

VIRTUALITY EVOLUTION OF JETS IN A QCD MEDIUM

Study of heavy-quark pairs within jets using QCD Factorization in heavy-ion collisions

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**WORK IN
PROGRESS!**

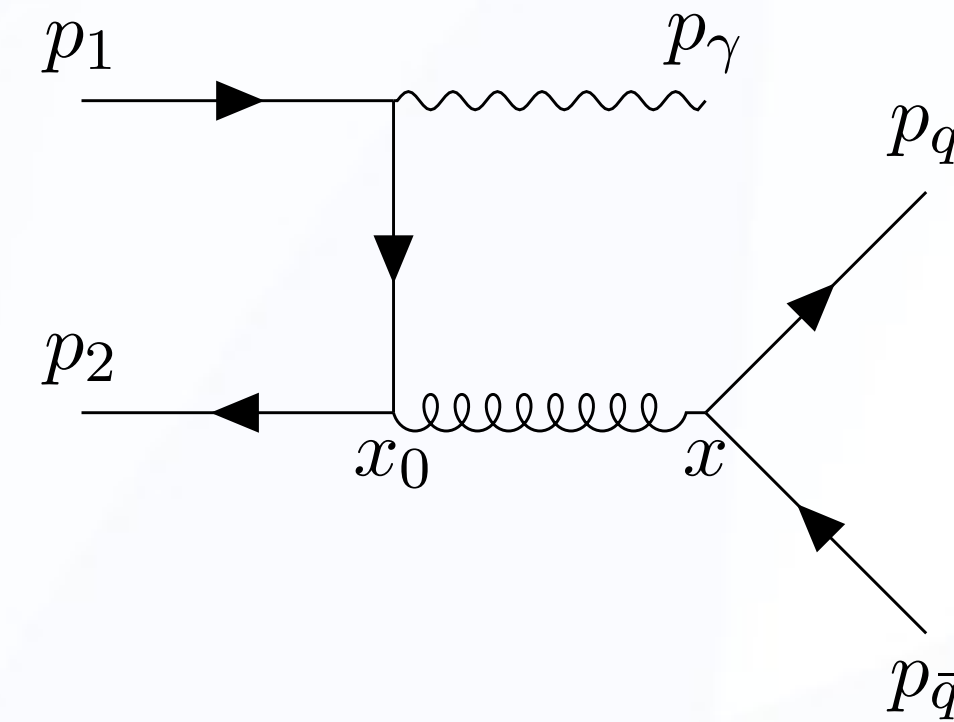
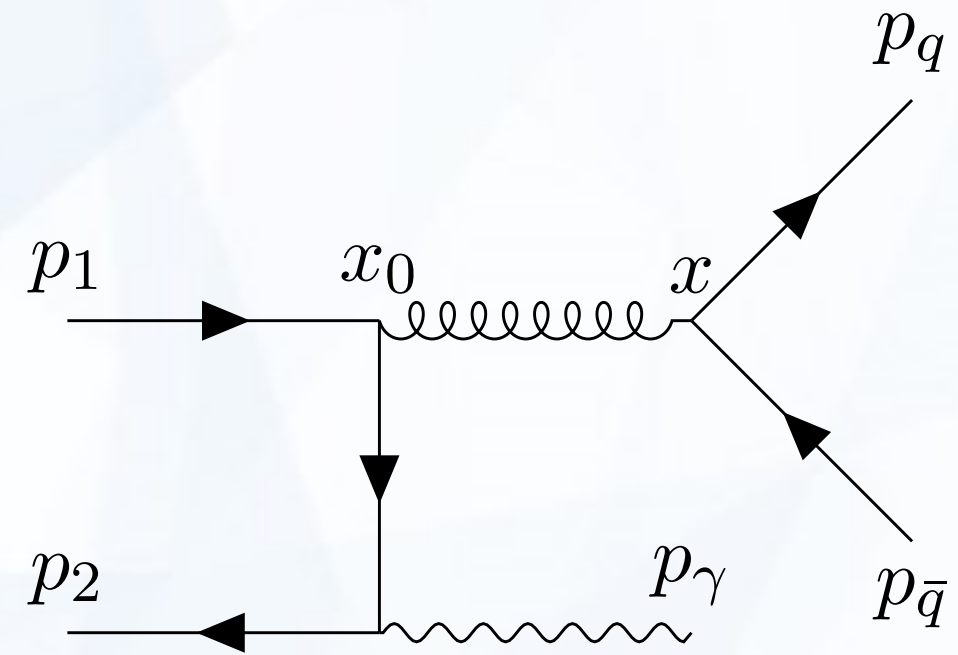
I. FACTORIZATION OF JET CROSS SECTIONS IN HEAVY-ION COLLISIONS

