iachello 75)*

G_{dyn} = U(6), G_{sym} = SO(3)

U(6) ⊃ U(5) ⊃ SO(5) ⊃ SO(3)	$\vert [N]\ n_d\ \tau\ n_\Delta\ L\rangle$	Spherical vibrator
U(6) ⊃ SU(3) ⊃ SO(3)	$\vert [N]\ (\lambda,\mu)\ K\ L\rangle$	Prolate-deformed rotor
U(6) ⊃ $\overline{\text{SU(3)}}$ ⊃ SO(3)	$\vert [N]\ (\bar{\lambda},\bar{\mu})\ \bar{K}\ L\rangle$	Oblate-deformed rotor
U(6) ⊃ SO(6) ⊃ SO(5) ⊃ SO(3)	$\vert [N]\ \sigma\ \tau\ n_\Delta\ L\rangle$	γ-unstable deformed rotor

Geometry

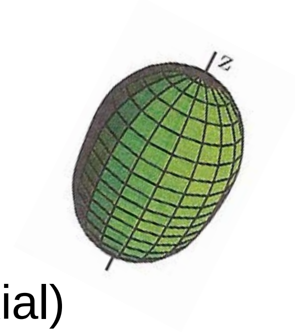
Coherent state

$$|\,\beta,\gamma;N\rangle \;=\; (N!)^{-1/2}(b_c^\dagger)^N\,|0\,\rangle$$

$$b_c^\dagger \;=\; (1+\beta^2)^{-1/2}\left[\beta\,\cos\gamma\,d_0^\dagger+\beta\,\sin\gamma\,\frac{1}{\sqrt{2}}\left(d_2^\dagger+d_{-2}^\dagger\right)+s^\dagger\right]$$

Energy surface

$$E_N(\beta,\gamma)=\langle\beta,\gamma;N|\hat{H}|\beta,\gamma;N\rangle$$



Global min: equilibrium shape (β_0,γ_0)

$\beta_0 = 0$ spherical

$\beta_0 > 0$ deformed: $\gamma_0 = 0$ (prolate), $\gamma_0 = \pi/3$ (oblate), $0 < \gamma_0 < \pi/3$ (triaxial)

Intrinsic state ground band $|\beta_0,\gamma_0;N\rangle$, L-projected states $|\beta_0,\gamma_0;N,x,L\rangle$

$U(6) \supset \mathbf{G_1} \supset G_2 \supset \ldots SO(3)$ $\qquad |N, \boldsymbol{\lambda_1}, \lambda_2,\ldots,L\rangle$

U(5)	$\beta_0 = 0$	$n_d = 0$
SU(3)	$(\beta_0 = \sqrt{2},\ \gamma_0 = 0)$	$(\lambda,\mu) = (2N,0)$
$\overline{SU(3)}$	$(\beta_0 = \sqrt{2},\ \gamma_0 = \pi/3)$	$(\bar{\lambda},\bar{\mu}) = (0,2N)$
SO(6)	$(\beta_0 = 1,\ \gamma_0$ arbitrary)	$\sigma = N$

- Dynamical symmetry corresponds to a particular shape (β_0,γ_0)
- $|\beta_0,\gamma_0;N\rangle$ lowest (highest) weight state in a particular irrep $\boldsymbol{\lambda_1}$ of leading subalgebra $\mathbf{G_1}$

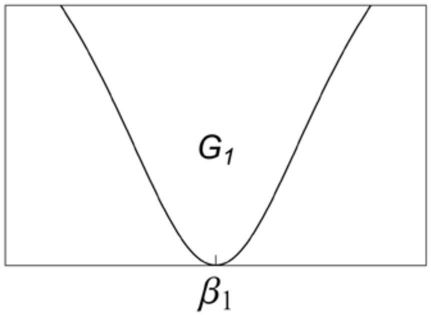
Dynamical Symmetry

$U(6) \supset G_1 \supset G_2 \supset \ldots SO(3)$ $\qquad$ $|N, \lambda_1, \lambda_2, \ldots, L\rangle$

- Complete solvability
- Good quantum numbers for all states
- DS: benchmark for a single shape

[G_1 = U(5), SU(3), $\overline{\text{SU(3)}}$, SO(6)]

Spherical, prolate- , oblate-, γ-unstable deformed



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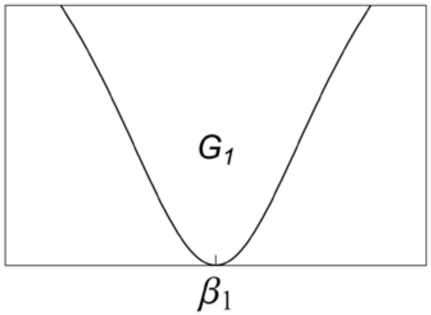
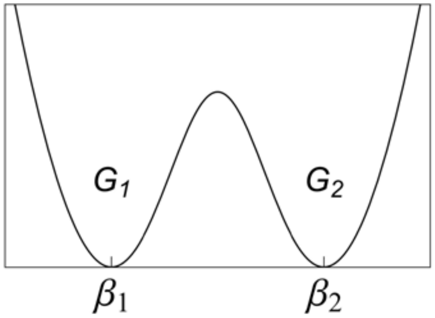
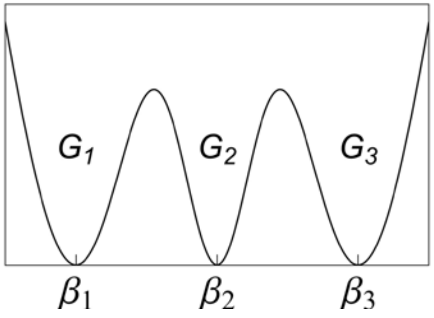


Partial Dynamical Symmetry

- Some states solvable and/or with good quantum numbers
- G_1, G_2 incompatible (non-commuting) symmetries
- PDS: benchmark for shape coexistence





Construction of Hamiltonians with a single PDS

$$G_{\rm dyn} \supset G \supset \cdots \supset G_{\rm sym}$$

$$[N] \qquad \langle\Sigma\rangle \qquad \qquad \Lambda$$

n-particle annihilation operator

$$\boxed{\hat{T}_{[n]\langle\sigma\rangle\lambda}\,|\,[N]\,\langle\Sigma_0\rangle\,\Lambda\rangle = 0}$$

for **all** possible Λ contained in the irrep $\langle\Sigma_0\rangle$ of G

Equivalently:

$$\boxed{\hat{T}_{[n]\langle\sigma\rangle\lambda}\,|\,[N]\,\langle\Sigma_0\rangle\,\rangle = 0}$$

| **Lowest weight state** $\rangle$

- Condition is satisfied if $\langle\sigma\rangle\otimes\langle\Sigma_0\rangle \not\ni [N-n]$

$$\hat{H} = \sum_{\alpha,\beta} u_{\alpha\beta}\hat{T}^{\dagger}_{\alpha}\hat{T}_{\beta}$$

DS is **broken** but
solvability of states with $\langle\Sigma\rangle = \langle\Sigma_0\rangle$ Is **preserved**

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$[N]$ $\langle\Sigma\rangle$ Λ

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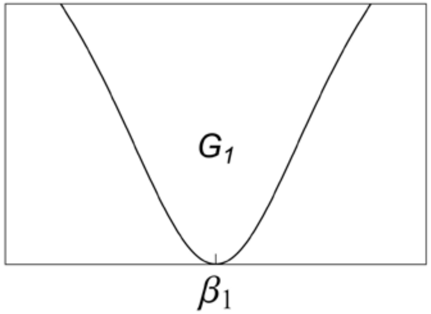
- PDS Hamiltonian $\hat{H}' = \hat{H} + \hat{H}_{\rm c}$ Intrinsic collective resolution

Intrinsic part: $H\,|[N]\,\langle\Sigma_0\rangle\,\Lambda\rangle = 0$

Collective part: H_c composed of Casimir operators of conserved $G_i \subset G$ in the chain

Multiple PDS and Shape Coexistence

$$\mathrm{U}(6) \supset \mathrm{G}_1 \supset \mathrm{G}_2 \supset \ldots \supset \mathrm{SO}(3) \qquad |N,\, \lambda_1,\, \lambda_2,\, \ldots,\, L\rangle \qquad (\beta_1,\gamma_1)$$



Single PDS
Single shape

$$\hat{H}|\beta_1,\gamma_1; N, \lambda_1 = \Lambda_0, \lambda_2, \ldots, L\rangle = 0$$

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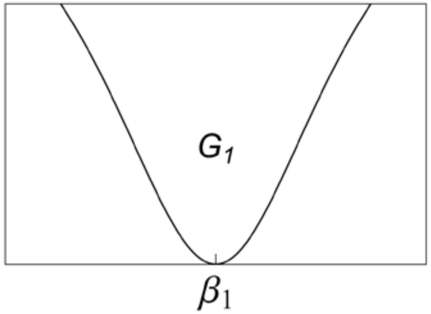

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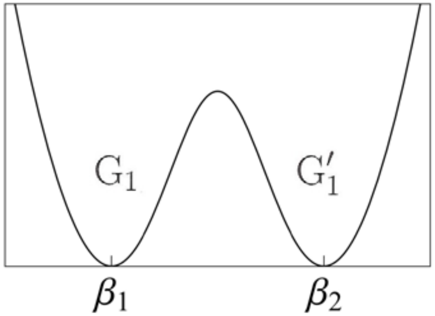
$$\mathrm{U}(6) \supset \mathrm{G}_1' \supset \mathrm{G}_2' \supset \ldots \supset \mathrm{SO}(3) \qquad |N,\, \sigma_1,\, \sigma_2,\, \ldots,\, L\rangle \qquad (\beta_2,\gamma_2)$$

Multiple PDS
Multiple shapes

$$\begin{cases} \hat{H}|\beta_1, \gamma_1; N, \lambda_1 = \Lambda_0, \lambda_2, \ldots, L\rangle = 0 \\ \hat{H}|\beta_2, \gamma_2; N, \sigma_1 = \Sigma_0, \sigma_2, \ldots, L\rangle = 0 \end{cases}$$



$$\mathrm{G}_1 \neq \mathrm{G}_1'$$

Critical-point Hamiltonian $\quad \hat{H}' = \hat{H} + \hat{H}_{\mathrm{c}}$

G_1 -PDS & G'_1 -PDS

Intrinsic part: $\hat{H}$ determines $E(\beta,\gamma)$ band structure

Collective part: $\hat{H}_{\mathrm{c}} = \sum_{\mathrm{G_i}} a_{\mathrm{G_i}} \hat{C}_{\mathrm{G_i}}$ rotational splitting

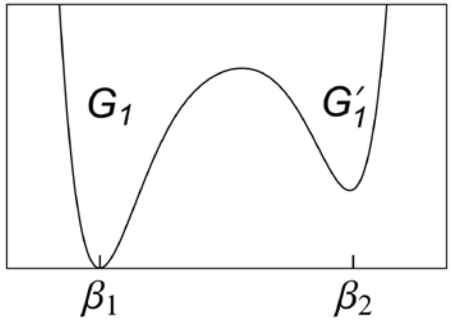
conserved G_i in both chains

Departure from the Critical Point

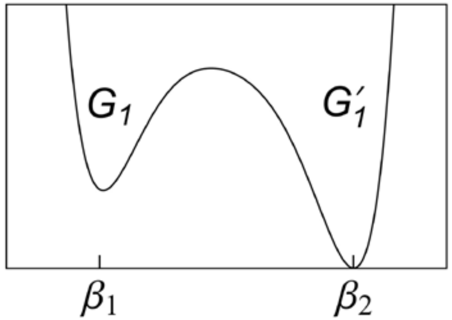
$\mathrm{U}(6) \supset \mathrm{G}_1 \supset \mathrm{G}_2 \supset \ldots \supset \mathrm{SO}(3) \qquad |N,\, \lambda_1,\, \lambda_2,\, \ldots,\, L\rangle \qquad (\beta_1,\gamma_1)$

$\mathrm{U}(6) \supset \mathrm{G}'_1 \supset \mathrm{G}'_2 \supset \ldots \supset \mathrm{SO}(3) \qquad |N,\, \sigma_1,\, \sigma_2,\, \ldots,\, L\rangle \qquad (\beta_2,\gamma_2)$

$\mathrm{G}_1 \neq \mathrm{G}'_1$



$$\hat{H}' = \hat{H}'_{\rm cp} + \alpha\, \hat{C}[\mathrm{G}_1]$$



$$\hat{H}' = \hat{H}'_{\rm cp} + \alpha\, \hat{C}[\mathrm{G}'_1]$$

Symmetry Approach to Shape-Coexistence

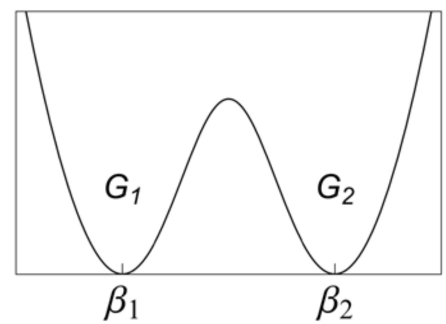
$U(6) \supset U(5) \supset SO(5) \supset SO(3)$	Spherical vibrator	$\beta = 0$
$U(6) \supset SU(3) \supset SO(3)$	Prolate-deformed rotor	$\beta = \sqrt{2},\ \gamma = 0$
$U(6) \supset \overline{SU(3)} \supset SO(3)$	Oblate-deformed rotor	$\beta = \sqrt{2},\ \gamma = \pi/3$
$U(6) \supset SO(6) \supset SO(5) \supset SO(3)$	γ-unstable deformed rotor	$\beta = 1$, γ arbitrary

