Mass, Asymmetry, and Isospin Dependence of Short-range Correlations

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- **Is there a comprehensive picture of nuclear SRC? (mass (A), isospin (pp/nn/pn), asymmetry (N − Z) dependence of SRC)?**
- **How to forge links between nuclear-structure theory (models) and observables sensitive to nuclear SRC?**

STRATEGY (and OUTLINE of this presentation)

1. Low-order correlation operator approximation (LCA): transition matrix elements between short-range correlated wave functions
2. Compute aggegrated effect of SRC for any $A(N,Z)(A \geq 4)$ (a_2 data from $A(e,e')$; magnitude of EMC effect)
3. Apply LCA to the computation of nuclear momentum distributions and find the driving physical mechanisms (Compare results to those of “ab-initio” approaches)
4. Compute isospin and asymmetry dependence of SRC ($A(e,e'pp)$, $A(e,e'pn)$, $A(e,e'p)$ data in selected kinematics)
5. Develop a proper reaction theory for SRC-driven two-nucleon knockout
   - proper factorization properties of cross sections (data for c.m. distributions of SRC pairs)
   - FSI corrections (elastic and charge-exchange)
   - $A(e,e'NN)$ for $N \gtrsim Z$ and $p(A,pNN(A-2))$ for $N > Z$

Universal physics from short-distance correlations

arXiv:1807.02468

1. Vicszek model for understanding emergent collective motion from local interactions: neighboring particles tend to align their velocities
2. Competition between an aligning force and a stochastic force
3. Two different energy (“time”) scales emerge:
   - Particles in high-density zones tend to align their velocities (liquid phase, SRC nucleons)
   - Particles in low-density zones move in a disorderly fashion (gas phase, IPM nucleons)

Nuclear transition matrix elements with SRC (I)

- Shift complexity from wave functions to operators

$$\mid \Psi\rangle = \frac{1}{\sqrt{\mathcal{N}}}\widehat{\mathcal{G}} \mid \Phi\rangle \qquad \text{with,} \qquad \mathcal{N} \equiv \langle \Phi \mid \widehat{\mathcal{G}}^{\dagger}\widehat{\mathcal{G}} \mid \Phi\rangle$$

$\mid \Phi\rangle$ is an IPM single Slater determinant

- Nuclear SRC correlation operator $\widehat{\mathcal{G}}$

$$\widehat{\mathcal{G}} \approx \widehat{\mathcal{S}} \left(\prod_{i<j=1}^{A} \left[1 + \hat{l}(i,j) \right] \right) ,$$

- Major source of correlations: central (Jastrow), tensor and spin-isospin (universal scaling functions in r_{ij})

$$\hat{l}(i,j) = -g_c(r_{ij}) + f_{t\tau}(r_{ij})\widehat{S}_{ij}\vec{\tau}_i \cdot \vec{\tau}_j \, . + f_{\sigma\tau}(r_{ij})\vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j$$

Nuclear transition matrix elements with SRC (II)

- Turn expectation values between correlated states Ψ into expectation values between uncorrelated states Φ

$$\langle \Psi \mid \widehat{\Omega} \mid \Psi \rangle = \frac{1}{\mathcal{N}} \langle \Phi \mid \widehat{\Omega}^{\text{eff}} \mid \Phi \rangle$$

- "Conservation Law of Misery": $\widehat{\Omega}^{\text{eff}}$ is an A-body operator

$$\widehat{\Omega}^{\text{eff}} = \widehat{\mathcal{G}}^{\dagger}\, \widehat{\Omega}\, \widehat{\mathcal{G}} = \left(\sum_{i<j=1}^{A} \left[1 - \hat{l}(i,j) \right] \right)^{\dagger} \widehat{\Omega} \left(\sum_{k<l=1}^{A} \left[1 - \hat{l}(k,l) \right] \right)$$

- Truncation procedure for short-distance phenomena

$$\text{SCALING:} \quad \Psi^{\dagger}(\vec{R} - \frac{\vec{r}}{2}) \Psi(\vec{R} + \frac{\vec{r}}{2}) \approx \sum_{n} c_n(\vec{r}) O_n(\vec{R}) \quad (\mid \vec{r} \mid \approx 0)$$

Low-order correlation operator approximation (LCA)

- LCA: N-body operators receive SRC-induced $(N+1)$-body corrections

Aggregated effect of SRC for any $A(N,Z)$ in LCA

- LCA expansion of the norm $\mathcal{N} \equiv \langle \Phi \mid \widehat{\mathcal{G}}^{\dagger}\widehat{\mathcal{G}} \mid \Phi \rangle$

$$\mathcal{N} = 1 + \frac{2}{A} \sum_{\alpha<\beta} {}_{\text{nas}}\langle \alpha\beta \mid \hat{l}^{\dagger}(1,2) + \hat{l}^{\dagger}(1,2)\hat{l}(1,2) + \hat{l}(1,2) \mid \alpha\beta \rangle_{\text{nas}}.$$

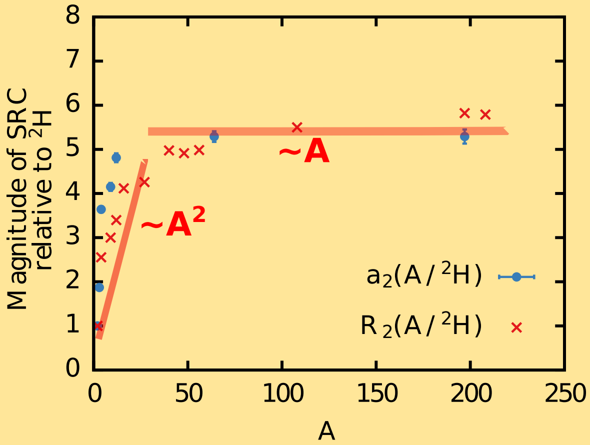
1. $\mid \alpha\beta\rangle_{\text{nas}}$: anti-symmetrized two-nucleon IPM-state
2. $\sum_{\alpha<\beta}$ extends over all IPM states $\mid \alpha \rangle \equiv \mid n_\alpha l_\alpha j_\alpha m_{j_\alpha} t_\alpha \rangle$,