Our HIC model

$$A_1 + A_2 \rightarrow \gamma + \text{Jet} + X$$

-  Nucleons inside nuclei are considered to be **uncorrelated**.
-  For high momentum scales the hard process is mostly initiated by the collision of **one pair of nucleons**

Parton-level diagrams



Other nucleons **only connect to the jet** in the hard process (see Florian's talk)

The impact parameter dependent cross section

- ★ Cross section **divided into jet and hard vertex sectors**

$$\frac{d\sigma}{d^2\mathbf{b}dO} = \int \prod_f \left[d\Gamma_{p_f} \right] \delta(O - O(\{p_f\})) \int d^2\mathbf{X} T_{A_1}(\mathbf{X}) T_{A_2}(\mathbf{X} - \mathbf{b}) \int \frac{d^4x_I d^4p_I}{(2\pi)^4} e^{ip_I \cdot x_I} J_{\mu\nu}(X_T + x_I/2, X_T - x_I/2)$$

$$\times \sum_{X_h} (2\pi)^4 \delta(P_{A_1} + P_{A_2} - p_\gamma - p_I - p_{X_h}) \frac{1}{2s_{NN}} M_h^{*\mu}(P_{A_1}, P_{A_2}; p_\gamma, p_{X_h}) M_h^\nu(P_{A_1}, P_{A_2}; p_\gamma, p_{X_h})$$

where $X = (x'_0 + x_0)/2$ and $x_I = x'_0 - x_0$

- 🏷 We defined the **jet tensor**

$$J_{\mu\nu}^{ab}(x'_0, x_0) \equiv \langle\langle\langle 0 | \bar{T}A_\mu^a(x'_0) | p_q p_{\bar{q}} \rangle \langle p_q p_{\bar{q}} | TA_\nu^b(x_0) | 0 \rangle \rangle \rangle = \frac{\delta^{ab}}{N_c^2 - 1} \langle\langle\langle 0 | \bar{T}A_\mu^c(x'_0) | p_q p_{\bar{q}} \rangle \langle p_q p_{\bar{q}} | TA_\nu^c(x_0) | 0 \rangle \rangle \rangle$$

Range of validity of factorization

 Cross section **still not factorized**



Introduce a light-like vector along p_I direction and define a **new set of LC-coordinates**

$$J_{\mu\nu}(X_T + x_I/2, X_T - x_I/2) \propto g_{\mu\nu} - p_I^+ \frac{p_{I\mu} \bar{n}_\nu + p_{I\nu} \bar{n}_\mu}{(p_I^+)^2 - p_I^2 \bar{n}^2} + p_I^2 \frac{\bar{n}_\mu \bar{n}_\nu}{(p_I^+)^2 - p_I^2 \bar{n}^2} + \bar{n}^2 \frac{p_{I\mu} p_{I\nu}}{(p_I^+)^2 - p_I^2 \bar{n}^2}$$

$$\simeq g_{\perp\mu\nu} + O\left(\frac{m_I^2}{(p_I^+)^2}\right)$$

 We must include a **factorization scale**.

Factorized cross section



In the phase space region where collinear factorization works

$$\frac{d\sigma}{d^2\mathbf{b}d\omega d^2\mathbf{p}_\gamma d\eta_\gamma} = \int d^2\mathbf{X} T_{A_1}(\mathbf{X}) T_{A_2}(\mathbf{X} - \mathbf{b}) \int d\Gamma_{p_I} \frac{d}{d\omega} J(\vec{n} \cdot p_I, \vec{n}; \mathbf{X}) \frac{d\sigma_h}{d\Gamma_{p_I} d^2\mathbf{p}_\gamma d\eta_\gamma}(P_{A_1}, P_{A_2}; p_I, p_\gamma)$$

The **hard-cross section** can be computed from the PDFs and tree level amplitude



