QGP Physics from Attractor Perturbations

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Based on work in progress with M. Spalinski

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Motivation

QGP evolution starts far from equilibrium

- Characteristics of heavy-ion collisions:

Speed ~ 1 fast

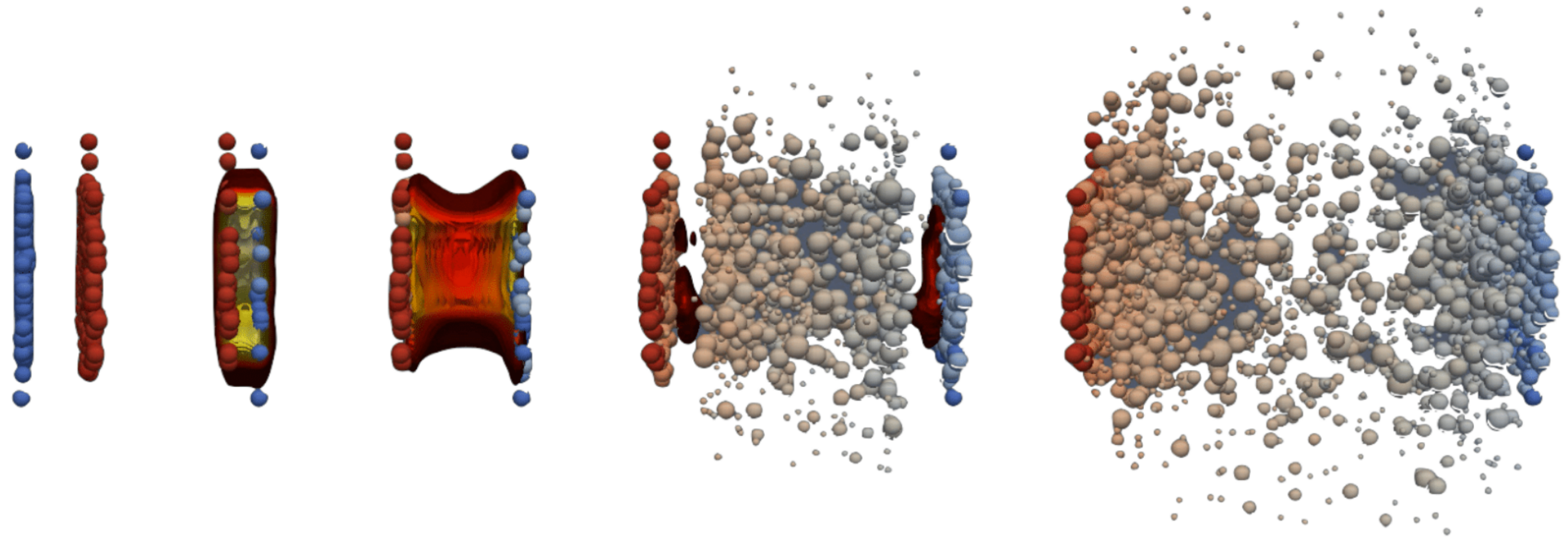
Energy $\sim 10 - 10^4$ GeV high

Collision time $\sim 0.01 - 1$ fm short

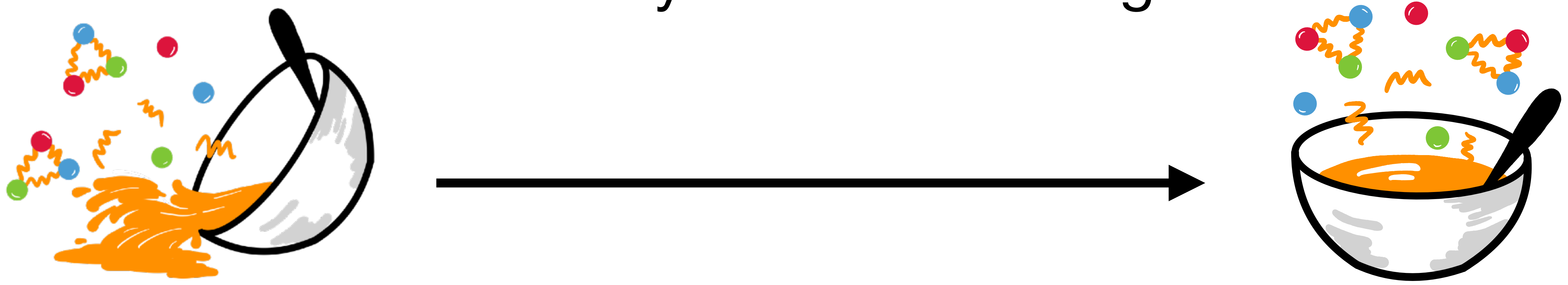
Size ~ 10 fm small

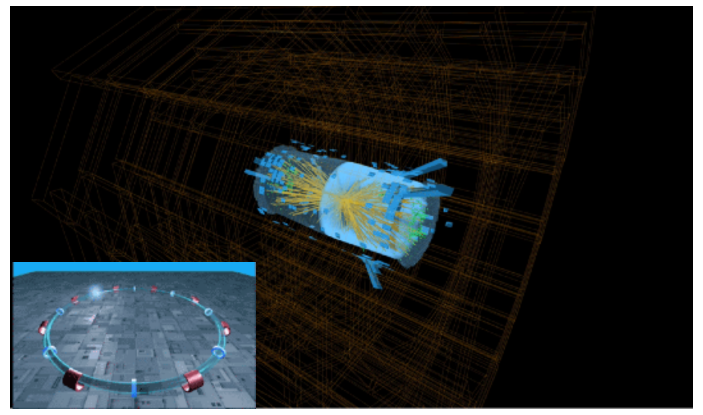
Particles $\sim 10^2 - 10^4$ few

initial stage QGP hadronic freezeout

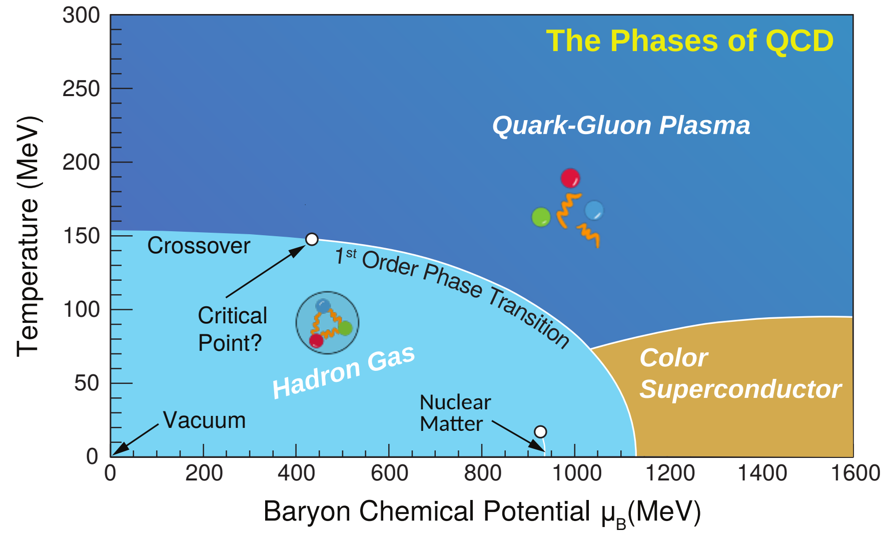


History of the little bang



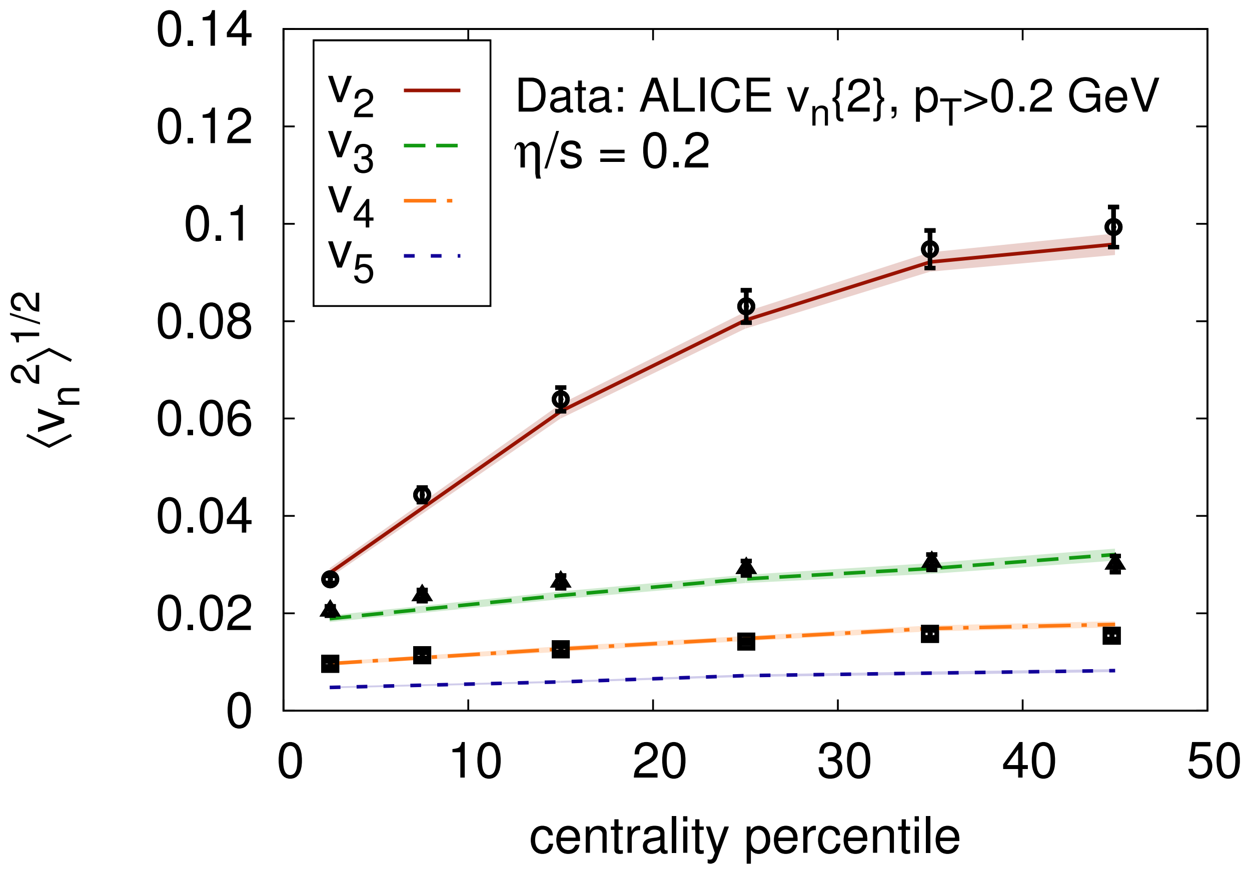


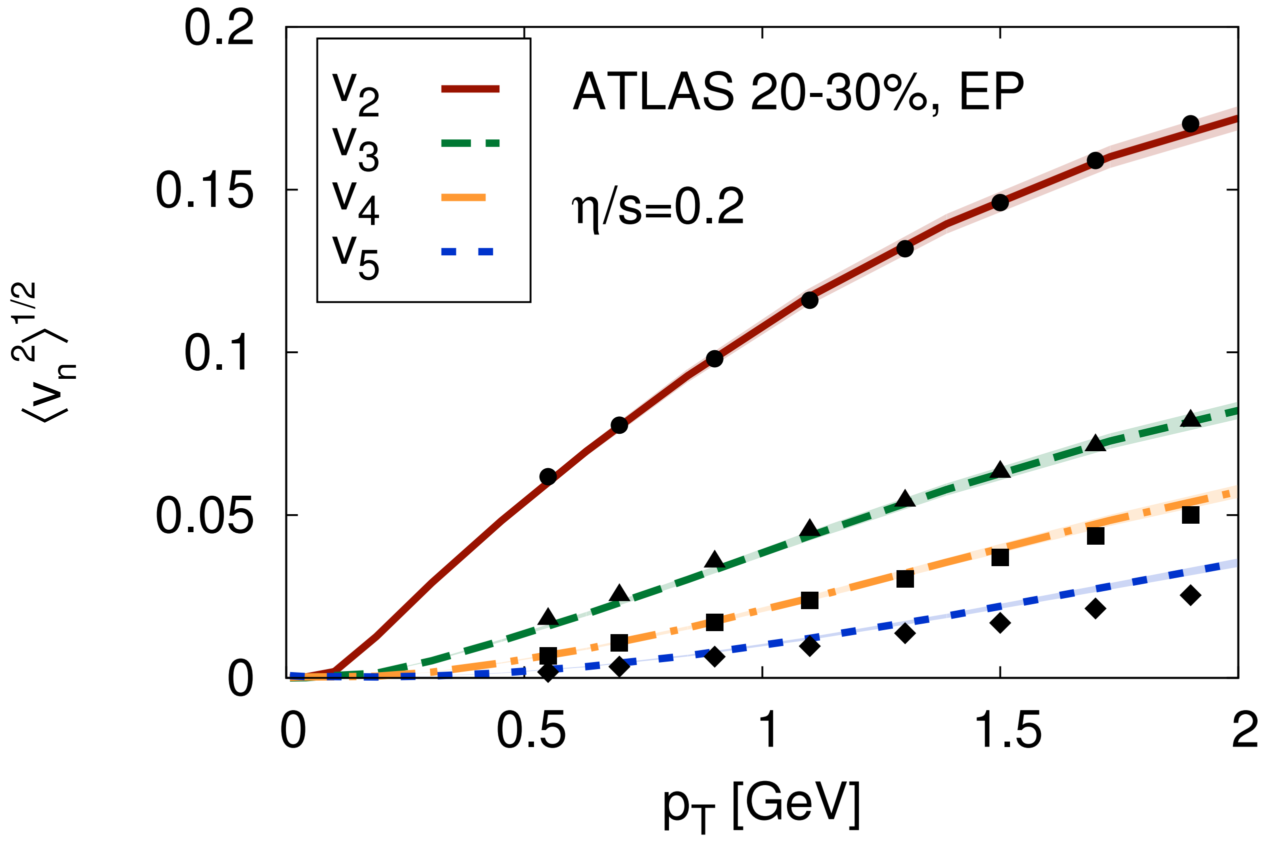
Collision events at LHC



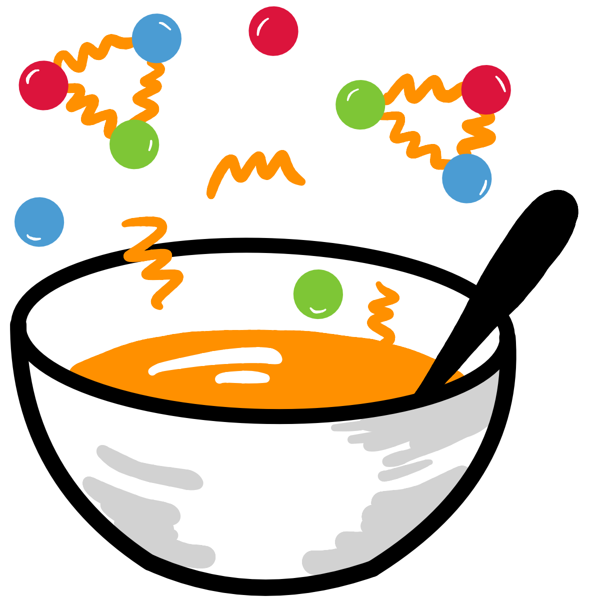
QGP is well described by hydrodynamics

- Flow collectivity manifests QGP as a *nearly perfect fluid*.



