Energy loss in chiral media

Jeremy Hansen

Iowa State University

Overview

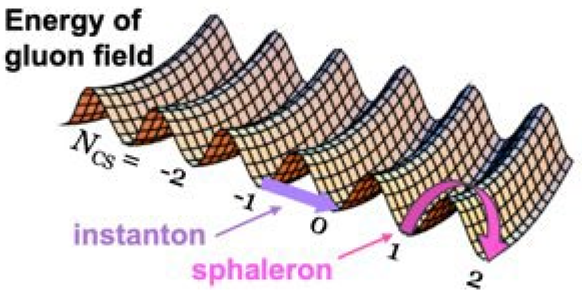
- What is chiral matter
- Sources of chiral imbalance
- Possible experimental applications
- Source of high-intensity radiation and leads to energy loss
- Collisional energy loss
- Brief discussion of radiative energy loss
- current and future work

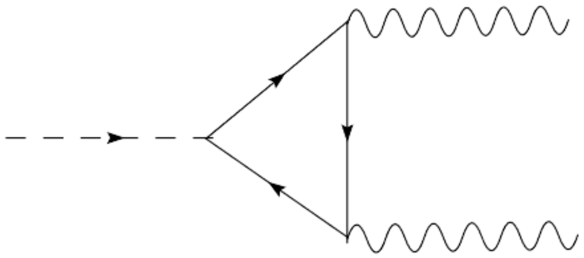
Chiral anomaly

Chiral anomaly breaks the axial symmetry ($\psi \to e^{i\gamma_5\theta}\psi$) of the system, creating a none conserved axial current.

$$\partial_\mu J_5^\mu = c_A \vec{E} \cdot \vec{B} \qquad \frac{d(N_R - N_L)}{dt} = c_A \int \vec{E} \cdot \vec{B} d^3x$$

QCD Vacuum





Sources of chiral imbalance may come from changes in N_{cs}

Weyl and Dirac Semimetals

The effects of the chiral anomaly have been seen in Weyl semimetals via the chiral magnetic effect

We are currently working on implications due to the quantum hall effect

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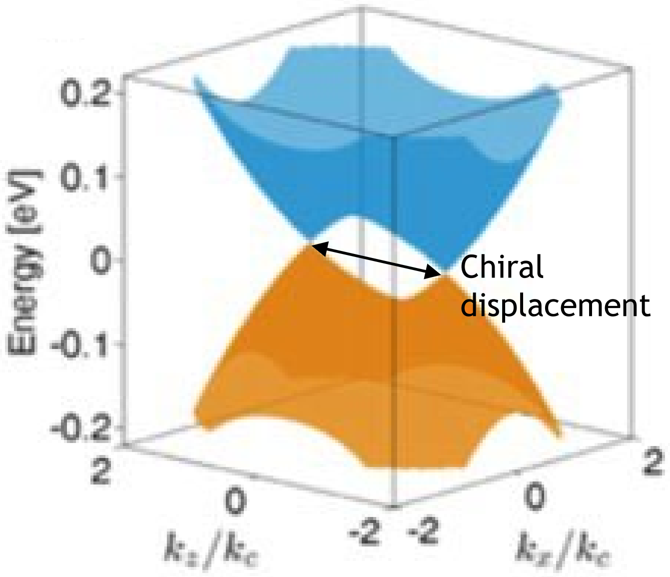
Chiral magnetic effect in $ZrTe_5$

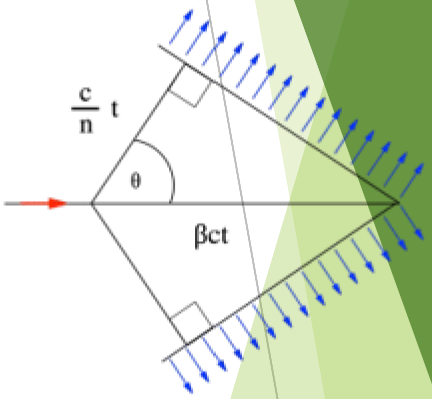
Qiang Li[1]*, Dmitri E. Kharzeev[2,3]*, Cheng Zhang[1], Yuan Huang[4], I. Pletikosić[1,5], A. V. Fedorov[6], R. D. Zhong[1], J. A. Schneeloch[1], G. D. Gu[1] and T. Valla[1]*

arXiv:1412.6543 [cond-mat.str-el]



Testing for the chiral anomaly

- One way of studying the effects for chiral media is through the interaction between a fast moving particle and a chiral medium.
- While traveling through the media the particle can lose energy via collisional or radiative energy loss
- We will focus mainly on the collisional energy loss for this presentation
- This has a number of advantages in various areas such as QGP and Axionic matter



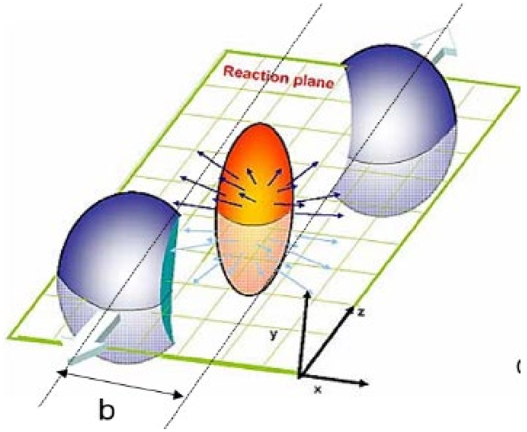
Cherenkov radiation

Quark-Gluon Plasma

One such advantage comes from globally chiral neutral media such as Quark-Gluon Plasma:

the enhancement due to the presence of any chiral magnetic effect for the rate of energy loss of a particle makes it sensitive to local chiral imbalances in the plasma.

overall the jets produced will be chiral neutral, however chirality of the radiated photons may oscillate depending on the local chiral imbalances in the plasma.



Axions and dark matter

Axions and axionic matter are highly related to the chiral anomaly and may serve as a dark matter candidate.

PHYSICAL REVIEW D 92, 103513 (2015)

Do dark matter axions form a condensate with long-range correlation?

Alan H. Guth,[1,*] Mark P. Hertzberg,[1,2,†] and C. Prescod-Weinstein[3,‡]

[1]*Center for Theoretical Physics & Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

[2]*Institute of Cosmology & Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155, USA*

[3]*Center for Theoretical Physics, MIT Kavli Institute for Astrophysics and Space Research & Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

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The study of the radiation produced in cosmic rays may shed light on the existence of such matter

Maxwell's equations

https://journals.aps.org/prl/issues/58/18

The effect of the anomaly in QED can phenomenologically described using the pseudoscalar field ϴ.

$$\mathcal{L}_{\rm MCS} = \mathcal{L}_{\rm QED} + c_A\theta(x)\vec{E}\cdot\vec{B}$$

In terms of $b^\mu = (b_0, -\boldsymbol{b}) = c_A\partial^\mu\theta = c_A(\dot{\theta}, -\boldsymbol{\nabla}\theta)$

arXiv:1206.1868v3

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = \rho - \boldsymbol{b}\cdot\boldsymbol{B}$$

$$\boldsymbol{\nabla}\times\boldsymbol{B} - \frac{\partial\boldsymbol{E}}{\partial t} = \boldsymbol{j} + b_0\boldsymbol{B} + \boldsymbol{b}\times\boldsymbol{E}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0$$

$$\boldsymbol{\nabla}\times\boldsymbol{E} + \frac{\partial\boldsymbol{B}}{\partial t} = 0$$

