Nonperturbative physics, chiral symmetry and EDM observables

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Flambaum, MP, Ritz, Stadnik, 1912.13129 (PRD2020)

Y. Ema, T. Gao, MP 2108.05398 (PRL2021)

Y. Ema, T. Gao, MP 2202.10524 (PRL2022)

Y. Ema, T. Gao, MP 2205.11532 (JHEP2022)

Y. Ema, T. Gao, MP, in progress

Plan

Part I, Recent results from our group (See also Ting Gao poster!):

- Paramagnetic EDMs from Hadronic CP violation.
- New result for EDMs (δ_{CKM})
- New constraints on EDMs of heavy particles.

Part II, Nonperturbative aspects of EDM calculations (QCD sum rules, lattice QCD)

- Chiral/$U_A(1)$ properties of nucleon interpolating currents
- "Standard" lattice current is problematic for $d_n(\theta)$.
- Revisiting QCD sum rule calculations: why would $\beta = +1$ interpolating current OPE "reproduce" naïve quark model?

Progress in paramagentic EDMs

$|d_e| < 1.6 \times 10^{-27}$e cm → $|d_e| < 4.1 \times 10^{-30}$ e cm (HfF+), 1.1×10^{-29}(ThO)

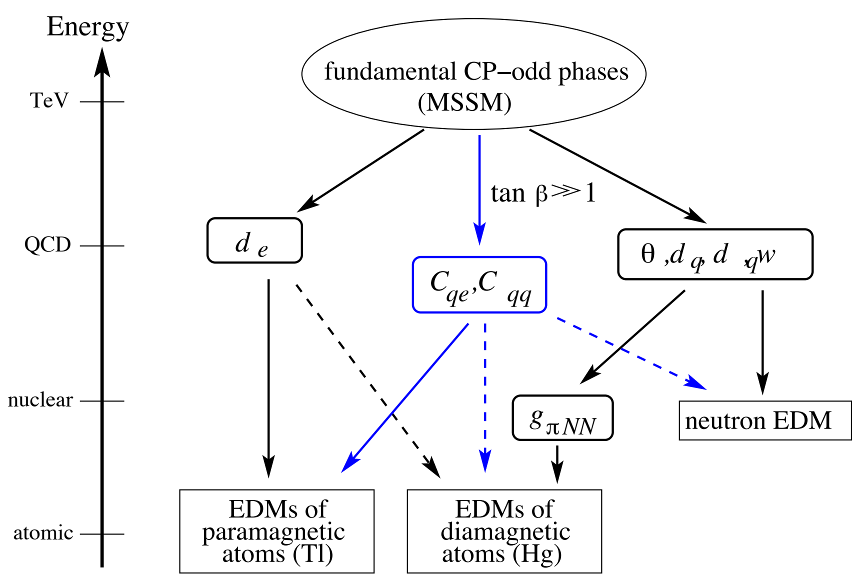
- In the last ~ 10 years, improved by a factor of ~ 400.
- Sensitivity is usually quoted as d_e. Relativistically enhanced as $d_{Atom} \sim Z^3 \alpha^2 d_e$. In reality, d_{Atom} is a linear combination of d_e and a semileptonic operator. Using most sensitive results from ThO and HfF+ molecules, one can limit both sources. Diatomic molecules have strong internal field and can effectively "enhance" modest external E field.
- More progress is real (e.g. ACME III). Most daring proposals want to go down to $d_e \sim 10^{-34}$ e cm.
- Theoretically is the cleanest. Atomic theory is under control at ~ 10% accuracy. In many models - minimum of QCD/nuclear input. SM contributions (θ_{QCD} and δ_{CKM}) were calculated in the last three years. Benchmark CKM value $d_e^{eq} = 1.0 * 10^{-35}$ e cm.

BSM physics and EDMs

$$\mathcal{L}_{eff}^{1\text{GeV}} = \frac{g_s^2}{32\pi^2}\,\theta_{QCD} G^a_{\mu\nu}\widetilde{G}^{\mu\nu,a}$$

$$-\frac{i}{2}\sum_{i=e,u,d,s} d_i\,\overline{\psi}_i(F\sigma)\gamma_5\psi_i - \frac{i}{2}\sum_{i=u,d,s}\widetilde{d}_i\,\overline{\psi}_i g_s(G\sigma)\gamma_5\psi_i$$

$$+\frac{1}{3}w\, f^{abc}G^a_{\mu\nu}\widetilde{G}^{\nu\beta,b}G_\beta{}^{\mu,c} + \sum_{i,j=e,d,s,b} C_{ij}\,(\bar{\psi}_i\psi_i)(\bar{\psi}_j i\gamma_5\psi_j) + \cdots$$



- One needs hadronic, nuclear, atomic matrix elements to connect Wilson coefficients to observables

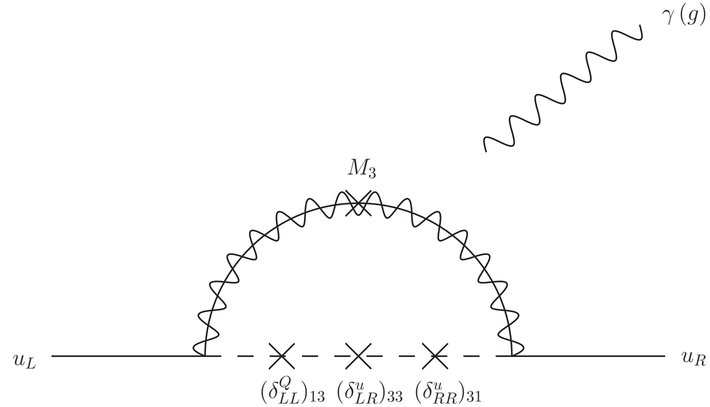
- Extremely high scales [10-100 TeV] can be probed if new physics generating EDMs violates CP maximally.