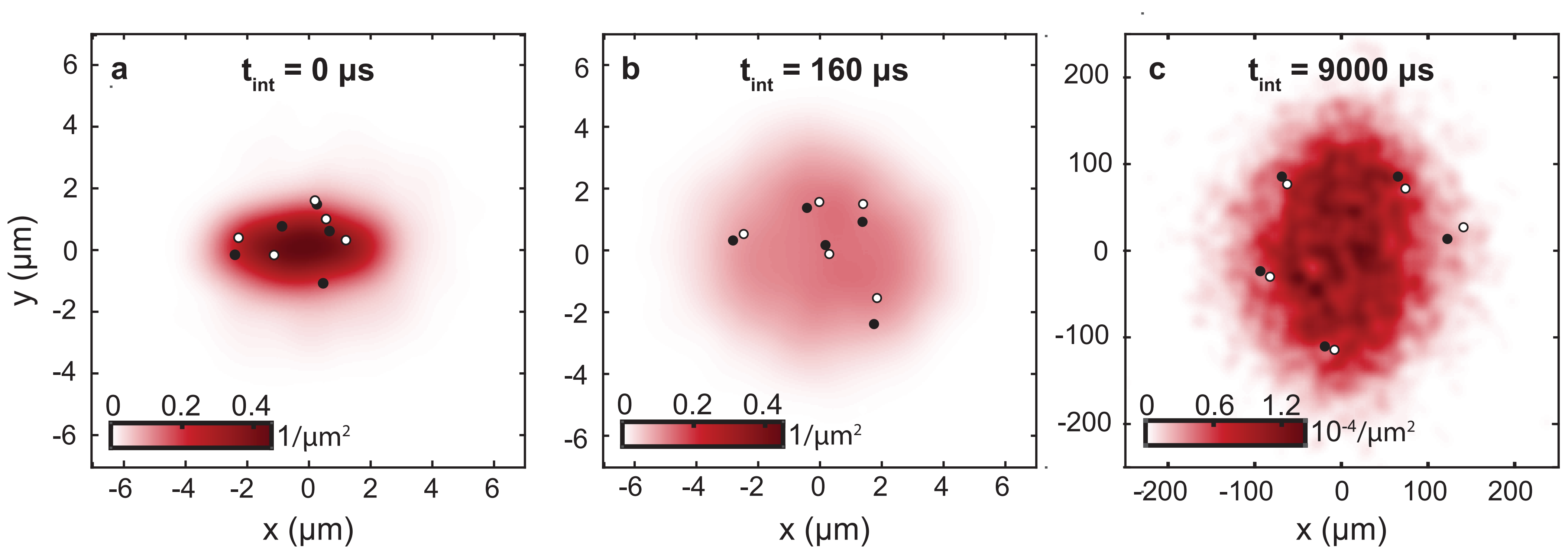


Gale et al, 1301.5893



Hydrodynamics is believed to be applicable near equilibrium

- And even more:



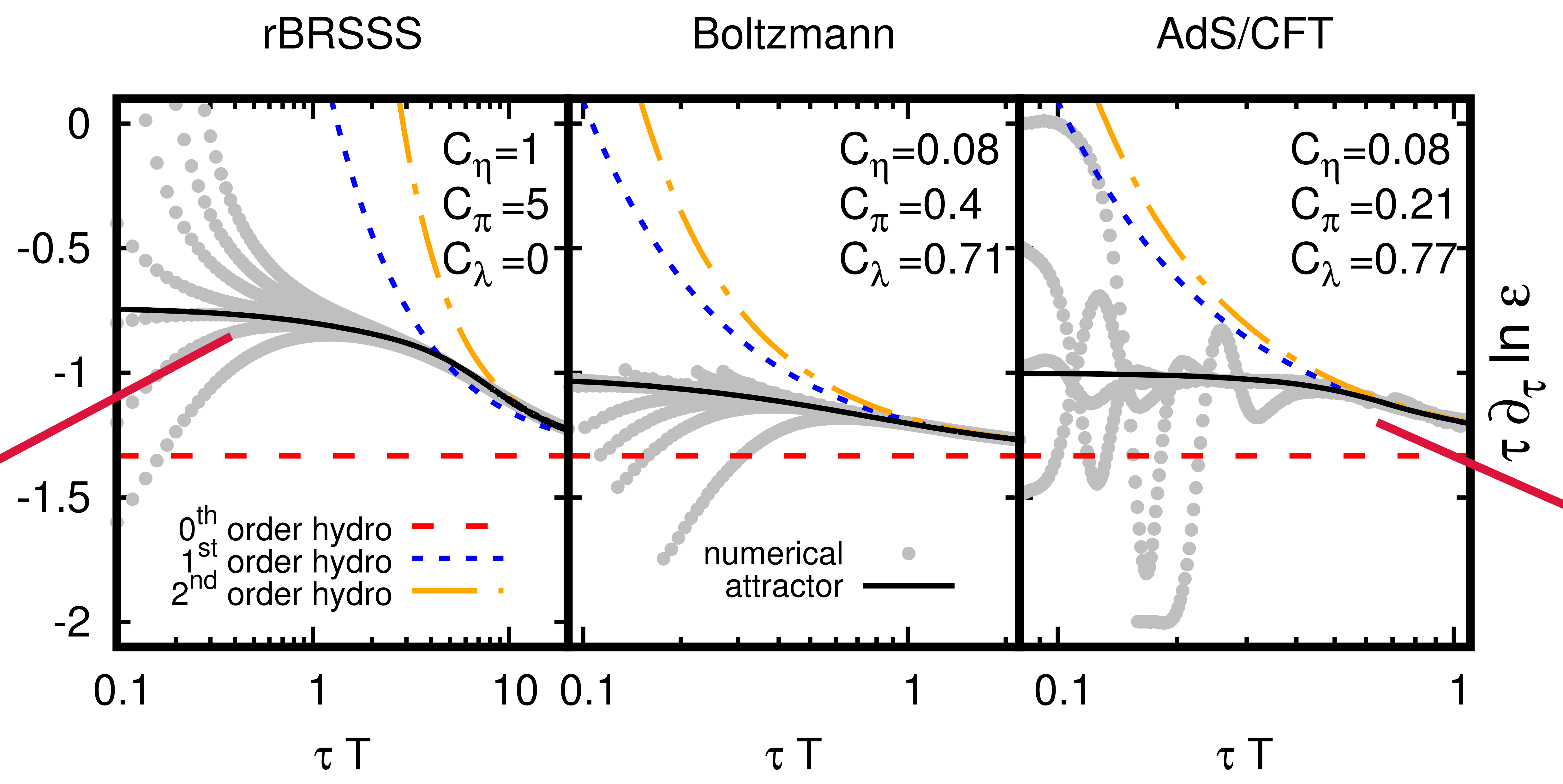
Density distribution in position space

5-particle hydrodynamics
Brandstetter et al, 2308.09699

Hydrodynamic attractor

- *Attractor* plays an important role to explain the success of hydrodynamics even far from equilibrium.

The onset of hydrodynamization starts at very early time



Initial information largely suppressed at later time

Florkowski et al, 1707.02282, Romatschke, 1712.05815

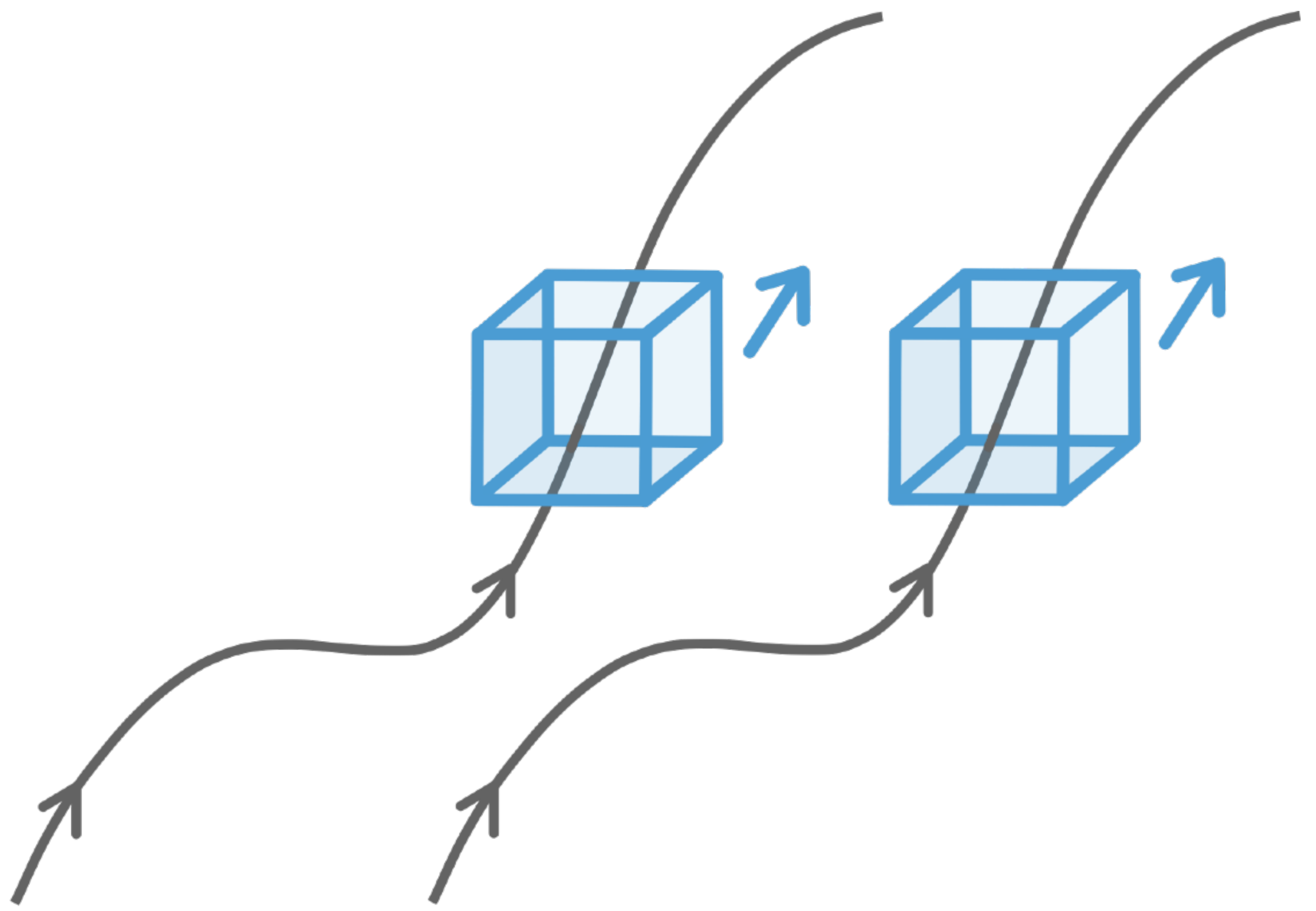
- Questions:
  - Would attractor wash out everything? No
  - Can attractor exist with less symmetries? Yes
  - How can we study jet with attractor? This work

Attractors

Fluids in equilibrium: Euler equation

- Stress tensor is homogeneous in LRF.

$$T^{\mu\nu}_{(0)\mathrm{LRF}} = \begin{pmatrix} \varepsilon & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} \xrightarrow{\text{boost}} T^{\mu\nu}_{(0)} = \varepsilon u^\mu u^\nu + p\Delta^{\mu\nu}$$



- Euler equation:

$$\partial_\mu T^{\mu\nu}_{(0)} = 0 \quad \Longrightarrow \quad \partial_t \psi = \nabla \cdot J_{(0)}[\psi] \quad \text{where} \quad \psi = (\mathrm{n}, \varepsilon, \pi, \ldots)$$

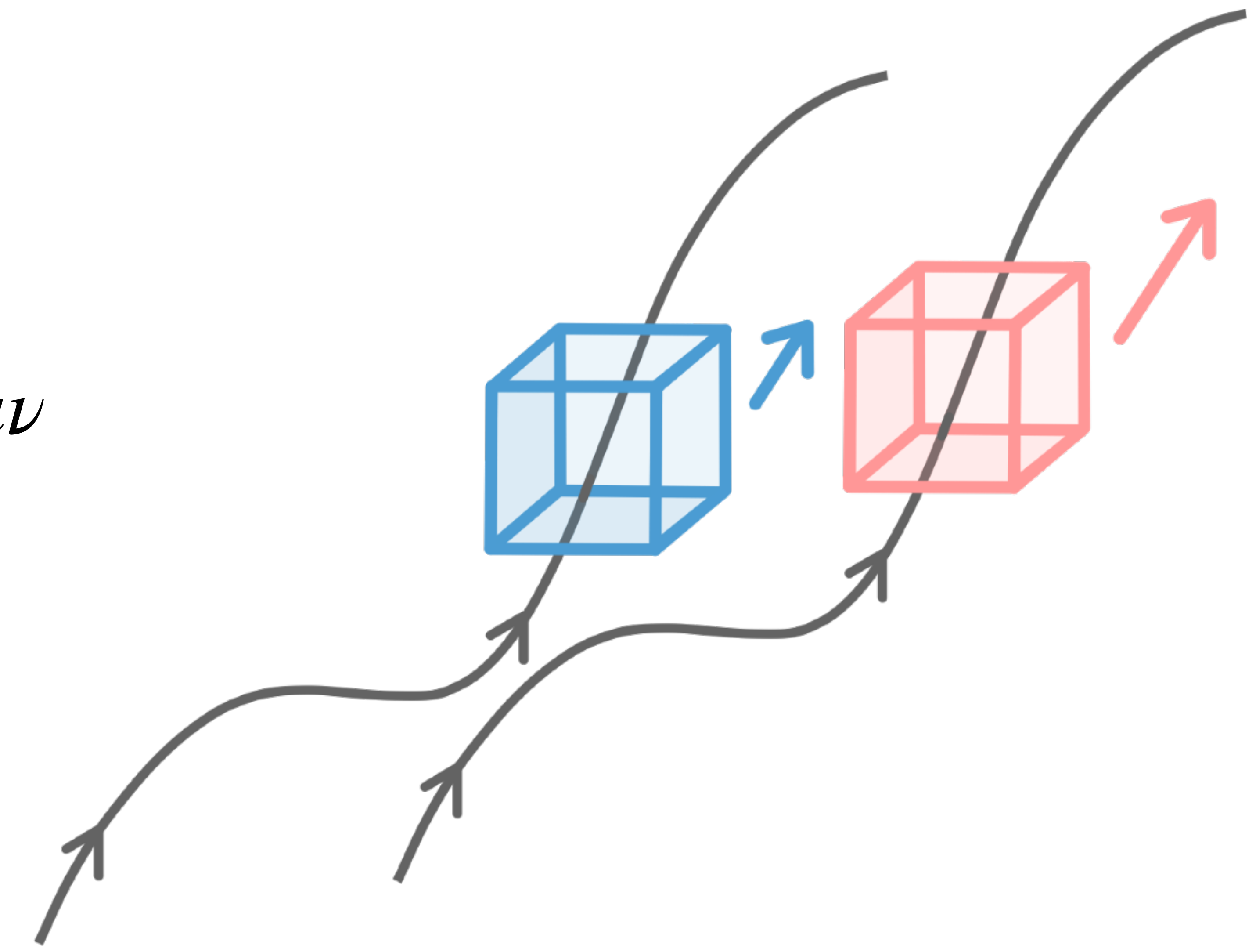
Conserved quantities evolve via advection & expansion.

Fluids near equilibrium: NS-like equations

- Stress tensor approximated by gradient expansion.

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + \dots$$

$$T^{\mu\nu}_{(1)} = -2\eta\sigma^{\mu\nu}, \quad \sigma^{\mu\nu} = \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(\partial_\alpha u_\beta + \partial_\beta u_\alpha) - \frac{1}{3}\partial \cdot u \Delta^{\mu\nu}$$



NB: there are infinite many equilibrium proxies for a non-equilibrium state.

- Navier-Stokes(NS)-like (e.g., Burnett, BRSSS, etc.) equations:

$$\partial_\mu T^{\mu\nu} = 0 \quad \Longrightarrow \quad \partial_t \psi = \nabla \cdot J[\psi, \nabla\psi, \dots] \quad \text{where} \quad \psi = (\mathrm{n}, \varepsilon, \pi_\mu, \dots)$$

Conserved quantities evolve via advection & expansion, as well as dissipation & diffusion.