Chiral magnetic effect

Quantum hall effect

We will focus on homogeneous media (**b**=0) in the next part of the talk

Collisional energy loss

we analyze the collisional energy loss via the Fermi model in our paper "Collisional energy loss and the chiral magnetic effect" https://arxiv.org/abs/2012.06089

Consider a point charge traveling in the **z**-direction perpendicular to impact parameter **b** through a medium with induced current j_A=σ_χ **B**, $(\boldsymbol{b_0} = \sigma_\chi)$ such that

$$= q\delta(z - vt)\delta(\mathbf{b}) \qquad = qv\hat{\boldsymbol{z}}\delta(z - vt)\delta(\mathbf{b}) \qquad b^\mu = (\sigma_\chi, 0)$$

We can solve the modified Maxwell equations for the individual electric and magnetic components

$$\boldsymbol{\nabla} \times \mathbf{B} = \partial_t \mathbf{D} + \sigma_\chi \mathbf{B} + qv\hat{\boldsymbol{z}}\delta(z - vt)\delta(\mathbf{b})$$

$$\boldsymbol{\nabla} \cdot \mathbf{D} = q\delta(z - vt)\delta(\mathbf{b})$$

$$\boldsymbol{\nabla} \times \mathbf{E} = -\partial_t \mathbf{B}$$

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0$$

Electromagnetic Components

https://arxiv.org/abs/2012.06089

- The solution is a superposition of helicity states λ = ±1

$$\mathbf{B}(\mathbf{r},t) = \int \frac{d^2k_\perp d\omega}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t} \sum_\lambda \boldsymbol{\epsilon}_{\boldsymbol{\lambda}\mathbf{k}} \frac{q\hat{\boldsymbol{z}}\cdot\boldsymbol{\epsilon}^*_{\boldsymbol{\lambda}\mathbf{k}}\lambda k}{k_\perp^2+\omega^2(1/v^2-\epsilon)-\lambda\sigma_\chi k}$$

$$\mathbf{E}(\mathbf{r},t) = \int \frac{d^2k_\perp d\omega}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t} \left(\sum_\lambda \boldsymbol{\epsilon}_{\boldsymbol{\lambda}\mathbf{k}} \frac{iq\hat{\boldsymbol{z}}\cdot\boldsymbol{\epsilon}^*_{\boldsymbol{\lambda}\mathbf{k}}\omega}{k_\perp^2+\omega^2(1/v^2-\epsilon)-\lambda\sigma_\chi k} - i\frac{\hat{\boldsymbol{k}}q}{vk_E}\right)$$

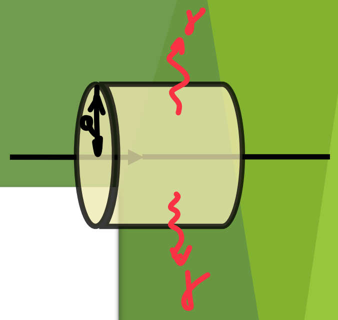
The integrals over angles can be cumbersome, however, an analysis after integration reveals an instability of the fields, here are two such examples.

$$E_{z\omega}(\mathbf{r}) = \frac{q}{2\pi}\frac{i\omega}{v^2\epsilon}\frac{e^{i\omega z/v}}{k_1^2-k_2^2}\sum_{\nu=1}^{2}(-1)^{\nu+1}\left[(v^2\epsilon-1)(k_\nu^2-s^2)-\sigma_\chi^2\right]K_0(bk_\nu) \qquad B_{\phi\omega}(\mathbf{r}) = \frac{q}{2\pi}\frac{e^{i\omega z/v}}{k_1^2-k_2^2}\sum_{\nu=1}^{2}(-1)^{\nu+1}k_\nu(k_\nu^2-s^2)K_1(bk_\nu)$$

For $s^2=\omega^2\left(\frac{1}{v^2}-\epsilon(\omega)\right), k_\nu^2 = s^2-\frac{\sigma_\chi^2}{2}+(-1)^\nu\sigma_\chi\sqrt{\omega^2\epsilon+\frac{\sigma_\chi^2}{4}}$

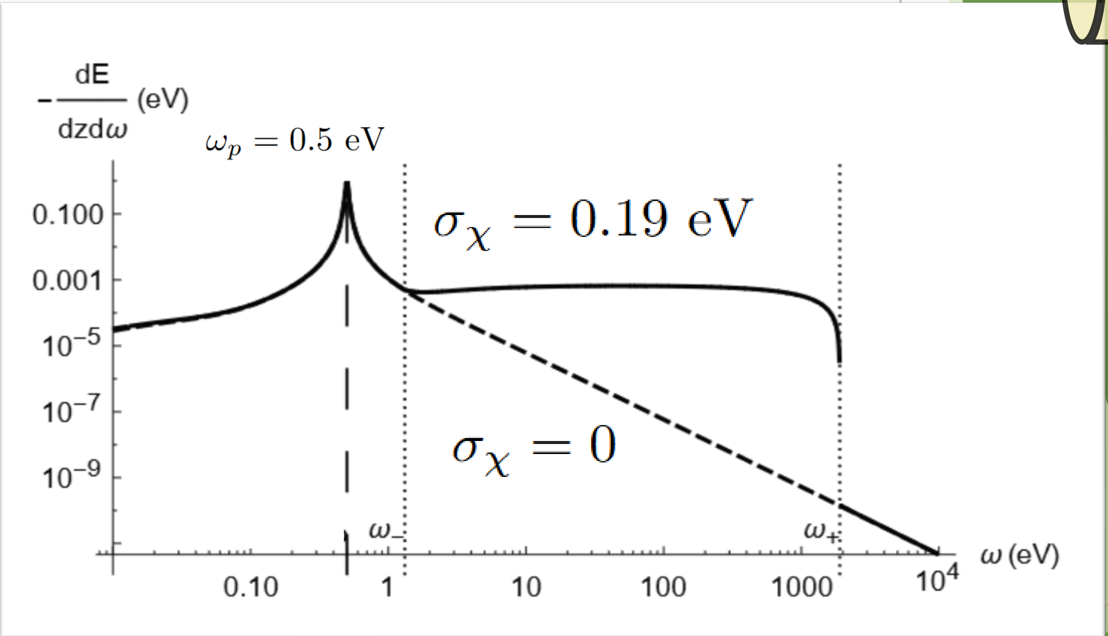
Collisional energy loss

https://arxiv.org/abs/2012.06089



The rate of energy loss for the particle can be computed from the modified Maxwell's equations as the flux of the Poynting vector out of a cylinder of radius "a" coaxial with the particle path.

$$-\frac{dE}{dz} = 2\pi a \int_{-\infty}^{\infty} (E_\phi B_z - E_z B_\phi) dt = 2a\mathbf{Re} \int_0^\infty (E_{\phi\omega} B^*_{z\omega} - E_{z\omega} B^*_{\phi\omega}) d\omega$$



The spectrum of electron with $\gamma = 100$ in a semimetal with parameters $\Gamma = 0.025$ eV and $m = 0.5$ MeV. $\omega_\pm$ give bounds to the enhancement due to the anomaly.

