EFT approach to light and heavy quenching in thin media

New jet quenching tools to explore equilibrium and non-equilibrium dynamics in heavy-ion collisions, ECT*, Trento, Italy, Feb 15, 2024

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Based on WK, I Vitev 2301.11940, works in preparation with I Vitev, and with WB Zhao and XN Wang

Different methodologies for multi-scale problems

Some scales in a brick

- Q
- ER
- $(E/L)^{1/2}, (\hat{q}L)^{1/2}$
- $T, \underbrace{m_D, \Lambda_{\rm QCD}}_{\text{Screening scales “}\xi\text{”}}$

Note that $\min\{E/L, \hat{q}L\} = \sqrt{E\hat{q}}$

◁ Parton propagation in a finite-size medium is characterized by many scales (not necessarily in this exact order).

• From dimensional analysis, any properly normalized observable will be a function of dimensionless control parameters

$$\text{Obs} = f\underbrace{\left(g_s,\ \frac{Q}{\xi},\ \frac{\hat{q}L^2}{E} \sim \frac{\xi^2}{E/L} \times \frac{L}{\lambda_g},\ R,\ \cdots\right)}_{\text{Phase space of dimensionless parameters}}$$

• One strategy is to understand this multi-dimensional function via comprehensive modeling, e.g., general-purpose Monte Carlo.

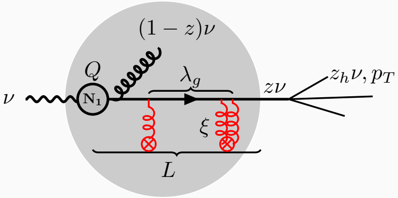
The effective theories approach

Rather than describing everything, focus on simplified problem in regions with scale separation.

- This talk: define scale separation for calculations in a thin medium in the context of $eA \to h$. But most discussions are similar to hardon production in small systems.
- Medium in eA is confined. Medium in small system can be strongly-coupled with strong off-eqilibirum effect.
- What is the minimal medium inputs to study high-E hadrons in such systems?

Scale separation in a thin medium (for collinear observable)

Semi-inclusive DIS in eA



$$\frac{d\sigma_{ep\to h}}{dx_B dQ^2 dz_h} = \frac{2\pi\alpha_e^2}{Q^4}\sum_{i,j}\underbrace{e_q^2 f_{i/A}(x_B)\otimes C_{ji}^h(x,z)}_{\color{red}{dN_{ij}/dz}}\otimes d_{h/j}(z_h)$$

$$\color{red}{z\frac{dN_{ji}}{dz}} \equiv \color{red}{F_{ij}}\left(z\,,\underbrace{\frac{Q^2}{\Lambda^2},\frac{E/L}{\xi^2},\lambda_g\xi,}_{\text{Large}}\underbrace{\frac{L}{\lambda_g},\frac{\xi}{\Lambda}}_{\text{Moderate}}\right)$$

Scale separation for the EFT, E/L is the semi-hard scale of the problem.

$$\Lambda^2 \sim \xi^2 \lesssim \xi^2\frac{L}{\lambda_g} \qquad E/L \qquad Q^2$$

Fragmentation outside of the medium (between $\xi^2 \frac{L}{\lambda_g}$ and E/L)

Hard production localized in medium (between E/L and Q^2)

Soft-Collinear Effective Theory for Jet Physics

The d.o.f of Soft Collinear Effective Theory (SCET) in the vacuum Bauer, Fleming, Luke, PRD63(2001)014006, Bauer, Fleming, Pirjol and Stewart, PRD63(2001)114020, $\cdots$

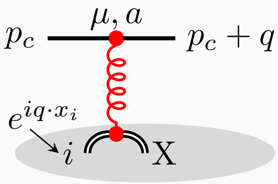
- Collinear mode $p_c \sim (1, \lambda^2, \lambda)E$.
- Soft mode $p_s \sim (\lambda^2, \lambda^2, \lambda^2)E$ (For $\mathrm{SCET_{II}}$ this is $p_s \sim (\lambda, \lambda, \lambda)E$).
- Multi-scale observables are factorized into single-scale sectors. RG/RRG equations seam different sectors to improve predictive power.