Fluids far from equilibrium: MIS-like equations

- Stress tensor involves non-hydrodynamic DOFs for UV completion.
  E.g., 0+1D boost-invariant conformal fluids:

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \pi^{\mu\nu} + \ldots = \begin{pmatrix} \varepsilon & & & \\ & p_T & & \\ & & p_T & \\ & & & p_L \end{pmatrix}$$

$$p_T = p - \pi/2,$$
$$p_L = p + \pi$$
$$A \equiv (P_T - P_L)/P$$

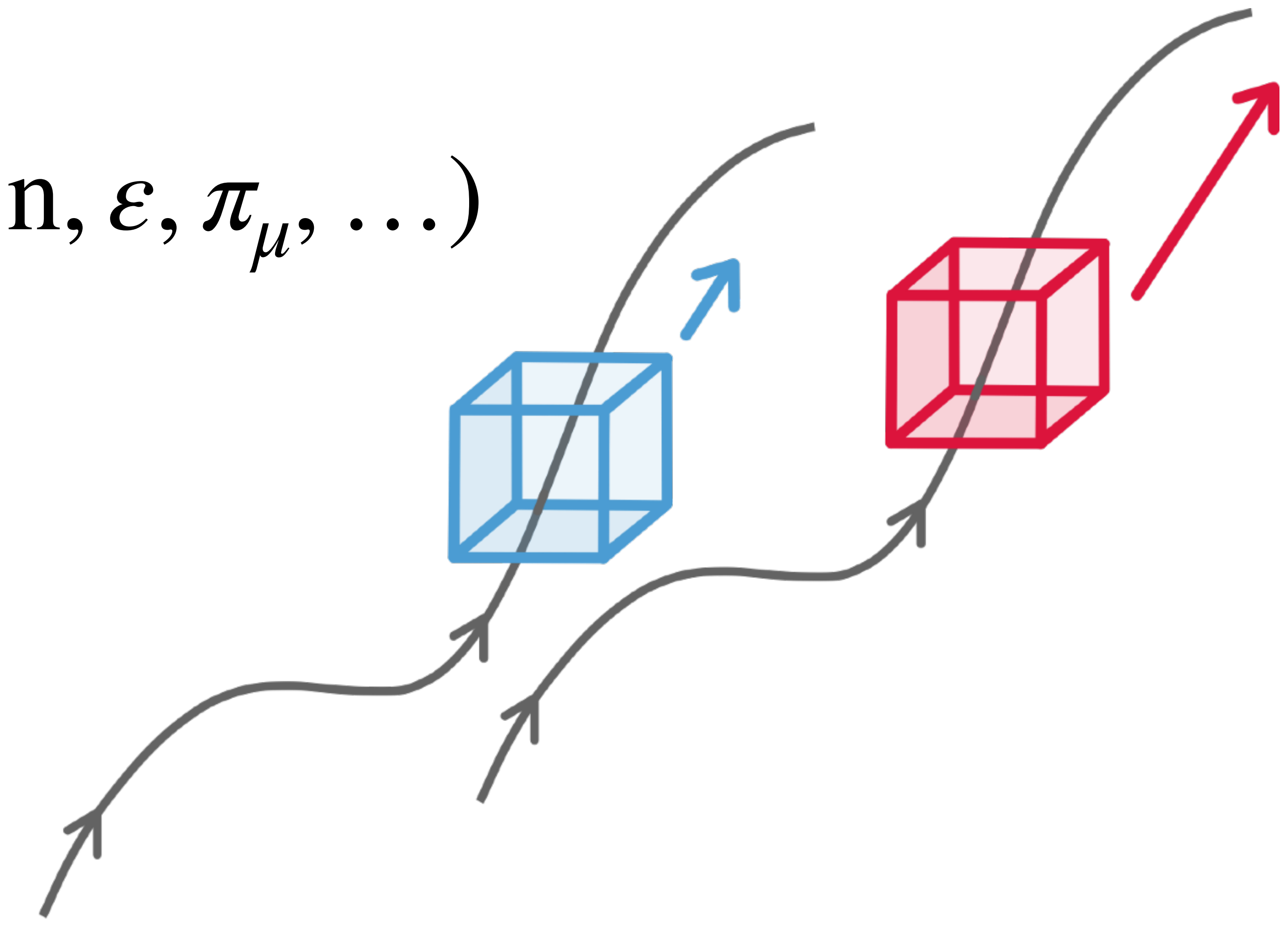
$$\pi \equiv \pi^\eta_\eta$$

NB: A vanishes in equilibrium

- MIS-like (e.g., Maxwell-Cattaneo, DNMR, BDNK etc.) equations:

$$\partial_\mu T^{\mu\nu} = 0 \quad \Longrightarrow \quad \partial_t \psi = \nabla \cdot J[\psi, \pi, \ldots] \quad \text{where} \quad \psi = (\mathrm{n}, \varepsilon, \pi_\mu, \ldots)$$

MIS $\qquad \tau_\pi \partial_\tau \pi = -(\pi - \pi^{(\mathrm{NS})}) + \ldots$

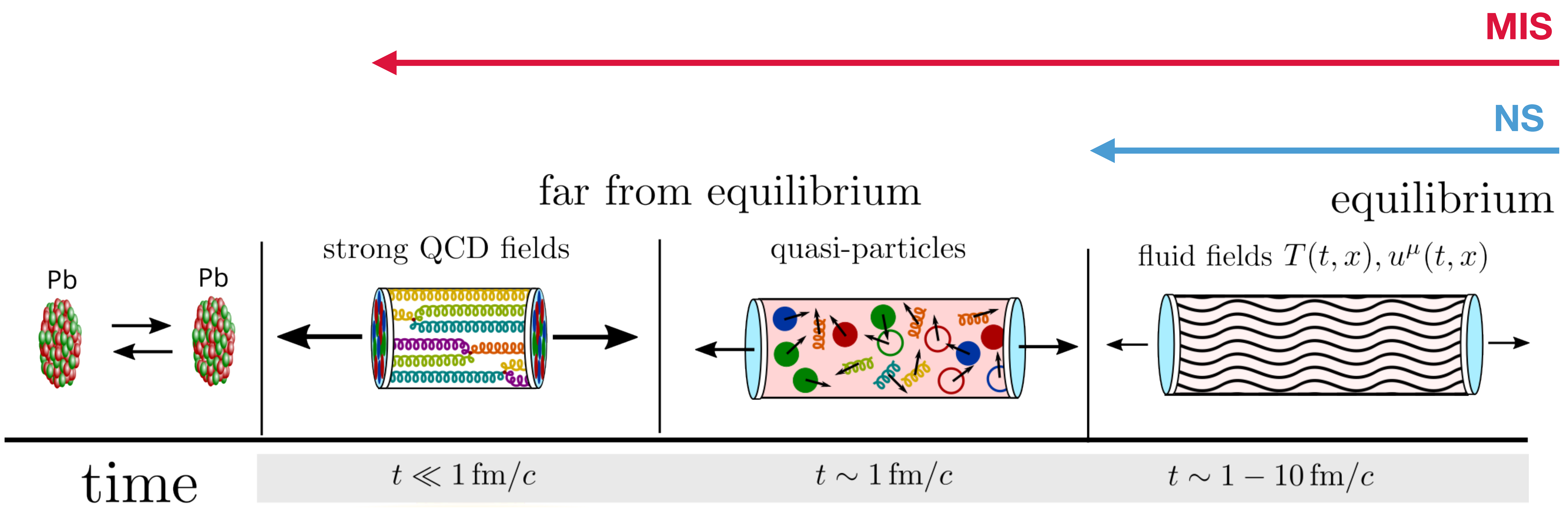


Fluids far from equilibrium: MIS-like equations

- Coupled equations for conformal system: $\varepsilon = 3p = C_e T^4, \eta = \frac{4}{3} C_e C_\eta T^3, \tau_\pi = C_\tau T^{-1}$

$$\tau T'(\tau) + T(\tau)\left(\frac{1}{3} - \frac{A(\tau)}{18}\right) = 0, \qquad C_\tau \tau A'(\tau) + \frac{2}{9} C_\tau A(\tau)^2 + \tau T(\tau) A(\tau) - 8 C_\eta = 0$$

- MIS-like theory does not necessarily capture the early-time QCD physics, but it is still a simple self-consistent theory valid at all times (as opposed to NS which is only valid at late times), thus can shed light on far-from-equilibrium dynamics.



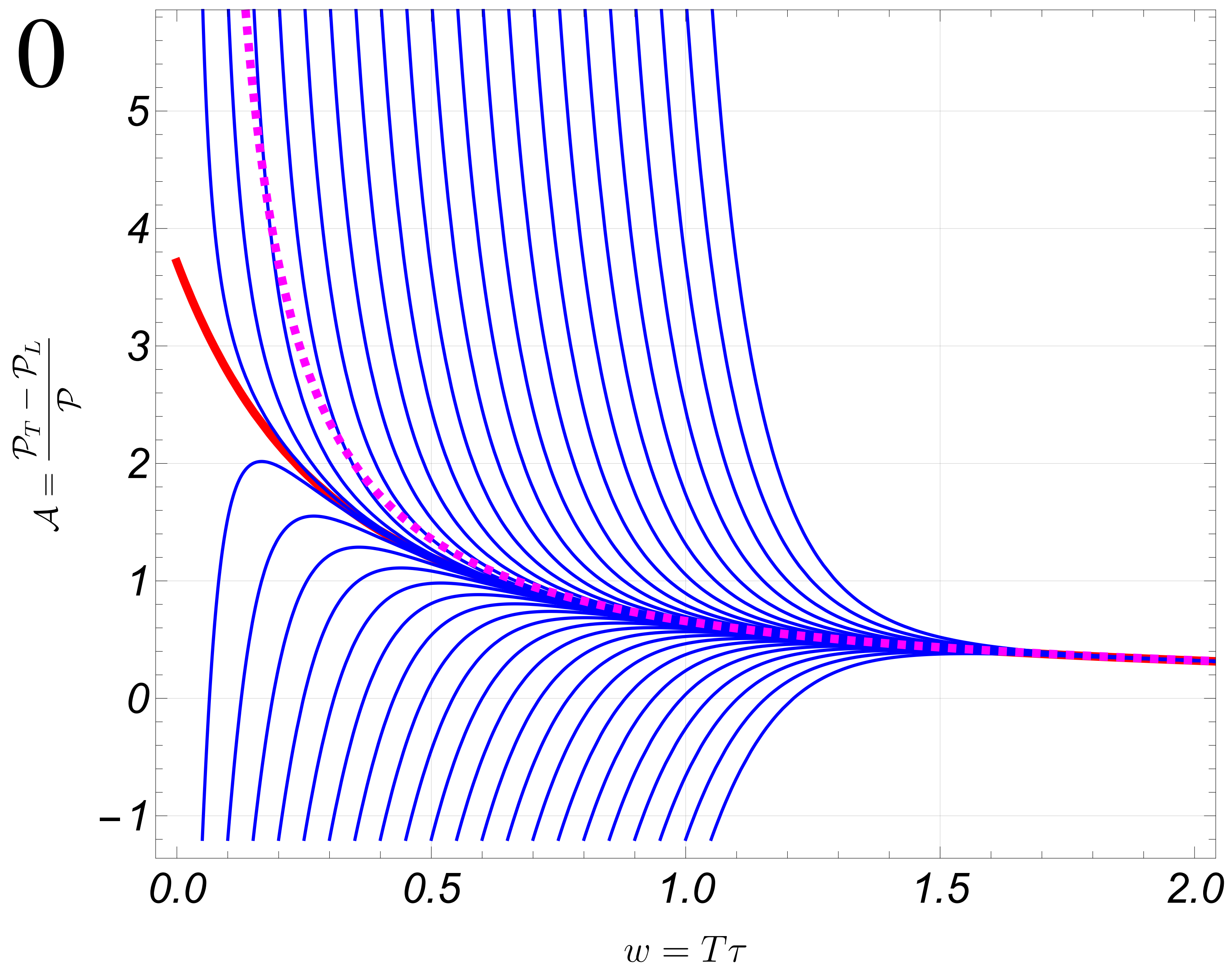
Courtesy of A. Mazeliauskas

Hydrodynamic attractors

- In terms of $w = \tau T$, equation for pressure anisotropy $A(w) \equiv (P_T - P_L)/P$ decouples:

$$C_\tau \left(1 + \frac{A(w)}{12}\right) wA'(w) + \frac{1}{3} C_\tau A(w)^2 + \frac{3}{2} wA(w) - 12 C_\eta = 0$$

decoupled 1st order *nonlinear & inhomogeneous* ODE



with asymptotic solutions

$$A(w) = \frac{C_0}{w^4}\left(1 + \mathcal{O}(w)\right) + 6\sqrt{C_\eta / C_\tau} + \mathcal{O}(w), \quad w \to 0$$

longitudinal expansion dominates + early-time attractor

Heller et al, 1503.07514; Jankowski et al, 2303.09414

$$A(w) = \frac{8C_\eta}{w}\left(1 + \frac{2C_\tau}{3w} + \mathcal{O}(w^{-2})\right) + C_\infty e^{-\frac{3w}{2C_\tau}} w^{\frac{C_\eta}{C_\tau}} \left(1 + \mathcal{O}(w^{-1})\right) + \ldots, \quad w \to \infty$$

late-time (hydrodynamic) attractor + non-hydrodynamic (transseries) modes.

Alternative formulation of attractors

- In the presence of additional scales other than T, τ is more convenient as dynamic variable than $w = \tau T$.

  - Early-time *attractor* solutions: μ: integration constant; $\alpha = \sqrt{C_\eta / C_\tau}$

$$T(\tau) \sim \mu(\mu\tau)^{-\frac{1-\alpha}{3}}\left(1 + \sum_{n=1}^{\infty} t_n^{(0)}(\mu\tau)^{\frac{n}{3}(2+\alpha)}\right), \qquad A(\tau) \sim 6\alpha\left(1 + \sum_{n=1}^{\infty} a_n^{(0)}(\mu\tau)^{\frac{n}{3}(2+\alpha)}\right)$$

  - Later-time asymptotic solutions Λ, C_∞: independent integration constant

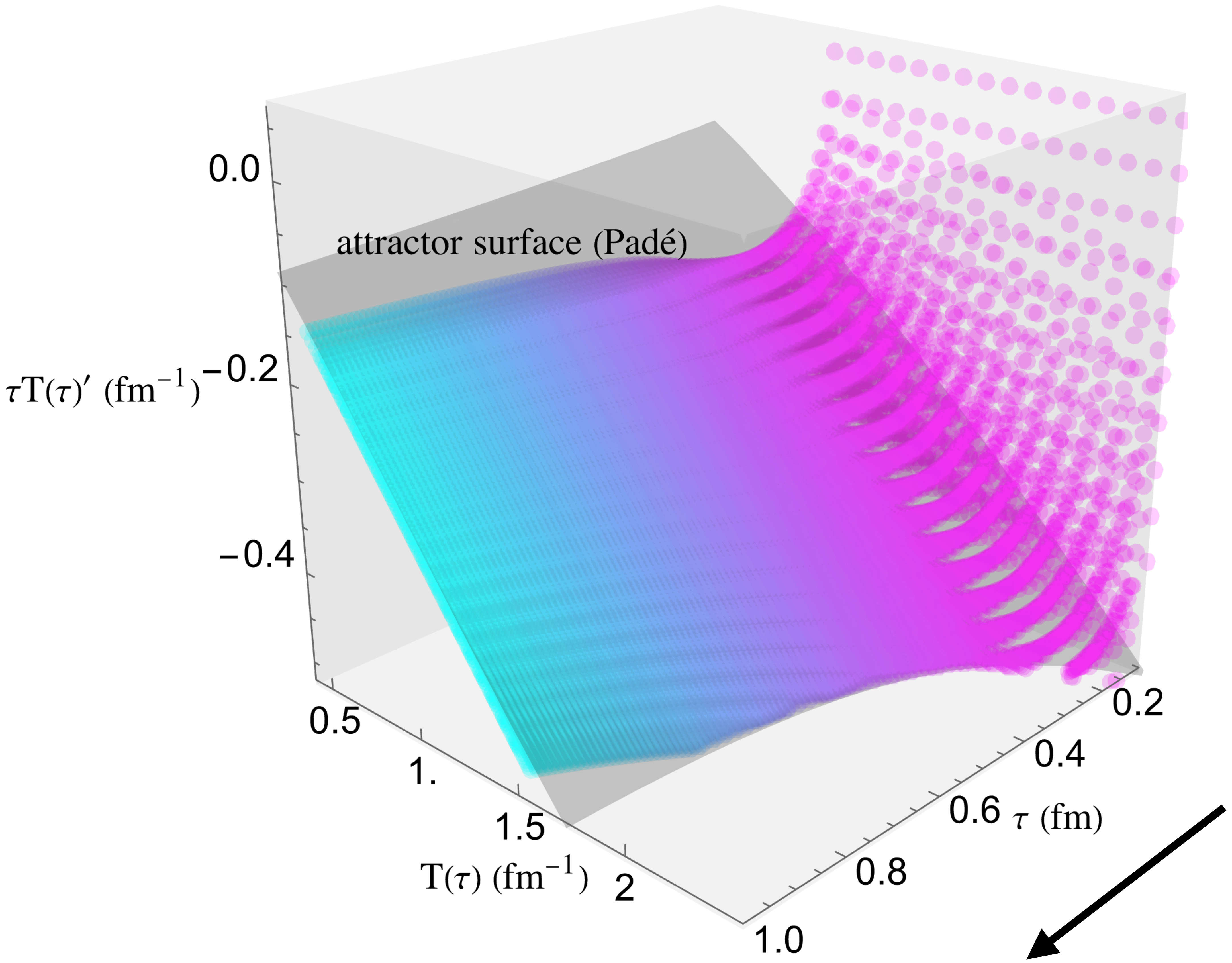
$$T(\tau) \sim \Lambda(\Lambda\tau)^{-\frac{1}{3}}\left(1 + \sum_{n=1}^{\infty} t_n^{(\infty)}(\Lambda\tau)^{-\frac{2}{3}n}\right) + C_\infty(\Lambda\tau)^{-\frac{2}{3}(1-\alpha^2)} e^{-\frac{3}{2C_\tau}(\Lambda\tau)^{2/3}}\left(1 + \mathcal{O}((\Lambda\tau)^{-2/3})\right) + \ldots$$