BSM: SUSY at 100 TeV and EDMs

(EDMs are not hopeless)

$$d_f \sim c_1 \frac{\delta m_f}{\Lambda^2_{\rm SUSY}} \theta_{\rm CP},$$

$$\tilde{d}_q \sim c_2 \frac{\delta m_q}{\Lambda^2_{\rm SUSY}} \ln\left(\frac{M_3^2}{\Lambda^2_{\rm SUSY}}\right) \theta_{\rm CP}$$



- Higgs mass point to a large scale of SUSY breaking, 10-100 TeV
- The requirements on approximately "flavor-aligned" scalar quark and scalar lepton sector are softened.
- LR mixing of quarks and leptons can get ~ m_t and m_τ instead of m_u and m_e. This can lead to a significant enhancement (McKeen, MP, Ritz, 2013)

$$\tilde{d}_u \simeq 5\times 10^{-26}\,{\rm cm}\left(\frac{4}{\tan\beta}\right)\left(\frac{\theta^2_{u13}M_3}{300\,{\rm GeV}}\right)\left(\frac{100\,{\rm TeV}}{\Lambda_{\rm SUSY}}\right)^3 \times\left[\ln\left(\frac{\Lambda^2_{\rm SUSY}}{M_3^2}\right)/10\right]\left(\frac{\sin\phi_{\tilde{u}\mu}}{1/\sqrt{2}}\right).$$

$$d_e \sim e f_e(r_1)\frac{\delta m_e}{\Lambda^2_{\rm SUSY}}\sin\phi_{\tilde{e}\mu}$$

$$\simeq 1\times 10^{-29}\,e\,{\rm cm}\left(\frac{\tan\beta}{4}\right)\left(\frac{\theta^2_{e13}M_1}{300\ {\rm GeV}}\right) \times\left(\frac{100\ {\rm TeV}}{\Lambda_{\rm SUSY}}\right)^3\left(\frac{\sin\phi_{\tilde{e}\mu}}{1/\sqrt{2}}\right),$$

Two sources of CP-violation in SM

- Theta term of QCD: too large EDMs if theta is arbitrary → new naturalness problem because of EDMs. ($d_n \sim \theta\, m_q/m_n^2$, $\theta < 10^{-10}$)

- Cabibbo-Kobayashi-Maskawa matrix and nearly maximal CP phase → still EDMs are too small to be observable in the next round of EDM experiments.

Our goal will be to examine paramagnetic EDM sensitivity to the SM CP violation, and to hadronic CP violation in general. In practice, how does the hadronic CP violation couple electron spin to electric field?

“Paramagnetic” EDMs:

- Paramagnetic EDM (EDM carried by electron spin) can be induced not only by a purely leptonic operator

$$d_e \times \frac{-i}{2}\, \overline{\psi}\sigma_{\mu\nu}\gamma_5 F_{\mu\nu}\psi$$

but by semileptonic operators as well:

$$C_S \times \frac{G_F}{\sqrt{2}}\, \overline{N}N\, \overline{\psi}i\gamma_5\psi$$

- Only a linear combination is limited in any single experiment. ThO 2018 ACME result is:

$|d_e| < 1.1\times10^{-29}$ e cm at $C_S = 0$

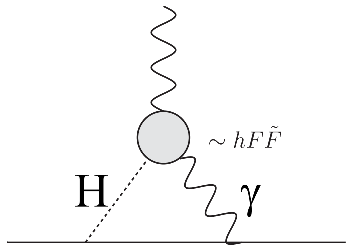
$|C_S^{singlet}| < 7.3\times10^{-10}$ at $d_e = 0$

$$d_e^{\text{equiv}} = d_e + C_S \times 1.5 \times 10^{-20}\, e\,\text{cm}$$ ← Specific for ThO

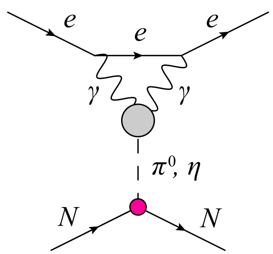
$$d_e^{equiv} = d_e + C_S{*}0.9{*}10^{-20}\ e\ cm$$ ← Specific for Hf F+

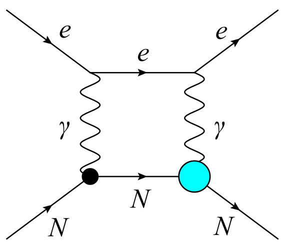
Hadronic CP violation → paramagnetic EDMs