After integrating over frequency in the ultrarelativistic limit

$$-\frac{dE}{dz} = \frac{q^2}{4\pi v^2}\left(\omega_p^2 \ln\frac{v}{a\omega_p} + \frac{1}{4}\gamma^2\sigma_\chi^2\right)$$

given plasma permittivity

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma}$$

Power and Recoil

https://arxiv.org/abs/2012.06089

Once the rate of energy loss is identified, it can be used to find the rate of chiral radiation emitted in the proper limit

$$\frac{dW}{d\omega} = -\frac{dE}{dz\omega d\omega}\Big|_{a\to\infty} = \frac{q^2}{4\pi}\left\{\frac{1}{2}\left(1-\frac{1}{v^2}\right)+\frac{\sigma_\chi}{2\omega}+\frac{(1+v^2)\sigma_\chi^2}{8v^2\omega^2}+\ldots\right\}$$

And the total radiated power $P = \frac{q^2}{4\pi}\frac{\sigma_\chi^2\gamma^2}{4}$

The previous results neglect recoil. Once this effect is taken into account, one obtains the modified results

$$\frac{dW^{\text{quant}}}{d\omega} = \frac{q^2}{(4\pi)2\omega}\left\{\sigma_\chi\left(\frac{x^2}{2}-x+1\right)-\frac{m^2}{E}x\right\}$$ and $$P^{\text{quant}} = \frac{q^2}{4\pi}\frac{\sigma_\chi E}{3}$$

Recoil therefore reduced the total radiated power’s dependence on energy

QCD and QGP

https://arxiv.org/abs/2012.06089

Our results can be readily generalized to the strong interactions which dominate the particle energy loss in QGP

$$-\frac{dE}{dz} = \frac{g^2 C_F}{4\pi v^2}\left(\frac{\tilde{\omega}_p^2}{2N_c^2}\ln\frac{v}{a\tilde{\omega}_p} + \frac{1}{4}\gamma^2\tilde{\sigma}_\chi^2\right)$$

where g is the strong coupling. The plasma frequency $\tilde{\omega}_p$ is obtained from ωp by replacing e → √Nc g. $\tilde{\sigma}_x$ is proportional to the QCD anomaly coefficient $\tilde{c}_A$ = N f $g^2/(16\pi^2)$.

When recoil is accounted for

$$-\frac{dE}{dz}\bigg|_{\text{anom}} = \frac{g^2 C_F}{4\pi}\frac{\tilde{\sigma}_\chi E}{3}$$

Inhomogeneous media

Jeremy Hansen, Kazuki Ikeda, Dmitri E. Kharzeev, Qiang Li, and Kirill Tuchin in preparation

- Up to this point have considered only the chiral magnetic effect, we will now consider the anomalous hall effect do the current

$$j_{\rm AH} = \vec{b} \times \vec{E}$$

- This gives rise to the rate of photon emission as computed by my collaborator

$$\frac{dW}{d\Omega d\omega} = \frac{\alpha}{16\pi} \sum_{\lambda} \delta(\omega + E' - E) \frac{\vec{k}^3}{EE'\omega^2 \varepsilon_{ij} e_i^* e_j}$$
$$4e_i^* e_j \left[p_i p_j' + p_j p_i' + \delta_{ij}(EE' - \boldsymbol{p} \cdot \boldsymbol{p}' - m^2)\right]$$

https://arxiv.org/abs/1809.08181

- An important part of the rate of emission stems from the effect on the photon's energy which obeys Fresnel's equation

$$\left|\vec{k}_i \vec{k}_j - \vec{k}^2 \delta_{ij} + \omega^2 \varepsilon_{ij}\right| = 0$$

Where $\varepsilon_{ij} = \varepsilon \delta_{ij} - i\epsilon_{ijk} b_k / \omega$

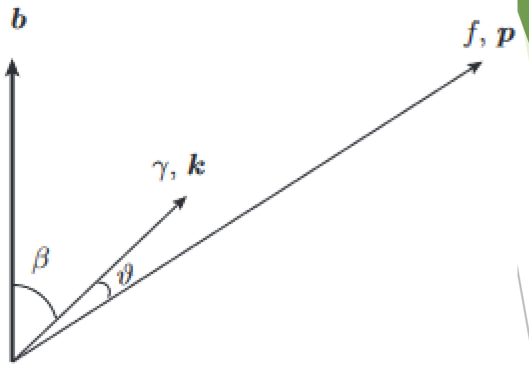
- The permittivity is described using the Fermi model

$$\varepsilon \approx 1/(1 + \omega_p^2/\omega^2)$$

Dispersion Relation

- the full solution to the Fresnel's equation gives

$$\omega^2-\vec{k}^2=\omega_p^2+\frac{b^2\omega^2\sin(\beta)^2-\lambda b\omega\sqrt{b^2\omega^2\sin(\beta)^4+4(\omega^2-\omega_p^2)^2\cos(\beta)^2}}{2(\omega^2-\omega_p^2)}$$



- Energy loss is maximum when the radiated particle (k) is aligned to the chiral displacement (b) such that

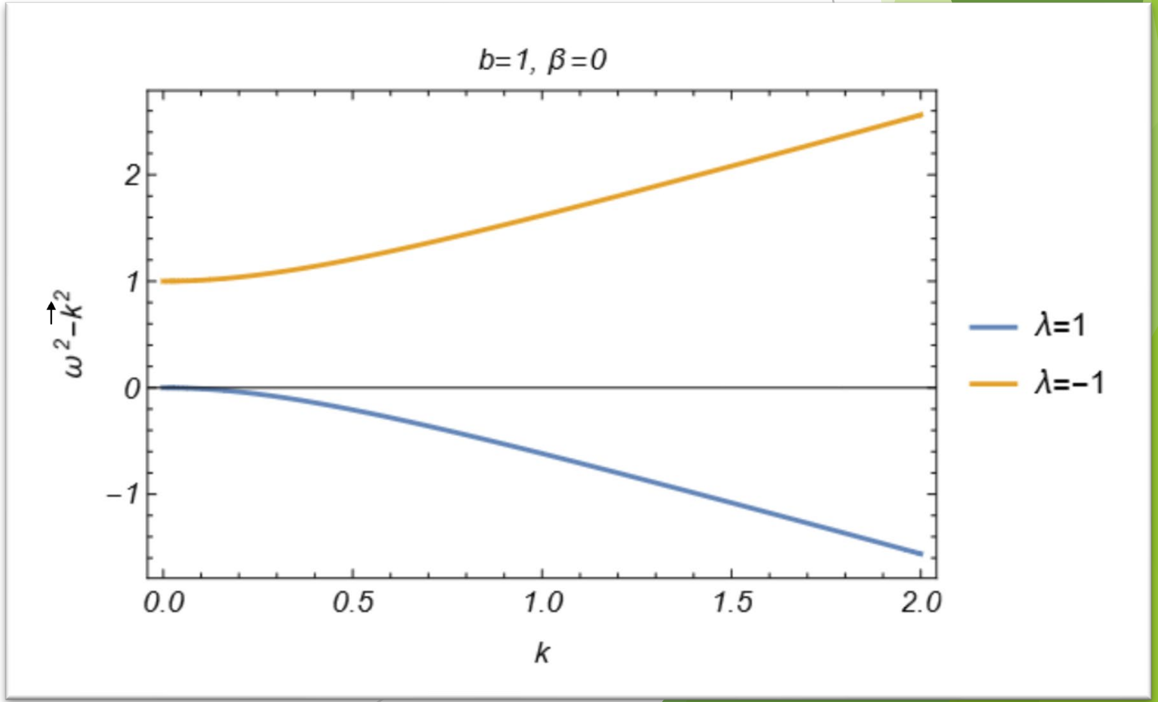
$$\omega^2-\vec{k}^2=\omega_p^2-\lambda b\omega,\ \beta=0$$

- When the anomaly is perpendicular to the outgoing radiation it gives no contribution for $\omega>\omega_p$ since

$$\omega^2-\vec{k}^2=\omega_p^2+\frac{(1-\lambda)b\omega^2}{2(\omega^2-\omega_p^2)}$$

