Attractor behavior of pre-equilibrium transport coefficients

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New jet quenching tools to explore equilibrium and non-equilibrium dynamics in heavy-ion collisions, ECT*, Feb 2024

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Outline

- Bottom-up thermalization, QCD kinetic theory
- Attractor behavior in pressure
- Calculating transport coefficients in QCD kinetic theory
- Attractor in transport coefficients

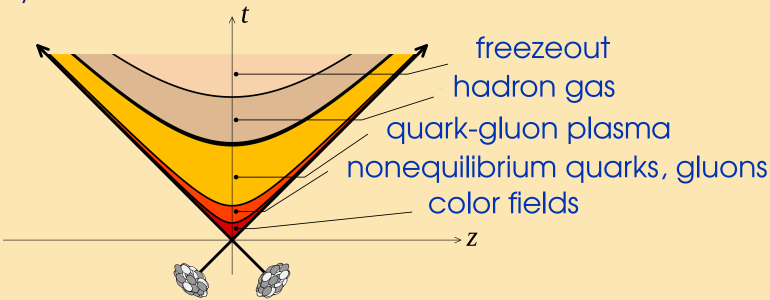
This talk:

- Heavy quark diffusion coefficient in heavy-ion collisions via kinetic theory, K. Boguslavski, A. Kurkela, T. L., F. Lindenbauer, J. Peuron, arXiv:2303.12520 [hep-ph]
- Jet momentum broadening during initial stages in heavy-ion collisions, K. Boguslavski, A. Kurkela, T.L., F. Lindenbauer, J. Peuron, arXiv:2303.12595 [hep-ph]
- Limiting attractors in heavy-ion collisions K. Boguslavski, A. Kurkela, T.L., F. Lindenbauer, J. Peuron, arXiv:2312.11252 [hep-ph]
- Jet quenching parameter in QCD kinetic theory K. Boguslavski, A. Kurkela, T.L., F. Lindenbauer, J. Peuron, arXiv:2312.00447 [hep-ph]
- 1+1D boost invariant expansion

Goal: calculate transport coefficients $\hat{q}$ and κ in pre-equilibrium phase

Heavy ion collision in spacetime

Stages of a heavy ion collision



- Timescales for hard $M \sim m_c, p_T$ probes:

$$1/M \ll 1/Q_s \ll t_{\text{therm}}$$

- Hard probes $M \sim m_c, p_T$ created first $\Longrightarrow$ cannot neglect pre-equilibrium
- Even if thermalization is quick, pre-equilibrium is hot, dense $\Longrightarrow$ large effect

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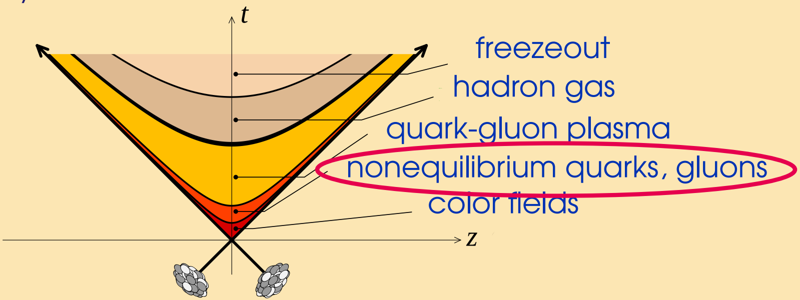

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Bottom-up thermalization

Bottom-up thermalization

Weak coupling QCD description of Glasma $\Longrightarrow$ QGP Baier, Mueller, Schiff, Son hep-ph/0009237

3 stages

1. Overoccupied, classical field stage ($0 \to \star$) : growing anisotropy of hard $\sim Q_s$ modes
2. Bath of soft particles develops ($\star \to \bullet$)
3. Radiative breakup of hard particles ($\bullet \to \blacktriangledown$)