Multiple PDS and Multiple Shapes

$G_1 = U(5)$ $G_2 = SU(3)$ spherical – prolate

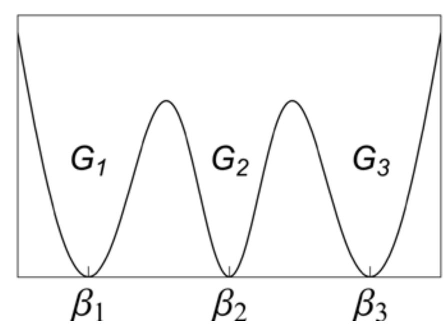
$G_1 = SU(3)$ $G_2 = \overline{SU(3)}$ prolate – oblate ♣

$G_1 = U(5)$ $G_2 = SO(6)$ spherical - γ-unstable ♣



Triple coexistence

$G_1 = U(5)$ $G_2 = SU(3)$ $G_3 = \overline{SU(3)}$ spherical-prolate-oblate ♣



♣ Leviatan, Shapira, PRC **93**, 051302(R) (2016)

♣ Leviatan, Gavrielov, Phys. Scr. **92**, 114005 (2017)
arXiv:1803.03982 [nucl-th] (2018)

SU(3) and $\overline{SU(3)}$ Dynamical Symmetries

$U(6) \supset SU(3) \supset SO(3)$ $\quad |[N]\,(\lambda,\mu)\,K\,L\rangle$ Prolate-deformed rotor

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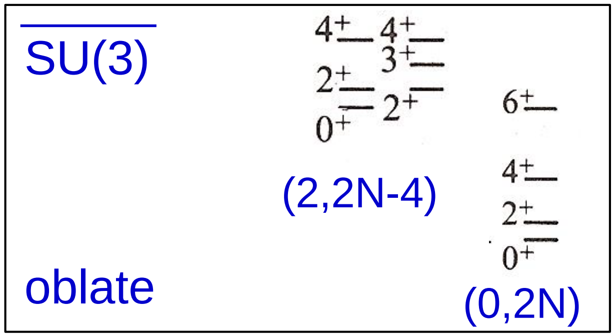
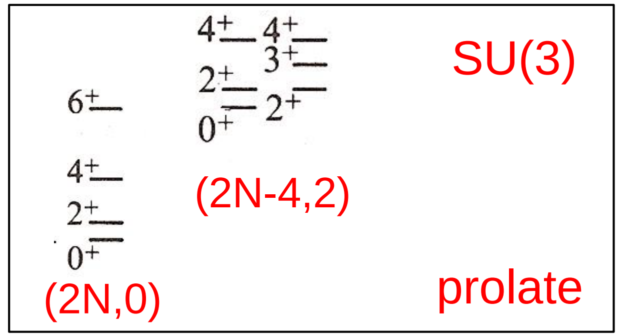




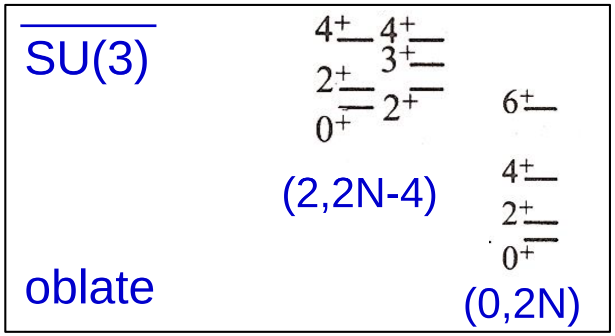
DS spectra are identical

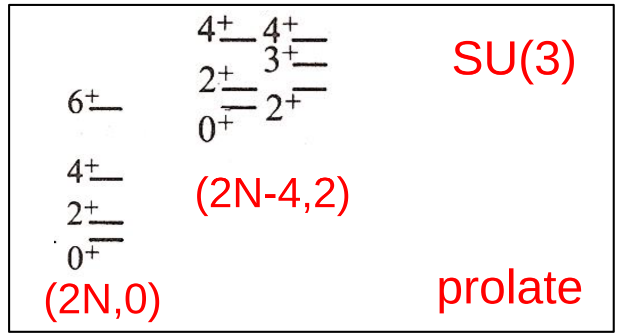
Quadrupole moments of corresponding states differ in sign

SU(3) and $\overline{\text{SU(3)}}$ Dynamical Symmetries

U(6) ⊃ SU(3) ⊃ SO(3) | [N] (λ ,μ) K L⟩ Prolate-deformed rotor

U(6) ⊃ $\overline{\text{SU(3)}}$ ⊃ SO(3) | [N] ($\bar{\lambda}$,$\bar{\mu}$) K L⟩ Oblate-deformed rotor





DS spectra are identical

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Prolate-Oblate Shape Coexistence

Intrinsic part of C.P. Hamiltonian

$$\begin{cases} \hat{H}|N,\,(\lambda,\mu)=(2N,0),\,K=0,\,L\rangle = 0 \\ \hat{H}|N,\,(\bar{\lambda},\bar{\mu})=(0,2N),\,\bar{K}=0,\,L\rangle = 0 \end{cases}$$

$$\hat{H} = h_0\,P_0^\dagger \hat{n}_s P_0 + h_2\,P_0^\dagger \hat{n}_d P_0 + \eta_3\,G_3^\dagger \cdot \tilde{G}_3 \qquad P_0^\dagger = d^\dagger \cdot d^\dagger - 2(s^\dagger)^2 \qquad G_{3,\mu}^\dagger = \sqrt{7}[(d^\dagger d^\dagger)^{(2)} d^\dagger]^{(3)}_\mu$$

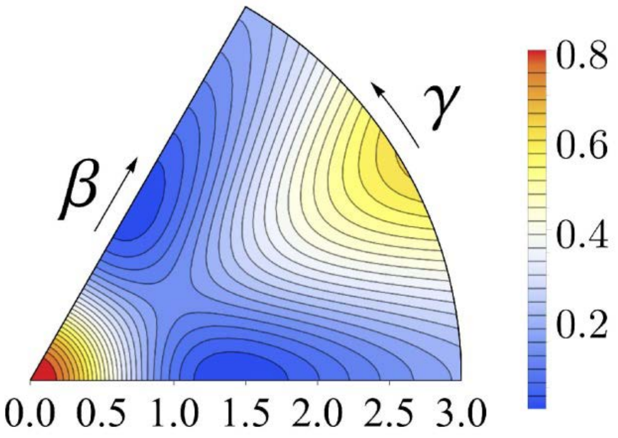
Energy Surface $\tilde{E}(\beta,\gamma) = (1+\beta^2)^{-3}\left\{(\beta^2-2)^2\left[h_0 + h_2\beta^2\right] + \eta_3\beta^6\sin^2(3\gamma)\right\}$

$$= z_0 + (1+\beta^2)^{-3}[A\beta^6 + B\beta^6\Gamma^2 + D\beta^4 + F\beta^2] \qquad \Gamma = \cos 3\gamma$$

- Two degenerate P-O global minima

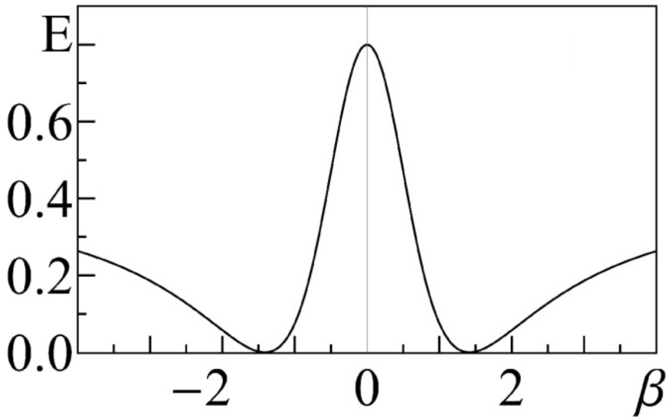
(β=√2,γ = 0) and (β=√2,γ = π/3) [or equivalently (β= -√2,γ 0)]

oblate-prolate



E(β,γ)

Saddle points support a barrier separating the various minima

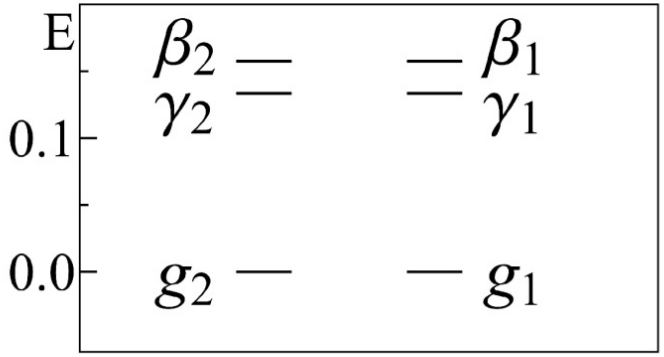


E(β,γ=0)

Normal modes:

$$\epsilon_{\beta 1} = \epsilon_{\beta 2} = \frac{8}{3}(h_0 + 2h_2)N^2$$

$$\epsilon_{\gamma 1} = \epsilon_{\gamma 2} = 4\eta_3 N^2$$

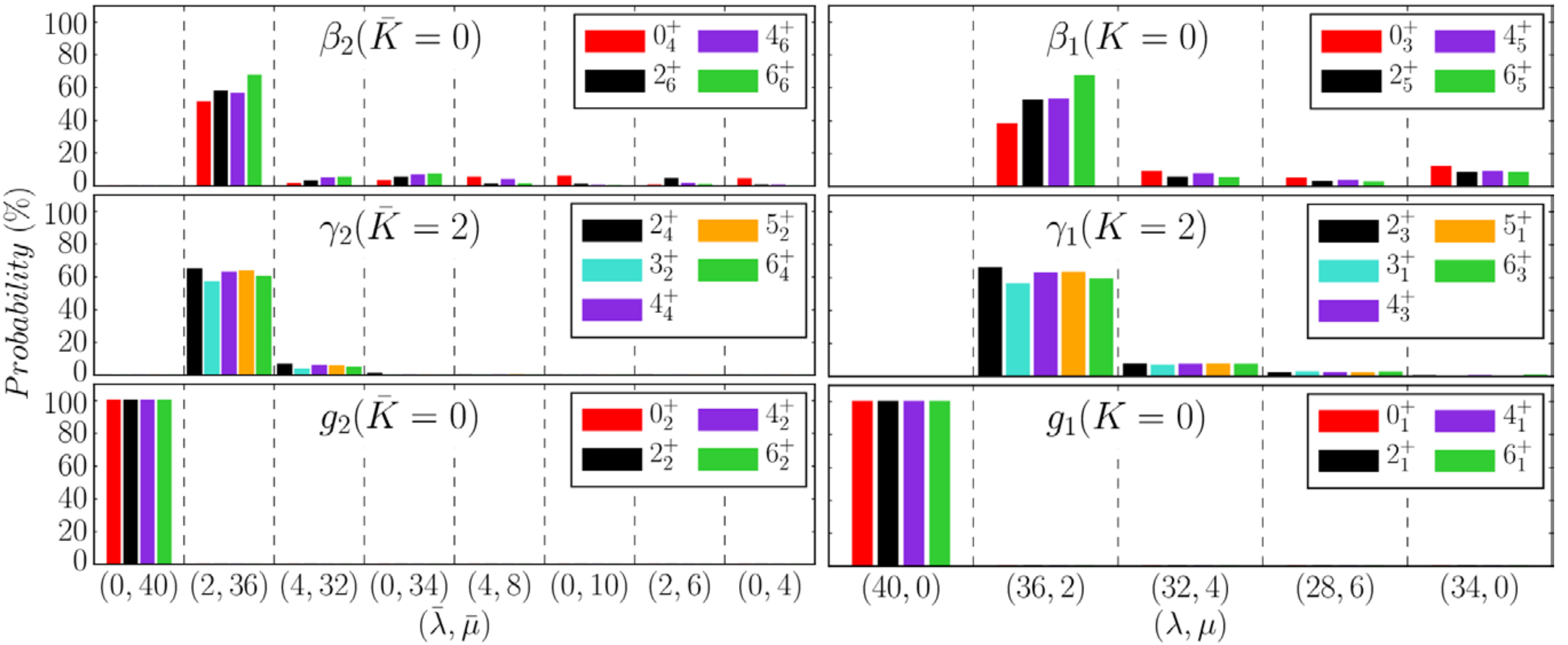


Complete Hamiltonian $\hat{H}' = \hat{H}(h_0, h_2, \eta_3) + \alpha\,\hat{\theta}_2 + \rho\,\hat{C}_2[\mathrm{SO}(3)]$

$$\alpha\,\hat{\theta}_2 = \alpha\,[-\hat{C}_2[SU(3)] + 2\hat{N}(2\hat{N}+3)]$$

$$\tilde{\alpha}(1+\beta^2)^{-2}[(\beta^2-2)^2+2\beta^2(2-2\sqrt{2}\beta\Gamma+\beta^2)] \qquad \tilde{\alpha} = \alpha/(N-2)$$