- This stems from the kinematic constraint

$$4m^2\omega^2+(\omega^2-\vec{k}^2)(4(EE'-m^2)+\omega^2-\vec{k}^2)=0$$



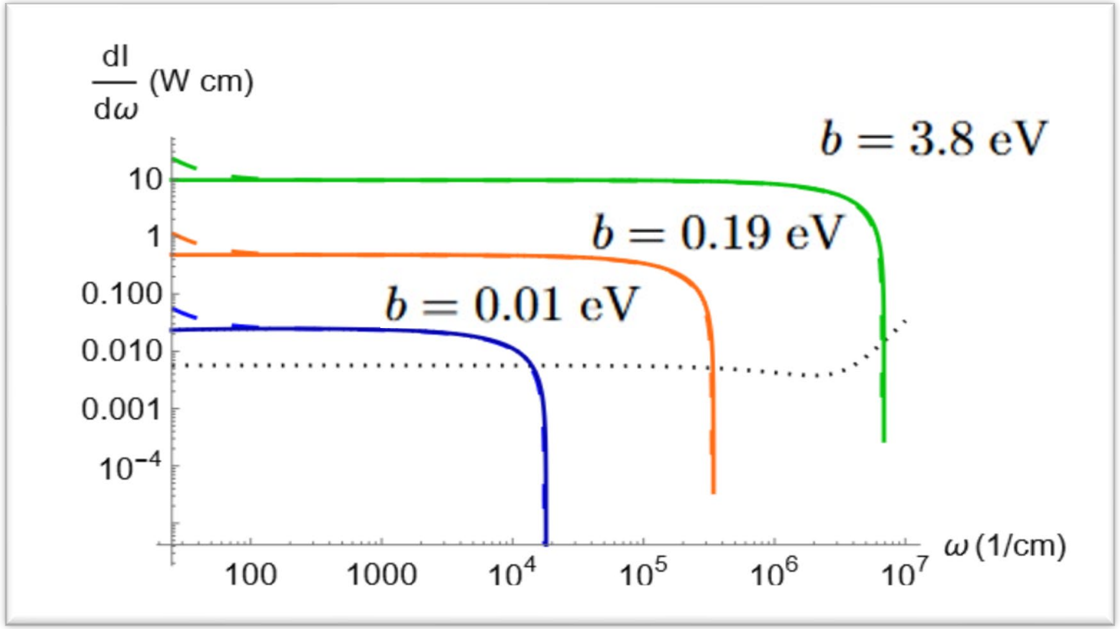
High energy

After integrating over angles one finds the intensity of radiation per frequency as an increasing function of chiral displacement

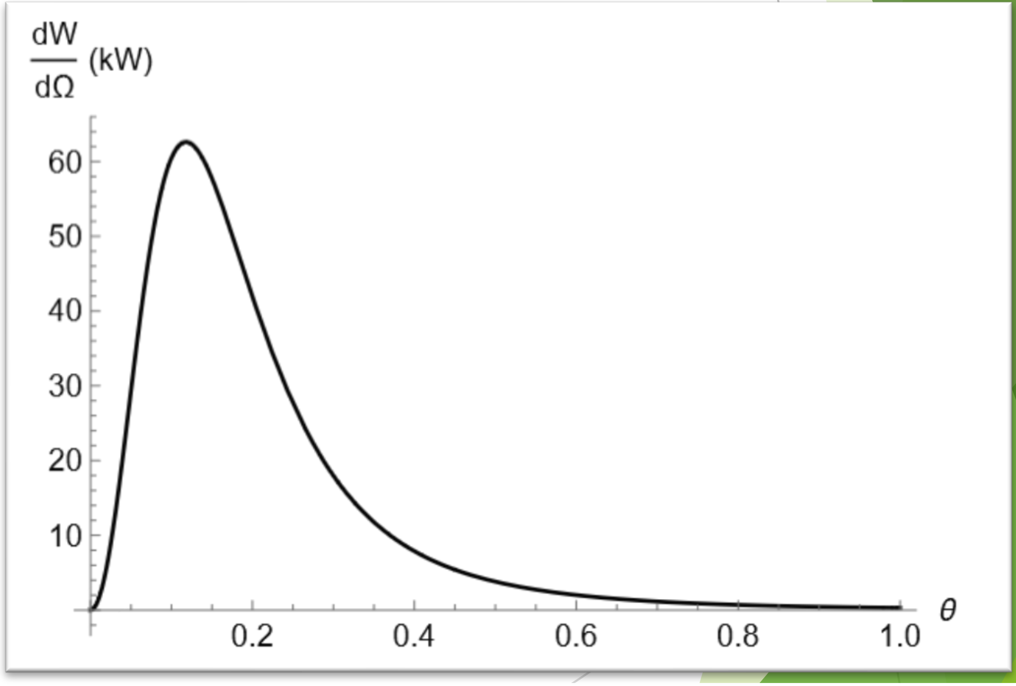
$$\frac{dP}{d\omega} = \frac{\alpha f}{2}\theta\left(\frac{E^2 f}{m^2} - \omega\right) \qquad \text{for} \qquad f = \lambda b \cos\beta$$

Alternatively, one can find the angular distribution for the intensity of radiation by integrating over frequencies

$$\frac{dW}{d\Omega} = \frac{\alpha f}{2\pi}\frac{1}{\vartheta^2 + m^2/E^2}$$



Intensity spectrum of Chiral Cherenkov radiation for electrons of energy $E = 3$ MeV, $\beta = 0$ in a Weyl semimetal, dotted line: bremsstrahlung in $Co_3Sn_2S_2$.



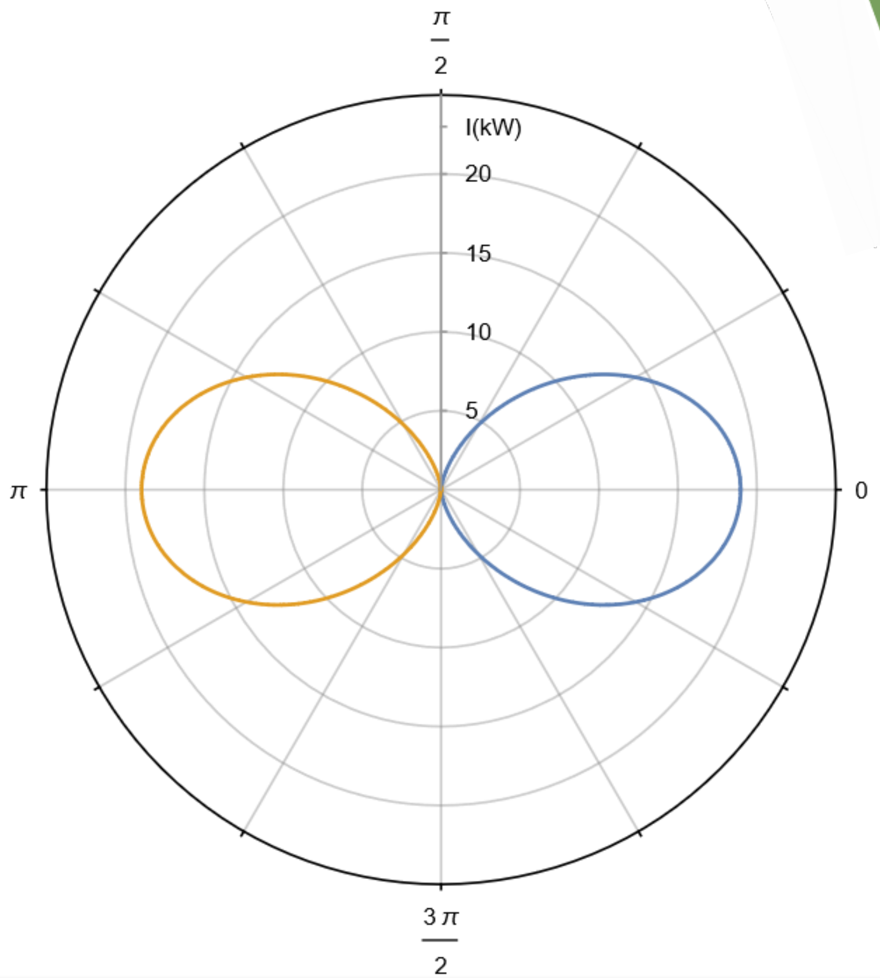
Angular distribution of Chiral Cherenkov radiation. $E = 3$ MeV, $\beta = 0$, $b = 3.9$ eV.