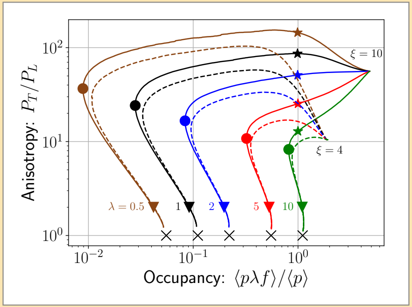
$$\tau_{\mathrm{BMSS}} = \alpha_s^{-13/5} Q_s^{-1}$$

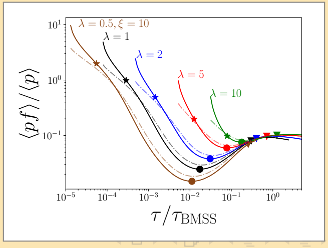
Can be tracked with AMY kinetic theory:

$$-\frac{d}{d\tau} f_{\mathbf{p}} = \mathcal{C}^{2\leftrightarrow 2}[f_{\mathbf{p}}] + \mathcal{C}^{1\leftrightarrow 2}[f_{\mathbf{p}}] + \mathcal{C}^{\exp}[f_{\mathbf{p}}].$$

Different initial conditions converge

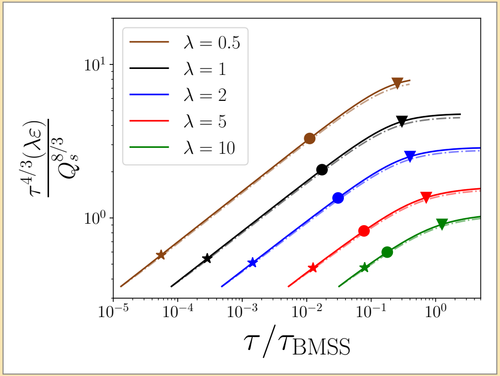
(ξ: initial anisotropy, $\lambda = 4\pi N_c \alpha_s$)

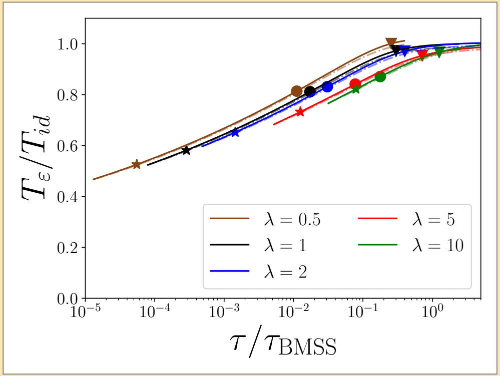




Approach to hydro

- Bjorken hydro $\varepsilon \sim 1/\tau^{4/3}$
- Most of pre-equilibrium: $\varepsilon \sim 1/\tau$





- $T_{\rm id}$= bkwd extrapolated ideal hydro
- $T_\varepsilon \sim \sqrt[4]{\varepsilon}$

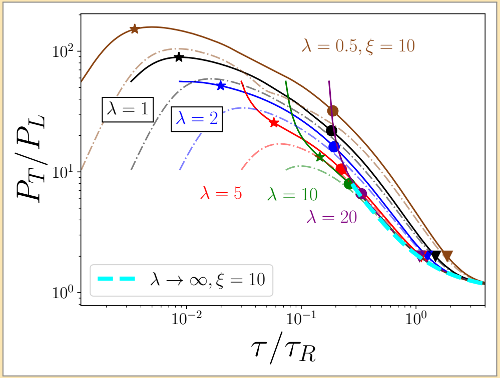
Attractors

Two "limiting attractors"

$$\tau_R(\lambda,\tau) = \frac{4\pi\frac{\eta}{s}}{T_\varepsilon}$$

(T_ε from energy density)

