Classical and quantum corrections in jet quenching

Jacopo Ghiglieri, SUBATECH, Nantes

Jet Quenching Workshop, ECT* Trento, February 14 2024

In this talk

- Introduction to classical and quantum physics in jet broadening
- Double-logarithmic quantum corrections
  - In the literature
  - In a weakly-coupled QGP, and their connection with classical physics
- Work done in collaboration with **Eamonn Weitz**, PhD@Nantes in late 2023
- No data (harmed) in (the making of) this talk

JG Weitz **JHEP11** (2022), E. Weitz's Ph.D. thesis **2311.04988**

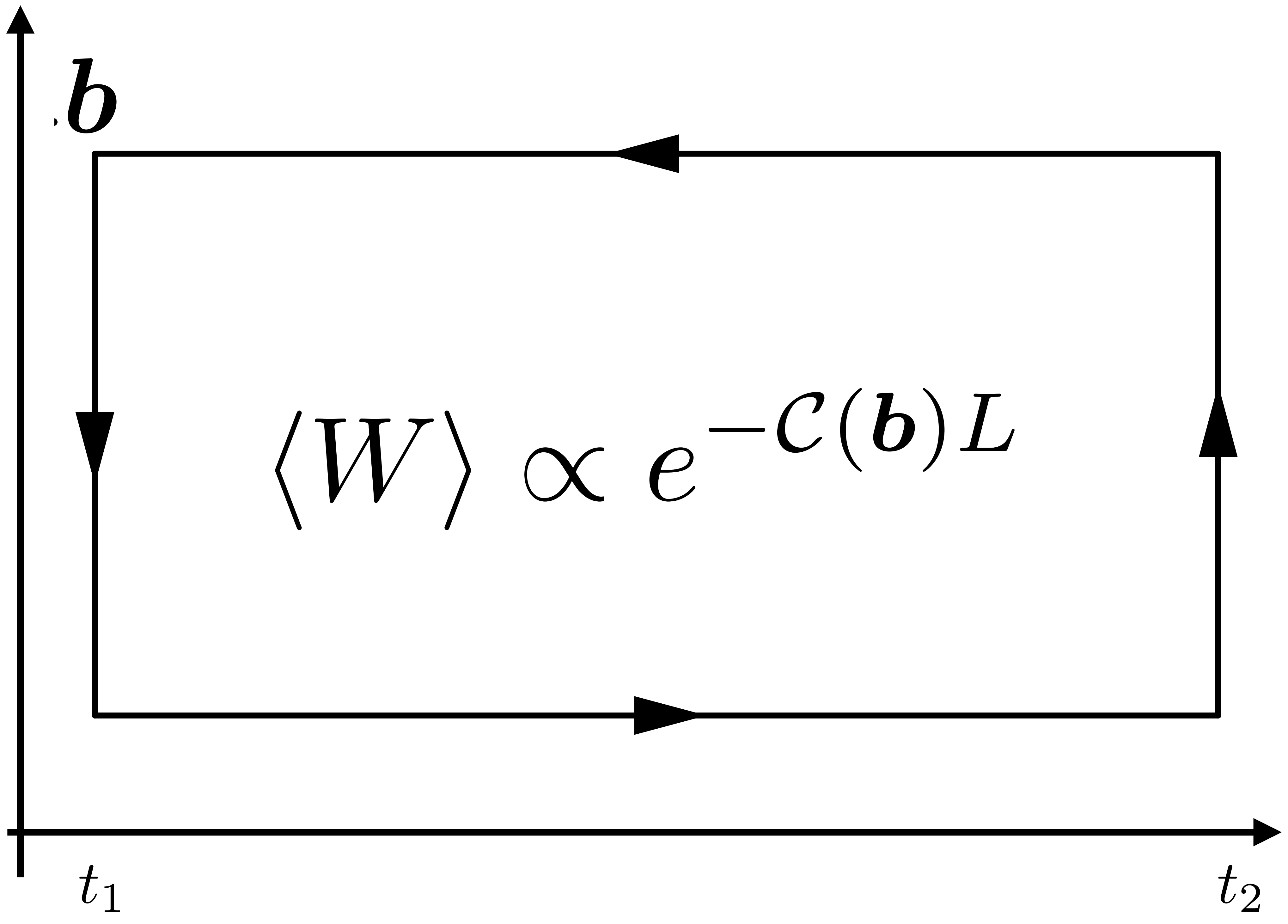
Transverse momentum broadening

- Consider the broadening of a single parton: $\hat{q}$ is given by the second moment of the broadening probability with μ process-dependent cutoff

$$\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L} = \frac{1}{L} \int^{\mu} \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 \mathcal{P}(k_\perp)$$

- $P(k_\perp)$ from a light-cone Wilson loop

$$\mathcal{P}(k_\perp) = \int_{\boldsymbol{b}} e^{-i\boldsymbol{k}_\perp \cdot \boldsymbol{b}} \exp\left[-\mathcal{C}(\boldsymbol{b}) L \right]$$



D'Eramo Liu Rajagopal **PRD84** (2011) Benzke Brambilla Escobedo Vairo **JHEP02** (2012)

Transverse momentum broadening

- Broadening probability

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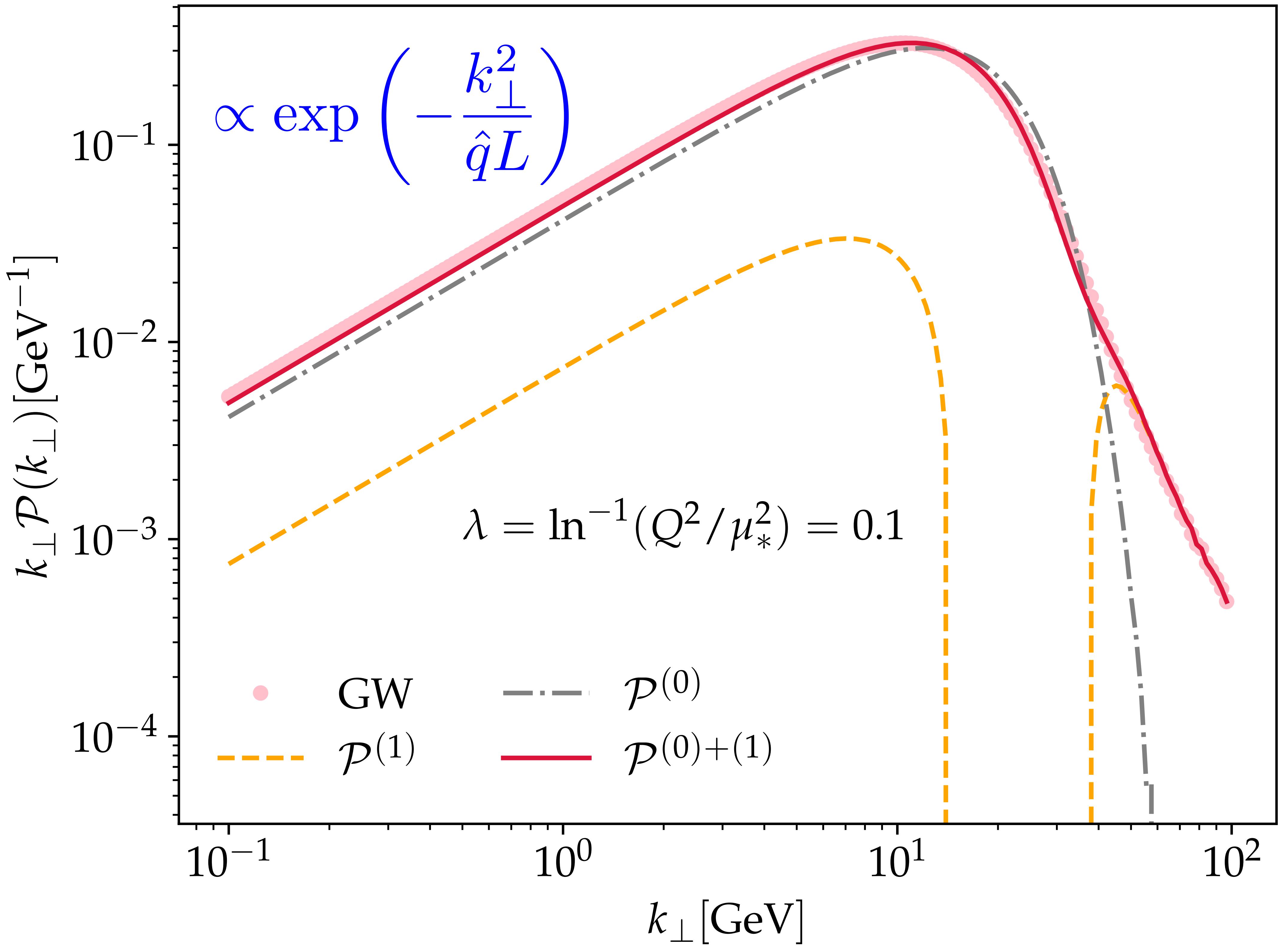
- IR Gaussian from multiple soft scatterings

$$\mathcal{P}(k_\perp)_{\rm HO} \propto \exp\left(-\frac{k_\perp^2}{\hat{q}L}\right)$$

harmonic oscillator (HO) approximation



Barata *et al* **PRD104** (2021)

Transverse momentum broadening

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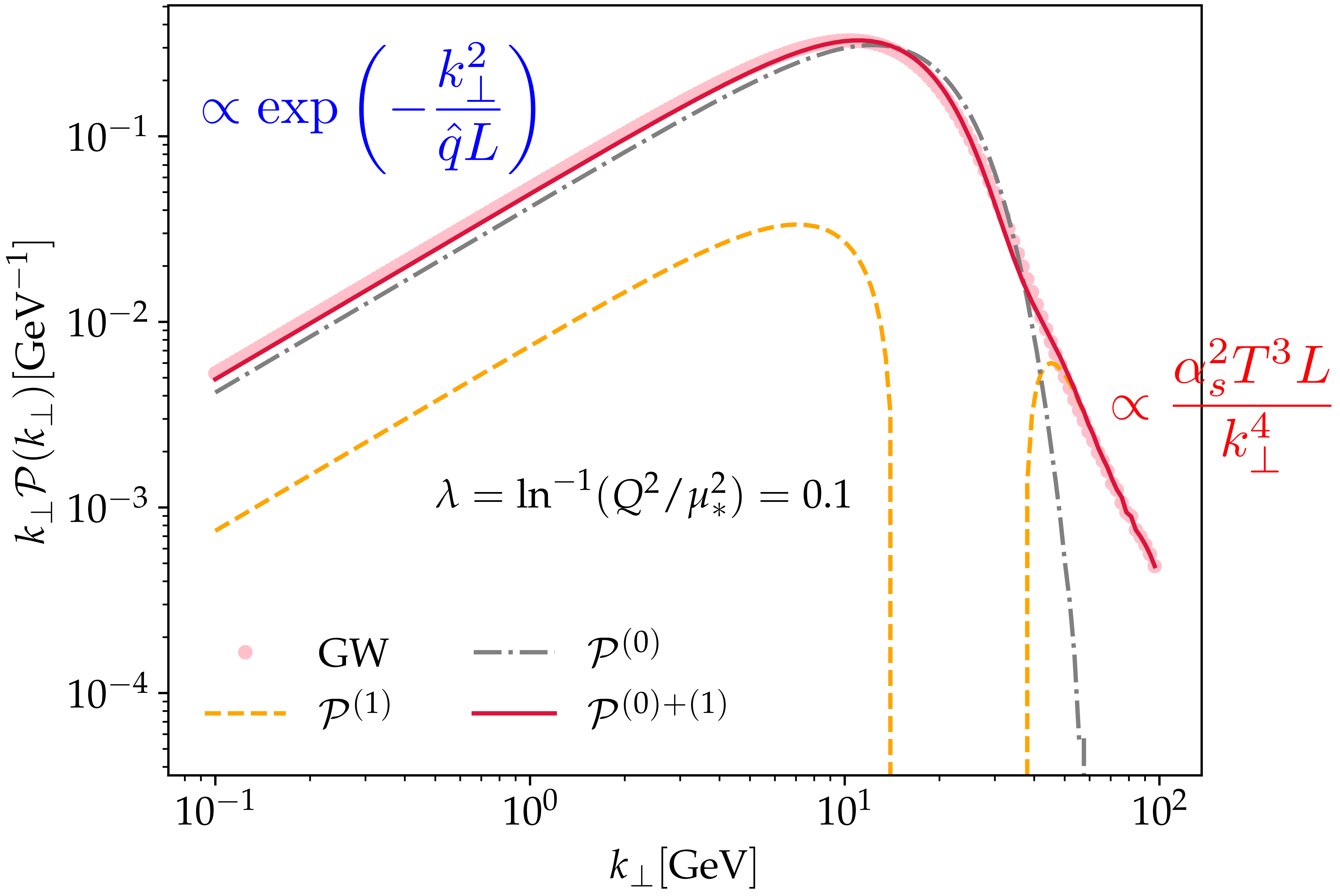
$$\mathcal{P}(k_\perp)_{\rm HO} \propto \exp\left(-\frac{k_\perp^2}{\hat{q}L}\right)$$

harmonic oscillator (HO) approximation

- **asymptotic freedom** $\Rightarrow$ it has to make way for the rare large momentum scatterings

$$\mathcal{P}(k_\perp)_{\rm Coulomb} \propto \frac{\alpha_s^2 T^3 L}{k_\perp^4}$$



Barata *et al* **PRD104** (2021)

Transverse momentum broadening

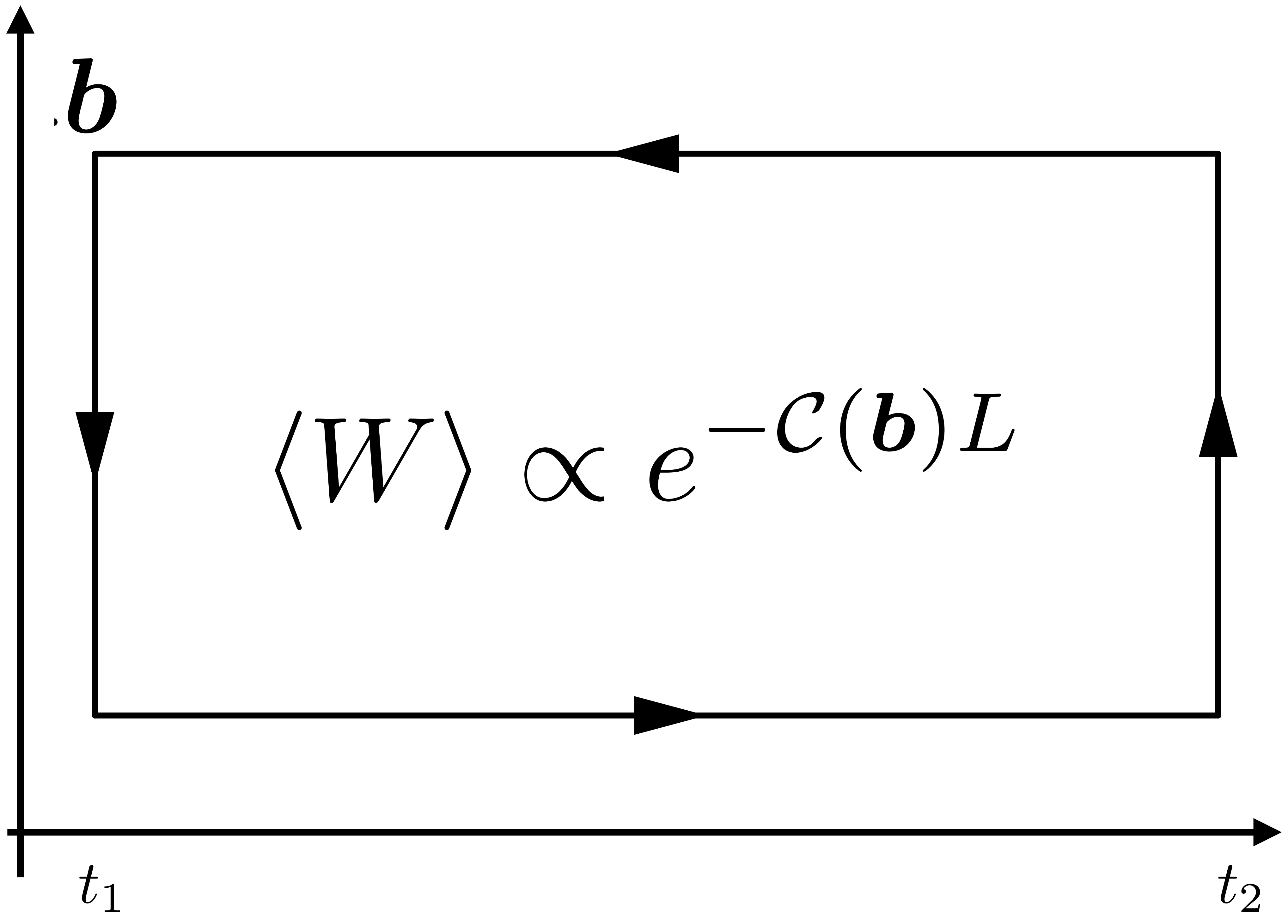
- $\hat{q}$ is also given by the second moment of the scattering kernel

$$\hat{q} = \int^{\mu} \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 \, \mathcal{C}(k_\perp)$$

- $C(k_\perp)$ from the light-cone Wilson loop

$$\mathcal{C}(\boldsymbol{b}) = \int \frac{d^2k_\perp}{(2\pi)^2} \left[1 - e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{b}} \right] \mathcal{C}(k_\perp)$$

real term and probability-conserving virtual term



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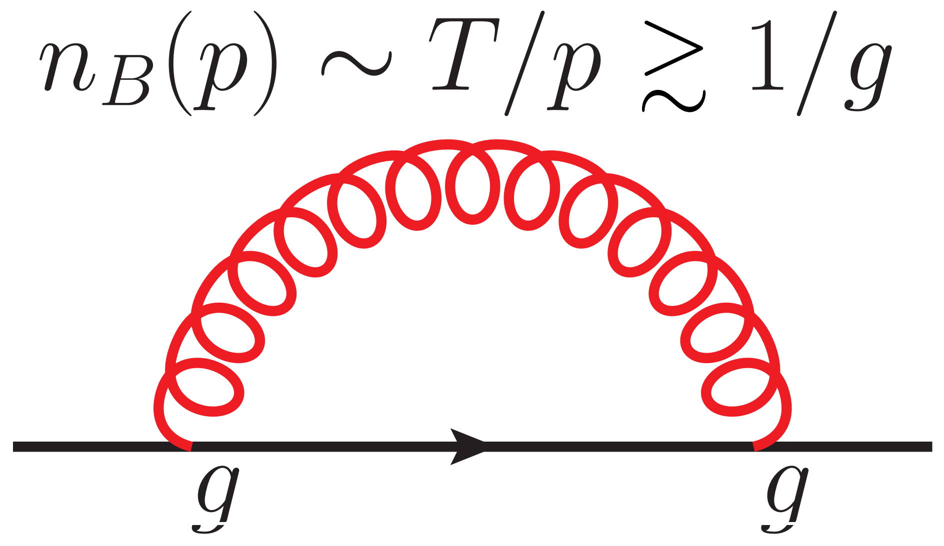
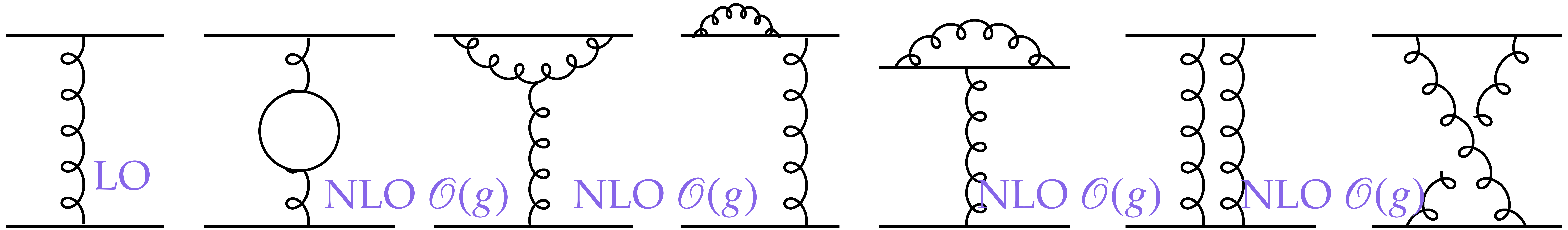
Classical gluons in the scattering kernel



- Classical (soft gluon) corrections to the scattering/broadening kernel can be problematic for perturbation theory, Linde problem
- Breakthrough: soft classical modes at space-like separations become **Euclidean** and **time-independent**
- Horrible HTL perturbative calculation or extremely challenging 4D lattice on the light-cone become **3D** Electrostatic QCD (EQCD).



Caron-Huot **PRD79** (2008)

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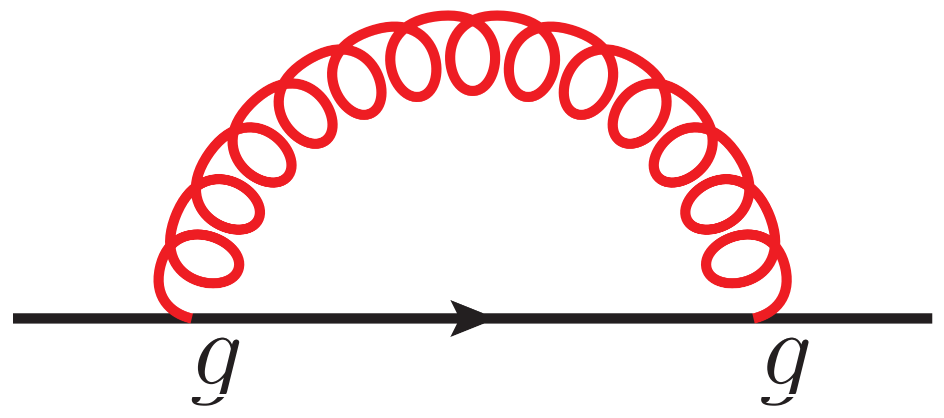
$n_B(p) \sim T/p \gtrsim 1/g$



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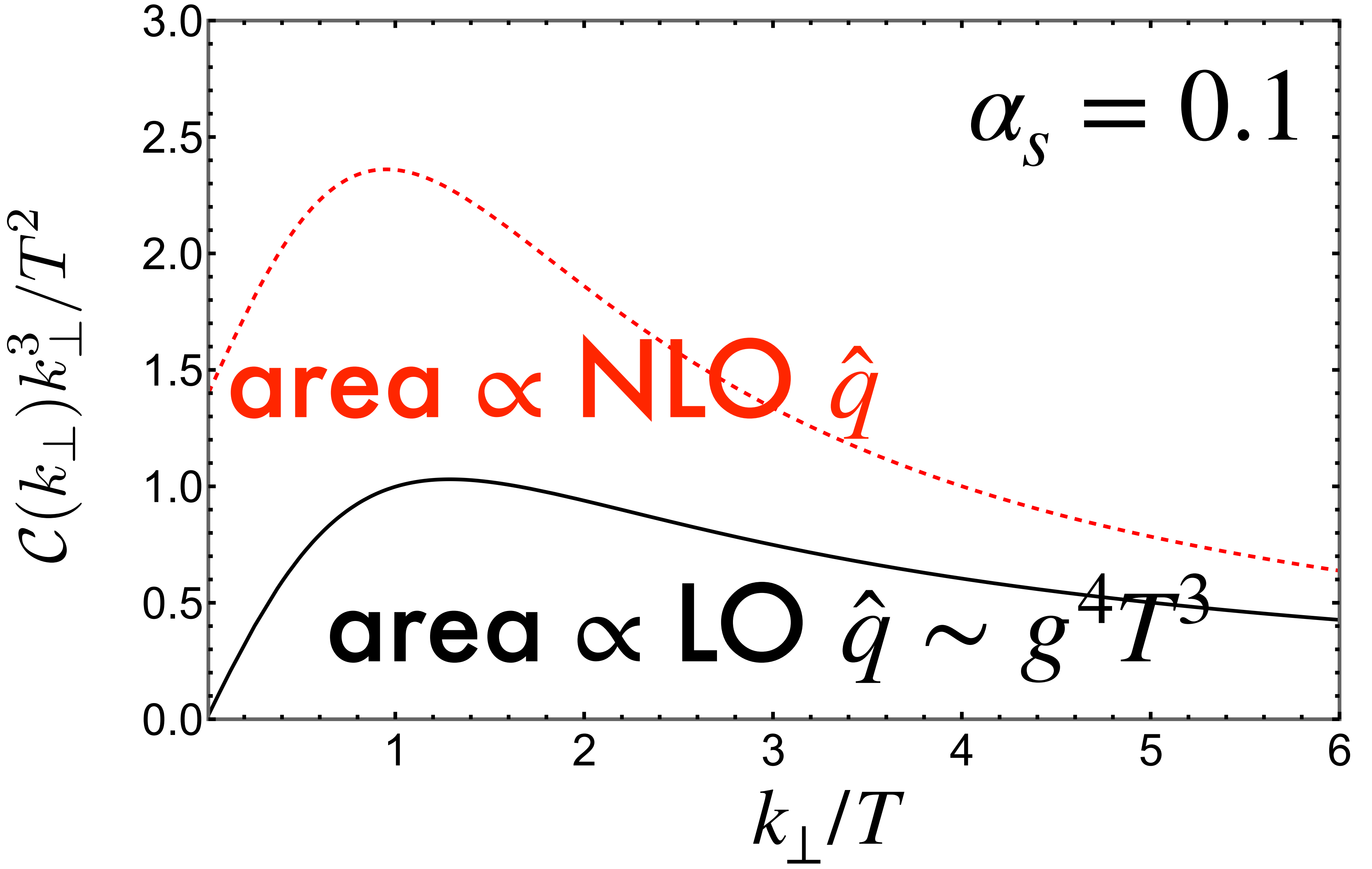

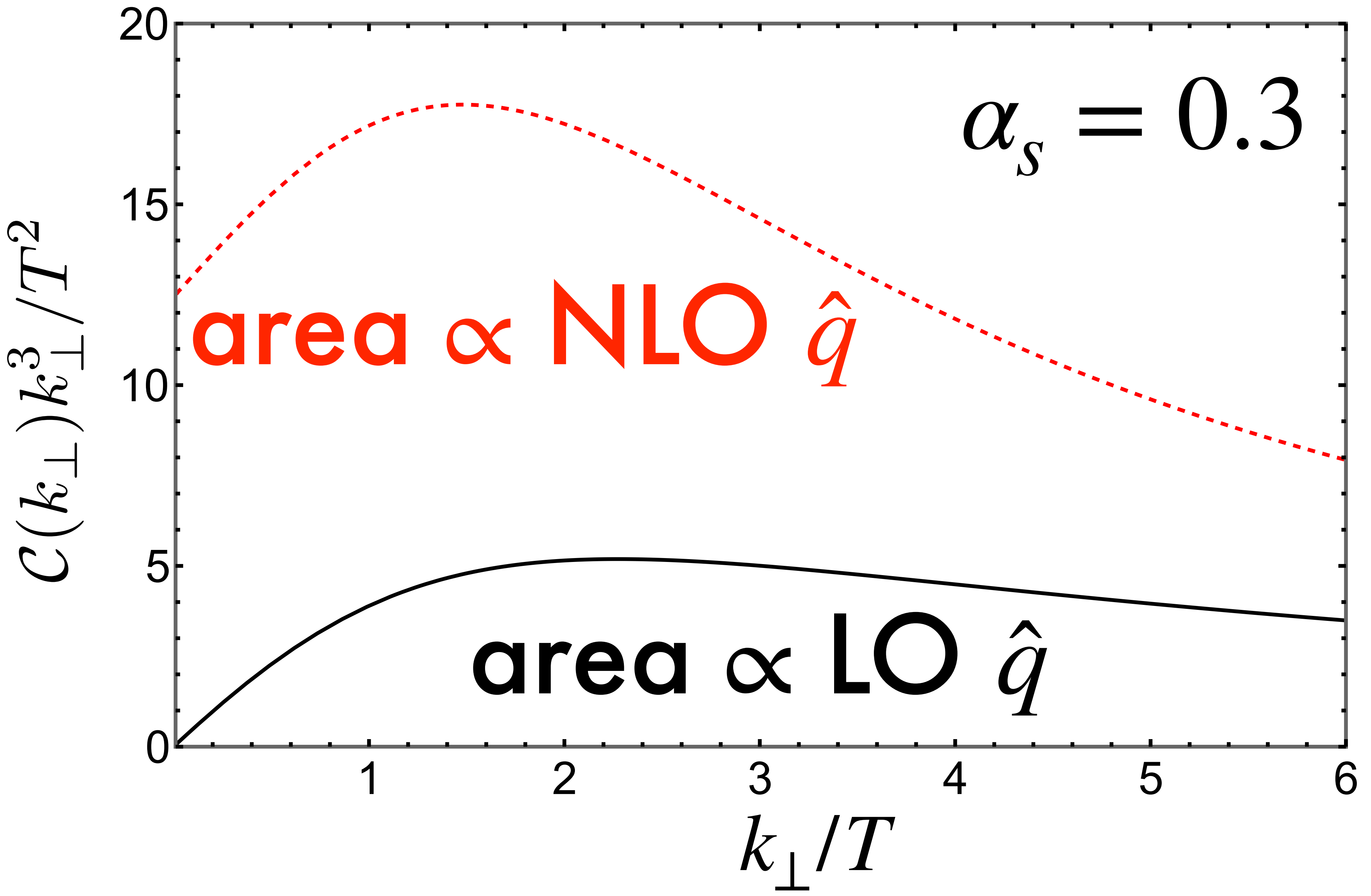

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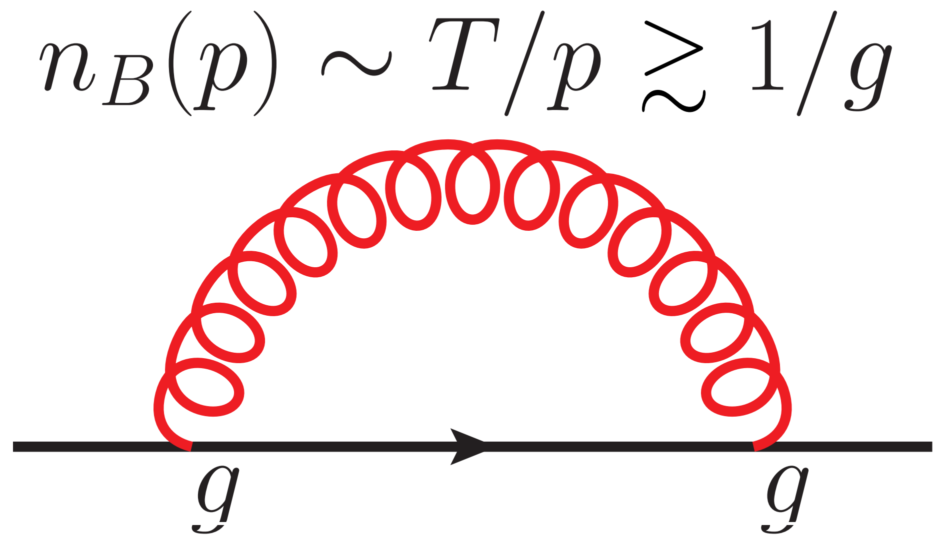