- CP violation in top-Higgs sector – Barr Zee diagrams, h-γ mediation

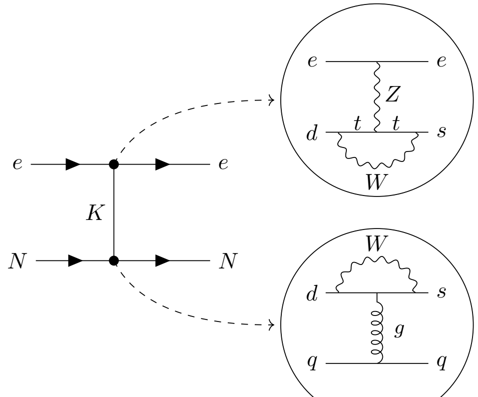


- Theta term, light quark, mu EDMs -- γ–γ mediation





- Kobayashi-Maskawa CP-violation – Z (and WW) mediation



Two-photon exchange induced C_S

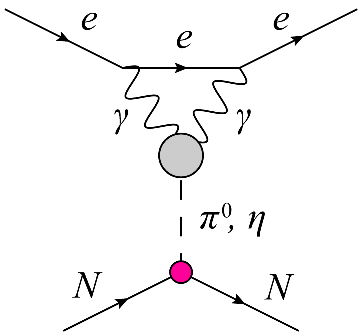
- Th used by ACME collaboration is a spin-less nucleus.
- ThO is mostly sensitive to CP violation in the lepton sector. If CP is broken in the strong interaction sector, *two photon exchange* can communicate it to the electron shells.
- Cutting across the two photons, the intermediate result can be phrased via *CP-odd nuclear polarizability*, **EB** δ(**r**), where E and B are created by an electron.
- Good scale separation is possible, $m_p >> p_F, \, m_\pi >> m_e \sim Z\alpha \, m_e$
- Nuclear uncertainties could be under control if the result is driven by "bulk" [as opposed to valence] nucleons.

LO chiral contribution:

- T-channel pion exchange gives

$$\mathcal{L} = \theta \times \frac{1}{m_\pi^2} \times 0.017 \times 3.5 \times 10^{-7} (\bar{e} i \gamma_5 e)(\bar{n}n - \bar{p}p)$$

$$= (\bar{e} i \gamma_5 e)(\bar{n}n - \bar{p}p) \times \frac{3.2 \times 10^{-13}\theta}{\text{MeV}^2}.$$



implying $|\theta| < 8.4 \times 10^{-8}$ sensitivity. However, adding exchange of η_8,

$$1 \to 1 - \frac{1}{3}\frac{f_\pi^2 m_\pi^2}{f_\eta^2 m_\eta^2} \times \frac{m_d - m_u}{m_d + m_u} \times \frac{A \times \sigma_N}{\frac{m_d - m_u}{2}\langle p|\bar{u}u - \bar{d}d|p\rangle \times (N - Z)}$$

$$1 \to 1 - 0.88 \simeq 0.12.$$

The effect can completely cancel within error bars on nucleon sigma term σ_N.

Photon box diagrams:

- Diagrams are IR divergent but regularized by Fermi momentum in the Fermi gas picture of a nucleus (intermediate N is above Fermi surface).

$$\mathcal{L} = \bar{e} i \gamma_5 e \bar{N} N \times \frac{2m_e \times 4\alpha \times \overline{d\mu} \times 6.2}{\pi p_F} = \bar{e} i \gamma_5 e \bar{N} N \times 2.4 \times 10^{-4} \times \overline{d\mu}$$

$$\overline{d\mu} = \frac{Z}{A}\mu_p d_p + \frac{A-Z}{A}\mu_n d_n = \frac{e}{2m_p} \times (1.08 d_p - 1.16 d_n)$$

- Nucleon EDM (theta) is very much a triplet, $d_p \simeq -d_n \simeq 1.6 \times 10^{-3} e\text{fm}\theta$

Full answer including chiral NLO. (accidental cancellation of π^0 and η)

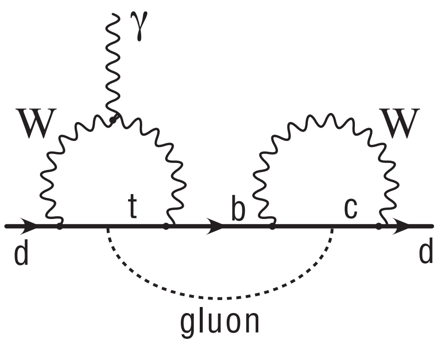
$$C_{SP}(\bar{\theta}) \approx \left[0.1_{\rm LO} + 1.0_{\rm NLO} + 1.7_{(\mu d)}\right] \times 10^{-2} \bar{\theta} \approx 0.03\,\bar{\theta}$$

Limit on theta term from ThO (electron EDM) experiment:

$$|\bar{\theta}|_{\rm ThO} \lesssim 3 \times 10^{-8}$$

* Improved by a factor of ~ 2 in Dec 2022, $\theta < 1.5 * 10^{-8}$

EDMs from SM sources: CKM



CKM phase generates tiny EDMs:

$$d_d \sim \mathrm{Im}(V_{tb}V_{td}^{*}V_{cd}V_{cb}^{*})\alpha_s m_d G_F^2 m_c^2 \times \text{loop suppression}$$
$$< 10^{-33} e\mathrm{cm}$$

- Quark EDMs identically vanish at 1 and 2 loop levels, $EW^2=0$

(Shabalin, 1981).

- 3-loop EDMs, EW^2QCD^1 are calculated by Khriplovich; Czarnecki, Krause.

- d_e vanishes at EW^3 level (Khriplovich, MP, 1991) < 10^{-38} e cm. It was calculated recently by Yamaguchi, Yamanaka to be 6 10^{-40} e cm

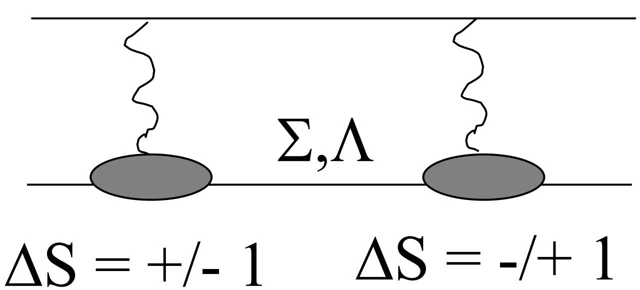
- Long distance effects give neutron EDM ~ 10^{-32} e cm; uncertain.