SCET with Glauber gluon from a background medium (SCET$_{\mathrm{G}}$)

A. Idilbi, A. Majumder PRD80(2009)054022, G. Ovanesyan, I. Vitev, JHEP06(2011)080.

- Interaction between collinear parton and medium is mediated by Glauber gluon $q \sim (\lambda^2, \lambda^2, \lambda)E$ generated by medium sources

$$A_G^{a-}(q^-, \mathbf{q}) = i g_s \int dx^+ d^2\mathbf{x} \frac{i e^{i q^- x^+} e^{-i \mathbf{q}\cdot\mathbf{x}}}{\mathbf{q}^2 + \xi^2} J^{a-}(x^- = 0, x^+, \mathbf{x}).$$



- At observable level, take ensemble average of a color-neutral medium

$$\langle\langle g_s A_G^a(q) e^{iq\cdot y} g_s A_G^b(k) e^{ik\cdot y}\rangle\rangle \propto \delta^{ab}\delta^{(2)}(\mathbf{k}+\mathbf{q}) \frac{\rho_G(x^+)}{(\mathbf{q}^2+\xi^2)^2} e^{iq^- x^+} e^{ik^- x^+}$$

  - In a weakly-coupled medium, $\rho_G = \sum_T g_s^2 \frac{C_T}{d_A} \rho_T$. ρ_T is parton density in representation T.

Relevant regions at opacity one in $\mathrm{SCET_G}$

- $e^{iq^- x^+}$ sets the power counting for collinear modes

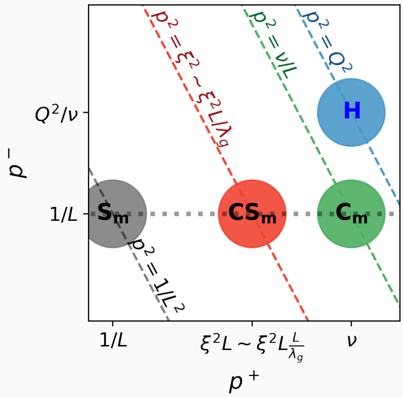
$$q^- x^+ \sim 1 \Rightarrow \lambda \sim \frac{1}{\sqrt{EL}},$$

$$p_c = (1, \lambda^2, \lambda)E, \text{ with } p_c^2 \sim q^2 \sim \frac{E}{L}$$

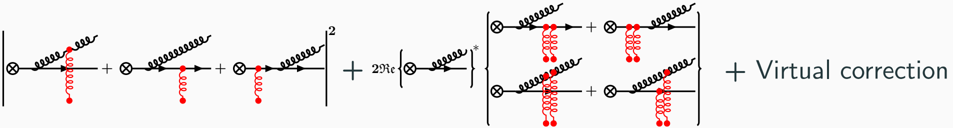
- Part of the problem is perturbative $E/L \gg \Lambda^2$. For $E = 60$ GeV and $L = 3$ fm, $E/L = 4$ GeV^2.

- The colliner-soft mode can be strongly coupled

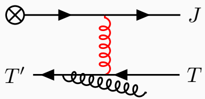
$$p_{cs} = (\eta, \lambda^2, \lambda\eta^{1/2})E, \quad p_{cs}^2 \sim \xi^2 ... \xi^2 \frac{L}{\lambda_g}.$$



- Correction to jet parton: $\Delta F_{ij}^{\mathrm{med}} = F_{ik}^{(0)} \otimes P_{kj}^{\mathrm{med}(1)}$



- What about NLO corrections to medium parton (anti-collinear mode)? $\Rightarrow$ For collinear observables these diagrams are scaleless, ρ_G remains as a single medium input. These diagrams become important for TMD observable.



