Heavy quark frag and diffusion coefficients in the pre-hydrodynamic QCD plasma

ECT* Jet and non-equilibrium workshop

Trento, Feb. 15th, 2024

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Based on

XD, PRC 109 (2024) 014901

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Heavy quarks

Thermalization of heavy quarks

- Distinguished separation of scales

Hard probe
(jet energy/heavy quark mass)

$E \gg T$

Medium
(Light parton energy in medium)

- Time scales in thermalization

Heavy quark production $\tau_O \sim 1/M$

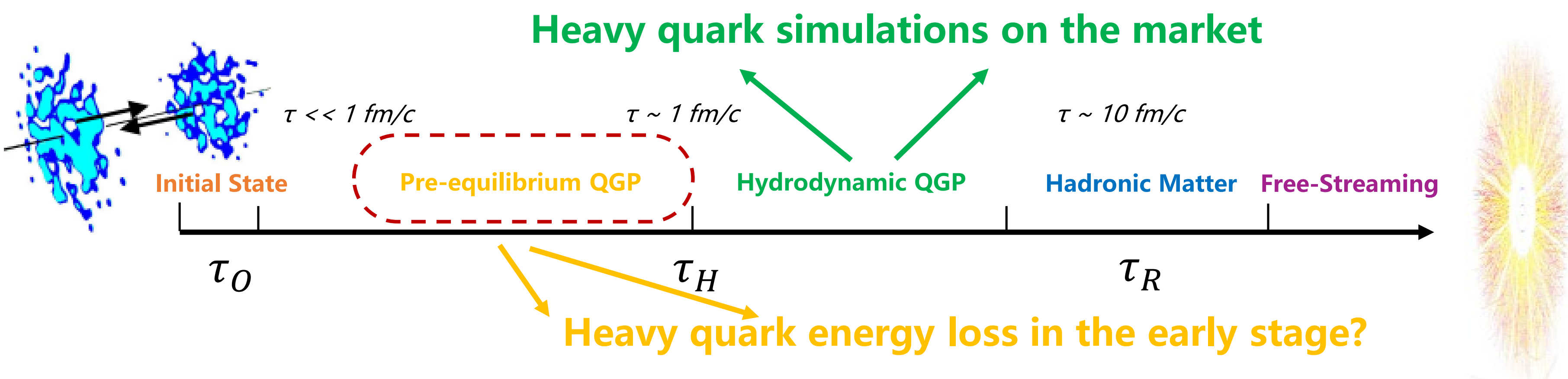
QGP thermalization $\tau_H \sim 1/T$

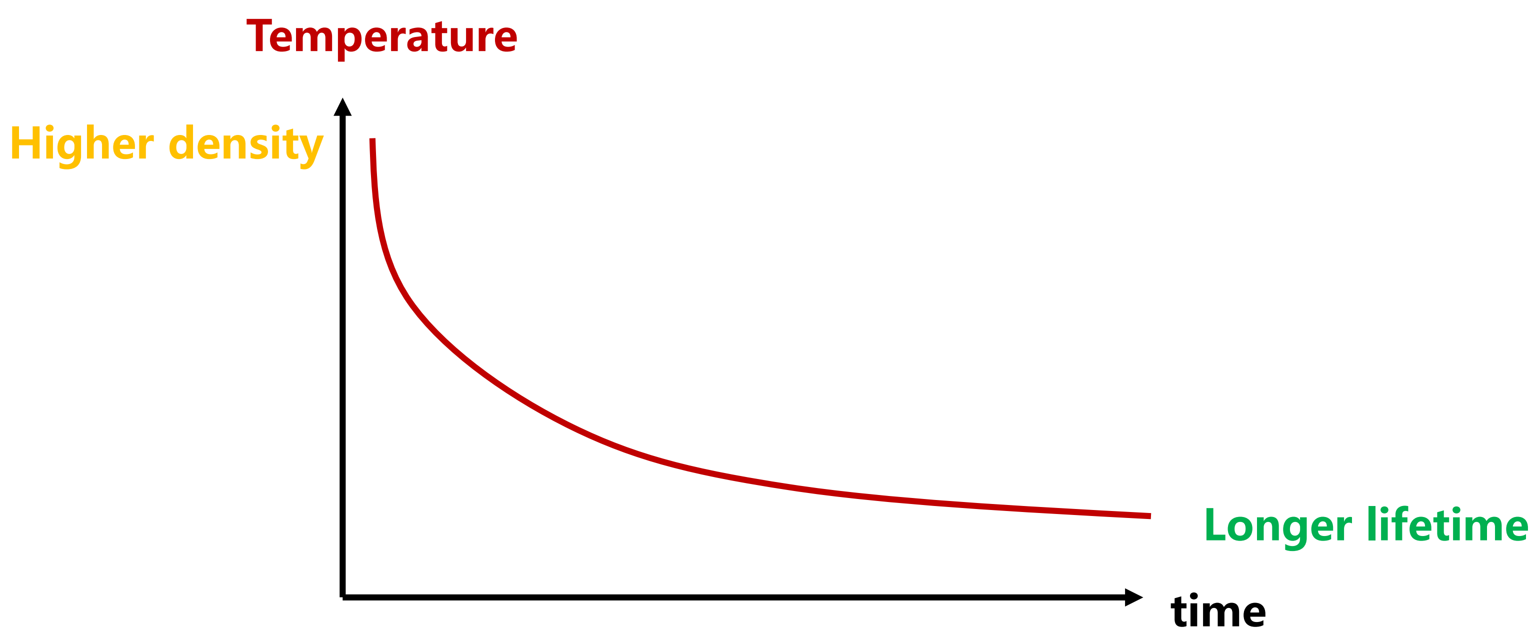
Heavy quark thermalization $\tau_R \sim M/T^2$

$\tau_O \ll \tau_H \ll \tau_R$

Pre-hydrodynamic stage

Thermalization of heavy quarks in heavy-ion collisions





Pre-hydrodynamic stage

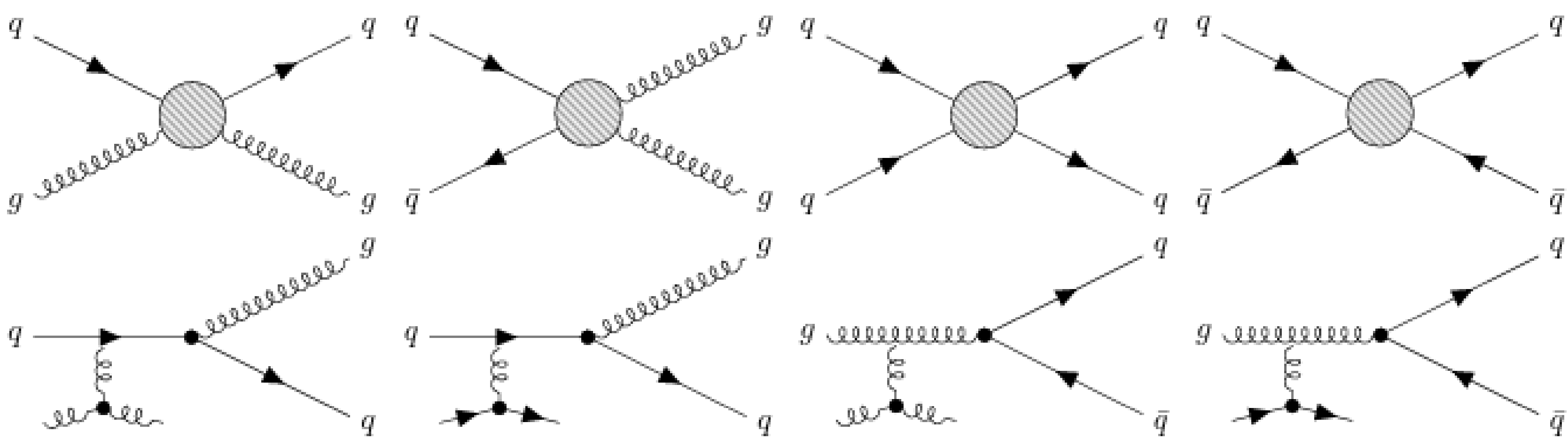
Simulation of non-equilibrium QCD matter