- $(\mathcal{N}-1)$: aggregated effect of SRC in g.s. of $A(N,Z)$
- Aggregated quantitative effect of SRC in A relative to ^{2}H

$$R_2(A/^2\text{H}) = \frac{\mathcal{N}(A)-1}{\mathcal{N}(^2\text{H})-1} = \frac{\text{measure for SRC effect in } A}{\text{measure for SRC effect in } ^2\text{H}}.$$

- Input to the calculations for $R_2(A/^2\text{H})$
  1. HO IPM states with $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$
  2. UNIVERSAL SCALING FUNCTIONS: $[g_c(r), f_{t\tau}(r), f_{\sigma\tau}(r)]$

$a_2(A/^2\mathrm{H})$ from $A(e,e')$ at $x_B \gtrsim 1.5$ and $R_2(A/^2\mathrm{H})$



- $\sim A^2$: local neighborhood gets filled
- $\sim A$: local neighborhood saturated

1. $A \lesssim 40$: strong mass dependence in SRC effect
2. $A \gtrsim 40$: soft mass dependence and SRC effect saturates
3. *$A \gtrsim 40$: aggregated SRC effect per nucleon is about $5\times$ larger than in ^2H*

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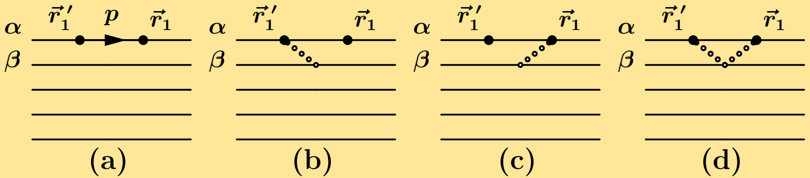
Single-nucleon momentum distribution $n^{[1]}(p)$

- Probability to find a nucleon with momentum p

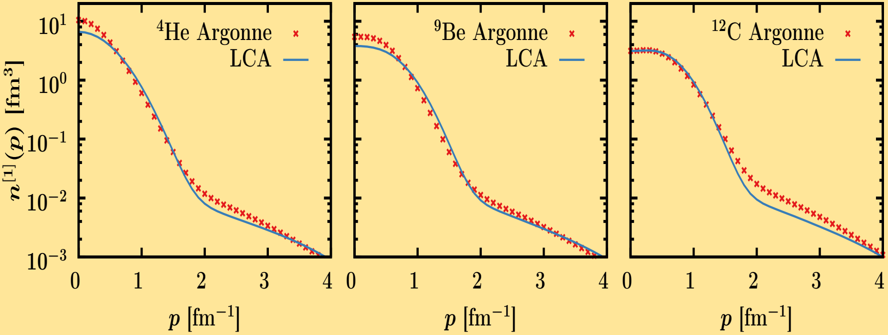
$$n^{[1]}(p) = \int \frac{d^2\Omega_p}{(2\pi)^3} \int d^3\vec{r}_1 \; d^3\vec{r}_1{}' \; d^{3(A-1)}\{\vec{r}_{2-A}\} e^{-i\vec{p}\cdot(\vec{r}_1{}'-\vec{r}_1)} \times \Psi^*(\vec{r}_1, \vec{r}_{2-A}) \Psi(\vec{r}_1{}', \vec{r}_{2-A}).$$

- SRC-induced corrections to IPM $n^{[1]}(p)$ are of two-body type
- Normalization property $\int dp\, p^2 n^{[1]}(p) = 1$ can be preserved



(a): IPM contribution **(b)-(d): SRC contributions in LCA**

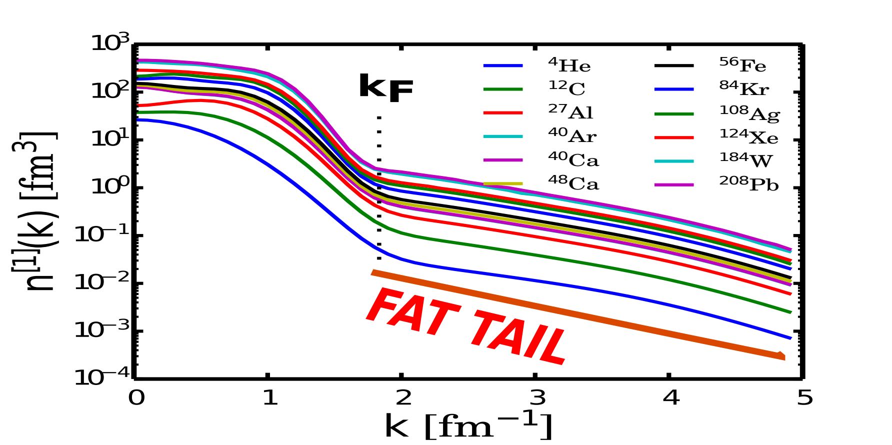
$n^{[1]}(p)$ for $A \leq 12$: LCA (Ghent) vs QMC (Argonne)



QMC: PRC89(2014)024305 LCA: JPG42(2015)055104

1. $p \lesssim p_F = 1.25 \text{ fm}^{-1}$: $n^{[1]}(p)$ is "Gaussian" (IPM PART)
2. $p \gtrsim p_F$: $n^{[1]}(p)$ has fat tail (CORRELATED PART)
3. fat tails of $n^{[1]}(p)$ in QMC and LCA are comparable
4. scale separation between "low" and "high" p

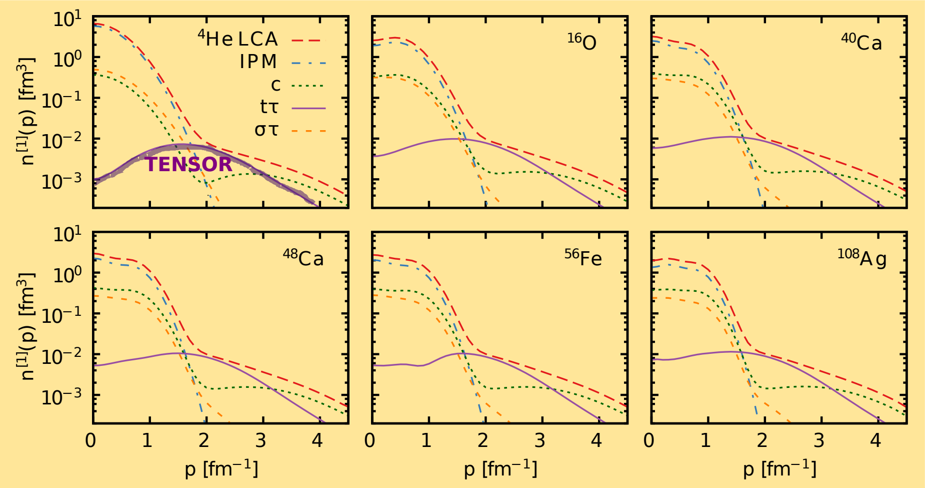
$n^{[1]}(k)$ in LCA: from light to heavy nuclei



LCA: JPG42(2015)055104

1. momentum dependence of fat tail of $n^{[1]}$ is "universal" (universal SRC 2N correlation functions)

Major source of correlated strength in $n^{[1]}(p)$?