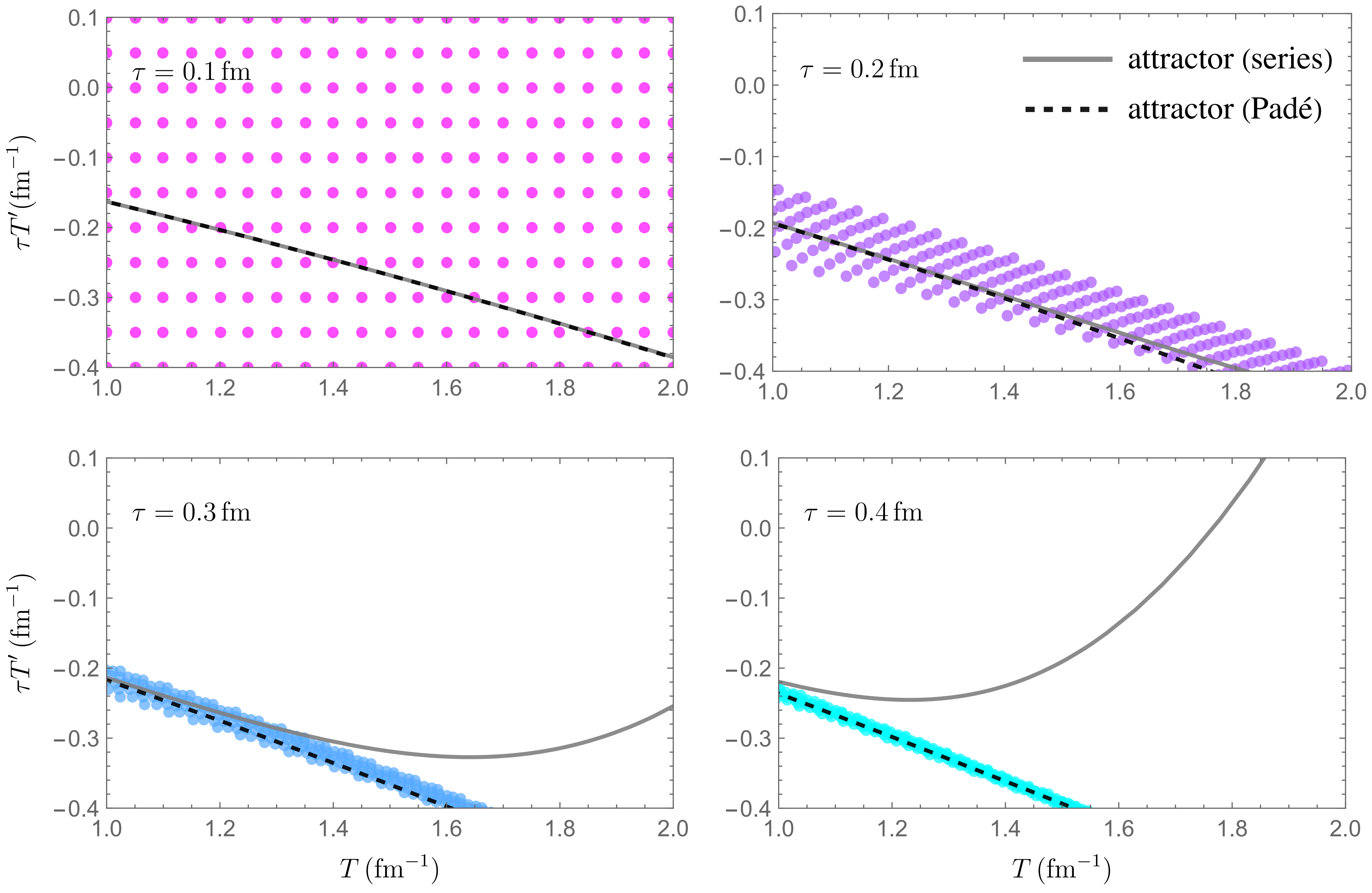
$$A(\tau) \sim 8C_\eta(\Lambda\tau)^{-\frac{2}{3}}\left(1 + \sum_{n=1}^{\infty} a_n^{(\infty)}(\Lambda\tau)^{-\frac{2}{3}n}\right) + C'_\infty(\Lambda\tau)^{-\frac{1}{3}+\alpha^2} e^{-\frac{3}{2C_\tau}(\Lambda\tau)^{2/3}}\left(1 + \mathcal{O}((\Lambda\tau)^{-2/3})\right) + \ldots$$

hydrodynamic attractor + non-hydrodynamic (transseries) modes.

Early-time attractor in phase space

- Generic solutions rapidly approach the *attractor surface* in phase space $(\tau T', T, \tau)$ at early time.





snapshot of $(\tau T', T)$ plane at different τ

Perturbations

Linearization

- Linearization of MIS theory around the attractor: XA et al, 2312.17237

$$\partial_\nu T^{\mu\nu} = \partial_\nu (T^{\mu\nu}_{\text{attractor}} + \delta T^{\mu\nu}) = 0 \quad \longrightarrow \quad \begin{cases} \partial_\nu T^{\mu\nu}_{\text{attractor}} = 0, \\ \partial_\nu \delta T^{\mu\nu} = 0. \end{cases}$$

- 6 independent fields:

$$\phi = (\delta T, \delta\theta, \delta\omega, \delta\pi_{11}, \delta\pi_{22}, \delta\pi_{12})(\tau, \mathbf{x})$$

where $\delta\theta \equiv \partial_i \delta u_i$ and $\delta\omega \equiv \epsilon_{ij}\partial_i \delta u_j$, $i = 1{,}2$. $\delta\omega$ decouples from $\delta\theta$ and δT.

The translation invariance symmetry in transverse plane is broken.

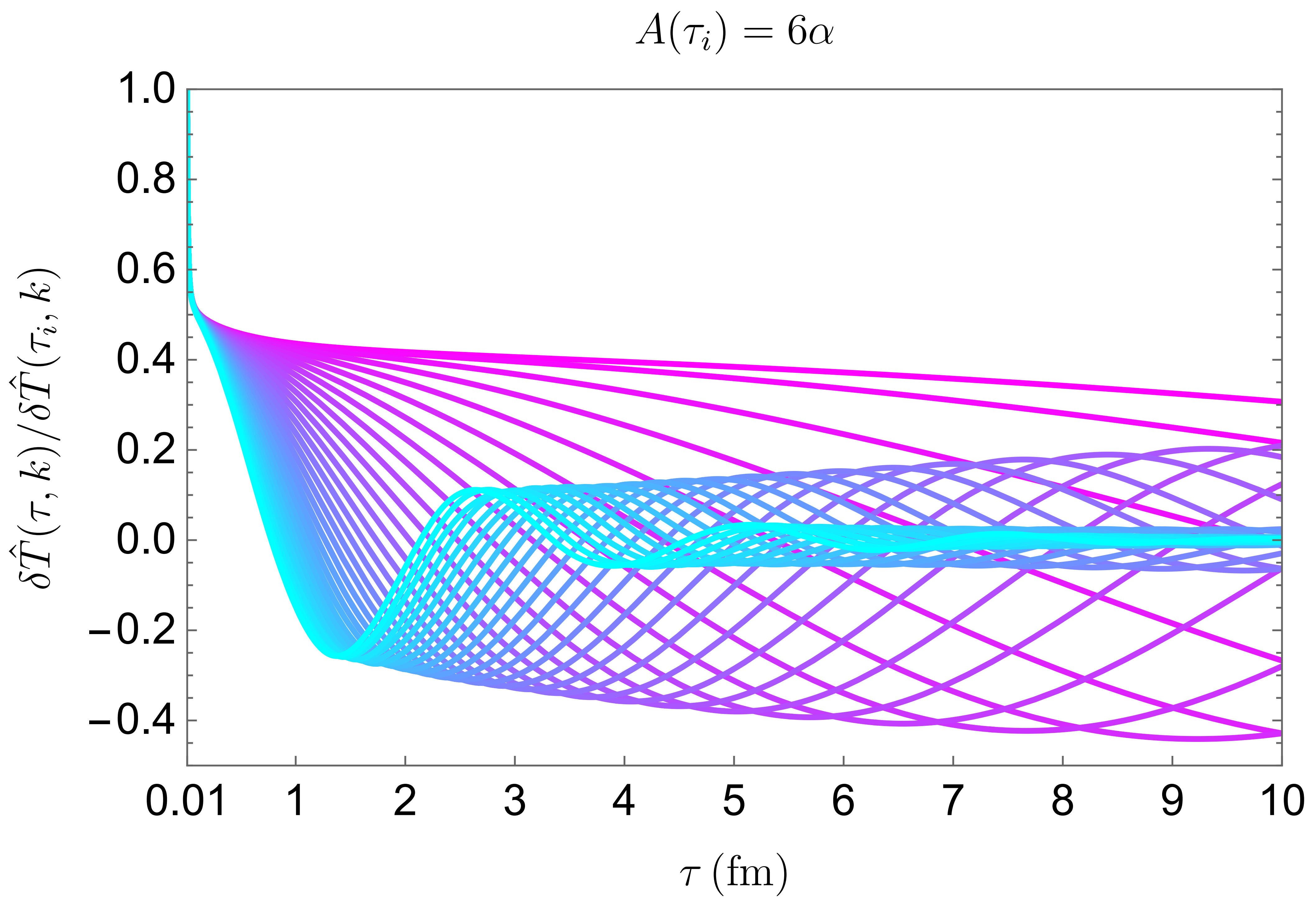
- The EOM for the dynamic system:

$$\partial_\tau \hat{\phi}_i(\tau, \mathbf{k}) = M_{ij} \hat{\phi}_j(\tau, \mathbf{k})$$

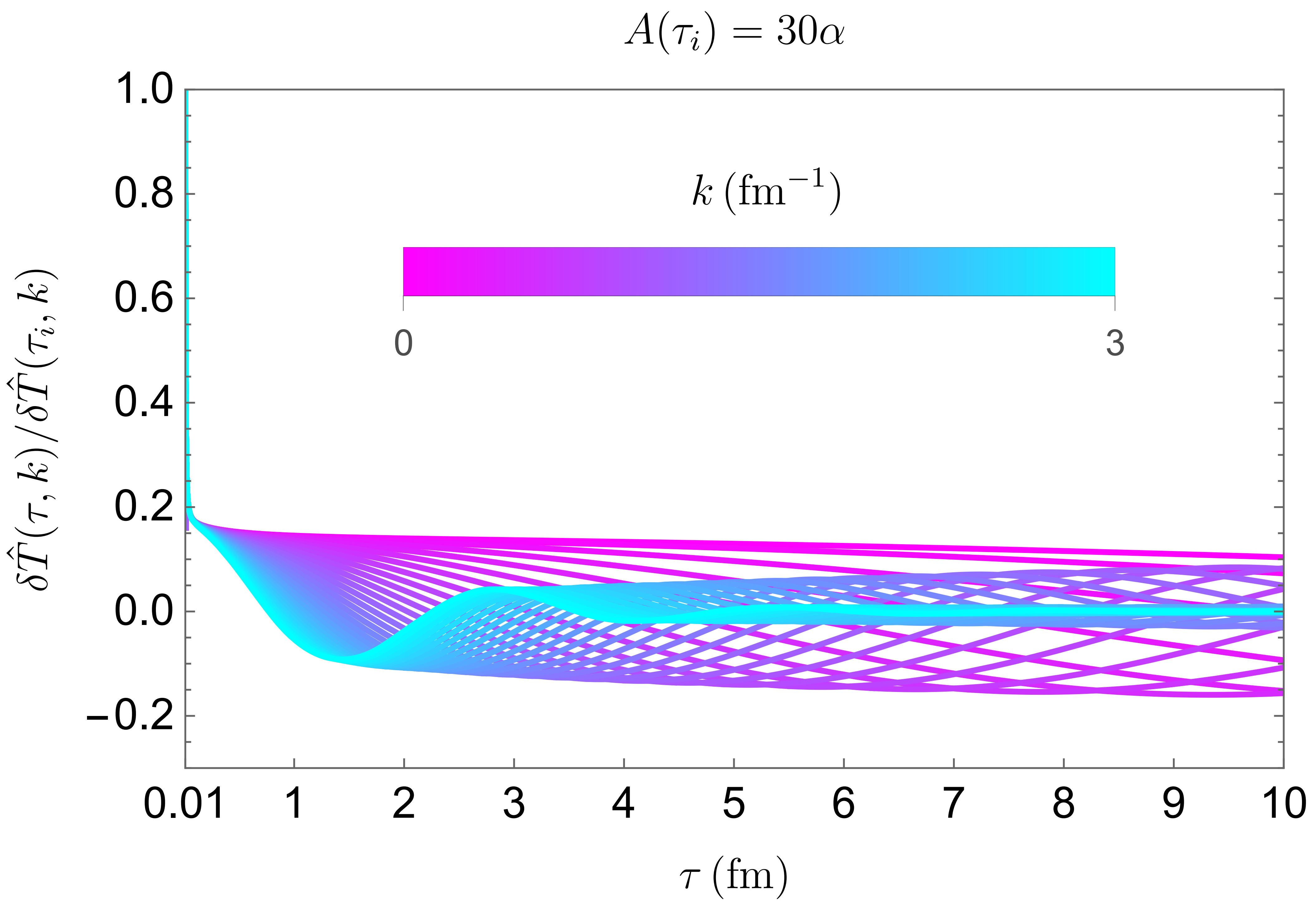
where $M = M(\tau, \mathbf{k})$.

1st-order linear homogeneous ODE system with nonconstant coefficients.