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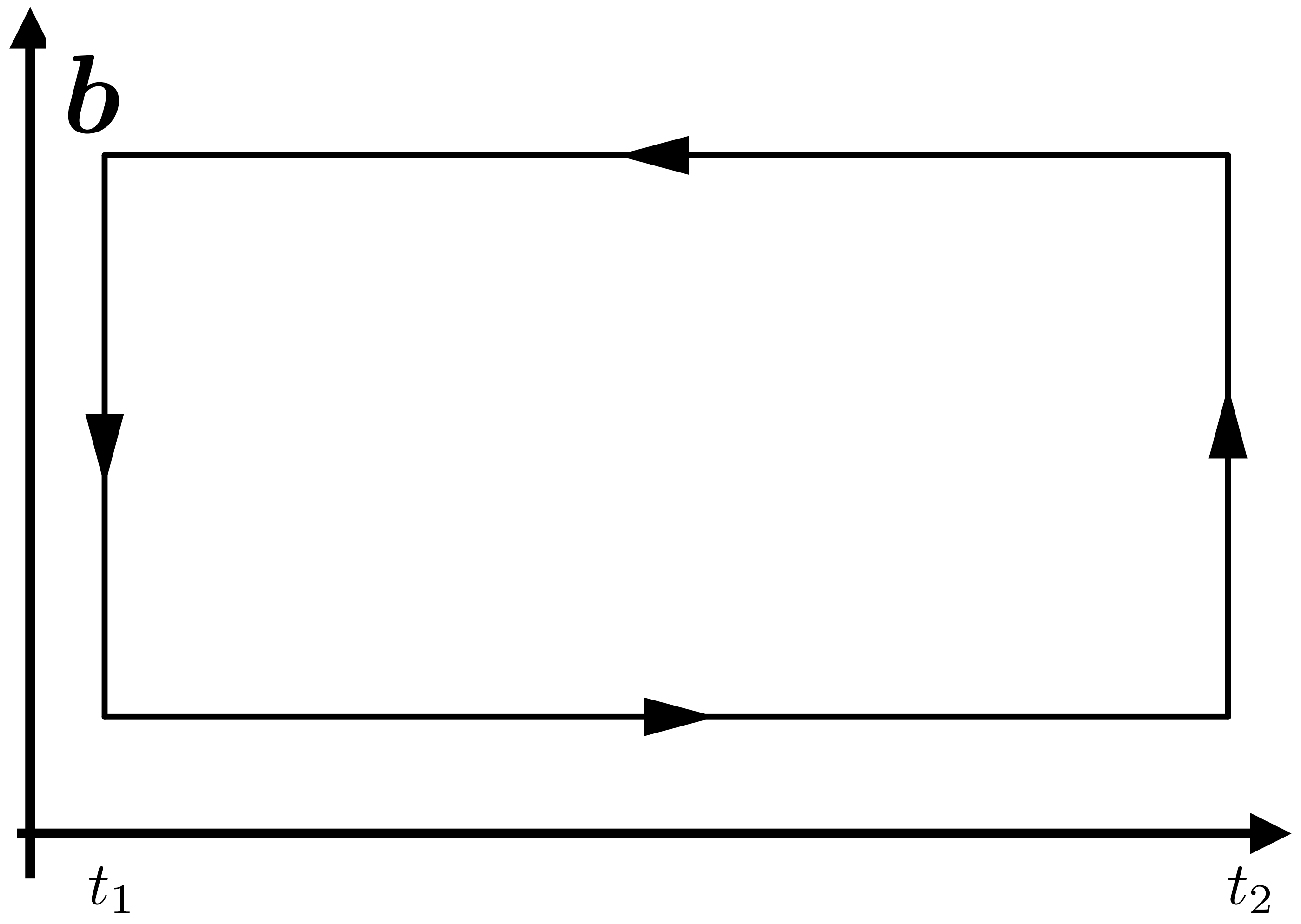
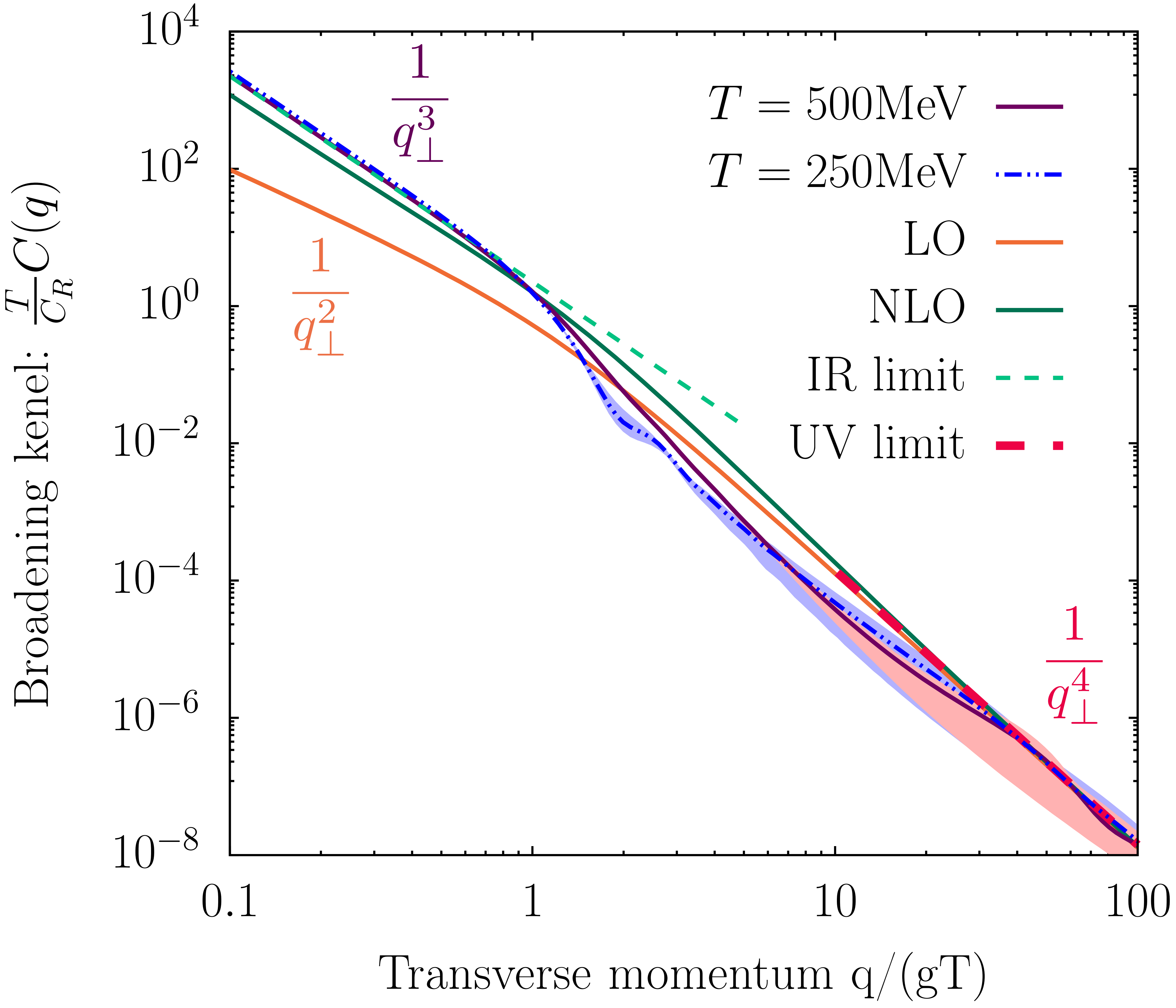
- Horrible HTL perturbative calculation or extremely challenging 4D lattice on the light-cone become **3D** Electrostatic QCD (EQCD).
  New strategy: lattice for $b \gtrsim 1/gT$, pQCD for $b \lesssim 1/gT$



- Recently: continuum-extrapolated EQCD lattice data for the scattering kernel and merging with pQCD
  Moore Schlusser **PRD101** (2020) Moore Schlichting Schlusser Soudi **JHEP2110** (2021)

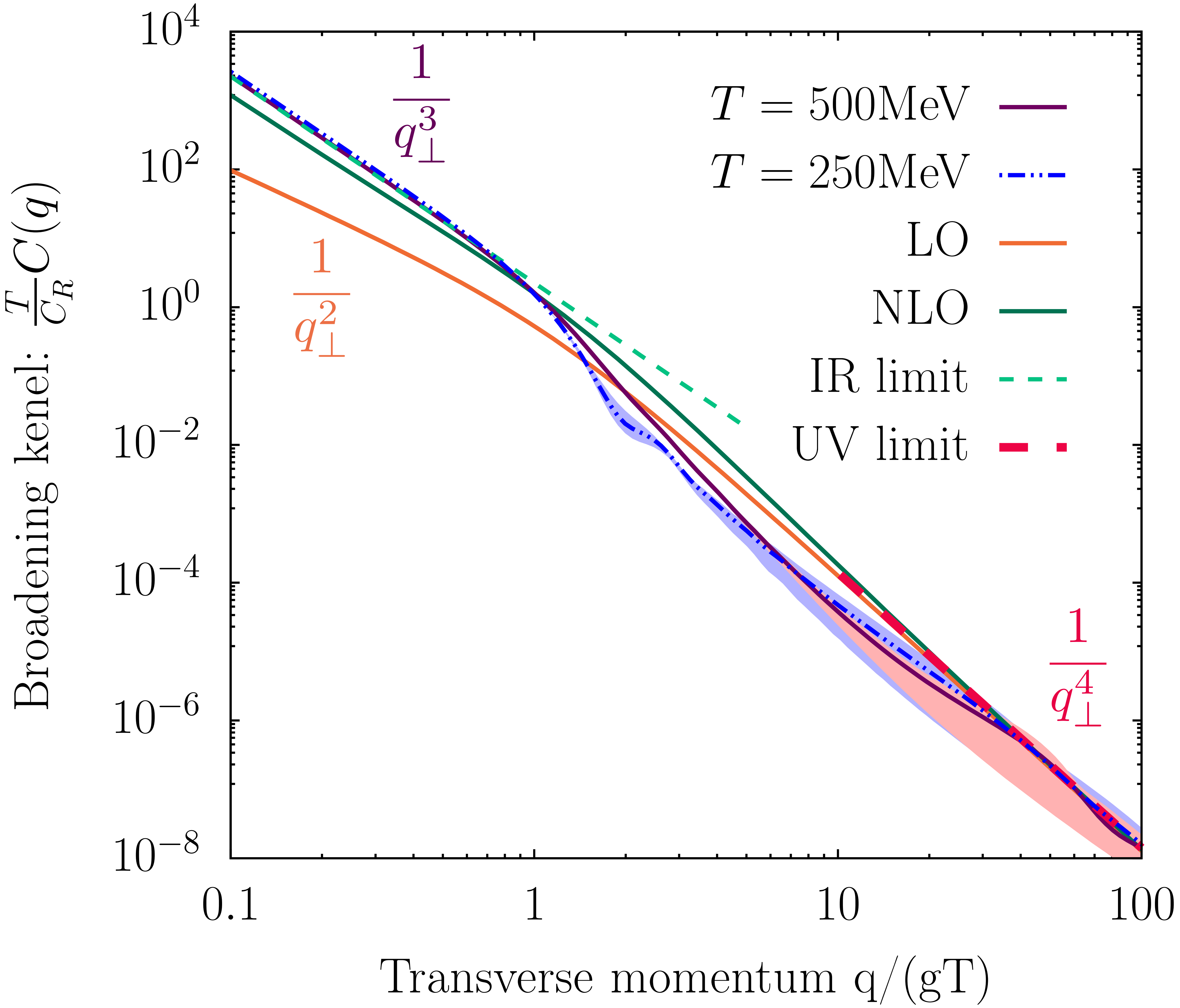
Non-perturbative classical contribution



- LO and NLO perturbative EQCD: Aurenche Gelis Zaraket (2002) Caron-Huot (2008)
  LO UV ($q_\perp > gT$) pQCD and matching: Arnold Xiao (2008) JG Kim (2018)
- Significant deviations from pQCD
- Non-perturbative magnetic "screening" means $q_\perp^{-3}$ instead of Molière $q_\perp^{-4}$

Schlichting Soudi **PRD105** (2022)

Non-perturbative classical contribution



- Only classical corrections here, what happens with **quantum corrections** for $q_\perp > gT$?
- Similar lattice EQCD+pQCD programme in progress for the in-medium jet mass
  Schlusser Moore **PRD102** (2020)
  JG Moore Schicho Schlusser **JHEP02** (2022)
  JG Schicho Schlusser Weitz **2312.11731**

Schlichting Soudi **PRD105** (2022)

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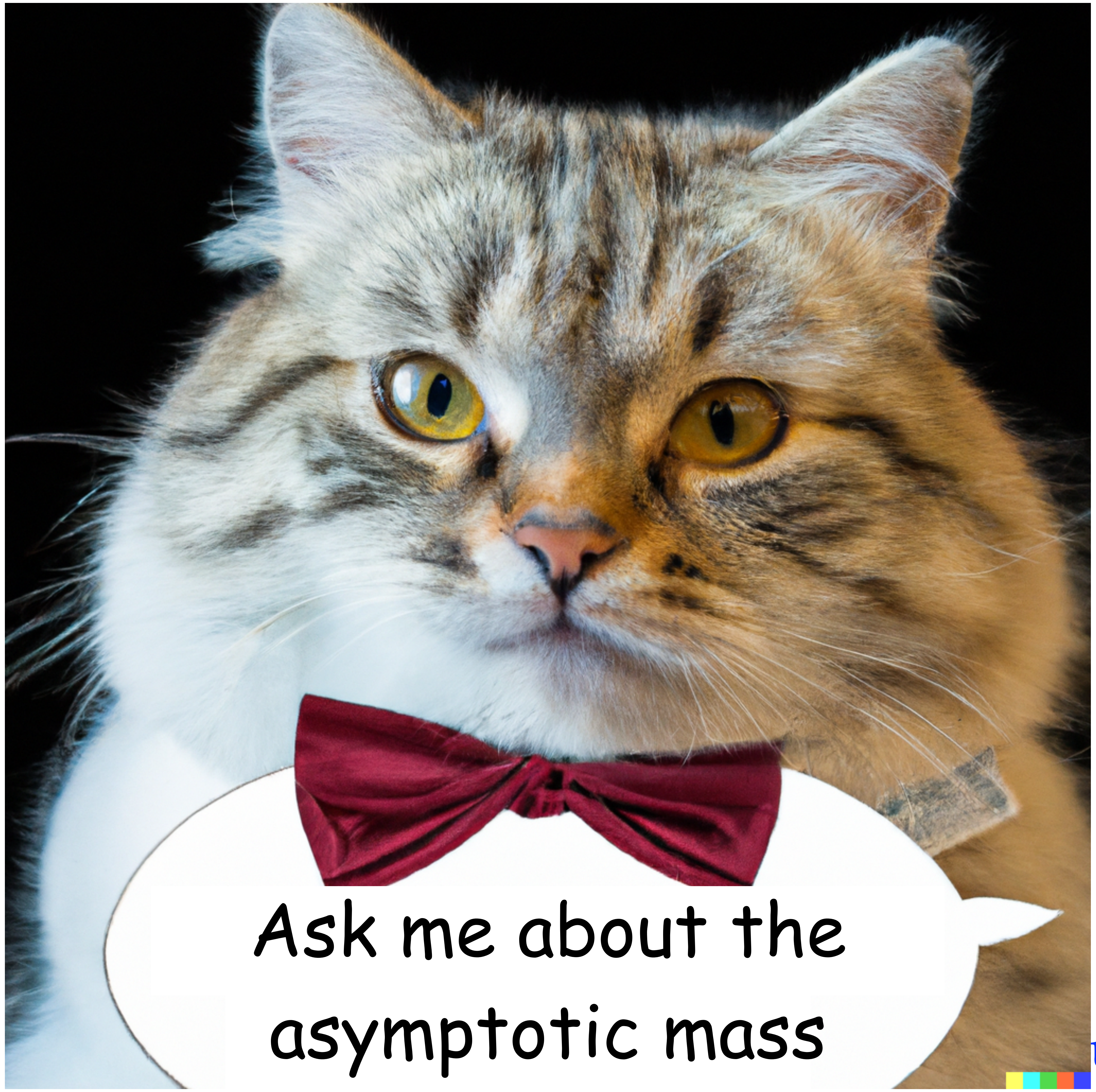

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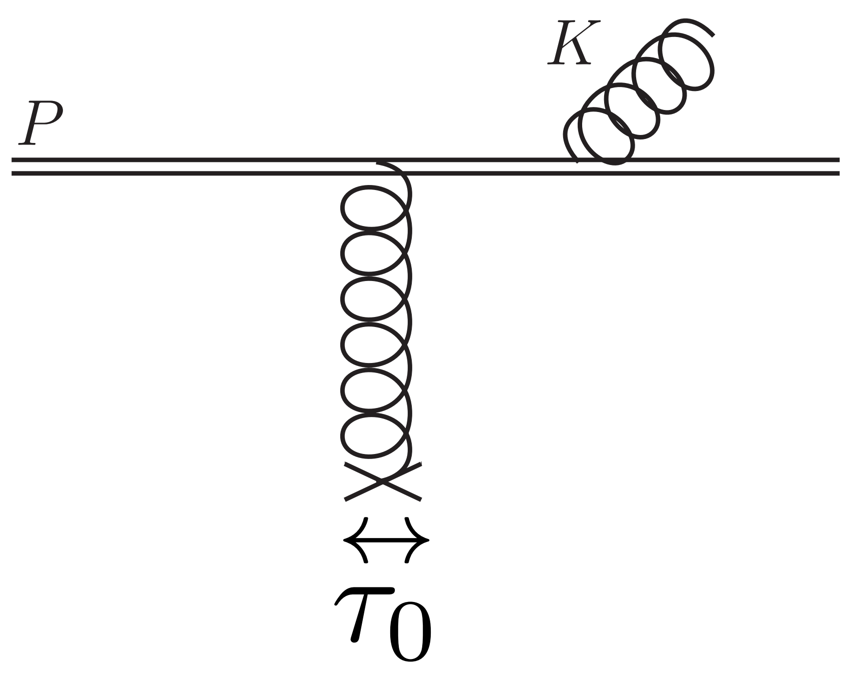
See backup slides

udi **PRD105** (2022)

The scattering kernel: quantum corrections

- Radiative corrections to momentum broadening are enhanced by soft and collinear logarithms in the single scattering regime ⇒ double logarithm



$$\delta\hat{q} = \frac{\alpha_s N_c}{\pi}\hat{q}_0 \int_{\text{single}} \frac{d\omega}{\omega}\frac{dk_\perp^2}{k_\perp^2} = \frac{\alpha_s N_c}{\pi}\hat{q}_0 \ln^2\left(\frac{L}{\tau_0}\right)$$

Liou Mueller Wu (2013) Blaizot Dominguez Iancu Mehtar-Tani (2013)

Caucal Mehtar-Tani **PRD106** (2022) **JHEP09** (2022)

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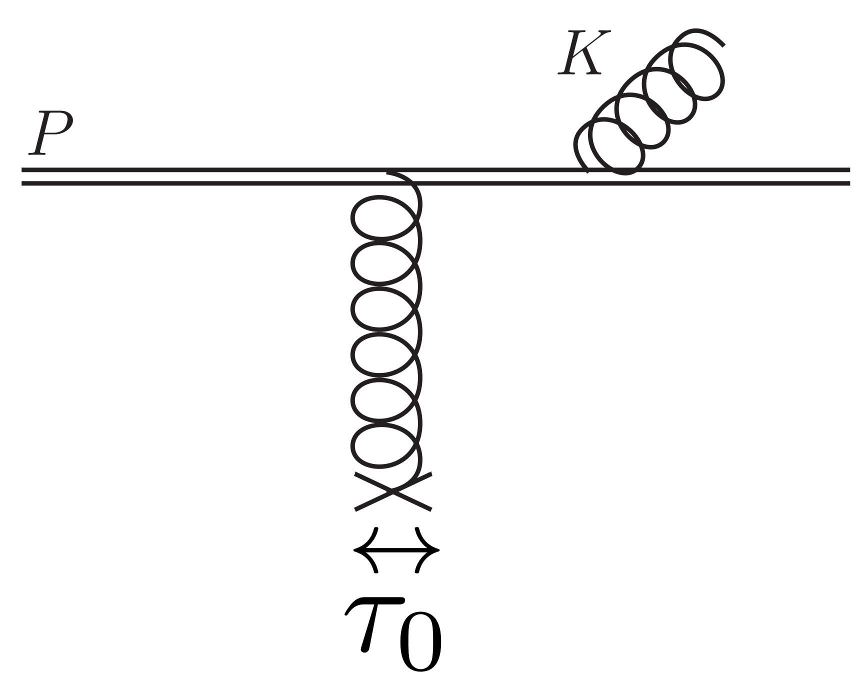
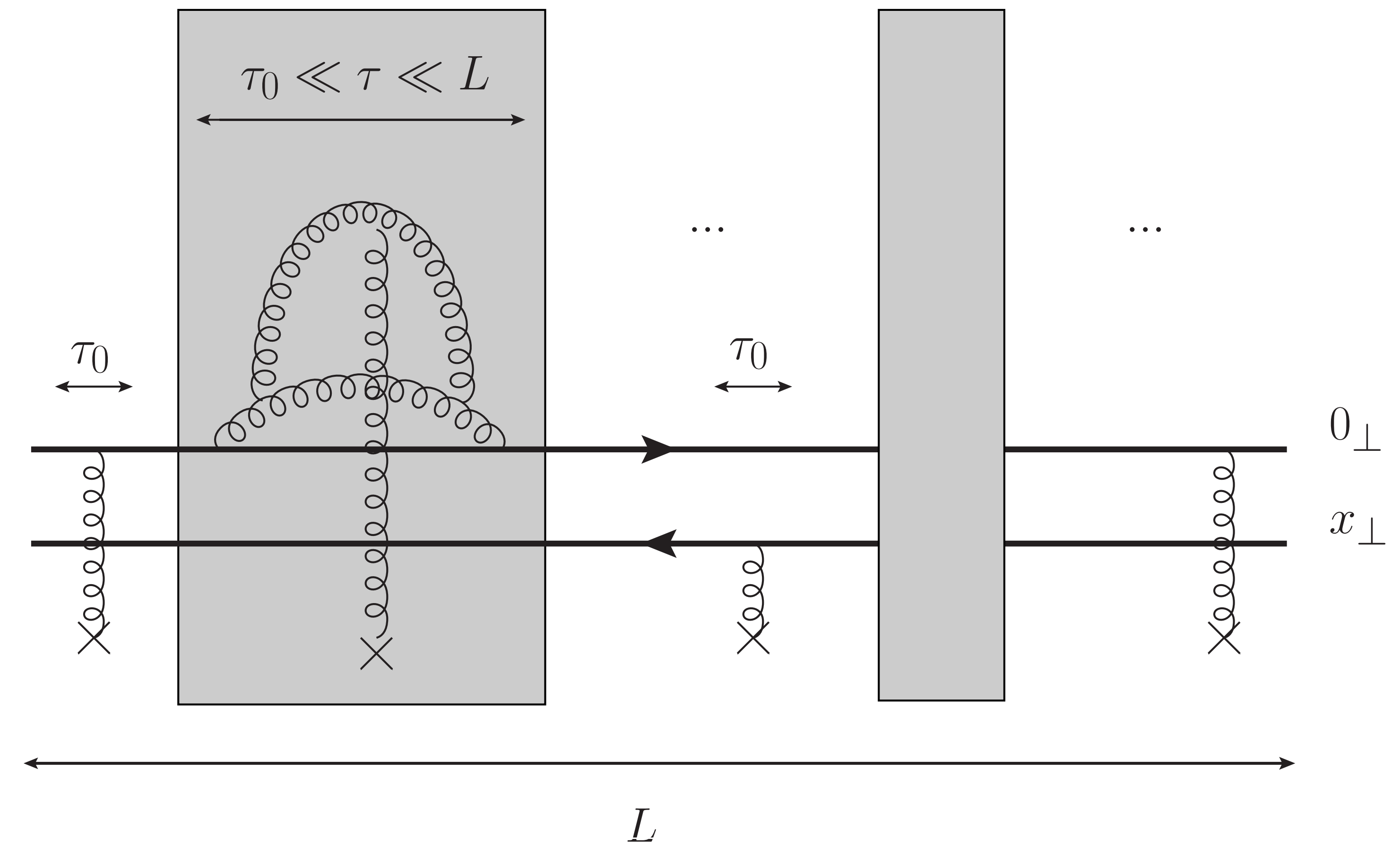


Liou Mueller Wu (2013) Blaizot Dominguez Iancu Mehtar-Tani (2013)

- This log² renormalises the LO qhat. *Resum* these logs

$$\hat{q}(\tau, \boldsymbol{k}_\perp^2) = \hat{q}^{(0)}(\tau_0, \boldsymbol{k}_\perp^2) + \int_{\tau_0}^{\tau} \frac{\mathrm{d}\tau'}{\tau'} \int_{Q_s^2(\tau')}^{\boldsymbol{k}_\perp^2} \frac{\mathrm{d}\boldsymbol{k}_\perp'^2}{\boldsymbol{k}_\perp'^2}\, \bar{\alpha}_s(\boldsymbol{k}_\perp'^2)\, \hat{q}(\tau', \boldsymbol{k}_\perp'^2)$$

$$Q_s^2(\tau) = \hat{q}(\tau, Q_s^2(\tau))\tau\,,$$



by solving the above numerically and semi-analytically

Caucal Mehtar-Tani **PRD106** (2022) **JHEP09** (2022)

Classical and quantum corrections

- Classical: large $\hat{q}_0(1 + \mathcal{O}(g))$ corrections, non-perturbative all-order determinations. Affect also NLO transport coefficients
- Quantum: large $\hat{q}_0(1 + \mathcal{O}(g^2 \ln^2(LT)))$ corrections, resummations and renormalisations. Affect also double splitting

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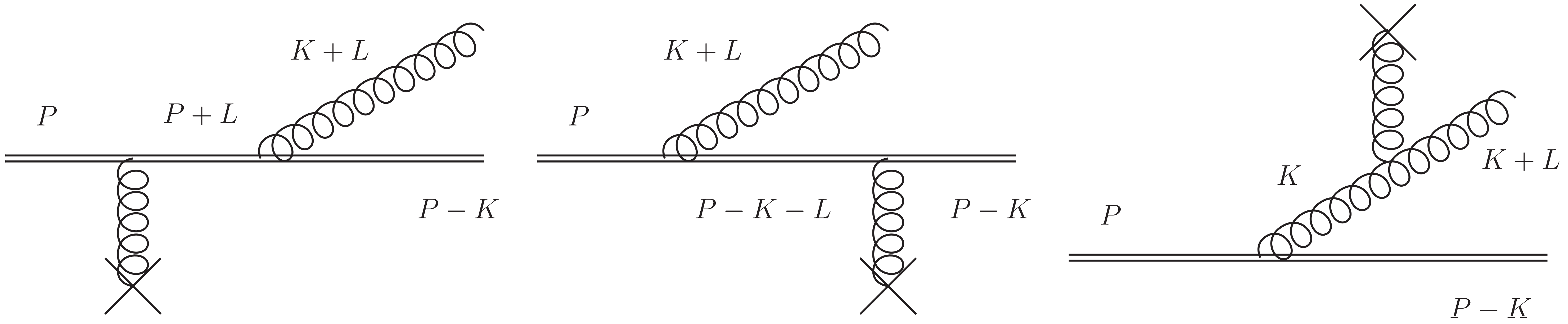
- Quantum: large $\hat{q}_0(1+\mathcal{O}(g^2\ln^2(LT)))$ corrections, resummations and renormalisations. Affect also double splitting

- Where do they meet in a weakly-coupled plasma? Is there a hierarchy or an interplay?