Set of coupled Boltzmann equations for quarks and gluon:

$$\left(\frac{\partial}{\partial \tau} - \frac{p_\parallel}{\tau}\frac{\partial}{\partial p_\parallel}\right) f_a(\tau, p_T, p_\parallel) = -C_a^{2\leftrightarrow 2}[f](\tau, p_T, p_\parallel) - C_a^{1\leftrightarrow 2}[f](\tau, p_T, p_\parallel)$$

$$a = gluon, quark$$

Including both elastic and inelastic scatterings in the QCD:



Thermalization of heavy quarks

Thermalization of heavy quarks in modelling

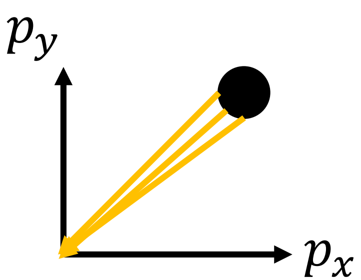
- Large mass, low velocity, elastic kicks from the medium dominate

Stochastic differential equation (SDE) for heavy quark dynamics:

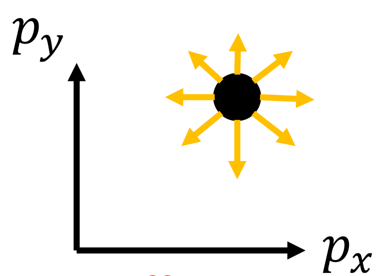
$$dp_i = -A_i dt + \sigma_{ij} dW_j$$

Stochastic term (elastic kicks)

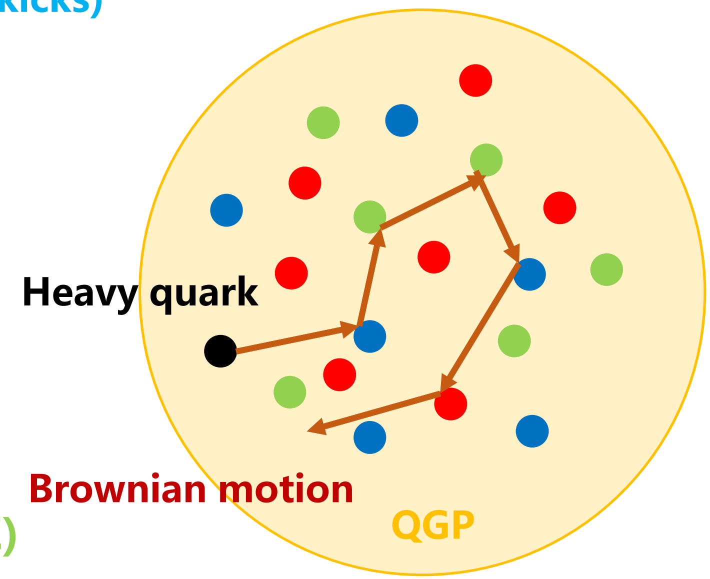
Drag — Diffusion



Drag



Diffusion



From the SDE to partial differential equation (PDE)

$$\partial_t f(p) = \partial_{p_i}[A_i f(p,t)] + \partial_{p_i}\partial_{p_j}[B_{ij} f(p,t)]$$

$$B_{ij} = \sigma_{ik}\sigma_{kj}/2$$

Drag — Dissipation/Energy loss

Diffusion — Momentum broadening

Thermalization
(Fluctuation-dissipation theorem)

Transport coefficients

Equilibrium transport coefficients

$$\partial_t f(p) = \partial_{p_i}[A_i f(p,t)] + \partial_{p_i}\partial_{p_j}[B_{ij} f(p,t)]$$

- Thermal partons as inputs, depending on temperature (a=gluon/quark, i=x,y,z)

$$A_{a,i}(p,T) = \frac{1}{2E}\int d\Pi\, \overline{\left|M_{aQ\to aQ}\right|^2} v_a f_a^{eq}(p,T)(1 \pm f_a^{eq}(p',T))(\vec{p}-\vec{p}')_i$$

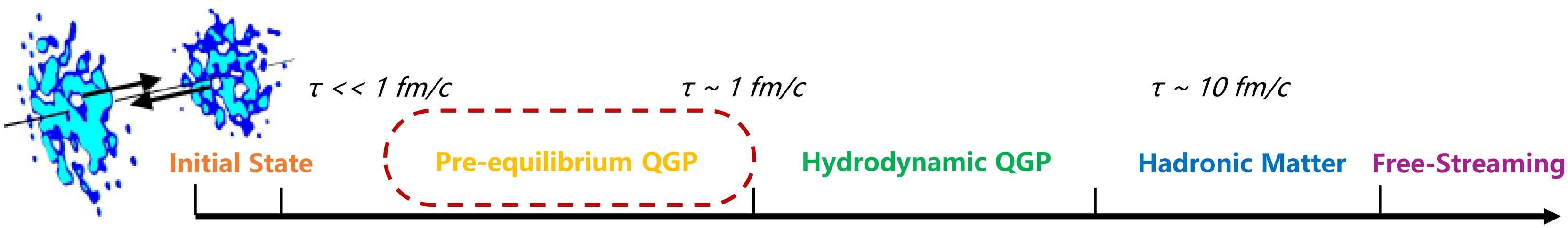
$$B_{a,ij}(p,T) = \frac{1}{4E}\int d\Pi\, \overline{\left|M_{aQ\to aQ}\right|^2} v_a f_a^{eq}(p,T)(1 \pm f_a^{eq}(p',T))(\vec{p}-\vec{p}')_i(\vec{p}-\vec{p}')_j$$

$$f_g^{eq}(p,T) = \frac{1}{e^{p/T}-1}$$

$$f_q^{eq}(p,T) = \frac{1}{e^{p/T}+1}$$

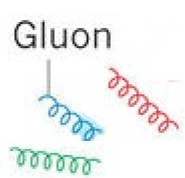
Pre-hydrodynamic stage

A more differentiated look

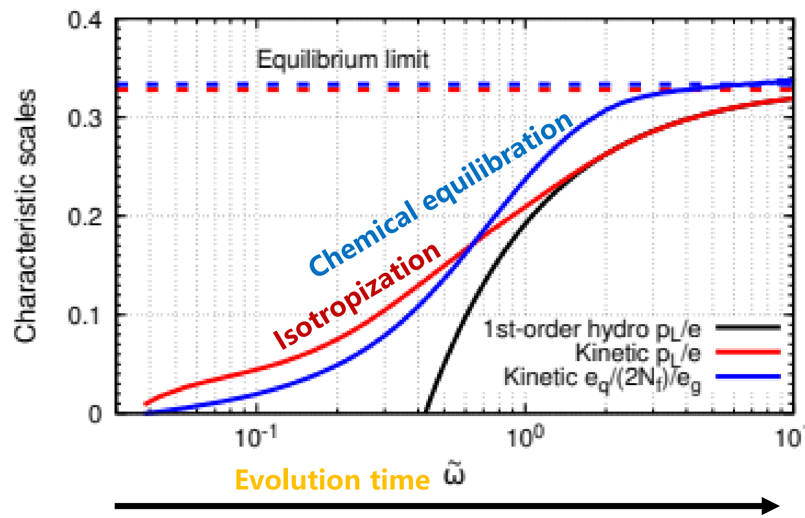


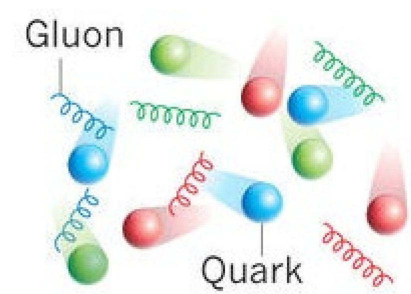
Pre-equilibrium QGP:

- Isotropizarion/chemical equilibration



Anisotropic
Gluon dominated





Thermal equilibrium
(gluon+quark)