1. $1.5 \lesssim p \lesssim 3 \text{ fm}^{-1}$ is dominated by tensor correlations
2. central correlations substantial at $p \gtrsim 3.5 \text{ fm}^{-1}$

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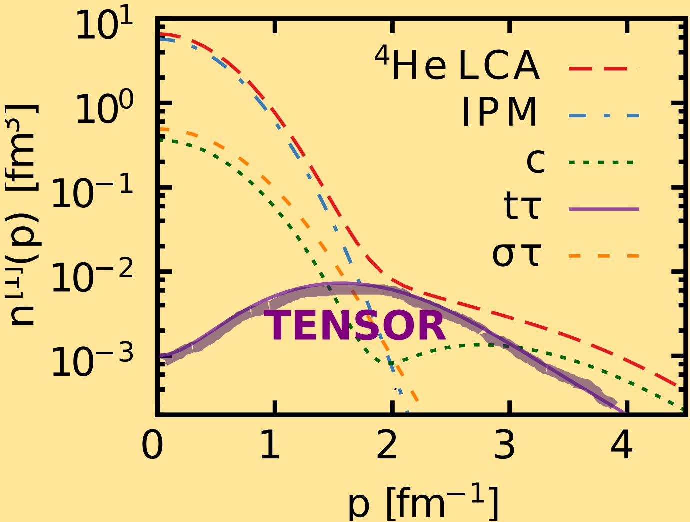
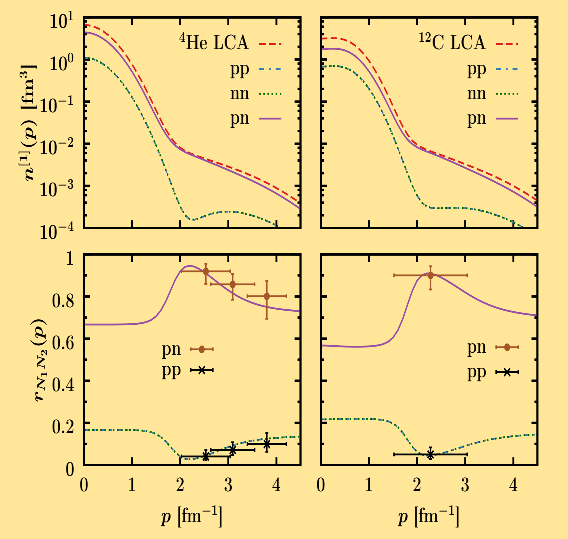


1. $1.5 \lesssim p \lesssim 3$ fm^{-1} is dominated by tensor correlations
2. central correlations substantial at $p \gtrsim 3.5$ fm^{-1}

Isospin dependence of SRC: pp, nn and pn

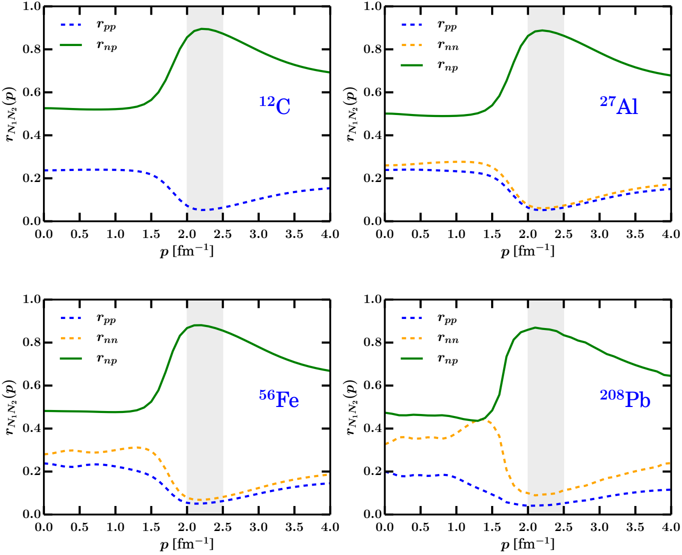
$r_{N_1N_2}(p)$: relative contribution of N_1N_2 pairs to $n^{[1]}(p)$



- Naive IPM:
  $r_{pp} = \frac{Z(Z-1)}{A(A-1)}$,
  $r_{nn} = \frac{N(N-1)}{A(A-1)}$,
  $r_{pn} = \frac{2NZ}{A(A-1)}$.
- Data extracted from $^4\mathrm{He}(e,e'pp)/(e,e'pn)$ (PRL 113, 022501) and $\frac{^{12}\mathrm{C}(p,ppn)}{^{12}\mathrm{C}(p,pp)}$ (Science 320, 1476) assuming that $r_{pp} \approx r_{nn}$
- **Fat tail is dominated by "pn" (momentum dependent)**

Nuclear momentum distribution: Pair composition

$r_{N_1N_2}(p)$: relative contribution of N_1N_2 pairs to n[1](p)



-The pp and pn SRC pair fractions are momentum dependent

-Semi-exclusive $A(e, e'pN)$ measurements mainly probe the low p part of the SRC region:

