A Global EDM Analysis with Theory Uncertainties

Nina Elmer

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with S. Degenkolb, M. Mühlleitner, T. Modak, T.Plehn

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

IMPRS
for Precision Tests of Fundamental Symmetries
INTERNATIONAL MAX PLANCK RESEARCH SCHOOL

SFitter Framework

- Easy to add new measurements from different experiments
- **Adaptable parameters** and predictions
- Comprehensive uncertainty treatment
- **Correlated systematic uncertainties** between measurements
- Markov chain to construct exclusive likelihood
- Use profiling and marginalization for likelihoods

➡ See Tilman’s talk yesterday

Electric dipole moments

- EDMs measured **below electroweak scale**
- Degrees of freedom: leptons, non-relativistic nucleons N and pions $\vec{\pi}$
- Possible Lagrangians: **weak-scale** and **hadronic-scale**
- **Matching** electron interactions between scales

Lagrangian(s)

- $\mathscr{L}_{\text{CPV}} = \mathscr{L}_{\text{CKM}} + \mathscr{L}_{\bar{\theta}} + \mathscr{L}_{\text{dipole}} + \mathscr{L}_{\text{Weinberg}} + \mathscr{L}_{\text{SMEFT}}$
- $\mathscr{L}_{\text{CKM}}, \mathscr{L}_{\bar{\theta}}$: **CP-violation** in CKM-matrix and gluon-field strength
- $\mathscr{L}_{had} \supset \mathscr{L}_{N,\text{sr}} + \mathscr{L}_{\pi N} + \mathscr{L}_{eN} - \frac{i}{2} F^{\mu\nu} d_e \, \bar{e} \sigma_{\mu\nu} \gamma_5 e$
- **short-range nucleon** EDM parameter $d_p^{sr} \approx - d_n^{sr}$
- effective **pion-nucleon interaction** $g_\pi^{(0)}, g_\pi^{(1)}$
- e - N interactions by isospin and tensor structure

Parameters

- Matching to weak-scale **further reduces number of parameters**
- $C_{S,P,T}^{(0,1)}$ expressed by three SMEFT Wilson coefficients
- Reduced to $C_{S,P,T}^{(0)}$ using hadronic matrix elements
- **Preserve full $C_{S,P,T}^{(0,1)}$ dependence**
- **7D parameter space:** $d_e, C_S^{(0)}, C_P^{(0)}, C_T^{(0)}, g_\pi^{(0)}, g_\pi^{(1)}, d_n^{sr}$

Measurements

Paramagnetic molecules [2212.11841, Nature 562 7727, Nature 473 493]

- ThO, HfF^+, YbF (constraints d_e, $C_S^{(0)}$)

Paramagnetic atoms [PhysRevLett.88.071805, PhysRevLett.63.965]

- ^{205}Tl, ^{133}Cs

Diamagnetic atoms [1601.04339, 1902.02864, 2207.08140, 1606.04931, PhysRevA.44.2783]

- ^{199}Hg, ^{129}Xe, ^{171}Yb, ^{225}Ra, TlF (constraints $C_P^{(0)}, C_T^{(0)}, g_\pi^{(0)}, g_\pi^{(1)}, d_n^{sr}$)

Nucleons [2001.11966]

- neutron (constraints $g_\pi^{(0)}, g_\pi^{(1)}, d_n^{sr}$)

EDMs from Lagrangian

- Relation of data and parameters: $d_i = \sum \alpha_{i,c_j}\, c_j$
- **Linearization** different from other global analyses
- Experimental uncertainties **uncorrelated Gaussians**