The double logarithm in a nutshell

$P = (p^+, 0, 0)$



- Radiative correction to the scattering kernel for a medium of scattering centers

$$\delta\mathcal{C}(k_\perp)^{N=1}_{\text{rad}} = 4\alpha_s C_R \int \frac{dk^+}{k^+} \int \frac{d^2 l_\perp}{(2\pi)^2} \mathcal{C}_0(l_\perp) \left[\frac{\boldsymbol{k}_\perp}{k_\perp^2} - \frac{\boldsymbol{k}_\perp + \boldsymbol{l}_\perp}{(\boldsymbol{k}_\perp + \boldsymbol{l}_\perp)^2} \right]^2$$

soft DGLAP ($k^+ \ll p^+$) x LO (elastic) scattering kernel x dipole factor

LMW: Liou Mueller Wu **NPA916** (2013) **BDIM**: Blaizot Dominguez Iancu Mehtar-Tani **JHEP06** (2013)

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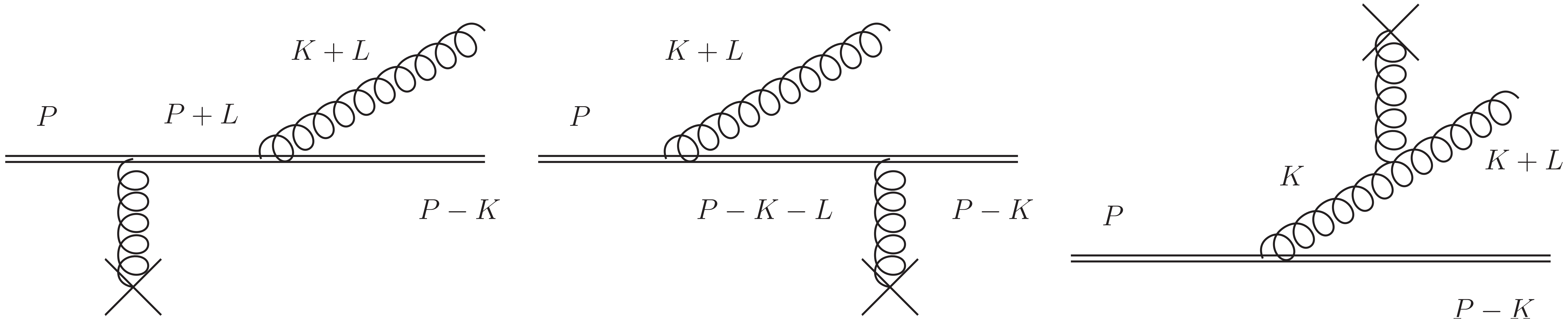

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soft DGLAP ($k^+ \ll p^+$) x LO (elastic) scattering kernel x dipole factor

- In principle just the first term in opacity series. If $k_\perp \gg l_\perp$ single-scattering regime

$$\delta\hat{q} = \int^\mu \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 \, \delta\mathcal{C}(k_\perp)^{\text{single}}_{\text{rad}} = 4\alpha_s C_R \int \frac{dk^+}{k^+} \int^\mu \frac{d^2 k_\perp}{(2\pi)^2 k_\perp^2} \overbrace{\int \frac{d^2 l_\perp}{(2\pi)^2} l_\perp^2 \, \mathcal{C}_0(l_\perp)}^{\hat{q}_0}$$

a **triple logarithm**. What are the boundaries?

LMW: Liou Mueller Wu **NPA916** (2013) **BDIM**: Blaizot Dominguez Iancu Mehtar-Tani **JHEP06** (2013)

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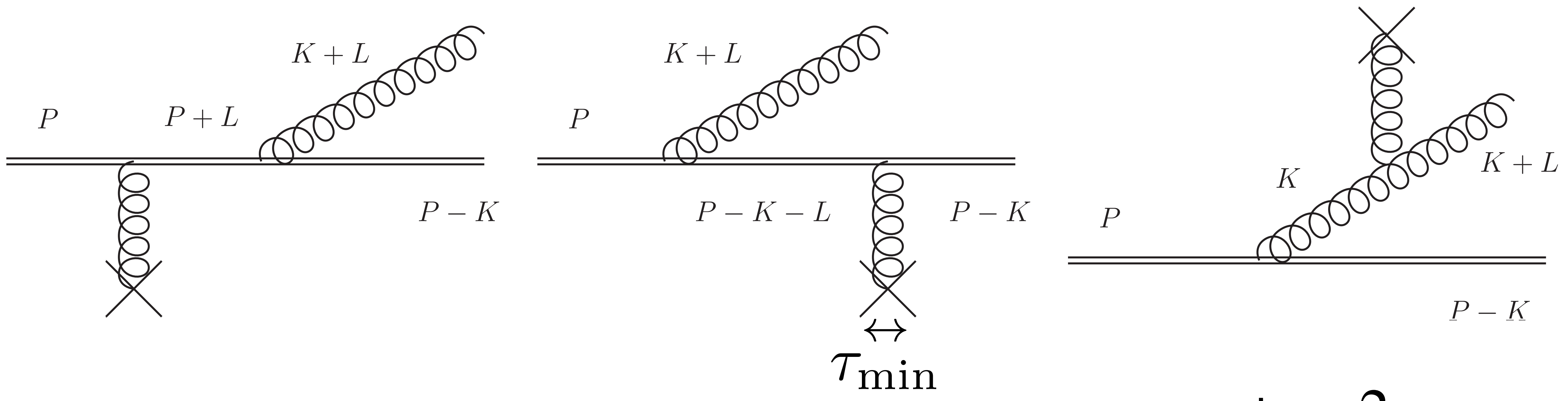

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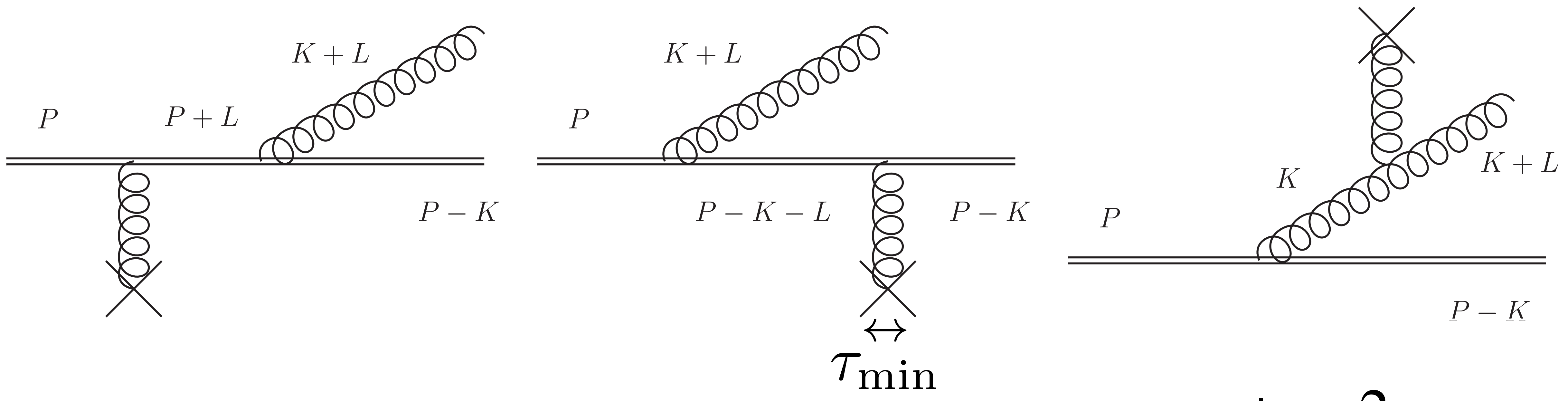
- Introduce formation time $\tau \equiv k^+/k_\perp^2$:

$$\delta\hat{q} = \frac{\alpha_s C_R}{\pi}\, \hat{q}_0 \int \frac{d\tau}{\tau} \int \frac{dk^+}{k^+}$$

The double logarithm in a nutshell



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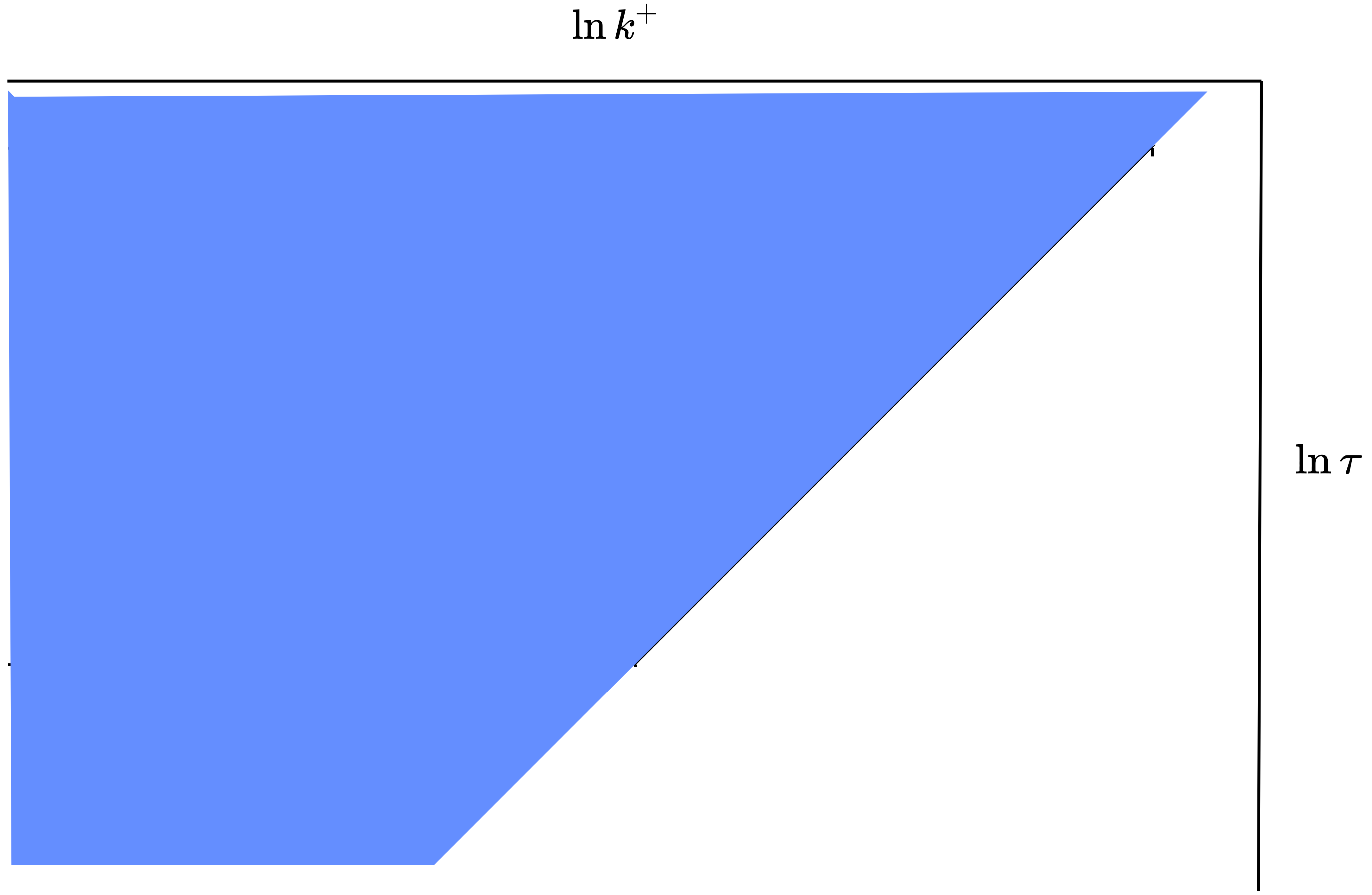
  - Require $\mu > k_\perp$: $\tau > k^+/\mu^2$



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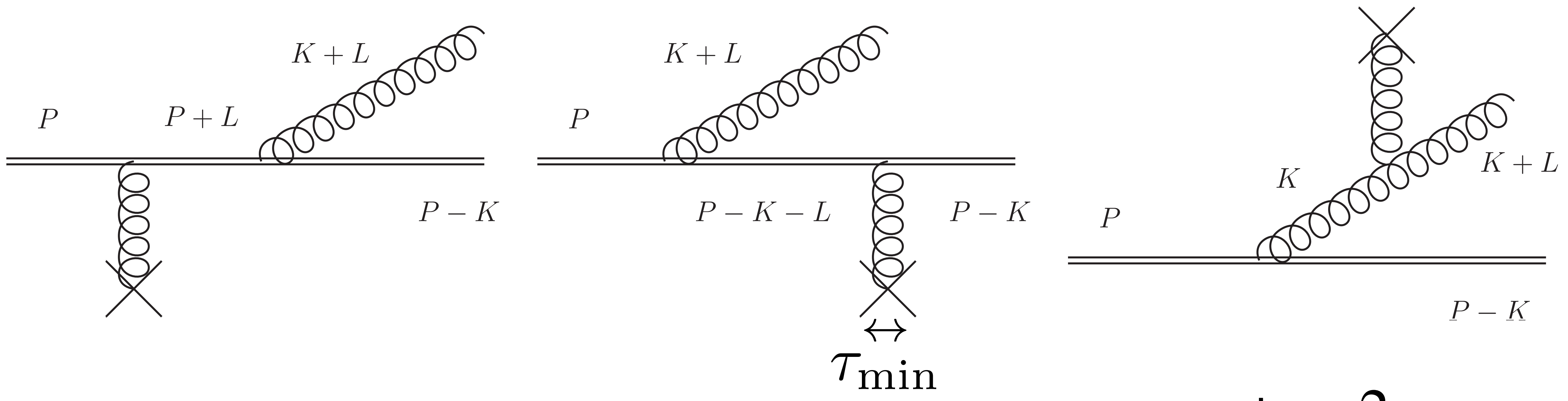

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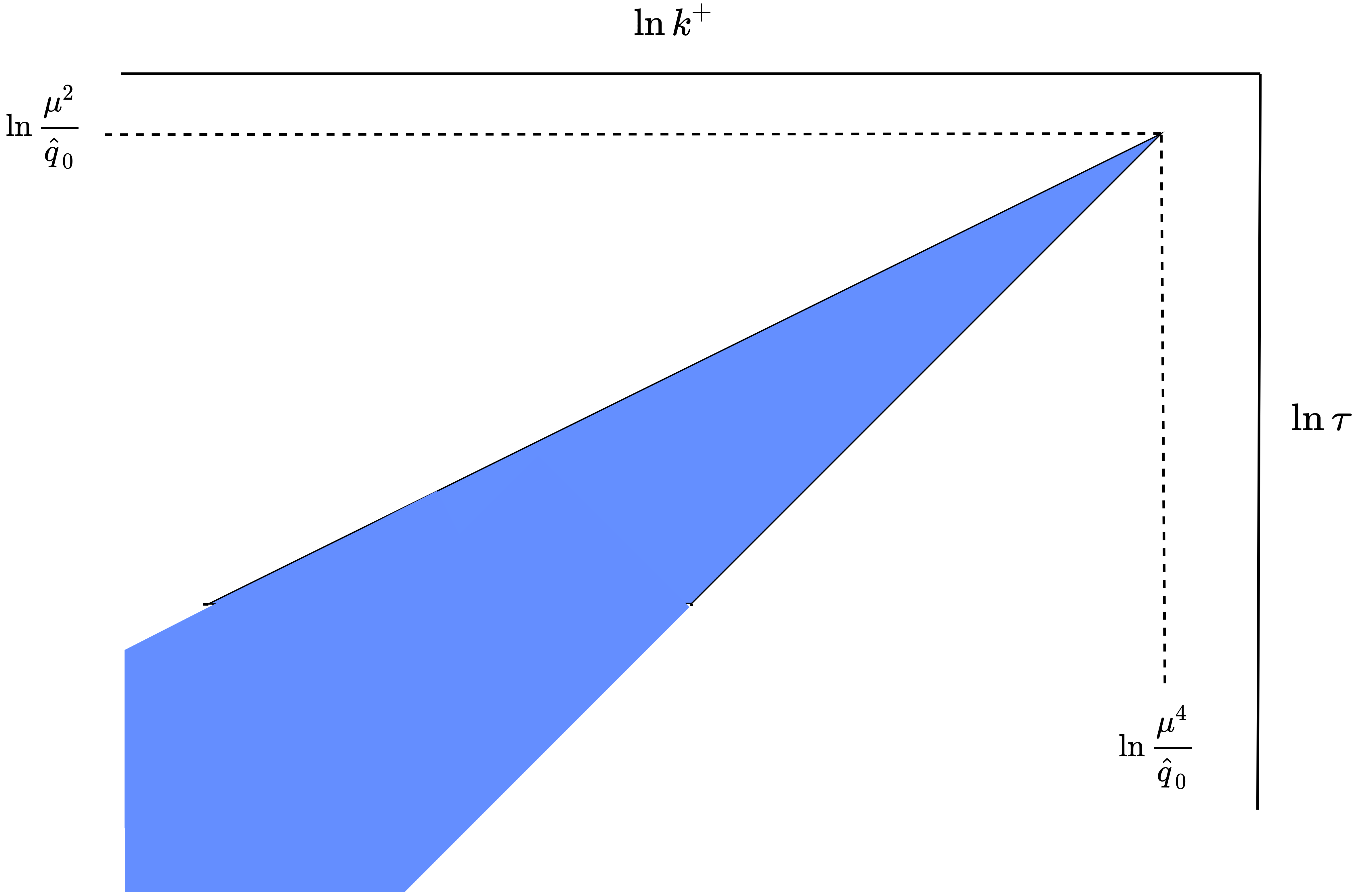

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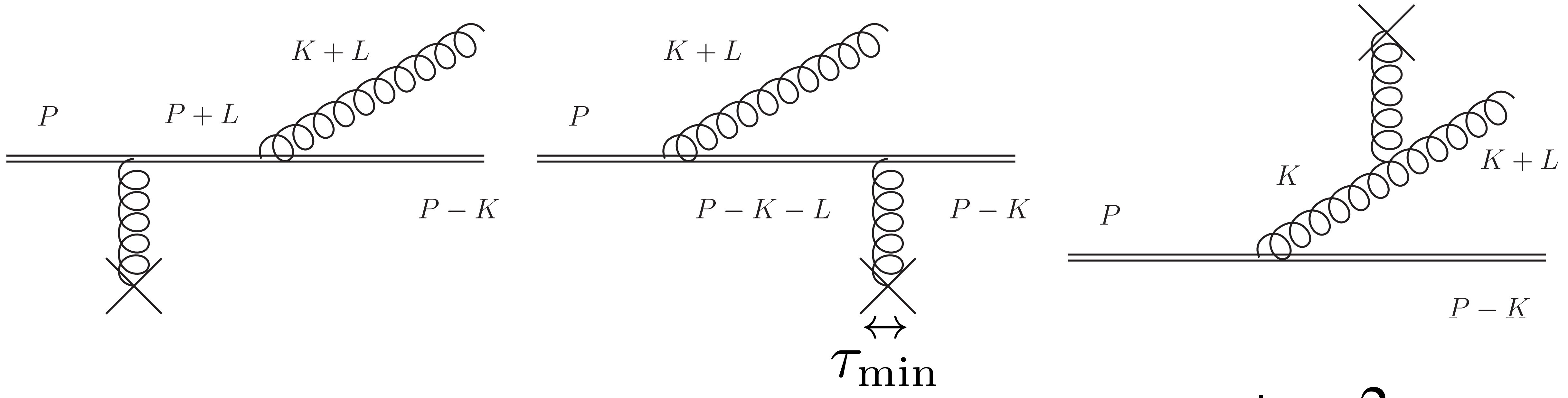

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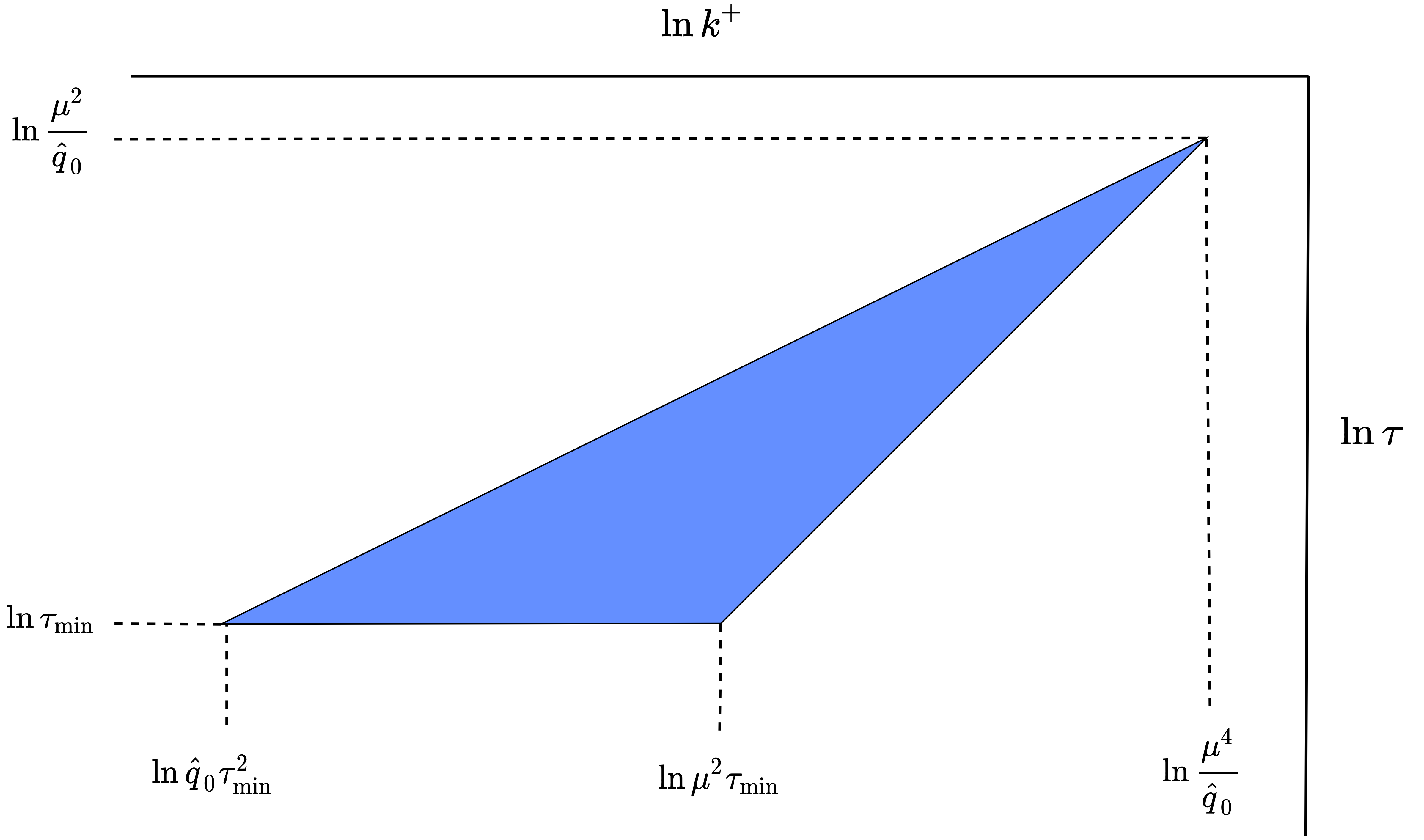

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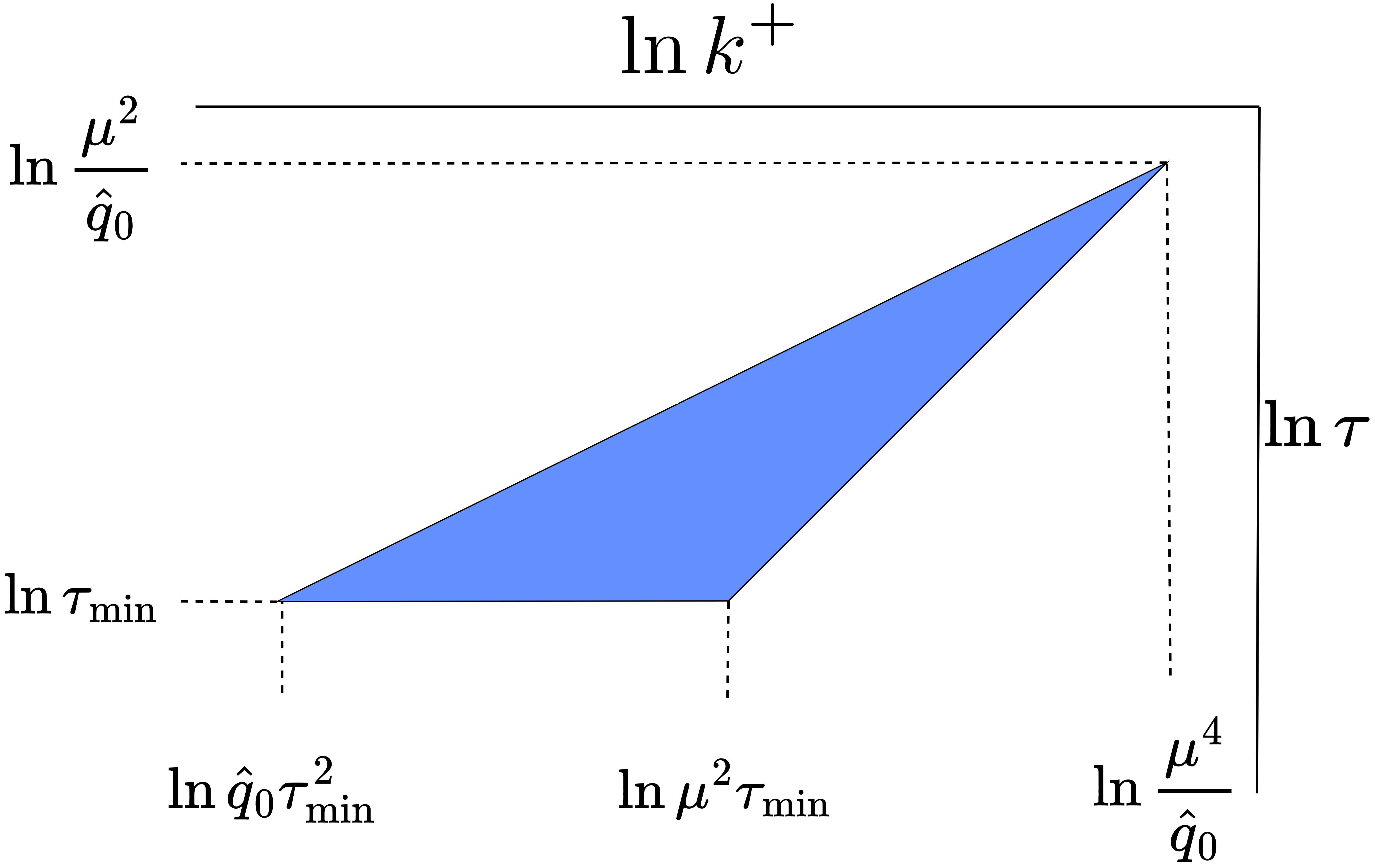

$$\delta \hat{q} = \frac{\alpha_s C_R}{\pi} \hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0 \tau^2}^{\mu^2 \tau} \frac{dk^+}{k^+} = \frac{\alpha_s C_R}{2\pi} \hat{q}_0 \ln^2 \frac{\mu^2}{\hat{q}_0 \tau_{\min}} \overset{\mu^2 = \hat{q}_0 L}{=} \frac{\alpha_s C_R}{2\pi} \hat{q}_0 \ln^2 \frac{L}{\tau_{\min}}$$

LMW: Liou Mueller Wu **NPA916** (2013) BDIM: Blaizot Dominguez Iancu Mehtar-Tani **JHEP06** (2013)

In a weakly coupled QGP

- In a weakly-coupled QGP at first order in the opacity one has

$$\delta\mathcal{C}(k_\perp)^{N=1}_{\mathrm{wQGP}} = 4\alpha_s C_R \int \frac{dk^+}{k^+}\left[1+2n_{\mathrm{B}}(k^+)\right]\int\frac{d^2l_\perp}{(2\pi)^2}\mathcal{C}_0(l_\perp)\left[\frac{\boldsymbol{k}_\perp}{k_\perp^2+m_\infty^2}-\frac{\boldsymbol{k}_\perp+\boldsymbol{l}_\perp}{(\boldsymbol{k}_\perp+\boldsymbol{l}_\perp)^2+m_\infty^2}\right]^2$$

obtained by explicit calculation in **Eamonn's thesis**, can be derived from **AMY**

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- Asymptotic mass $m_\infty^2 \sim g^2T^2$ in the dipole factor for the jet partons
  - $k_\perp \gtrsim gT$ and the dipole factor suppresses $l_\perp \ll k_\perp \Rightarrow l_\perp \gtrsim gT$
  - $\tau_{\text{min}} \sim 1/l_\perp \lesssim 1/gT$ and these soft scatterings happen at a rate $\Gamma_{\text{soft}} \sim g^2T$
  - LPM regime when to $\tau_{\text{LPM}} \gtrsim 1/g^2T$. Indeed $\sqrt{k^+/\hat{q}_0} \sim \sqrt{k^+/T} \times 1/g^2T$
  - m_∞^2 irrelevant in the double-log region where $k_\perp \gg l_\perp$ and $k_\perp \gg gT$