Mode-by-mode analysis



large k suppression

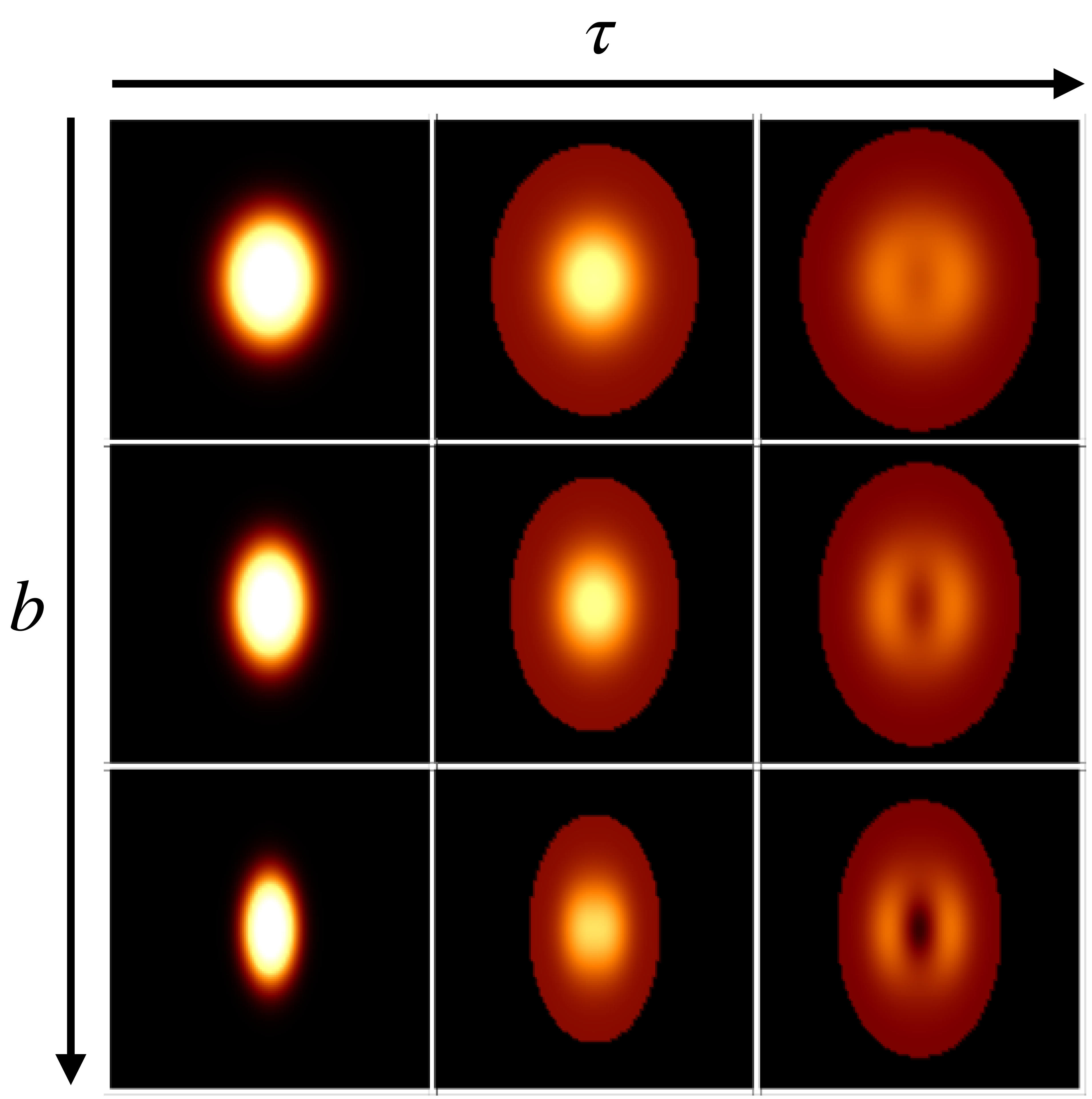


off-attractor suppression

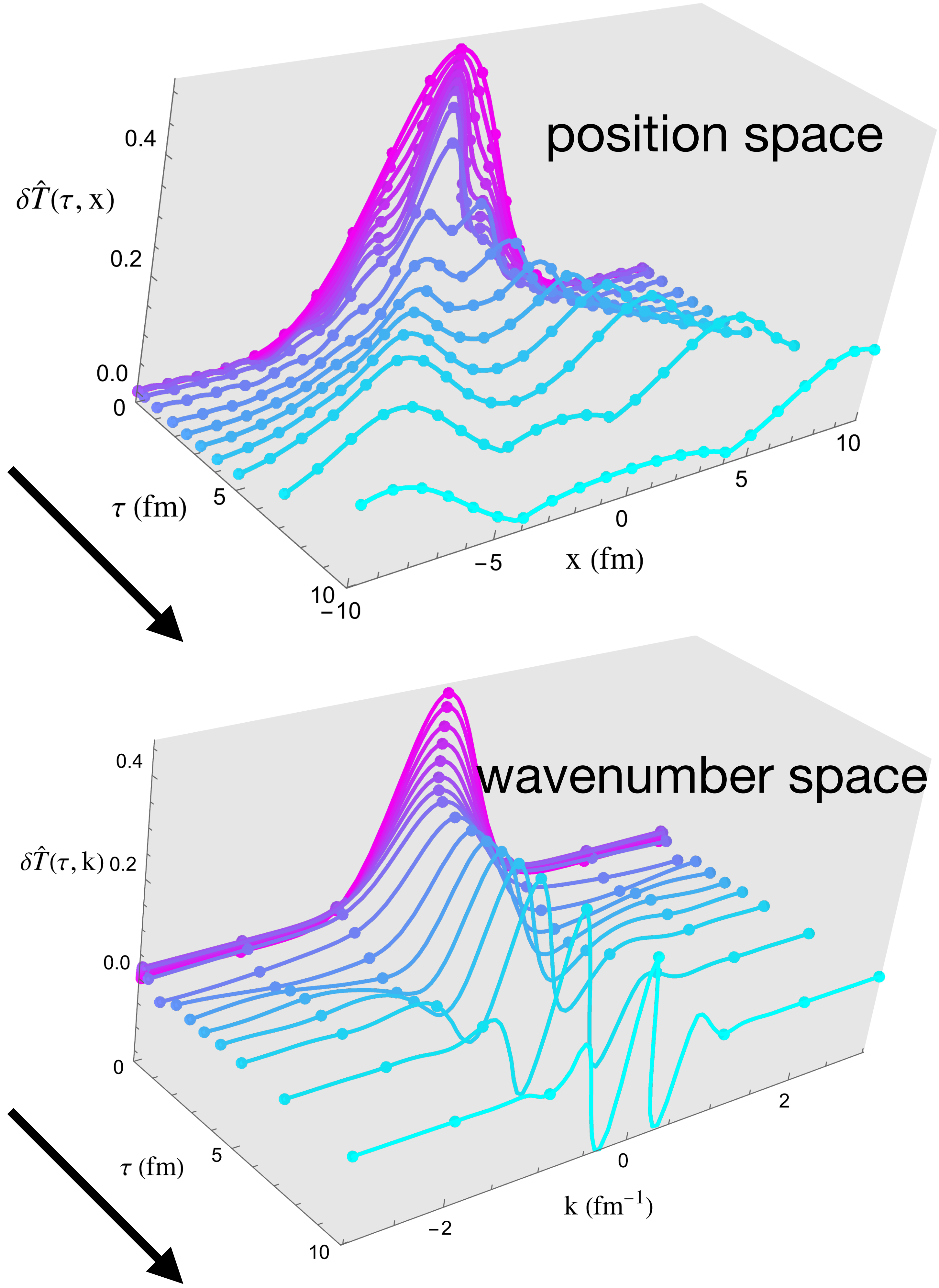
- Suppression for large k modes and off-attractor perturbations.
- Upper cutoff of k set by suppression, lower cutoff of k set by system size.

Transverse dependence

- Transverse information is encoded in a finite set of Fourier modes (FFT).



temperature density plot



Observables

Late-time asymptotics

- Late-time asymptotic solutions perturbed around attractor: XA et al, 2312.17237

$$\delta\hat{T} = C_1(\Lambda\tau)^{a_1}\, e^{-\frac{3}{2C_\tau}(\Lambda\tau)^{2/3}} + C_2(\Lambda\tau)^{a_2}\, e^{-\frac{1}{2c_\alpha^2 C_\tau}(\Lambda\tau)^{2/3}} + e^{-\frac{\alpha^2}{c_\alpha^2 C_\tau}(\Lambda\tau)^{2/3}}(\Lambda\tau)^{a_3}\left(C_3\, e^{-ic_\alpha k\tau} + C_4\, e^{ic_\alpha k\tau}\right)$$

$$\delta\hat{\theta} = C_1'(\Lambda\tau)^{a_1-1}\, e^{-\frac{3}{2C_\tau}(\Lambda\tau)^{2/3}} + C_2'(\Lambda\tau)^{a_2-\frac{1}{3}}\, e^{-\frac{1}{2c_\alpha^2 C_\tau}(\Lambda\tau)^{2/3}} + e^{-\frac{\alpha^2}{c_\alpha^2 C_\tau}(\Lambda\tau)^{2/3}}(\Lambda\tau)^{a_3}\left(C_3'\, e^{-ic_\alpha k\tau} + C_4'\, e^{ic_\alpha k\tau}\right)$$

$$\delta\hat{\omega} = e^{-\frac{3}{4C_\tau}(\Lambda\tau)^{2/3}}(\Lambda\tau)^{a_4}\left(C_5\, e^{-i\alpha k\tau} + C_6\, e^{i\alpha k\tau}\right)$$

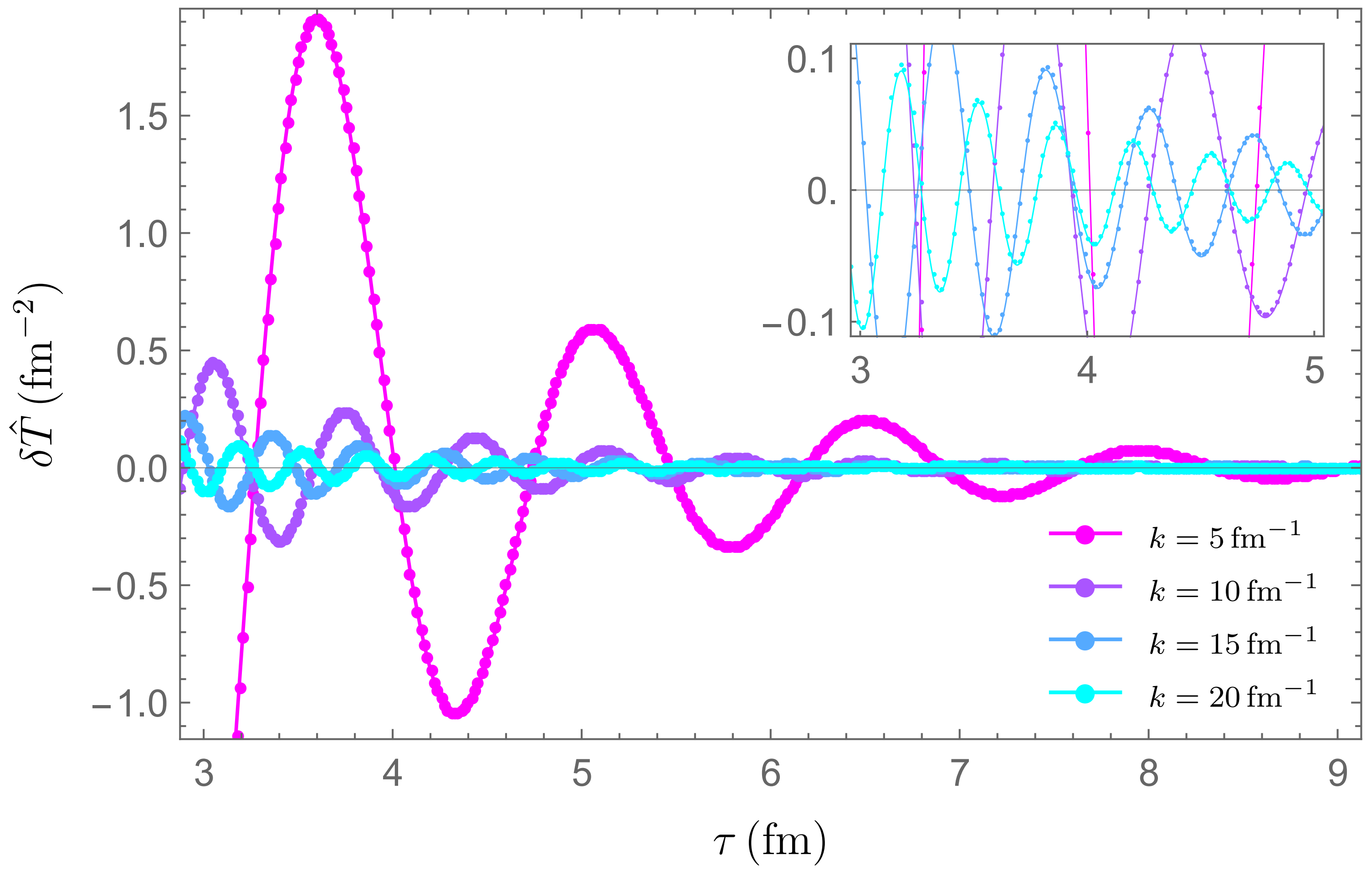
$\Lambda, C_1, \ldots, C_6$: integration constants

$$c_\alpha = \sqrt{(1+4\alpha^2)/3}$$

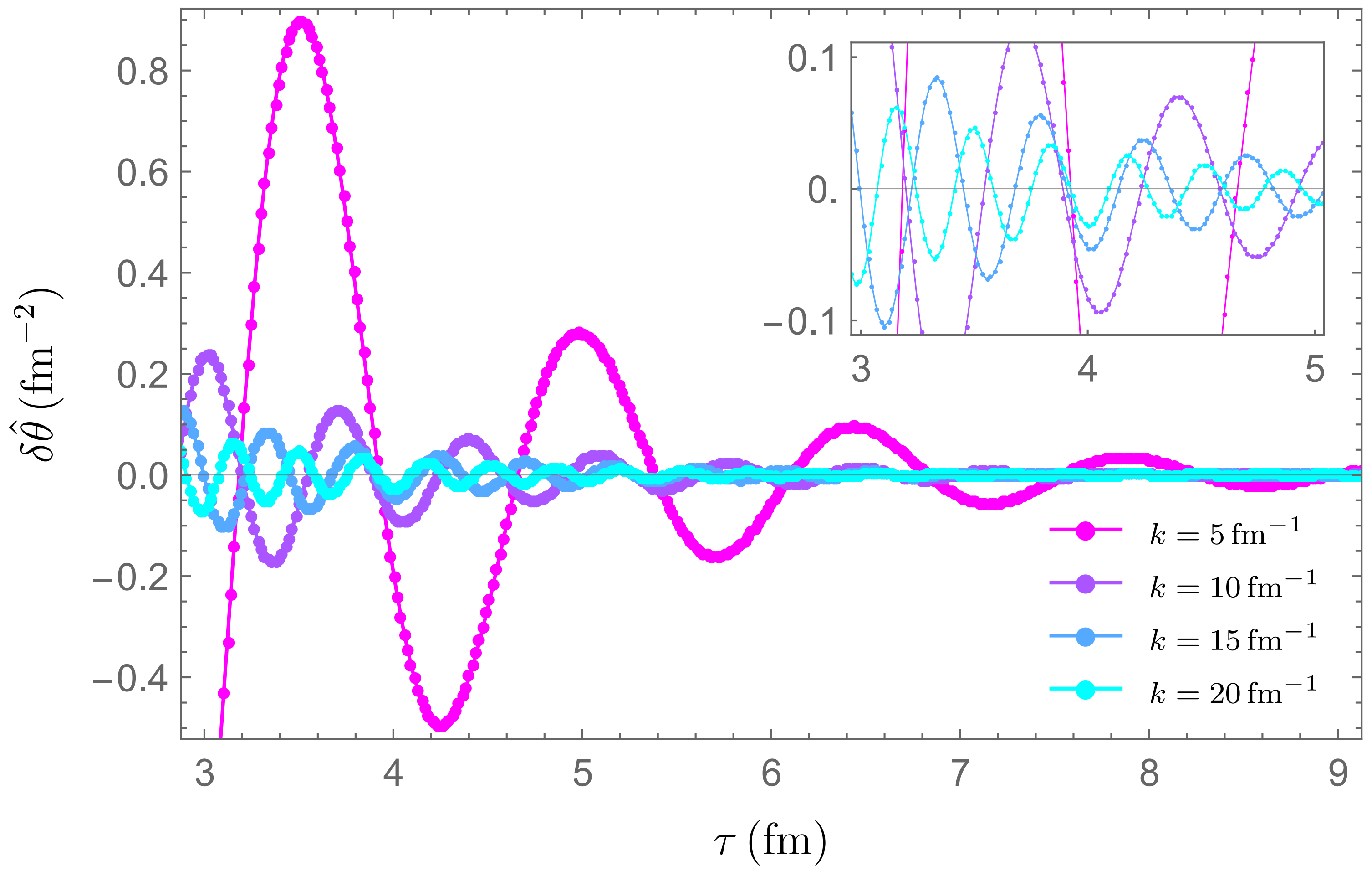
$$a_1 = -\frac{2}{3}(1-\alpha^2), \quad a_2 = \frac{2\alpha^2}{27c_\alpha^4}\left(1 - 16\alpha^2 - \frac{2\Lambda^2}{C_\tau^3 c_\alpha^4 k^2}\right), \quad a_3 = \frac{1}{54c_\alpha^4}\left(1 + 8\alpha^2 + 64\alpha^4 + 32\alpha^6 + \frac{4\alpha^2\Lambda^2}{C_\tau^3 c_\alpha^4 k^2}\right)$$

- The attractor is *stable* against transverse dynamics;
- Observables are extracted from the asymptotic data of $(\delta\hat{T}, \delta\hat{\theta}, \delta\hat{\omega}, \delta\hat{\pi}_{ij})$ determined by $(C_1, \ldots, C_6)(\mathbf{k})$.