From now on we will focus on the calculation of the **jet function**

The jet function

$$\frac{d}{do} J(\bar{n} \cdot p_I, \vec{n}; \mu^2, \mathbf{X}) \equiv \int_0^{\mu^2} \frac{dm_I^2}{2\pi} \int d^4 x_I e^{\frac{i}{2} \bar{n} \cdot p_I n \cdot x_I + \frac{i}{2} \frac{m_I^2}{\bar{n} \cdot p_I} \bar{n} \cdot x_I} \left(\sum_{m=1}^{\infty} \prod_{j=1}^m \int d\Gamma_{p_j} \right) \delta(o - o(\{p_j\}))$$

$$\times \frac{-g_{\perp}^{\mu\nu}}{2(N_c^2 - 1)} \langle \langle \langle 0 | \bar{T} A_{\mu}^a(X + x_I/2) | \{p_j\} \rangle \langle \{p_j\} | T A_{\nu}^a(X - x_I/2) | 0 \rangle \rangle \rangle$$



Only valid in **LC-gauge**


For a general gauge we must construct the S-matrix element as a **gauge-invariant object**

II. THE JET FUNCTION FOR $Q\bar{Q}$ PRODUCTION AT LO

The coordinates for the jet function

 The hard collision takes place around $X^\mu = (0, \mathbf{X}, 0)$


$$t_h \equiv \bar{n} \cdot X$$

 $x_I^+ \sim 1/p_I^- = 2p_I^+/m_I^2$ needed to account for the **uncertainty of jet production** time around t_h .

 \underline{x}_I will account for the **initial position** of the jet

The general formula for $Q\bar{Q}$ spectrum

The **main formula** on our model


$$\frac{dJ}{dz dm_I^2}(p_I^+, \vec{n}; m_I^2, X) = \frac{1}{2\pi} \int d^2 \underline{x}_I dx_I^+ e^{\frac{i}{2} \frac{m_I^2}{p_I^+} x_I^+} \int \frac{d^2 p_{-q}}{(2\pi)^2} \frac{d^2 p_{-\bar{q}}}{(2\pi)^2} \frac{1}{8\pi z(1-z)p_I^+}$$

$$\times \frac{-1}{2(N_c^2 - 1)} \langle \langle \langle 0 | \bar{T} A_{a\mu}(X_T + \tilde{x}_I/2) | p_q p_{\bar{q}} \rangle \langle p_q p_{\bar{q}} | T A^{a\mu}(X_T - \tilde{x}_I/2) | 0 \rangle \rangle \rangle$$

-  If we integrate in m_I^2 up to ∞ we go to the **usual limit**

III. SOME PRELIMINAR RESULTS

A. The gluon jet function for $Q\bar{Q}$ in vacuum

B. $g \rightarrow g$ due to single scattering

C. $g \rightarrow Q\bar{Q}$ due to single scattering

Our model for calculations...



BDMPS-Z formalism

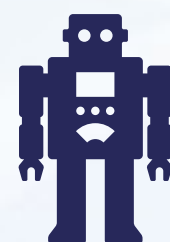


Model the effect of other nucleons by **classical fields**

$$\langle A^{a-}(t_1, \underline{x}) A^{b-}(t_2, \underline{y}) \rangle = \delta(t_1 - t_2) \delta^{ab} 2n(t) \sigma(\underline{x} - \underline{y})$$