Transport coefficients

Non-equilibrium transport coefficients

$$\partial_t f(p) = \partial_{p_i}[A_i f(p,t)] + \partial_{p_i}\partial_{p_j}[B_{ij} f(p,t)]$$

- Non-equilibrium parton distribution (a=gluon/quark, i=x,y,z)

B. Svetitsky, PRD37 (1988) 2484

$$A_{a,i}(p,\tau) = \frac{1}{2E}\int d\Pi\, \overline{|M_{aQ\to aQ}|^2} v_a f_a(p,\tau)(1 \pm f_a(p',\tau))(\vec{p}-\vec{p}')_i$$

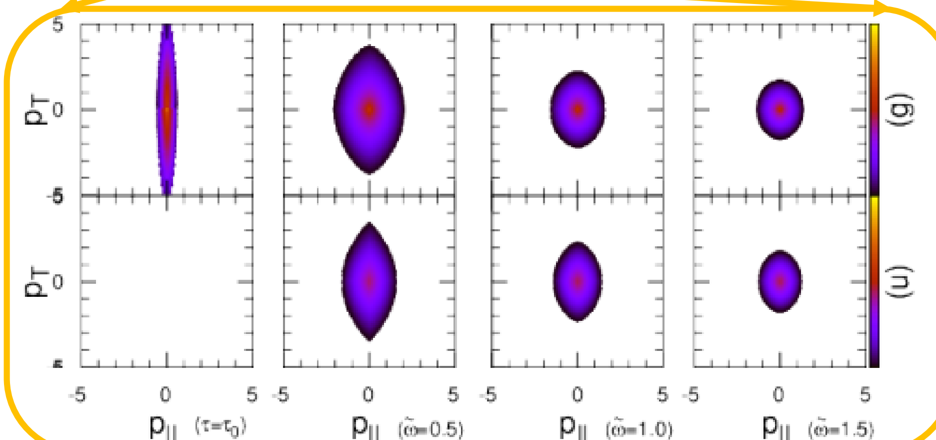
$$B_{a,ij}(p,\tau) = \frac{1}{4E}\int d\Pi\, \overline{|M_{aQ\to aQ}|^2} v_a f_a(p,\tau)(1 \pm f_a(p',\tau))(\vec{p}-\vec{p}')_i(\vec{p}-\vec{p}')_j$$

- Kinetically:

Anisotropic -> Isotropic

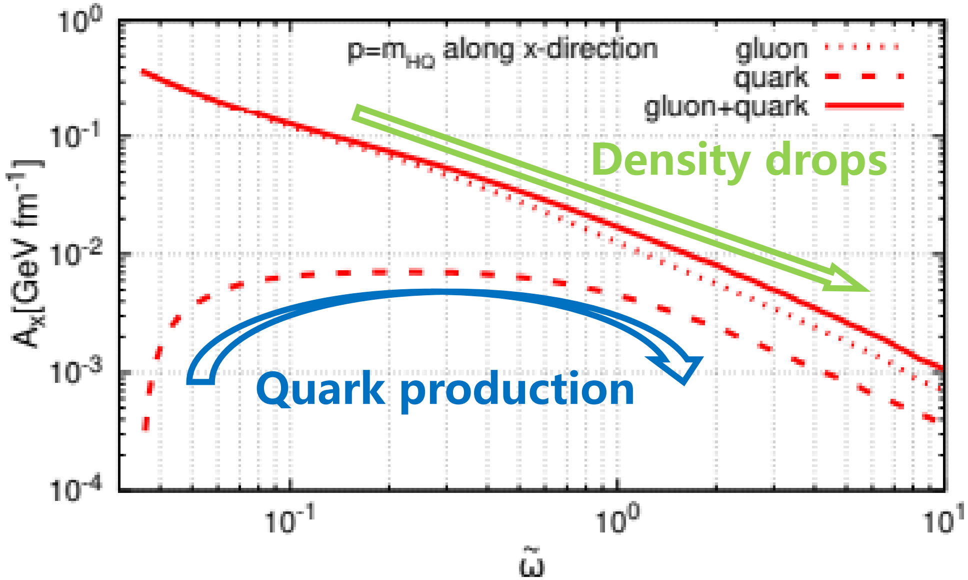
- Chemically:

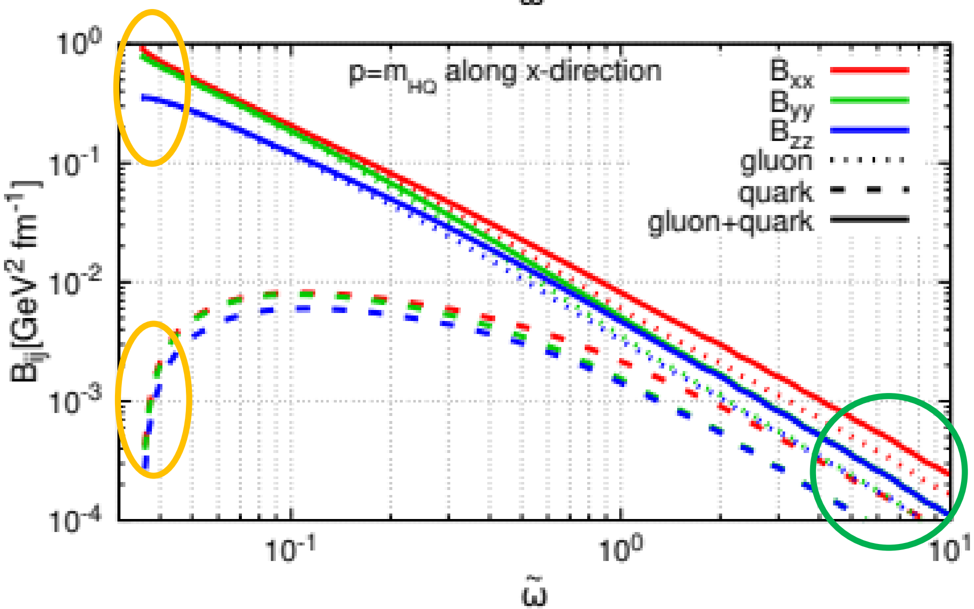
Gluon -> Gluon+Quark



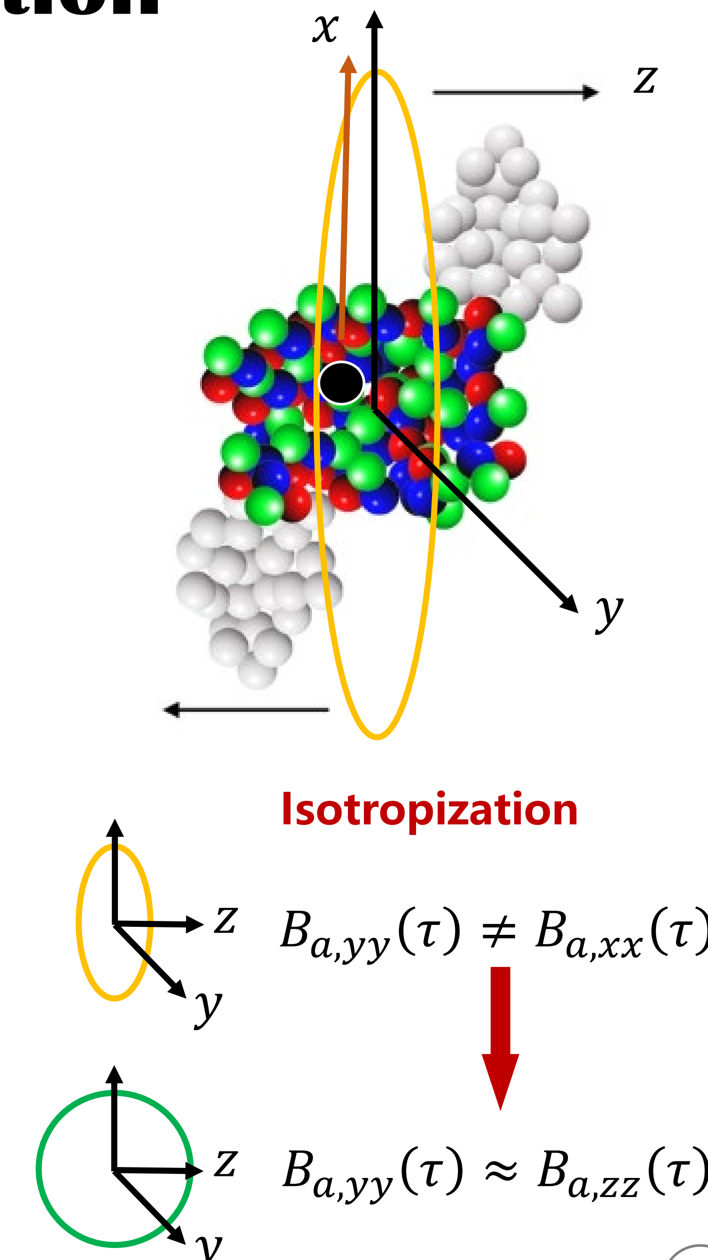
XD, S. Schlichting, PRD104 (2021) 0540111