CKM CP-violation and paramagnetic EDMs

- Several groups attempted to calculate d_e (MP, Khriplovich; …)

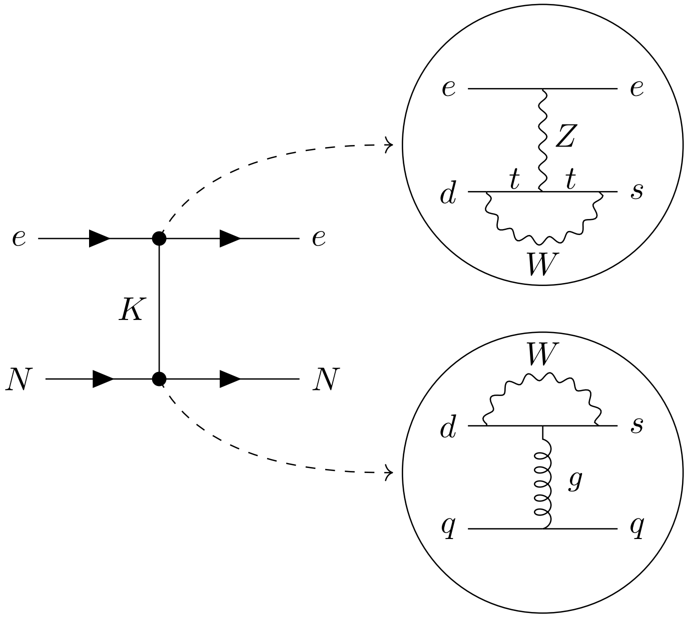
- The result is small to the point of being not interesting (e.g. 10 orders of magnitude below current bounds)

- Semileptonic (C_S) operator is more important. MP and Ritz (2012) estimated two-photon mediated EW^2EM^2 effects and found that CS is induced at the level equivalent to ~ 10^{-38} e cm



It turns out that there are much larger contributions at EW^3 order

Semileptonic CP operator at EW^3 order



- The induced semileptonic operator is calculable in chiral perturbation theory (in m_s expansion)
- The result is large, d_e(equiv) = + 1.0 10^{-35} e cm
- Same EW penguin that is responsible for $B_s \rightarrow \mu\mu$, Re $K_L \rightarrow \mu\mu$

Ema, Gao, MP, PRL2022

Semileptonic Electroweak Penguin

- The upper part: EW penguin $\mathcal{L}_{\rm EWP} = \mathcal{P}_{\rm EW} \times \bar{e}\gamma_\mu\gamma_5 e \times \bar{s}\gamma^\mu(1-\gamma_5)d + (h.c.)$

$$\mathcal{L}_{Uee} = -\frac{if_0^2}{2}\mathcal{P}_{\rm EW} \times \bar{e}\gamma_\mu\gamma_5 e \times {\rm Tr}\left[h^\dagger\left(\partial^\mu U\right)U^\dagger\right] + (h.c.),$$

In the leading order, the dominant diagram is K_S exchange.

$$\mathcal{L}_{Kee} = -2\sqrt{2}f_0 m_e \bar{e}i\gamma_5 e\left(K_S \times {\rm Im}\mathcal{P}_{\rm EW} + K_L \times {\rm Re}\mathcal{P}_{\rm EW}\right)$$

- Lower part: EW[1] B-B-M coupling is related by flavor SU(3) to the s-wave amplitudes of the non-leptonic hyperon decays. Theory fit to decay amplitudes is [surprisingly] good (~5-10%):

$$\mathcal{L}_{\rm SP} = -a{\rm Tr}(\bar{B}\{\xi^\dagger h\xi, B\}) - b{\rm Tr}(\bar{B}[\xi^\dagger h\xi, B]) + (h.c.).$$

contains $2^{1/2}f_0^{-1}((b-a)\bar{p}p + 2b\bar{n}n)K_S$

LO kaon exchange result

- Using EW penguin and strong penguin below,

$$\mathcal{L}_{KNN} \simeq -\frac{\sqrt{2}\, G_F \times [m_{\pi^+}]^2 f_\pi}{|V_{ud}V_{us}| f_0} \times 2.84(0.7\bar{p}p + \bar{n}n)$$
$$\times \left(\mathrm{Re}(V_{ud}^* V_{us}) K_S + \mathrm{Im}(V_{ud}^* V_{us}) K_L\right).$$

We calculate C_S

$$C_S \simeq \mathcal{J} \times \frac{N + 0.7Z}{A} \times \frac{13[m_{\pi^+}]^2 f_\pi m_e G_F}{m_K^2} \times \frac{\alpha_{\mathrm{EM}} I(x_t)}{\pi \sin\theta_W^2}$$

$$\mathcal{J} = \mathrm{Im}(V_{ts}^* V_{td} V_{ud}^* V_{us}) \simeq 3.1 \times 10^{-5}$$