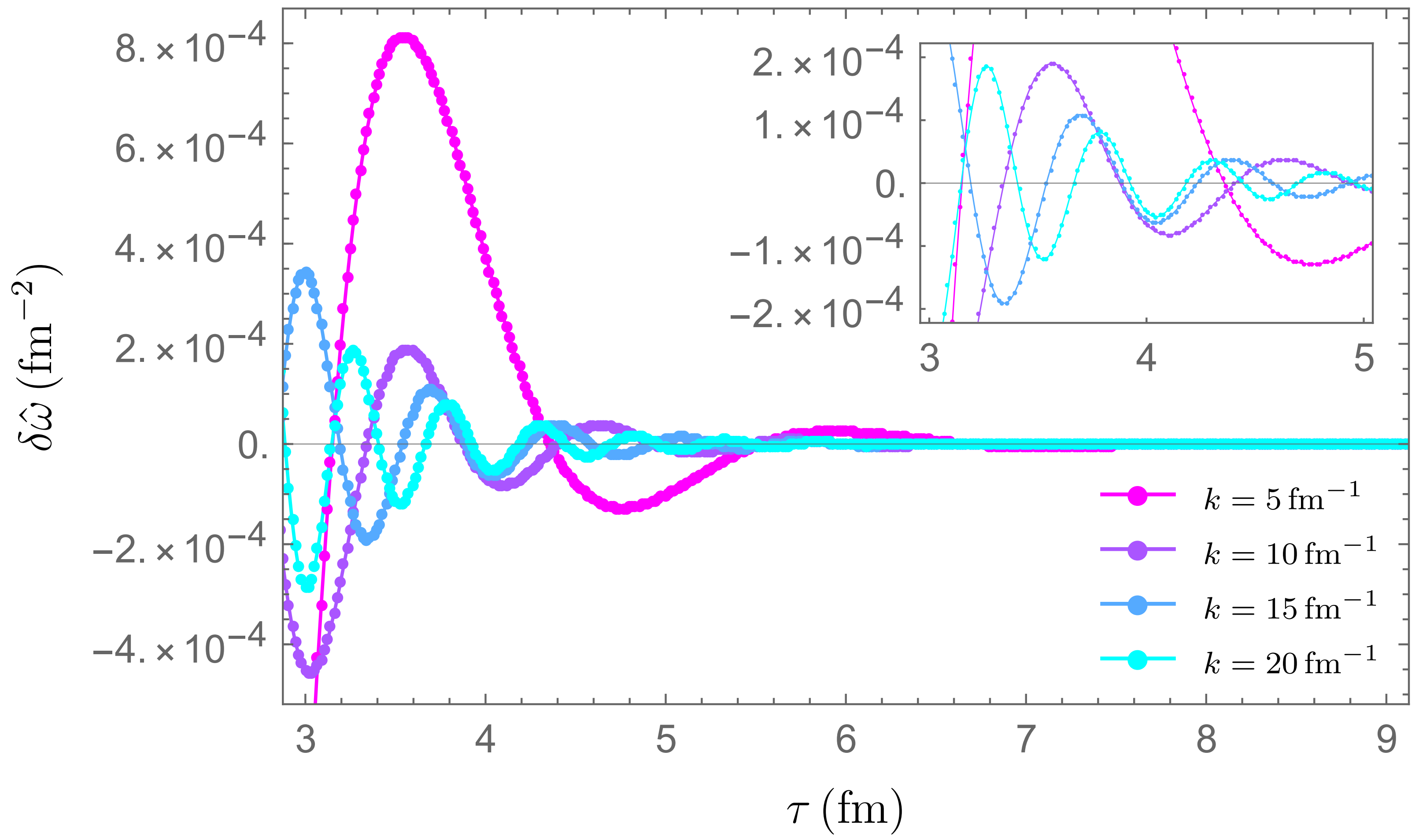
Late-time asymptotics



temperature



fluid divergence



vorticity

The analytic solutions fit the numerics in a wide range of time.

Observables

- Momentum anisotropy

$$A_T \equiv \frac{\langle T_{11} - T_{22}\rangle_\perp}{\langle T_{11} + T_{22}\rangle_\perp} = \frac{12\langle \delta u_1^2 - \delta u_2^2\rangle_\perp + 9\langle \delta\hat{\pi}_{11} - \delta\hat{\pi}_{22}\rangle_\perp}{2(3+A)}\,.$$

- Cooper-Frye formula

$$\hat{m}_\perp \equiv \frac{\sqrt{m^2 + p_\perp^2}}{T}, \quad \hat{p}_\perp \equiv \frac{p_\perp}{T}, \quad K_n$$: Bessel function

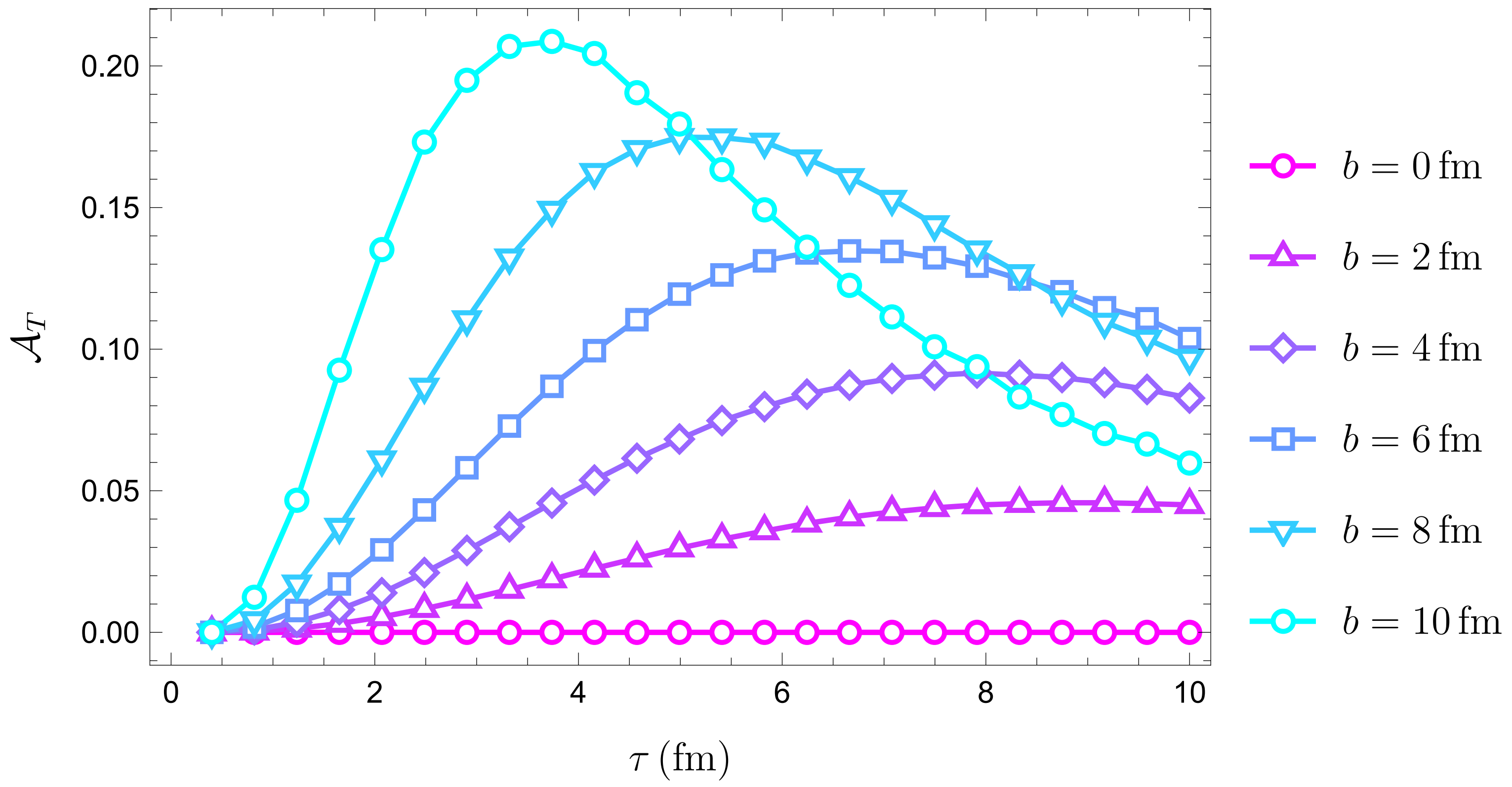
$$\frac{dN}{p_\perp dp_\perp d\phi dy} = \frac{1}{(2\pi)^3}\int d^3\sigma_\mu p^\mu f(x,p)$$

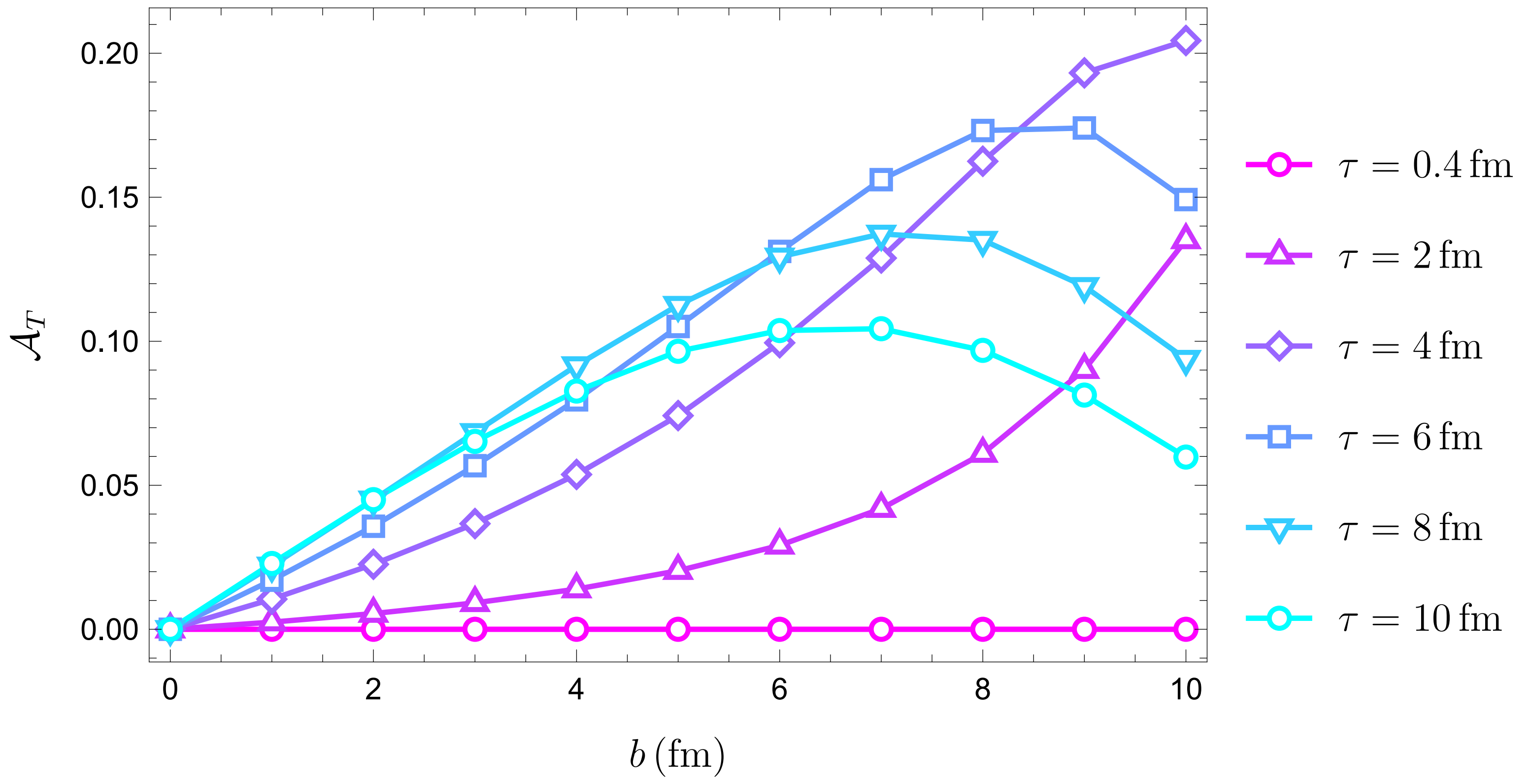
$$= \frac{m_\perp \tau_f R^2}{8\pi^2}\left\{2K_1(\hat{m}_\perp) + \frac{1}{12}\left[\hat{p}_\perp^2 K_1(\hat{m}_\perp) - 2\hat{m}_\perp K_2(\hat{m}_\perp)\right] A \; + \; \text{perturbations}\right\}$$

where $f(x,p) = e^{u\cdot\hat{p}}(1 + \epsilon_{\mu\nu}\hat{p}^\mu\hat{p}^\nu)$ with $\epsilon_{\mu\nu} = \pi_{\mu\nu}/2(\epsilon + p)$.