Time evolution

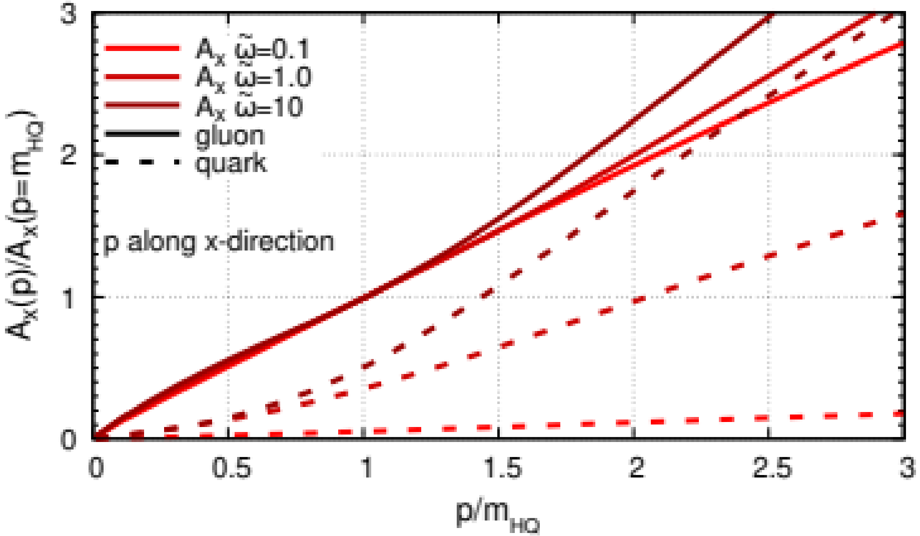


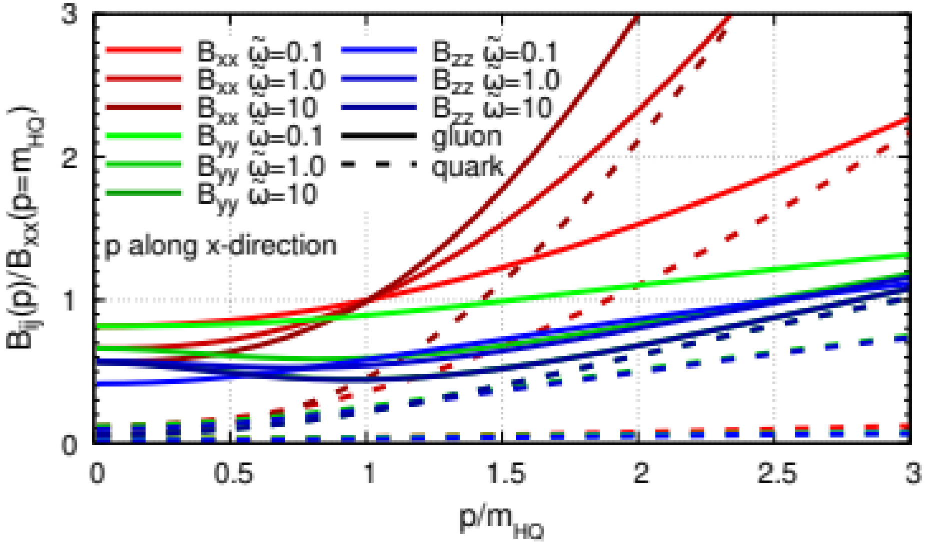


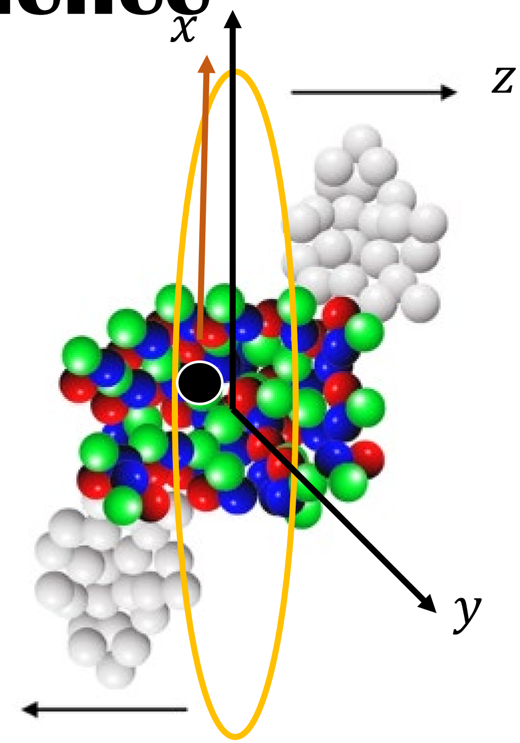
Fixed coupling $\lambda = 10$, LO pQCD



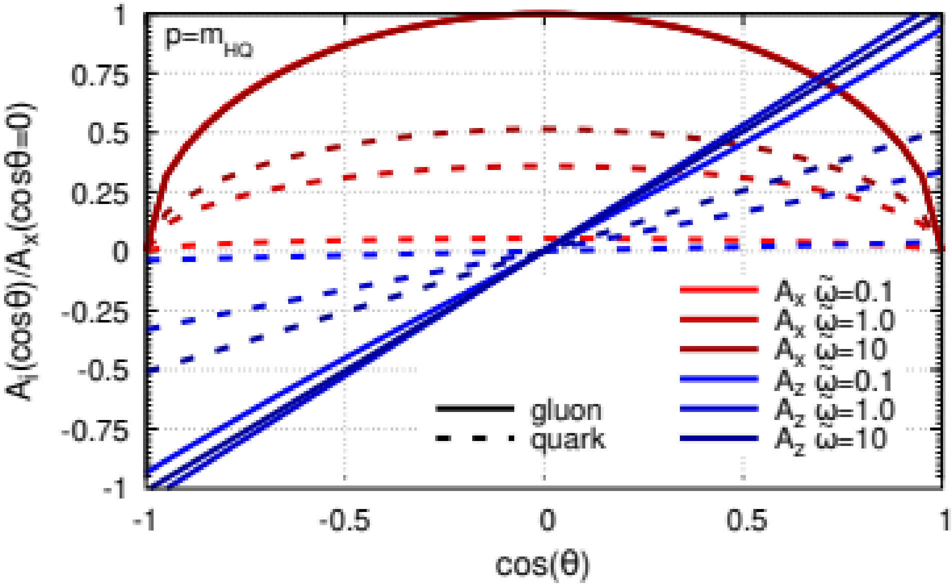
Momentum dependence

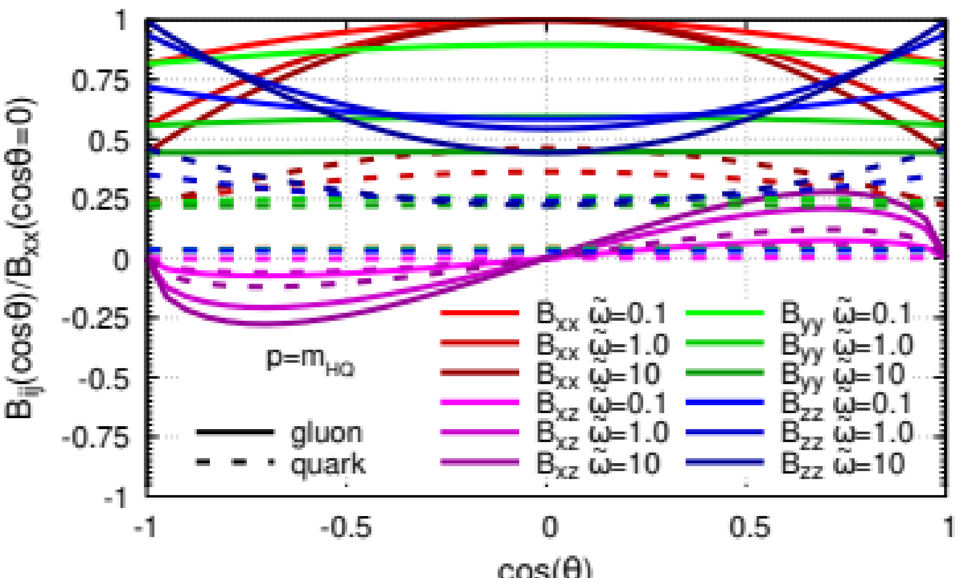


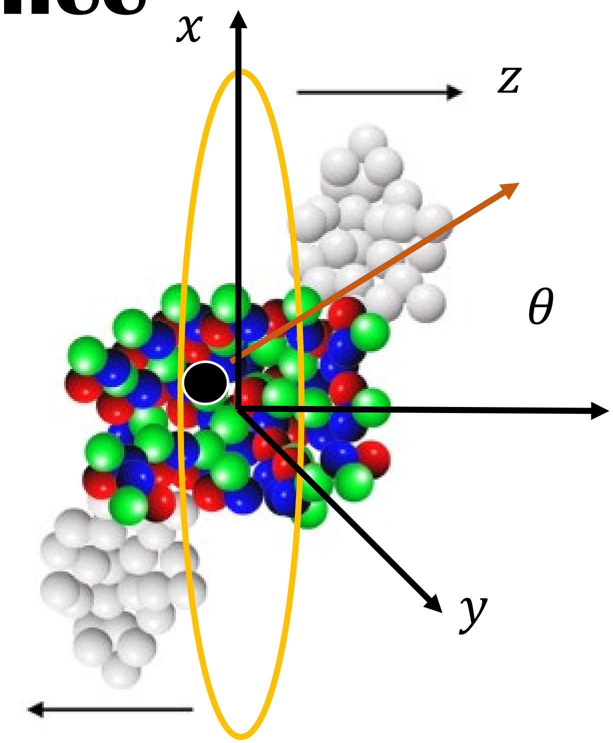




Angular dependence



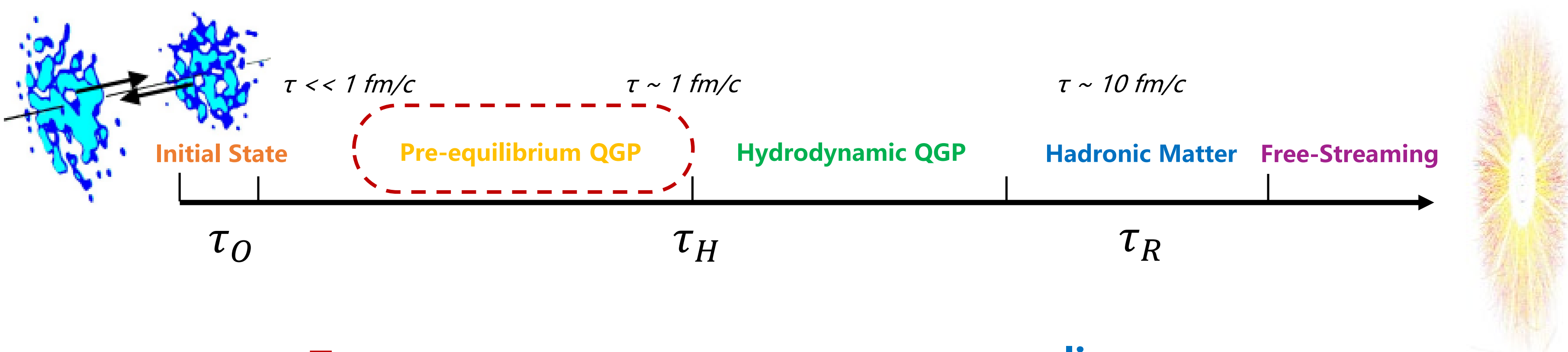