System i	α_{i,d_e}	$\alpha_{i,C_S^{(0)}}$ [e cm]	$\alpha_{i,C_P^{(0)}}$ [e cm]	$\alpha_{i,C_T^{(0)}}$ [e cm]	$\alpha_{i,g_\pi^{(0)}}$ [e cm]	$\alpha_{i,g_\pi^{(1)}}$ [e cm]	$\alpha_{i,d_n^{sr}}$	$\alpha_{i,d_p^{sr}}$
n	–	–	–	–	$1.38^{\pm 0.02} \cdot 10^{-14}$	$2.73^{\pm 0.02} \cdot 10^{-16}$	1	−1
^{205}Tl	$-558^{\pm 28}$ [74]	$-6.77^{\pm 0.34} \cdot 10^{-18}$	$1.5^{+2}_{-0.7} \cdot 10^{-19}$	$8.8^{\pm 0.9} \cdot 10^{-21}$	n/a	n/a	n/a	n/a
^{133}Cs	$123^{\pm 4}$	$7.80^{+0.2}_{-0.8} \cdot 10^{-19}$	$-1.4^{+0.8}_{-2} \cdot 10^{-20}$	$1.7^{\pm 0.2} \cdot 10^{-20}$	–	–	–	–
^{199}Hg	$-0.012^{+0.0094}_{-0.002}$ [75, 76]	$-1.26^{+0.7}_{-1.2} \cdot 10^{-21}$	$6.6^{+1.2}_{-0.3} \cdot 10^{-23}$	$-6.4^{+3}_{-4} \cdot 10^{-21}$	$-1.18^{+0.19}_{-2.62} \cdot 10^{-17}$	$1.6^{+0}_{-6.5} \cdot 10^{-17}$	$-1.56^{\pm 0.39} \cdot 10^{-4}$	$-1.56^{\pm 0.39} \cdot 10^{-5}$
^{129}Xe	$-8^{+0}_{-8} \cdot 10^{-4}$ [76, 77]	$-2.1^{+1.2}_{-2.5} \cdot 10^{-22}$	$1.7^{+0.5}_{-0.4} \cdot 10^{-23}$	$1.24^{+0.78}_{-0.61} \cdot 10^{-21}$	$-0.4^{+1.2}_{-23} \cdot 10^{-19}$	$-2.2^{+1.1}_{-17} \cdot 10^{-19}$	$1.7^{+0.7}_{-0} \cdot 10^{-5}$	$3.51^{\pm 0.88} \cdot 10^{-6}$
^{171}Yb	$(-0.012^{+0.01145}_{-0.002})$ [78]	$-9.1^{+5}_{-11} \cdot 10^{-22}$	$4.5^{+1.8}_{-1.1} \cdot 10^{-23}$	$-4.4^{+2.2}_{-2.9} \cdot 10^{-21}$	$-9.5^{\pm 2.4} \cdot 10^{-18}$	$1.3^{\pm 0.33} \cdot 10^{-17}$	$-1.13^{\pm 0.28} \cdot 10^{-4}$	$-1.13^{\pm 0.28} \cdot 10^{-5}$
^{225}Ra	$-0.054^{\pm 0.002}$ [76]	$8.6^{+9.5}_{-4.5} \cdot 10^{-21}$	$-7.0^{+1.7}_{-2.6} \cdot 10^{-22}$	$-4.5^{+2.0}_{-2.5} \cdot 10^{-20}$	$1.7^{+5.2}_{-0.8} \cdot 10^{-15}$	$-6.9^{+3.1}_{-21} \cdot 10^{-15}$	$-5.36^{\pm 1.34} \cdot 10^{-4}$	$-1.11^{\pm 0.28} \cdot 10^{-4}$
TlF	$81^{\pm 20}$ [50, 70]	$5.6^{+4.9}_{-2.5} \cdot 10^{-18}$	$2.4^{+1.0}_{-1.9} \cdot 10^{-19}$	$4.8^{+1.2}_{-1.1} \cdot 10^{-16}$	$1.9^{+0.1}_{-1.4} \cdot 10^{-14}$	$-1.6^{\pm 0.4} \cdot 10^{-13}$	$-9.47^{\pm 2.37} \cdot 10^{-2}$	$-4.59^{\pm 1.15} \cdot 10^{-1}$
HfF^+	1	$9.17^{\pm 0.06} \cdot 10^{-21}$	–	–	–	–	–	–
ThO	1	$1.51^{+0}_{-0.2} \cdot 10^{-20}$	–	–	–	–	–	–
YbF	1	$8.99^{\pm 0.70} \cdot 10^{-21}$	–	–	–	–	–	–
	$\eta^{(m)}_{i,d_e} \left[\frac{\text{mrad}}{\text{s}\ e\ \text{cm}}\right]$	$k^{(m)}_{i,C_S} \left[\frac{\text{mrad}}{\text{s}}\right]$	α_{i,C_P}	α_{i,C_T}	$\alpha_{i,g_\pi^{(0)}}$	$\alpha_{i,g_\pi^{(1)}}$	$\alpha_{i,d_n^{sr}}$	$\alpha_{i,d_p^{sr}}$
HfF^+	$3.49^{\pm 0.14} \cdot 10^{28}$ [75, 79–82]	$3.2^{+0.1}_{-0.2} \cdot 10^{8}$ [75, 79, 80]	–	–	–	–	–	–
ThO	$-1.21^{+0.05}_{-0.39} \cdot 10^{29}$ [75, 83–85]†	$-1.82^{+0.42}_{-0.27} \cdot 10^{9}$ [75, 83, 85–87]†	–	–	–	–	–	–
YbF	$-1.96^{\pm 0.15} \cdot 10^{28}$ [75, 86–89]	$-1.76^{\pm 0.2} \cdot 10^{8}$ [75, 86–88]	–	–	–	–	–	–

Single-parameter ranges

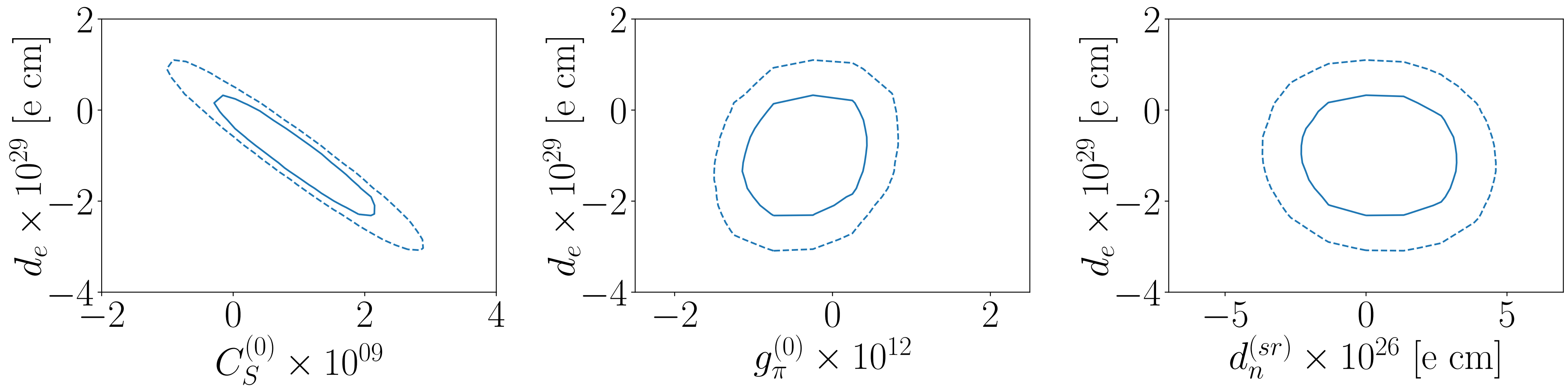
System i	d_e [e cm]	C_S	C_P	C_T
Tl	$(7.2 \pm 7.7) \cdot 10^{-28}$	$(5.9 \pm 6.4) \cdot 10^{-8}$	$(-2.7 \pm 3.0) \cdot 10^{-6}$	$(-4.5 \pm 4.9) \cdot 10^{-5}$
Cs	$(-1.4 \pm 5.6) \cdot 10^{-26}$	$(-2.3 \pm 8.9) \cdot 10^{-6}$	$(1.3 \pm 5) \cdot 10^{-4}$	$(-1.1 \pm 4.2) \cdot 10^{-4}$
^{199}Hg	$(-1.8 \pm 2.6) \cdot 10^{-28}$	$(-1.7 \pm 2.5) \cdot 10^{-9}$	$(3.4 \pm 4.8) \cdot 10^{-8}$	$(-3.4 \pm 4.9) \cdot 10^{-10}$
^{129}Xe	$(2.2 \pm 2.3) \cdot 10^{-25}$	$(8.3 \pm 8.7) \cdot 10^{-7}$	$(-1.0 \pm 1.1) \cdot 10^{-5}$	$(-1.4 \pm 1.5) \cdot 10^{-7}$
^{171}Yb	$(5.7 \pm 4.4) \cdot 10^{-25}$	$(7.5 \pm 5.7) \cdot 10^{-6}$	$(-1.5 \pm 1.2) \cdot 10^{-4}$	$(1.6 \pm 1.2) \cdot 10^{-6}$
^{225}Ra	$(-7.4 \pm 1.1) \cdot 10^{-23}$	$(4.7 \pm 7) \cdot 10^{-4}$	$(-5.7 \pm 8.5) \cdot 10^{-3}$	$(-8.9 \pm 13) \cdot 10^{-5}$
TlF	$(-2.1 \pm 3.6) \cdot 10^{-25}$	$(-3.0 \pm 5.1) \cdot 10^{-6}$	$(-7.1 \pm 12) \cdot 10^{-5}$	$(-3.6 \pm 6.1) \cdot 10^{-8}$
HfF^+	$(-1.3 \pm 2.1) \cdot 10^{-30}$	$(-1.4 \pm 2.3) \cdot 10^{-10}$		
ThO	$(4.3 \pm 4.1) \cdot 10^{-30}$	$(2.9 \pm 2.7) \cdot 10^{-10}$		
YbF	$(-2.4 \pm 5.9) \cdot 10^{-28}$	$(-2.7 \pm 6.6) \cdot 10^{-8}$		

