Evolution of QCD jets in non-equilibrium plasma

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in collaboration with

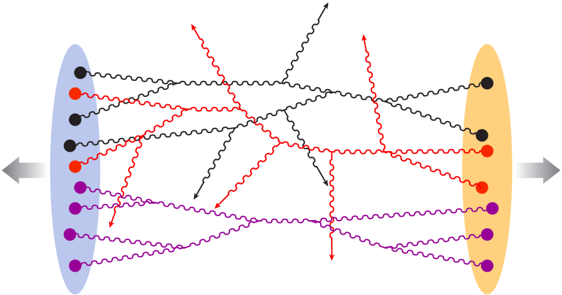
Xiaojian Du, Carlos A. Salgado and Bin Wu

February 2023

- After a heavy-ion collision, an out-of-equilibrium high-populated system of gluons is produced.

Phys. Rev. D 55 (1997). Jalilian-Marian et al.

Nucl. Phys. B 529 (1998). Kovchegov and Mueller



Ann. Rev. Nucl. Part. Sci. 60 (2010). Gelis et al.

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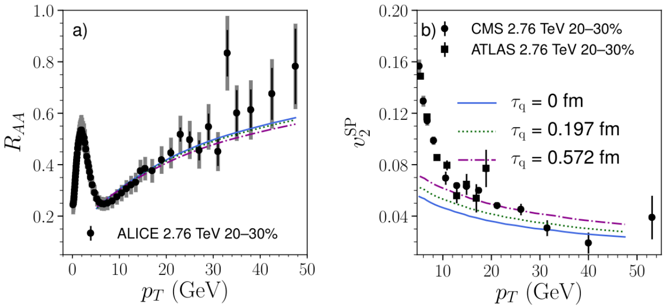
- A negligible energy loss at initial times is necessary in order to reproduce experimental data.



Physics Letters B 803 (Apr. 2020). Andres et al.

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How can we study the evolution?

- In the weak coupling limit, the bulk thermalization follows a bottom-up fashion.

  Phys. Lett. B 502 (2001). Baier et al.

- The only tool used for a quantitative study of these systems before is the Effective Kinetic Theory (EKT).

  JHEP 01 (2003). Arnold, Moore, and Yaffe

- Our study uses the Boltzmann Equation in Diffusion Approximation (BEDA) as an alternative approach.

  Physics Letters B 834 (2022). SBC, Salgado, and Wu

  ((Nov. 2023) [arXiv:2206.12376]). SBC, Salgado, and Wu

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- The QCD Boltzmann equation at leading order:

$$(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{x}})f^a = C^a_{2\leftrightarrow 2}[f] + C^a_{1\leftrightarrow 2}[f]\,, \quad f^a = \{f^g, f^q, f^{\bar{q}}\}$$

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- We consider a spatially homogeneous system.

$$f^a(\mathbf{p}) = f^a(p)$$

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- We consider a spatially homogeneous system.

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- The thermalization can be studied following the time evolution of the screening mass, the jet quenching parameter and the effective temperature and net quark chemical potential[1].

$$m_D^2 = m_D^2[f] \qquad \hat{q} = \hat{q}[f]$$

$$T_*(t) \equiv \frac{\hat{q}}{2\alpha_s N_c m_D^2 \ln\frac{\langle p_t^2\rangle}{m_D^2}} \qquad \mu_* = \mu_*[f]$$

[1] All quarks are assumed to have identical distribution. In general each flavour would have its own μ_* associated.

- In diffusion approximation, the $2 \leftrightarrow 2$ collision kernel can be expressed as a Fokker-Planck equation plus an additional source term.