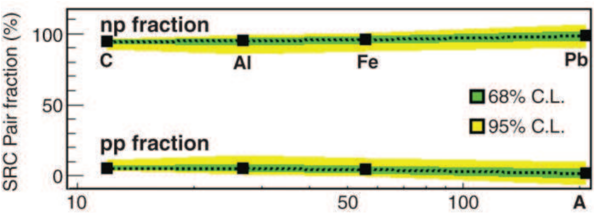
$r_{np} \sim 90\%$ and

$r_{nn} + r_{pp} \sim 10\%$

Science express

Momentum sharing in imbalanced Fermi systems

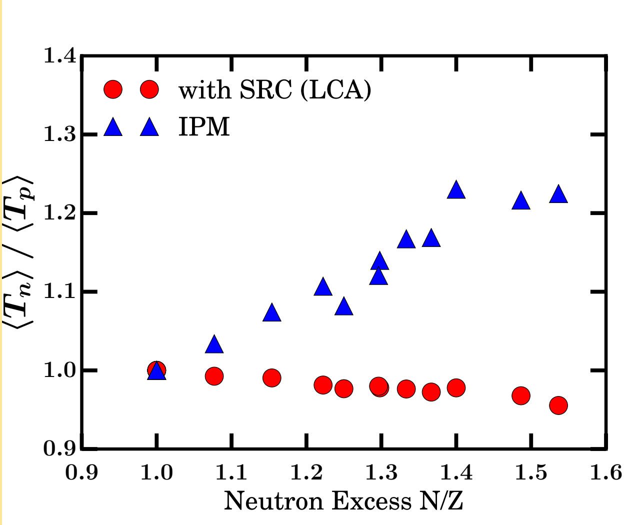
O. Hen,[1*] M. Sargsian,[2] L. B. Weinstein,[3] E. Piasetzky,[1] H. Hakobyan,[4,5] D. W. Higinbotham,[6] M



LCA predicts that ≈90% of correlated pairs is "pn", and ≈5% is "pp" (UNIVERSAL: A independent)

SRC induce inversion of kinetic energy sharing

Ratio $\langle T_n = p_n^2/(2M_n) \rangle \, / \, \langle T_p = p_p^2/(2M_p) \rangle$ from computed $n^{[1]}(p)$



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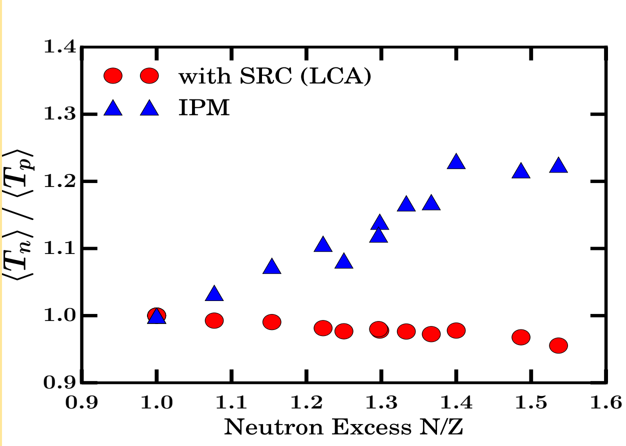
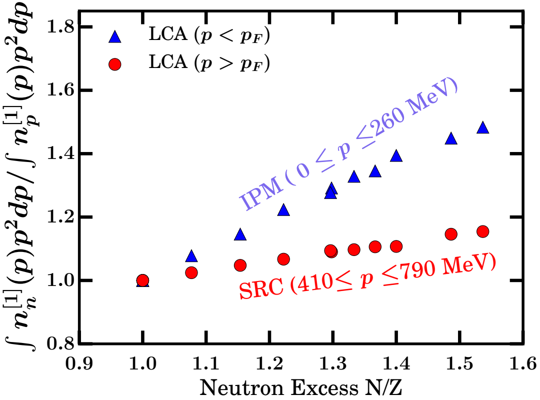


After correcting for SRC in LCA, minority component has largest kinetic energy (strongly depends on N/Z)

Asymmetry dependence of the SRC?

Asymmetry dependence of the relative weight of neutrons and protons in the $n^{[1]}(p)$



Can be "measured": ratio $\left(A(e,e'n)\sigma_{en}^{-1}\right) / \left(A(e,e'p)\sigma_{ep}^{-1}\right)$ at "low" and "high" missing momenta (Data mining @JLAB)

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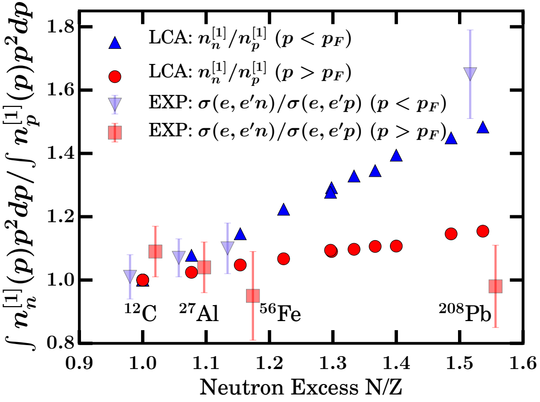


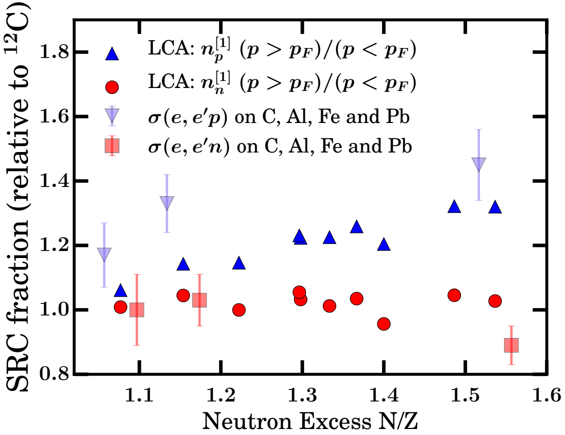
Relative weight of the protons and neutrons is very different in "IPM" and "SRC" regions of the nuclear momentum distributions!

(Expt.: Data Mining Collaboration @JLAB)

Asymmetry dependence of the SRC?