That has the following LO scaling

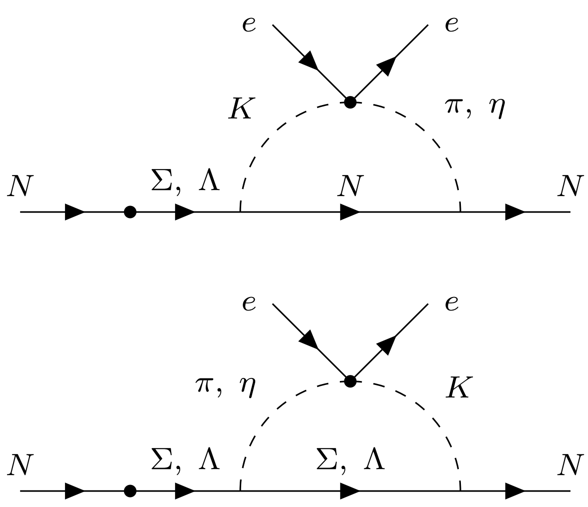
$$G_F C_S \propto \mathcal{J} G_F^3 m_t^2 m_e m_s^{-1} \Lambda_{\mathrm{hadr}}^2$$

Numerically, it is

$$C_S(\mathrm{LO}) \simeq 5 \times 10^{-16}.$$

NLO kaon-pion loop

- We calculate leading order corrections that have $(m_s)^{-1/2}$ scaling



- The loop itself is proportional to ~ m_K, but there is a baryonic pole that brings $1/m_s$.

The NLO brings positive contribution of ~ 30%.

$$\frac{C_{S,NLO}(p)}{C_{S,LO}(p)} = \frac{m_K^3(0.77D^2 + 2.7DF - 2.3F^2)}{24\pi f_0^2(m_{\Sigma^+} - m_p)}$$

$$\frac{C_{S,NLO}(n)}{C_{S,LO}(n)} = \frac{m_K^3}{24\pi f_0^2}\left(\frac{(a/b+3)}{2\sqrt{6}(m_\Lambda - m_n)}\right. \times(-0.44D^2 + 3.2DF + 1.3F^2) \left. + \frac{a/b - 1}{2\sqrt{2}(m_{\Sigma^0} - m_n)}\left(-0.53D^2 - 1.9DF + 1.6F^2\right)\right).$$

Final result

- Combining $(m_s)^{-1}$ and $(m_s)^{-1/2}$ effects, we get

$$C_S(\mathrm{LO}+\mathrm{NLO}) \simeq 6.9\times 10^{-16}$$
$$\Longrightarrow \; d_e^{\mathrm{equiv}} \simeq 1.0\times 10^{-35}\, e\,\mathrm{cm}.$$

- The result EW^3 much larger than the EW^2EM^2 estimate by ~1000.

- Note that actually establishing the correct sign is tricky.

- The result is under "best possible" theoretical control, and can be improved on the lattice

$$\langle N|i(\bar{s}\gamma_\mu(1-\gamma_5)d-\bar{d}\gamma_\mu(1-\gamma_5)s)|N\rangle_{\mathrm{EW}^1}$$
$$=\frac{f_S}{m_N}iq_\mu\bar{N}N+\frac{f_T}{m_N}q_\nu\bar{N}\sigma_{\mu\nu}\gamma_5 N.$$

EDMs of heavy flavors

- Among Wilson coefficients of different kind, EDMs of heavy flavours d_i are interesting. i = muon, tau, charm, bottom, top.

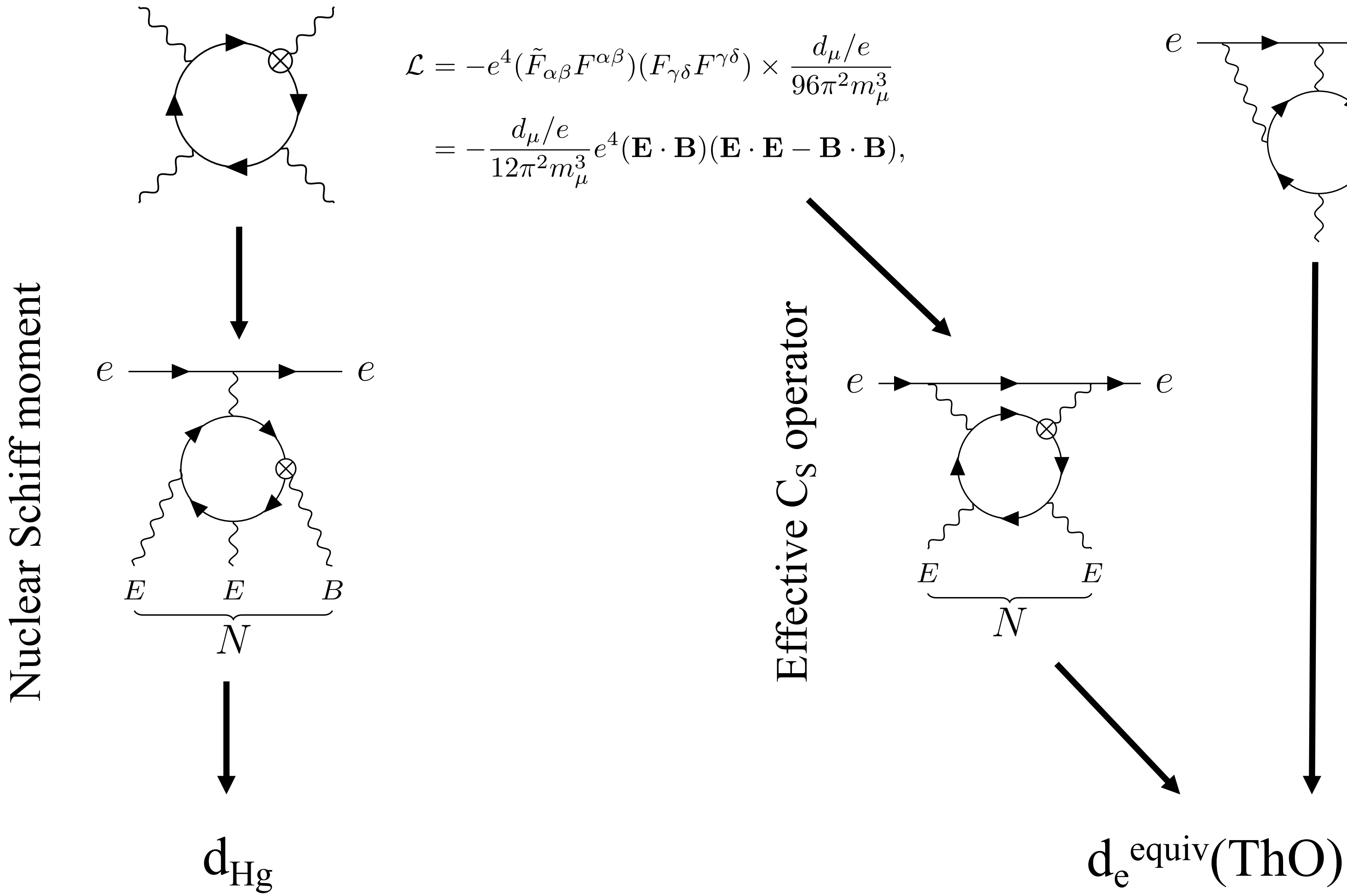
- Muon EDM is limited as a biproduct of BNL g-2 experiment. Can be significantly improved in dedicated beam experiments (PSI, Fermilab, J-Parc)