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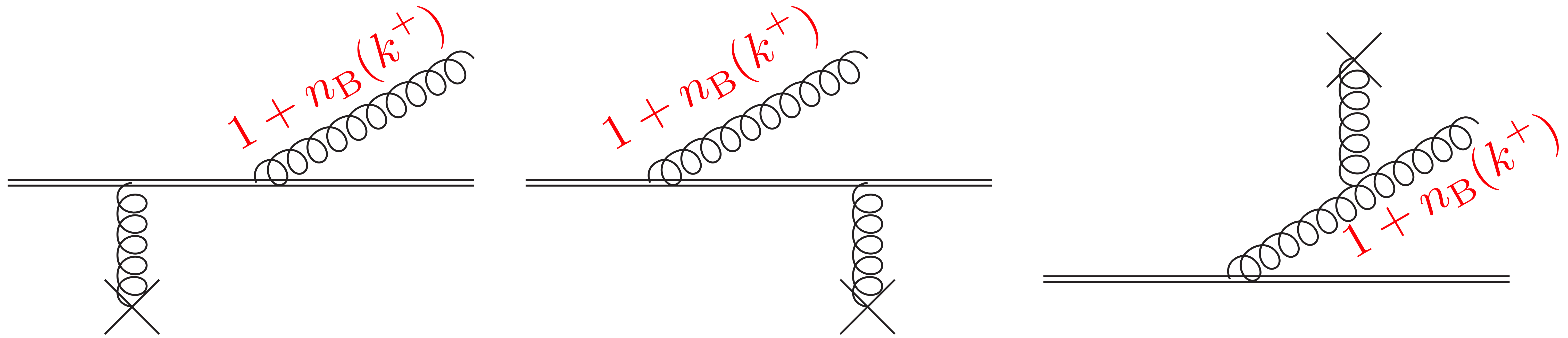
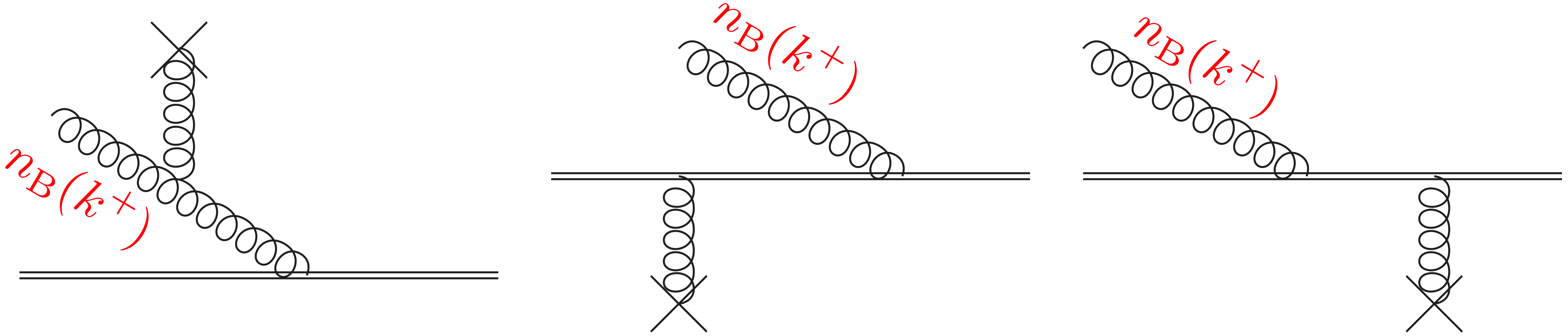
- Bose-Einstein distribution $n_{\mathrm{B}}(k^+)$: not just scattering centers in the medium
  - Stimulated emission



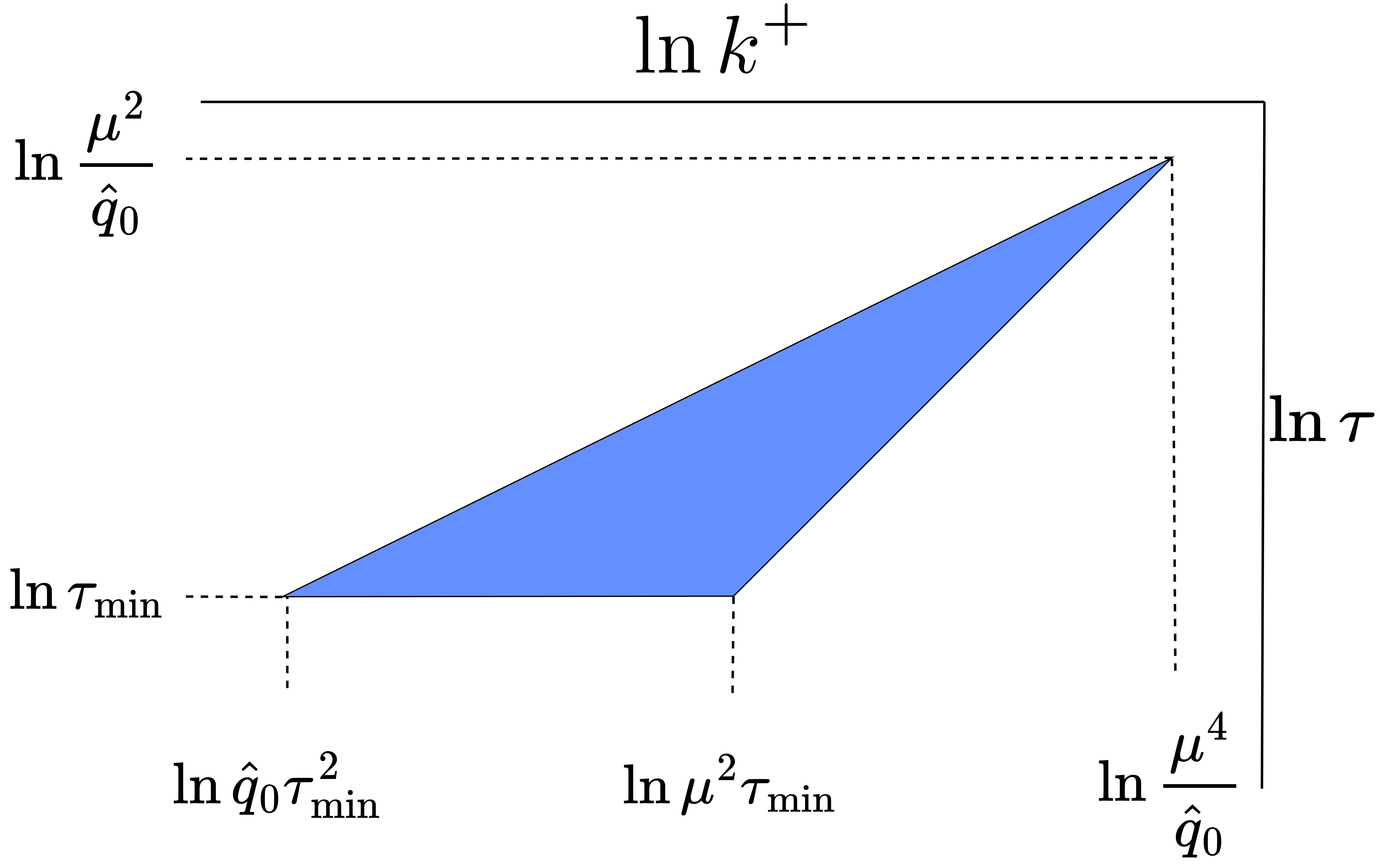
  - Absorption



Double logs in a weakly coupled QGP

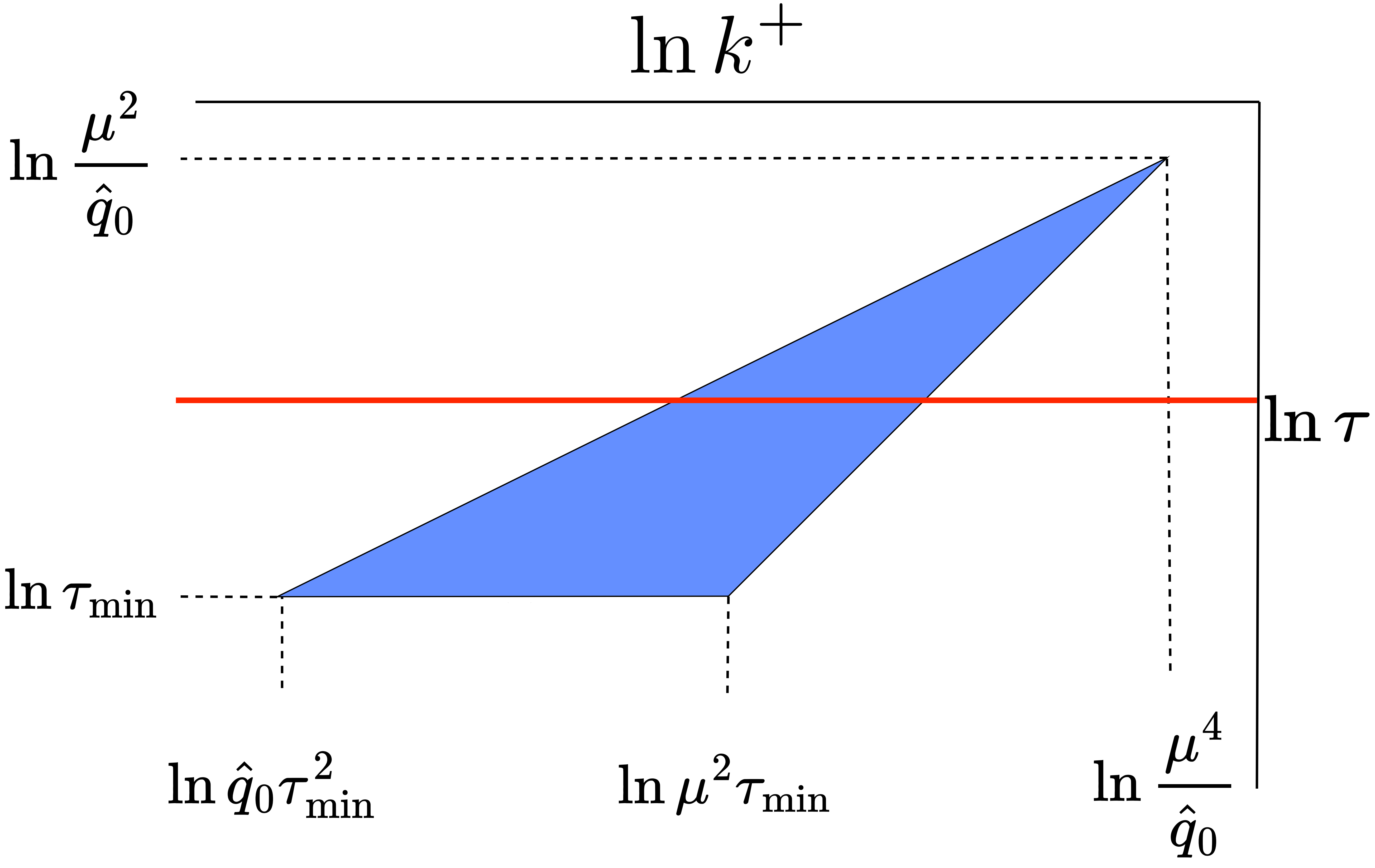
Double logs in a weakly coupled QGP

- Taking LMW/BDIM at face value



Double logs in a weakly coupled QGP

- Taking LMW/BDIM at face value
- $1/g^2T$ minimum LPM time going through the triangle



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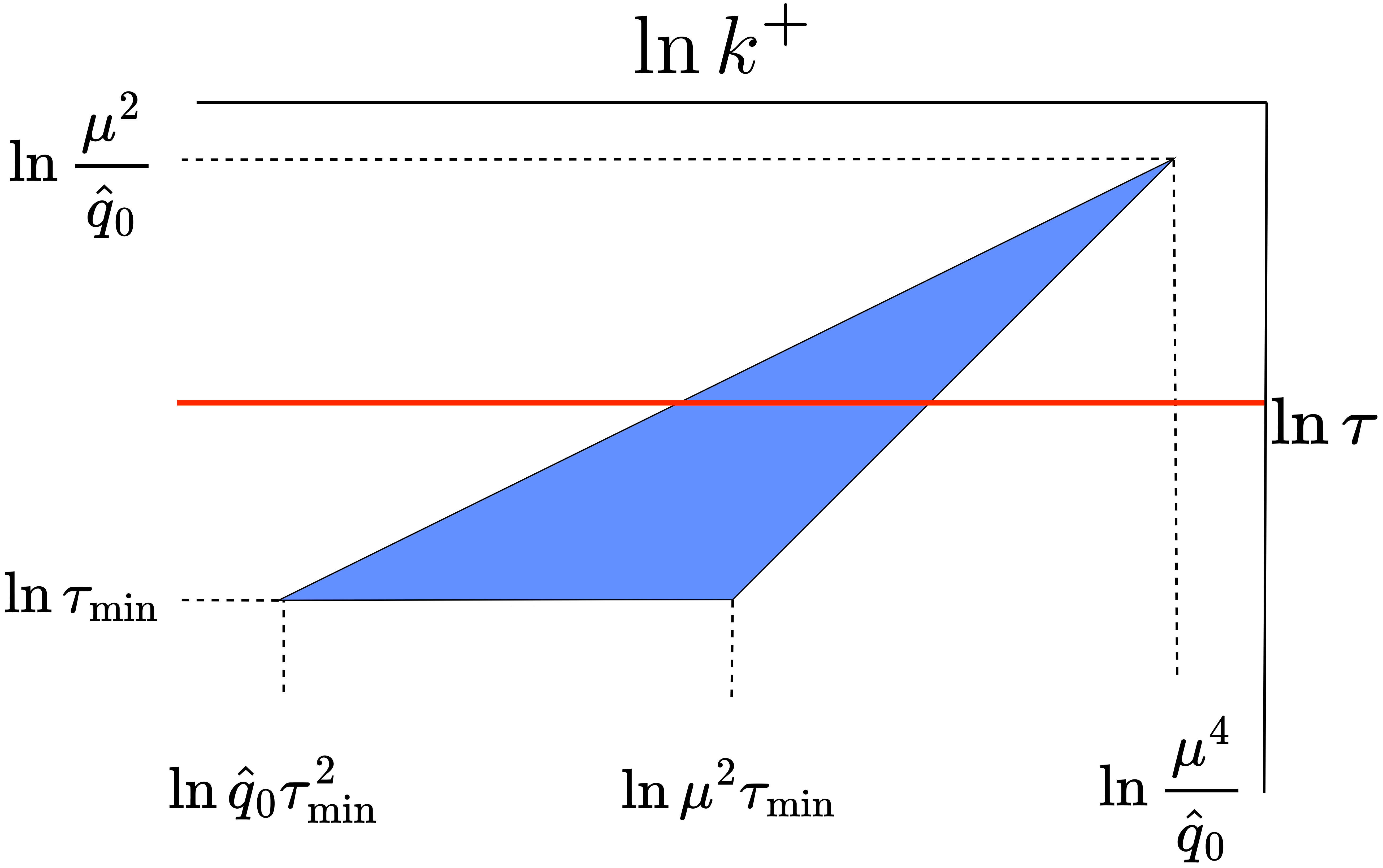

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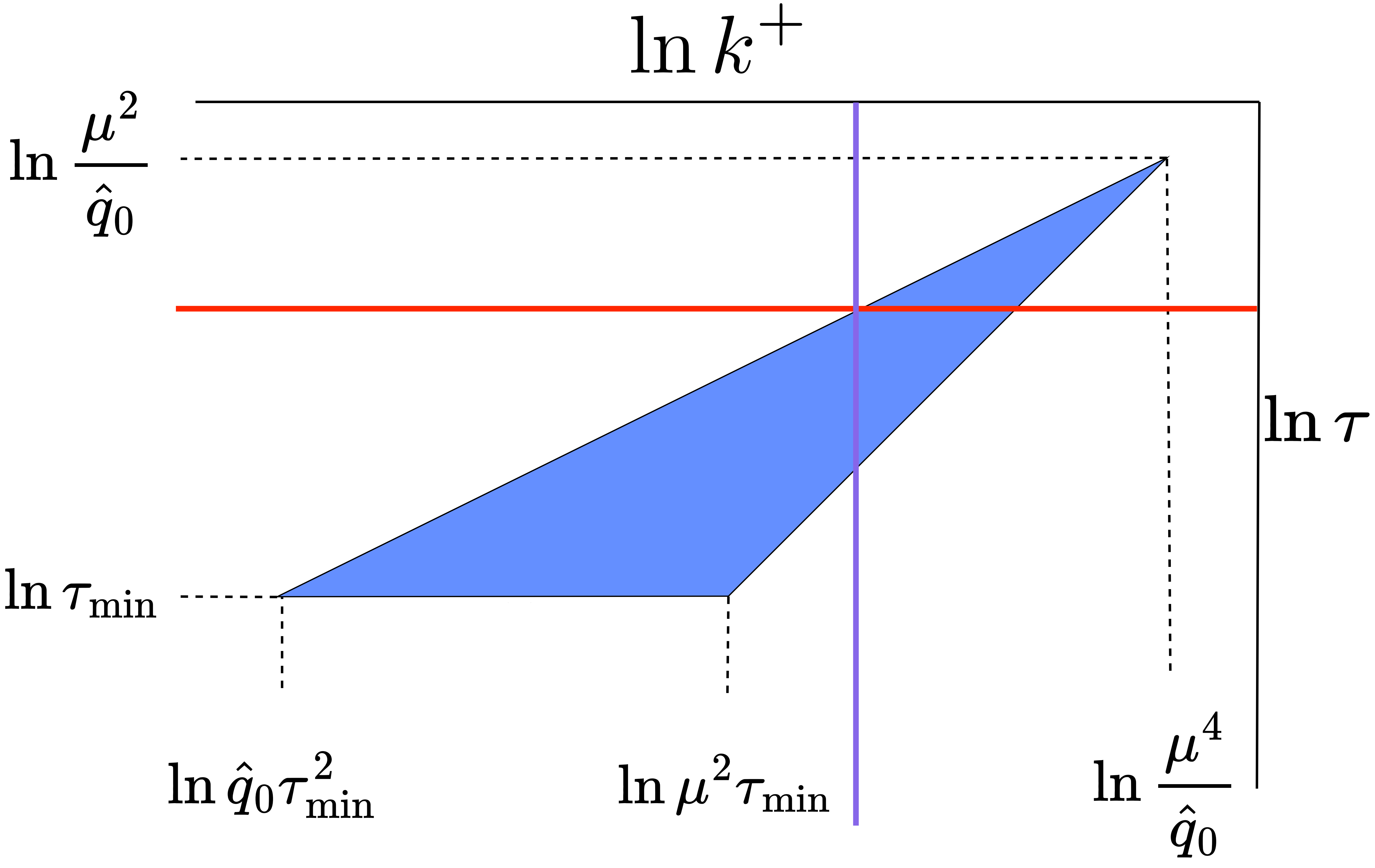

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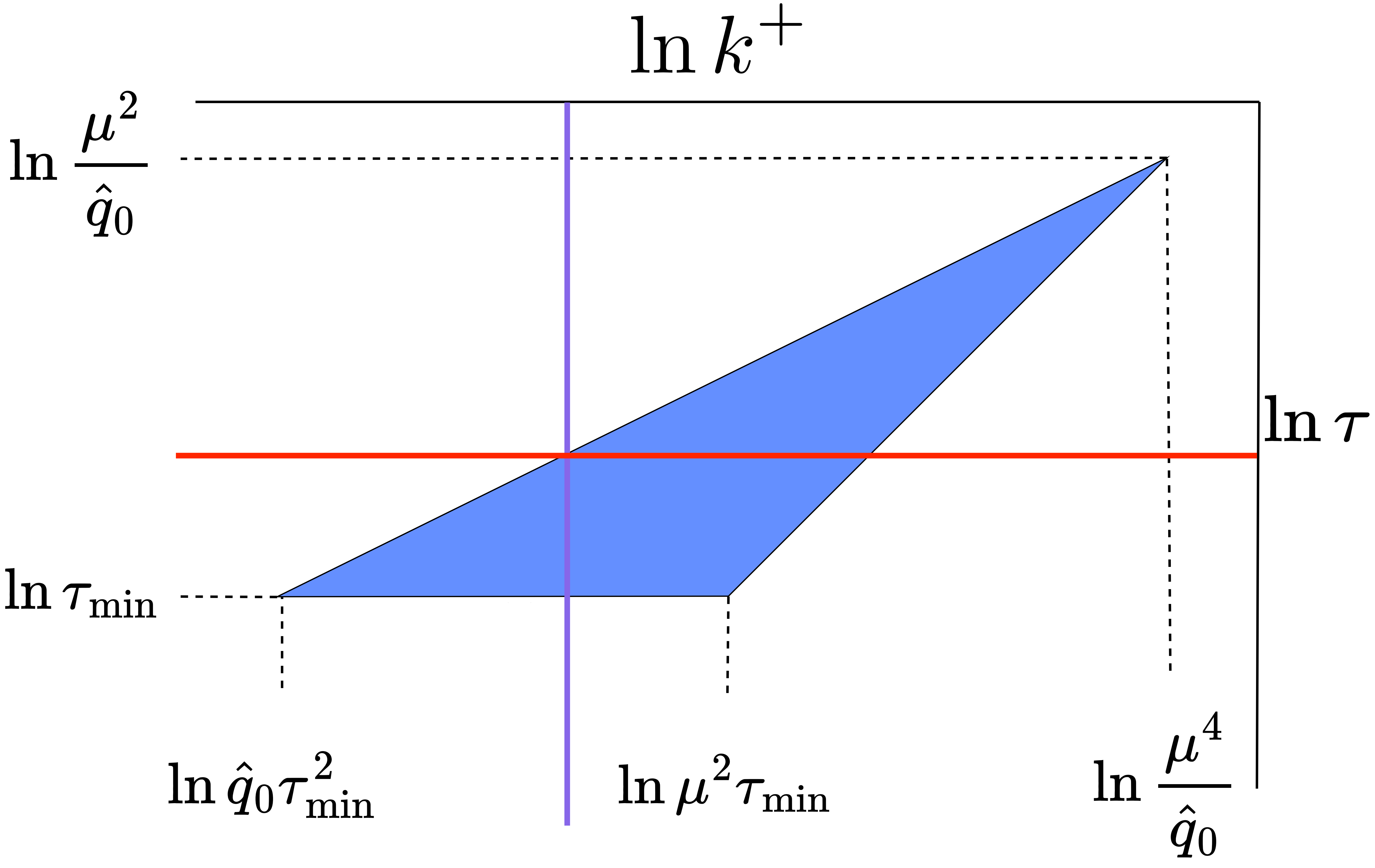
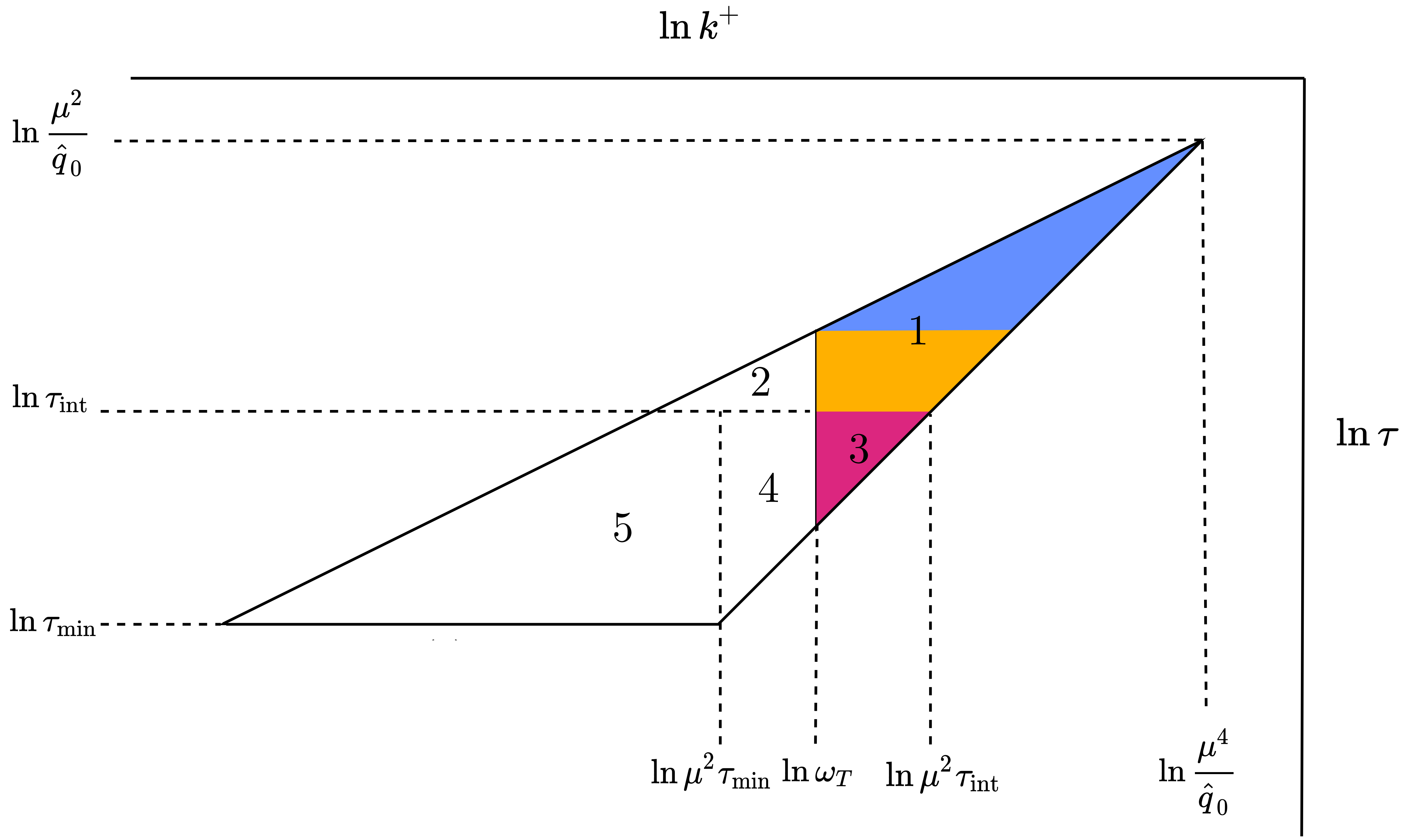
    - $k^+ = T$ for $\mu > T$



The few-scattering regime

- Consider for illustration $gT < \mu < T$
- Blue: $\tau > 1/g^2T$ and $k^+ > T$. $n_B(k^+)$ irrelevant, **few-scattering regime** single << few << many (deep LPM)
- Ochre: $\tau_{\rm int} < \tau < 1/g^2T$ with $1/gT < \tau_{\rm int} < 1/g^2T$ **intermediate regulator** to separate the few and single scattering regimes



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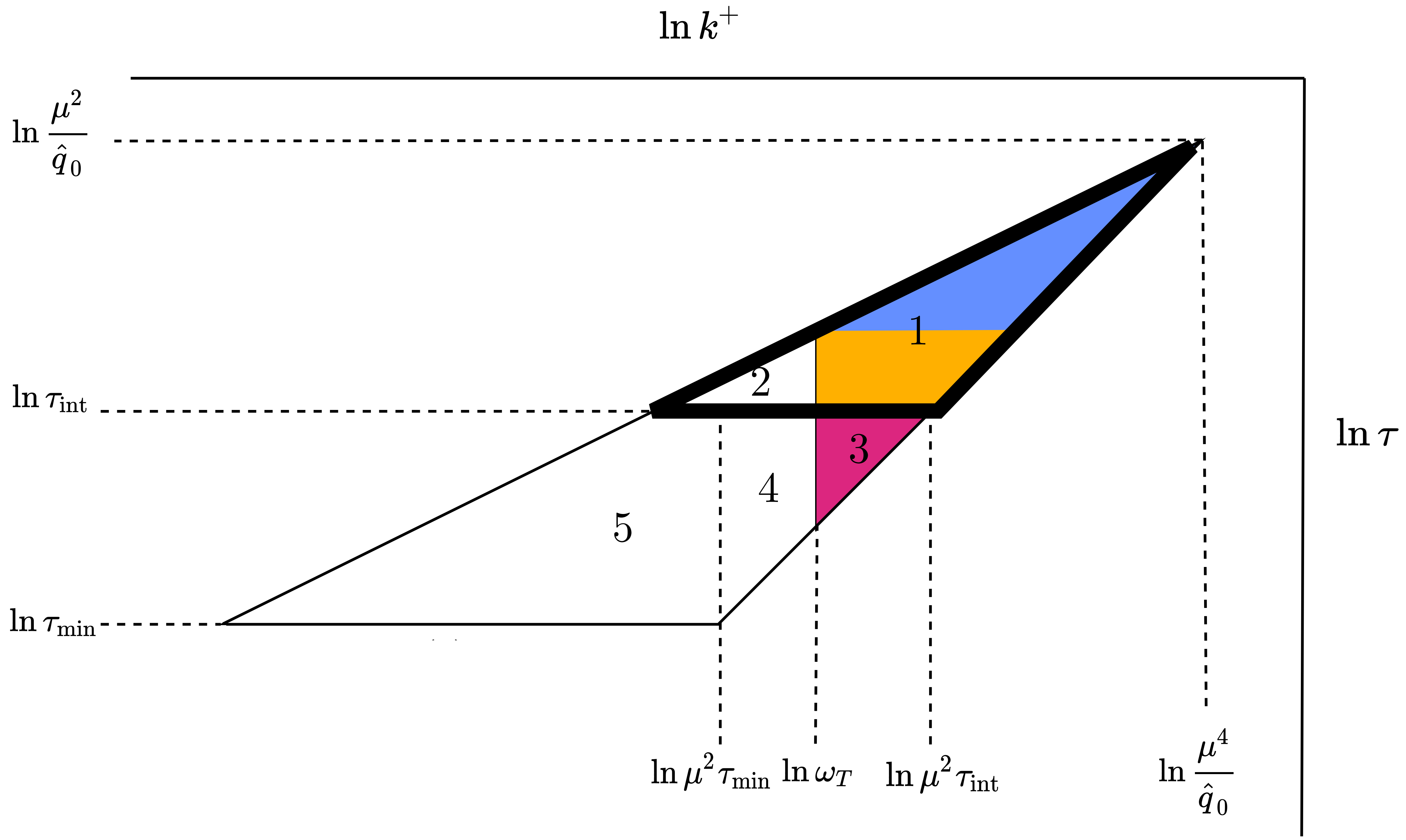