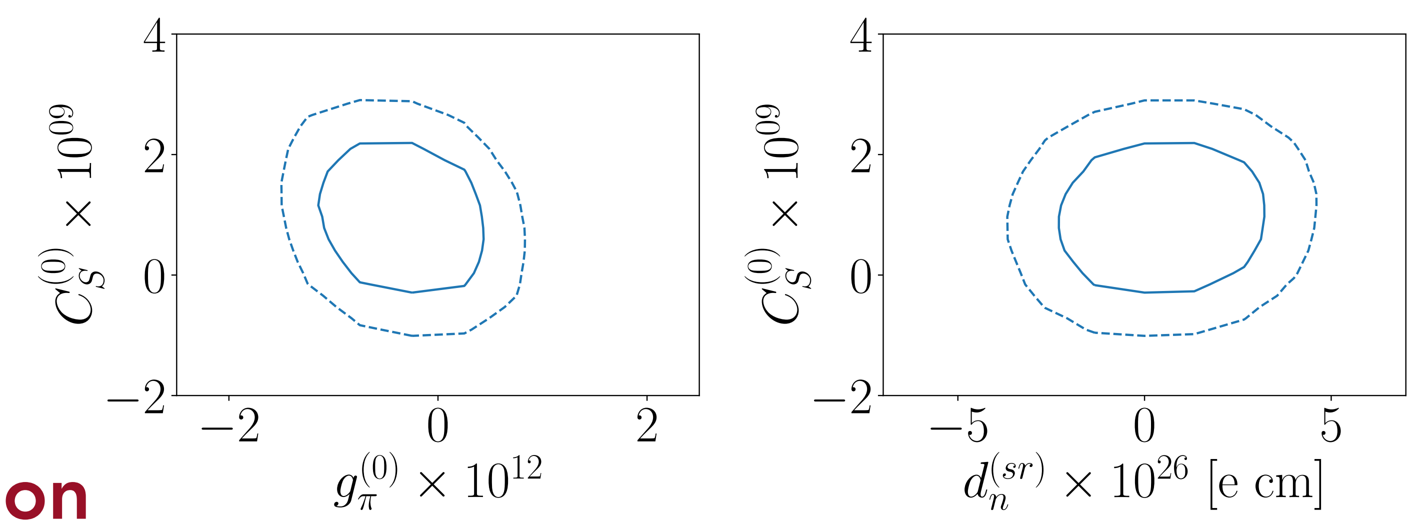
	$g_\pi^{(0)}$	$g_\pi^{(1)}$	d_n^{sr}	d_p^{sr}
n	$(0 \pm 8.1) \cdot 10^{-13}$	$(0 \pm 4.1) \cdot 10^{-11}$	$(0 \pm 1.1) \cdot 10^{-26}$	$(0 \pm 1.1) \cdot 10^{-26}$
^{199}Hg	$(-1.9 \pm 2.7) \cdot 10^{-13}$	$(1.4 \pm 2.0) \cdot 10^{-13}$	$(-1.4 \pm 2.0) \cdot 10^{-26}$	$(-1.4 \pm 2.0) \cdot 10^{-25}$
^{129}Xe	$(4.4 \pm 4.6) \cdot 10^{-9}$	$(8 \pm 8.3) \cdot 10^{-10}$	$(-1.0 \pm 1.1) \cdot 10^{-23}$	$(-5.0 \pm 5.2) \cdot 10^{-23}$
^{171}Yb	$(7.2 \pm 5.5) \cdot 10^{-10}$	$(-5.2 \pm 4.0) \cdot 10^{-10}$	$(6.0 \pm 4.6) \cdot 10^{-23}$	$(6.0 \pm 4.6) \cdot 10^{-22}$
^{225}Ra	$(2.4 \pm 3.5) \cdot 10^{-9}$	$(-5.8 \pm 8.7) \cdot 10^{-10}$	$(-7.5 \pm 11) \cdot 10^{-21}$	$(-3.6 \pm 5.4) \cdot 10^{-20}$
TlF	$(-9.0 \pm 15) \cdot 10^{-10}$	$(1.1 \pm 1.8) \cdot 10^{-10}$	$(1.8 \pm 3.1) \cdot 10^{-22}$	$(3.7 \pm 6.3) \cdot 10^{-23}$

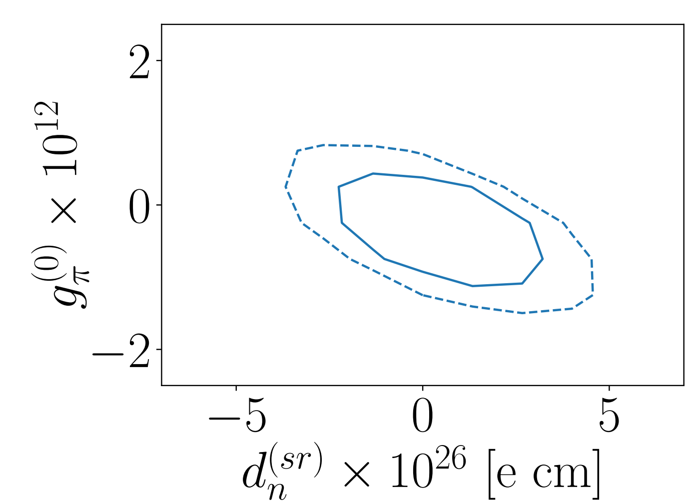
Strongest measurements

- Strongest measurements: HfF^{+}, ThO, n, Hg
- **4D sub-space** to understand correlations
- Well-constrained (hopefully) parameters: $d_e, C_S^{(0)}, g_\pi^{(0)}, d_n^{sr}$
- $d_e - C_S^{(0)}$ constrained by ThO and HfF^{+}
- $g_\pi^{(0)} - d_n^{sr}$ constrained by Hg and n