Superratio of A=Al, Fe, Pb relative to C

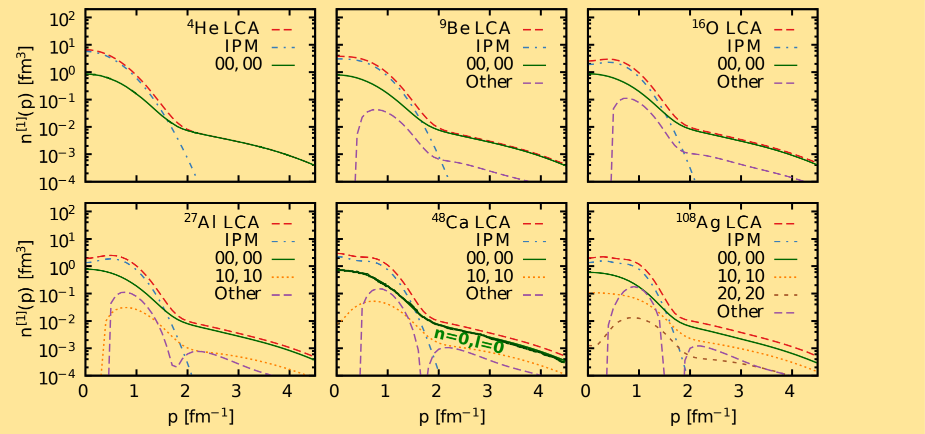
$$\text{EXPT.:}\quad \frac{A(e,e'N)|_{p>p_F}}{A(e,e'N)|_{p<p_F}} \quad \text{TH:}\quad \frac{\int_{400\text{ MeV}}^{800\text{ MeV}} dp\,p^2 n_N^{[1]}(p)}{\int_0^{250\text{ MeV}} dp\,p^2 n_N^{[1]}(p)} \qquad (N=p,n)$$



The weight of the minority component in the tail (SRC) part of $n^{[1]}(p)$ increases with the asymmetry N/Z

Quantum numbers of SRC-susceptible IPM pairs?

- ⋆ **Fat tails: correlation operators acting on IPM pairs.**
- ⋆ **Quantum number of IPM pairs that contribute most?**



$\sum_{nl} \sum_{n'l'} n^{[1],\text{corr}}_{nl,n'l'}(p) = n^{[1],\text{corr}}(p)$ *(relative motion of IPM pairs)*

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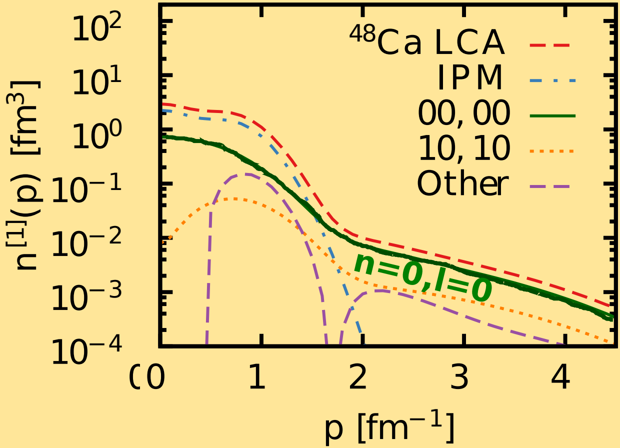


IPM pairs with relative $(n = 0, l = 0)$ very susceptible to SRC

Nucleon knockout data and nuclear models (I)

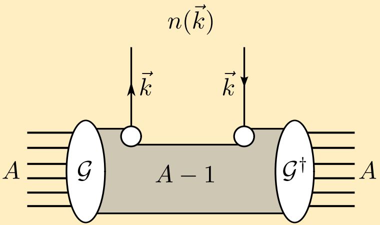
The quasi-free one-nucleon knockout case

- Link between $A(e,e'N)$ cross section and single-nucleon spectral function can be derived

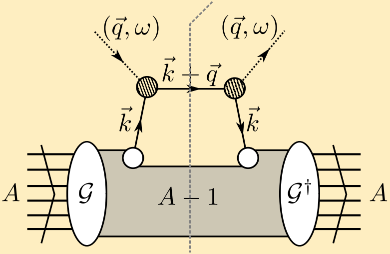
$$\frac{d^5\sigma}{d\epsilon' d\Omega_{\epsilon'} d\Omega_N dE_N}(e,e'N) = K\sigma_{eN} S\left(E_m, p_m\right)$$

- Factorization is approximate: relativity, final state interactions, spin effects, . . .

NUCLEAR STRUCTURE



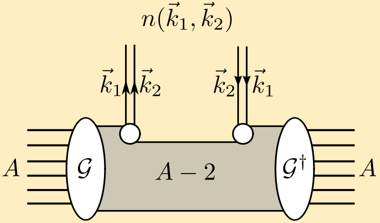
$A(e,e'p)$ OBSERVABLES



The quasi-free two-nucleon knockout case

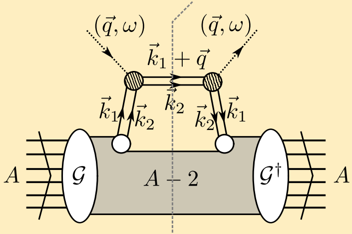
- Connection between SRC driven $A(e, e'NN)$ observables and high-momentum part of two-nucleon momentum distribution $n^{[2]}\left(\vec{p}_1, \vec{p}_2\right)$ is not trivial
(*dominated by three-nucleon correlations*)

NUCLEAR STRUCTURE



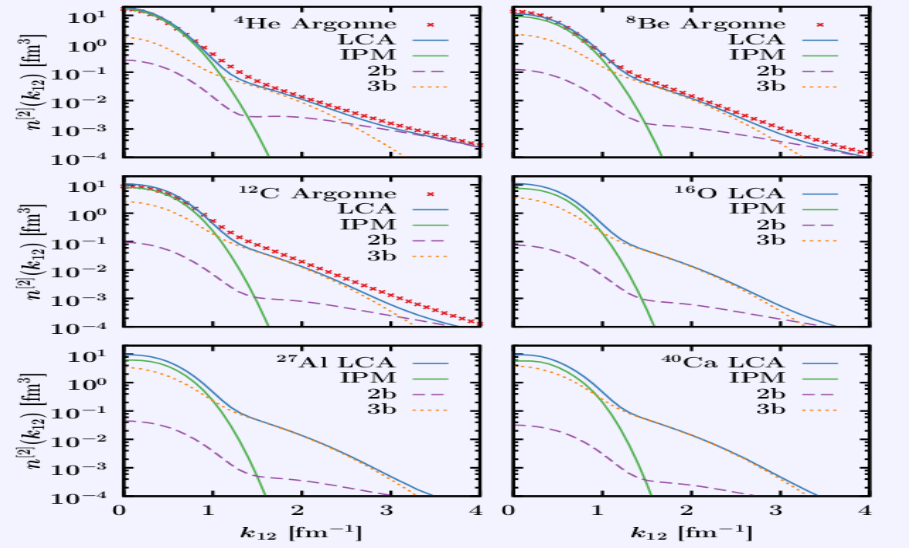
Two-nucleon momentum distribution

$A(e, e'NN)$ OBSERVABLES



Measurements with specific kinematic cuts

Relative two-nucleon momentum distribution in LCA: tail is dominated by "3-nucleon" SRC effects