- Hence **regions 1+2** give at double-log accuracy

$$\delta\hat{q}^{\rm few} = \frac{\alpha_s C_R}{\pi}\hat{q}_0 \int_{\tau_{\rm int}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{dk^+}{k^+}\left[1+2n_{\rm B}(k^+)\right] = \frac{\alpha_s C_R}{2\pi}\hat{q}_0 \left\{ \frac{2T}{\hat{q}_0\tau_{\rm int}^2} + \ln^2\frac{\mu^2}{\hat{q}_0\tau_{\rm int}} - \frac{1}{2}\ln^2\frac{\omega_T}{\hat{q}_0\tau_{\rm int}^2} \right\}$$

$$\omega_T = 2\pi e^{-\gamma_E} T$$

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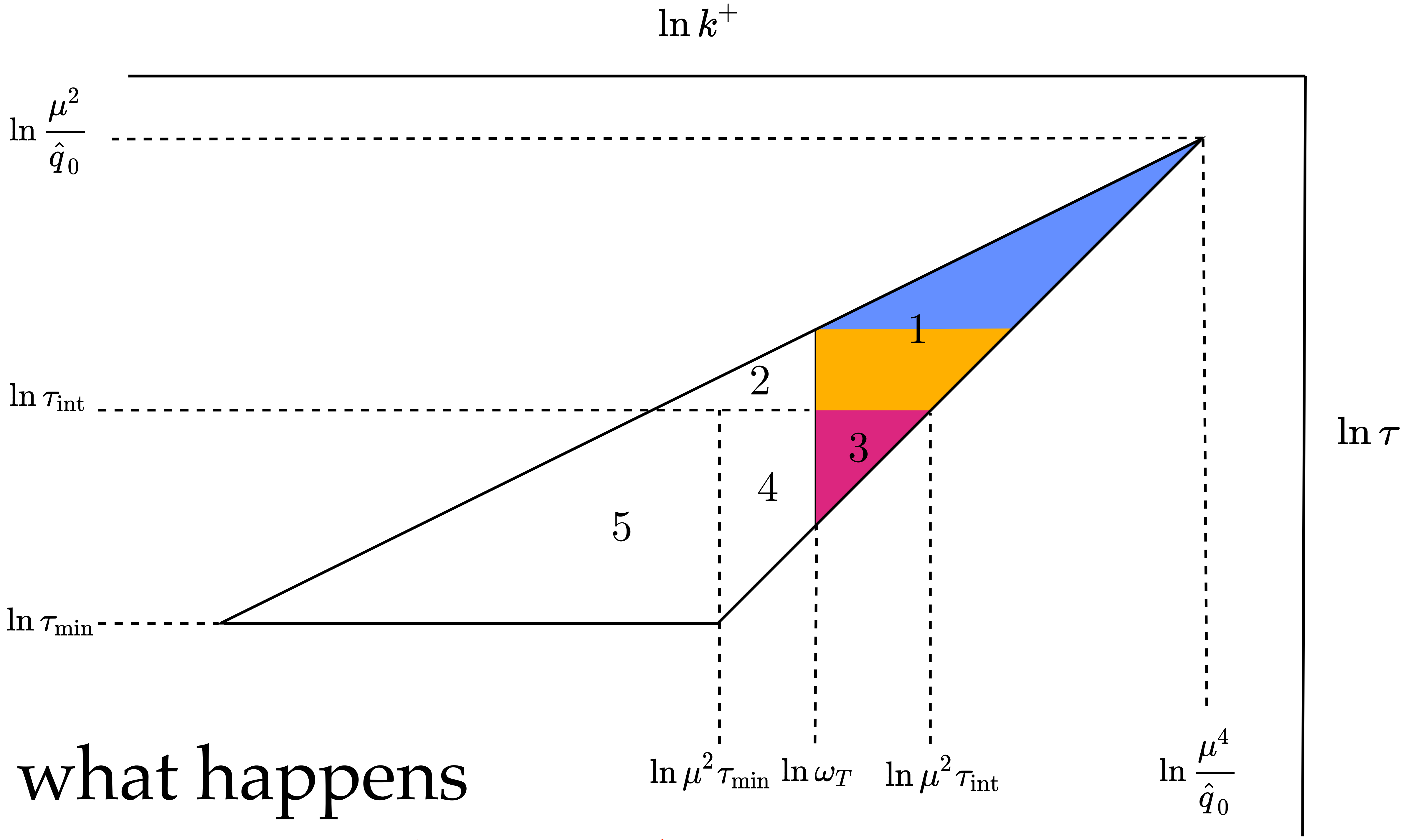
$$\omega_T = 2\pi e^{-\gamma_E} T$$

- When $k^+ < T$ $\quad n_{\text{B}}(k^+ \ll T) \approx \frac{T}{k^+} - 1/2$
  - **log gets replaced** by power-law in τ_{int} from classical term
  - The contribution from the 2 triangle gets subtracted off from the 1+2 triangle



- At DLA all still a matter of areas of triangles, what happens left of $\omega_T \sim T$ is not double-log enhanced but power-law $(1/g)$ enhanced
- Need to sort out regulator dependence and classical terms

The few-scattering regime

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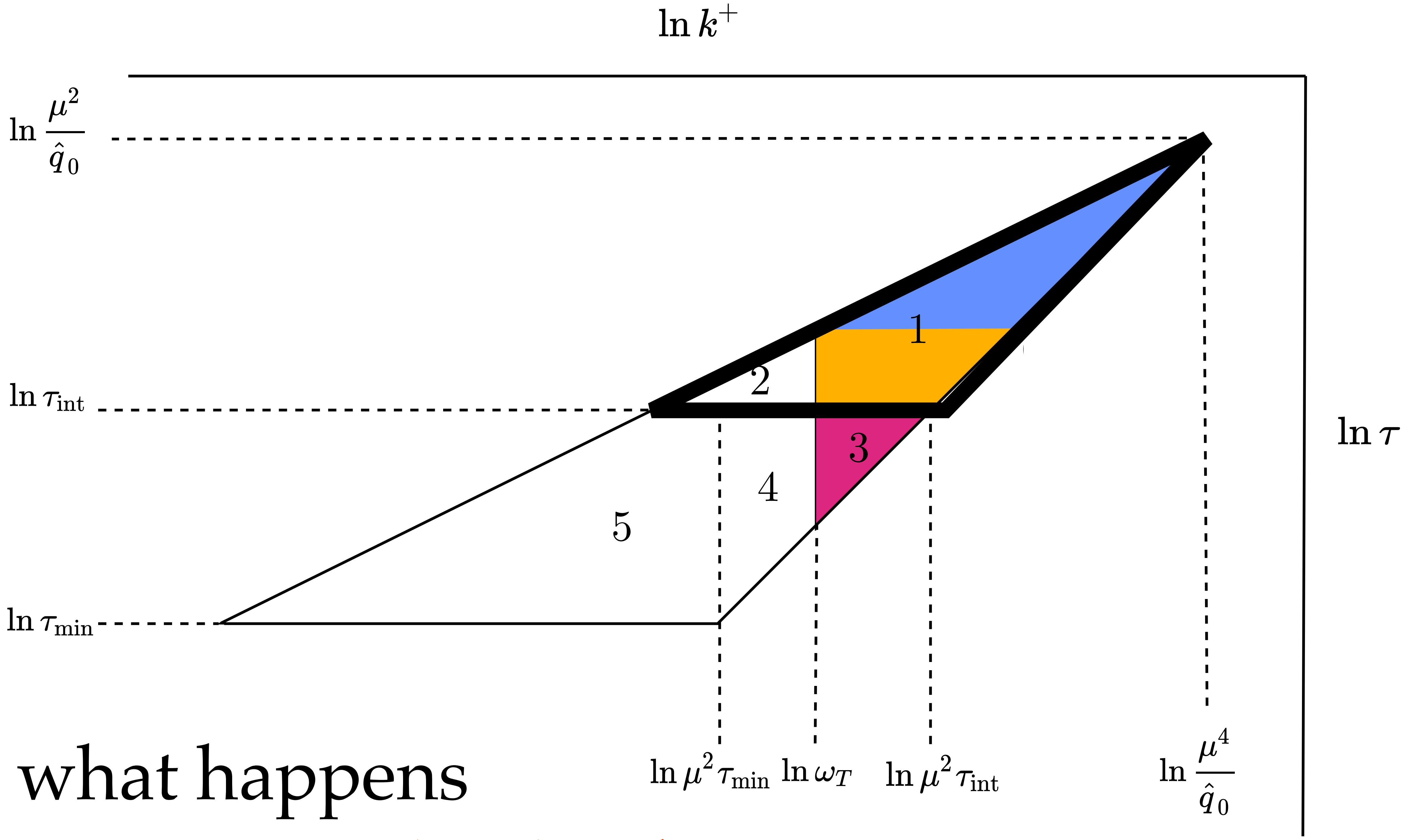
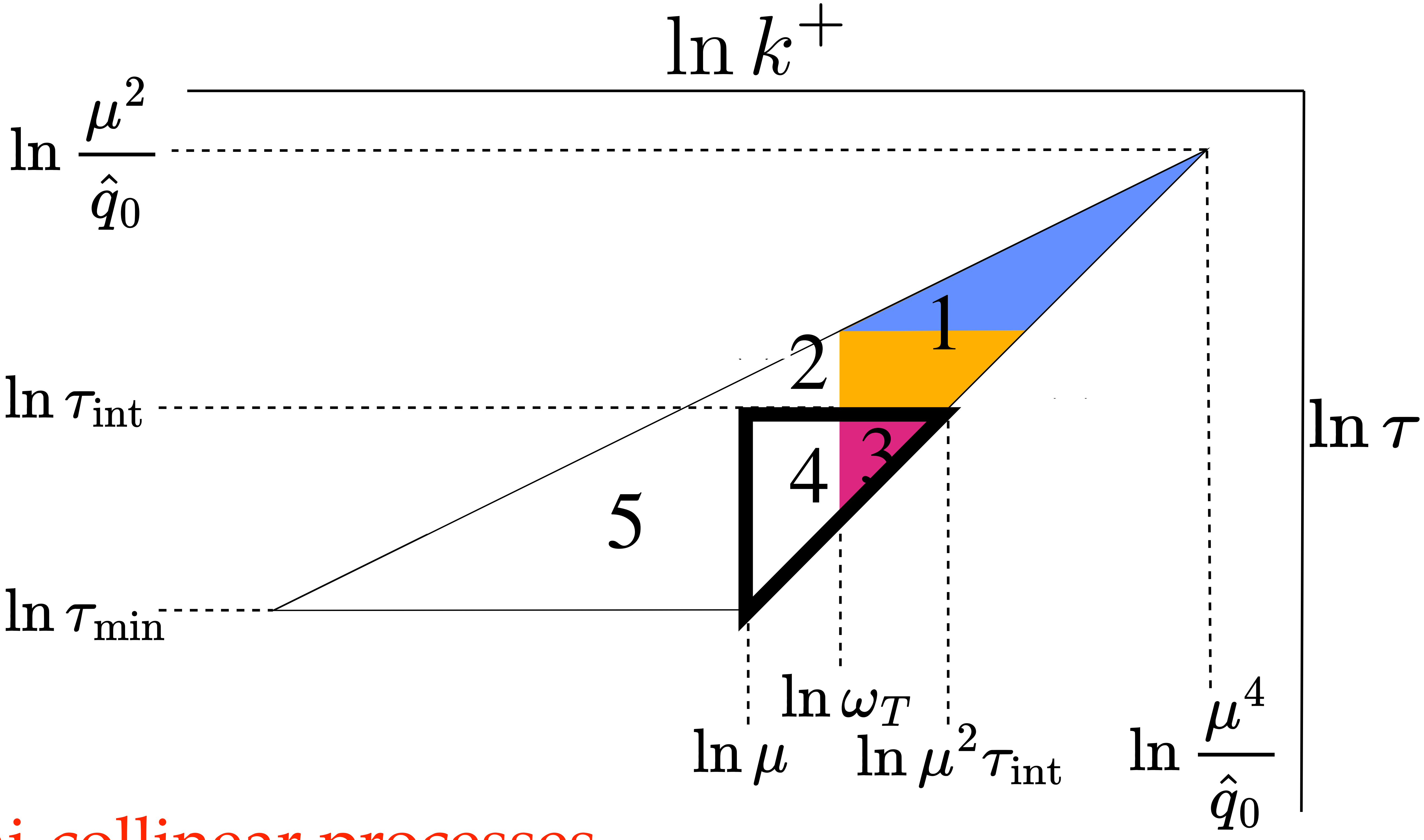


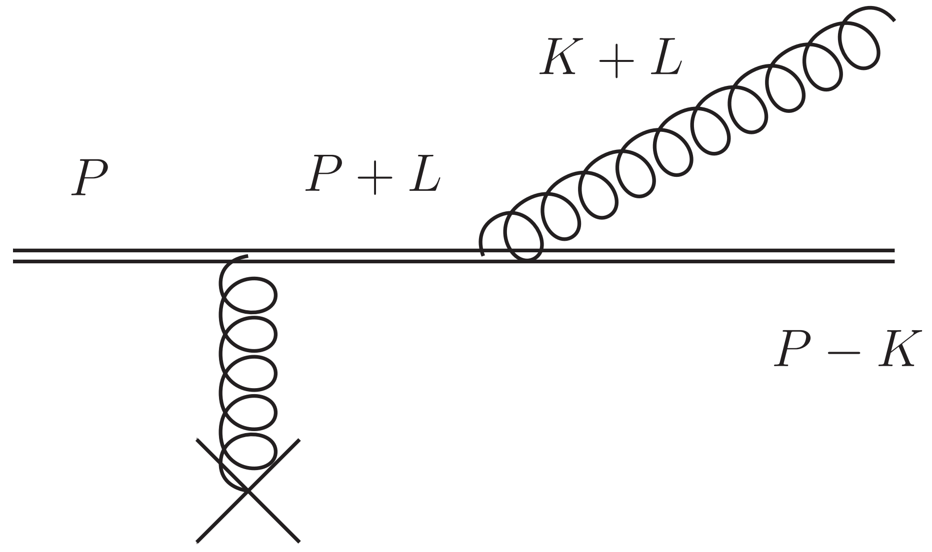
- At DLA all still a matter of areas of triangles, what happens left of $\omega_T \sim T$ is not double-log enhanced but power-law $(1/g)$ enhanced
- Need to sort out regulator dependence and classical terms

The single-scattering regime

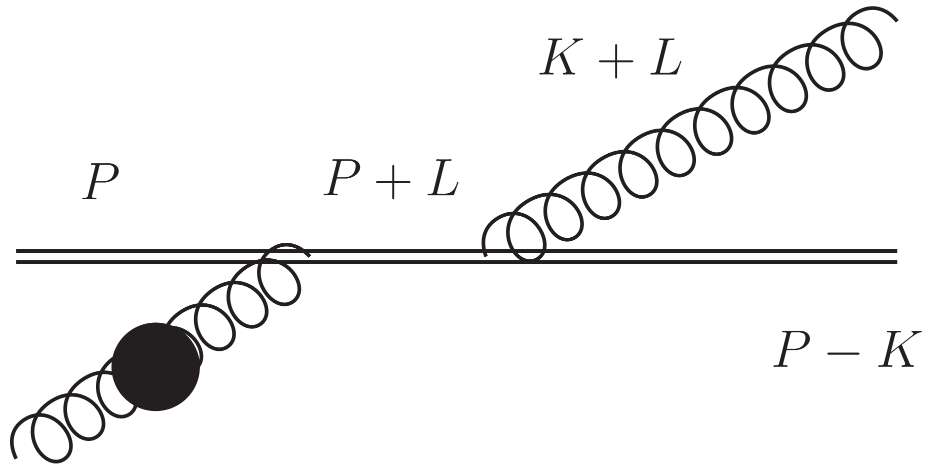
- Consider for illustration $gT < \mu < T$
- Magenta: $\tau < \tau_{\rm int} < 1/g^2T$, genuine single soft scattering regime
- Here the **formation time overlaps with the duration** ($\sim 1/gT$) of the soft scattering. Need to go beyond instantaneous approximation



- **Regions 3+4** can be dealt with using semi-collinear processes



soft scattering

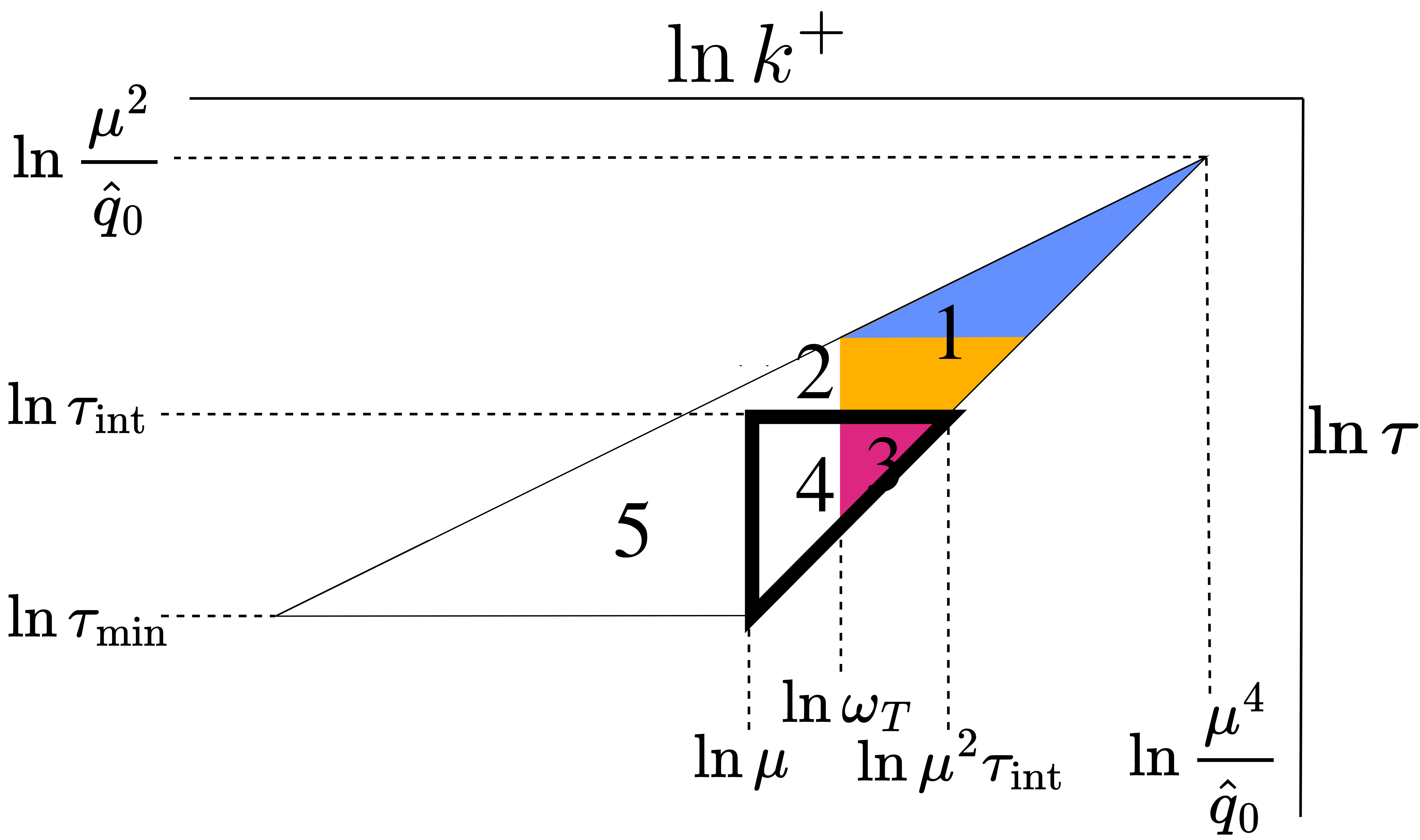


soft time-like plasmon

JG Hong Kurkela Lu Moore Teaney **JHEP1305** (2013)
JG Moore Teaney **JHEP1603** (2015)

The single-scattering regime

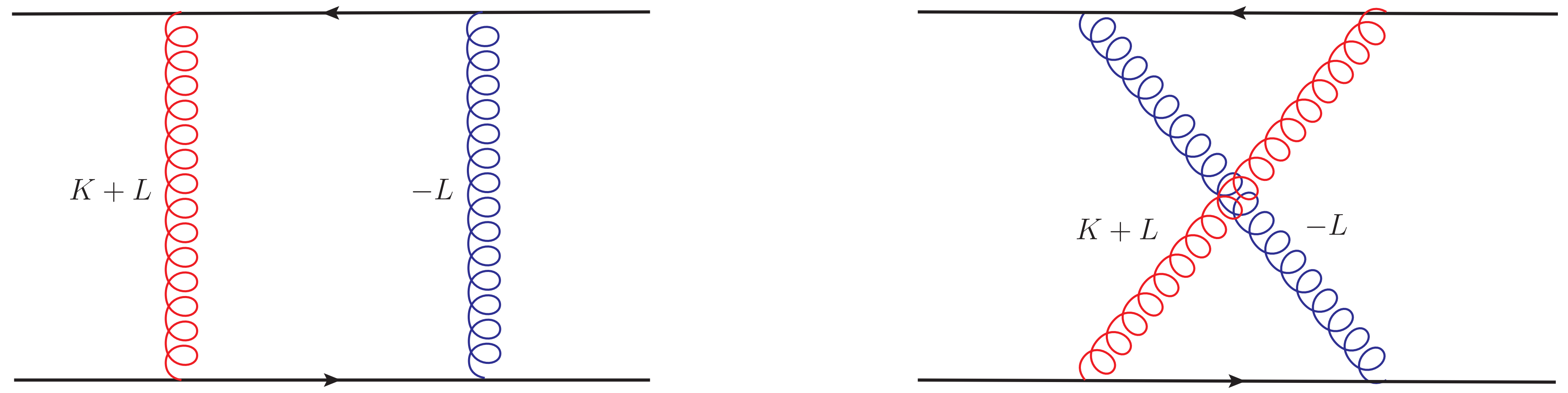
- **Regions 3+4** can be dealt with using semi-collinear processes, reduce again to EQCD for L integration JG Hong Kurkela Lu Moore Teaney **JHEP1305** (2013)
- Regulator-dependent (k^+_{IR}) classical contribution
- Double-log is area of triangle 3, corresponds to **instantaneous approx**
- Non harmonic, non-instantaneous subleading terms. First appearance of **Debye mass**

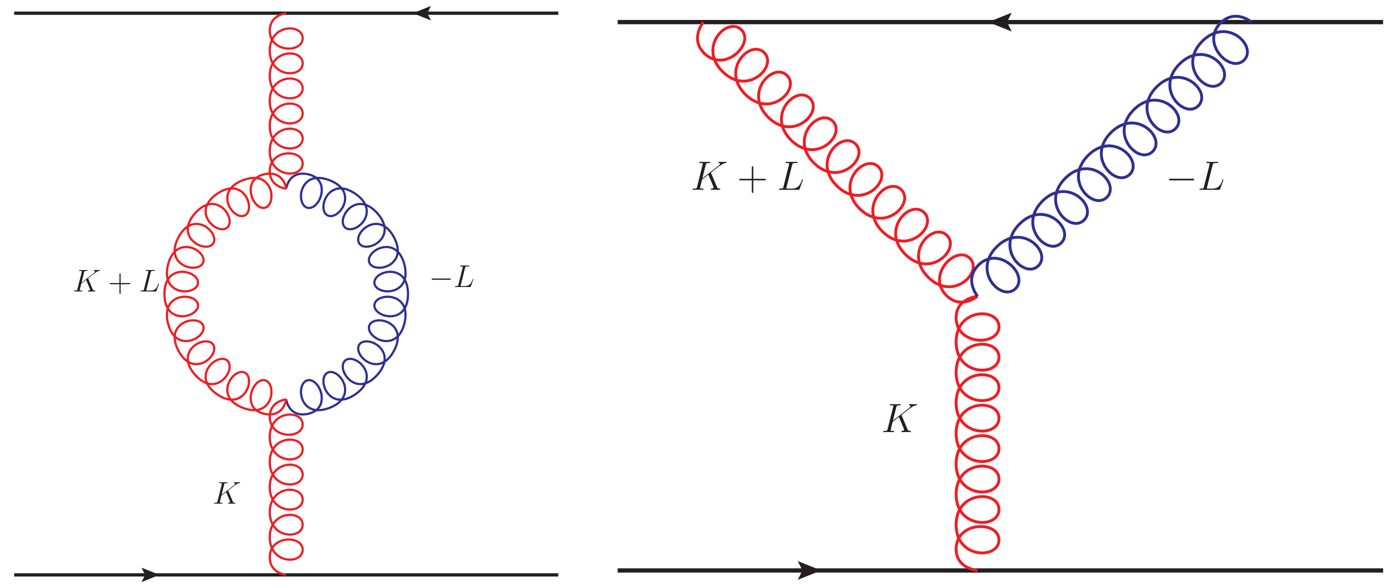


$$\delta\hat{q}^{\mathrm{single}} = \frac{\alpha_s C_R}{2\pi}\hat{q}_0\left\{\frac{4Tf(k^+_{\mathrm{IR}})}{k^+_{\mathrm{IR}}} + \ln^2\frac{\mu^2\tau_{\mathrm{int}}}{\omega_T}\right\} + \mathcal{O}\left(\alpha_s^3T^3\ln^3\frac{\mu^2}{m_DT}\right)$$

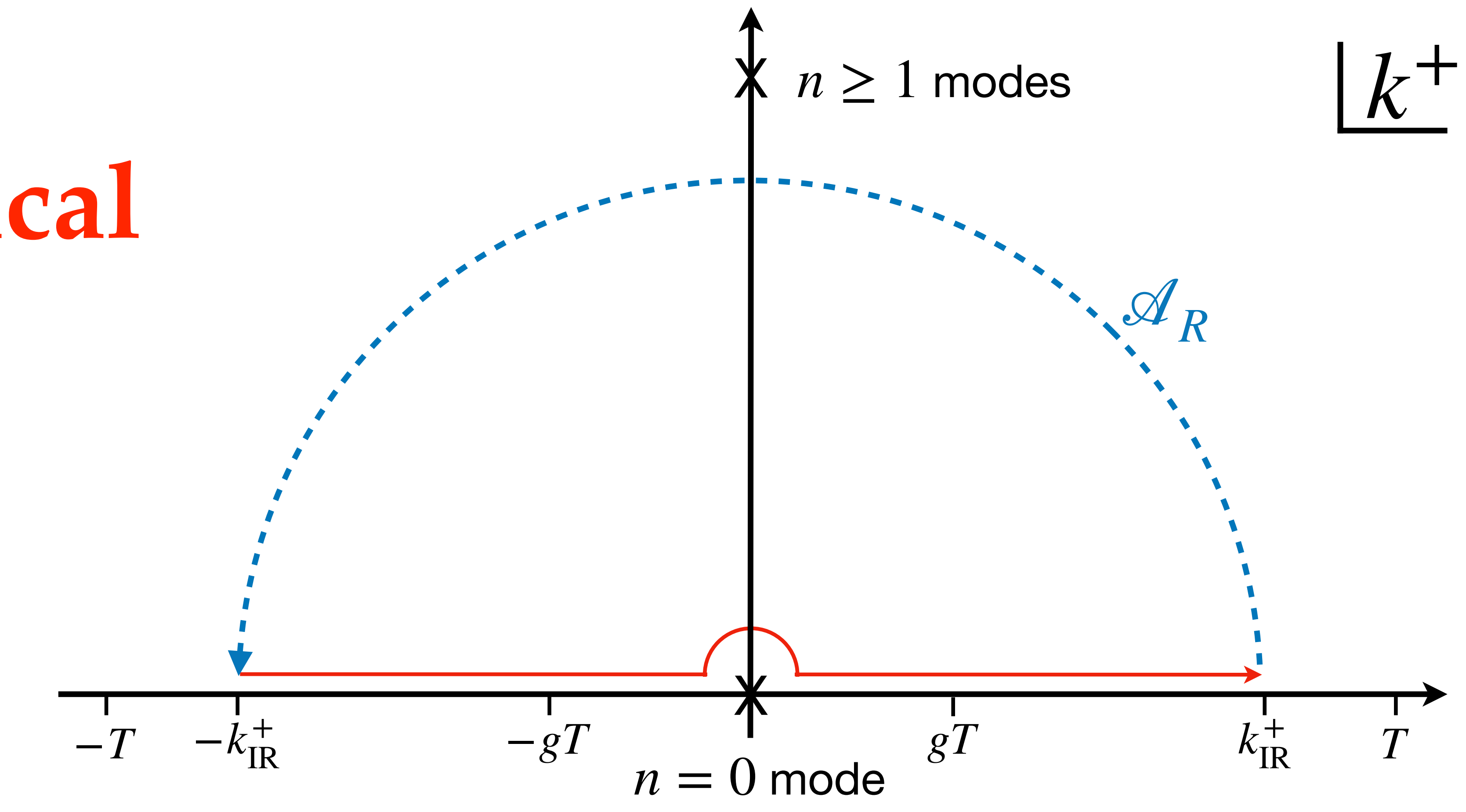
Connection to classical regime

- We computed these diagrams for $K \gtrsim T, K \gg L$





- Caron-Huot computed the same diagrams for $K \sim L \sim gT$
- $1/k^+_{\mathrm{IR}}$ regulator dependence cancels at the boundary. No double counting
- $n_{\mathrm{B}}(k^+ \ll T) \approx T/k^+ - 1/2$ naturally **switches off quantum corrections** and **turns them into the classical ones** *within the same diagrams*