Well-constrained 4D-space



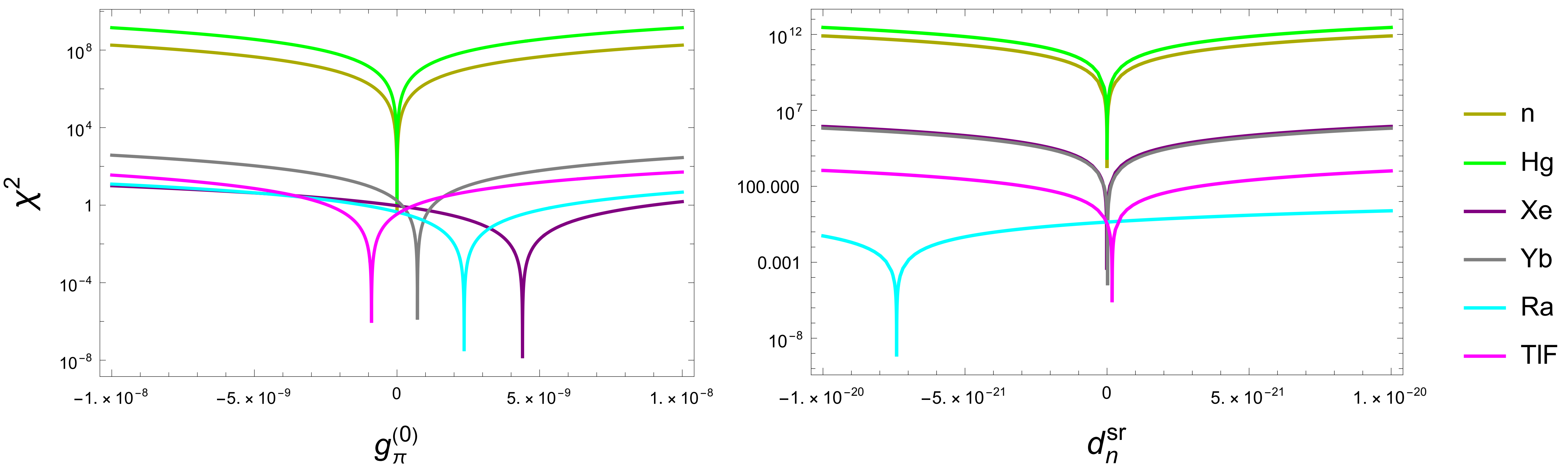




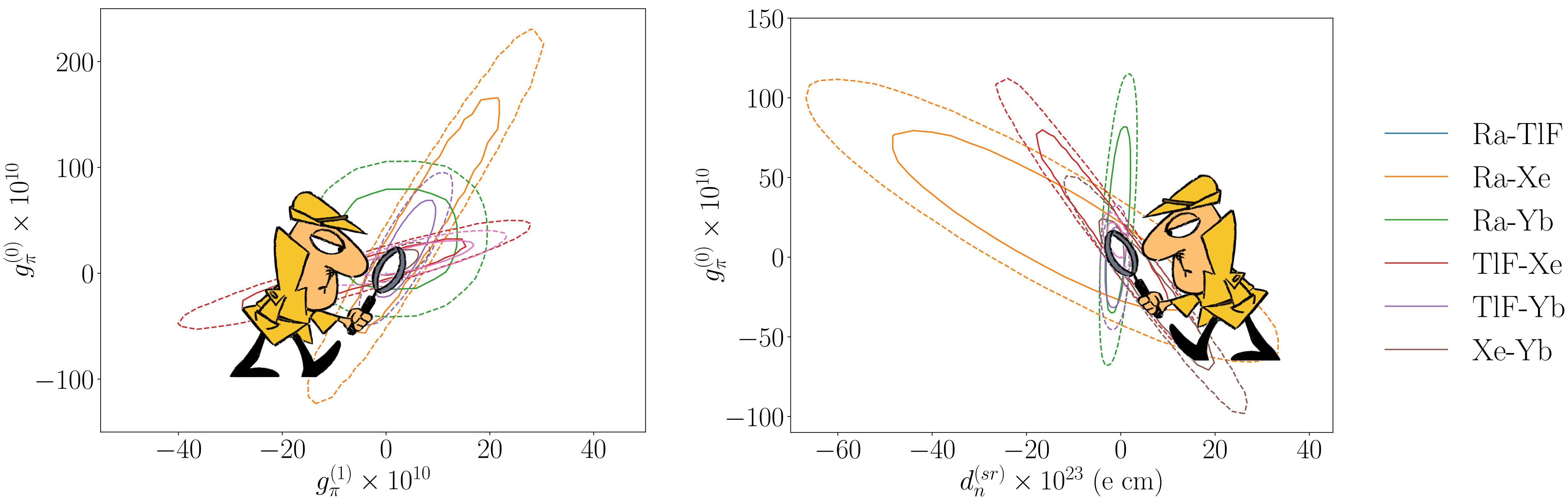
- $d_e - C_S^{(0)}$: strong **anti-correlation**
- $g_\pi^{(0)} - d_n^{sr}$: weaker **anti-correlation**
- **no correlation** between sub-sets → factorization

Hadronic parameters

- Expand hadronic analysis to $g_\pi^{(0)}, g_\pi^{(1)}, d_n^{sr}$
- Hg, neutron measurements: **strongest constraints**
- single-parameter χ^2 values
- Difference of $\mathcal{O}(10^4)$ between measurements