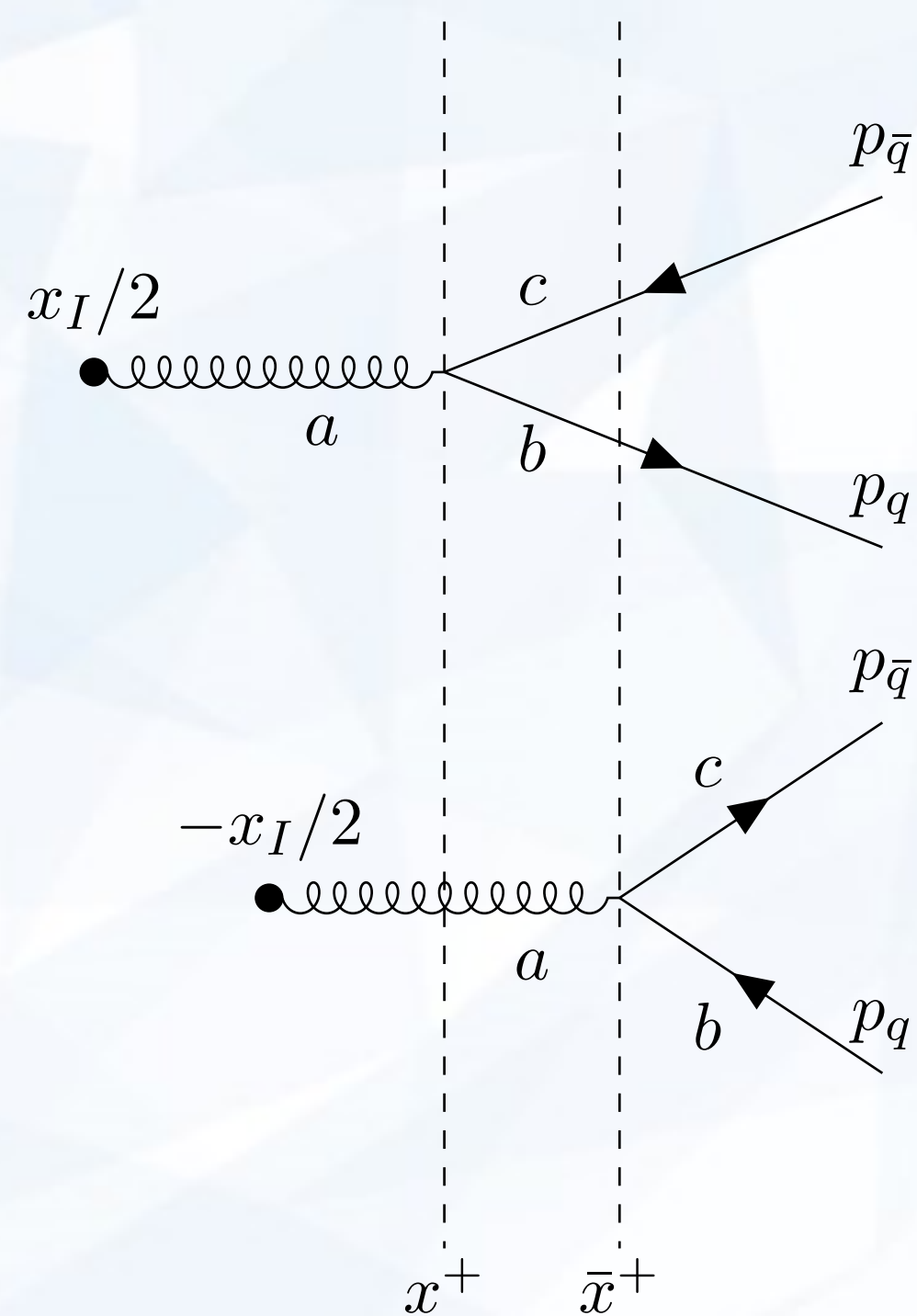


The gluon jet can extract **as much virtuality from the medium as desired** in a single scattering.



We will still use it as a **toy model**

A. The gluon jet function for $Q\bar{Q}$ in vacuum



[Ellis, Stirling, Webber (2011)]

$$\frac{dJ}{dz dm_I^2}(p_I^+, \vec{n}; m_I^2, X) = \frac{\alpha_s}{2\pi} \frac{1}{m_I^2} P_{Q \leftarrow g}(z, m, m_I^2) \theta(z(1-z)m_I^2 - m^2)$$