High energy limit

- After integrating over angles and frequencies, and converting the rate of radiation produced to the rate of energy loss on obtains the total radiated power

$$P = \frac{\alpha f^2 E^2}{2m^2} = \frac{\alpha b^2 \cos^2 \beta E^2}{2m^2}$$

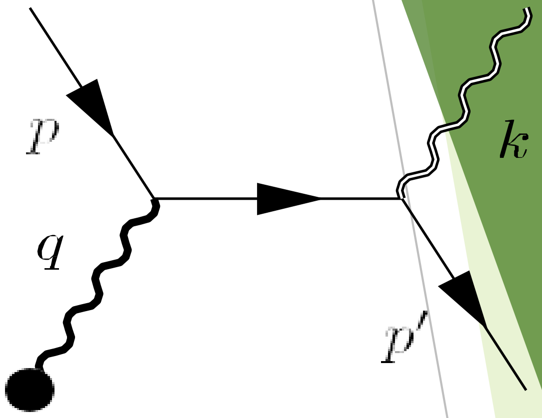
- The two loops have opposite polarization



Chiral Cherenkov radiation intensity for incidence angle (β). $E = 3$ MeV, $b = 3.9$ eV.

Radiative energy loss

In addition to the collision energy loss, the anomaly may impact radiative energy loss as well



For an in-depth discussion refer to our papers

https://doi.org/10.1103/PhysRevD.105.116008

https://doi.org/10.1103/PhysRevD.108.076007

In particular, we consider how the anomaly modifies the photon propagator such that

$$D_{\mu\nu}(q) = -i\frac{q^2 g_{\mu\nu} + i\epsilon_{\mu\nu\rho\sigma} b^\rho q^\sigma + b_\mu b_\nu}{q^4 + b^2 q^2 - (b \cdot q)^2}$$

https://arxiv.org/abs/hep-ph/0406128

And the impact of the anomaly on the corresponding dispersion for a homogeneous medium

$$\omega^2 = \boldsymbol{k}^2 + k^2 = \boldsymbol{k}^2 - \lambda b_0 |\boldsymbol{k}|$$

In case we find that under the right conditions, we find that the rate of energy loss may be enhanced by the chiral anomaly

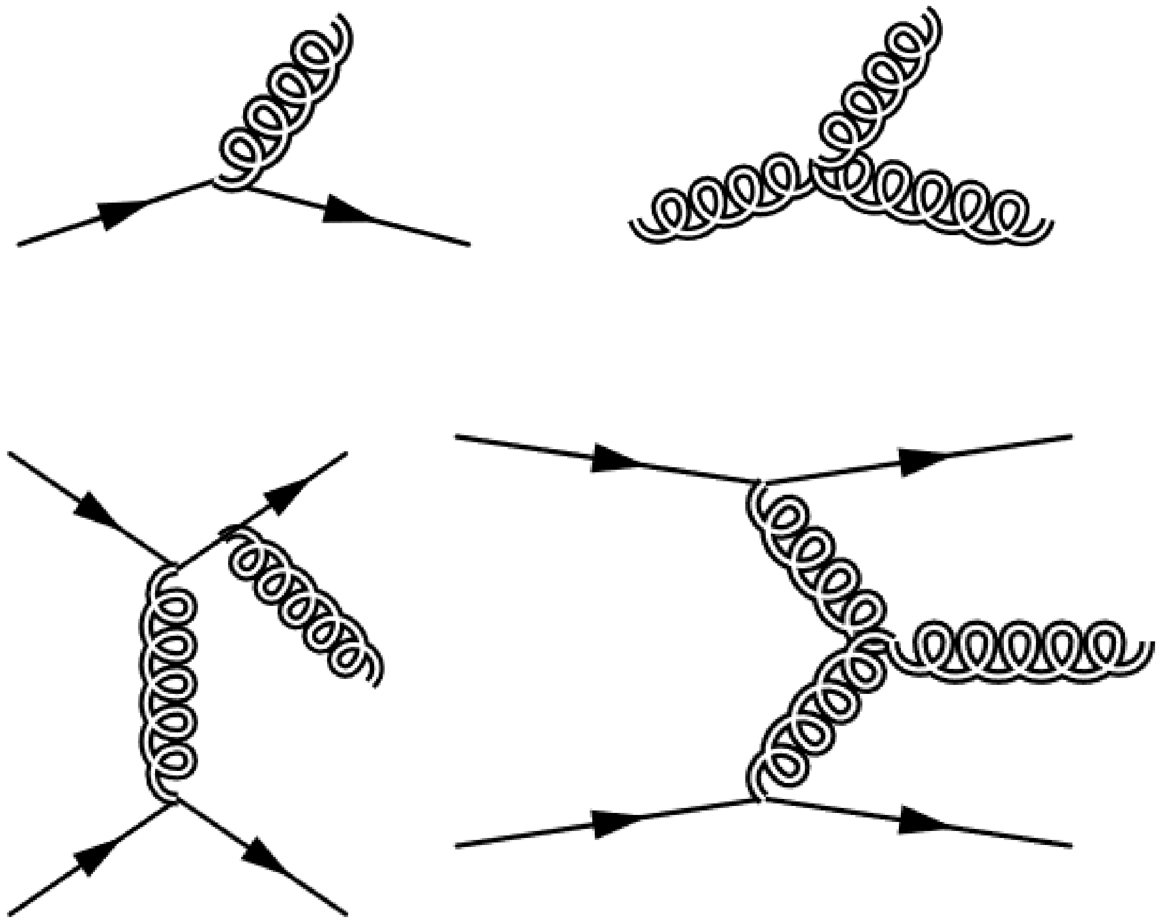
Multiple Scattering

- For the Radiative energy loss we have neglected coherence effects due to multiple scattering, however for an accurate description of energy loss this can not be ignored
- For multiple scattering medium effects such as Cherenkov radiation is often neglected as small

arXiv:hep-ph/9604327v1

- Our study, however, shows that medium effects due to the chiral anomaly can be significant given the right conditions and should not be neglected

QCD



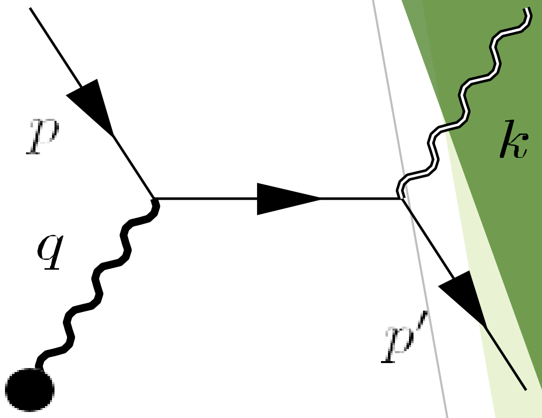
- We wish to extend our treatment to the realm of QCD. In which we consider collisional and radiative energy loss for quark scattering and gluon radiation.
- In particular we consider the impact of the anomaly on a quark radiating a gluon, and a gluon radiating a gluon

Summary

- The presence of a finite chiral conductivity has a potentially significant effect on the collisional and radiative energy loss including Weyl semimetals and QGP.
- In both the collisional and radiative energy losses chiral magnetics and hall effects can have interesting and varying impacts, most notable in the dispersion relation for produced radiation
- Currently we are expanding our treatment to QCD. This may be seen most notably in the dispersion relation of the gluon, and three gluon vertex in the ultra-relativistic limit.