Phys. Lett. B 475 (2000). Mueller

Nucl. Phys. A 930 (2014). Blaizot, Wu, and Yan

$$C^a_{2\leftrightarrow 2} = \frac{1}{4}\hat{q}_a(t)\nabla_{\mathbf{p}} \cdot \left[\nabla_{\mathbf{p}} f^a + \frac{\mathbf{v}}{T^*(t)} f^a(1+\epsilon_a f^a)\right] + \mathcal{S}_a$$

$$\mathcal{S}_q = \frac{2\pi\alpha_s^2 C_F^2 \ln\frac{\langle p_t^2\rangle}{m_D^2}}{p}\left[\mathcal{I}_c f(1-F) - \bar{\mathcal{I}}_c F(1+f)\right],$$

$$\mathcal{S}_{\bar{q}} = \mathcal{S}_q\big|_{F\leftrightarrow\bar{F}}, \qquad \mathcal{S}_g = -\frac{N_f}{2C_F}(\mathcal{S}_q + \mathcal{S}_{\bar{q}}),$$

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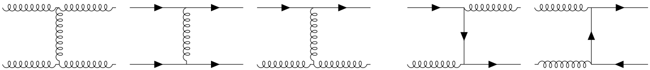
Phys. Lett. B 475 (2000). Mueller

Nucl. Phys. A 930 (2014). Blaizot, Wu, and Yan

Fokker-Planck term Source term



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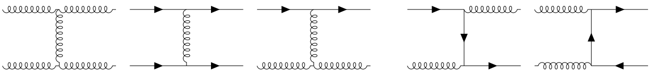

- The gluon distribution function is known to diverge at small p, $f \propto 1/p$, for over-occupied systems, which is interpreted as the onset of Bose-Einstein Condensation (BEC).

Nucl. Phys. A 920 (2013). Blaizot, Liao, and McLerran

- The presence of BEC can be study numerically by choosing the appropriate boundary conditions with[2] $\dot{n}_c \propto (\lim_{p\to 0} pf - T_*)$.

Nucl. Phys. A 930 (2014). Blaizot, Wu, and Yan

[2] $n_c \equiv$ number density of the BEC.

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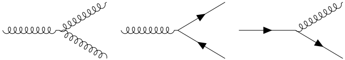
- The $1 \leftrightarrow 2$ kernel can be computed in the deep LPM regime

Nucl. Phys. B 483 (1997). Baier et al.

Phys. Rev. D 78 (2008). Arnold and Dogan

$$C^a_{1\leftrightarrow 2} = \int_0^1 \frac{dx}{x^3} \sum_{b,c} \left[\frac{\nu_c}{\nu_a} C^c_{ab}(\mathbf{p}/x; \mathbf{p}, \mathbf{p}(1-x)/x) - \frac{1}{2} C^a_{bc}(\mathbf{p}; x\mathbf{p}, (1-x)\mathbf{p}) \right]$$

- The $C^a_{bc}(\mathbf{p}; x\mathbf{p}, (1-x)\mathbf{p})$ describes the collinear splitting $a \leftrightarrow bc$.
- The three possible processes involved are the three QCD interaction vertices.



- Will the BEC still appear in initially over-populated system after including inelastic collisions?

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Rapid thermalization of the soft sector

- At small p, the $g \leftrightarrow gg$ and $g \leftrightarrow q\bar{q}$ are the dominant processes in the production of gluons and (anti)quarks, respectively.
- The distributions of gluons and quarks quickly fill a thermal distribution up to small soft momentum p_s

$$f^g(p) \approx \frac{T_*}{p} \qquad \text{for } p \lesssim p_g$$

$$f^q(p) \approx \frac{1}{e^{-\frac{\mu_*}{T_*}} + 1} \qquad \text{for } p \lesssim p_q$$

At early times, p_s is given by $(\mathcal{I}_c = \mathcal{I}_c[f])$

$$p_g \equiv (\hat{q}_A m_D^4 t^2/2)^{\frac{1}{5}} \qquad p_q \equiv [\alpha_s C_F \pi(\mathcal{I}_c + \bar{\mathcal{I}}_c)t]^{\frac{2}{5}} \hat{q}_F^{\frac{1}{5}}$$

- This behavior implies that $\dot{n}_c = 0$, so no BEC is observed as in the pure gluon case.