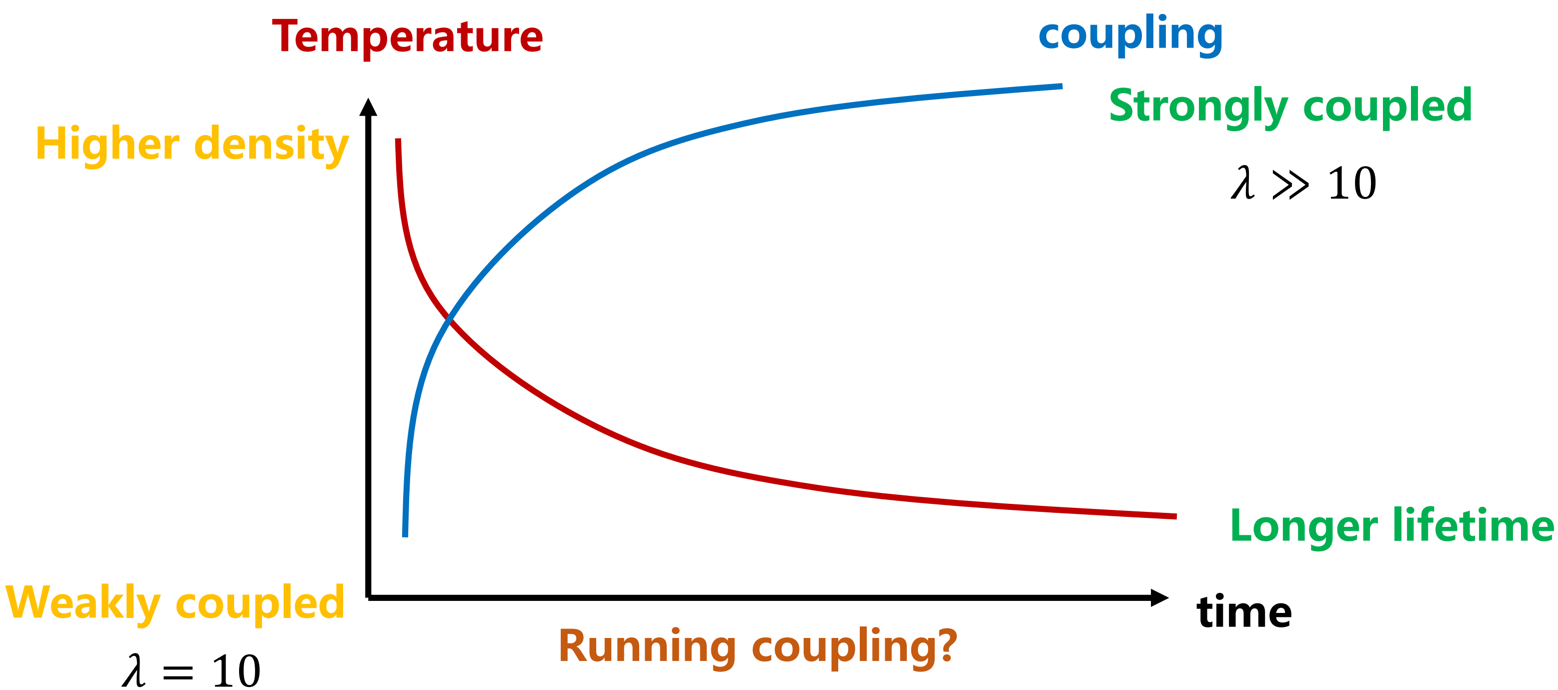


Heavy quark in x-z plane

Pre-hydrodynamic stage

The QCD plasma is more complicated…





Attractor solution for the QGP

Based on energy conservation

$$\tau^{\frac{4}{3}} e(\tilde{w}) = \left(4\pi \frac{\eta}{s}\right)^{\frac{4}{9}} \left(\frac{\pi^2 \nu_{\mathrm{eff}}}{30}\right)^{\frac{1}{9}} (\tau_0 e_0)^{\frac{8}{9}} C_\infty \mathcal{E}(\tilde{w})$$