Radiative energy loss

For the radiative energy loss we consider this process



With corresponding matrix element and differential cross section

$$\mathcal{M} = e^2 \overline{u}(p') \left(\not{\epsilon}^*_{k\lambda} \frac{\not{p}' + \not{k} + m}{(p'+k)^2 - m^2} \not{A}(q) + \not{A}(q) \frac{\not{p} - \not{k} + m}{(p-k)^2 - m^2} \not{\epsilon}^*_{k\lambda} \right) u(p)$$

https://arxiv.org/abs/2203.13134

The anomaly modifies the photon propagator in the following way

$$D_{\mu\nu}(q) = -i \frac{q^2 g_{\mu\nu} + i\epsilon_{\mu\nu\rho\sigma} b^\rho q^\sigma + b_\mu b_\nu}{q^4 + b^2 q^2 - (b \cdot q)^2}$$

https://arxiv.org/abs/hep-ph/0406128

https://arxiv.org/abs/2203.13134

Source current

Using the photon propagator in chiral medium in the static limit

$$D_{00}(\boldsymbol{q}) = \frac{i}{\boldsymbol{q}^2}\,,$$

$$D_{0i}(\boldsymbol{q}) = D_{0i}(\boldsymbol{q}) = 0\,,$$

$$D_{ij}(\boldsymbol{q}) = -\frac{i\delta_{ij}}{\boldsymbol{q}^2 - b_0^2} - \frac{\epsilon_{ijk}q^k}{b_0(\boldsymbol{q}^2 - b_0^2)} + \frac{\epsilon_{ijk}q^k}{b_0\boldsymbol{q}^2}$$

Given how the source current couples to the propagator, the anomaly most apparently impacts the energy loss via magnetic moment

To find the vector potential we use the static current

$$J^0(\boldsymbol{x}) = e'\delta(\boldsymbol{x})\,, \qquad \boldsymbol{J}(\boldsymbol{x}) = \boldsymbol{\nabla} \times (M\delta(\boldsymbol{x}))$$

Electric Monopole Magnetic Moment

Giving rise to

$$A^0(\boldsymbol{q}) = e'/\boldsymbol{q}^2$$

$$A^\ell(\boldsymbol{q}) = -\frac{1}{\boldsymbol{q}^2 - b_0^2}\left[i(M \times \boldsymbol{q})^\ell + \frac{b_0}{\boldsymbol{q}^2}(M\cdot \boldsymbol{q}q^\ell - \boldsymbol{q}^2 M^\ell)\right]$$

Magnetic moment

https://arxiv.org/abs/2203.13134

In the a material a good approximation that is often assumed is that the magnetic moment is stochastically oriented such that

$$\langle M_i \rangle = 0,\ \langle M_i M_j \rangle = \frac{M^2}{3}\delta_{ij}$$

This allows us to separate the contributions of due the electric charge and magnetic moment such that

$$|\mathcal{M}|^2 = |\mathcal{M}_e|^2 + |\mathcal{M}_M|^2$$

Focusing on the contribution due to the magnetic moment, one will find that the cross-section depends on the averaged product of vector potentials

$$\langle A^i(\boldsymbol{q})A^{j*}(\boldsymbol{q})\rangle = \frac{M^2}{3(\boldsymbol{q}^2 - b_0^2)^2}\left[\left(\delta^{ij} - \frac{q^i q^j}{\boldsymbol{q}^2}\right)(\boldsymbol{q}^2 + b_0^2) - 2ib_0\epsilon^{ijk}q_k\right]$$

Which gives a resonance like behavior a at **q**=b_0

https://arxiv.org/abs/2203.13134

Magnetic moment

The problem with the expression for the differential cross-section is that it diverges at **q** = b_0. As such its behavior must be regulated in some way. This can be accomplished by taking account of the finite resonance width.

$$\frac{1}{q^4 + b^2 q^2 - (b \cdot q)^2} \to \frac{1}{q^4 + b^2 q^2 - (b \cdot q)^2 + iq^2\Gamma^2}$$

This is related to the photon decay width W as $\Gamma^2 = 2\sqrt{b_0|\boldsymbol{q}|}W \approx 2b_0 W$

https://arxiv.org/abs/2203.13134

Magnetic moment

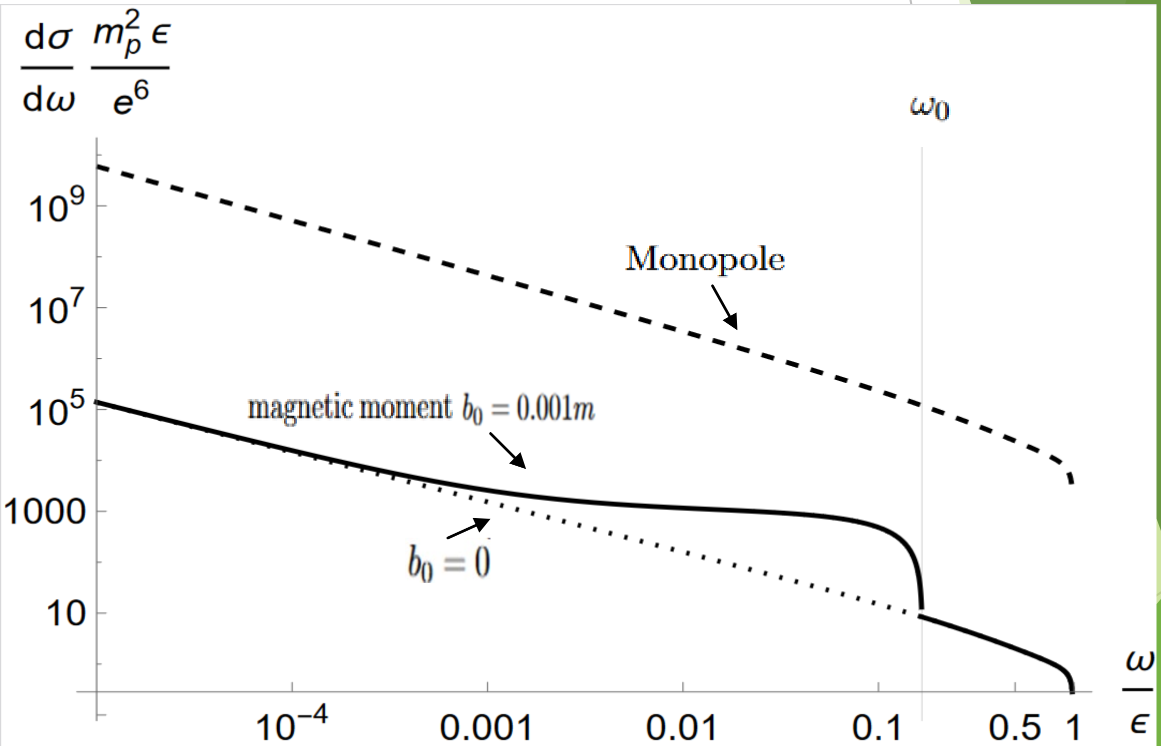
Regulating the divergence at **q**=b_0 with the finite resonate width Γ in the ultra-relativistic semi-soft photon limit, we can derive the following approximation for the differential cross section integrated over directions.