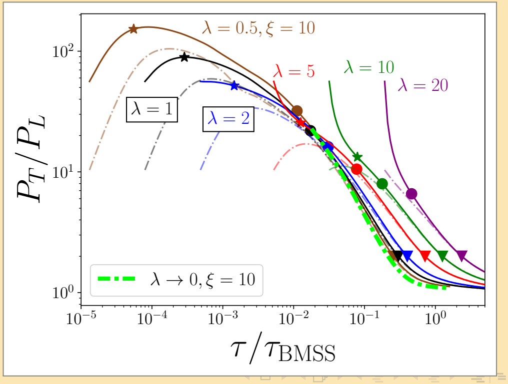
- Isotropization rate near equilibrium
- "Hydro attractor" in literature



$$\tau_{\text{BMSS}} = \alpha_s^{-13/5}/Q_s$$

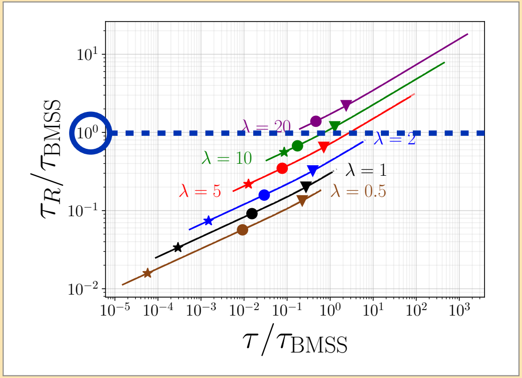
- Weak coupling QCD thermalization
- Timescale for rough isotropy

(Then hydro attractor takes over)



How different are the timescales?

- Weak coupling: timescales different
- Viscous hydro (relevant scale τ_R) follows from EKT (relevant scale τ_{BMSS}) Contradiction? No!
- $\lambda \ll 1 \implies \tau_R \ll \tau_{\text{BMSS}}$ First spend **long** time in BMSS regime then short time on hydro attractor



(τ_R depends on τ, because $\varepsilon(\tau)$ changes)

BMSS regime can matter more than hydro attractor for hard probes.

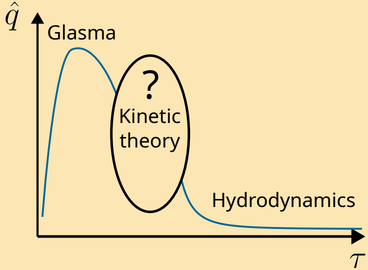
We plot on log scale. E.g. if $\hat{q} \sim \epsilon(\tau) \sim 1/\tau$
$0.1\text{fm} < \tau < 1\text{fm}$ and $1\text{fm} < \tau < 10\text{fm}$ contribute equally to $\int d\tau \hat{q}(\tau)$

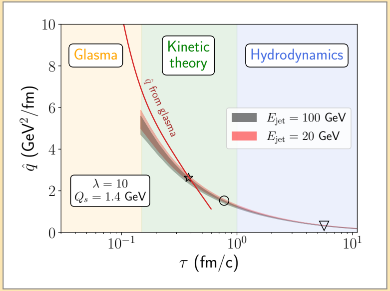
Transport coefficients $\hat{q}$ and κ

Transport coefficients pre-equilibrium

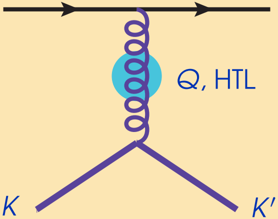
$$\left.\begin{matrix}\hat{q}\\ \kappa\end{matrix}\right\} = \frac{d\langle q_\perp^2\rangle}{dt} \quad \left\{\begin{matrix}\text{jet } (p=\infty)\\ \text{H.Q. } (m=\infty)\end{matrix}\right.$$

- Standard for a long time:
  $\hat{q}$, κ in thermal system
  $\Longrightarrow$ Input for jet quenching, H.Q. diffusion
- Recent interest: glasma phase
  E.g. A. Ipp et al 2001.10001, 2009.14206
  Avramescu et al 2303.05599
  Carrington et al 2112.06812, 2202.00357, 2304.03241, 2001.05074
  Pooja Khowal et al 2110.14610
  M. Ruggieri et al 2203.06712
  Y. Sun et al. 1902.06254
  K. Boguslavski et al 2005.02418
- Aim: complete the picture
  from the glasma to hydrodynamics
  More details in F. Lindenbauer's talk