$\overline{\mathrm{SU(3)}}$ decomposition SU(3) decomposition

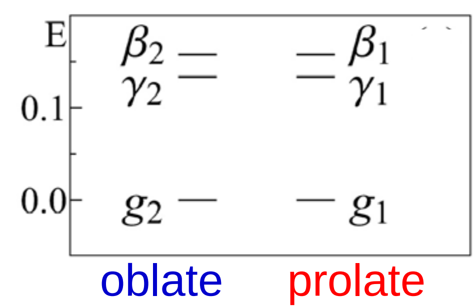


Ground g_1 band: pure SU(3)-DS states (2N,0)

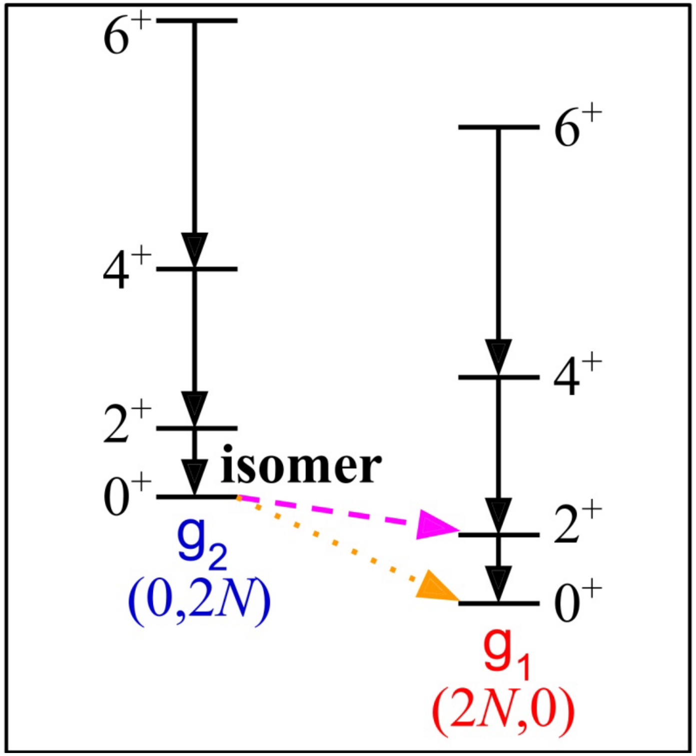
Ground g_2 band: pure $\overline{\mathrm{SU(3)}}$-DS states (0,2N)

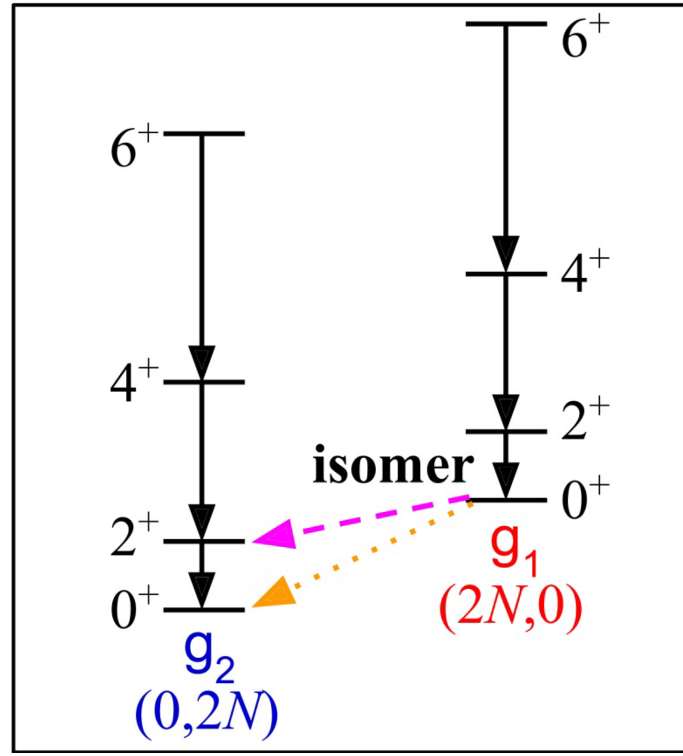
Excited β and γ bands: considerable mixing

$\Rightarrow$ SU(3)-PDS coexisting with $\overline{\mathrm{SU(3)}}$-PDS



P-O coexistence





$T(E2) = e_B(d^\dagger s + s^\dagger \tilde{d})$ (1,1) ⊕ (2,2) tensor

$T(E0) \propto \hat{n}_d$ (0,0) ⊕ (2,2) tensor

E2 selection rule: $g_1 \not\leftrightarrow g_2$

E0 selection rule: $g_1 \not\leftrightarrow g_2$

$$Q_L = \mp e_B \sqrt{\frac{16\pi}{40}} \frac{L}{2L+3} \frac{4(2N-L)(2N+L+1)}{3(2N-1)}$$

ANALYTIC expressions !

$$B(E2; g_i,\ L+2 \rightarrow g_i,\ L) =$$

$$e_B^2 \frac{3(L+1)(L+2)}{2(2L+3)(2L+5)} \frac{(4N-1)^2(2N-L)(2N+L+3)}{18(2N-1)^2}$$

U(5), SU(3) and $\overline{SU(3)}$ Dynamical Symmetries

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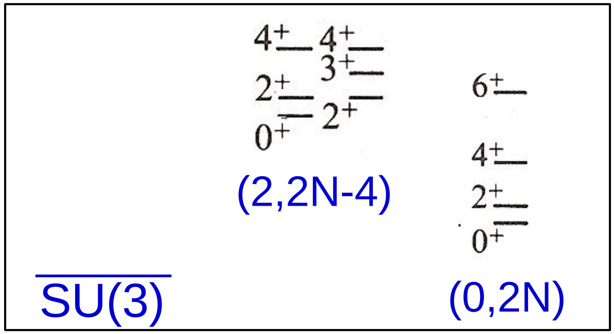
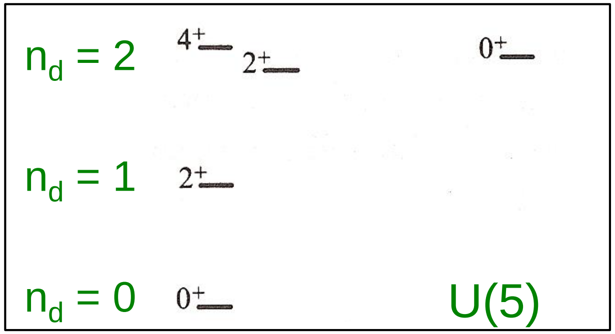
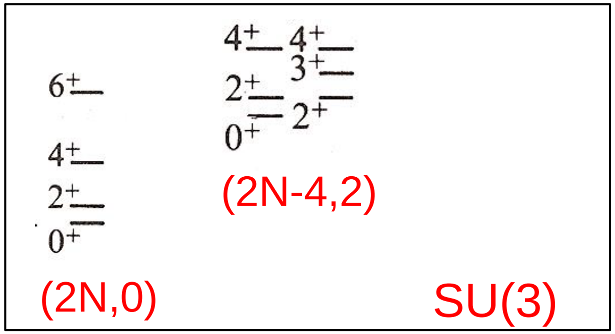
$U(6) \supset SU(3) \supset SO(3)$	$\vert [N]\ (\lambda, \mu)\ K\ L \rangle$	Prolate-deformed rotor
$U(6) \supset \overline{SU(3)} \supset SO(3)$	$\vert [N]\ (\overline{\lambda}, \overline{\mu})\ K\ L \rangle$	Oblate-deformed rotor







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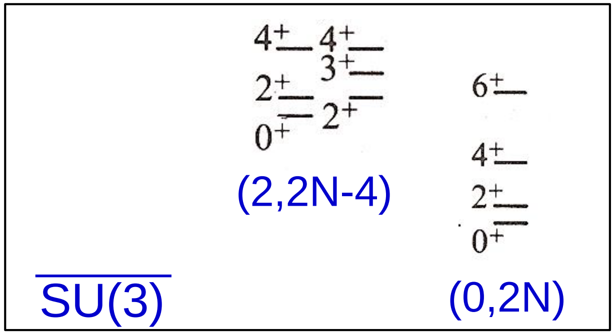
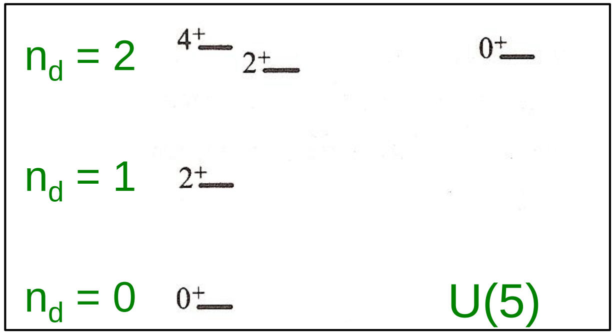
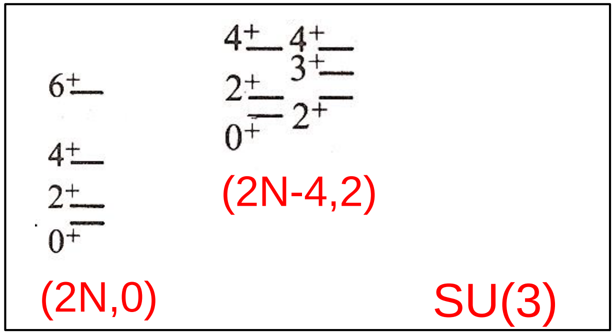






Spherical-Prolate-Oblate Shape Coexistence

Intrinsic part of C.P. Hamiltonian

$$\begin{cases} \hat{H}|N,\ (\lambda,\mu) = (2N,0),\ K=0,\ L\rangle = 0 \\ \hat{H}|N,\ (\bar{\lambda},\bar{\mu}) = (0,2N),\ \bar{K}=0,\ L\rangle = 0 \\ \hat{H}|N,\ n_d=0,\ \tau=0,\ L=0\rangle = 0 \end{cases}$$

$$\hat{H} = h_2\, P_0^\dagger \hat{n}_d P_0 + \eta_3\, G_3^\dagger \cdot \tilde{G}_3$$

$$P_0^\dagger = d^\dagger \cdot d^\dagger - 2(s^\dagger)^2 \qquad G_{3,\mu}^\dagger = \sqrt{7}[(d^\dagger d^\dagger)^{(2)} d^\dagger]_\mu^{(3)}$$

Energy Surface

$$\tilde{E}(\beta,\gamma) = \beta^2[\, h_2(\beta^2-2)^2 + \eta_3\beta^4 \sin^2(3\gamma)\,](1+\beta^2)^{-3}$$

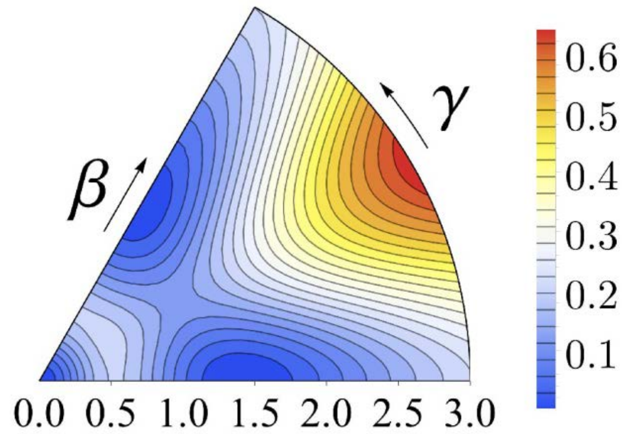
- Three degenerate S-P-O global minima: $\beta=0$, ($\beta = \pm\sqrt{2}, \gamma = 0$)

Complete Hamiltonian

$$\hat{H}' = h_2\, P_0^\dagger \hat{n}_d P_0 + \eta_3\, G_3^\dagger \cdot \tilde{G}_3 + \alpha\, \hat{\theta}_2 + \rho\, \hat{C}_2[\mathrm{SO(3)}]$$