$$\frac{d\sigma_M}{d\omega} \approx \frac{2e^4{}_M{}^2}{3(2\pi)^3\omega}\left[\frac{3b_0^2}{m^2}\ln\left(\frac{4\epsilon^4}{m^2\omega^2}\right) + \ln^2\frac{4\epsilon^2}{m^2} + \frac{2b_0^4\pi}{m^2\Gamma^2}\Theta(\omega_0-\omega)\right]$$

where $\omega_0 = \dfrac{2\epsilon^2 b_0}{2\epsilon b_0 + m^2}$

This can be compared to the traditional expression from the mono-pole contribution in this limit

$$\frac{d\sigma_e}{d\omega} \approx \frac{Z^2e^6}{12(2\pi)^3 m^2\omega}\left(\ln\frac{2\epsilon^2}{m\omega} - \frac{1}{2}\right)$$



photon bremsstrahlung cross section for $m = m_p, \Gamma = 0.01b_0, \epsilon = 100m$, and $M = 5M_N$

Electric Charge

https://arxiv.org/abs/2307.05761

The coulomb contribution gains a resonant behavior due to the effect of the anomaly on the dispersion relation of the photon (photon gains an effective mass)

$$\omega^2 = \boldsymbol{k}^2 + k^2 = \boldsymbol{k}^2 - \lambda b_0 |\boldsymbol{k}|$$

This effect can be seen in the equation for the matrix element

$$\mathcal{M} = e^2 \overline{u}(p') \left(\not{\epsilon}^*_{k\lambda} \frac{\not{p}' + \not{k} + m}{2p' \cdot k + k^2} \not{A}(\boldsymbol{q}) - \not{A}(\boldsymbol{q}) \frac{\not{p} - \not{k} + m}{2p \cdot k - k^2} \not{\epsilon}^*_{k\lambda} \right) u(p)$$

This resonant behavior has two significant cutoffs, the Debye mass μ, and the chiral relaxation time τ of the medium

Regulating the divergences

https://arxiv.org/abs/2307.05761

- The divergence in the fermion propagator can be regulated by the chiral relaxation time τ. For κ equal to the four dot between the fermion and the out going radiation we have

$$\frac{1}{2\omega\kappa - k^2} \to \frac{1}{2\omega\kappa - k^2 + iE/\tau},$$

- The photon propagator is also effected by the chiral anomaly via the dispersion relation for the out going radiation. In which case the divergence can be screened through the Debye mass

$$\frac{1}{\boldsymbol{q}^2} \to \frac{1}{\boldsymbol{q}^2 + \mu^2}$$

Momentum transfer at high energies

https://arxiv.org/abs/2307.05761

- At high energies the resonance in the fermion propagator can be expressed in the following way

$$\frac{1}{2p\cdot k - k^2 + iE/\tau} = \frac{1}{\omega E(\frac{m^2}{E\omega}\frac{\omega-\omega^*}{E-\omega^*} + \theta^2 + \frac{i}{\omega\tau})}$$

- The minimum momentum transfer can be approximated similarly and inherits the same resonant behavior as the fermion propagator

$$\boldsymbol{q}^2|_{\phi=0,E\theta=E'\theta'} \approx \frac{1}{4}\left[\frac{m^2(\omega-\omega^*)}{E'(E-\omega^*)} + \frac{\omega E}{E'}\theta^2\right]^2 = \frac{1}{4}\frac{\omega^2E^2}{E'^2}\left[\frac{m^2(\omega-\omega^*)}{\omega E(E-\omega^*)} + \theta^2\right]^2$$

- This suggests that the photon propagator can be regulated similarly to the fermion propagator

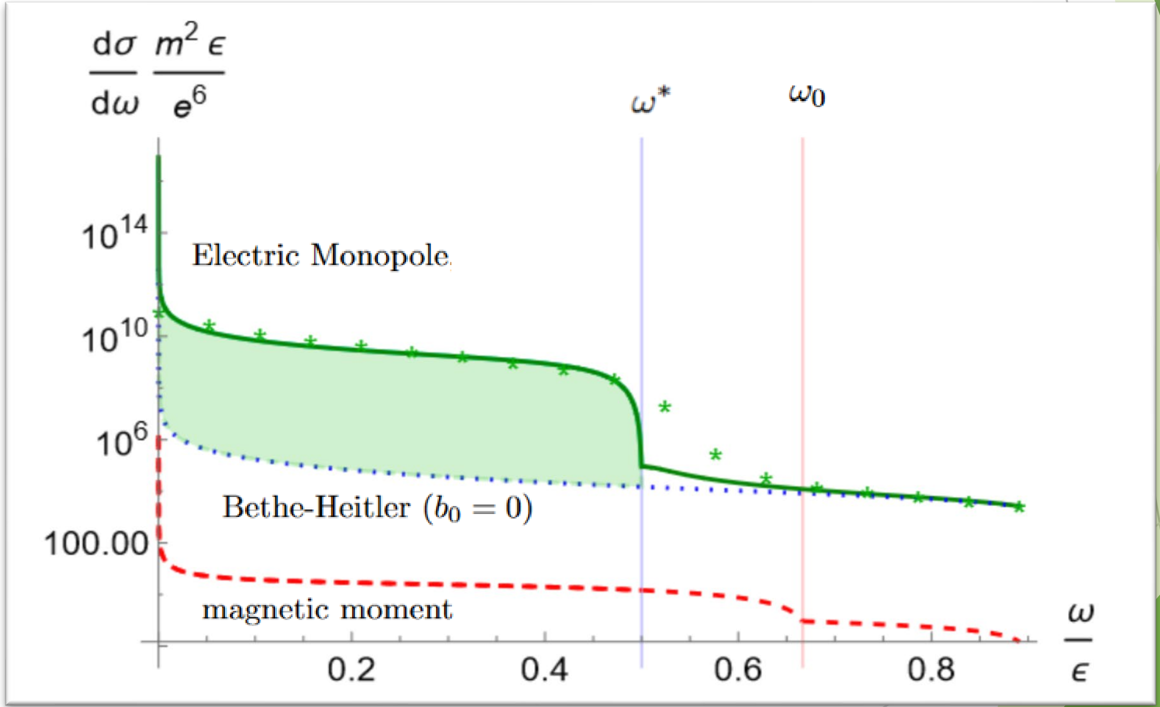
$$\boldsymbol{q}^2 \to \boldsymbol{q}^2 + \frac{E^2}{4E'^2\tau^2} + \mu^2$$

Electric Charge

https://arxiv.org/abs/2307.05761

The effect of the anomaly can vary dramatically depending on the size of the Debye mass. Most notably the effect is strongest when μ is much smaller than the mass of the incoming particle. When compared to radiation due to Bethe-Heitler, the anomaly acts by enhancing the amount of radiation such that

$$\frac{d\sigma(b_0\lambda > 0)}{d\omega} \approx \frac{3\pi}{2}\frac{m^2\tau^3 b_0}{\ln\frac{2E^2}{m\omega}}\frac{d\sigma_{\rm BH}}{d\omega}$$