- There is a creative proposal to measure MDMs and limit EDMs of charmed baryons using thin fixed target and bent crystal technology just before the LHCb experiment (E. Bagli et al, 2017).

- Heavy flavors contribute to observable EDMs via loops. Top quark EDM is limited indirectly by electron EDM via a two-loop (top-Higgs-gamma) Barr-Zee diagrams. The result is stronger than the direct measurements at LHC.

Muon EDM inside a loop

- Muon loop induces E^3B effects, and electron EDM at 3-loops.



$$\mathcal{L} = -e^4(\tilde{F}_{\alpha\beta}F^{\alpha\beta})(F_{\gamma\delta}F^{\gamma\delta}) \times \frac{d_\mu/e}{96\pi^2 m_\mu^3}$$
$$= -\frac{d_\mu/e}{12\pi^2 m_\mu^3} e^4 (\mathbf{E}\cdot\mathbf{B})(\mathbf{E}\cdot\mathbf{E} - \mathbf{B}\cdot\mathbf{B}),$$

New indirect constraints on muon EDM

- Owing to the fact that the electric field inside a large nucleus is not that small $eE \sim Z\alpha R_N^{-1} \sim 30$ MeV compared to m_μ, effects formally suppressed by higher power of m_μ win over three-loop electron EDM.

- New results:

Hg EDM experiment: $S_{^{199}\mathrm{Hg}}/e \simeq (d_\mu/e) \times 4.9 \times 10^{-7}\,\mathrm{fm}^2, \quad |d_\mu| < 6.4 \times 10^{-20}\,e\,\mathrm{cm}$

ThO EDM experiment: $d_e^{\mathrm{equiv}} \simeq 5.8 \times 10^{-10}\,d_\mu \implies |d_\mu| < 1.9 \times 10^{-20}\,e\,\mathrm{cm}.$

New limit from Boulder HfF: $\sim 9*10^{-21}$

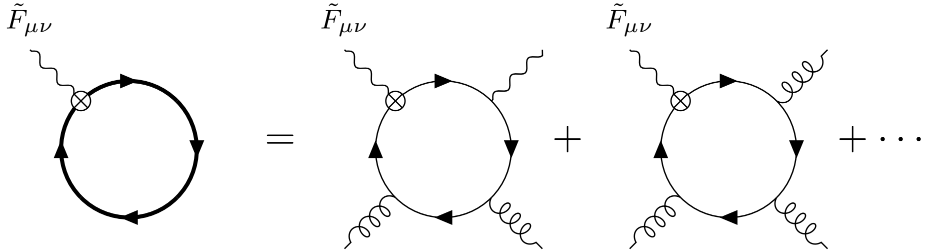
- Factor of 20 improvement over the BNL constraint, $|d_\mu| < 1.8 \times 10^{-19}$
- New benchmark for the muon beam EDM experiments.

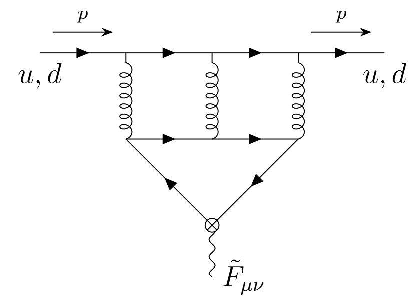
NB: 3-loop contributions calculated by Grozin et al. will be revised

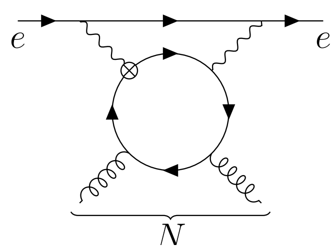
- Tau EDM is constrained by three-loop induced d_e.