Putting everything together

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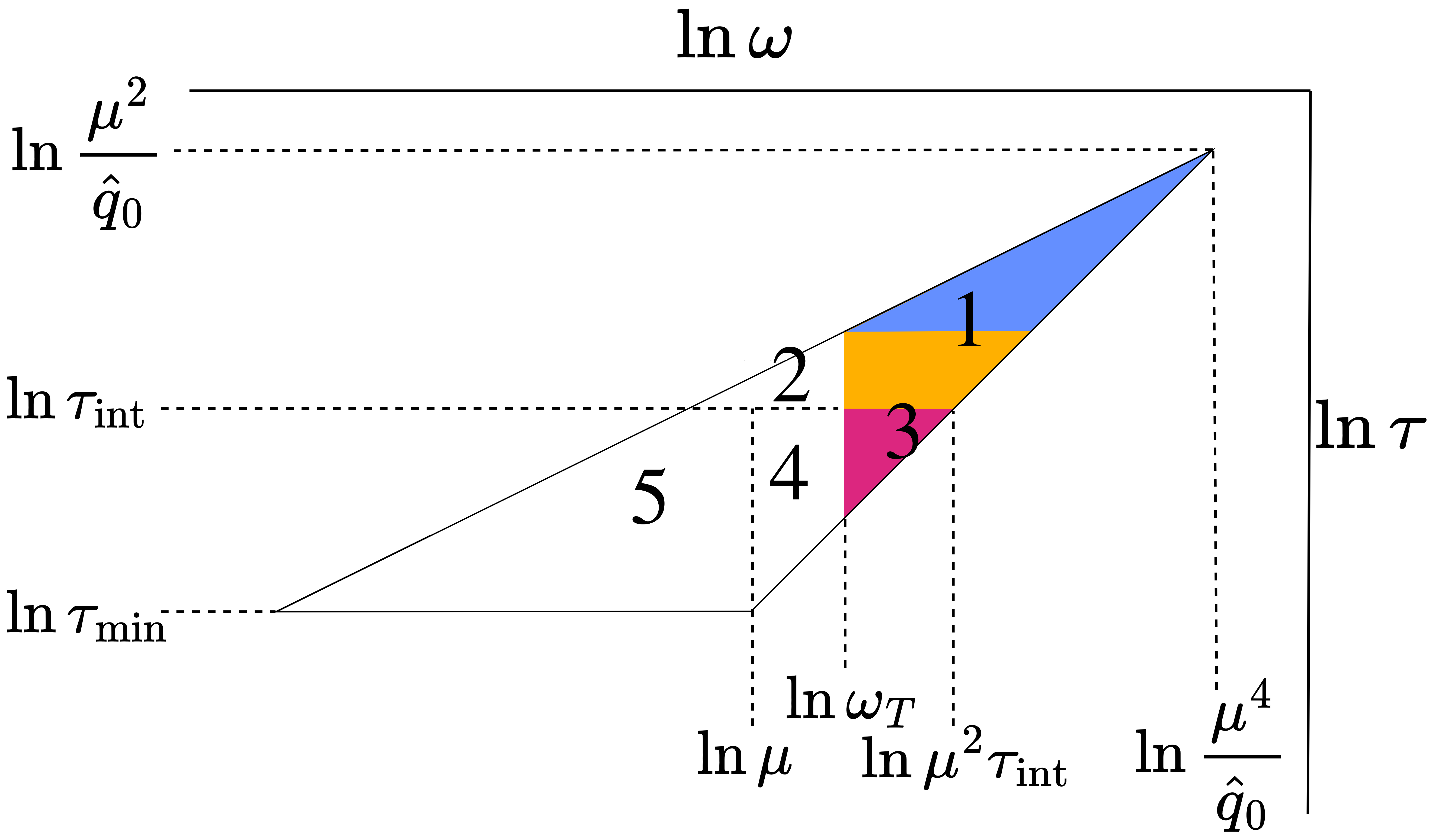

- To **double-log accuracy**

$$\delta\hat{q} = \delta\hat{q}^{\text{few}} + \delta\hat{q}^{\text{single}} = \frac{\alpha_s C_R}{4\pi}\hat{q}_0\ln^2\frac{\mu^4}{\hat{q}_0\omega_T}$$

- This corresponds to the area of 1+3, significant reduction from the original triangle

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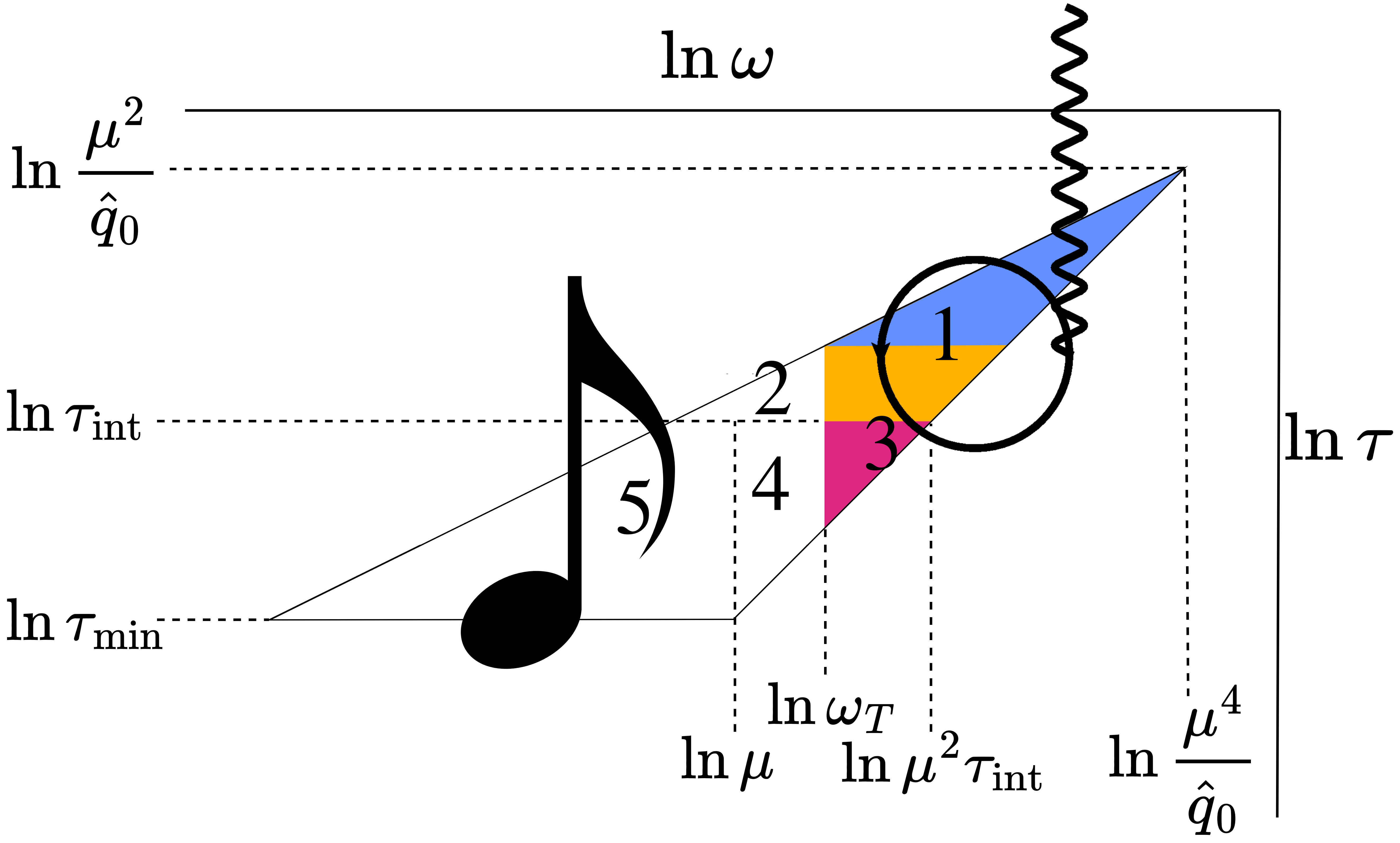

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Higher $\langle k_\perp^2 \rangle$: $\mu > T$

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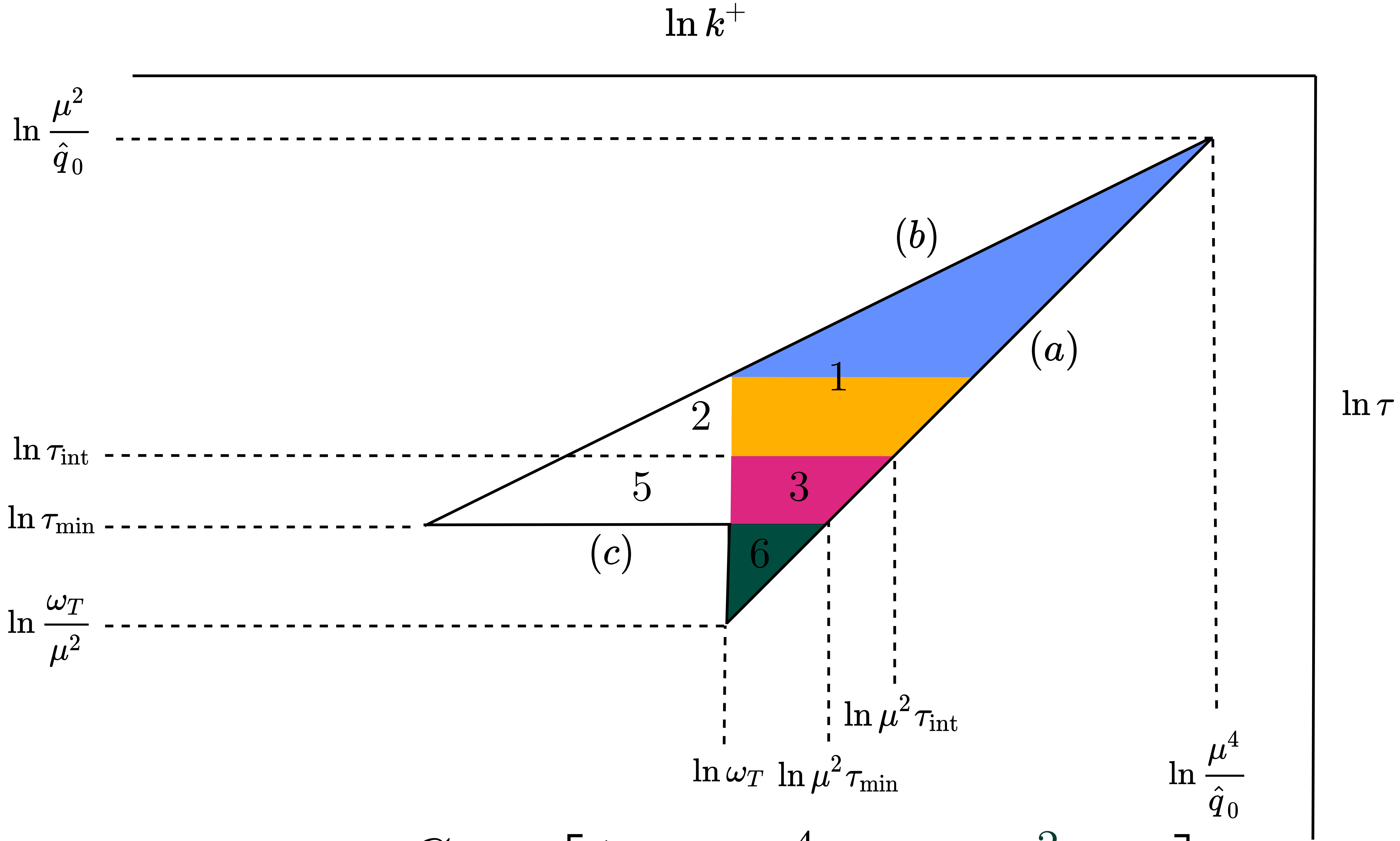
- Our approach can be extended here
- Larger $\langle l_\perp^2 \rangle$ semi-collinear rate unavailable
- Previous calculation still valid to DLA if we subtract triangle below $\tau_{\min}$



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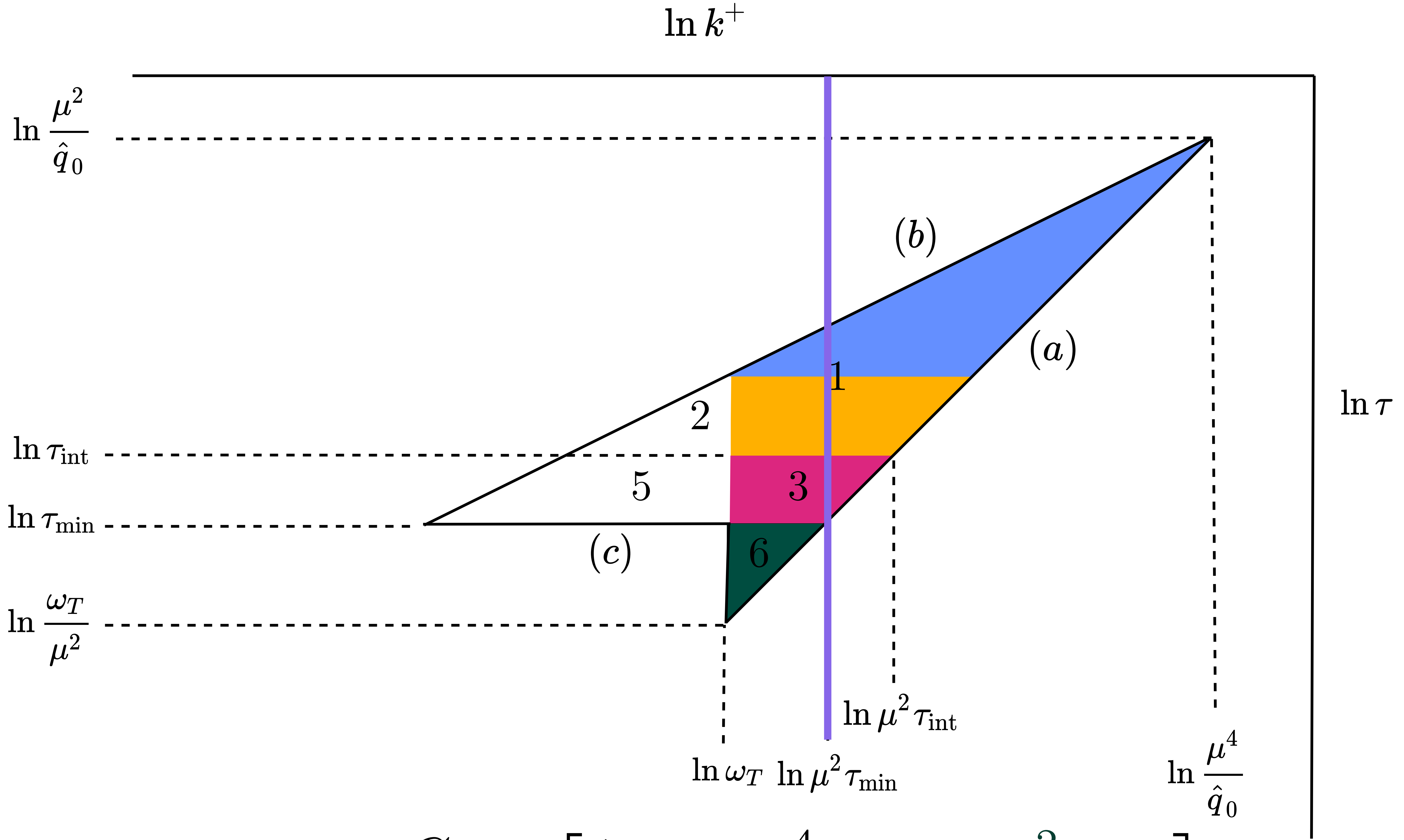
Higher $\langle k_\perp^2 \rangle$: $\mu > T$

- Our approach can be extended here
- Larger $\langle l_\perp^2 \rangle$ semi-collinear rate unavailable
- Previous calculation still valid to DLA if we subtract triangle below $\tau_{\rm min}$
- Difference with LMW / BDIM smaller. Vertical line cuts the original triangle in two halves of equal surface



$$\delta \hat{q} = \frac{\alpha_s C_R}{2\pi} \hat{q}_0 \left[\frac{1}{2} \ln^2 \frac{\mu^4}{\hat{q}_0 \omega_T} - \ln^2 \frac{\mu^2 \tau_{\rm min}}{\omega_T} \right]$$

Outlook: beyond DLA

$$\delta\hat{q} = \frac{\alpha_s C_R}{4\pi}\,\hat{q}_0 \ln^2 \frac{\mu^4}{\hat{q}_0\omega_T} + \dots$$

- Difficult to gauge impact of these double logs when single logs or smaller double logs are unavailable and the scale of $\hat{q}_0$ is unclear
- Way forward: we present a resummation equation for $\delta C(k_\perp)$, including all needed thermal effects, generalizing LMW and Iancu **JHEP10** (2014)
- Its solution would **smoothly interpolate** between single, few and many scatterings, shedding light on these issues by going beyond the harmonic oscillator approx
- Methods such as improved opacity expansion (Barata Mehtar-Tani Soto-Ontoso Tywoniuk **JHEP09** (2021)) or numerics of Andres *et al* **JHEP07** (2020), **JHEP03** (2021) Isaksen Tywoniuk **JHEP09** (2023) could be used

Conclusions

- The emergence of statistical functions in a weakly-coupled QCD seals off the low-frequency slice of the original LMW triangle to double logs
- There, double-log-enhanced quantum physics makes way to power-law enhanced classical physics
- These results can be used as low τ seed to the long-τ resummations of Caucal and Mehtar-Tani
- Evaluations beyond DLA could shed light on the hierarchy of classical and quantum corrections

Extra slides

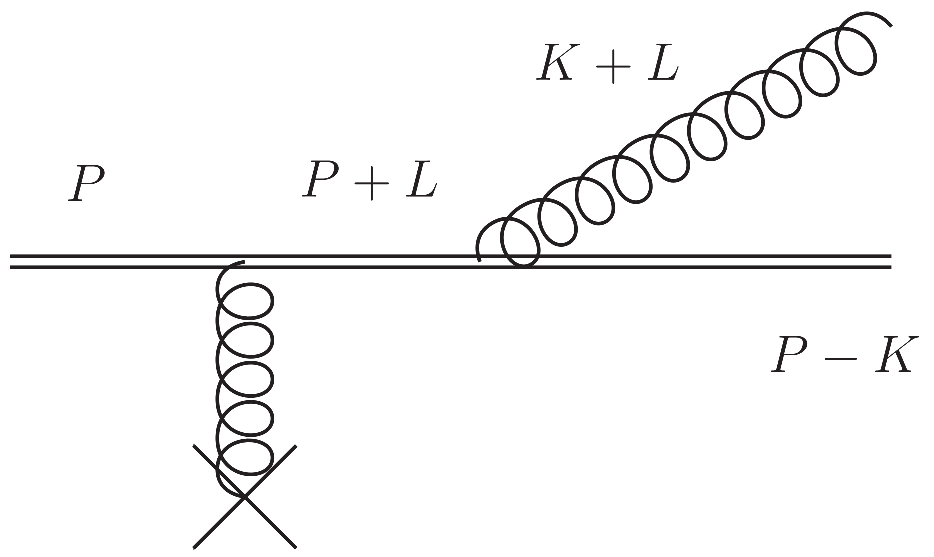
Vacuum-thermal cancellation

$$\nu_{\mathrm{IR}} \ll T \ll \nu_{\mathrm{UV}}$$

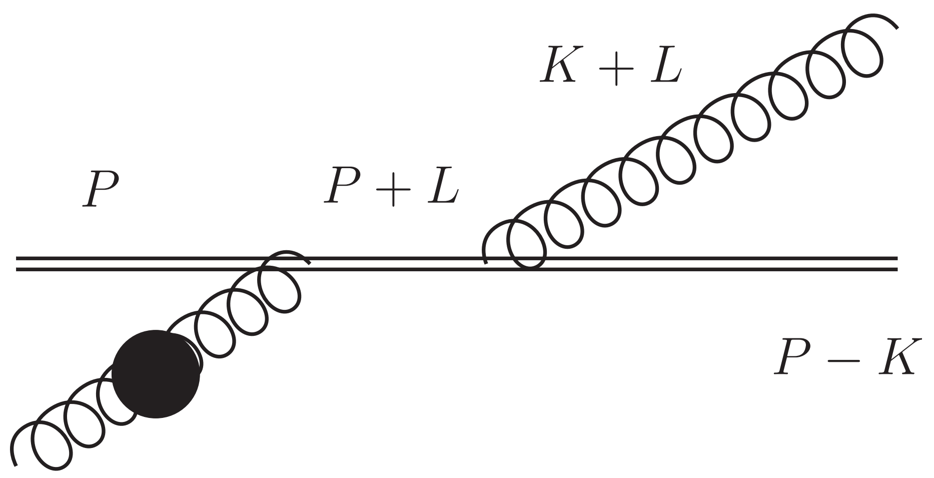
$$\int_{\nu_{\mathrm{IR}}}^{\nu_{\mathrm{UV}}} \frac{dk^+}{k^+}\Big(\underbrace{1}_{\text{vacuum}} + \underbrace{2n_{\mathrm{B}}(k^+)}_{\text{thermal}}\Big) = \underbrace{\ln\frac{\nu_{\mathrm{UV}}}{\nu_{\mathrm{IR}}}}_{\text{vacuum}} + \underbrace{\frac{2T}{\nu_{\mathrm{IR}}} - \ln\frac{2\pi T}{\nu_{\mathrm{IR}} e^{\gamma_E}} + \mathcal{O}\left(\frac{\nu_{\mathrm{IR}}}{T}, \exp(-\nu_{\mathrm{UV}}/T)\right)}_{\text{thermal}}$$

$$= \frac{2T}{\nu_{\mathrm{IR}}} + \ln\frac{\nu_{\mathrm{UV}} e^{\gamma_E}}{2\pi T} + \mathcal{O}\left(\frac{\nu_{\mathrm{IR}}}{T}, \exp(-\nu_{\mathrm{UV}}/T)\right)$$

Semi-collinear processes



soft scattering



soft time-like plasmon

$$\delta\mathcal{C}(k_\perp)_{\rm semi} = \frac{g^2 C_R}{\pi k_\perp^4}\int \frac{dk^+}{k^+}(1+n_{\rm B}(k^+))\hat{q}\left(\rho;\frac{k_\perp^2}{2k^+}\right)$$

$$\hat{q}(\rho;l^-) = g^2 C_A T\int^\rho \frac{d^2l_\perp}{(2\pi)^2}\frac{m_D^2 l_\perp^2}{(l_\perp^2+l^{-2})(l_\perp^2+l^{-2}+m_D^2)},$$

$$\hat{q}(\rho;l^-)_{\rm subtr} = \alpha_s C_A T\Big\{\underbrace{m_D^2\ln\left(\frac{\rho^2}{m_D^2}\right)}_{\rm HO}\underbrace{-l^{-2}\ln\left(1+\frac{m_D^2}{l^{-2}}\right)-m_D^2\ln\left(1+\frac{l^{-2}}{m_D^2}\right)}_{l^- -\text{dependent}}\Big\}$$

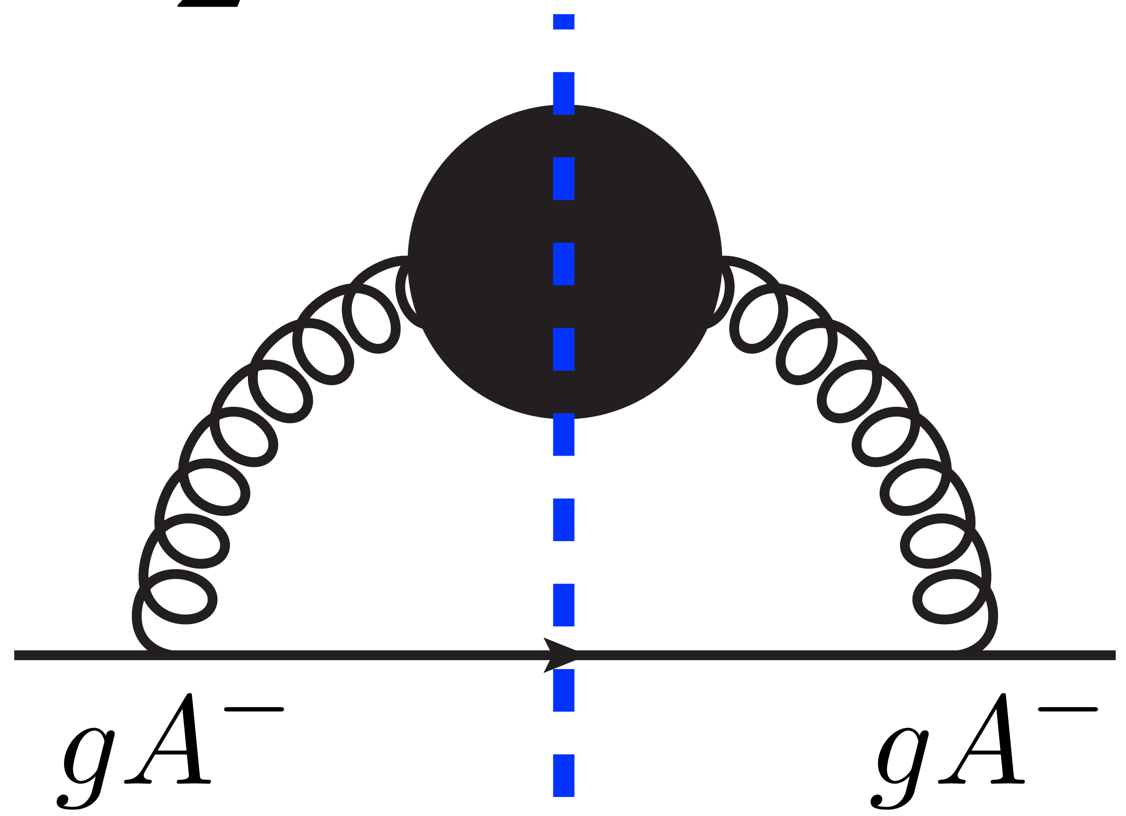
The resummation equation

$$\delta\mathcal{C}(x_\perp) = -2\alpha_s C_R \mathrm{Re}\int \frac{dk^+}{k^{+3}}\left(\frac{1}{2}+n_\mathrm{B}(k^+)\right)\int_0^{L_\mathrm{med}} d\tau\,\nabla_{\boldsymbol{B}_{2\perp}}\cdot\nabla_{\boldsymbol{B}_{1\perp}}\left[\tilde{G}(\boldsymbol{B}_{2\perp},\boldsymbol{B}_{1\perp};\tau)-\mathrm{vac}\right]\Bigg|^{\boldsymbol{B}_{2\perp}=\boldsymbol{x}_\perp,\boldsymbol{B}_{1\perp}=0}_{\boldsymbol{B}_{2\perp}=0,\boldsymbol{B}_{1\perp}=0}$$

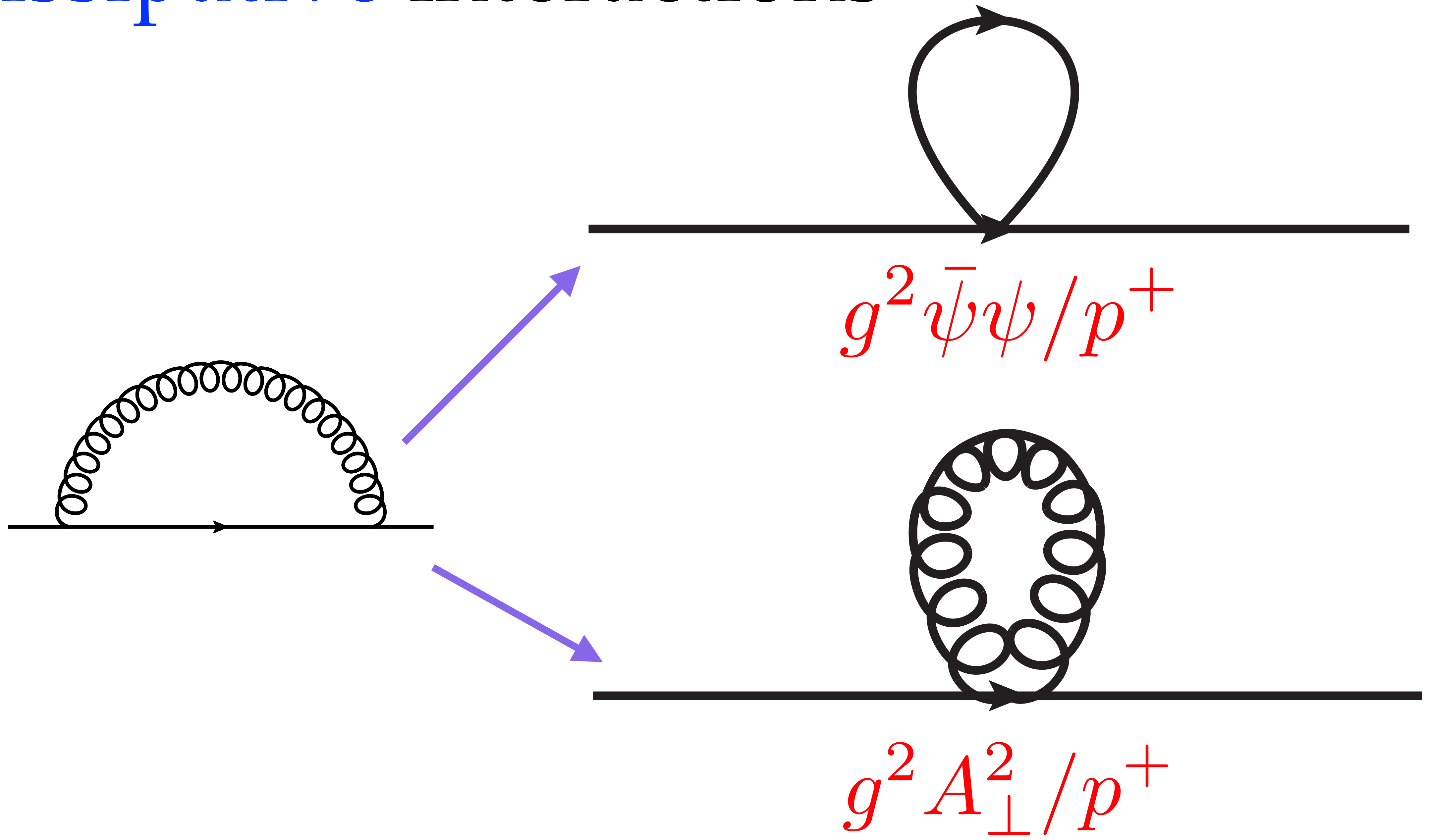
$$\left\{i\partial_\tau + \frac{\nabla^2_{B_\perp}-m^2_{\infty\,g}}{2k^+} + \frac{i}{2}\big(\mathcal{C}_g(B_\perp)+\mathcal{C}_g(|\boldsymbol{B}_\perp-\boldsymbol{x}_\perp|)-\mathcal{C}_g(x_\perp)\big)\right\}\tilde{G}(\boldsymbol{B}_\perp,\boldsymbol{B}_{1\perp};\tau)=0$$