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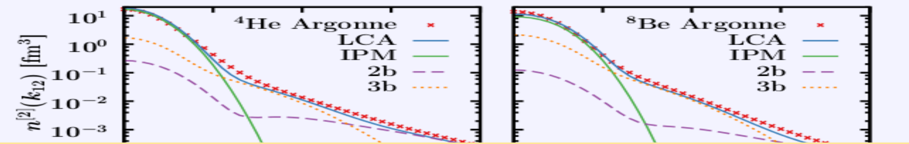
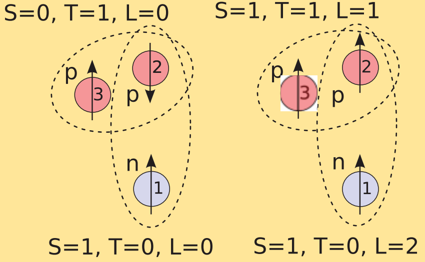


Correlations through the mediation of a third particle:

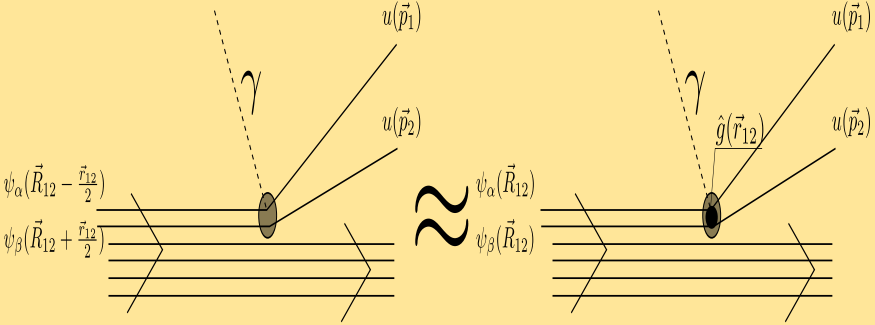


uncorrelated correlated

Feldmeier *et al.*, PRC 84 (2011), 054003

Exclusive SRC-driven $A(e, e'NN)$ (I)

- SRC-prone IPM pairs: close-proximity ($n_{12} = 0, l_{12} = 0$) state
- The EXCLUSIVE $A(e, e'NN)$ cross sections can be factorized [PLB383,1 ; PRC89,024603 ; PRC96,034608]



ZRA: Zero-range approximation

Reaction theory for SRC-driven $2N$ knockout

1. **$A(e,e'NN)$ cross section factorizes**

$$\frac{d^8\sigma}{d\epsilon' d\Omega_{\epsilon'} d\Omega_1 d\Omega_2 dT_{p_2}}(e,e'NN) = K\sigma_{eNN}(k_+,k_-,q)F^{(D)}(P)$$

- factorization in (relative pair motion) $\times$ (c.m. pair motion)
- $F^{(D)}(P)$: FSI corrected conditional probability to find a dinucleon with c.m. momentum P in a relative $(n_{12}=0, l_{12}=0)$ state

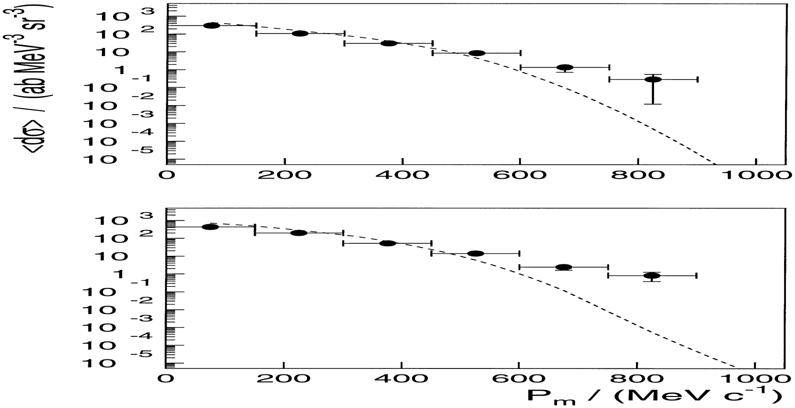
2. **A dependence of the $A(e,e'pp)$ cross sections is soft**

(much softer than predicted by naive $Z(Z-1)$ counting)

$$\frac{A(e,e'pp)}{{}^{12}\mathrm{C}(e,e'pp)} \approx \frac{N_{pp}(A)}{N_{pp}\left({}^{12}\mathrm{C}\right)} \times \left(\frac{T_A(e,e'p)}{T_{{}^{12}\mathrm{C}}(e,e'p)}\right)^{1-2}$$

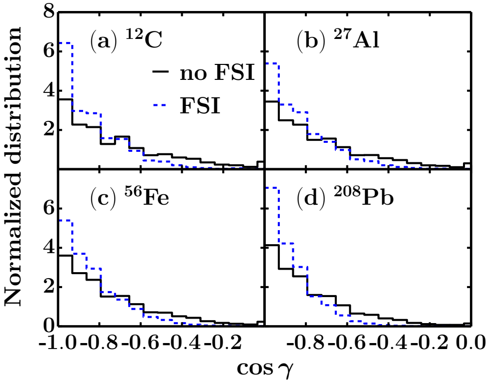
Factorization of the $A(e, e'pp)$ cross sections

$^{12}C(e, e'pp)$ @ MAMI (Mainz) (Physics Letters B **421** (1998) 71.)



For P $\lesssim$ 0.5 GeV c.m. motion of correlated pairs in ^{12}C is mean-field like (Gaussian $\left(\exp \frac{-P^2}{2\sigma^2_{c.m}}\right)$)!
Data prove the proposed factorization in terms of $F^{(D)}(P)$.