Calculating transport coefficients



Momentum broadening from interactions with medium particles:

$$\begin{matrix}\hat{q} \\ \kappa\end{matrix} \sim \int_{\mathbf{k}\mathbf{k}'\mathbf{p}'} \frac{q_T^2}{E_{\mathbf{p}}} (2\pi)^4 \delta^4(P + K - P' - K') \, |\mathcal{M}|^2 \, f(\mathbf{k}) \, (1 + f(\mathbf{k}')) \,,$$

- κ: heavy quark $P = (M, \mathbf{0})$, $M \to \infty$
- $\hat{q}$: energetic jet $P^2 = 0, p \to \infty$ (need cutoff $\hat{q} \sim \ln \Lambda_\perp$)

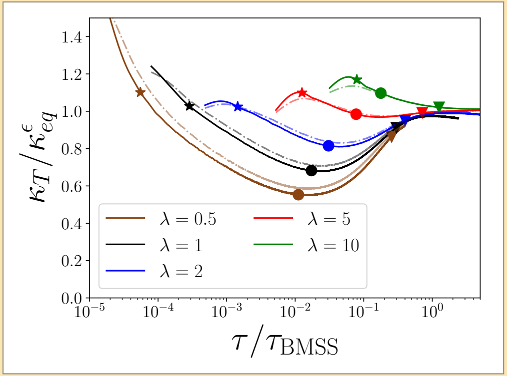
These limits: **medium properties**, not probe

Result: κ

Compare to thermal system with same ε
(Landau matching,

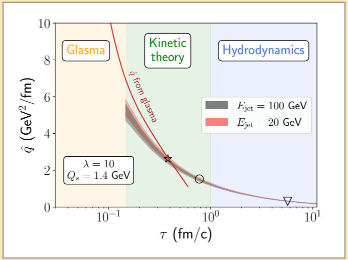
thermal with same m_D or T_* is much further)

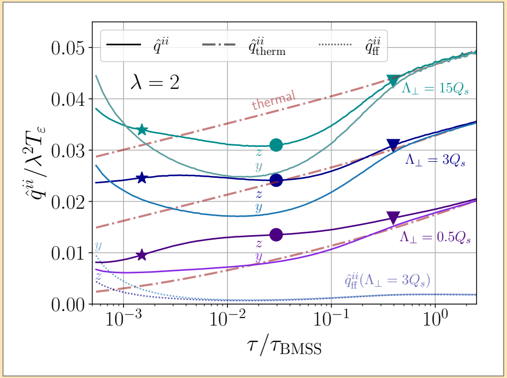
- Enhancement first (overoccupied)
- Then suppression (underoccupied)
- Larger $\lambda = 4\pi N_c \alpha_s$:
  behavior smoothed out



Result: $\hat{q}$

- Large cutoff $\Lambda_\perp$:
  Enhancement first, then suppression
- Smaller $\Lambda_\perp$:
  smoother, overall enhancement