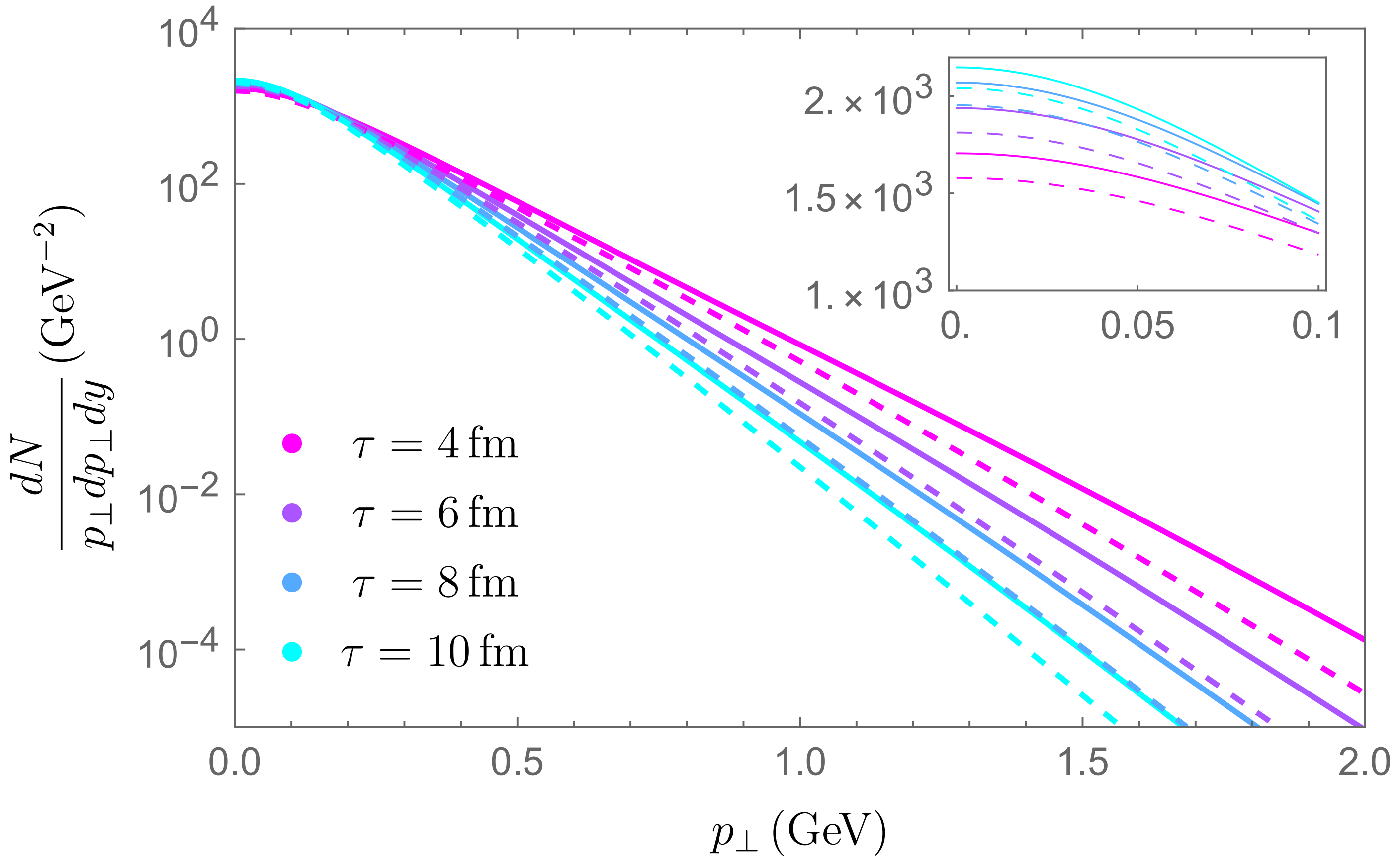
Observables

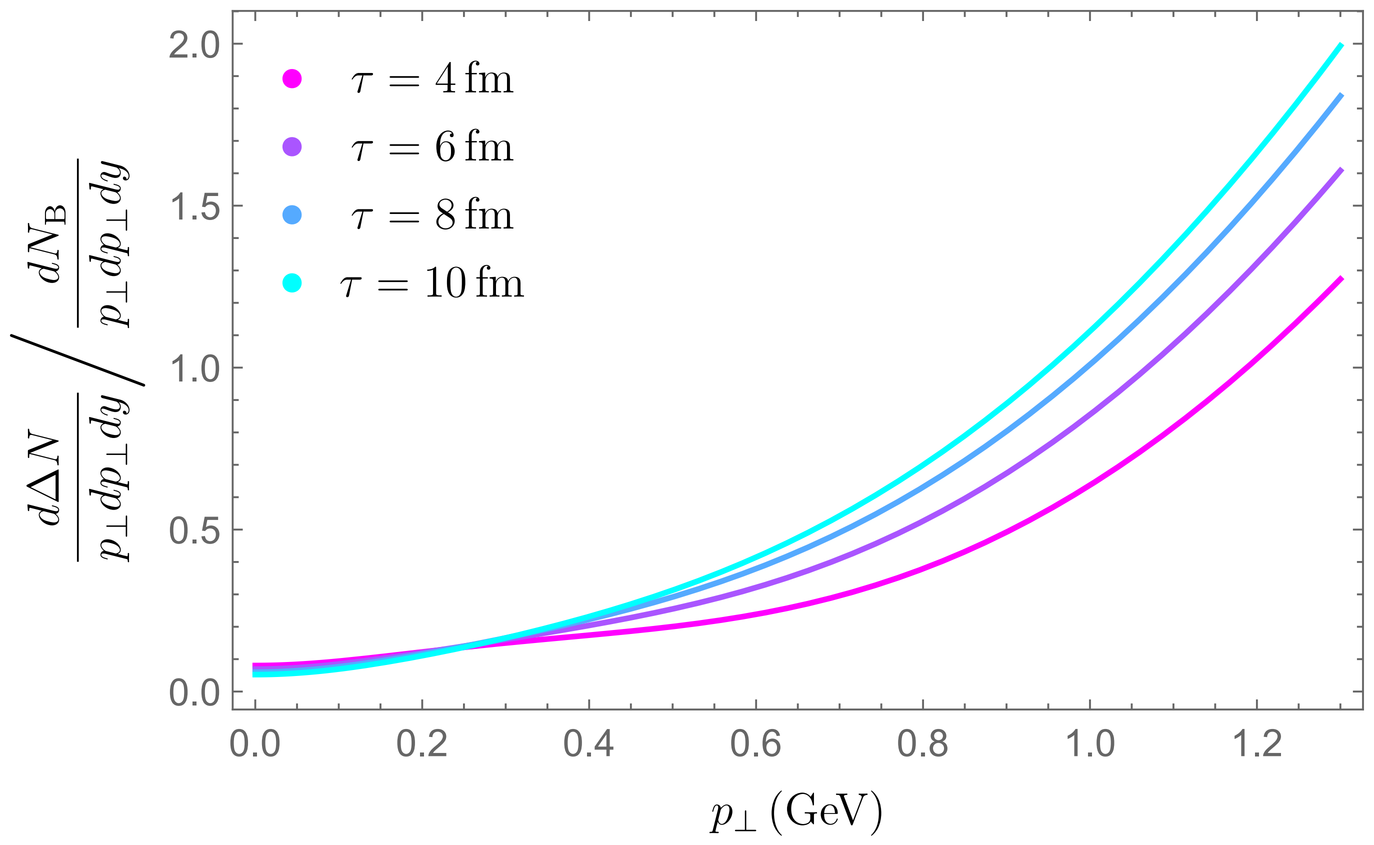
- Momentum anisotropy

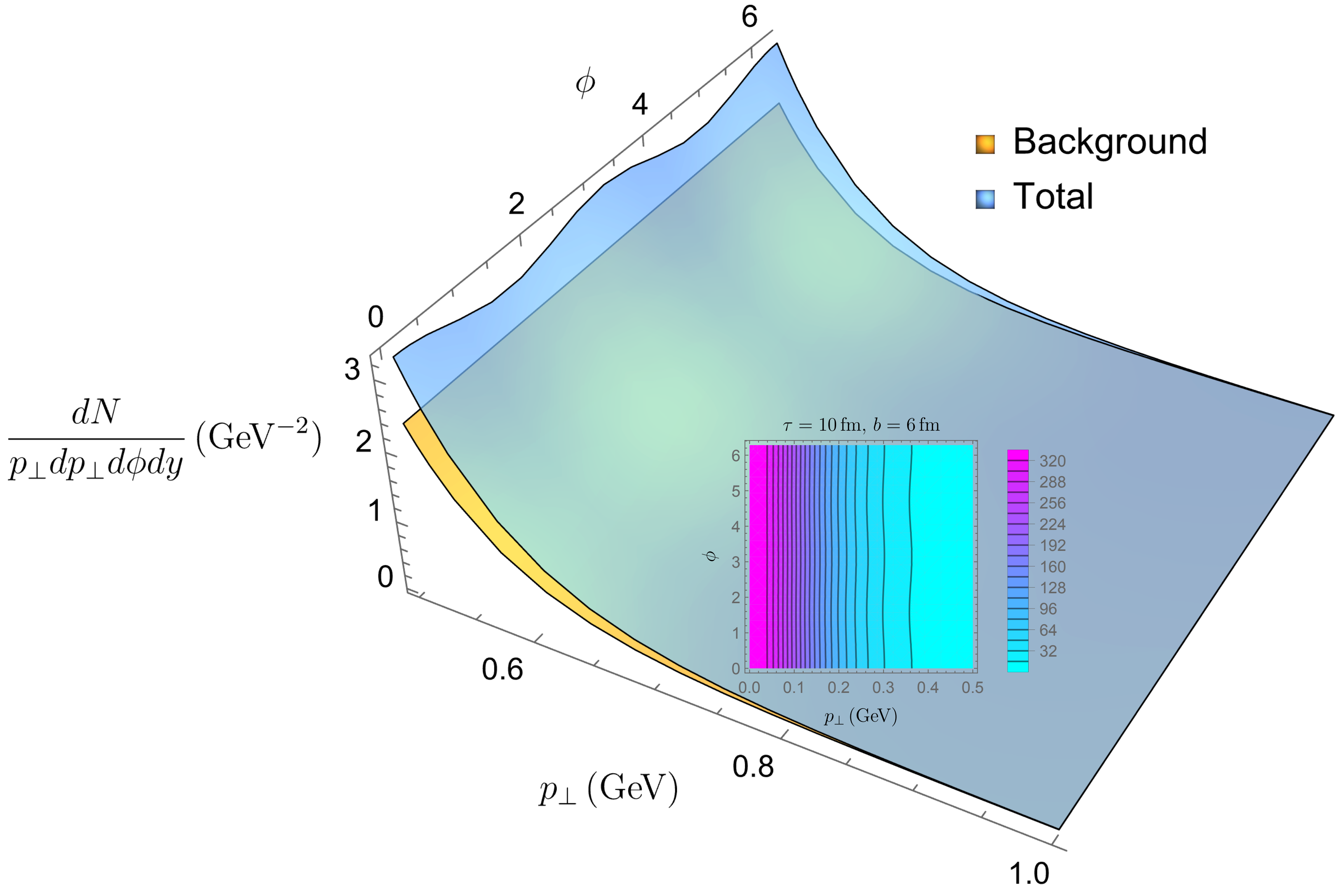




- Multiplicity distribution







Jets

Jet-medium interaction

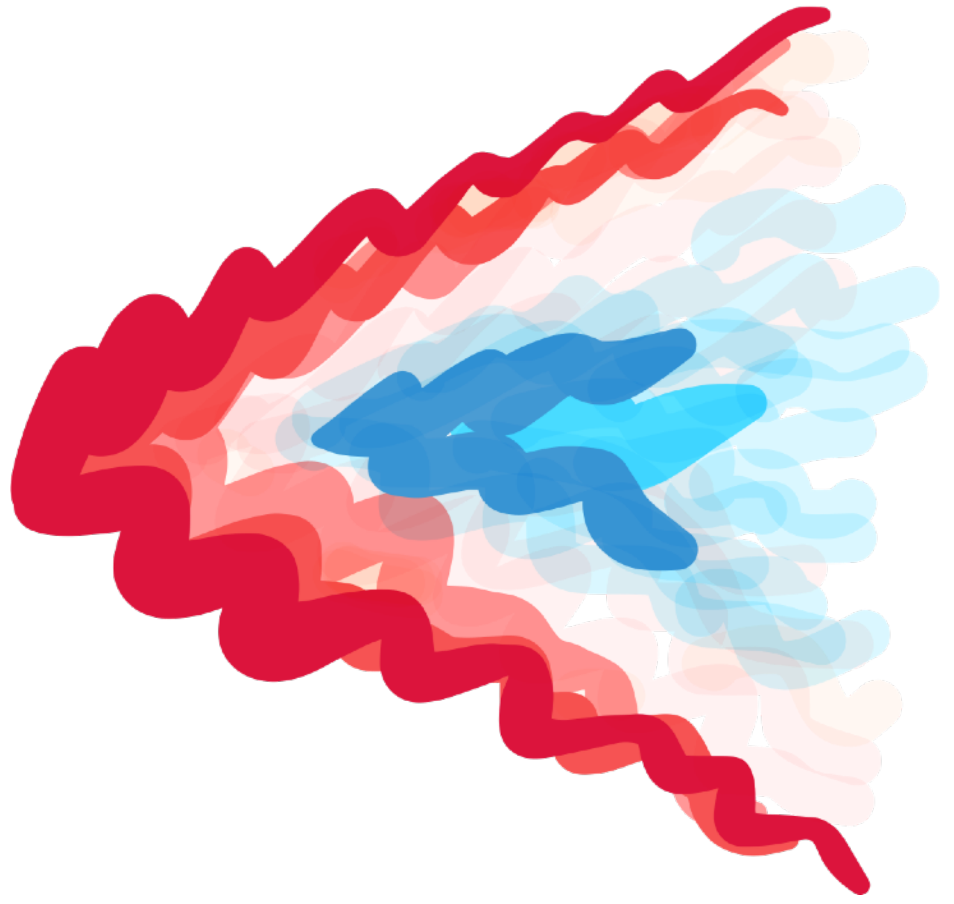
- The total energy of jet and fluid system is conserved:

$$\partial_\nu T^{\mu\nu} = \partial_\nu \left(T^{\mu\nu}_{\text{attractor}} + \delta T^{\mu\nu} + T^{\mu\nu}_{\text{jet}} \right) = 0$$

- Effect of jet-medium interaction described by perturbations: XA et al, in progress

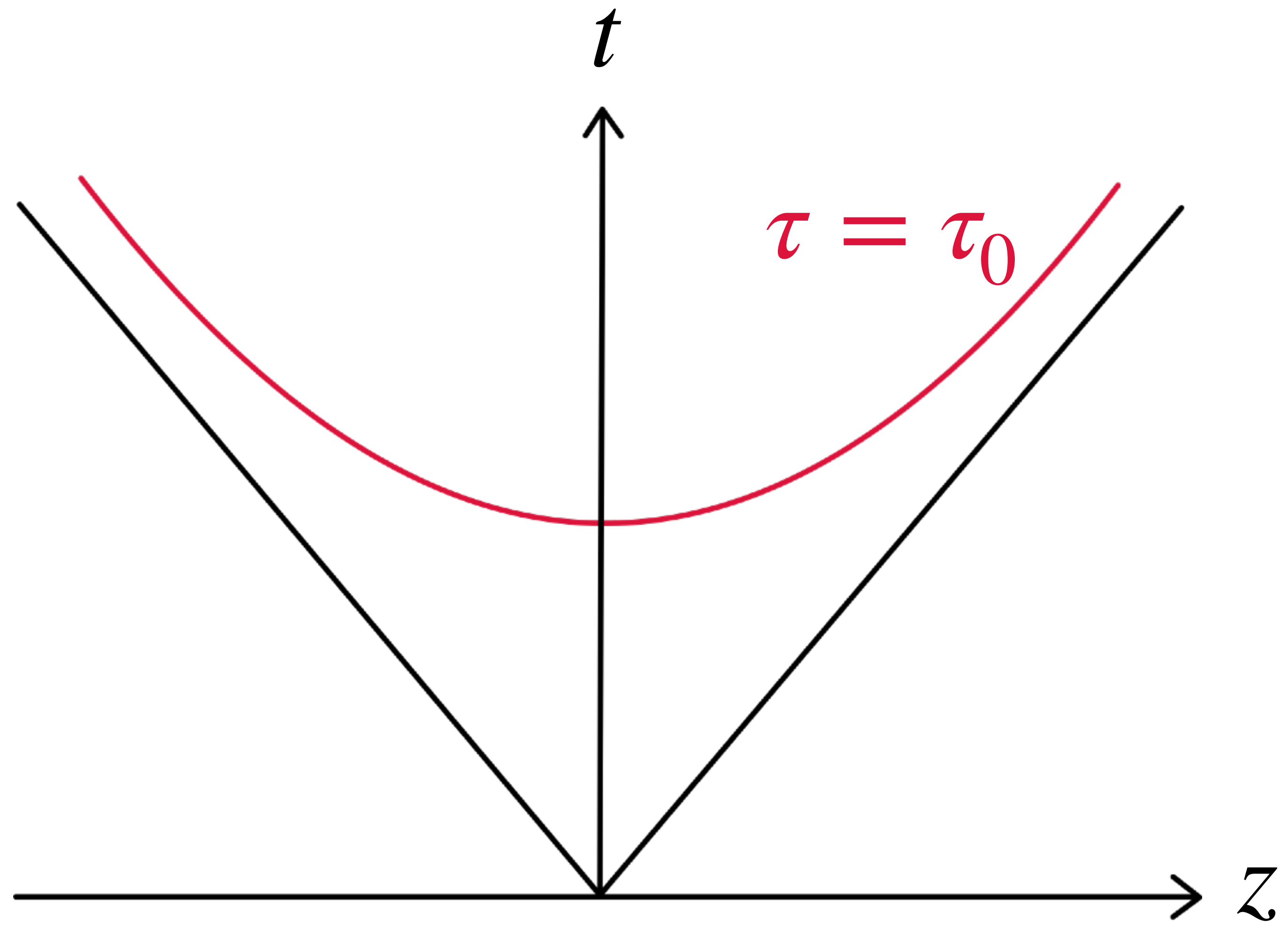
$$\begin{cases} \partial_\nu T^{\mu\nu}_{\text{attractor}} = 0, \\ \partial_\nu \delta T^{\mu\nu} = -\partial_\nu T^{\mu\nu}_{\text{jet}} = J^\mu \,. \end{cases}$$