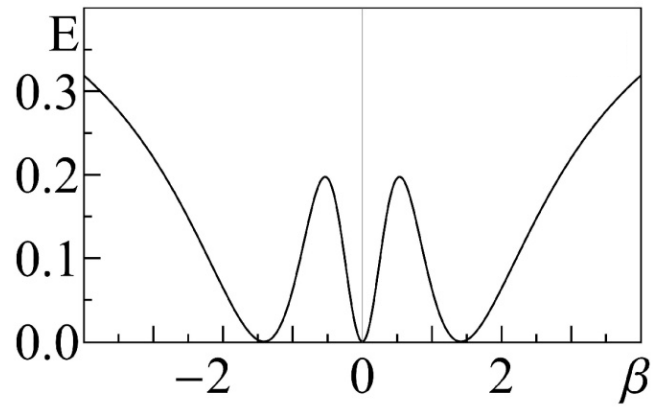
oblate-spherical-prolate

Triple coexistence



E(β,γ)

Saddle points support a barrier separating the various minima



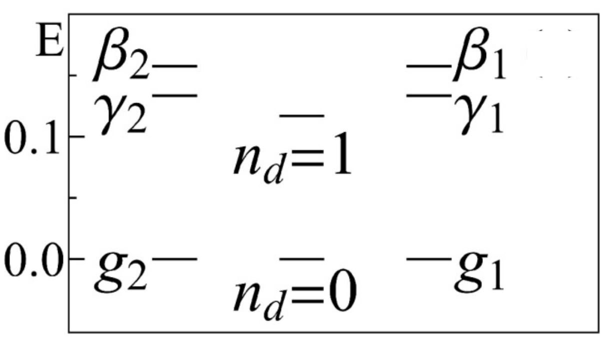
E(β,γ=0)

Normal modes:

$$\epsilon_{\beta 1} = \epsilon_{\beta 2} = \frac{16}{3} h_2 N^2$$

$$\epsilon_{\gamma 1} = \epsilon_{\gamma 2} = 4\eta_3 N^2$$

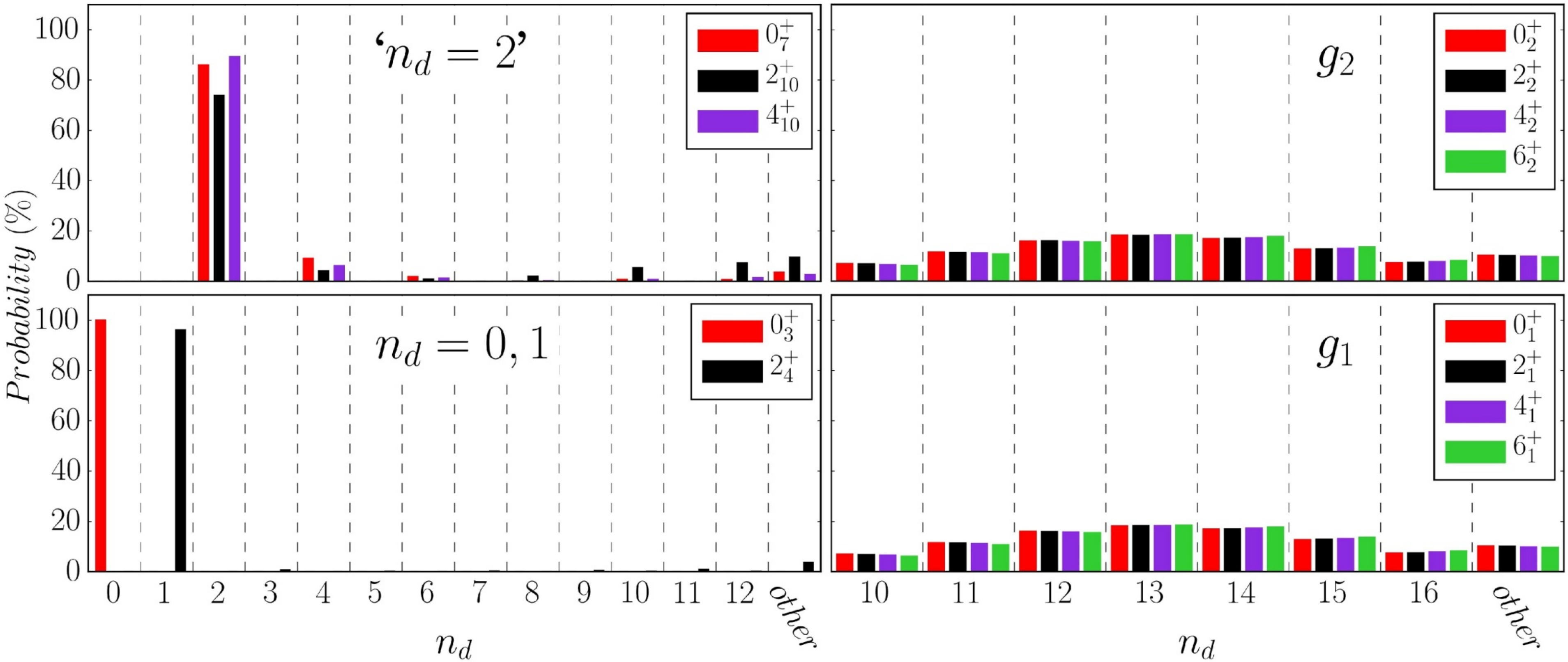
$$\epsilon = 4h_2 N^2$$



bandhead spectrum

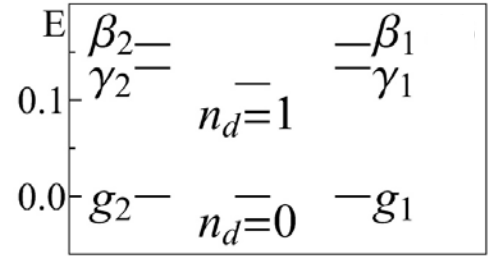
Triple Spherical-Prolate-Oblate Coexistence

U(5) decompostion



P-O bands show similar behavior as in P-O coexistence

New aspect: occurrence of spherical type of states
(n_d=L=0) and (n_d=1,L=2) pure U(5)-DS
Higher spherical states: pronounced (~70%) n_d=2



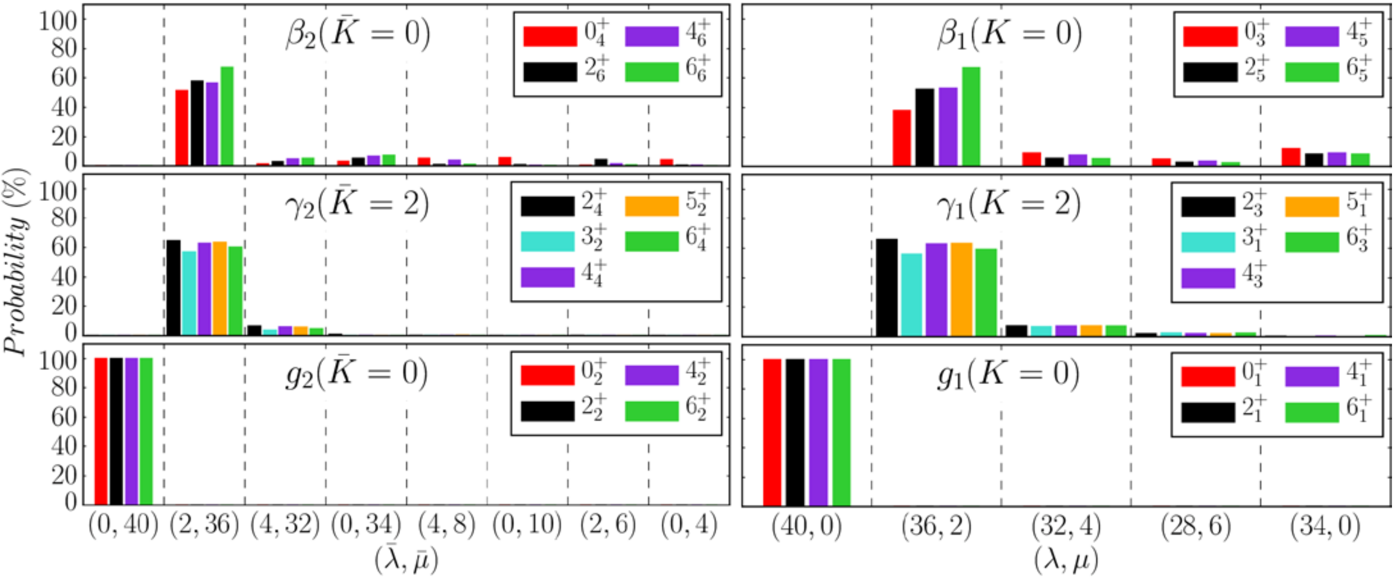
oblate spherical prolate

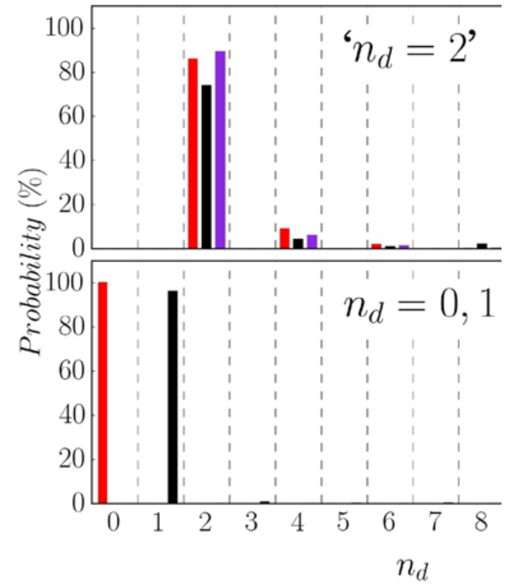
Coexisting Partial Dynamical Symmetries

$\overline{\text{SU(3)}}$ decomposition

SU(3) decompostion

U(5) decompostion



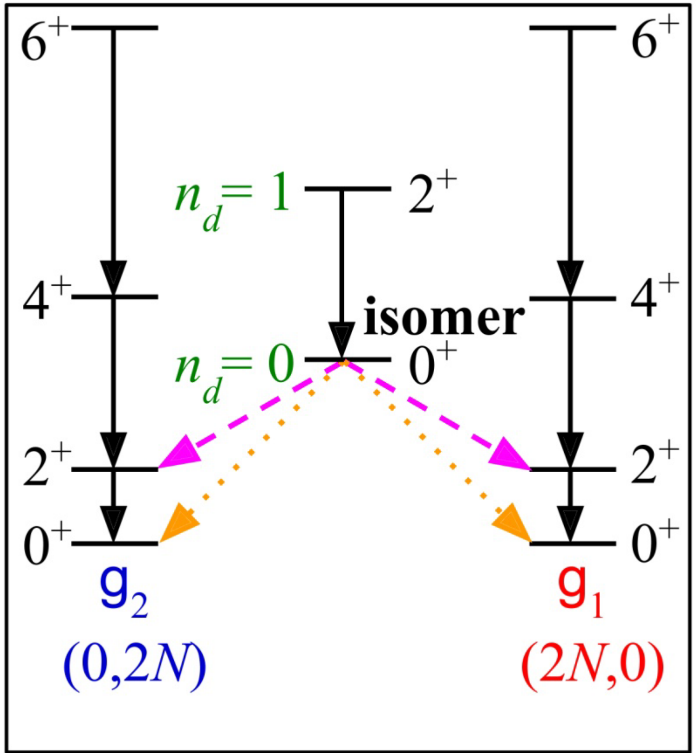


oblate

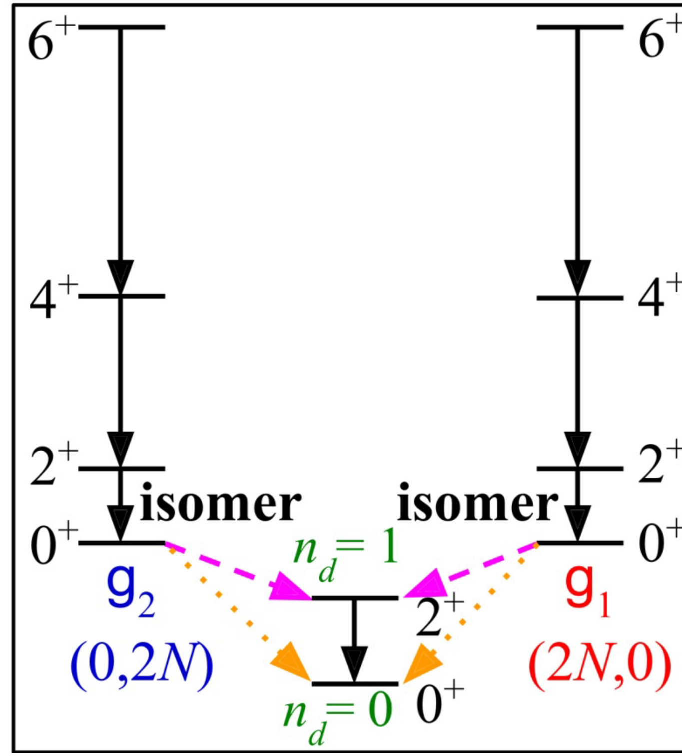
prolate

spherical

The purity of selected sets of states with respect to
SU(3), $\overline{\text{SU(3)}}$ and U(5), in the presence of other mixed states,
are the hallmarks of coexisting SU(3)-PDS, $\overline{\text{SU(3)}}$-PDS and U(5)-PDS



S-P-O coexistence



$T(E2) = e_B(d^\dagger s + s^\dagger \tilde{d})$ $\Delta n_d = \pm 1$

Spherical → deformed E2 rates very weak

Deformed SU(3) & SU(3) DS states

($g_1 \to g_1$, $g_2 \to g_2$) Q_L & B(E2) KNOWN!