Simplify the calculation

$$\boldsymbol{A}_\perp = \boldsymbol{k}_\perp,\ \boldsymbol{B}_\perp = \boldsymbol{k}_\perp + x\boldsymbol{q}_\perp,\ \boldsymbol{C}_\perp = \boldsymbol{k}_\perp - (1-x)\boldsymbol{q}_\perp,\ \boldsymbol{D}_\perp = \boldsymbol{k}_\perp - \boldsymbol{q}_\perp, \tag{2.44}$$

where again x and $\boldsymbol{k}_\perp$ are the longitudinal momentum fraction and the transverse momentum of the emitted parton relative to the parent parton respectively. Furthermore, $\boldsymbol{q}_\perp$ is the transverse momentum introduced by the Glauber gluon exchange. In addition, we have the following phases

$$\begin{aligned}
&\Omega_1 - \Omega_2 = \frac{\boldsymbol{B}_\perp^2}{p_0^+ x(1-x)},\ \Omega_1 - \Omega_3 = \frac{\boldsymbol{C}_\perp^2}{p_0^+ x(1-x)},\ \Omega_2 - \Omega_3 = \frac{\boldsymbol{C}_\perp^2 - \boldsymbol{B}_\perp^2}{p_0^+ x(1-x)},\\
&\Omega_4 = \frac{\boldsymbol{A}_\perp^2}{p_0^+ x(1-x)},\ \Omega_5 = \frac{\boldsymbol{A}_\perp^2 - \boldsymbol{D}_\perp^2}{p_0^+ x(1-x)}.
\end{aligned} \tag{2.45}$$

We reproduce the light parton in-medium splitting functions for reference and subsequent comparison. The result for $q \to qg$ is given by

$$\begin{aligned}
\left(\frac{dN^{\text{med}}}{dx\, d^2\boldsymbol{k}_\perp}\right)_{q\to qg} = {}& \frac{\alpha_s}{2\pi^2} C_F \frac{1+(1-x)^2}{x} \int \frac{d\Delta z}{\lambda_g(z)} \int d^2\boldsymbol{q}_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\text{med}}}{d^2\boldsymbol{q}_\perp} \left[\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2} \cdot \left(\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2} - \frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2} \right) \right.\\
& \times (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) + \frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2} \cdot \left(2\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2} - \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2} - \frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2} \right) (1 - \cos[(\Omega_1 - \Omega_3)\Delta z])\\
& + \frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2} \cdot \frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2} (1 - \cos[(\Omega_2 - \Omega_3)\Delta z]) + \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2} \cdot \left(\frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2} - \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2} \right) (1 - \cos[\Omega_4 \Delta z])\\
& \left. - \frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2} \cdot \frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2} (1 - \cos[\Omega_5 \Delta z]) + \frac{1}{N_c^2} \frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2} \cdot \left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2} - \frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) \right].
\end{aligned} \tag{2.46}$$

[Can be found in many literature, this one taken from Kang, Felix, and Vitev JHEP 1703 (2017) 146]

- The opacity-one results $P_{kj}^{\text{med}(1)}$ are complicated, previous studies often rely on Monte Carlo / numerical mDGLAP.

$$\partial_{\ln \mu^2} F = \left(P^{\text{vac}} + P^{\text{med}}\right) \otimes F$$

- Analytic insights from a region analysis

$$\Delta F_{ji}^{\text{med}}(z) \approx \Delta F_{ji}^{\text{med,c}}(z) + \Delta F_{ji}^{\text{med, cs}}(z)$$

Each sector is obtained by expand the full calculation with the respective power counting.