Hard partons through the medium

- Imagine a hard quark propagating through a medium with $p^+ \equiv \dfrac{p^0 + p^z}{2} \gg T$. Dispersive and dissipative interactions



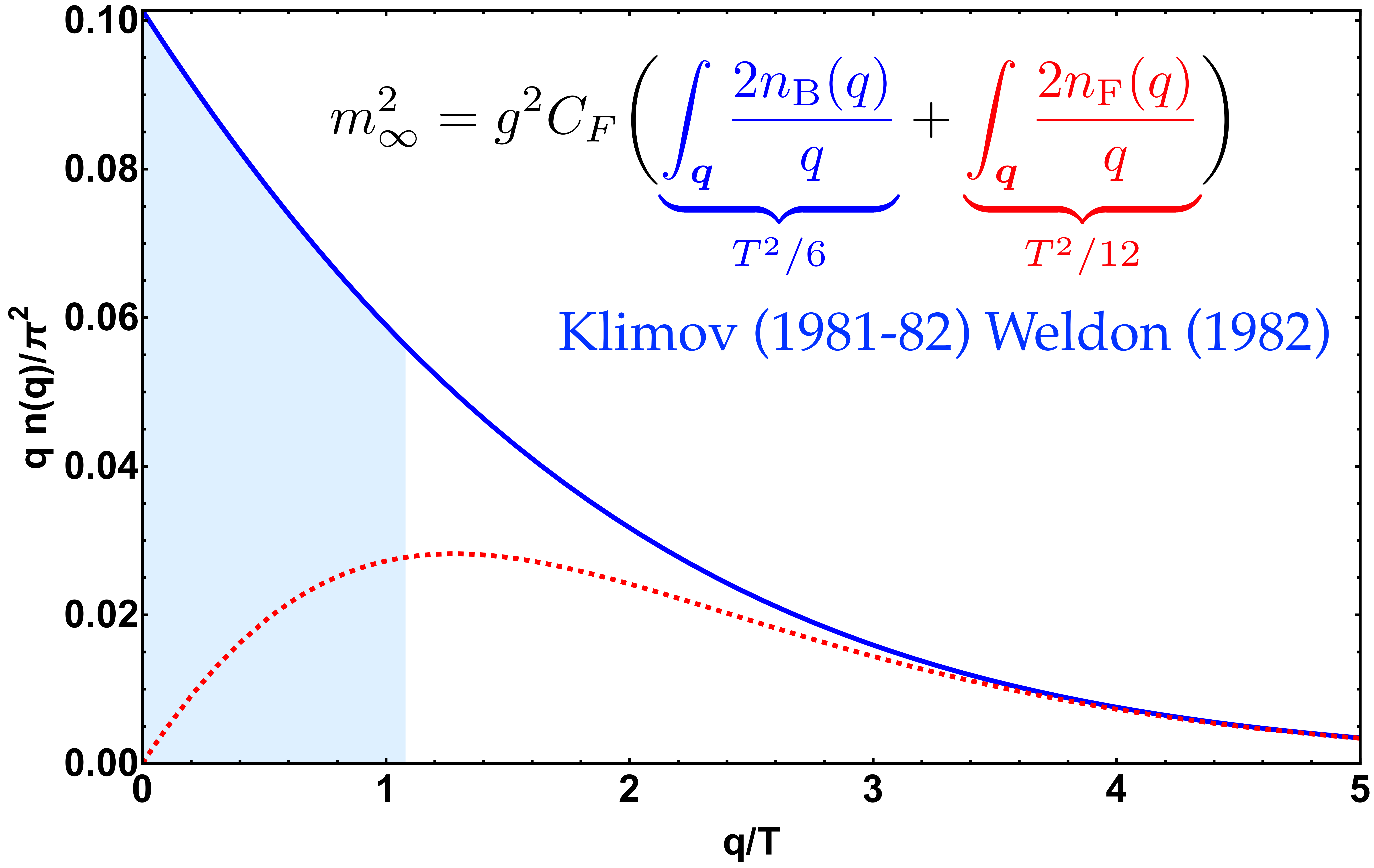
$$\mathcal{C}(k_\perp) \sim g^2 \int_Q G^{--}(Q)\delta(q^-)\delta^{(2)}(\boldsymbol{q}_\perp - \boldsymbol{k}_\perp)$$

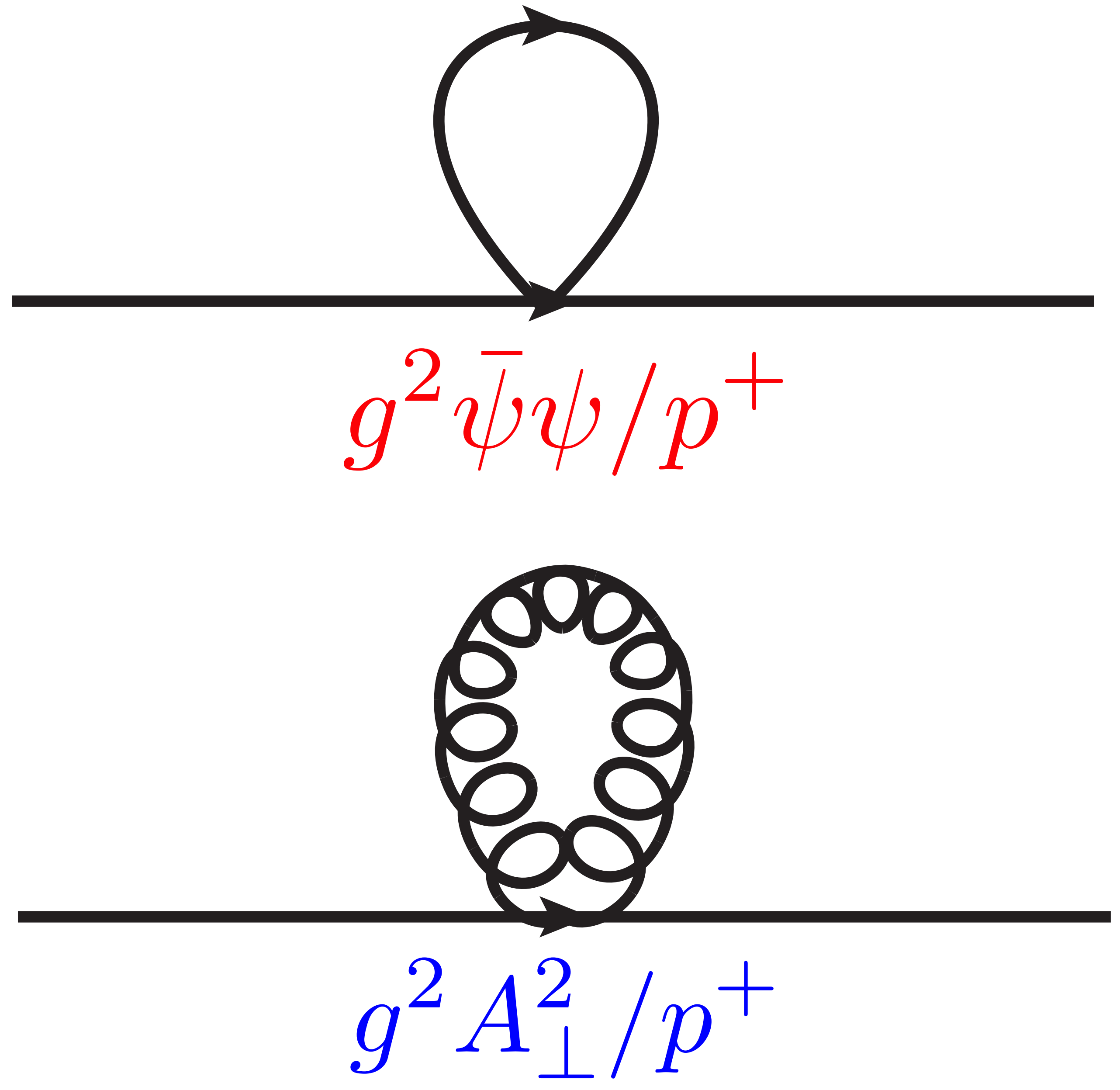


- The mass shift is then $m_\infty^2 = g^2T^2/3$ for a hard quark close to the mass shell

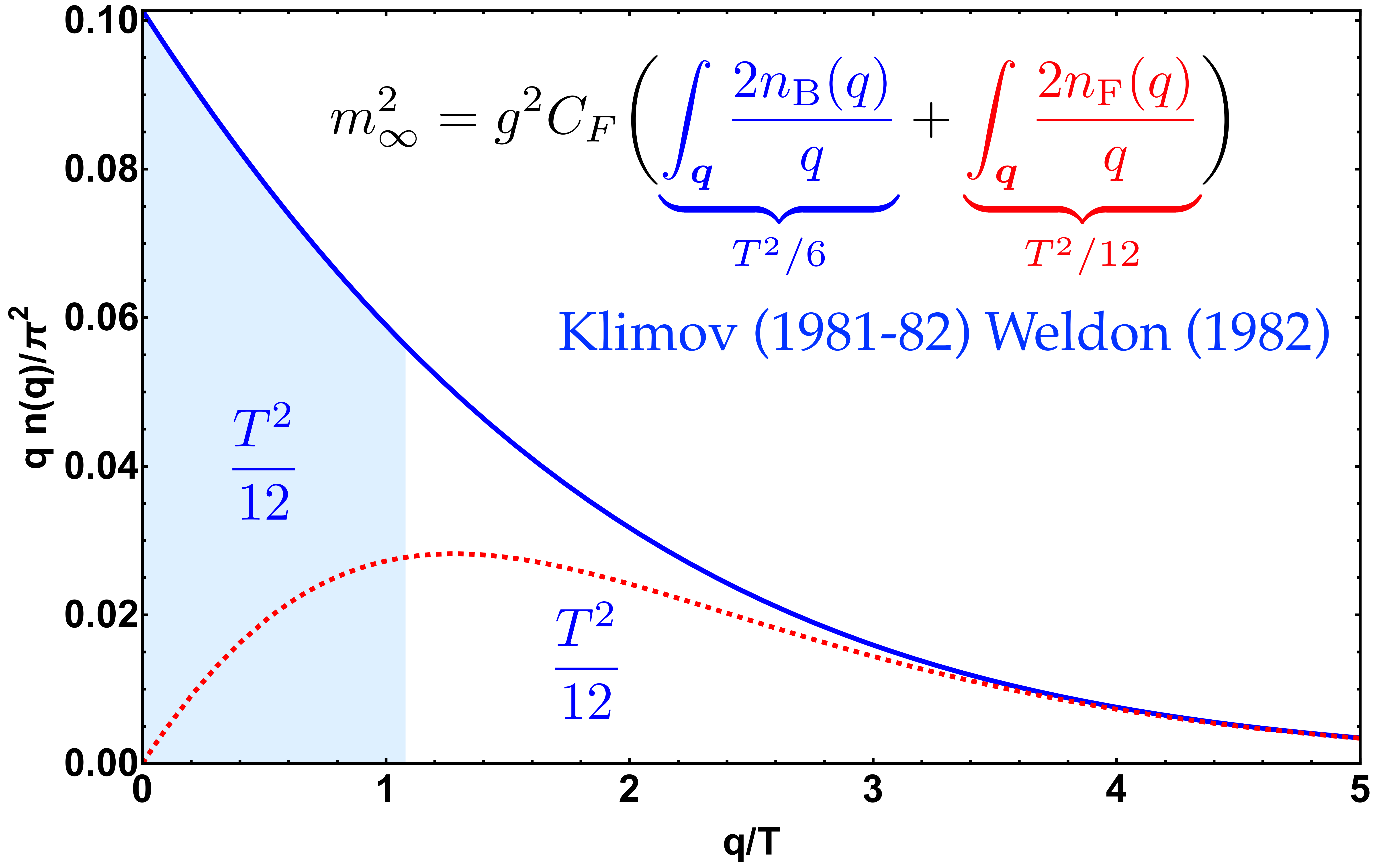
Klimov (1981-82) Weldon (1982)

The asymptotic mass





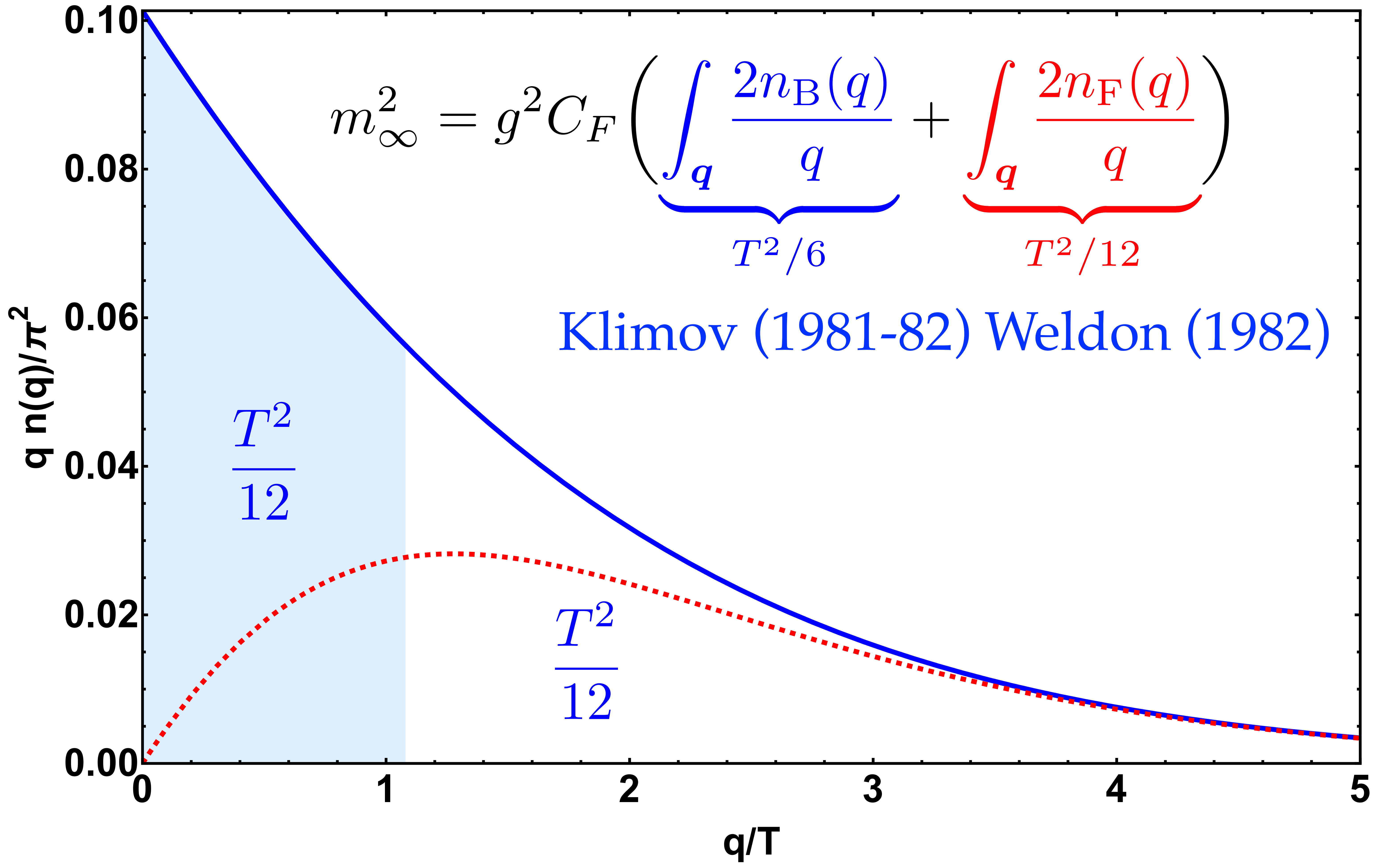
Classical gluons and the asymptotic mass



$$n_B(q \ll T) \approx \frac{T}{q}$$

- Half of the bosonic integral comes from the $q \lesssim T$ region

Classical gluons and the asymptotic mass



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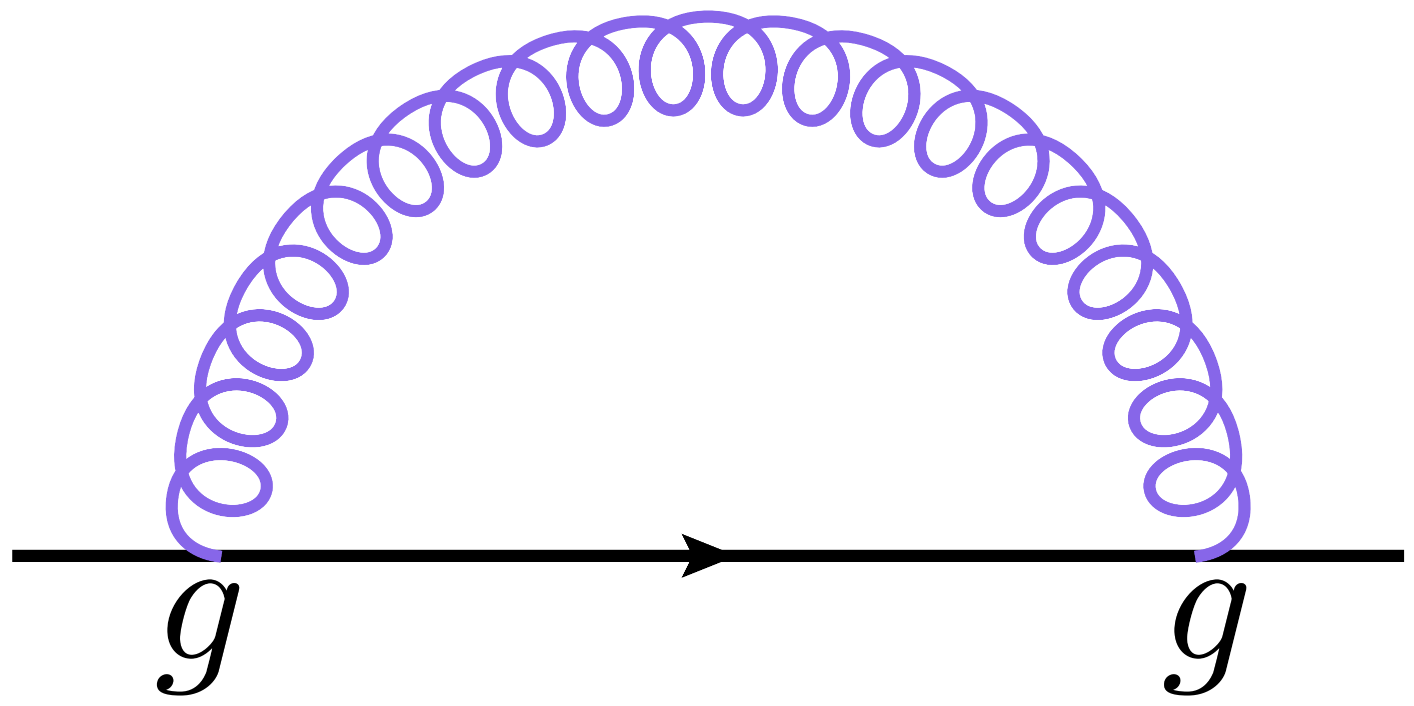
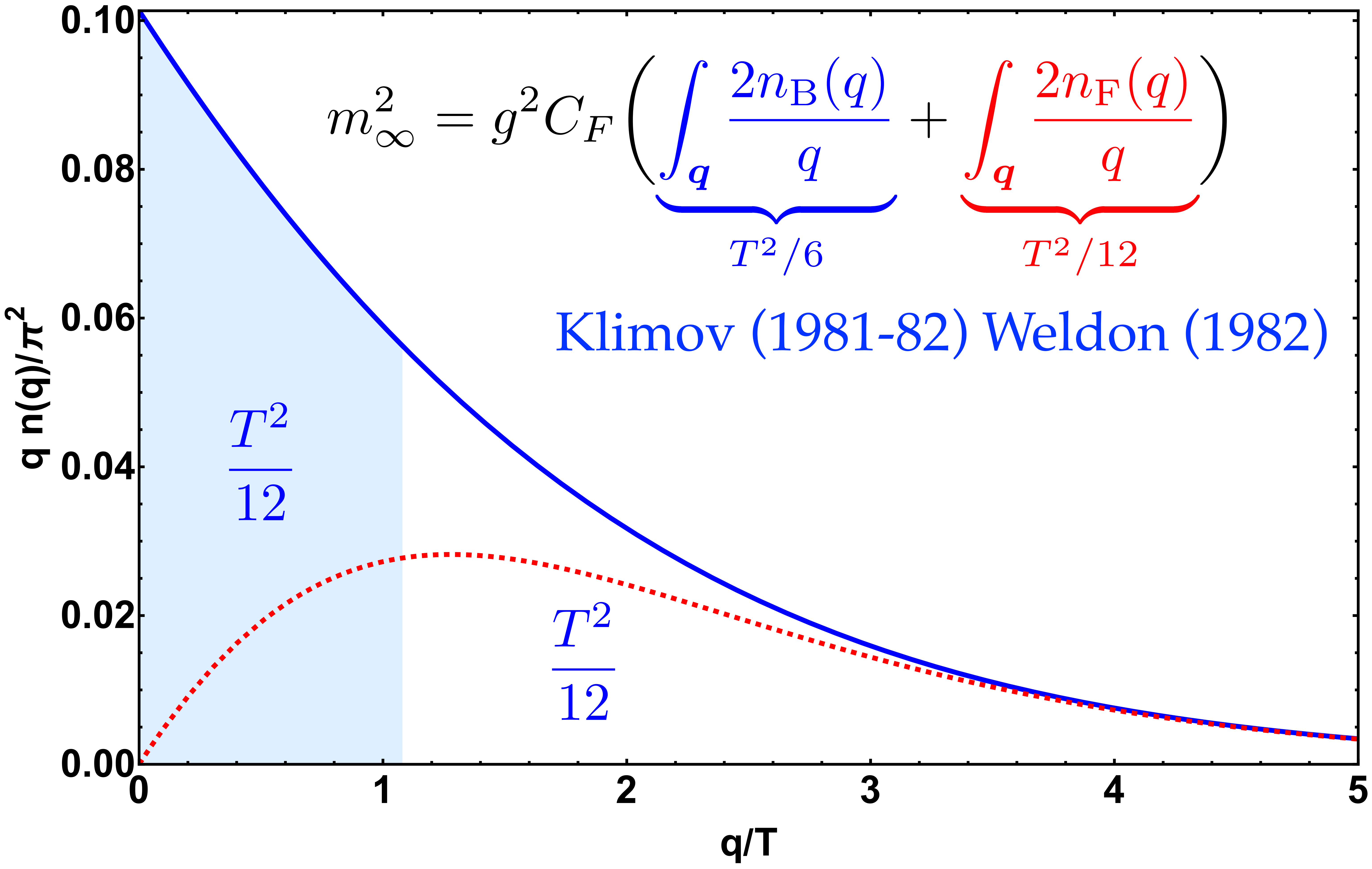
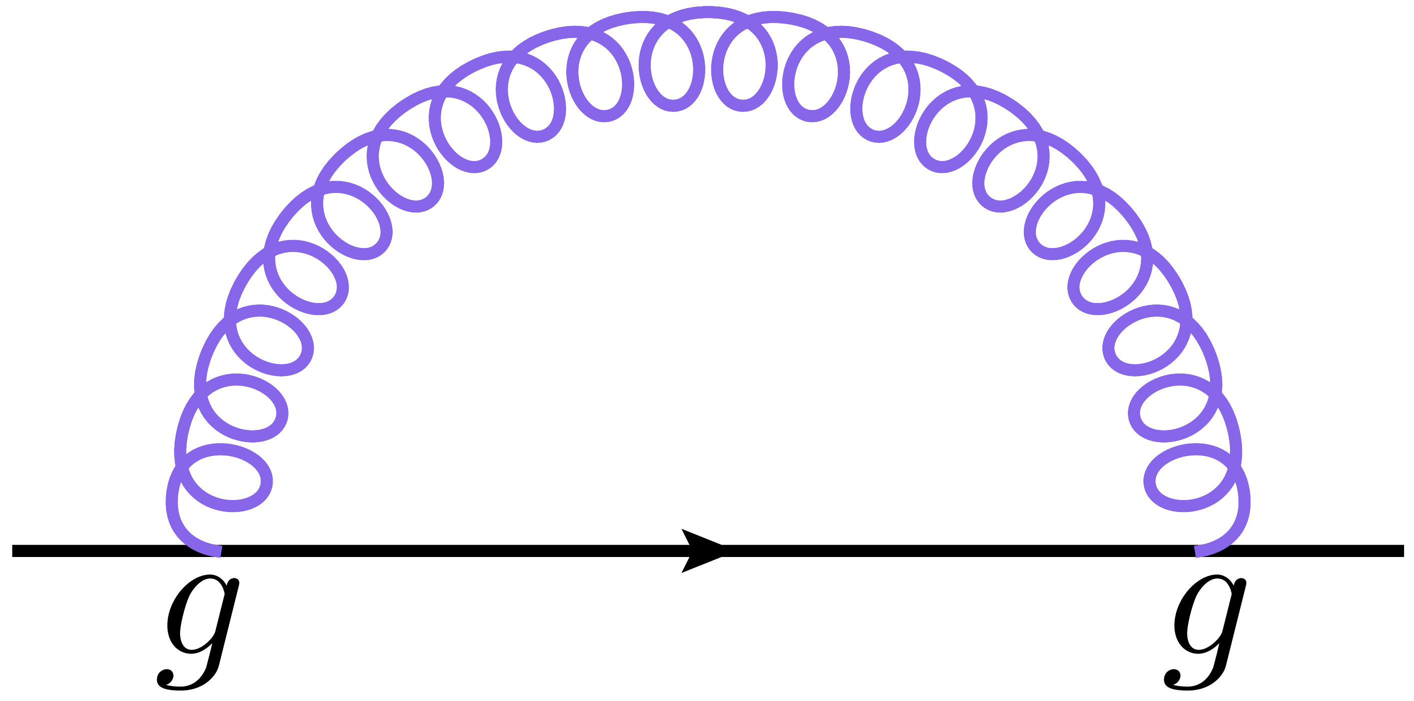