Charm and bottom EDMs

Charm loop gives (γ)^2(gluon)^2 and (γ)^1(gluon)^3 effective operators

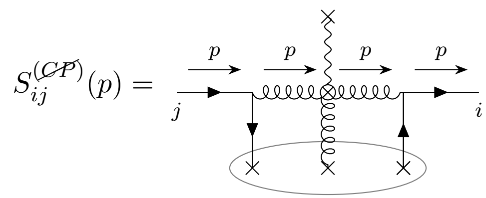






$$\langle N|\frac{g_s^2}{32\pi^2}G^a_{\mu\nu}G^{a\mu\nu}|N\rangle = -\frac{m_N}{9}\bar{N}N,$$

d_e^{equiv}(ThO)



Nonperturbative 3-gluon
induced tensor charge

d_n , d_{Hg}

- All EDMs are induced by charm and bottom EDMs.

New indirect constraints on c-, b- quarks EDMs

- New results:

Neutron EDM experiment: $|d_c| < 6 \times 10^{-22}\, e\,\mathrm{cm}, \quad |d_b| < 2 \times 10^{-20}\, e\,\mathrm{cm},$

ThO EDM experiment: $|d_c| < 1.3 \times 10^{-20}\, e\,\mathrm{cm}, \quad |d_b| < 7.6 \times 10^{-19}\, e\,\mathrm{cm},$

- Neutron EDM estimates have uncertainty ~ up to a factor of O(few) due to limitation of QCD sum rule method in this channel. C_S derived limits have *minimal* uncertainty, O(10%).
- Independent of (similar order of magnitude) bounds based on RG running of operators, and contribution to the GGGdual Weinberg operator.
- The strength of these limits on charm EDM points to the conclusion that future charmed baryon EM moment proposal should focus on MDM.

Conclusions, part I

- In *lots of hadronic CP violation models*, including the SM, the paramagnetic EDMs (*experiments looking for* d_e) are induced by the semi-leptonic operators of (electron pseudoscalar)*(nucleon scalar) type.
- C_S is induced by theta term via a two-photon exchange resulting in sensitivity $|\theta| < 1.5\times10^{-8}$. Further progress by O(100) for d_e type of experiments will bring the sensitivity to hadronic CP violation on par with current d_n limits.
- CKM CP violation induces C_S. The result is large and calculable and is dominated by the EW^3 order. The *equivalent* d_e (ThO) is found to be $+1.0 \times 10^{-35}$ e cm. This is 1000 times larger than previously believed.
- New indirect limits on muon, charm and bottom provide new target for the EDM beam experiments: $d_\mu < 9*10^{-21}\ e\ cm$

Revisiting nonperturbative calculations of d_n

- Use chiral PT, rely on IR enhanced contributions, use some pheno input (or lattice input) to infer π-NN CP-odd couplings. (Crewther, DiVecchia, Veneziano, Witten, ++, 1980++)

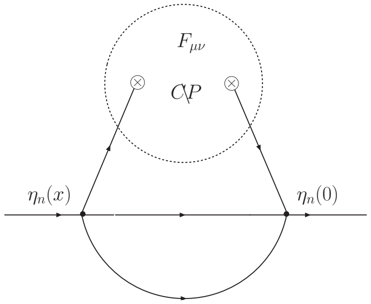
- MP, A. Ritz: 1999-2002: apply QCD sum rules to estimate the OPE coefficients in the external CP-violating and EM backgrounds. Some intriguing parallels to the naïve quark model (NQM) answers are established.

- Preferable direction: set up proper lattice QCD calculations. Tensor charges are calculated, but observables that are very sensitive to the quark mass, such as d_n(theta) prove to be difficult.

- Ema, Gao, MP – ongoing. Investigate chiral properties of the correlator of nucleon interpolating currents. Explore SR ←→ NQM

Nonperturbative calculations of nucleon (hadronic) observables

$$\Pi(Q^2) = i\int d^4x e^{ip\cdot x}\langle 0|T\{\eta_N(x)\overline{\eta}_N(0)\}|0\rangle_{C\!\!\!/P,F,\pi},$$



- Interpolating η currents can be formulated in terms of 3 quarks with appropriate quantum numbers.

- $\Pi(x)$ can be calculated at short distances, using perturbative QCD + nonperturbative condensates. On the other hand, due to quark-hadron duality, we expect that $\Pi(Q^2)$ has also representation in terms of the hadronic resonances and their matrix elements. QCD sum rules *hopes* to match the two at some intermediate/borderline scale, $Q^2 \sim GeV^2$.

- Lattice QCD can perform these calculations "honestly", $x \rightarrow$ large

Nucleon Interpolating Currents

$$\eta_n = j_1^{(n)} + \beta j_2^{(n)}.$$

$$j_1^{(n)} = 2\epsilon_{ijk}\left(d_i^T \mathcal{C}\gamma_5 u_j\right) d_k, \quad j_2^{(n)} = 2\epsilon_{ijk}\left(d_i^T \mathcal{C} u_j\right)\gamma_5 d_k,$$

- $\beta = 0$, $\eta = j_1$, is the so-called QCD current i.e. the current used the most in the lattice QCD community. It takes its origin in the NQM, because it is j_1 that has a NR limit.

- $\beta = -1$ can be called "Ioffe current", and it has been used the most in various QCD SR literature of 1980s-1990s.

- $\beta = +1$ found to be the most convenient choice (MP and Ritz) for the neutron EDM calculations created by external sources.

Recap of d_n results (QCD SR, $\beta = 1$)

- Use odd-number of γ-matrices for the SR, and spurious phases of the 2-point functions will never appear

- Simple estimate based on the leading term of the OPE has a strong correspondence with the NQM (according to "Ioffe formula", the coefficient outside the square brackets below = 1).

$$d_n^{\text{est}} = \frac{8\pi^2 |\langle \overline{q} q \rangle|}{m_n^3} \Big[-\frac{2\chi m_*}{3} e(\bar{\theta} - \theta_{\text{ind}}) + \frac{1}{3}(4d_d - d_u) + \frac{\chi m_0^2}{6}(4e_d \tilde{d}_d - e_u \tilde{d}_u) \Big],$$

- Why such a correspondence; what is so special about β=1 current?