G. Giacalone, A. Mazeliauskas, S. Schlichting, PRL 123 (2019) 262301

Energy density

$$e(\tilde{w}) = \frac{\pi^2 \nu_{\mathrm{eff}}}{30} T^4$$

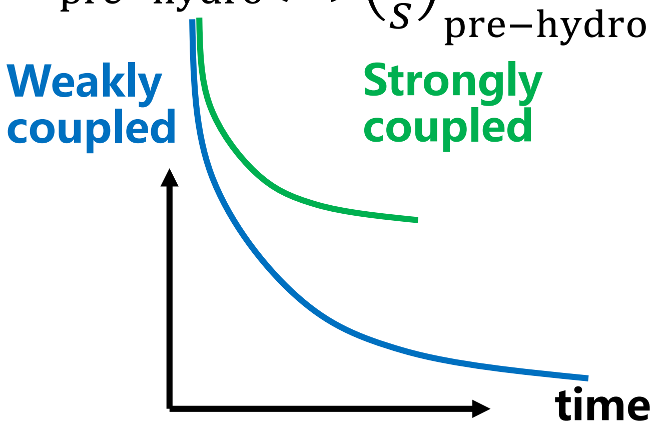
Universal time

$$\tilde{w} = \frac{\tau T s}{4\pi\eta}$$

With fixed initial condition, simple rescaling formula for specific $\tilde{w}$

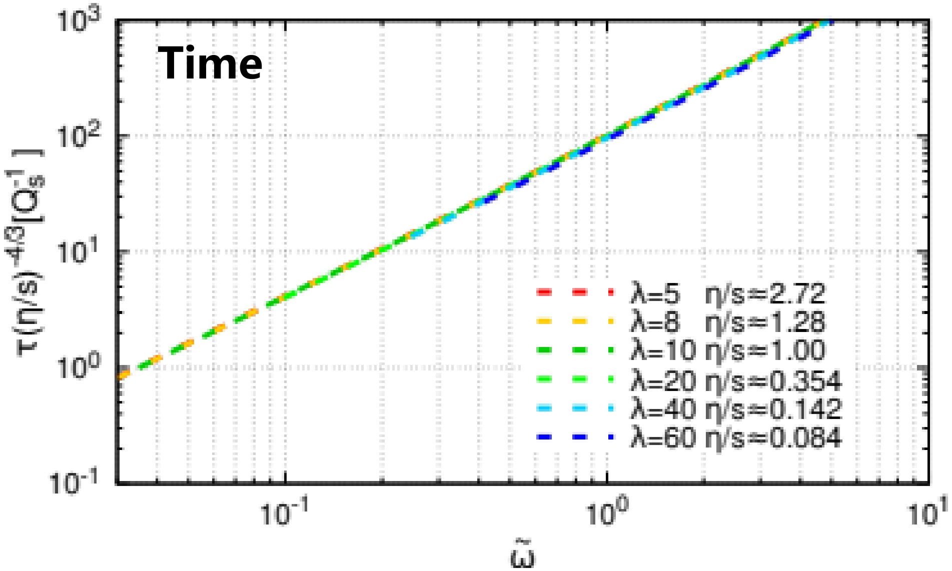
$$\tau_{\mathrm{pre-hydro}}(\tilde{w}) \left(\frac{\eta}{s}\right)_{\mathrm{pre-hydro}}^{-\frac{4}{3}} \approx \text{coupling independent}$$

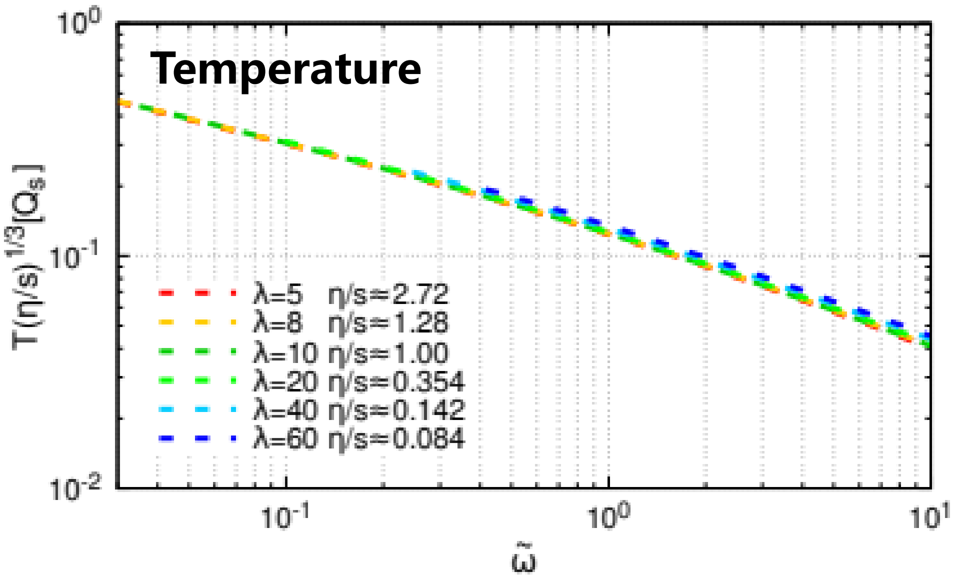
$$T_{\mathrm{pre-hydro}}(\tilde{w}) \left(\frac{\eta}{s}\right)_{\mathrm{pre-hydro}}^{\frac{1}{3}} \approx \text{coupling independent}$$



Shorter time, faster thermalization

Rescaling of the soft QGP





Remark

Viscosity η/s fitted to 1st order hydrodynamics close to the hydrodynamic limit

Nice rescaling for the QGP simulated from the kinetic theory, even at large couplings

Connection between weakly & strongly coupled plasmas

Transport coefficients

Transport coefficients at different coupling

$$\partial_t f(p) = \partial_{p_i}[A_i f(p,t)] + \partial_{p_i}\partial_{p_j}[B_{ij} f(p,t)]$$

- Non-equilibrium parton distribution (a=gluon/quark, i=x,y,z)

$$A_{a,i}(p,\tau) = \frac{1}{2E}\int d\Pi \overline{\left|M_{aQ\to aQ}\right|^2} \nu_a f_a(p,\tau)(1 \pm f_a(p',\tau))(\vec{p}-\vec{p}')_i$$

$$B_{a,ij}(p,\tau) = \frac{1}{4E}\int d\Pi \overline{\left|M_{aQ\to aQ}\right|^2} \nu_a f_a(p,\tau)(1 \pm f_a(p',\tau))(\vec{p}-\vec{p}')_i(\vec{p}-\vec{p}')_j$$