Spherical U(5)-DS states (n_d=1 → n_d=0)

Q(n_d=1,L=2) = 0

$B(E2; n_d = 1, L = 2 \to n_d = 0, L = 0) = e_B^2 N$

$T(E0) \propto \hat{n}_d$ diagnal in n_d

No E0 transitions involving these spherical states

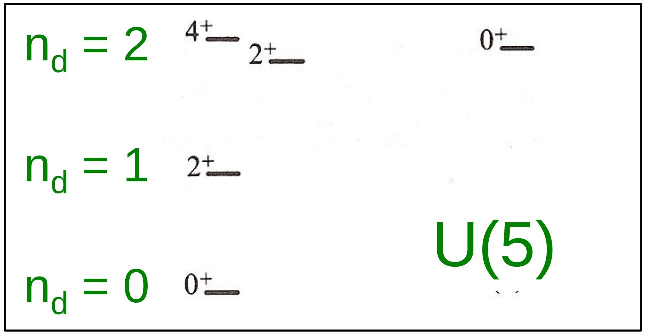
The spherical states exhaust the (n_d=0,1) irreps of U(5)

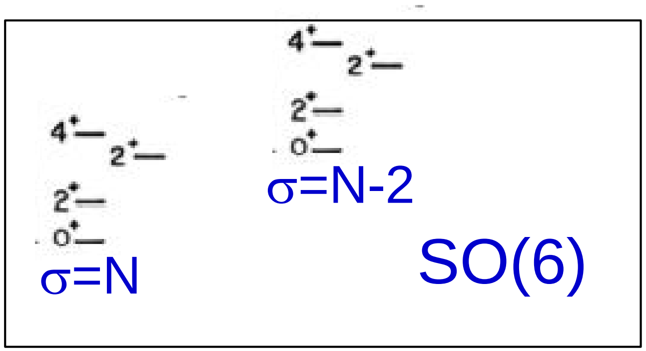
The n_d=2 component in the (L=0,2,4) states of the g_1 and g_2 bands is extremely small

U(5) and SO(6) Dynamical Symmetries

$U(6) \supset U(5) \supset SO(5) \supset SO(3)$ $\quad |[N]\, n_d\, \tau\, n_\Delta\, L\rangle$ Spherical vibrator

$U(6) \supset SO(6) \supset SO(5) \supset SO(3)$ $\quad |[N]\, \sigma\, \tau\, n_\Delta\, L\rangle$ γ-unstable rotor





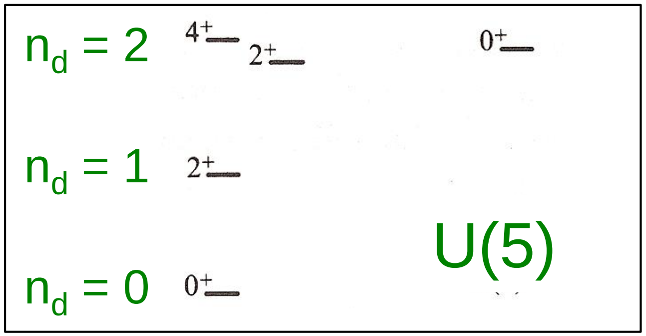
common segment

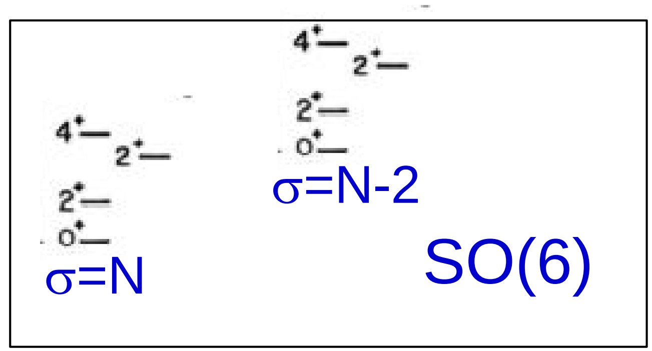
$SO(5) \supset SO(3)$

U(5) and SO(6) Dynamical Symmetries

$U(6) \supset U(5) \supset SO(5) \supset SO(3)$ $\quad |[N]\, n_d\, \tau\, n_\Delta\, L\rangle$ Spherical vibrator

$U(6) \supset SO(6) \supset SO(5) \supset SO(3)$ $\quad |[N]\, \sigma\, \tau\, n_\Delta\, L\rangle$ γ-unstable rotor





common segment

$SO(5) \supset SO(3)$

Spherical and γ-unstable deformed Shape Coexistence

Intrinsic part of C.P. Hamiltonian

$$\begin{cases} \hat{H}|N,\, \sigma = N,\, \tau,\, L\rangle = 0 \\ \hat{H}|N, n_d = \tau = L = 0\rangle = 0 \end{cases}$$

$$\hat{H} = r_2\, R_0^\dagger \hat{n}_d R_0 \qquad R_0^\dagger = d^\dagger \cdot d^\dagger - (s^\dagger)^2$$

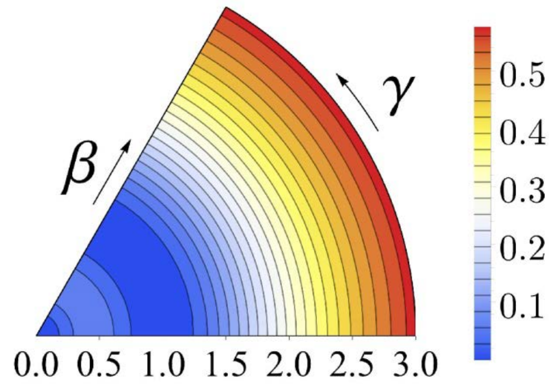
Energy Surface

$$\tilde{E}(\beta) = r_2 \beta^2 (\beta^2 - 1)^2 (1 + \beta^2)^{-3}$$
$$= (1 + \beta^2)^{-3} [\, A\beta^6 + D\beta^4 + F\beta^2\,]$$

- Two degenerate spherical and γ-unstable deformed global minima: β=0 and β=1

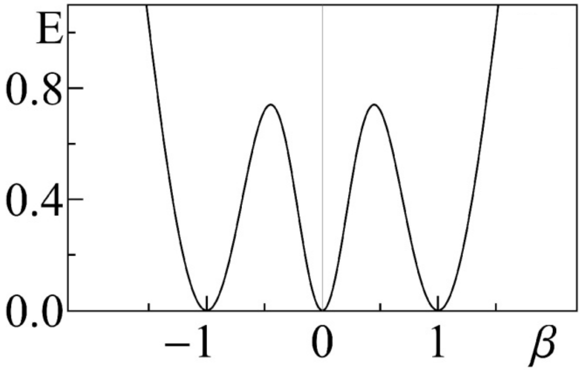
Spherical & γ-unstable deformed

Energy surface independent of γ
SO(5) symmetry



$E(\beta,\gamma)$

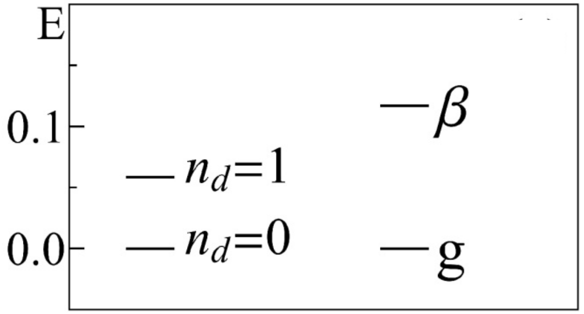
a barrier separates the spherical and γ-unstable deformed minima



$E(\beta,\gamma=0)$

Normal modes:

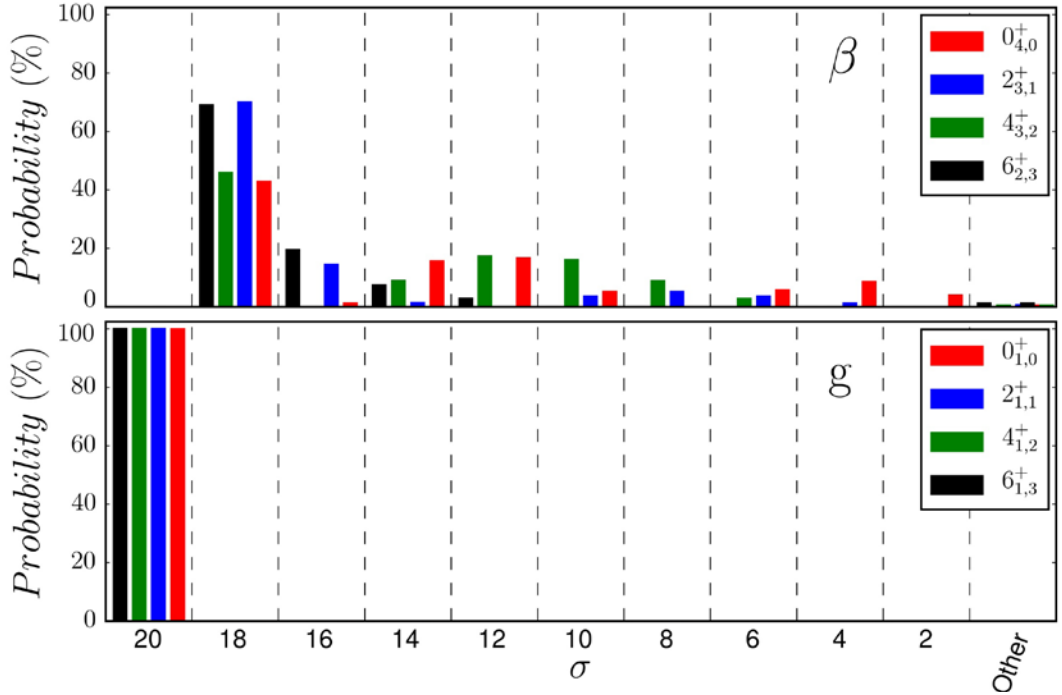
$$\epsilon_\beta = 2r_2 N^2$$
$$\epsilon = r_2 N^2$$



bandhead spectrum

Complete Hamiltonian

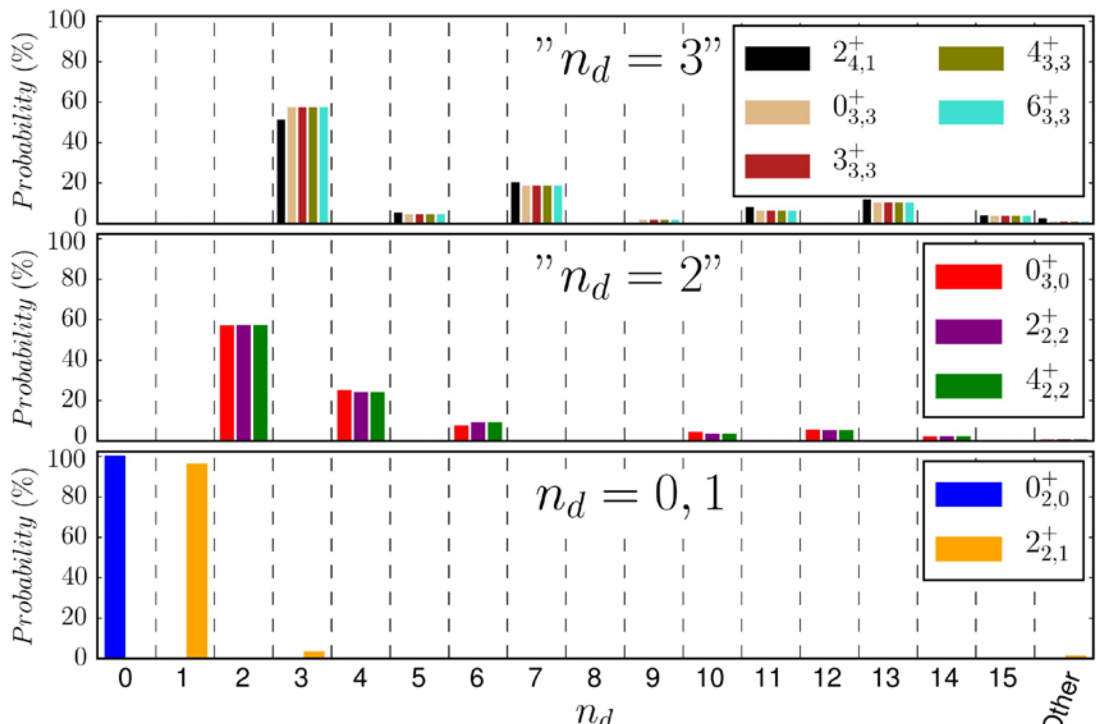
$$\hat{H}' = r_2\, R_0^\dagger \hat{n}_d R_0 + \rho_5\, \hat{C}_2[\mathrm{SO}(5)] + \rho_3\, \hat{C}_2[\mathrm{SO}(3)]$$