Correlations in hadronic sector



- Rich **correlations** for $g_\pi^{(0)}, g_\pi^{(1)}$
- Aligned **correlations** for $g_\pi^{(0)}, d_n^{sr}$
- Combination Xe with Ra, Yb, TlF: constraining just d_n^{sr}

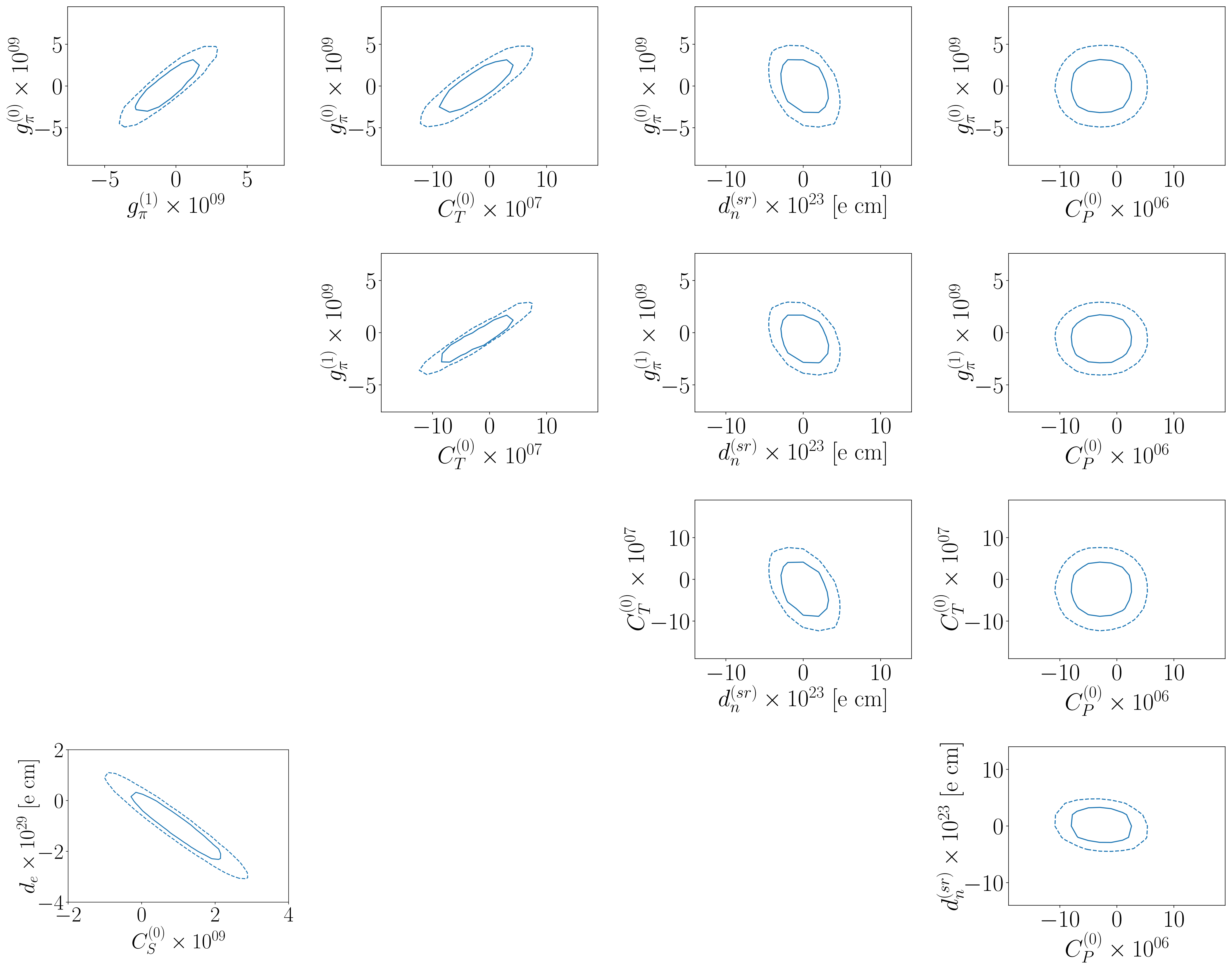
Simplified profile likelihood

- Interested in 2D correlations and 1D profile likelihoods
- Typical measurements in 5D space: Tl, Cs, TlF, Xe, Ra, Yb
- Strongest measurements in 5D space: **narrow correlations**
- **Not enough strong measurements** means profile likelihood dominated by typical measurements
- n and Hg without impact and numerically hard → remove from dataset

Global analysis

- 2D-subspace $d_e - C_s^{(0)}$ from ThO and HfF^+ **factorized**
- 5D parameter-set: $C_P^{(0)}, C_T^{(0)}, g_\pi^{(0)}, g_\pi^{(1)}, d_n^{sr}$
- Ignoring 10^{-3}-correlations from Hg and n
- 2D-parameter correlations first