A. Kinetic simulation at running coupling?

$$\left|M_{aQ\to aQ}\right|^2, f_a(p,\tau), \ldots$$

B. Rescaling of the coefficients with medium profile (time, temperature) provided by the attractor?

$$A_{a,i}^{\text{stronger}} = K A_{a,i}^{\text{weaker}}$$

$$B_{a,ij}^{\text{stronger}} = K B_{a,ij}^{\text{weaker}}$$

Rescaling of the hard HQ

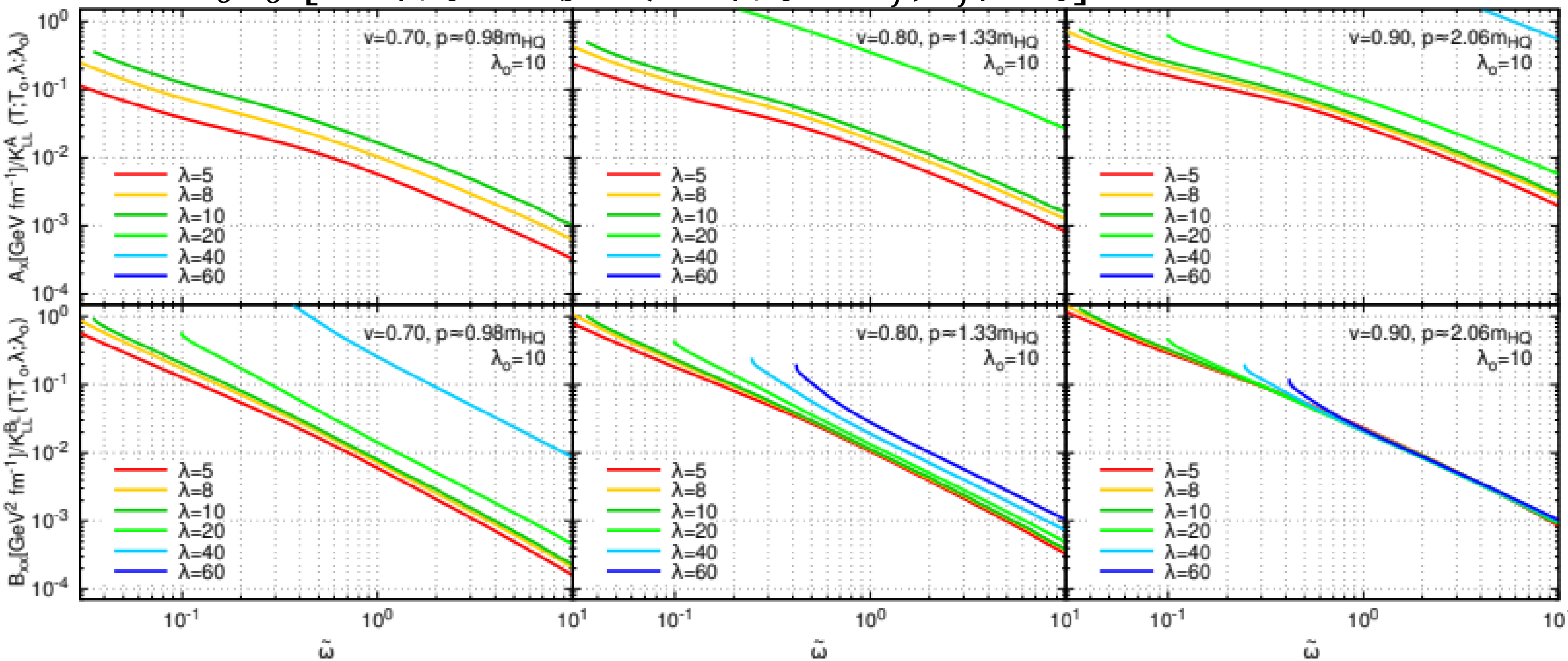
Leading-log factors (LO pQCD) from thermal medium

$$K_{LL}^{A} = \frac{\lambda^2}{\lambda_o^2}\frac{T^2}{T_o^2}\left[\frac{\ln(1/\mu) + a_b + (\ln(1/\mu) + a_f)N_f/2N_c}{\ln(1/\mu_o) + a_b + (\ln(1/\mu_o) + a_f)N_f/2N_c}\right]$$

$$K_{LL}^{B} = \frac{\lambda^2}{\lambda_o^2}\frac{T^3}{T_o^3}\left[\frac{\ln(1/\mu) + b_b + (\ln(1/\mu) + b_f)N_f/2N_c}{\ln(1/\mu_o) + b_b + (\ln(1/\mu_o) + b_f)N_f/2N_c}\right]$$

G. Moore, D. Teaney, PRC71 (2005) 064904

S. Caron-Huot, G. Moore, PRL100 (2008) 052301



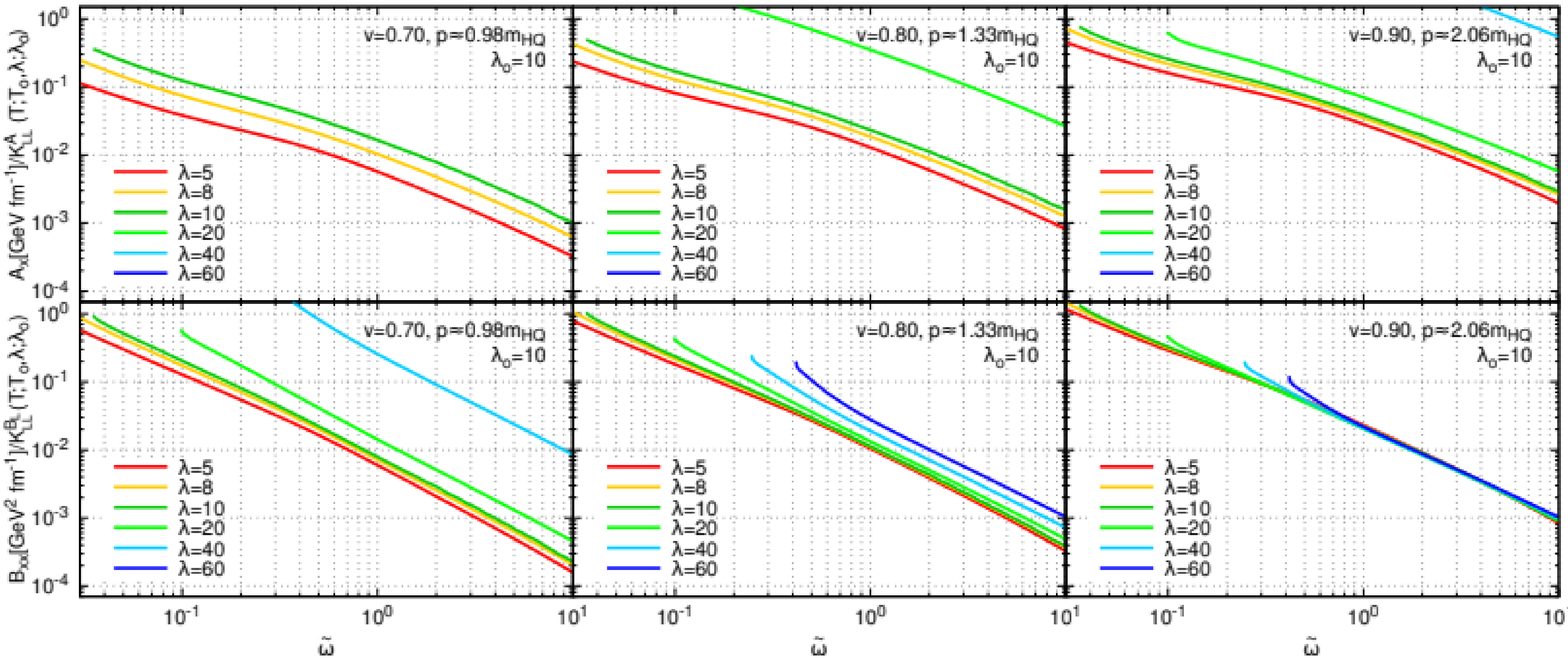
Rescaling of the hard HQ

Leading-log factors (LO pQCD) from thermal medium

Rescaling breaks down at large coupling (negative log term)