Collinear divergences at opacity one (flavor non-singlet)

- For collinear sector, expand in the small parameter $\frac{\xi^2}{E/L}$. Divergences can be cured by dimensional regularization (DR, $d = 4 - 2\epsilon$):

$$\Delta F^{\mathrm{med}}_{\mathrm{NS}}(z) = \int_z^1 dx F_{\mathrm{NS}}(\frac{z}{x}) P^{\mathrm{med}(1)}_{qq}(x) + \text{virtual correction.} + \mathcal{O}\left(\frac{\xi^2}{E/L}\right)$$

$$P^{\mathrm{med}(1)}_{qq}(x) = \frac{\alpha_s B}{8} \cdot \frac{\alpha_s \rho_G L}{E/L} \cdot \frac{P^{vac(0)}_{qq}(x)}{[x(1-x)]^{1+2\epsilon}} \cdot \left[\frac{\mu^2 L}{\chi z E}\right]^{2\epsilon} \cdot C_n \Delta_n(x)$$

- Qualitatively different collinear splitting spectra from vacuum splittings

$$\frac{P^{\mathrm{vac}(0)}_{qq}(x)}{[x(1-x)]^{1+2\epsilon}} \sim \frac{1}{x^{1+2\epsilon}(1-x)^{2+2\epsilon}}, \quad \frac{P^{\mathrm{vac}(0)}_{gg}(x)}{[x(1-x)]^{1+2\epsilon}} \sim \frac{1}{x^{2+2\epsilon}(1-x)^{2+2\epsilon}}$$

- The natural scale is zE/L. Q^2 only appears in coefficients $B\left(\frac{Q^2 L}{2E}\right)$ and $\chi\left(\frac{Q^2 L}{2E}\right)$.

Separate and renormalize the divergences

- How to separate the $1/\epsilon$ poles? For non-singlet sector:

$$\int_0^1 \frac{g(x)}{x^{1+2\epsilon}(1-x)^{2+2\epsilon}}dx = \int_0^1 \frac{g(x)-(1-x)^2 g(0) - x(2-x)g(1) - x(x-1)g'(1)}{x^{1+2\epsilon}(1-x)^{2+2\epsilon}}dx$$
$$- \frac{g(0)}{2\epsilon} + \frac{g'(1)}{2\epsilon} - g(1)\left(\frac{1}{2\epsilon}+2\right) + \mathcal{O}(\epsilon)$$

- The leading-log contribution contains the derivative of the parton spectra

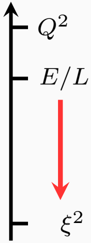
$$\Delta F_{\rm NS}(z) = \frac{\alpha_s(\mu^2)}{8}\frac{\alpha_s(\mu^2)B\rho_G L}{\nu/L}\left(\frac{1}{2\epsilon} + \ln\frac{\mu^2 L}{\chi z \nu}\right) 2C_F\left[2C_A\left(-\frac{d}{dz}+\frac{1}{z}\right)+\frac{C_F}{z}\right]F_{\rm NS}(z) + \cdots$$

- This IR divergence is canceled by the UV in the collinear-soft sector ($p_{\rm cs}^2 \sim \xi^2 ... \xi^2 L/\lambda_g$).

- For realistic medium, physics at the collinear-soft scale is already strongly coupled. So we simply absorb the divergence by a renormalization $F_{ij} \longrightarrow \left(\delta_{ik} + \frac{1}{\epsilon}M_{ik}^{(1)}\right) \otimes F_{kj}$.

The renormalization group equations

Define the evolution variable $\tau(\mu^2) = \frac{\rho_G L^2}{\nu} \frac{\pi B}{2\beta_0} \left[\alpha_s(\mu^2) - \alpha_s\left(\chi \frac{z\nu}{L}\right)\right]$:



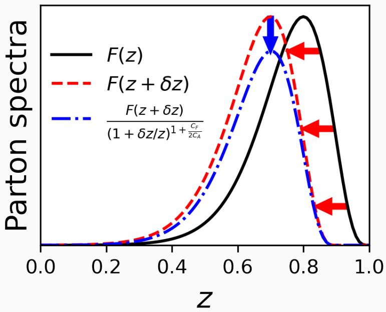
$$\frac{\partial F_{\mathrm{NS}}}{\partial \tau} = \left(4 C_F C_A \frac{\partial}{\partial z} - \frac{4 C_F C_A + 2 C_F^2}{z}\right) F_{\mathrm{NS}}$$

$$\frac{\partial F_f}{\partial \tau} = \left(4 C_F C_A \frac{\partial}{\partial z} - \frac{4 C_F C_A + 2 C_F^2}{z}\right) F_f + 2 C_F T_F \frac{F_g}{z},$$

$$\frac{\partial F_g}{\partial \tau} = \left(4 C_A^2 \frac{\partial}{\partial z} - \frac{2 N_f C_F}{z}\right) F_g + 2 C_F^2 \sum_f \frac{F_f}{z}.$$

The scale evolution equation is independent of the details at scale Q^2 and ξ^2.