Nucl. Phys. A 961 (2017). Blaizot, Liao, and Mehtar-Tani

Physics Letters B 834 (2022). SBC, Salgado, and Wu

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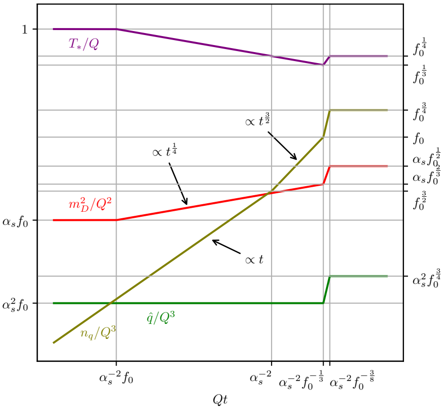
Three different stages for thermalization as in pure gluon scenario.

Physics Letters B 834 (2022). SBC, Salgado, and Wu



Parametric estimation for $f_0 \ll 1$

1. Soft gluon radiation and overheating.
   - T_* is almost constant since both m_D^2 and $\hat{q}$ are dominated by the hard sector.

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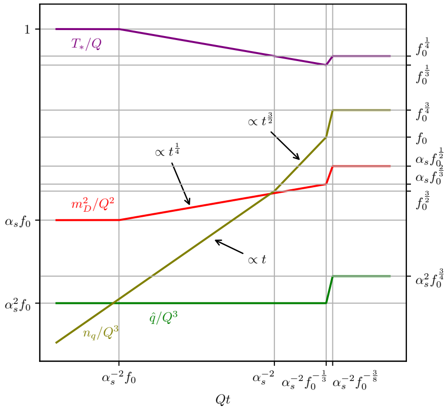
Physics Letters B 834 (2022). SBC, Salgado, and Wu



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2. Cooling and overcooling of soft gluons.
   - Soft gluons dominate the screening $\Rightarrow m_D^2 \uparrow \Rightarrow T_* \downarrow$.
   - $n_q \propto t$ lead by hard gluons, until $Qt \sim \alpha_s^{-2}$, when $n_q \propto t^{\frac{3}{2}}$ when the soft sector takes control.

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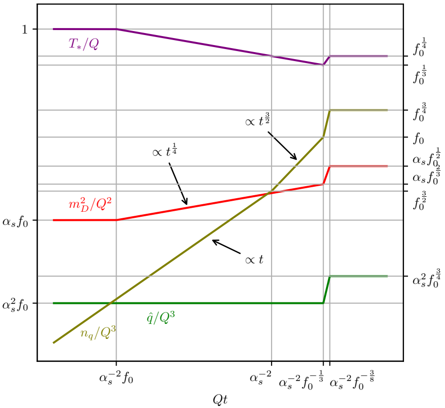

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3. Reheating and mini-jet quenching in a QGP with T_*.
   - $\hat{q}$ receives dominant contribution from $g, q, \bar{q}$.
   - T_* increases until it reaches T_{eq}.

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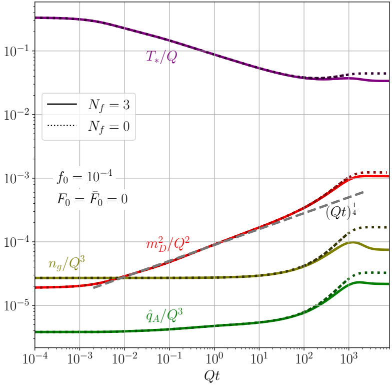
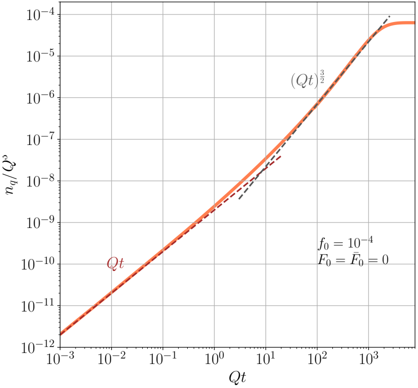
f, F and $\bar{F}$ are gluon, quark and antiquark distributions function. Initial condition is inspired by CGC: $f = f_0\theta(Q - p)$.