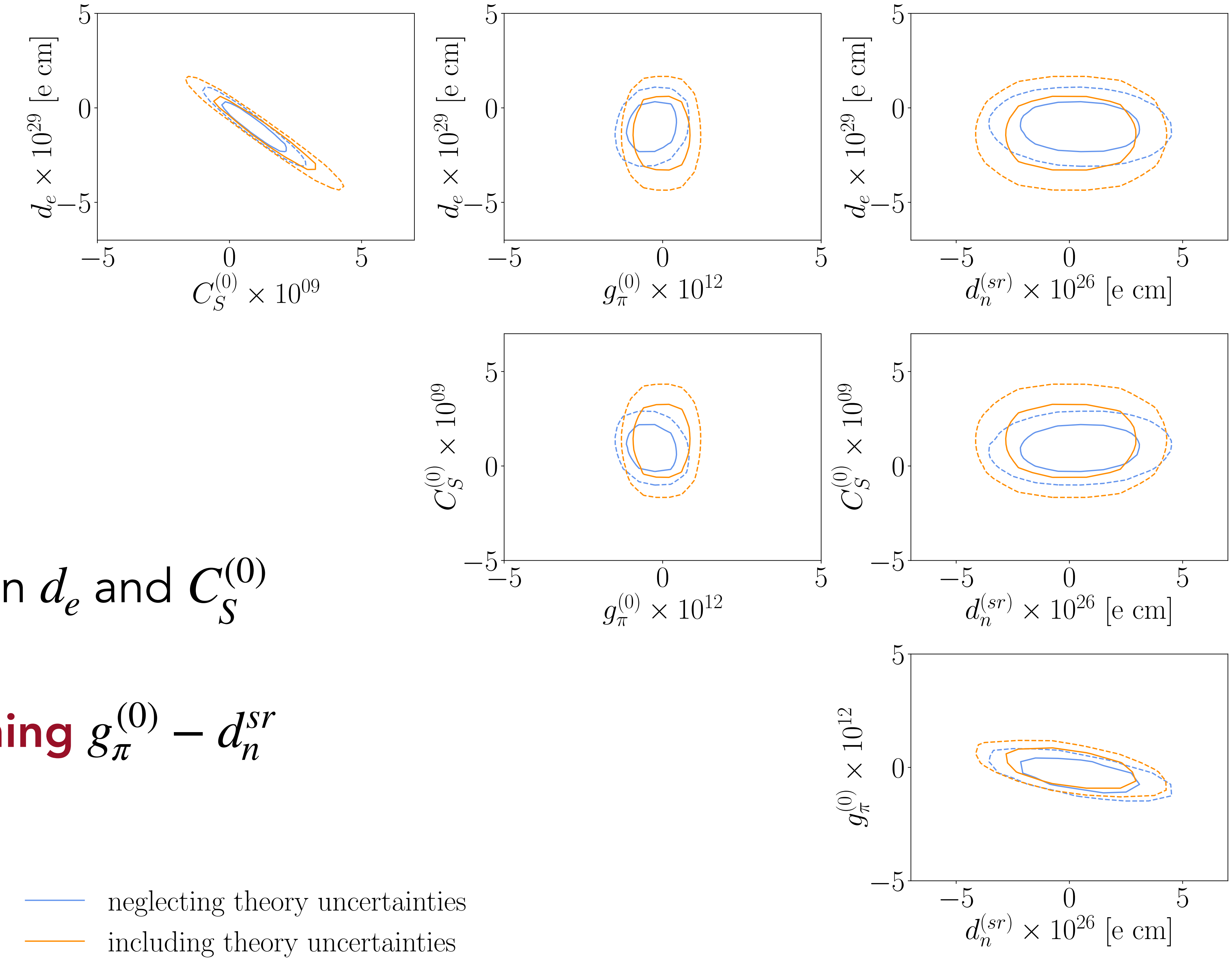
Global analysis



Theory uncertainties

- Reminder: $d_i = \sum \alpha_{i,c_j} \, c_j$
- Theory uncertainties on each α_{i,c_j}
- **Uncorrelated** between measurements and parameters
- Profiling **flat likelihood** from nuisance parameters
  (1) error bars added linearly
  (2) dependencies consistent with $\alpha = 0$ removed
- Applied to every measurement and model parameter

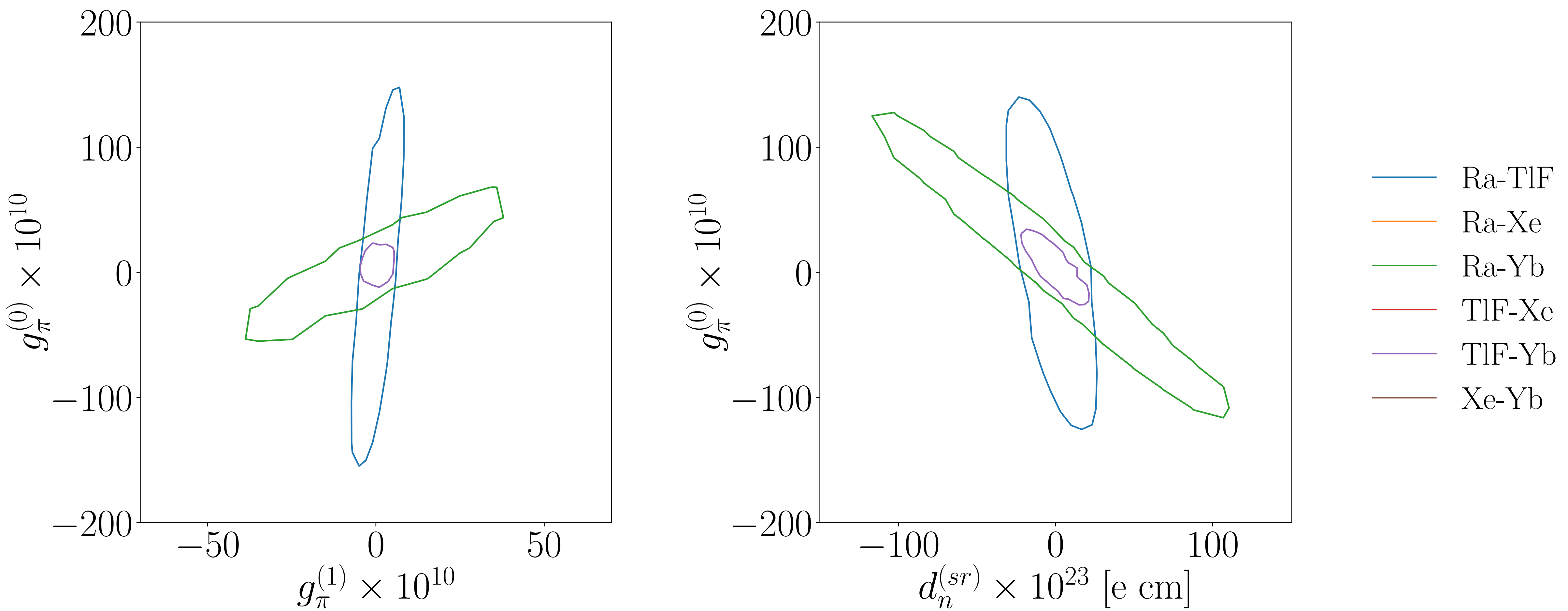
Theory uncertainties in 4D space



- Little **impact** on d_e and $C_S^{(0)}$
- slight **broadening** $g_\pi^{(0)} - d_n^{sr}$