Energy loss of collinear partons



A "traveling wave" solution for F_{NS}

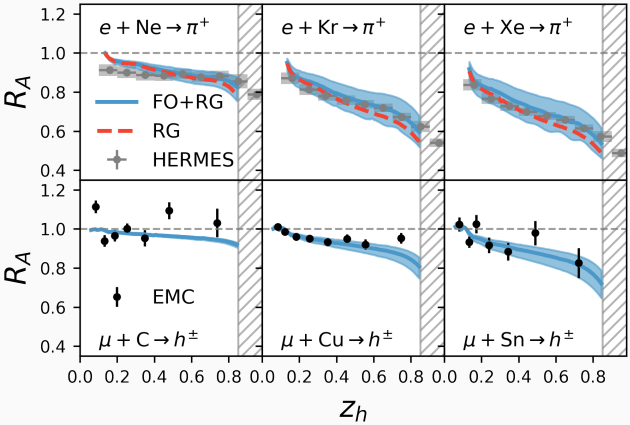
$$\frac{\partial F_{\mathrm{NS}}(\tau, z)}{\partial \tau} = \left(4 C_F C_A \frac{\partial}{\partial z} - \frac{4 C_F C_A + 2 C_F^2}{z}\right) F_{\mathrm{NS}}$$

$$F_{\mathrm{NS}}(\tau, z) = \frac{F_{\mathrm{NS}}\,(0, z + 4 C_F C_A \tau)}{(1 + 4 C_F C_A \tau / z)^{1 + C_F/(2C_A)}}$$

The primary effect: shift spectra by $\delta z = -4 C_F C_A \tau$.

This is the energy loss $\Delta E = E \delta z \propto L^2$.

Apply to quenching in eA



- $R_A = D_{eA}(z_h)/D_{ed}(z_h)$,
  $D_{eA} = \sigma_{eA\to h+X}/\sigma_{eA\to X}$
- NLO cross-section with NNFF1.0LO and (n)NNPDF3.0 (n)PDF.
- For HERMES: $\langle Q^2\rangle \approx 2.25$ GeV, $\langle \nu \rangle = 12$ GeV [NPB780(2007)1-27]
- For EMC: $\langle Q^2\rangle = 11$ GeV^2 and $\langle \nu \rangle = 62$ GeV [EMC ZPC52(1991)1–11] .

Good description with $\xi = 0.35$ GeV, $\rho_G = 0.4\ \mathrm{fm}^{-3}$, excluding the threshold region $z_h \to 1$.

Relation to modified DGLAP equation & anomalous dimensions

- It can be shown that the modified DGLAP equation contains the same leading-log physics. But the RG approach is much more efficient.
- Extract medium-induced anomalous dimensions (preliminary)

$$\int_0^1 dx \int d\mathbf{k}^2 \int d\mathbf{q}^2 \frac{dN_{qq}^{(1)}}{dx d^2\mathbf{k} d^2\mathbf{q}}(x^2-1)$$
$$= \underbrace{\frac{\alpha_s^2(\mu^2) C_F}{2\pi} \frac{\alpha_s \rho_G L}{E/L} (8C_A + 2C_F) C_{\frac{Q^2 L}{2E}, \frac{\theta^2 EL}{8}}}_{\Delta\gamma(3)} \left[\frac{1}{2\epsilon} + \ln\frac{\mu^2}{E/L} + \mathcal{O}(1)\right]$$