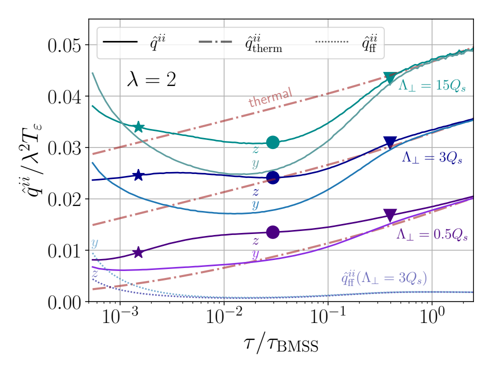


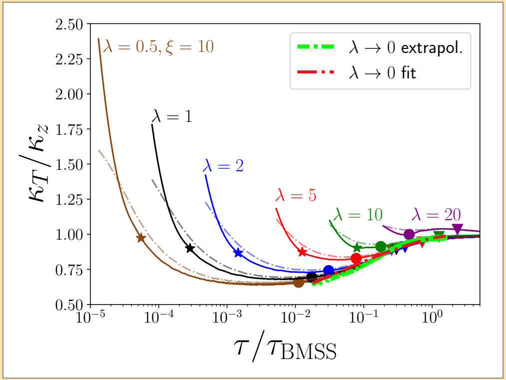


- $\varepsilon \sim 1/\tau$ large
- At end of BMSS ▼: $\hat{q} \approx$ JETSCAPE estimate (can match by tuning $\Lambda_\perp$)

Anisotropy

- Inital overoccupied: $\kappa_T > \kappa_L$, $\hat{q}_T > \hat{q}_L$ $\implies$ Bose enhancement, Glasma
- Then underoccupied $\kappa_T < \kappa_L$, $\hat{q}_T < \hat{q}_L$ $\implies$ Anisotropy of f