Theory uncertainties in hadronic sector

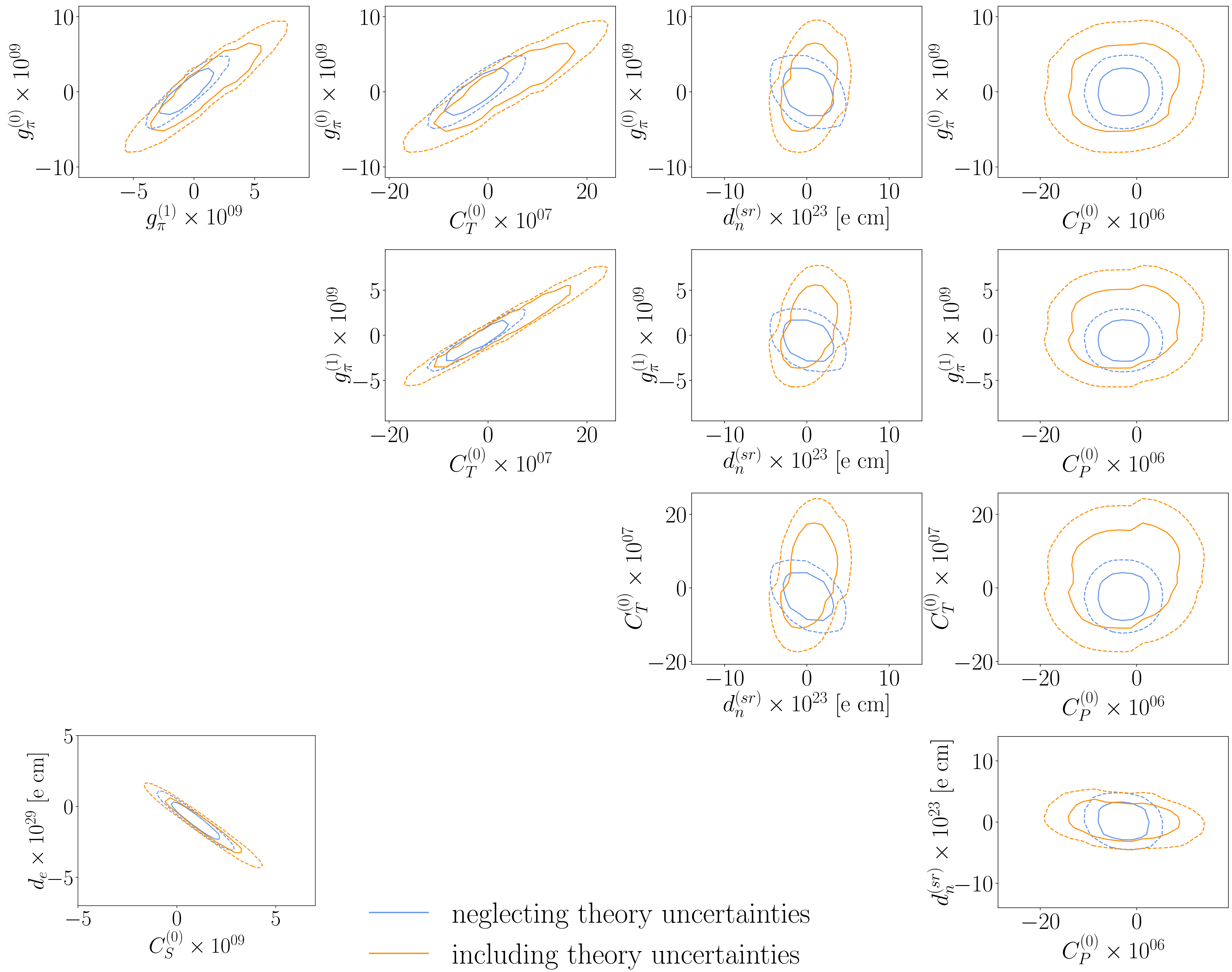
- No impact of Xe on $g_\pi^{(0)}$ (range of theory uncertainties)
- **Different** correlation patterns for $g_\pi^{(0)} - g_\pi^{(1)}$
- Correlation patterns **aligned** for $g_\pi^{(0)} - d_n^{sr}$



Theory uncertainties in global analysis

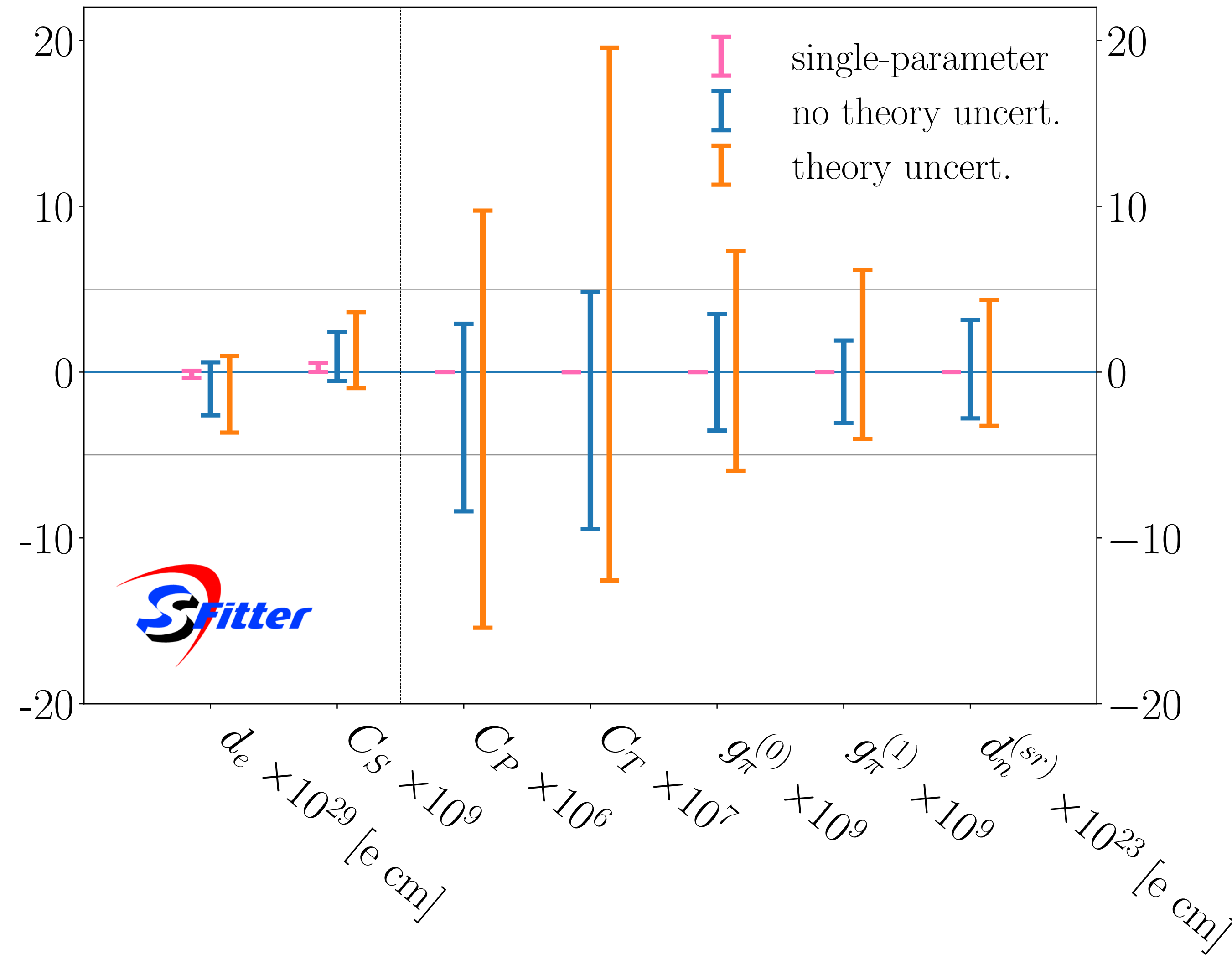
- Same considerations as before
  - Not including Hg and n
  - Factorizing out $d_e - C_S^{(0)}$ plane
- **Weaker constraints** on hadronic model parameter
  - exception d_n^{sr}: only hardly correlated, small theory uncertainties