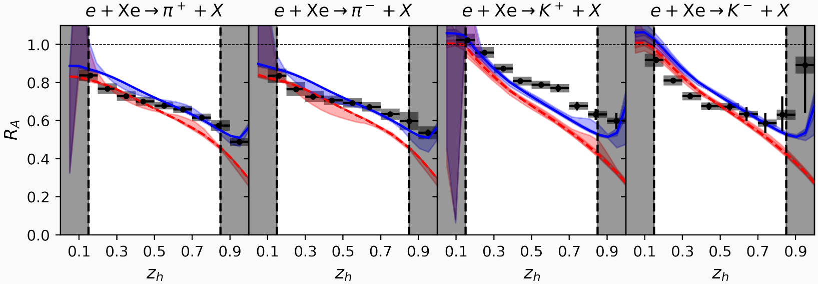
for the region $\xi^2 \ll \mu^2 \ll E/L$.

Opacity expansion $\Longrightarrow$ gradient expansion (preliminary)

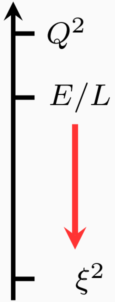
One can further organize higher-opacity correction in this approach.

$$\frac{\partial F_{\rm NS}}{\partial \tau} = \underbrace{2C_F\left[2C_A\frac{\partial F_{\rm NS}}{\partial z} - (2C_A + C_F)\frac{F_{\rm NS}}{z}\right]}_{\rm N=1} + \underbrace{B_2(\tau, \alpha_s, \frac{E}{L\xi^2})\left[\frac{\partial^2 F_{\rm NS}}{\partial z^2} - \frac{2}{z}\frac{\partial F_{\rm NS}}{\partial z} + \frac{2F_{\rm NS}}{z^2}\right]}_{\rm N=2,\ \ SGA} + \cdots$$

- Opacity-two leading-log contribution appears as a diffusion term.
- Opacity expansion $\Rightarrow$ Extract LL terms after expansion in $\frac{\xi^2}{E/L}$ $\Rightarrow$ Gradient expansion.



What is the perturbative controllable part of energy loss?



- RG analysis of shows that (light) parton energy loss is sensitive to physics at two scales in a thin static medium:

$$\Delta E_q = C_F C_A \frac{4\pi}{\beta_0} \frac{B\rho L}{2E/L} \left[\alpha_s(\xi^2) - \alpha_s\left(\frac{\chi E}{L}\right) \right]$$

- Physics at medium screening scale ξ^2 is strongly-coupled. Only scale E/L is perturbative for energetic partons.

Mass effect in energy loss

- The off-shellness of a heavy quark is different from a light quark.

$$\frac{1}{\mathbf{k}^2 + x^2 M^2} \approx \frac{1}{\mathbf{k}^2} \frac{\theta^2}{\theta^2 + \theta_D^2}, \quad \theta = \frac{\mathbf{k}}{\omega}, \quad \theta_D = \frac{M}{E}$$

- For energy loss, it is sufficient to investigate the soft-gluon approximation of $Q \to Q + g$

$$\frac{dP_{QQ}^{\mathrm{med}(1)}}{d\omega} \otimes F_Q(E+\omega) = \int_0^\infty d\omega \frac{2C_F}{\omega} F_Q(E+\omega) \int \frac{d^{2-2\epsilon}\mathbf{k}}{(2\pi)^{-2\epsilon}} \frac{\alpha_s^{(0)}}{2\pi^2} \frac{\Phi_{\mathrm{LPM}}\left(\frac{\mathbf{k}^2 + \omega^2\theta_D^2}{2\omega/L}\right)}{(\mathbf{k}^2 + \omega^2\theta_D^2)}$$
$$\int \frac{d^{2-2\epsilon}\mathbf{q}}{(2\pi)^{-2\epsilon}} \frac{\alpha_s^{(0)}}{\pi} \frac{C_A \rho_G L}{(\mathbf{q}^2} \frac{2\mathbf{q}\cdot(\mathbf{k}+\mathbf{q})}{(\mathbf{k}+\mathbf{q})^2 + \omega^2\theta_D^2} + \mathcal{O}\left(\frac{\xi^2}{E/L}\right)$$

- Without $\omega^2\theta_D^2$, this is the same as light parton.