SO(6) decompostion

- g-band: pure SO(6)-DS (σ=N)
- Excited β bands: mixed

$\Rightarrow$ SO(6)-PDS

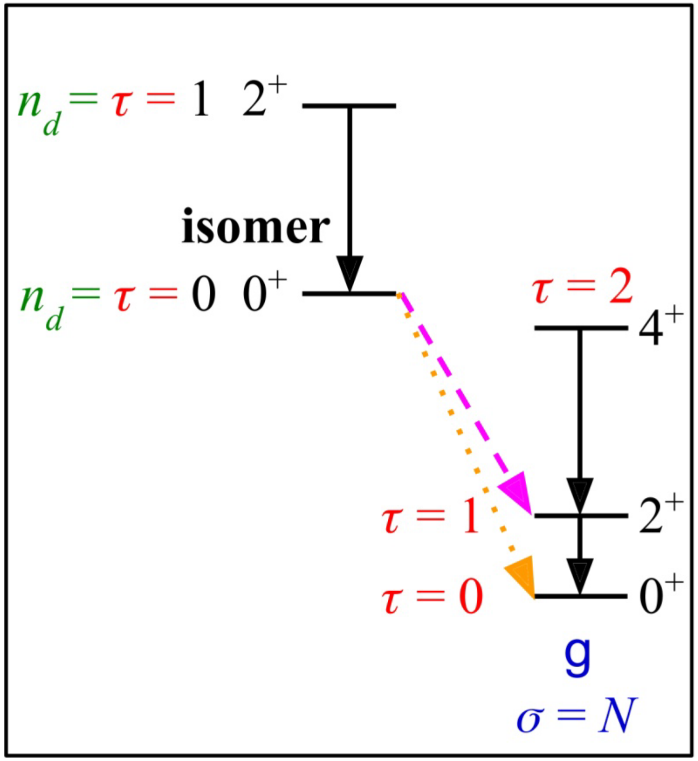


U(5) decompostion

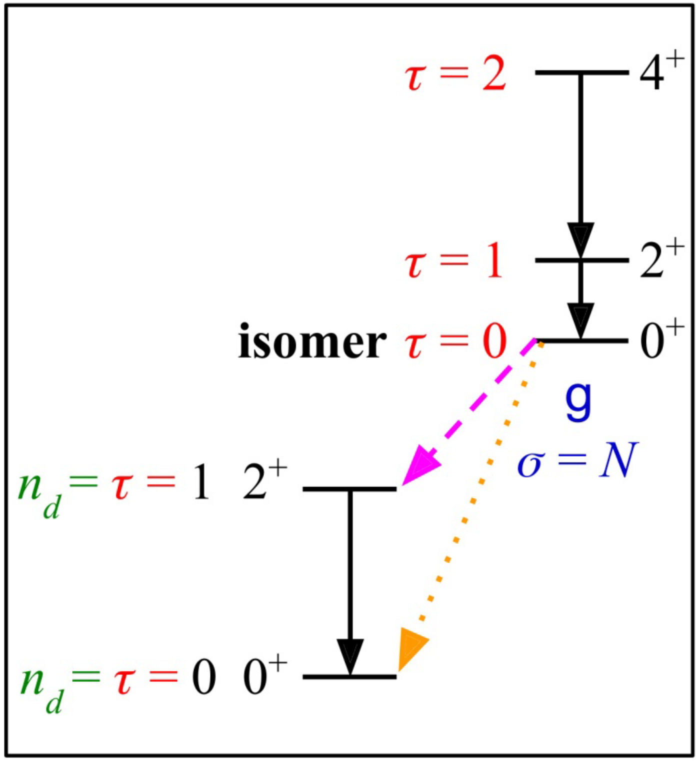
- Spherical states: pure U(5)-DS with (n_d=τ=L=0) & (n_d=τ=1,L=2)
- Higher spherical states: pronounced & coherent mixing

$\Rightarrow$ U(5)-PDS

Coexisting U(5)-PDS & SO(6)-PDS



Spherical and γ-unstable deformed coexistence



$T(E2) = e_B(d^\dagger s + s^\dagger \tilde{d})$ $\Delta\sigma = 0$, Δn_d & $\Delta\tau = \pm 1$

$T(E0) \propto \hat{n}_d$ diagnal in n_d

deformed → spherical E2 rates very weak

g-band exhausts the σ=N irrep of SO(6)

No E0 transitions involving these spherical states

Deformed SO(6)-DS states (g → g)

$Q(\sigma=N,\tau) = 0$

$$B(E2; g; \tau+1, L' = 2\tau+2 \rightarrow g; \tau, L = 2\tau)$$

$$= e_B^2 \frac{\tau+1}{2\tau+5}(N-\tau)(N+\tau+4)$$

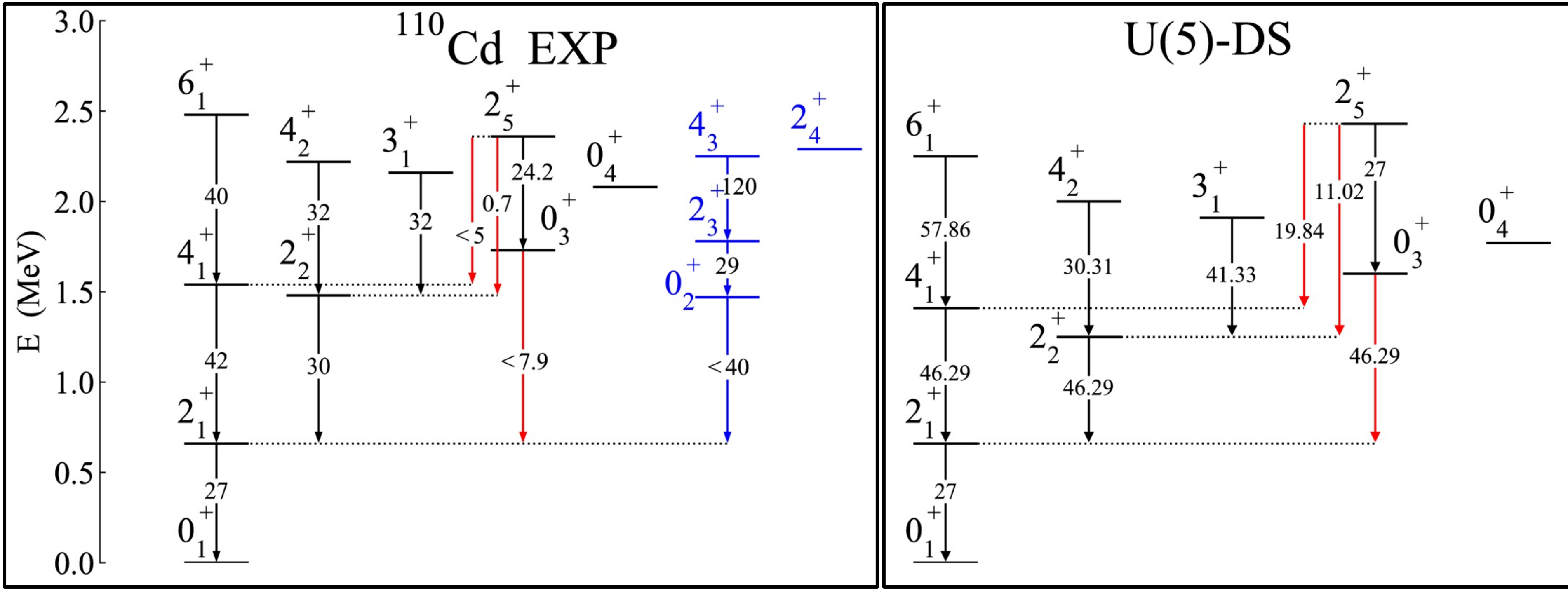
Spherical U(5)-DS states (n_d=1 → n_d=0)

$Q(n_d=1, L=2) = 0$

$$B(E2; n_d = 1, L = 2 \rightarrow n_d = 0, L = 0) = e_B^2 N$$

KNOWN !

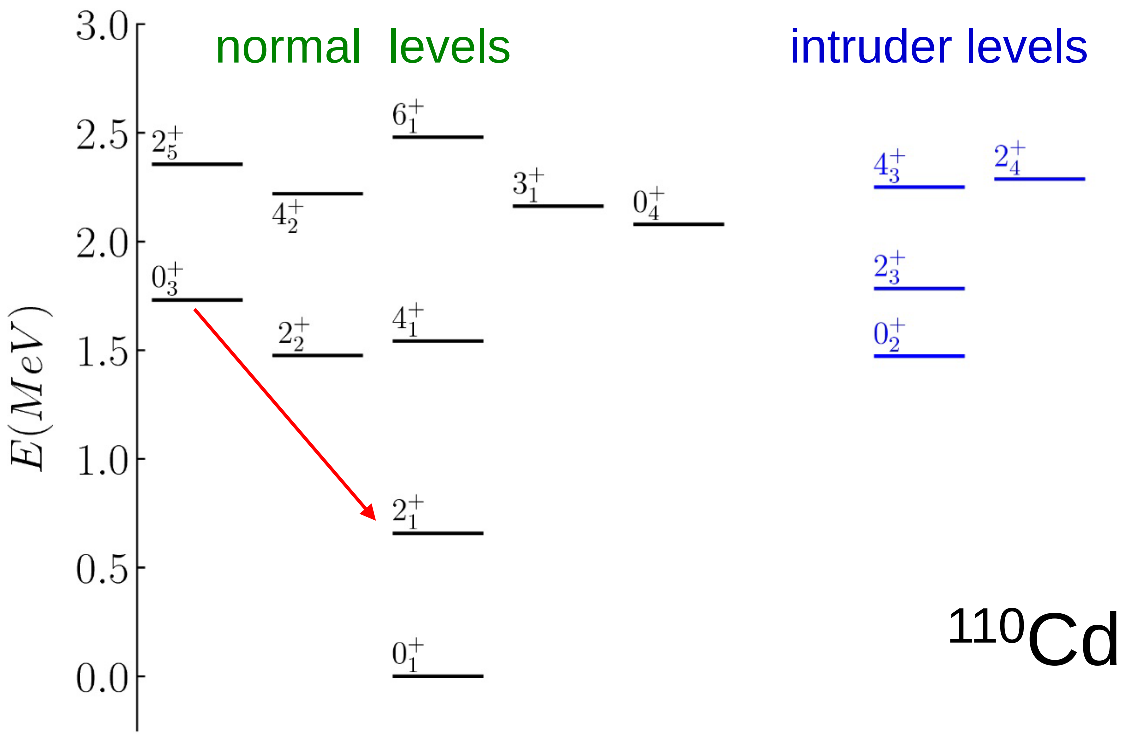
The Cd problem



Most states good spherical vibrator $B(E2; 2_1 \rightarrow 0_1) = 27.0\ (8)$ W.u.