Numerical results for $f_0 = 10^{-4}$



Quark number density for $f_0 = 10^{-4}$

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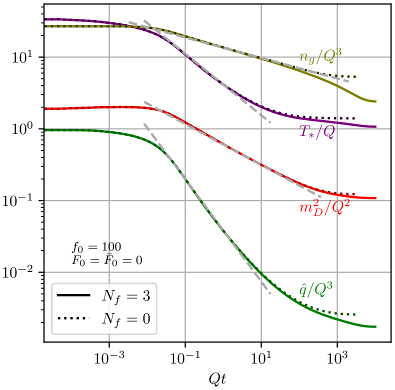
Two-stage thermalization, as in pure gluon scenario.

Physics Letters B 834 (2022). SBC, Salgado, and Wu



Numerical results for $f_0 = 100$

1. Soft gluon radiation and overheating.
   - T_* is almost constant since the soft gluons do not play an important role.
2. Momentum broadening and cooling (no overcooling)
   - T_* starts to decrease until it reaches thermal equilibrium.
   - All the quantities evolve according the universal scaling / self similar solution (dashed lines).

See also:

Phys. Rev. D 86 (2012). Kurkela and Moore

Phys. Rev. D 89.7 (2014). Abraao York et al.

Phys. Rev. D 86 (2012). Berges, Schlichting, and Sexty

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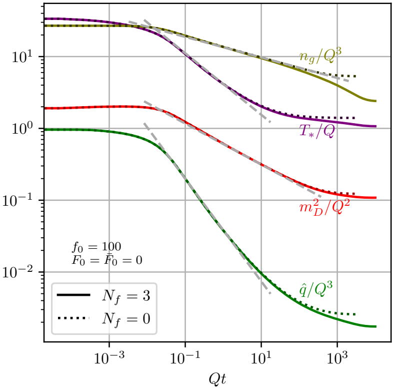
Physics Letters B 834 (2022). SBC, Salgado, and Wu



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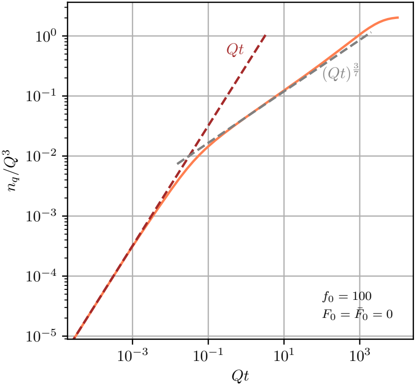
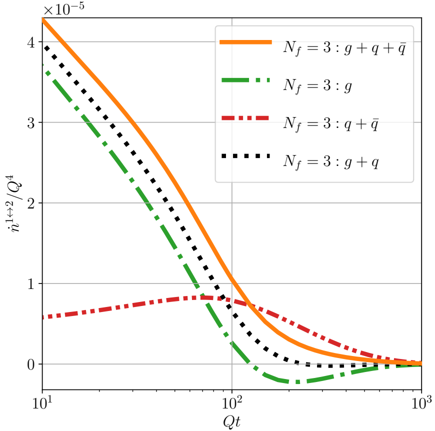


Quark number evolution for $f_0 = 100$

- At later times, the $g \leftrightarrow gg$ processes nearly establish detailed balance.
- The decrease in gluon number is mainly due to $gg \to q\bar{q}$ and $g \to q\bar{q}$.



Variation in number density due to $1 \leftrightarrow 2$ processes for $f_0 = 0.01$.

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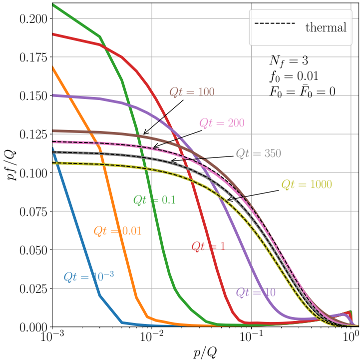
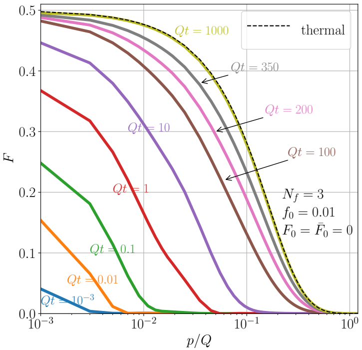
- This consistent with the picture of the subsystem of gluons achieving thermal equilibrium among itself, while the quark sector still needs time to have a Fermi-Dirac profile.