Opening-angle distribution $A(e, e'pp)$

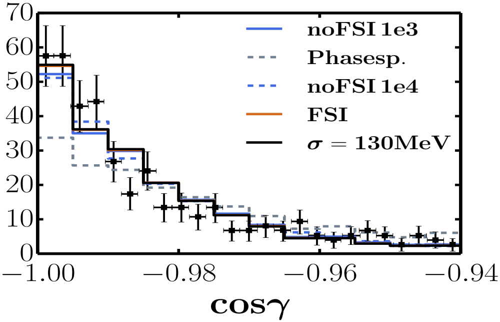


γ: angle between the correlated nucleons

1. **Attenuation (in eikonal model) reduces the cross sections**
2. **Attenuation and charge-exchange marginally affects the angular distributions *(FSI preserves factorization properties like back-to-back emission)***

$A(e, e'NN)$: Effect of the final-state interactions?

Opening-angle distribution ^{4}He$(e, e'pp)$



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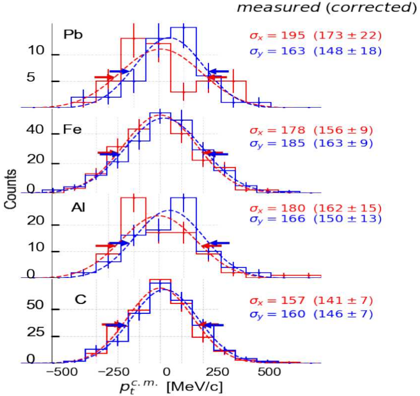
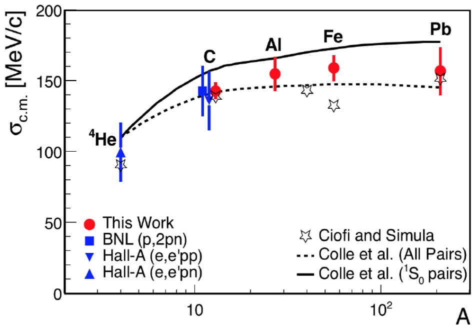
C.m. motion of SRC correlated pairs is Gaussian!

Mass dependence of the extracted widths of c.m. momentum distributions

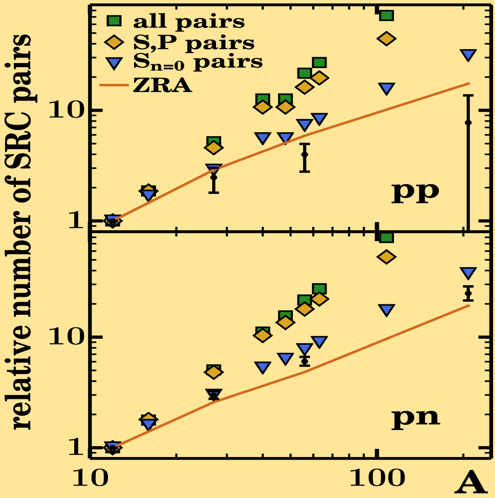




Observations are in line with predictions

arXiv:1805.01981 (Data Mining Collaboration @JLAB)

A dependence of number of pp and pn SRC pairs

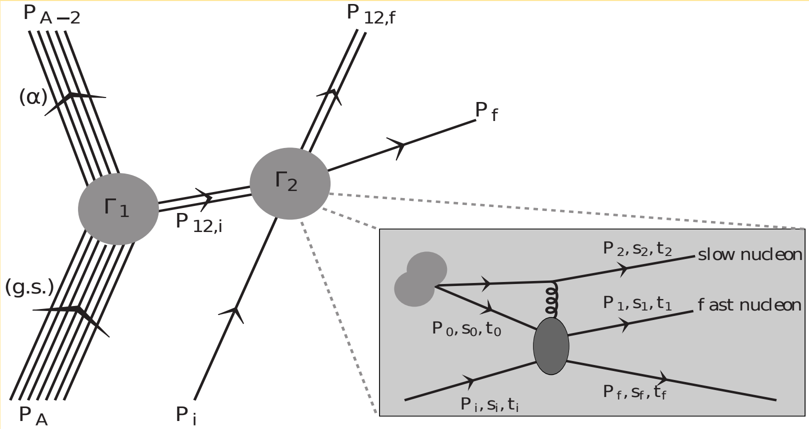


PRC92, 024604 (2015)

- Analysis of $A(e, e'pp)$ and $A(e, e'p)$ (A=^{12}C, ^{27}Al, ^{56}Fe, ^{208}Pb) in "SRC" kinematics (Data Mining Collaboration @JLAB)
- FSI corrections applied to the data
- **THEORY: Monte Carlo reaction-model calculations in the large phase space**
- **THEORY:** $(n = 0, l = 0)$ **pair counting**
- SRC: Selectivity in quantum numbers!

$p(A, pNN\ A-2)$ with radioactive beams

SRC in neutron-rich matter? Success of program partially hinges on a proper factorization expression for cross section.



S. Stevens *et al.*, PLB777 (2018) 374

$p(A, pNN\ A-2)$ with radioactive beams

SRC in neutron-rich matter? Success of program partially hinges on a proper factorization expression for cross section.

$$\frac{\mathrm{d}\sigma^{(pN_1N_2)}}{\mathrm{d}\Omega_f\,\mathrm{d}E_1\,\mathrm{d}\Omega_1\,\mathrm{d}E_2\,\mathrm{d}\Omega_2} = 2^{2|M_T|-1}\,\mathcal{S}(J_A,\beta\gamma)\,\mathcal{K}\,\frac{\mathrm{d}\sigma^{pN_1}}{\mathrm{d}t}\left\{\frac{E_2}{E_m+m_{N_1}}\sum_J\frac{1}{2J+1}\sum_M\sum_T\frac{1}{2T+1}F^{\beta\gamma}_{JM,T}(\vec{P},\vec{k})\right\}_{\mathrm{PF}}, \tag{11}$$

with $\mathcal{K}$ a kinematic factor

$$\mathcal{K} = \frac{1}{(2\pi)^8}\,\frac{(P_f\cdot P_1)^2-m_p^2m_{N_1}^2}{\sqrt{(P_i\cdot P_A)^2-m_p^2m_A^2}}\,m_Am_R^*\,\frac{p_fp_1p_2}{E_R}\left|1-\frac{E_f}{E_R}\,\frac{\vec{p}_R\cdot\vec{p}_f}{p_f^2}\right|^{-1} \tag{12}$$

$$F^{\beta\gamma}_{JM,T}(\vec{P},\vec{k}) = \sum_{\mu=T-1}^{1-T}\Bigg|\mathcal{F}^{(0)}_\nu[f_c-3f_{\sigma\tau}](k)\,\mathcal{P}^{\varepsilon\beta\gamma}_{JMT\mu}(\vec{P}) - \delta_{T,0}\,12\sqrt{2\pi}\,\mathcal{F}^{(2)}_\nu[f_{t\tau}](k)\sum_{m_l=-2}^{2}\langle 2m_l1\mu|1(m_l+\mu)\rangle\,\mathcal{P}^{\varepsilon\beta\gamma}_{JMT(m_l+\mu)}(\vec{P})\,Y_{2,m_l}(\Omega_k)\Bigg|^2.$$