BUT: [W.u.]	EXP	U(5)	
$B(E2; 0_3 \rightarrow 2_1)$	< 7.9	46.29 .	$(n_d = 2 \rightarrow n_d = 1)$
$B(E2; 2_5 \rightarrow 4_1)$	< 5	19.84	$(n_d = 3 \rightarrow n_d = 2)$
$B(E2; 2_5 \rightarrow 2_2)$	$< 0.7^{+0.5}_{-0.6}$	11.02	
$B(E2; 0_4 \rightarrow 2_2)$	small BR	57.86	

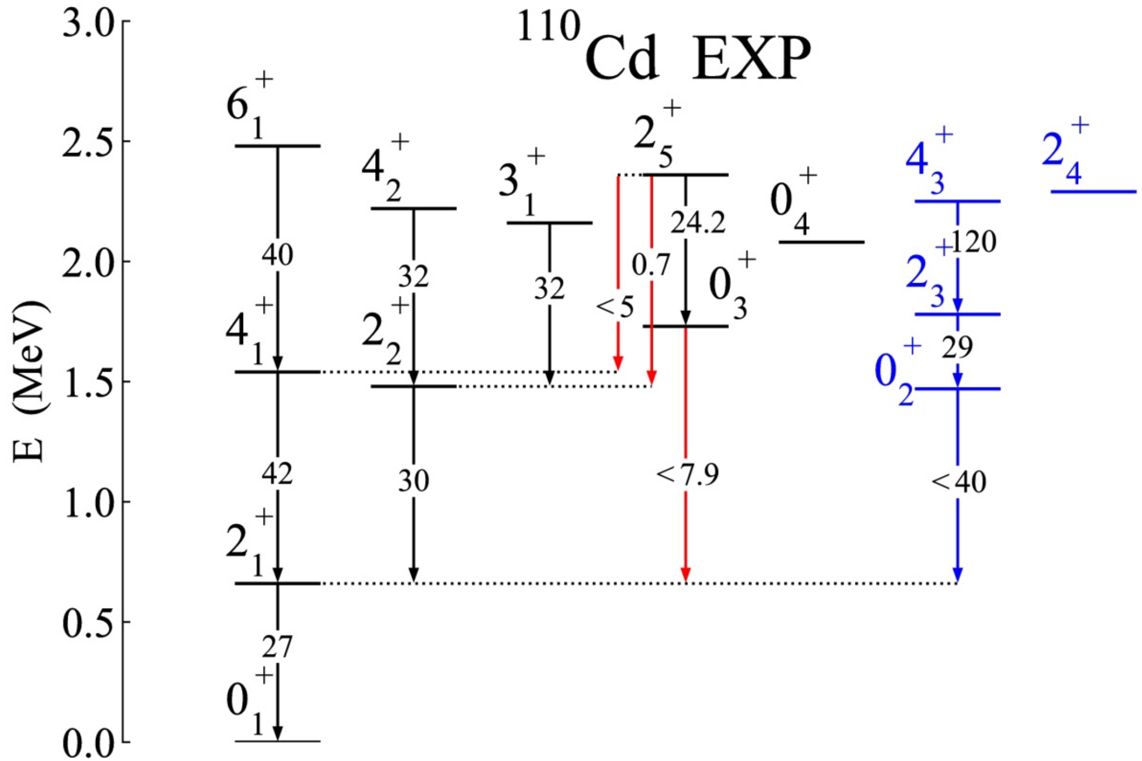
Garret *et al.* PRC (2012)



- Attempted solution: normal-intruder mixing

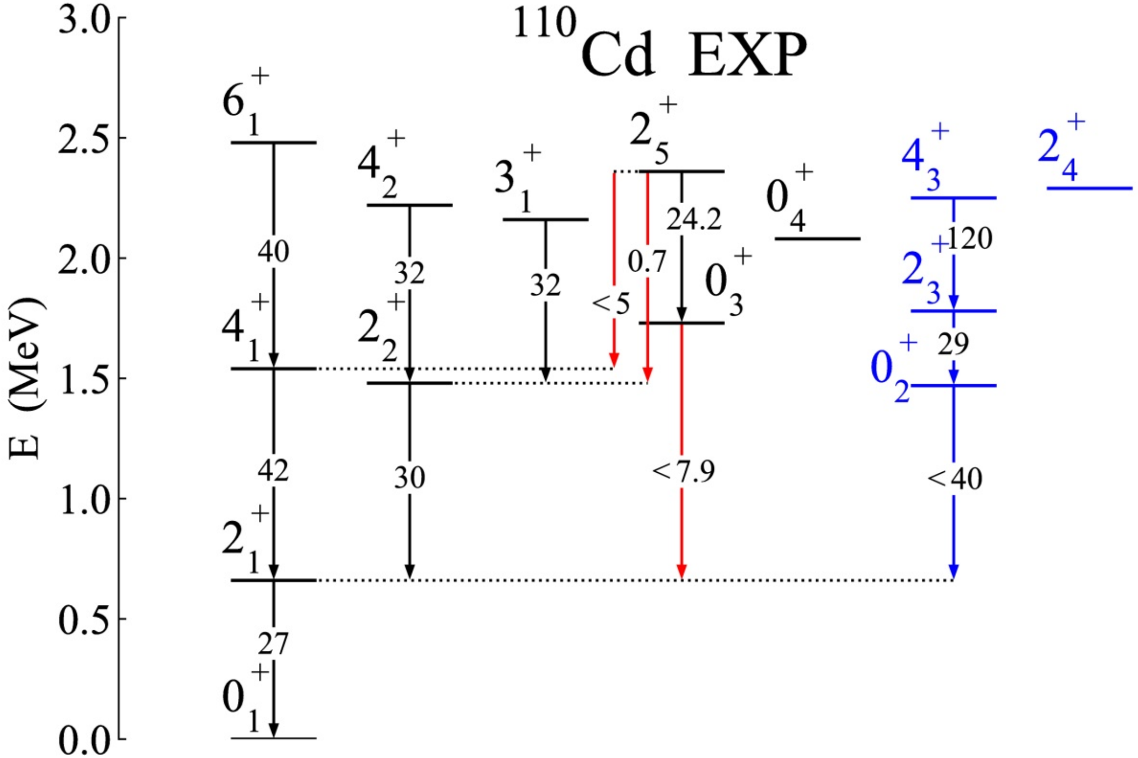
Requires **strong** (maximal) **mixing** to reproduce the observed pattern

$B(E2; 0_3 \rightarrow 2_1) < 7.9$ W.u. $(n_d = 2 \rightarrow n_d = 1)$



Strong normal-intruder mixing is unsatisfactory

- It results in discrepancy in the decay pattern of n_d=3 states (enhanced intruder-normal E2 decays in contrast to exp)
- Unmixed IBM calculations agree with data for (n_d=3, L=6,4,3) yrast states, but seriously disagree for non-yrast states (n_d=2,L=0) and (n_d=3,L=0,2)
- E(intruder) rises away from neutron midshell (^{114}Cd) $\Rightarrow$ smaller mixing. In contrast, experimentally (Garrett, Batchelder PRC 2008, 2010, 2012, 2014) the discrepancy in two- & three-phonon states persists for ^{A}Cd (A=110-126)



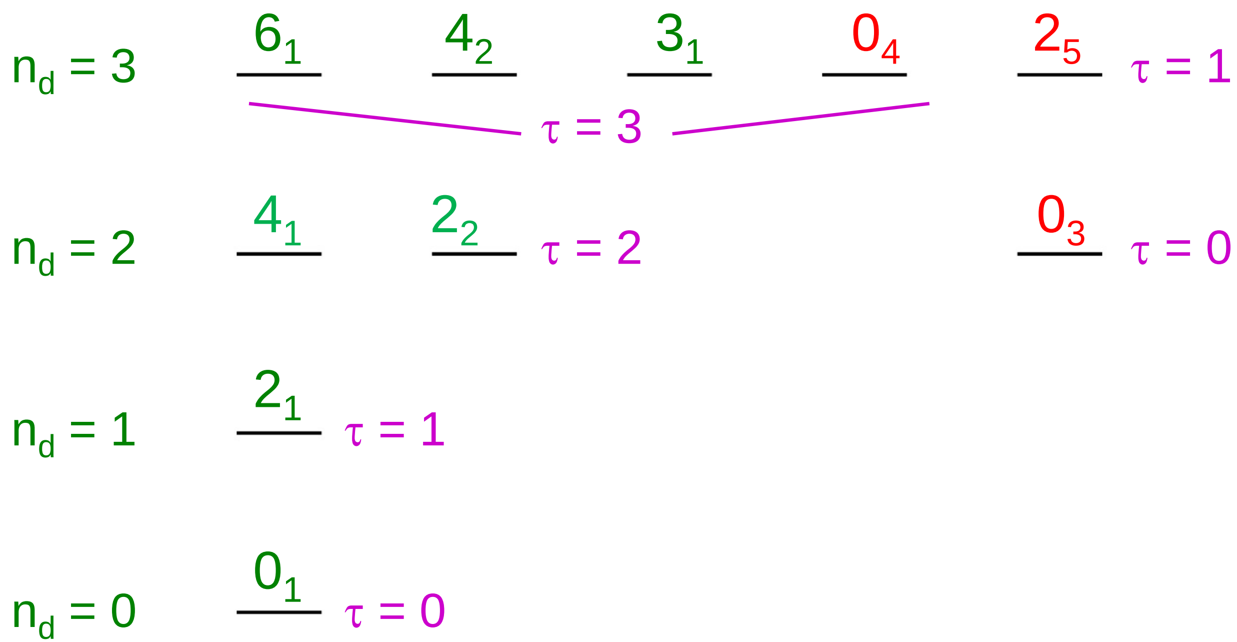
Strong normal-intruder mixing refuted

- Claims: “Breakdown of vibrational motion in Cd isotopes” (Garrett PRC 2008)
  “Need for a paradigm change”
  “Serious questioning on the validity of the multi-phonon interpretation”

- Alternatives: γ-soft rotor (Garrett, PRC 2012), “Tidal wave” (Frauendorf 2011), EFT (Papenbrock PRC 2015)

- This talk: an approach to the problem based on partial dynamical symmetry

$U(6) \supset U(5) \supset SO(5) \supset SO(3) \qquad |[N]\, n_d\, \tau\, n_\Delta\, L\rangle$



good U(5) — Class A: $n_d = \tau = 0,1,2,3$ ($n_\Delta = 0$) — $0_1(0)$, $2_1(658)$, $4_1(1542)$, $2_2(1476)$, $6_1(2480)$, $4_2(2220)$, $3_1(2163)$

broken U(5):

- Class B: $n_d = \tau+2 = 2,3$ ($n_\Delta = 0$) — $0_3(1731)$, $2_5(2356)$
- Class C: $n_d = \tau = 3$ ($n_\Delta = 1$) — $0_4(2079)$

- Some states with good U(5) symmetry
- Some states break U(5) symmetry

$\Longrightarrow$ Partial Dynamical Symmetry

U(5) PDS

U(6) ⊃ U(5) ⊃ SO(5) ⊃ SO(3) $\quad |[N]\, n_d\, \tau\, n_\Delta\, L\rangle$

$$G_0^\dagger = [(d^\dagger d^\dagger)^{(2)} d^\dagger]^{(0)}$$

$$K_0^\dagger = s^\dagger (d^\dagger d^\dagger)^{(0)}$$

$$G_0|[N], n_d = \tau, \tau, n_\Delta = 0, L\rangle = 0 \qquad L = \tau, \tau+1, \ldots, 2\tau-2, 2\tau$$ (Talmi 2004)

$$K_0|[N], n_d = \tau, \tau, n_\Delta, L\rangle = 0$$

$$\boxed{\hat{V}_0 = r_0\, G_0^\dagger G_0 + e_0 \left(G_0^\dagger K_0 + K_0^\dagger G_0 \right)}$$

$$\hat{H}_{PDS} = \hat{H}_{DS} + \hat{V}_0$$ U(5)-PDS Hamiltonian

Class A: Solvable $|N, n_d = \tau, \tau, n_\Delta = 0, L\rangle$ $0_1(0)$, $2_1(658)$, $4_1(1542)$, $2_2(1476)$ $6_1(2480)$, $4_2(2220)$, $3_1(2163)$

Class B:
Class C: } Mixed $0_3(1731)$, $2_5(2356)$ $0_4(2079)$

Normal and intruder levels

$$\hat{H} = \hat{H}^{(N)}_{\text{normal}} + \hat{H}^{(N+2)}_{\text{intruder}} + \hat{V}_{\text{mix}}$$

IBM with configuration mixing (CM)
(Duval, Barrett, Van Isacker, Garcia Ramos,…)

$$\hat{H} = \hat{H}^{(N)}_{\text{normal}} + \hat{H}^{(N+2)}_{\text{intruder}} + \hat{V}_{\text{mix}}$$

$$\hat{H}^{(N)}_{\text{normal}} = \hat{H}_{\text{PDS}}$$

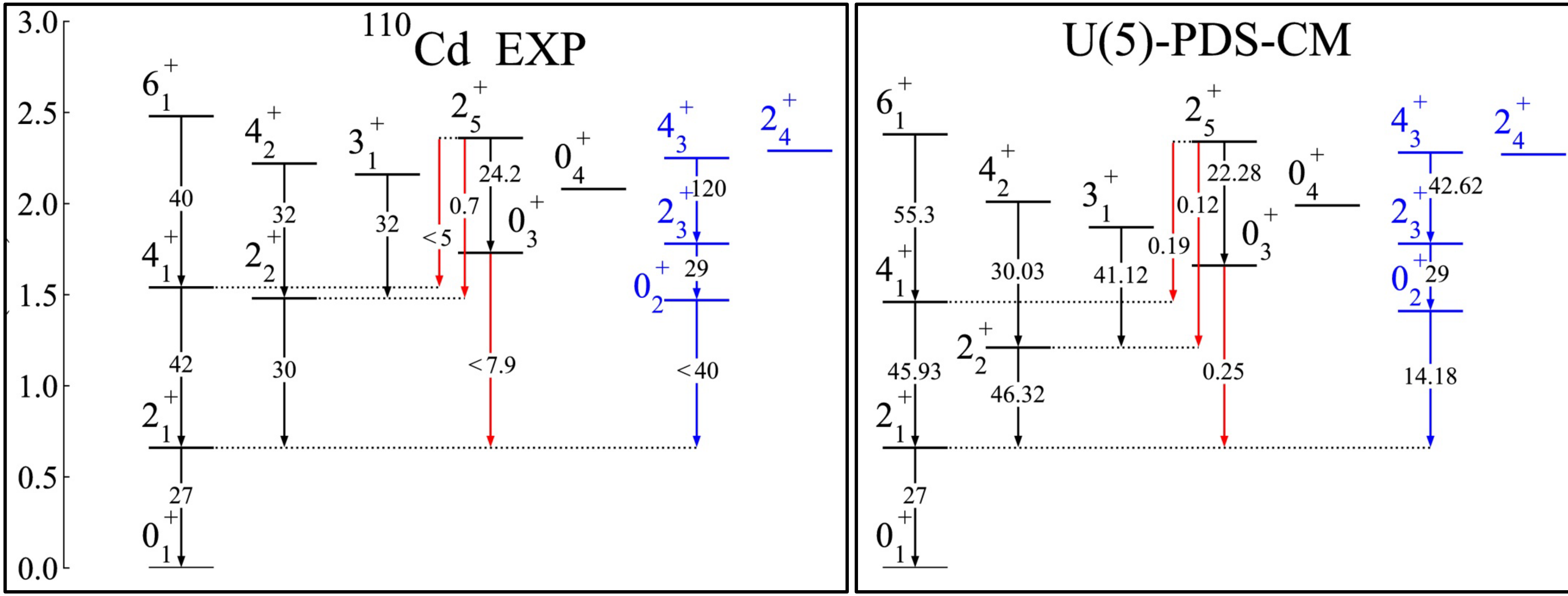
$$\hat{H}^{(N+2)}_{\text{intruder}} = \kappa \hat{Q} \cdot \hat{Q} + \Delta$$