Phys. Rev. Lett. 122 (2019). Kurkela and Mazeliauskas



Gluon distribution function.



Quark distribution function.

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- We can explore the jet as a perturbation to an spatially homogeneous medium.

$$f(\mathbf{p},\mathbf{x},t) = f_{back}(\mathbf{p},t) + \delta f(\mathbf{p},\mathbf{x},t)$$

- After linearizing the BEDA, the background will evolve by itself

$$\partial_t f_{back} = C^a_{2\leftrightarrow 2}[f_{back}] + C^a_{1\leftrightarrow 2}[f_{back}],$$

meanwhile the perturbation

$$(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{x}})\delta f^a(\mathbf{p},\mathbf{x},t) = \delta C^a_{2\leftrightarrow 2}[f,\delta f] + \delta C^a_{1\leftrightarrow 2}[f,\delta f].$$

$$C^a_i[f] = C^a_i[f_{back}] + \underbrace{\tilde{C}^a_i[f_{back}]\delta f(\mathbf{p},\mathbf{x},t)}_{\equiv \delta C^a_i[f,\delta f]} + \ldots$$

Studying the jet as a perturbation I

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- This allows us to integrate out the spatial dependence

JHEP 07 (2021). Schlichting and Soudi

$$\partial_t \bar{\delta f}^a(\mathbf{p},t) = \delta C^a_{2\leftrightarrow 2}[f,\bar{\delta f}] + \delta C^a_{1\leftrightarrow 2}[f,\bar{\delta f}],$$

$$\bar{\delta f}^a(\mathbf{p},t) \equiv \int d^3\mathbf{x}\,\delta f^a(\mathbf{p},\mathbf{x},t)$$

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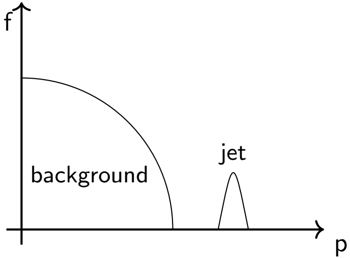
- Previous works have studied the behaviour of this jets in EKT for (non-evolving) thermal backgrounds.

JHEP 07 (2021). Schlichting and Soudi

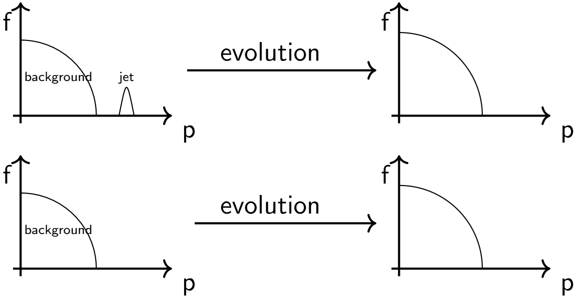
JHEP 05 (2023). Mehtar-Tani, Schlichting, and Soudi

- We will explore the evolution of jets in both thermal and non thermal backgrounds.



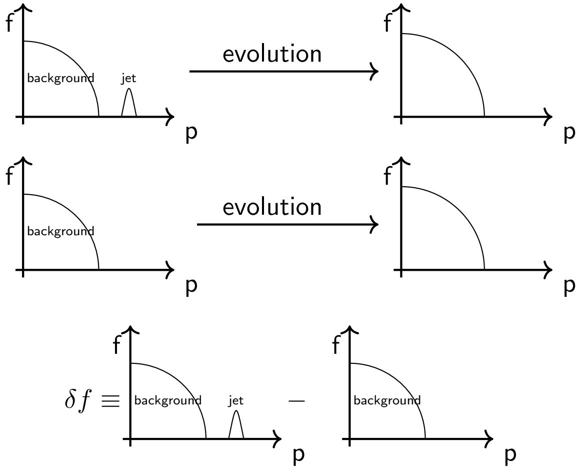
- In the numerical results, the perturbation will contain 2% of the total energy of the system.