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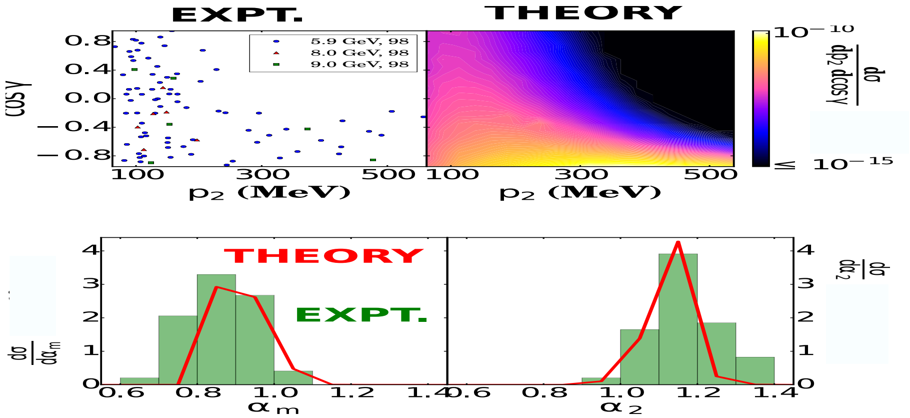
Postdictions for $^{12}C(p, ppn)$ from BNL



- DATA: A. Tang *et al.*, Phys. Rev. Lett. **90**, 042301 (2003)
- Calculations based on a factorized form of the cross section

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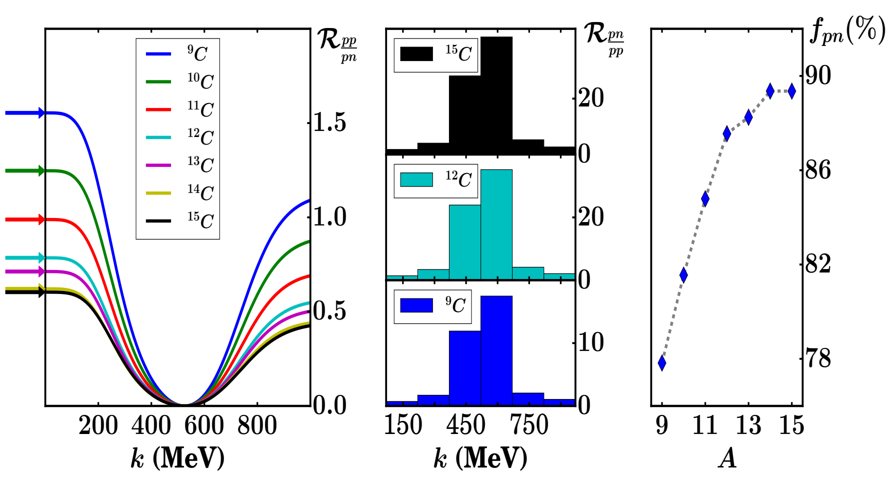
$p(A, pNN\ A-2)$ with radioactive beams: asymmetry dependence of nuclear SRC

Ratios of SRC pp to pn pairs for various carbon isotopes



CONCLUSIONS (I) - Nuclear Structure Theory

- **Nuclear SRC can be captured by general and robust principles**
- **LCA: efficient way of computing the SRC contributions to NMDs**
  1. **Magnitude of EMC effect and $A(e,e')/D(e,e')$ scaling factor ($x_B \gtrsim 1.5$) can be predicted in LCA**
  2. **$A \leq 12$: LCA predictions for fat tails are in line with those of QMC**
  3. **Systematic studies of isospin and asymmetry dependence of SRC**
  4. **Natural explanation for the universal behavior of the NMD tails**
- **MAJOR contribution to SRC strength: correlation operators acting on IPM pairs in a nodeless relative S state**

CONCLUSIONS (II)- - Nuclear Reactions Theory

- **Insights from study of SRC contribution to NMD has implications for SRC-driven $A(e, e'NN)A-2$ and $p(A, pNN\ A-2)$**
  1. **Scaling behavior of cross section ($\sim F(P)$) (CONFIRMED)**
  2. **Very soft mass dependence of cross section (CONFIRMED)**
  3. **Peculiar asymmetry dependence of SRC pairs (CONFIRMED)**
- **Generally applicable techniques for quantifying SRC: two-body effects in neutrino reactions, role of SRC in exotic forms of hadronic matter, . . .**
- **SRC induced spatio-temporal fluctuations are measurable, are significant and are quantifiable**


THANK YOU!

Selected publications

- S. Stevens, J. Ryckebusch, W. Cosyn, A. Waets
  "Probing short-range correlations in asymmetric nuclei with quasi-free pair knockout reactions" arXiv:1707.05542 and PLB **B777** (2018), 374.
- C. Colle, W. Cosyn, J. Ryckebusch
  "Final-state interactions in two-nucleon knockout reactions"
  arXiv:1512.07841 and PRC **93** (2016) 034608.
- J. Ryckebusch, M. Vanhalst, W. Cosyn
  "Stylized features of single-nucleon momentum distributions"
  arXiv:1405.3814 and JPG **42** (2015) 055104.
- C. Colle, O. Hen, W. Cosyn, I. Korover, E. Piasetzky, J. Ryckebusch, L.B. Weinstein
  "Extracting the Mass Dependence and Quantum Numbers of Short-Range Correlated Pairs from $A(e, e'p)$ and $A(e, e'pp)$ Scattering" arXiv:1503.06050 and PRC **92** (2015), 024604.
- C. Colle, W. Cosyn, J. Ryckebusch, M. Vanhalst
  "Factorization of electroinduced two-nucleon knockout reactions"
  arXiv:1311.1980 and PRC **89** (2014), 024603.