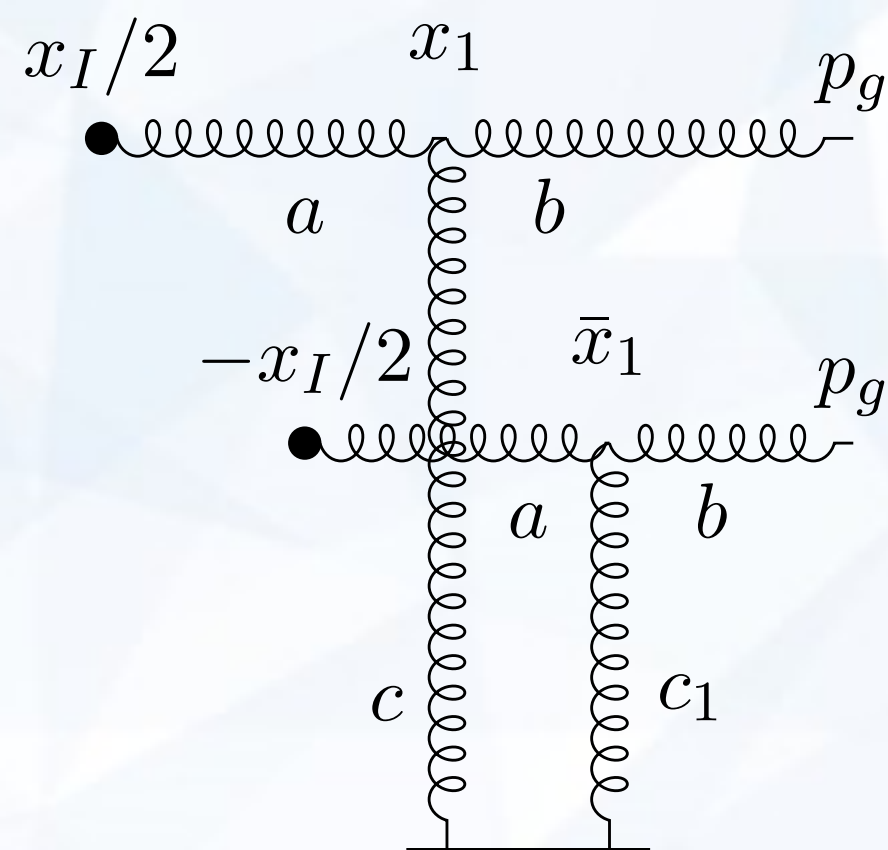
where $P_{Q \leftarrow g}(z, m, m_I^2) \equiv T_F \left[z^2 + (1-z)^2 + \frac{2m^2}{m_I^2} \right]$. Integrating in z and m_I^2

$$J(p_I^+, \vec{n}; \mu^2, X) = \frac{\alpha_s}{6\pi} \left[-\frac{\sqrt{1 - \frac{4m^2}{\mu^2}}}{3\mu^2} + 2 \log \left(\sqrt{1 - \frac{4m^2}{\mu^2}} + 1 \right) + \log \left(\frac{\mu^2}{4m^2} \right) \right]$$



The factorization scale **regularizes** the UV divergence

B. $g \rightarrow g$ due to single scattering



$$\frac{dJ}{dm_I^2}(p_I^+, \vec{n}; m_I^2, X) = \frac{1}{2\pi} \int dx_1^+ [g^2 2n(x_1^+) C_A \sigma_T] \frac{4p_I^+}{m_I^2} \sin\left(\frac{m_I^2}{p_I^+} x_1^+\right)$$