Theory uncertainties in correlations



Conclusion and Outlook

- 11 measurement for 7 parameters $d_e, C_S^{(0)}, C_P^{(0)}, C_T^{(0)}, g_\pi^{(0)}, g_\pi^{(1)}, d_n^{sr}$
- Four strongest measurements constraining $d_e, C_S^{(0)}, g_\pi^{(0)}, d_n^{sr}$
- Strong limits on $d_e - C_S^{(0)}$ factorizing
- Hadronic sector **highly correlated**
- Theory uncertainties
  (1) semileptonic: **mild impact**
  (2) hadronic: **weaker constraints**
- Comments over email or coffee?



Backup slides

Table with measurements

System i	Measured d_i [e cm]	Upper limit on $\lvert d_i \rvert$ [e cm]
n	$(0.0 \pm 1.1_{\mathrm{stat}} \pm 0.2_{\mathrm{syst}}) \cdot 10^{-26}$	$2.2 \cdot 10^{-26}$
^{205}Tl	$(-4.0 \pm 4.3) \cdot 10^{-25}$	$1.1 \cdot 10^{-24}$
^{133}Cs	$(-1.8 \pm 6.7_{\mathrm{stat}} \pm 1.8_{\mathrm{syst}}) \cdot 10^{-24}$	$1.4 \cdot 10^{-23}$
HfF^+	$(-1.3 \pm 2.0_{\mathrm{stat}} \pm 0.6_{\mathrm{syst}}) \cdot 10^{-30}$	$4.8 \cdot 10^{-30}$
ThO	$(4.3 \pm 3.1_{\mathrm{stat}} \pm 2.6_{\mathrm{syst}}) \cdot 10^{-30}$	$1.1 \cdot 10^{-29}$
YbF	$(-2.4 \pm 5.7_{\mathrm{stat}} \pm 1.5_{\mathrm{syst}}) \cdot 10^{-28}$	$1.2 \cdot 10^{-27}$
^{199}Hg	$(2.20 \pm 2.75_{\mathrm{stat}} \pm 1.48_{\mathrm{syst}}) \cdot 10^{-30}$	$7.4 \cdot 10^{-30}$
^{129}Xe	$(-1.76 \pm 1.82) \cdot 10^{-28}$	$4.8 \cdot 10^{-28}$
^{171}Yb	$(-6.8 \pm 5.1_{\mathrm{stat}} \pm 1.2_{\mathrm{syst}}) \cdot 10^{-27}$	$1.5 \cdot 10^{-26}$
^{225}Ra	$(4 \pm 6_{\mathrm{stat}} \pm 0.2_{\mathrm{syst}}) \cdot 10^{-24}$	$1.4 \cdot 10^{-23}$
TlF	$(-1.7 \pm 2.9) \cdot 10^{-23}$	$6.5 \cdot 10^{-23}$

2212.11841, Nature 562 7727, Nature 473 493, PhysRevLett.88.071805, PhysRevLett.63.965, 1601.04339, 1902.02864, 2207.08140, 1606.04931, PhysRevA.44.2783, 2001.11966

SMEFT Wilson coefficients

$$C_S^{(0)} = -\, g_S^{(0)} \frac{v^2}{\Lambda^2}\,\mathrm{Im}\left(C_{\ell edq} - C_{\ell equ}^{(1)}\right) \qquad C_S^{(1)} = \; g_S^{(1)} \frac{v^2}{\Lambda^2}\,\mathrm{Im}\left(C_{\ell edq} + C_{\ell equ}^{(1)}\right)$$

$$C_T^{(0)} = -\, g_T^{(0)} \frac{v^2}{\Lambda^2}\,\mathrm{Im}\left(C_{\ell equ}^{(3)}\right) \qquad C_T^{(1)} = -\, g_T^{(1)} \frac{v^2}{\Lambda^2}\,\mathrm{Im}\left(C_{\ell equ}^{(3)}\right)$$

$$C_P^{(0)} = \; g_P^{(0)} \frac{v^2}{\Lambda^2}\,\mathrm{Im}\left(C_{\ell edq} + C_{\ell equ}^{(1)}\right) \qquad C_P^{(1)} = -\, g_P^{(1)} \frac{v^2}{\Lambda^2}\,\mathrm{Im}\left(C_{\ell edq} - C_{\ell equ}^{(1)}\right)$$

Which parameter from which system?

parameter	experimental system
d_e	paramagnetic molecules
C_S	paramagnetic molecules
C_T	diamagnetic systems (Hg, Xe)
C_P	diamagnetic systems (Hg, Xe)
$g_\pi^{(0)}$	neutron, Hg
$g_\pi^{(1)}$	Hg, neutron, other diamagnetic systems
d_n^{sr}	neutron, Hg