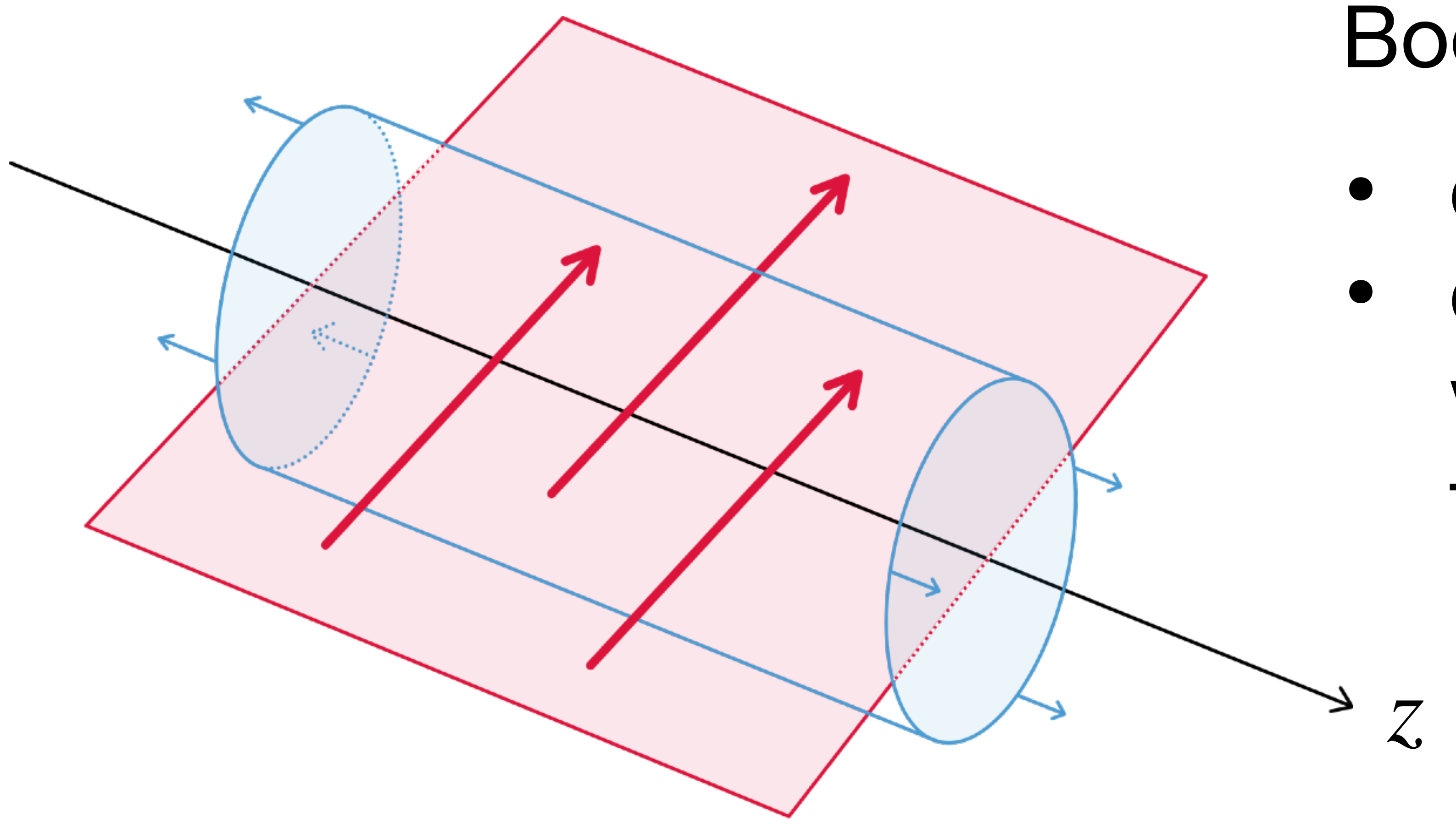
Chaudhuri et al, 0503028
Casalderrey-Solana et al, 0602183
Chesler et al, 0712.0050
Neufeld et al, 0802.2254
Qin et al, 0903.2255
Yan et al, 1707.09519
Casalderrey-Solana et al, 2010.01140
…



Jet parton as a source

- Boost-invariant and knife-shape jet parton





Boost-invariant assumption

- captures main effects qualitatively
- corresponds to the longest wavelength modes along rapidity that are more relevant

$$J^\mu = f^\mu \, (\tau\gamma_s)^{-1} \delta^{(2)}(\mathbf{x} - \mathbf{x}_s(\tau))$$

effective drag force $f^\mu = \frac{dE}{dt} u^\mu$

parton trajectories (example)

$$\mathbf{x}_s(\tau) = (\mathbf{x}_0 + \mathbf{v}_s(\tau - \tau_0))\Theta(\tau - \tau_0)$$

- Energy loss of a quark in a strongly coupled plasma

heavy quark: $\frac{dE}{dt} = \frac{\pi}{2}\sqrt{\lambda}\gamma_s v_s^2 T^2 \, .$

Herzog et al, 0605158

light quark: $\frac{dE}{d\tau} = \frac{4E_{\text{in}}\tau^2}{\pi \ell_{\text{stop}}^2 \sqrt{\ell_{\text{stop}}^2 - \tau^2}} \, .$

Chesler et al, 1402.6756

Particular solutions due to jet

- Inhomogeneous EOM

$$\partial_\tau \hat{\phi}_i(\tau, \mathbf{k}) = M_{ij}\hat{\phi}_j(\tau, \mathbf{k}) + J_i(\tau, \mathbf{k})$$

- The late-time solutions can be found by Wronskian:

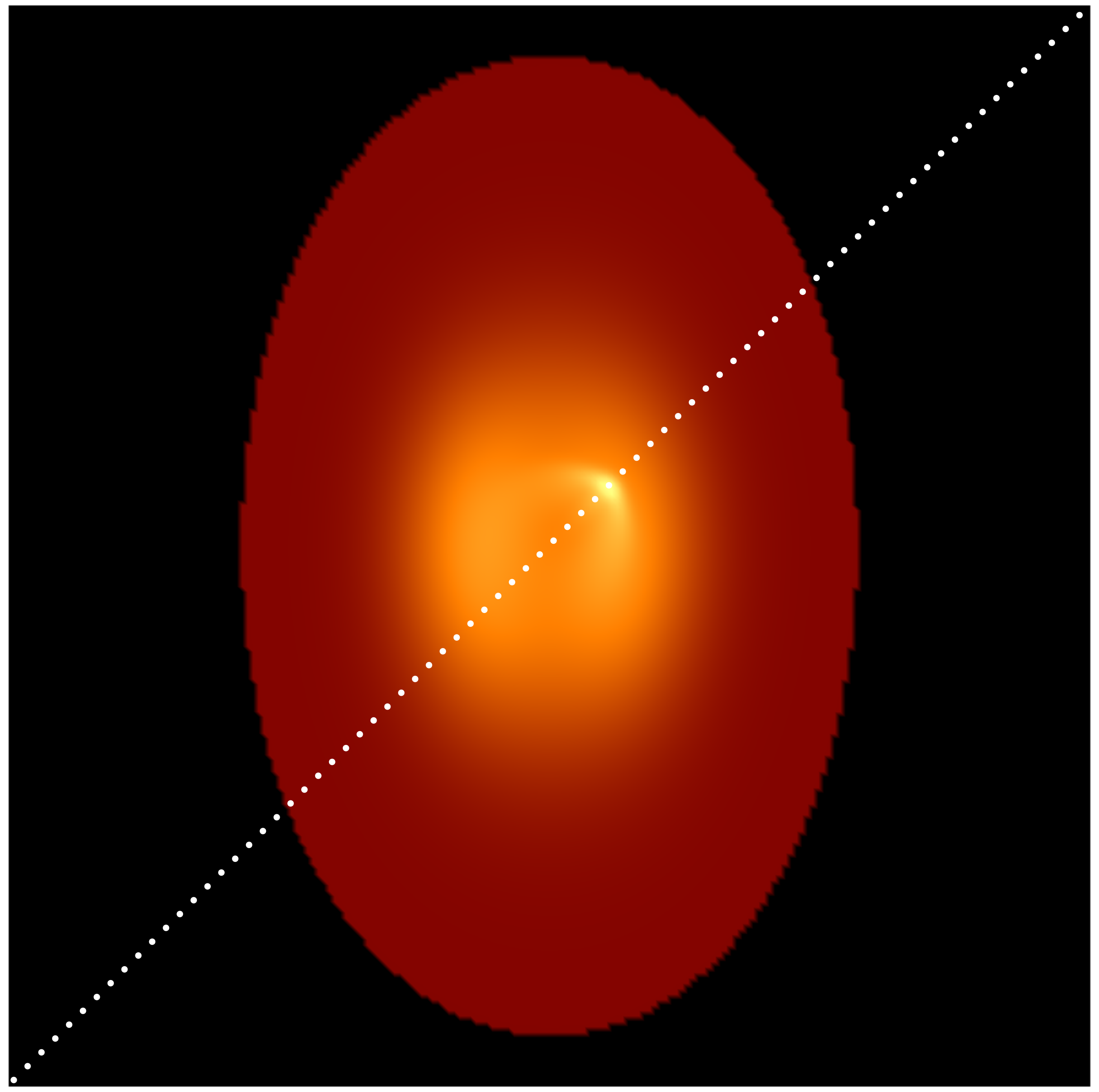
$$\delta\hat{\phi}(\tau, \mathbf{k}) = \sum_{i=1}^{4} C_i(k)\,\delta\hat{\phi}_i(\tau, \mathbf{k}) + \delta\hat{\phi}_p(\tau, \mathbf{k})$$

One can show the particular solutions have the *power-law* behavior, e.g.,

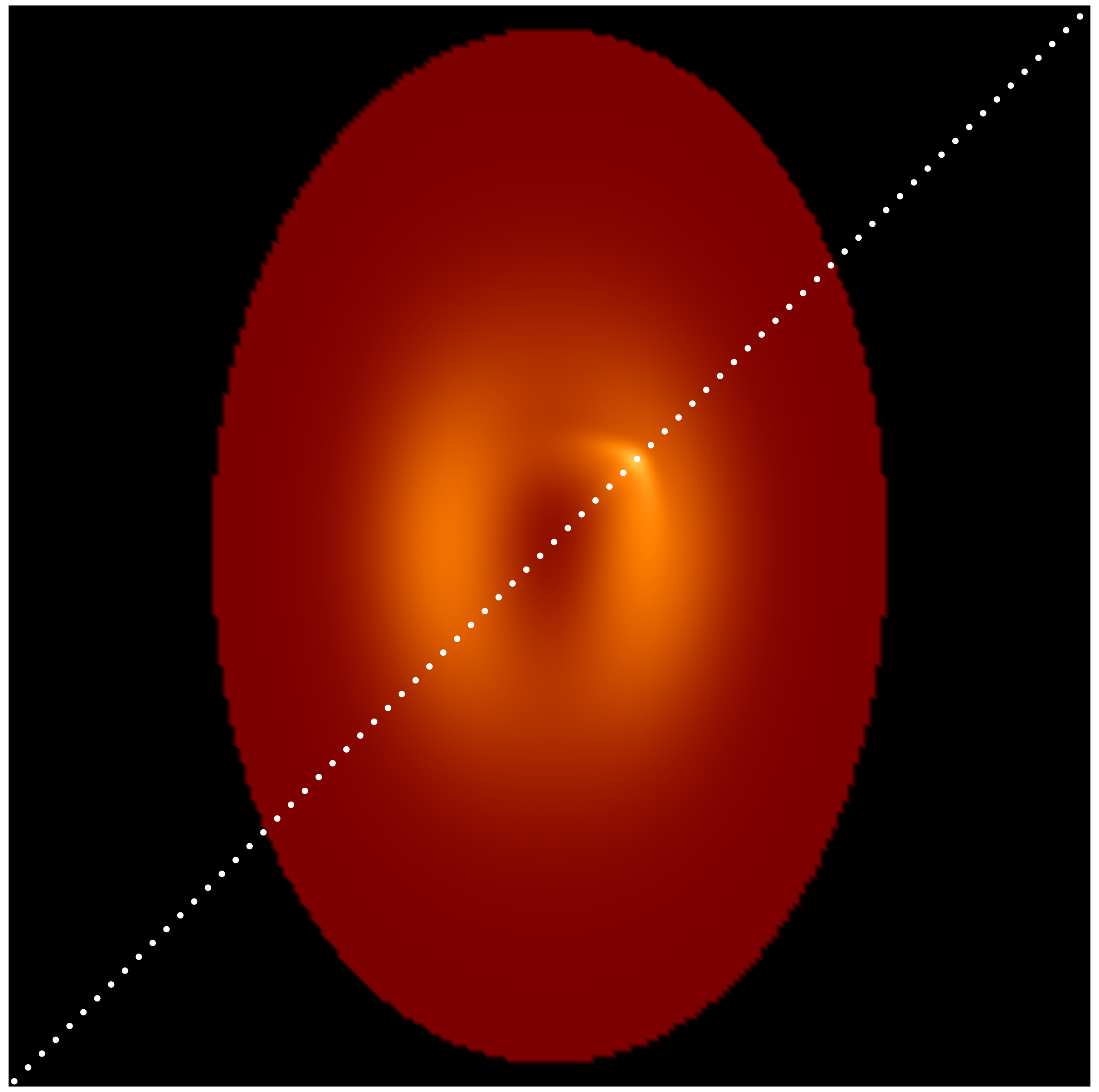
$$\delta\hat{T}_p(\tau, \mathbf{k}) \propto \frac{i\sqrt{\lambda}v_s^2 C_\eta e^{-i\mathbf{k}\cdot\mathbf{x}_s(\tau)}}{(4C_\eta + C_\tau(1 - 3(\hat{k}\cdot v_s)^2))k}\tau^{-1/3}(1 + \mathcal{O}(\tau^{-1/3})) \qquad \text{(heavy quark)}$$

Jet wake

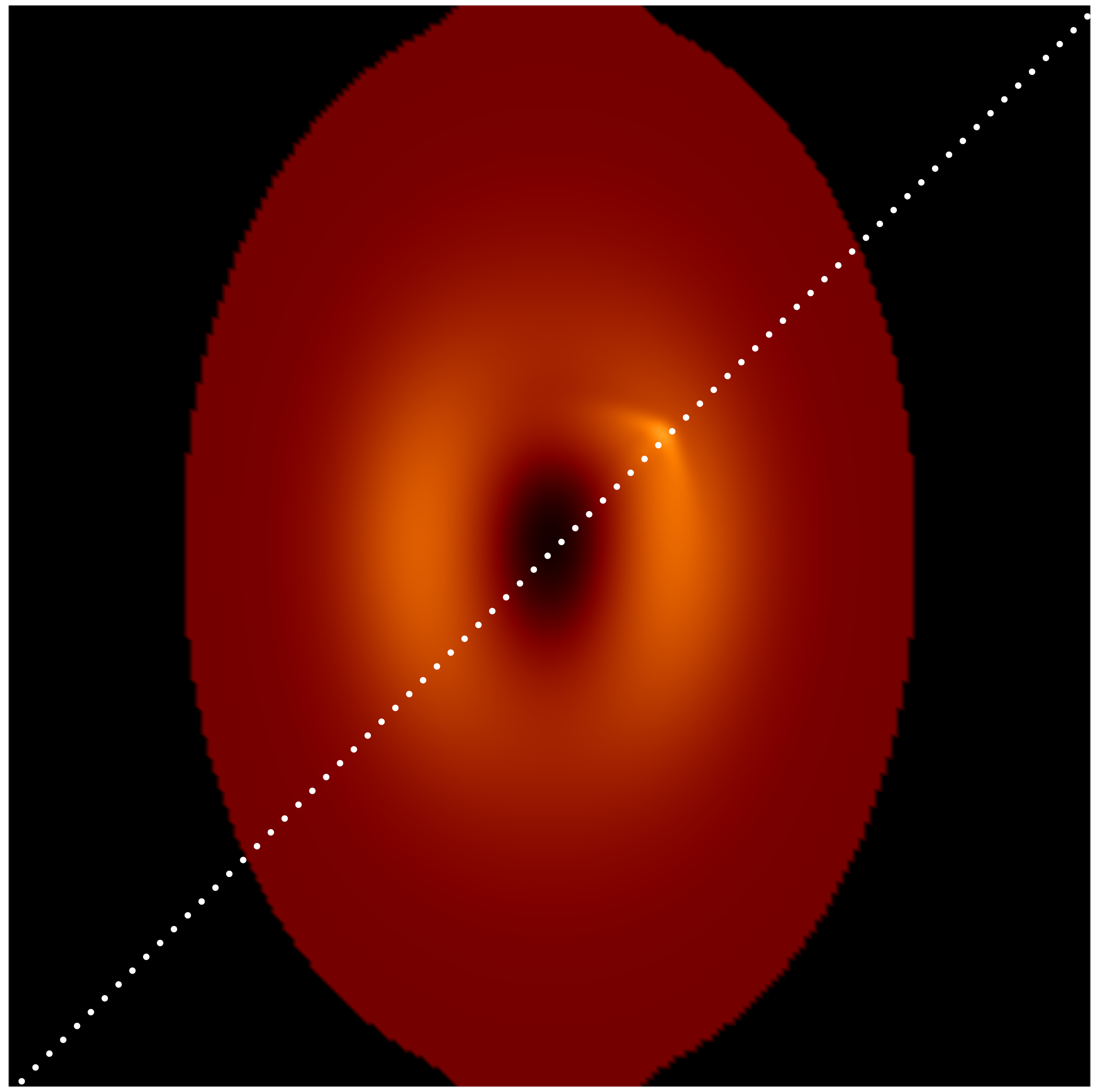
- The transverse tomography with jet wake based on FFT:



6 fm



8 fm



10 fm

Conclusion

Recap

- Transverse dynamics and jet-medium interaction are described by perturbations around attractor.

- Problem reduces to a set of linear ODEs which can be analyzed semi-analytically.

- Physical observables (including jet) are captured by a finite set of asymptotic data.

Outlook

- Jet physics after freezeout. see all talks on observables on Monday

- Stochastic fluctuations.

- Other approaches, e.g., kinetic approach. see also Aleksas's talk