Phase space region for leading-log contribution

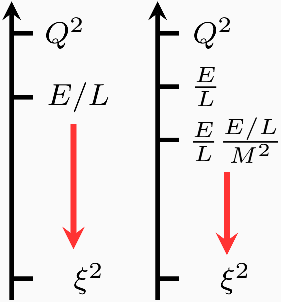
- The phase-space region where dead-cone scale can be neglected:

$$k^2 \sim \frac{\omega}{L} \gg \omega^2\theta_D^2 \Longrightarrow \omega \ll \frac{1}{\theta_D^2 L}$$

- An approximation treatment to identify leading-log contribution: dropping $\omega^2\theta_D^2$ from the matrix-element, but impose a reduced integration limit $\Theta\left(\omega < \frac{1}{\theta_D^2 L}\right)$

$$\xi^2 L \ll \omega \ll \min\left\{\frac{1}{\theta_D^2 L}, E\right\}$$

Mass modifies the perturbative region of energy loss calculations



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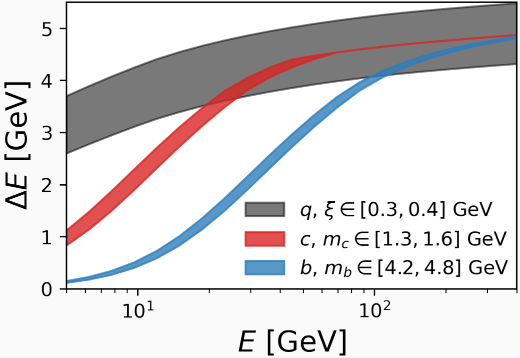
- The quark mas modifies/lowers the scale on the perturbative side.

$$\Delta E_Q = C_F C_A \frac{4\pi}{\beta_0} \frac{B\rho L}{2E/L} \left[\alpha_s(\xi^2) - \alpha_s\left(\frac{\chi E}{L} \min\left\{\frac{E/L}{M^2}, 1\right\}\right) \right]$$

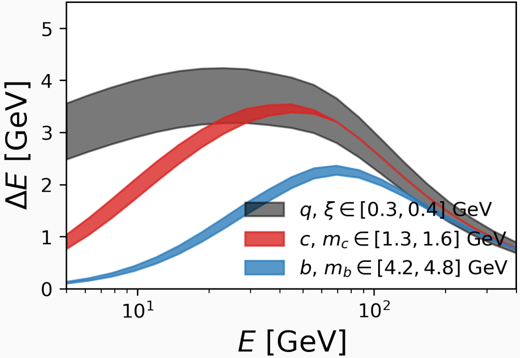
- NP effect is even more dominant for heavy quark energy loss.

Heavy vs light quark energy loss in *eA*

For $Q^2 \gg E/L,\;\; L = 0.75 R_{\mathrm{Pb}}$.



For $Q^2 = 25$ GeV^2, $\;\; L = 0.75 R_{\mathrm{Pb}}$.



Summary

- We studied parton propagation in a thin medium scenario defined by the scale separation

$$Q^2 \gg E/L \gg \xi^2, \quad L/\lambda_g \sim \xi/\Lambda \sim 1$$

- After power expansion in $\frac{\xi^2}{E/L}$, collinear observable acquires an IR divergences. Its renormalization lead to a set of PDE type RG equations.
- Equations encode energy loss. Anomalous dimension at opacity one can be extracted. Can be extended to higher-order in opacity as a gradient expansion.
- A pocket formula for Q^2, E, L, ξ^2, M dependent energy loss in thin medium is obtained.
- Parton energy loss is sensitive to physics at both screening scale and semi-hard scale $\frac{E}{L}$.
- Heavy quark mass further reduces the semi-hard scale.

Questions