$$\mu \sim \# \sqrt{\lambda}$$

Rescaling works better at larger velocity



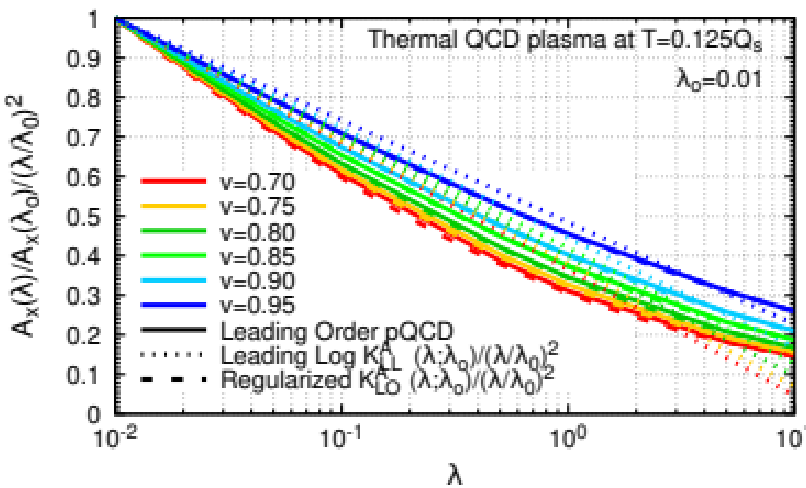
Rescaling of the hard HQ

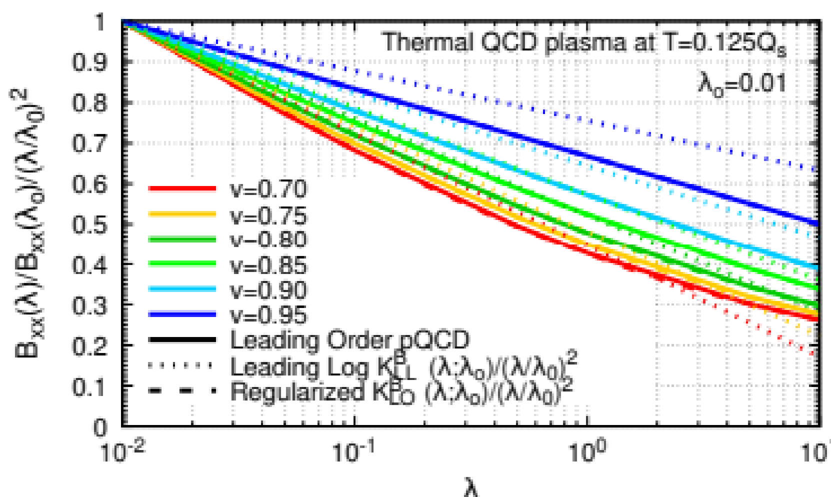
Regularized Leading-log factors from thermal medium

$$K^A_{RLL} = \frac{\lambda^2}{\lambda_o^2}\frac{T^2}{T_o^2}\left[\frac{\ln(1/\mu + a) + a_b + (\ln(1/\mu + c) + a_f)N_f/2N_c}{\ln(1/\mu_o + a) + a_b + (\ln(1/\mu_o + c) + a_f)N_f/2N_c}\right]$$

$$K^B_{RLL} = \frac{\lambda^2}{\lambda_o^2}\frac{T^3}{T_o^3}\left[\frac{\ln(1/\mu + d) + b_b + (\ln(1/\mu + e) + b_f)N_f/2N_c}{\ln(1/\mu_o + d) + b_b + (\ln(1/\mu_o + e) + b_f)N_f/2N_c}\right]$$

Effectively equivalent to removing the infrared cut of parton momentum at screening mass in calculating the factors





Fitting parameters to kinetic theory + LO pQCD calculation

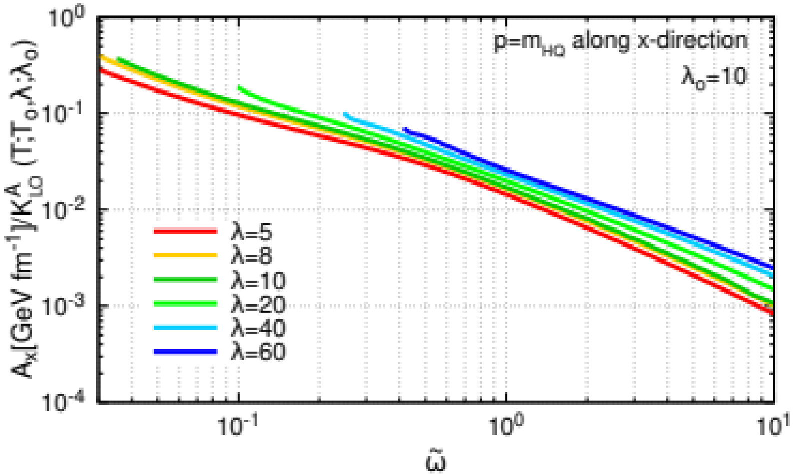
Rescaling of the hard HQ

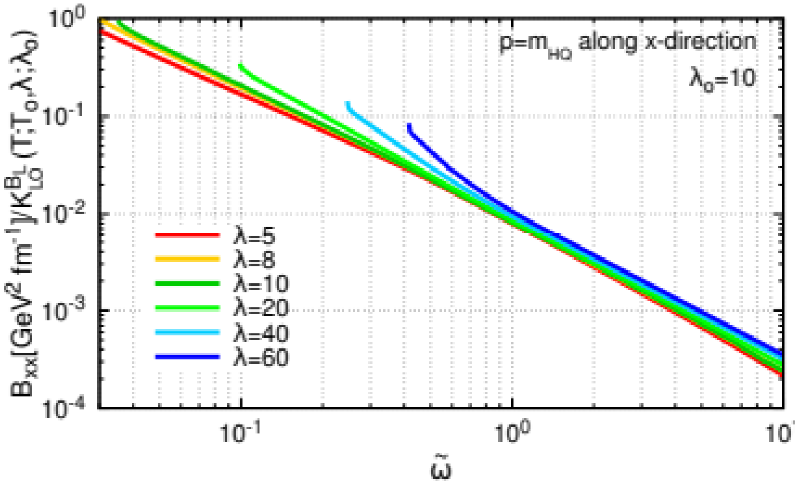
Regularized Leading-log factors from thermal medium

$$K_{RLL}^{A} = \frac{\lambda^2}{\lambda_o^2}\frac{T^2}{T_o^2}\left[\frac{\ln(1/\mu + a) + a_b + (\ln(1/\mu + c) + a_f)N_f/2N_c}{\ln(1/\mu_o + a) + a_b + (\ln(1/\mu_o + c) + a_f)N_f/2N_c}\right]$$

$$K_{RLL}^{B} = \frac{\lambda^2}{\lambda_o^2}\frac{T^3}{T_o^3}\left[\frac{\ln(1/\mu + d) + b_b + (\ln(1/\mu + e) + b_f)N_f/2N_c}{\ln(1/\mu_o + d) + b_b + (\ln(1/\mu_o + e) + b_f)N_f/2N_c}\right]$$

Effectively equivalent to removing the infrared cut of parton momentum at screening mass in calculating the factors





Much better rescaling, but of course pQCD is not enough for strong coupling
Non-perturbative methods in need for the rescaling factor

HQ energy loss from rescaling

With proper rescaling factor $K^A(\lambda)$

$$\left\langle \frac{\Delta p_x}{p_0} \right\rangle = \int_{\tau_0}^{\tau} -\frac{A_i}{p_0}\frac{T^2}{T_o^2} d\tau \left(\frac{\eta}{s}\right)^{4/3} K^A \approx 1 - e^{-0.38(\eta/s)^{2/3} K^A}$$

Charm, pre-hydrodynamic period up to $\tilde{w} = 1$

One can simply rescale the medium profile to estimate the energy loss

Heavy quark phenomenological data studies suggest a factor $K \geq 5$, with pre-hydrodynamic effect, would that be smaller?

See report and review: Rapp, et al. 1803.03824

Summary and Outlook

- Calculated heavy quark (HQ) drag and diffusion coefficients in the **weakly coupled pre-hydrodynamic QCD plasma**, including **anisotropization/chemical equilibration** of the plasma

- Discussed **rescaling for the QCD plasma** from attractor to connect weak/strong couplings

- Discussed **rescaling for the HQ** and tested LL factor, seen breakdown of the pQCD at larger coupling. (Should benefit calculations with running couplings, as well as matching CGC to hydrodynamics)

- **Non-perturbative** calculation and rescaling factor for HQ is needed (and a smooth transition between weakly coupled to strongly coupled)

pQCD

$$B \sim \lambda^2 T^3 \ln(1/\lambda)$$

In between?

e.g. AdS/CFT?

$$B \sim \sqrt{\lambda} T^3$$

Other non-perturbative frameworks…

- Non-equilibrium medium effect and running coupling should be implemented in simulations on the market