- Since we do not have access to linearized BEDA, we can compute the perturbation as sketched:



Studying the jet as a perturbation III

- Since we do not have access to linearized BEDA, we can compute the perturbation as sketched:



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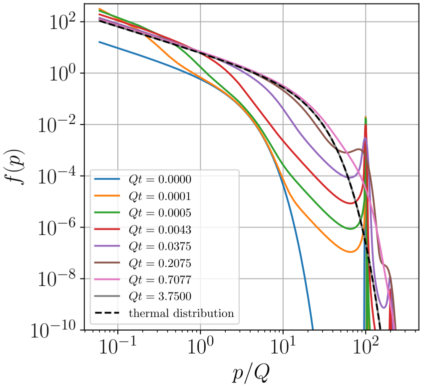
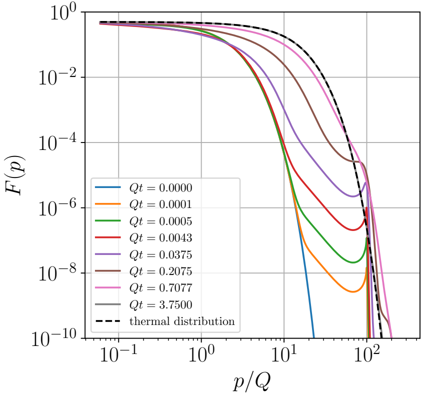
- We have a thermal background of quarks and gluon plus a gluon perturbation.



Gluon distribution function



Quark distribution function

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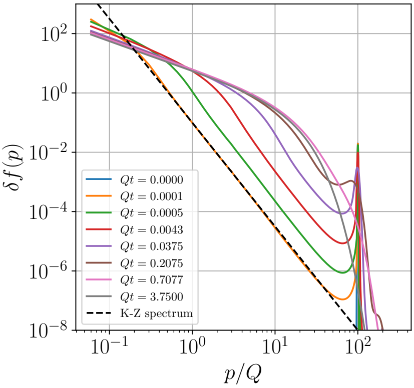
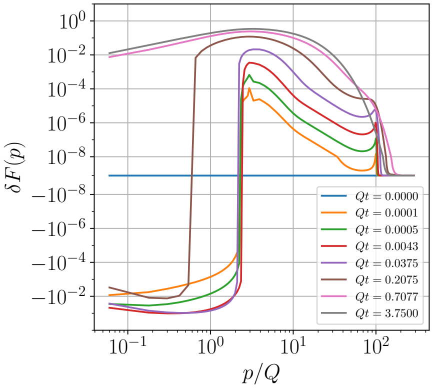
Results for thermal background II

- In the gluon perturbation we can see the Kolmogorov-Zhakarov spectrum.

JHEP 07 (2021). Schlichting and Soudi



Gluon perturbation



Quark perturbation

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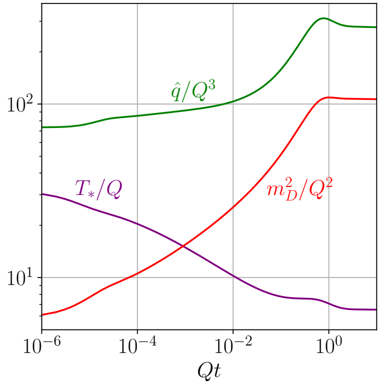
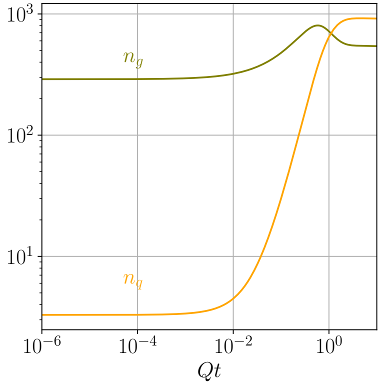