- We can then expect large contributions from soft classical gluons

Classical gluons and the asymptotic mass



$$n_B(q \ll T) \approx \frac{T}{q}$$

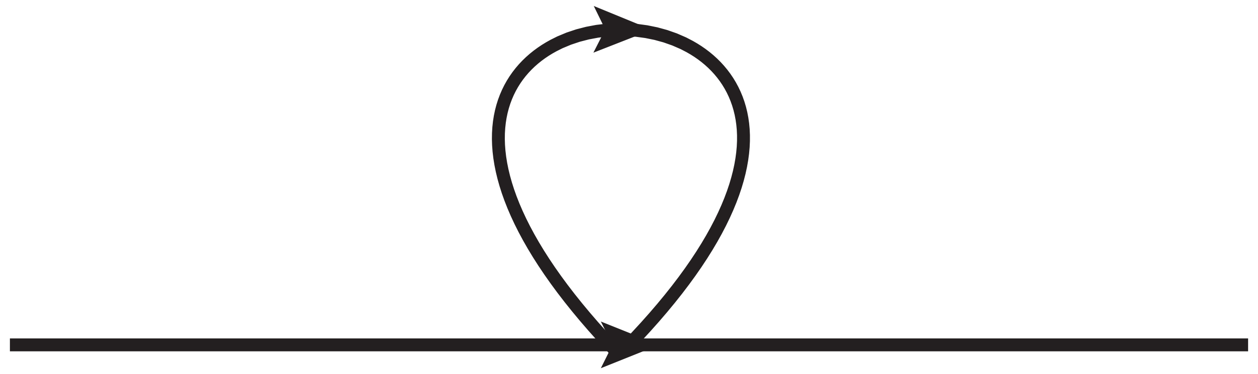


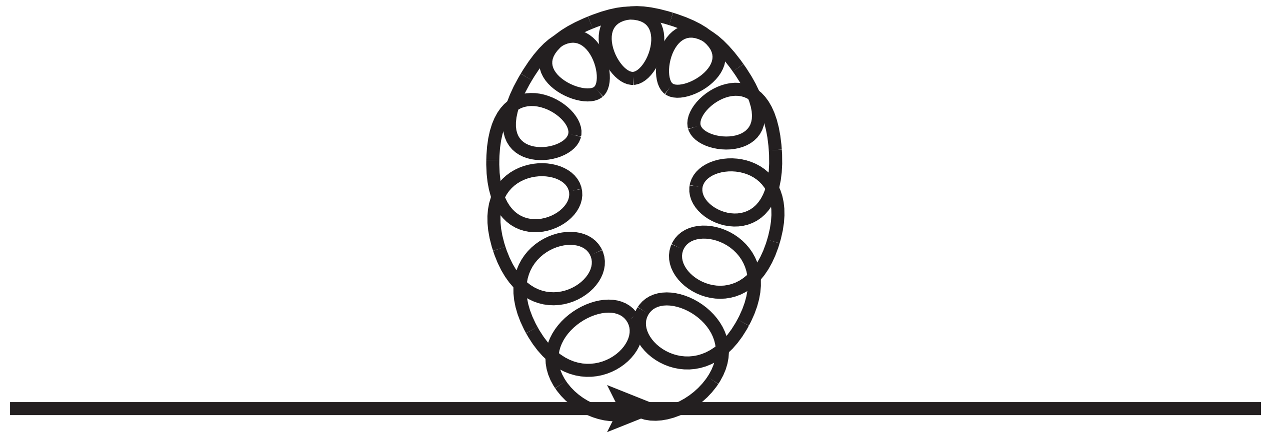
- For $q \lesssim gT$ this contribution becomes non-perturbative, $g^2 n_B(q) \sim 1$

The asymptotic mass, non-perturbatively

$$m_\infty^2 = g^2 C_F \Bigg(\underbrace{\int_{\boldsymbol{q}} \frac{2n_{\rm B}(q)}{q}}_{T^2/6} + \underbrace{\int_{\boldsymbol{q}} \frac{2n_{\rm F}(q)}{q}}_{T^2/12} \Bigg)$$

$$= g^2 C_F \left(Z_g + Z_f \right) + \mathcal{O}(1/p^+)$$



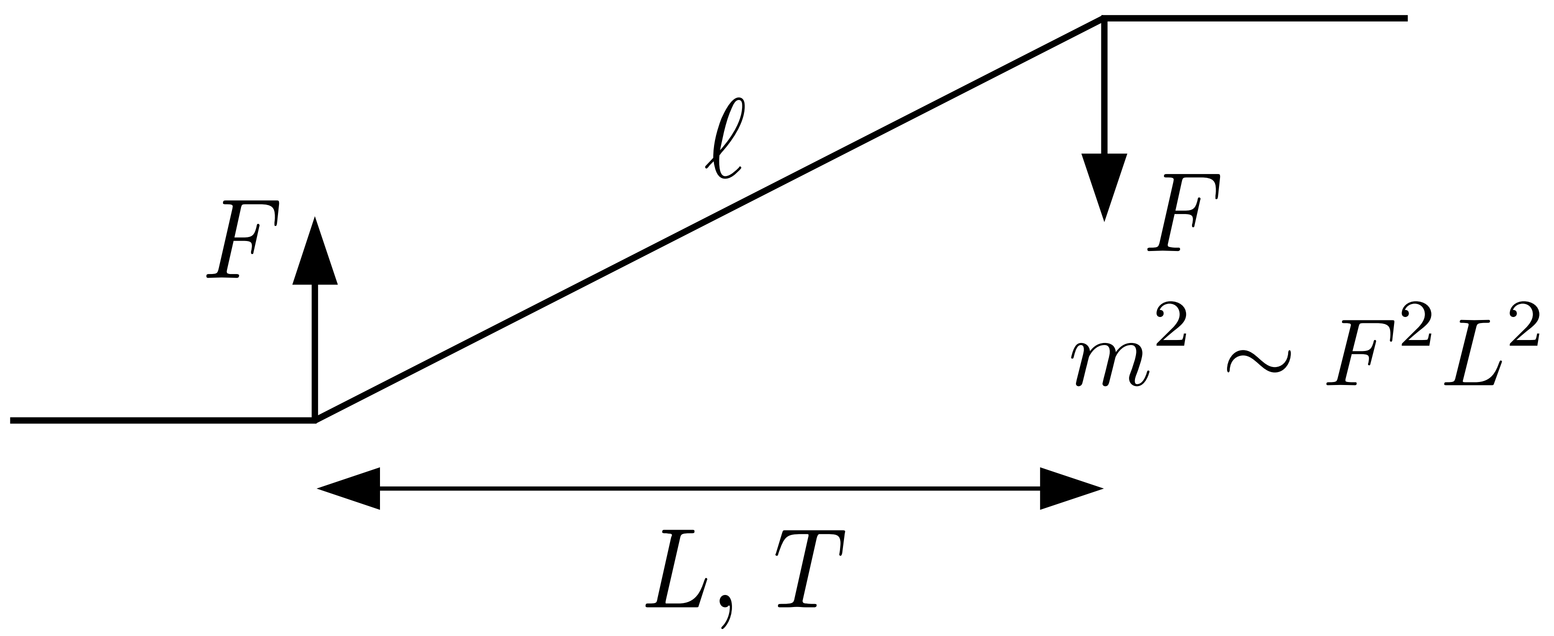


- From Feynman diagrams to EFT operators, concentrate on $Z_{\rm g}$

$$Z_{\rm f} \equiv \frac{1}{2d_R} \left\langle \bar{\psi} \frac{\not{v}}{v \cdot D} \psi \right\rangle \quad \text{with } v^\mu = (1,0,0,1)$$

$$Z_{\rm g} \equiv \frac{1}{d_A} \left\langle v_\alpha F^{\alpha\mu} \frac{1}{(v \cdot D)^2} v_\nu F^\nu{}_\mu \right\rangle$$

Caron-Huot (2008)



Moore Schlusser (2020)

The asymptotic mass, non-perturbatively

- From Feynman diagrams to EFT operators, concentrate on $Z_{\rm g}$

$$Z_{\rm g} \equiv \frac{1}{d_A}\left\langle v_\alpha F^{\alpha\mu} \frac{1}{(v\cdot D)^2} v_\nu F^{\nu}{}_{\mu}\right\rangle$$
$$= \frac{2}{d_A}\int_0^\infty \mathrm{d}L\,L\,\mathrm{Tr}\left\langle U(-\infty;L)v_\alpha F^{\alpha\mu}(L)\,U(L;0)\,v_\nu F^{\nu}{}_{\mu}(0)U(0;-\infty)\right\rangle$$

- Breakthrough: soft classical modes at space-like separations become **Euclidean** and **time-independent**. Light-like limit possible, see main talk before for caveats in the case of $\hat{q}$.

- Horrible HTL perturbative calculation or extremely challenging 4D lattice on the light-cone become 3D Electrostatic QCD (EQCD). NLO $\delta Z_{\rm g} = -\dfrac{Tm_D}{2\pi}$

Caron-Huot (2008)

The asymptotic mass, non-perturbatively

- From Feynman diagrams to EFT operators, concentrate on Z_g

$$Z_g \equiv \frac{1}{d_A}\left\langle v_\alpha F^{\alpha\mu}\frac{1}{(v\cdot D)^2}v_\nu F^{\nu}{}_{\mu}\right\rangle$$
$$= \frac{2}{d_A}\int_0^\infty \mathrm{d}L L\,\mathrm{Tr}\left\langle U(-\infty;L)v_\alpha F^{\alpha\mu}(L)\,U(L;0)\,v_\nu F^{\nu}{}_{\mu}(0)U(0;-\infty)\right\rangle$$

- **Our strategy**: lattice EQCD for $L \gtrsim 1/m_D$, pQCD for $L \lesssim 1/m_D \sim 1/gT$
What does it mean in practice?

- Recently: continuum-extrapolated EQCD lattice data for the scattering kernel and merging with pQCD Moore Schlusser **PRD101** (2020) Moore Schlichting Schlusser Soudi **JHEP2110** (2021) Schlichting Soudi **PRD105** (2022)

EQCD

$$Z_{\rm g} = \frac{2}{d_A}\int_0^\infty {\rm d}L L\, {\rm Tr}\left\langle U(-\infty;L)v_\alpha F^{\alpha\mu}(L)\,U(L;0)\,v_\nu F^{\nu}{}_{\mu}(0)U(0;-\infty)\right\rangle$$

- EQCD is the *dimensionally-reduced* (3D) EFT for the classical modes, which correspond to the Euclidean zero modes. 3D SU(3) + adjoint Higgs ($A_0 \rightarrow \Phi$)

$$S_{\rm EQCD} = \int_{\vec{x}} \left\{ \frac{1}{2}{\rm Tr}\, F_{ij}F_{ij} + {\rm Tr}\,[D_i,\Phi][D_i,\Phi] + m_D^2 {\rm Tr}\,\Phi^2 + \lambda_E ({\rm Tr}\,\Phi^2)^2 \right\}$$

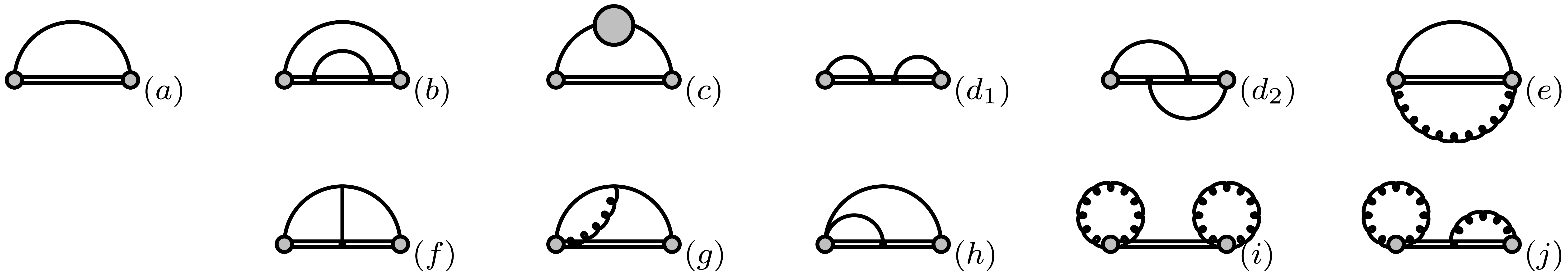
Kajantie Laine Rummukainen Shaposhnikov (1995-97) Braaten Nieto (1994-95)

- By putting EQCD on the lattice we can get the classical contribution non-perturbatively at all orders. But how?

EQCD

$$Z_{\rm g} = \frac{2}{d_A}\int_0^\infty \mathrm{d}L\,L\,\mathrm{Tr}\left\langle U(-\infty;L)v_\alpha F^{\alpha\mu}(L)\,U(L;0)\,v_\nu F^{\nu}{}_{\mu}(0)U(0;-\infty)\right\rangle$$

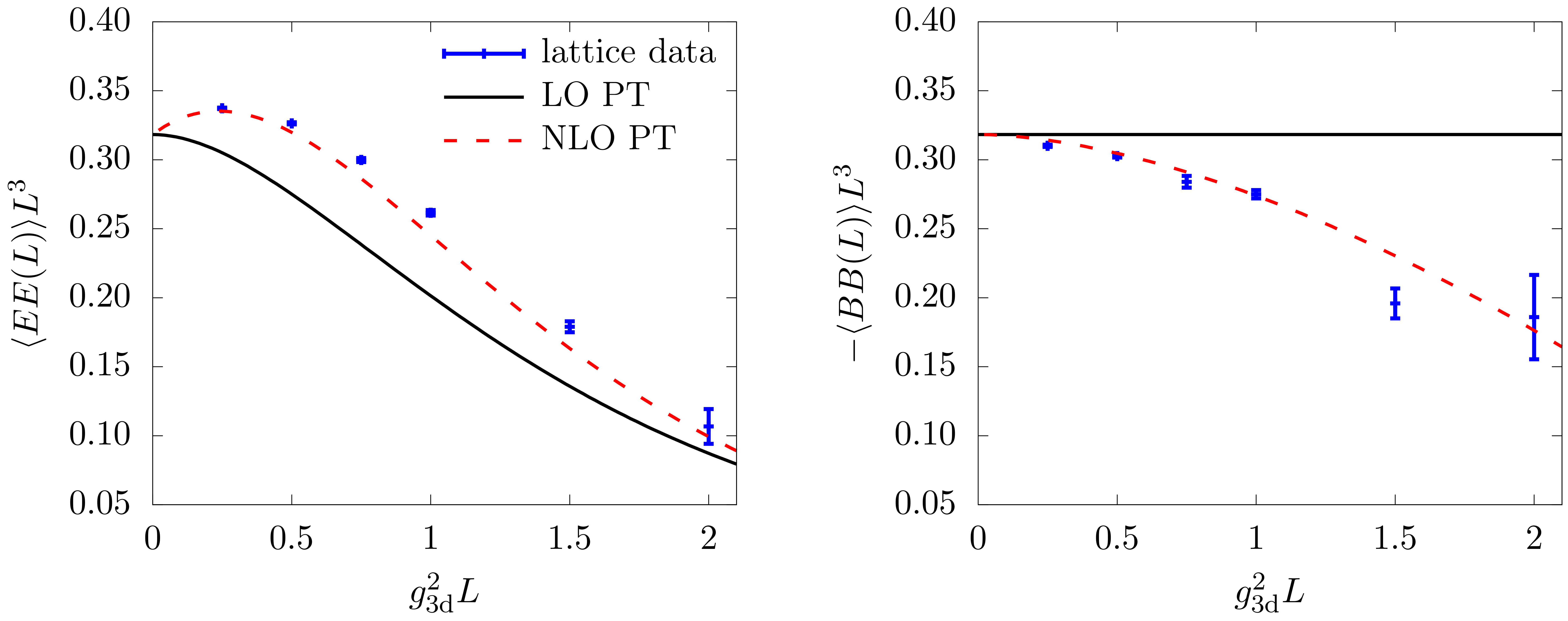
- In practice, we get continuum-extrapolated results for $\mathrm{Tr}\left\langle U(-\infty;L)F(L)\,U(L;0)\,F(0)U(0;-\infty)\right\rangle_{\rm EQCD}$ at a few discrete values of L.
  Moore Schlusser **PRD102** (2020) JG Moore Schicho Schlusser **JHEP02** (2021)
- We need to match to the 4D continuum, since EQCD has the wrong UV
- Start by computing the EQCD correlator to NLO



EQCD results

- Good agreement in the UV, excellent at high $T = 100$ GeV

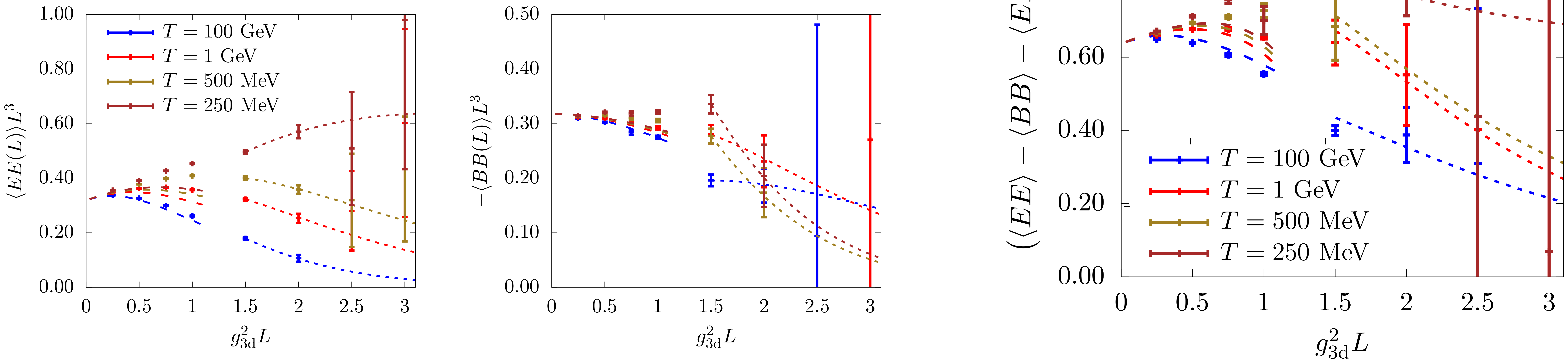
$$Z_g^{\mathrm{EQCD}} = \frac{T}{2}\int_0^\infty dL\, L\,\left(\langle EE\rangle - \langle BB\rangle - \langle EB\rangle\right)$$



JG Moore Schicho Schlusser (2021)

EQCD results

$$Z_{\rm g}^{\rm EQCD} = \frac{T}{2}\int_0^\infty dL\, L\left(\langle EE\rangle - \langle BB\rangle - \langle EB\rangle\right)$$



- IR tails modeled by non-perturbative exp. falloff (magnetic screening)
- UV tails handled by perturbative EQCD

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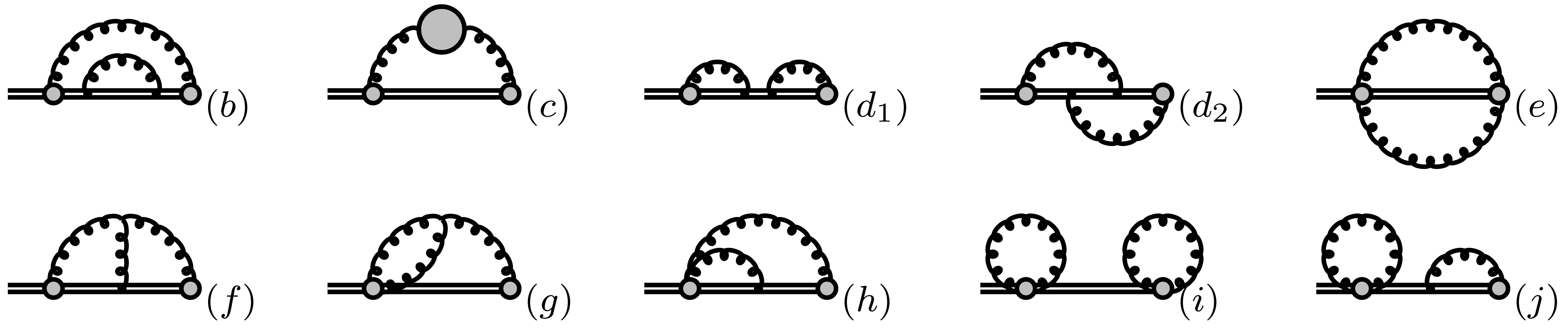
Matching to full QCD

- Integration UV-divergent ($L \to 0$) $\quad Z_{\rm g}^{\rm EQCD} = \frac{T}{2}\int_0^\infty dL\, L\left(\langle EE\rangle - \langle BB\rangle - \langle EB\rangle\right)$
- EQCD super-renormalizable, $\langle FF(L \to 0)\rangle = c_0 \frac{1}{L^3} + c_2 \frac{g^2 T}{L^2} + \ldots$
- Only the first two terms give rise to power-law and log divergences. They must cancel with the IR limits of a bare calculation in full thermal QCD. This is easily verified for the power law, that can simply be subtracted
- For the log in a first stage we introduce an intermediate cutoff regulator $-c_2 \frac{g^2 T}{L^2}\theta(L_0 - L)$ and integrate numerically the UV-subtracted EQCD data

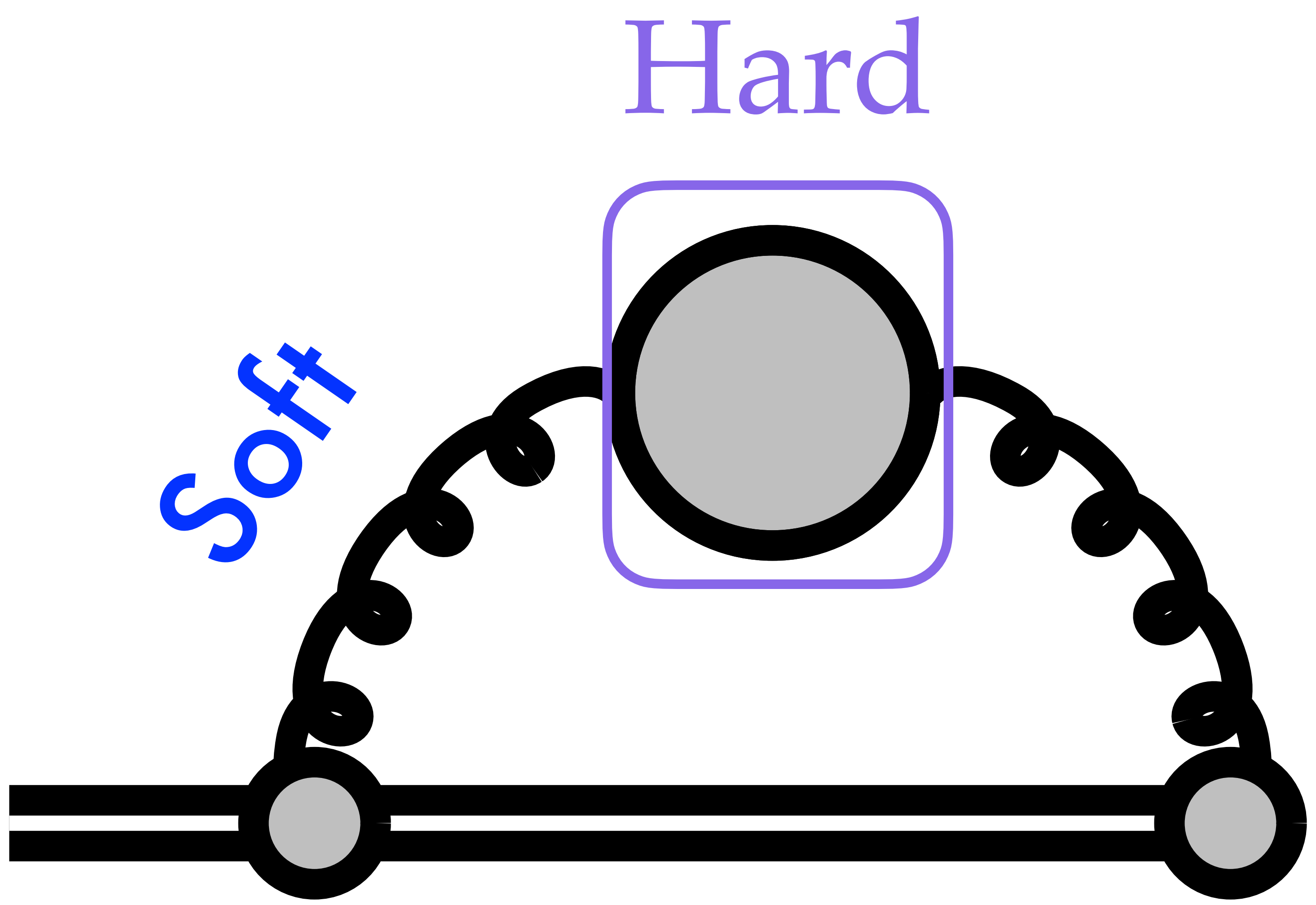
JG Moore Schicho Schlusser (2021)

Matching to full QCD

- Proper handling of the log divergence requires the two-loop calculation in thermal QCD



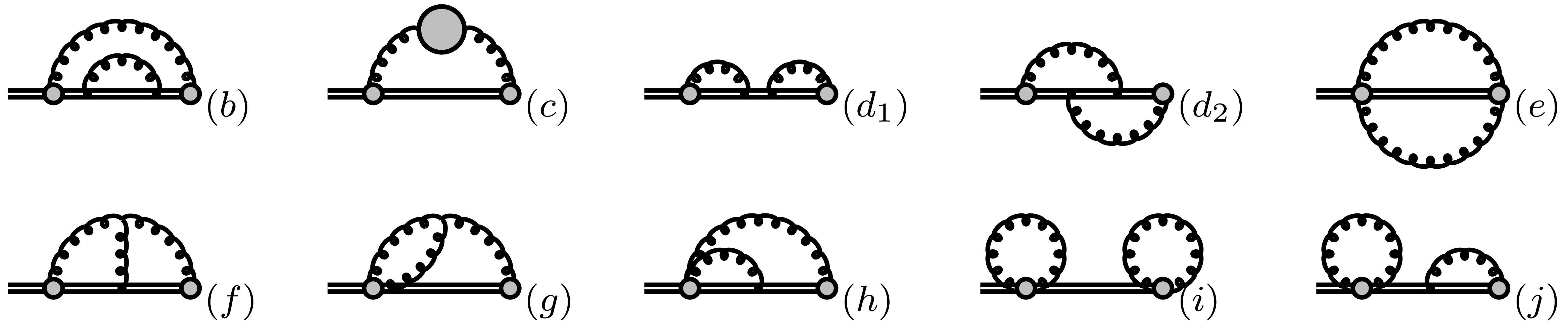
- Only diagram *c* matters in Feynman gauge
- Translated the cutoff to dimensional regularisation. UV pole of EQCD cancels IR pole of QCD, leaving behind a $g^2T^2\ln(T/m_D)$ term. Regulator dependence gone! Regulator-independent classical contribution negative



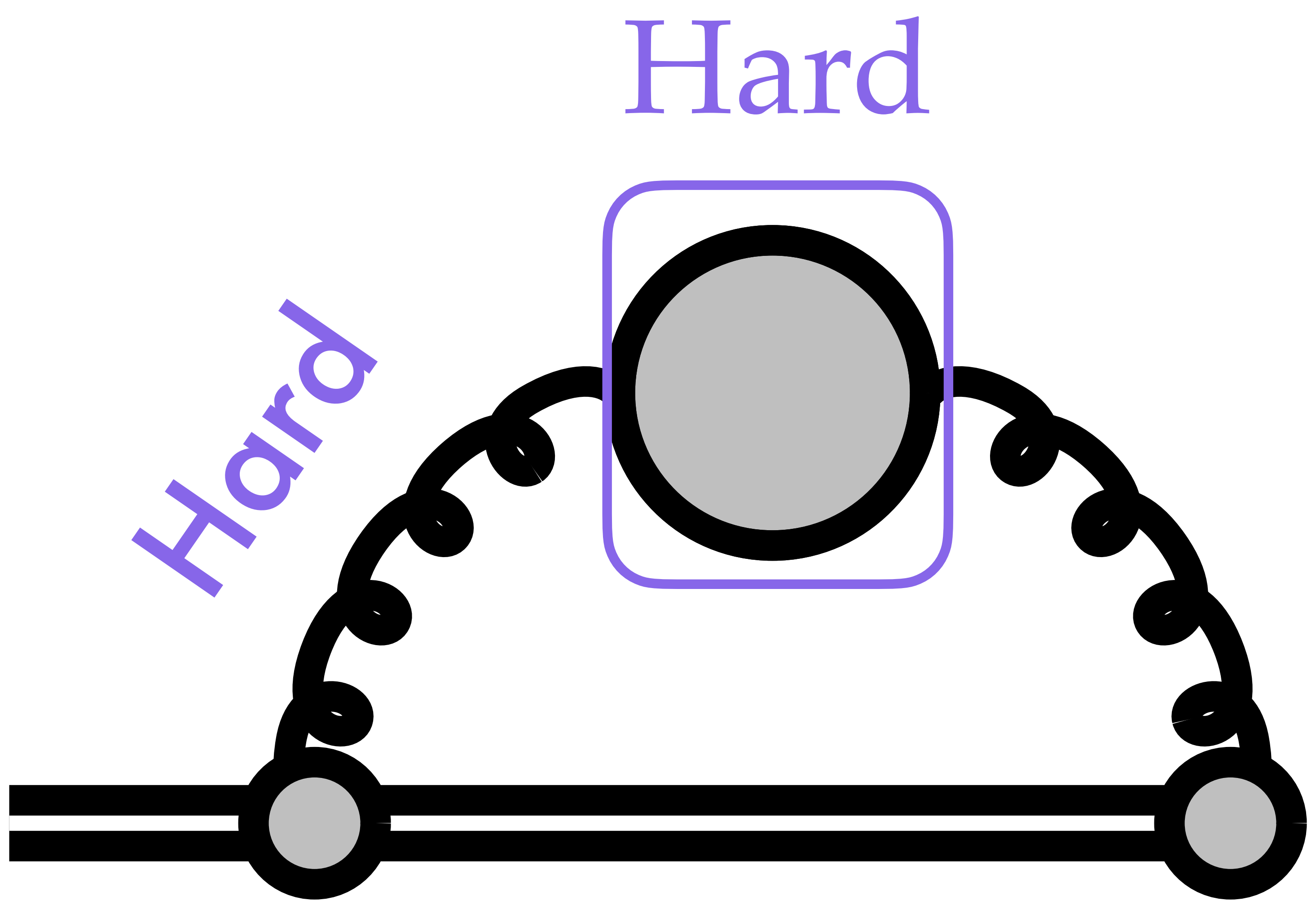
JG Schicho Schlusser Weitz **2312.11731**

Matching to full QCD

- Proper handling of the log divergence requires the two-loop calculation in thermal QCD



- Only diagram *c* matters in Feynman gauge
- Remainder of the calculation suggests emergence of double-logarithmic enhancements in the jet's energy



JG Schicho Schlusser Weitz **2312.11731**