$$\hat{Q} = d^\dagger s + s^\dagger \tilde{d}$$

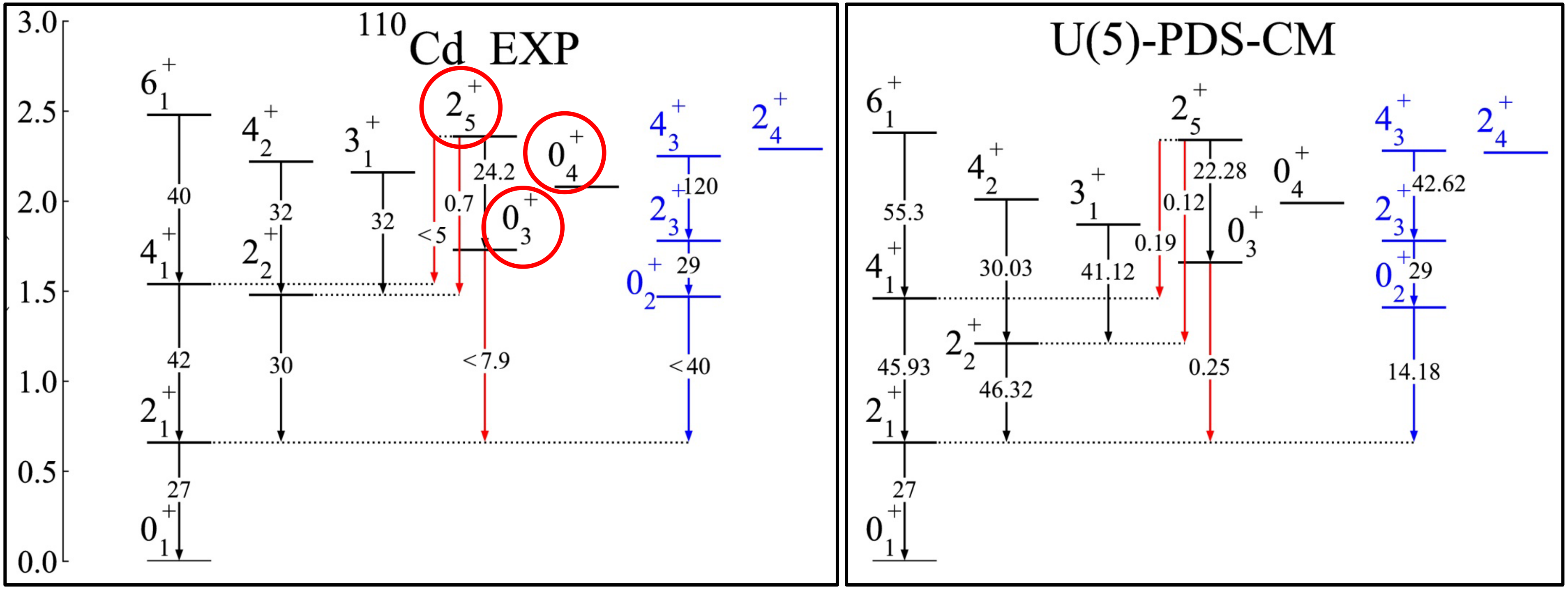
$$\hat{V}_{\text{mix}} = \alpha \left[(d^\dagger d^\dagger)^{(0)} + (s^\dagger)^2 + \text{H.c.} \right]$$

$$\hat{T}(E2) = e_B^{(N)}\, \hat{Q}^{(N)} + e_B^{(N+2)}\, \hat{Q}^{(N+2)}$$

Normal and intruder levels in ^{110}Cd



Normal and intruder levels in ^{110}Cd



Majority of normal states are pure wrt U(5) (> 97%) with weak normal-intruder mixing

0_3(1731): (0.9% $n_d = 2$) , (94% $n_d = 3$) , (5.1% intruder)

0_4(2079): (79.8% $n_d = 2$) , (2% $n_d = 3$), (18% intruder)

2_5(2356): (1.2% $n_d = 3$) , (95.8% $n_d = 4$) , (2.9% intruder)

L_i	L_f	EXP	U(5)-DS	U(5)-PDS-CM
2_1^+	0_1^+	27.0 (8)	27.00	27.00
4_1^+	2_1^+	42 (9)	46.29	45.93
2_2^+	2_1^+	30 (5)	46.29	46.32
	0_1^+	1.35 (20); 0.68 (14)^a	0.00	0.00
0_3^+	2_2^+	$< 1680^a$	0.00	55.95
	2_1^+	$< 7.9^a$	46.29	0.25
6_1^+	4_1^+	40 (30); 62 (18)^a	57.86	55.30
	4_2^+	$< 5^a$	0.00	0.00
	$4_{3;i}^+$	14 (10); 36 (11)^a		2.39
4_2^+	4_1^+	12_{-6}^{+4}	27.55	27.45
	2_2^+	32_{-14}^{+10}	30.31	30.03
	2_1^+	$0.20_{-0.09}^{+0.06}$	0.00	0.00
	$2_{3;i}^+$	$< 0.5^a$		0.005
3_1^+	4_1^+	$5.9_{-4.6}^{+1.8}$	16.53	16.48
	2_2^+	32_{-24}^{+8}	41.33	41.12
	2_1^+	$1.1_{-0.8}^{+0.3}$; 0.85 (25)^a	0.00	0.00
	$2_{3;i}^+$	$< 5^a$		0.012
0_4^+	2_2^+	[$< 0.65^a$]	57.86	1.24
	2_1^+	[0.010^a]	0.00	31.76
	$2_{3;i}^+$	[100^a]		16.32
2_5^+	0_3^+	24.2 (22)^a	27.00	22.28
	4_1^+	$<5^a$	19.84	0.19
	2_2^+	$^a0.7_{-0.6}^{+0.5}$	11.02	0.12
	2_1^+	$2.8_{-1.0}^{+0.6}$	0.00	0.00
	$2_{3;i}^+$	$< 5^a$		0.002
	$0_{2;i}^+$	$< 1.9^a$		0.20

Normal and intruder levels in ^{110}Cd

[W.u.]	EXP	U(5)-PDS-CM
$B(E2; 0_3 \to 2_1)$	< 7.9	0.25
$B(E2; 2_5 \to 4_1)$	< 5	0.19
$B(E2; 2_5 \to 2_2)$	$< 0.7^{+0.5}_{-0.6}$	0.12

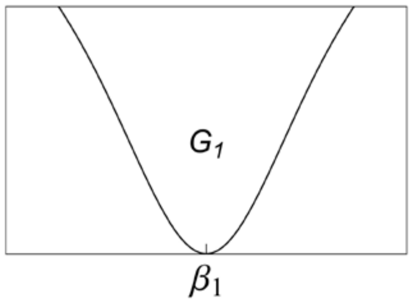
L_i	L_f	EXP	U(5)-PDS-CM
$0_{2;i}^+$	2_1^+	$< 40^a$	14.18
$2_{3;i}^+$	$0_{2;i}^+$	29 (5)^a	29.00
	0_1^+	$0.31_{-0.12}^{+0.08}$	0.08
	2_1^+	$0.7_{-0.4}^{+0.3}$	0.00
	2_2^+	$< 8^a$	0.96
$2_{4;i}^+$	2_1^+	$0.019_{-0.019}^{+0.020}$	0.10
$4_{3;i}^+$	2_1^+	$0.22_{-0.19}^{+0.09}$	0.49
	2_2^+	$2.2_{-2.2}^{+1.4}$	0.00
	$2_{3;i}^+$	120_{-110}^{+50}	42.62
	4_1^+	$2.6_{-2.6}^{+1.6}$	0.00

PDS and coexisting normal and intruder states

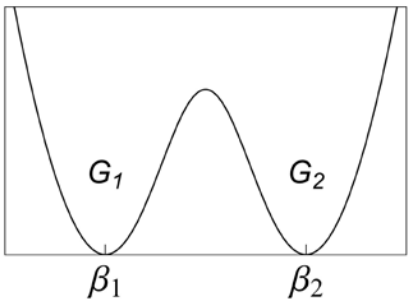
- Vibrational structure of ^{110}Cd by means of **U(5) PDS**
- The PDS Hamiltonian **retains** good U(5) symmetry for **yrast** states, but **breaks** it in selected **non-yrast** states
- The **mixing** with the **intruder** levels is **weak**, and affects mainly the broken U(5)-DS states
- Most low-lying normal levels **maintain** the **vibrational character**.
  Only particular states exhibit a departure from this behavior,
  in line with the empirical data
- Calculations are underway (Gavrielov, Garcia-Ramos, Van Isacker, A.L.) to see if this approach can be implemented in other neutron-rich Cd isotopes

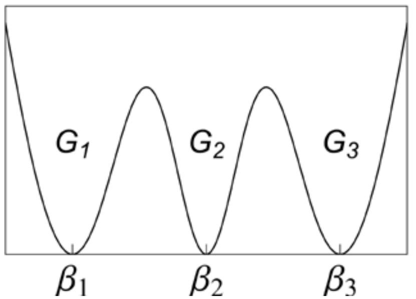
Concluding Remarks

Single DS
or PDS



Multiple PDS





- A symmetry-based approach to shape coexistence
  Ingredients: spectrum generating algebra with several DS chains
  geometry: coherent states
  intrinsic-collective resolution of the Hamiltonian

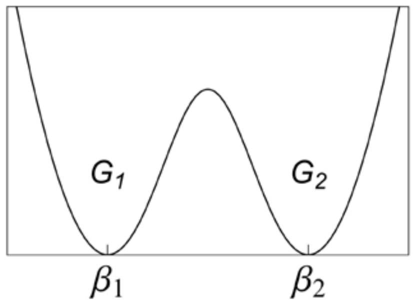
- A single number-conserving rotational invariant H which conserves the dynamical symmetry for selected bands
  Multiple Partial Dynamical Symmetries relevant for shape-coexistence

U(5) and SU(3) PDS	spherical-prolate
SU(3) and $\overline{SU(3)}$ PDS	prolate-oblate
U(5), SU(3) and $\overline{SU(3)}$ PDS	spherical-prolate-oblate
U5) and SO(6) PDS	spherical - γ-unstable deformed

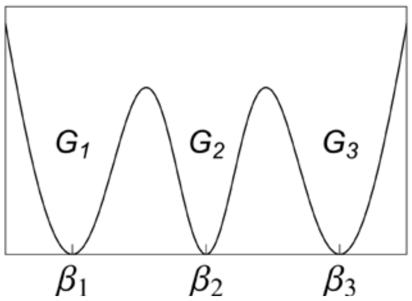
- Closed expressions for quadrupole moments and B(E2) values;
  selection rules for E2 & E0 transitions and isomeric states

Concluding Remarks

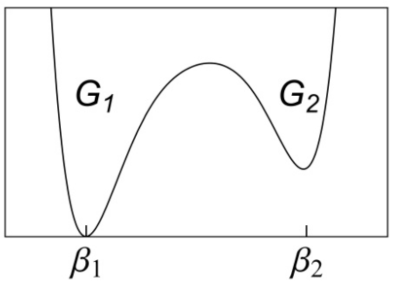
G_1 -PDS
G_2 -PDS



G_1 -PDS
G_2 -PDS
G_3 -PDS



- Structure away from the critical point, can be studied by adding the Casimir operator of a particular DS chain



- PDS: solvable bands are unmixed.
  Band mixing can be incorporated by including in H kinetic terms which do not affect $E(\beta,\gamma)$ but, if strong, may destroy the PDS

- Coexisting normal and intruder states in nuclei can exibit PDS

- Study of shape-coexistence and exotic structure in nuclei provides a fertile ground for exploring the role of competing and persisting symmetries and for the development of generalized notions of symmetries

Thank you