Back to basics: QCD + theta term

$$\mathcal{L}_{QCD} = -\frac{1}{4}(G^a_{\mu\nu})^2 + \sum_{u,d,} \bar{q}(iD_\mu\gamma_\mu - m_q)q + \frac{\theta g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

Do a standard iso-singlet quark chiral rotation to eliminate θGGdual.

$$\to m_*(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d)\theta + m_*(\bar{u}u + \bar{d}d)\theta^2/2 + \ldots$$

m_* is the reduced quark mass, $m_u m_d(m_u+m_d)$. The expectation value of the second term over the vacuum here is the vacuum energy dependence on the theta angle (and upon the rescaling the axion mass squared.) *We assume that U(1) problem is solved somehow, and the mass of the singlet is lifted. Otherwise, pole diagram with the singlet will cancel theta dependence.* Expectation value of the second term over nucleon, gives theta-dependence of nucleon mass.

All observables that depend on q should also depend on m_* and vanish in the chiral limit! Also, observables do not depend on how you distribute θ, putting some parts to quark mass, and some to GGdual.

QCD + theta term + Nucleon Source

$$\mathcal{L} = -\frac{1}{4}(G^a_{\mu\nu})^2 + \sum_{u,d,} \bar{q}(iD_\mu\gamma_\mu - m_q)q + \frac{\theta g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} + \text{Source} \times (j_1 + \beta j_2) + h.c.$$

- This is the basis for studying nucleon properties. It is almost QCD, but not quite!

- Let us perform a chiral rotation, as on the previous slide. If this transformation would lead to

$$\text{Source} \times (j_1 + \beta j_2) \;\rightarrow\; \text{Source} \times e^{i\alpha\gamma_5} \times (j_1 + \beta j_2)$$

then it is an innocent transformation, and the new phase can be reabsorbed into the source. Otherwise, θ dependence will persist even in $m_q \rightarrow 0$ limit.

- This is true only for β=1 and β=-1 current choices. It is specifically not true for the lattice current β=0. It has unphysical L←→R quark transitions.

Unphysical θ dependence of some correlators

- Under the iso-singlet chiral transformation,

$$j_1 = 2\epsilon^{abc} d^a (d^{bT} C\gamma_5 u^c) \rightarrow 2\epsilon^{abc} e^{i\theta\gamma_5} d^a (d^{bT} C\gamma_5 e^{i2\theta\gamma_5} u^c)$$

- This results in a rephasing-invariant theta-dependent pieces in the OPE:

$$\Pi(x)/24 = b_2^3 \langle \bar{q} q \rangle^3 (4\cos(4\theta) e^{i2\theta\gamma_5} + e^{i6\theta\gamma_5}) \\ + b_2 b_1^2 \langle \bar{q} q \rangle x^2 (6 e^{i2\theta\gamma_5} + e^{-i2\theta\gamma_5})$$

- If the correlator Π(*x*) is matched to physical observables (e.g. hadron masses, they will acquire *θ*-dependence in the strict chiral limit.)

- Absolutely same problems will persist in the $d_n(\theta)$ calculation performed with the "lattice current". There will be dependences, in general, on unphysical phases, related to the chirality breaking built into the interpolating current itself.

Chiral currents and limited duality with NQM

- Let us return to β=1 current. We can show that there is a strong (but not absolute) correspondence between isospin structure of the first term in the OPE and the naïve quark model (for the vector, scalar, and tensor charges). Also, anomalous magnetic moments reproduce NQM, i.e. famous relation: $\mu_p/\mu_n = (4Q_u - Q_d)/(4Q_d - Q_u) = -3/2$.
- The current can be rewritten as the combination of purely L and purely R current. In the OPE calculations these two parts connect only at very high order. It is then $\Pi \sim \langle LLL, LLL \rangle + \langle RRR, RRR \rangle$

$$\eta_n(\beta=1) = 4\epsilon_{ijk}\left[\left(d^T_{Ri}\mathcal{C}u_{Rj}\right)d_{Rk} - \left(d^T_{Li}\mathcal{C}u_{Lj}\right)d_{Lk}\right]$$

- Thus, in the calculations, effectively, all propagators are sandwiched between projectors, $S_{dLL} \sim (1-\gamma_5)/2\, S_d\, (1+\gamma_5)/2$. NRQM, on the other hand, technically is the application of $(1+\gamma_0)/2$ projector.
- At the technical level, SR vs NRQM correspondence comes as γ_5 vs γ_0 duality. In many calculations, you can replace the one by another. Also “dimensionality” is reduced from 4 to 2 in both cases.

Conclusions for Part II

- Chiral properties of the nucleons interpolating currents, under $U(1)_A$ rotations, are crucial for obtaining observables such as those dependent on θ, and vanishing in $m_* \to 0$ limit.
- The “lattice currents” do not transform covariantly under $U(1)_A$ rotations, leading to spurious dependences of correlators on unphysical angles.
- The physical behavior of nucleon correlators is guaranteed with $\beta=1$ and $\beta= -1$ current choices. We suggest that Lattice QCD community uses those for e.g. d_n(theta), as well as explores different chirality channels (pointed out in Pospelov and Ritz) that are guaranteed to have no unphysical phases.