Brick approximation and integration over virtuality

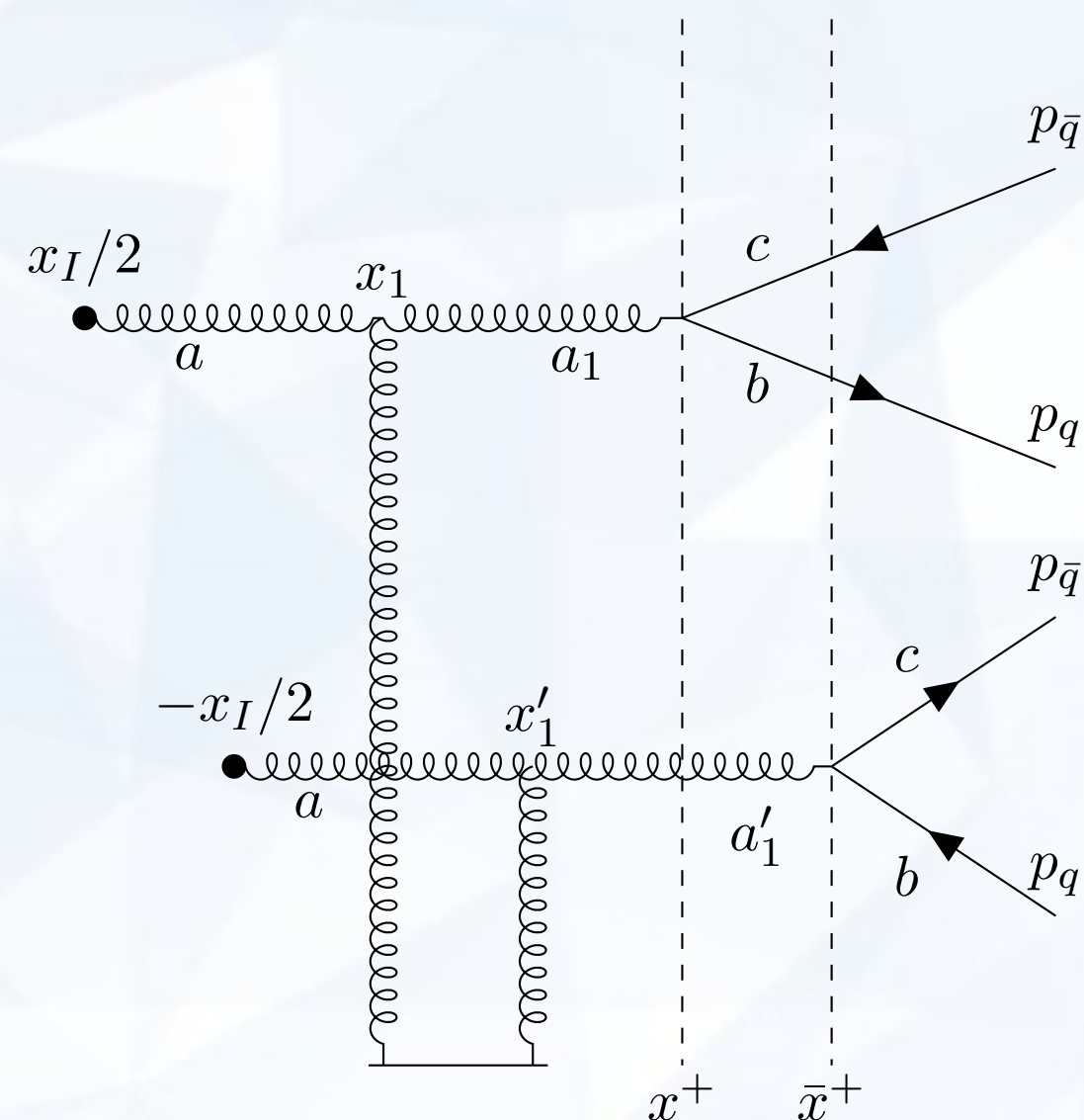
$$J(p_I^+, \vec{n}; \mu^2, X) = \frac{\alpha_s}{2\pi} (2n C_A \sigma_T) 4p_I^+ \left[\frac{L}{p_I^+} \text{Si}\left(\frac{L}{p_I^+} \mu^2\right) + \frac{\cos\left(\frac{L}{p_I^+} \mu^2\right) - 1}{\mu^2} \right]$$



In the $\mu \rightarrow \infty$ limit

$$J(p_I^+, \vec{n}; \mu^2 \rightarrow \infty, X) = \alpha_s (2n C_A \sigma_T) L = 2C_A \frac{L}{\lambda}$$

C.3. $g \rightarrow Q\bar{Q}$ due to single scattering



$$\frac{dJ}{dx dm_I^2}(p_I^+, \vec{n}; m_I^2, X) = \alpha_s(\alpha_s 2C_A n\sigma_T) \frac{4p_I^+}{(m_I^2)^2} \left[1 - \cos\left(\frac{m_I^2}{p_I^+} L\right) \right] \int \frac{d^2p}{2\pi} \frac{m^2 + (z^2 + (1-z)^2)p^2}{[p^2 + m^2]^2}$$

Virtuality integration

$$\frac{dJ}{dz}(p_I^+, \vec{n}; \mu^2, X) = \alpha_s(\alpha_s 2C_A n\sigma_T) 4p_I^+ \left[\frac{L}{p_I^+} \text{Si}\left(\frac{L}{p_I^+} \mu^2\right) + \frac{\cos\left(\frac{L}{p_I^+} \mu^2\right) - 1}{\mu^2} \right] \int \frac{d^2p}{2\pi} \frac{m^2 + (z^2 + (1-z)^2)p^2}{[p^2 + m^2]^2}$$



The factorization scale does **no longer regularize** the UV divergence (limitation of our model)

IV. CONCLUSIONS AND OUTLOOK

 We must introduce a **factorization scale** in order to ensure that the jet function can be factorized from the hard process

 Our toy model that allows us to understand some **basic features** of this separation of scales

 Future **improved model** that allows us to keep trace of the virtuality at each step on the process

 Future **resummation** at all orders in opacity