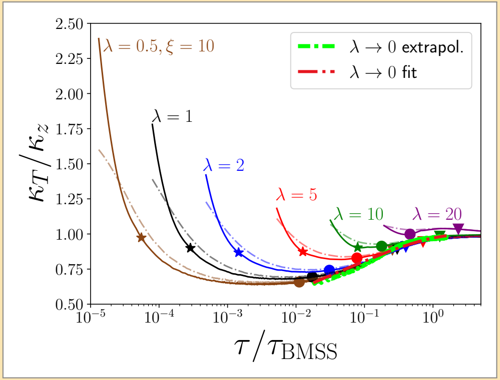


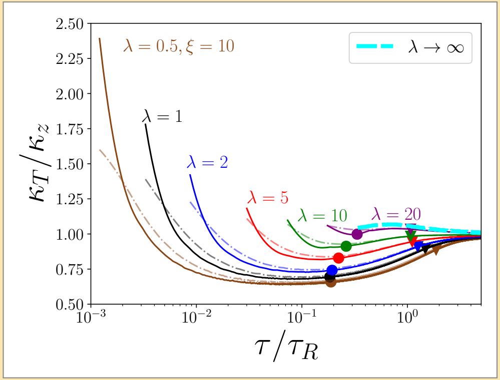


Attractors for $\hat{\mathrm{q}}$ and κ

κ anisotropy, 2 attractors

Anisotropy of κ, scaling with the two attractor timescales

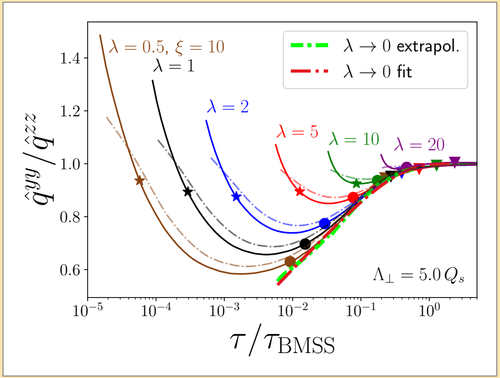


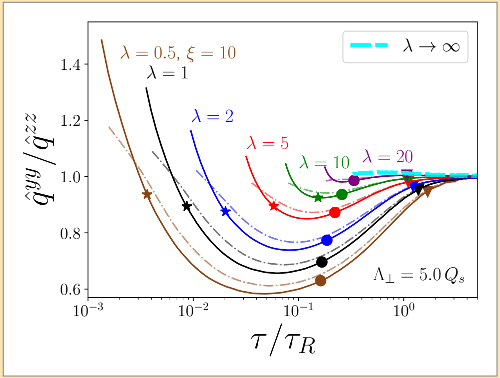


Weak coupling BMSS is a better description, over larger range in τ

$\hat{q}$ anisotropy, 2 attractors

Anisotropy of $\hat{q}$, scaling with the two attractor timescales

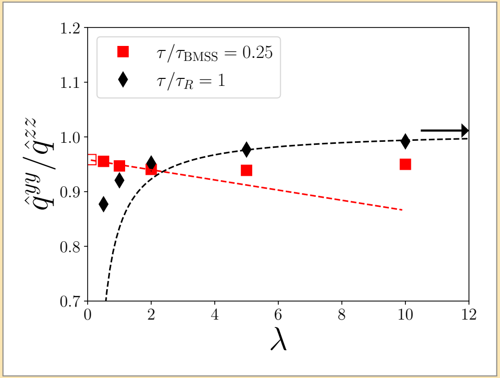


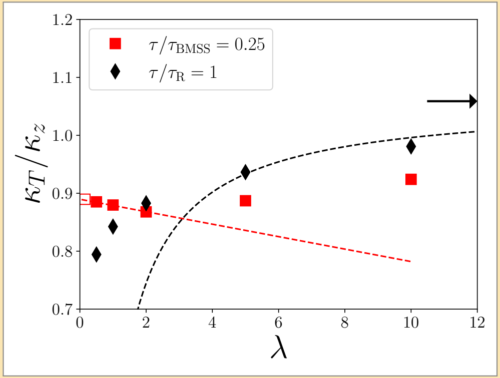


Weak coupling BMSS is a better description, over larger range in τ

Extrapolating to weak and strong coupling

How do we construct the attractor curves?

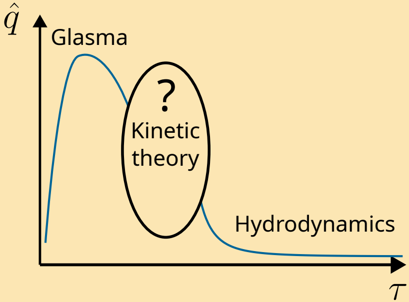




- Take fixed value of τ/τ_{BMSS} or τ/τ_R
- Linear fit in λ or $1/\lambda$, separately for each τ.
- For BMSS also provide a parametrization of the τ-dependence ("$\lambda \to 0$ fit" in plot)

Conclusions

- Pre-equilibrium stage short, but hot
  $\Longrightarrow$ Effect on hard observables?
- QCD kinetic theory:
  trace system from glasma to hydro
  — and calculate transport coefficients
- Introduced concept of **limiting attractors**
  - Hydro attractor: close to equilibrium, works better at strong coupling
  - BMSS attractor: most of pre-equilibrium weak coupling



$\hat{q}$, κ parametrizations available

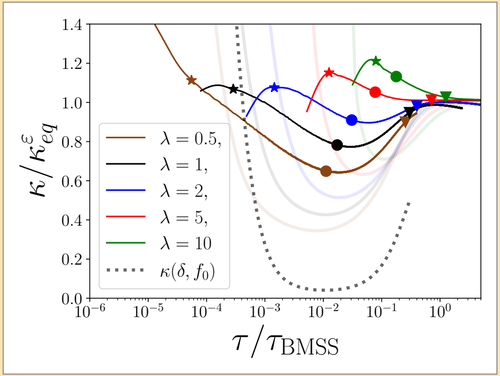
Relevant microscopic scales

- Occupation number f
- Coupling α_s
- Anisotropy $\delta \sim \sqrt{\frac{\langle p_z^2 \rangle}{\langle p_T^2 \rangle}}$
- Hard scale $p_T^2 \sim Q_s^2$

From these estimate

- Energy density $\varepsilon \sim \delta Q_s^4 f$
- Debye scale $m_D^2 \sim \alpha_s \delta\, Q_s^2 f$
- Soft mode eff. temperature $T_* \sim Q_s(f+1)$
- $\kappa \sim m_D^2 T_*$

Understanding the systematics



(Light: T_*, m_D from EKT, dashed: f, δ from EKT)