Evolution of macroscopic quantities



Number density of quarks and gluons

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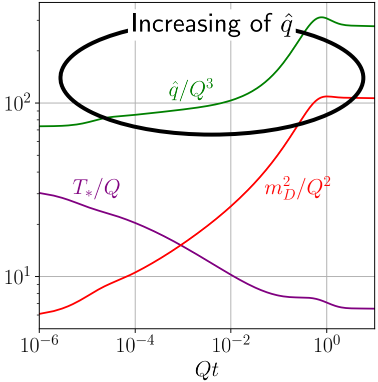
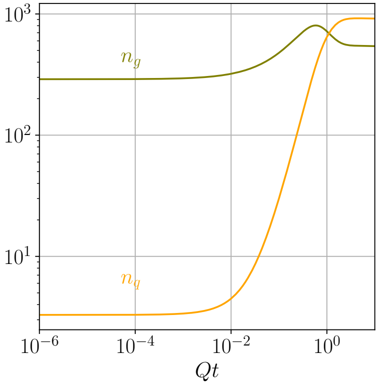


Evolution of macroscopic quantities



Number density of quarks and gluons

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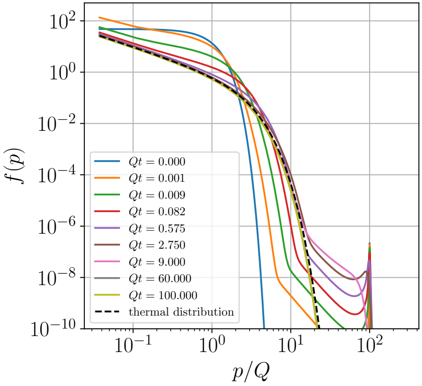
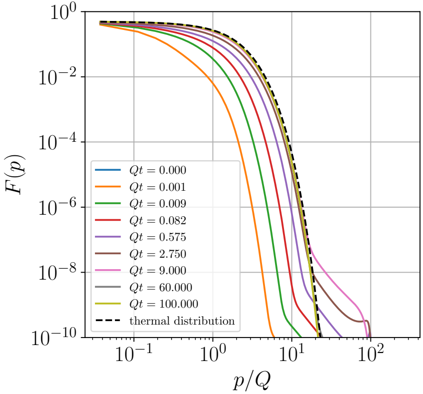
- The initial distribution is a highly populated gluon background plus a gluon perturbation.



Gluon distribution function



Quark distribution function

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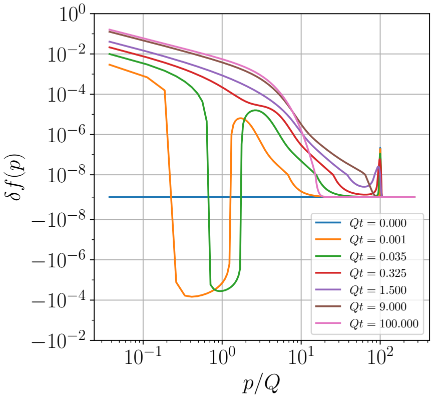
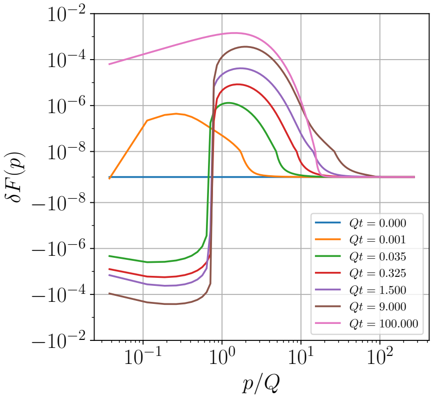
- The negativity in the gluon perturbation only appears for initially overpopulated backgrounds. This deplection might be related with jet wake.



Gluon perturbation



Quark perturbation

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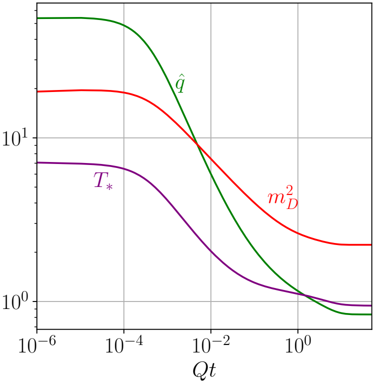
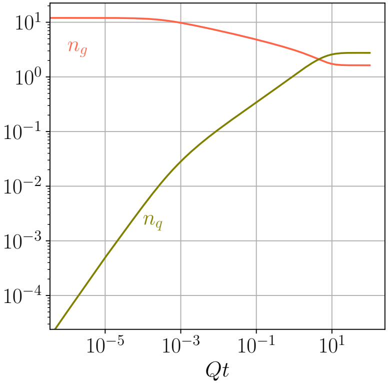
Results for out of equilibrium background III



Evolution of macroscopic quantities



Number density of quarks and gluons

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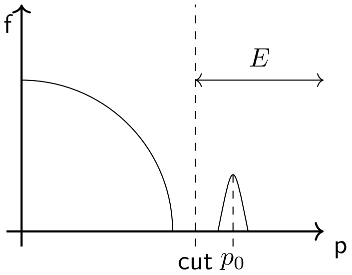
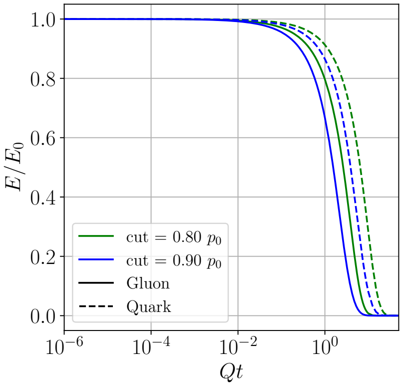
- We can get an estimation of the energy loss of the jet computing the energy up to some momentum cut.





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- The Boltzmann Equation in Diffusion Approximation (BEDA) provides a framework to study the thermalization of a system of quarks and gluons.
- The soft sector of gluons and (anti)quarks quickly achieves a thermal distribution due to inelastic processes and independently of the initial condition.
- Both under and over-occupied limits have been parametrically understood in accordance with numerical simulations.
- The BEDA is also suitable to use as a tool for exploring jet evolution in a medium, identifying the former as a perturbation of the latter.

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- Future work in this direction will study the behaviour of more realistic jets, including more dimensions in the system evolution.
- Also, the study of the jet wake as a perturbation is possible in this framework.

$$(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{x}}) \underbrace{\delta f^a(\mathbf{x},\mathbf{p},t)}_{\text{Recover } \mathbf{x} \text{ dependence}} = \delta C^a_{2\leftrightarrow 2}[f] + \delta C^a_{1\leftrightarrow 2}[f]$$

- Determining the evolution of δf allows us to compute $\delta T^{\mu\nu}(\mathbf{x},t)$ and related with the wake.
- It is also possible to study the wake with the k-modes after using a Fourier Transform.

$$(\partial_t + \mathbf{v}\cdot\mathbf{k})\tilde{\delta f}^a(\mathbf{k},\mathbf{p},t) = \tilde{\delta C}^a_{2\leftrightarrow 2}[f] + \tilde{\delta C}^a_{1\leftrightarrow 2}[f]$$

$$\tilde{\delta f}(\mathbf{k},\mathbf{p},t) \rightarrow \tilde{\delta T}^{\mu\nu}(\mathbf{k},\mathbf{p},t) \rightarrow \delta T^{\mu\nu}(\mathbf{x},\mathbf{p},t)$$

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Thanks!

Back-up

- Jet quenching parameter

$$\hat{q}_a = 8\pi\alpha_s^2 C_a \ln\frac{\langle p_t^2\rangle}{m_D^2}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left[N_c f\,(1+f) + \frac{N_f}{2}F(1-F) + \frac{N_f}{2}\bar{F}(1-\bar{F})\right]$$

- Screening mass

$$m_D^2 = 8\pi\alpha_s\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{|\mathbf{p}|}\left(N_c f + \frac{N_f}{2}F + \frac{N_f}{2}\bar{F}\right)$$

- Integrals $\mathcal{I}_c$

$$\mathcal{I}_c = \int\frac{d^3p}{(2\pi)^3}\frac{1}{p}[f + F + f(F-\bar{F})], \qquad \bar{\mathcal{I}}_c = \mathcal{I}_c|_{F\leftrightarrow\bar{F}}$$

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