For frequencies less than

$$\omega^* = \frac{\lambda b_0 E^2}{\lambda b_0 E + m^2}$$

in the high energy limit

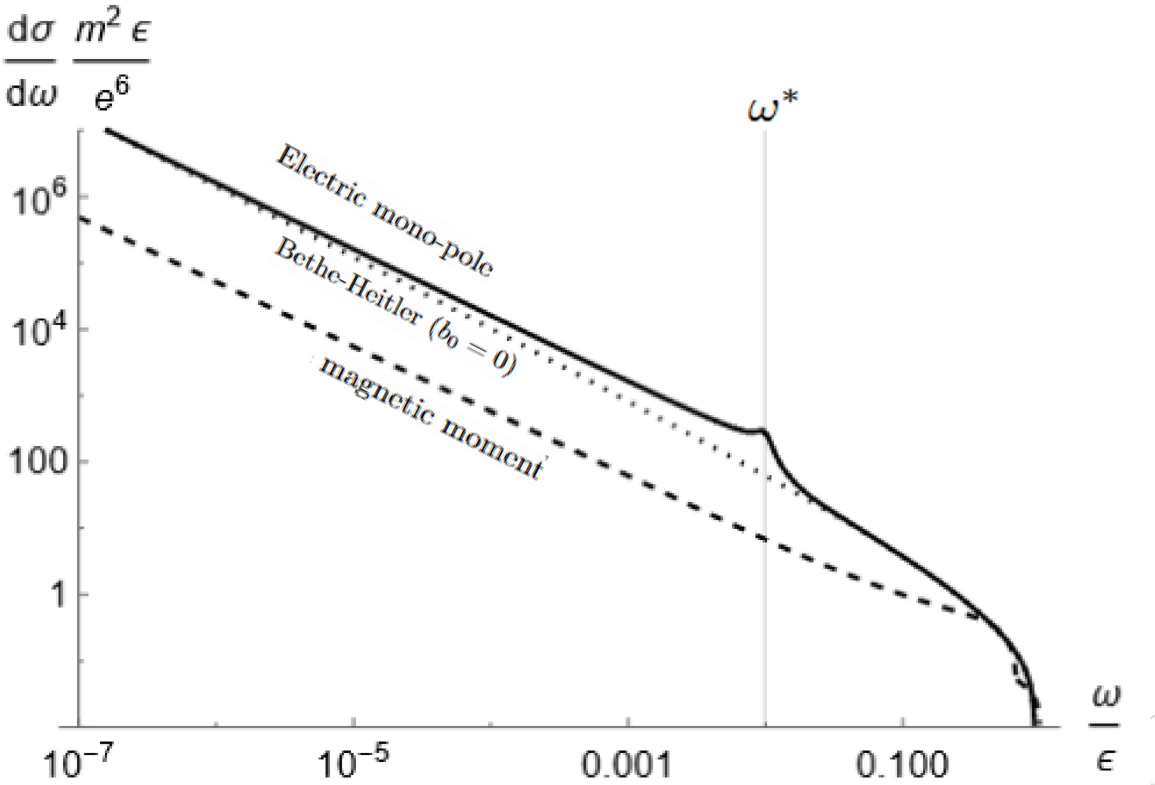
$b_0 = 0.1m,\ \mu = 10^{-3}b_0,\ \tau^{-1} = 0.1b_0,\ E = 10m,\ Z = 33,\ M = M_N,\ \Gamma = 0.1b_0$

Debye Mass

https://arxiv.org/abs/2307.05761

In the high-temperature limit such that μ is much larger than the mass of the incoming particle, that effect of the anomaly can be screened out

$$\frac{d\sigma(b_0\lambda > 0)}{d\omega} \approx \frac{\pi}{2}\frac{\mu}{E}\frac{b_0\tau}{\ln\frac{2E^2}{\mu\omega}}\frac{d\sigma_{\rm BH}}{d\omega}$$



contribution: $b_0 = 10^{-2}m$, $E = 10^2 m$, $\mu = 10m$, $\tau^{-1} = \Gamma = 0.1 b_0$, $Z = 33$.

Energy Loss

https://arxiv.org/abs/2307.05761

For our calculation, we assume that the photon formation time is much shorter than the mean free path ℓ. This allows us to neglect quantum interference and express the rate of energy loss per length in terms number of scatterers per unit volume

$$n = 1/\ell\sigma^{eZ\to eZ}$$

where σ is the elastic scattering cross-section. The rate of energy loss can be expressed as

$$-\frac{dE}{dz} = n\int_0^E \omega \frac{d\sigma^{eZ\to eZ\gamma}}{d\omega} d\omega$$

In the high energy limit neglecting the magnetic moment contribution, one can obtain an expression for the rate of energy loss in the unscreened and screened limits

$$-\frac{dE}{dz} \approx \frac{e^2E}{8\pi^2\ell}\left\{\ln\frac{2E}{m} - \frac{1}{3} + \tau E \arctan 2b_0\tau\right\}$$

$$-\frac{dE}{dz} \approx \frac{e^2E}{16\pi^2\ell}\left\{\ln\frac{2E}{\mu} + \frac{2b_0\mu\tau}{3E}\right\}$$

Unscreened vs screened

Note of interest

For the polarization sum

$$d_k^{ij} = \sum_{\text{pol}} e_k^i e_k^{*j} = \delta^{ij} - \frac{k^i k^j}{\boldsymbol{k}^2}$$

This is due to the fact that the dispersion relation for the photon in the media is given by

$$\omega^2 = \boldsymbol{k}^2 + k^2 = \boldsymbol{k}^2 - \lambda b_0 |\boldsymbol{k}|$$

There is no remaining gauge freedom to transform it to any other form. In particular, d_{ij} cannot be replaced with $-g_{\mu\nu}$. As an illustration, consider the amplitude $e_k \cdot M$. Let k be in the z-direction, then using the current conservation $k \cdot M = 0$ we can write

$$\sum_{\text{pol}} |e_k \cdot \mathcal{M}|^2 = |\mathcal{M}_x|^2 + |\mathcal{M}_y|^2 = |\mathcal{M}_x|^2 + |\mathcal{M}_y|^2 + |\mathcal{M}_z|^2 - \frac{\omega^2}{\boldsymbol{k}^2}|\mathcal{M}_0|^2 \neq -g^{\mu\nu}\mathcal{M}_\mu \mathcal{M}_\nu^*$$

None chiral limit

A good model for the plasma permittivity is given by

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma}$$

Where ω_p is the plasma frequency, and Γ is a damping term related to the electrical conductivity much smaller than ω_p.

This gives an overall contribution to the rate of energy loss at the pole ϵ=0 as Γ goes to zero.

$$-\frac{d\varepsilon^{\text{pole}}}{dz} = \frac{q^2\omega_p^2}{4\pi v^2} K_0\left(a\omega_p/v\right) \mathbf{Re}\left\{a\sqrt{\omega_p^2/v^2 - \sigma_\chi^2} K_1\left(a\sqrt{\omega_p^2/v^2 - \sigma_\chi^2}\right)\right\}$$

Other sources

- On of the sources of the chiral relaxation time stems from spontaneous photon emission

$$W=\frac{e^2}{8\pi}\int\frac{d\omega}{\omega}\left[b_0\left(\frac{\omega^2}{2E^2}-\frac{\omega}{E}+1\right)-\frac{m^2\omega}{E^2}\right]\Theta(\omega^*-\omega)\Theta(\lambda b_0)$$

for $$\omega^* = \frac{\lambda b_0 E^2}{\lambda b_0 E + m^2}$$

- In particular when $m^2 \gg \lambda b_0 E$ we find that $W \approx \alpha b_0/2$ acting as a lower bound for the chiral relaxation time

$$\tau^{-1} > \alpha b_0/2.$$

Energy Loss polarization dependence

Neglecting the magnetic moment contribution, one can obtain an expression for the rate of energy loss in the unscreen limit

$$-\frac{dE(b_0\lambda<0)}{dz}\approx\frac{e^2E}{16\pi^2\ell}\left[\ln\frac{2E}{m}-\frac{1}{3}-\frac{2b_0E}{9m^2}\left(\pi+2\ln\frac{E}{b_0}\right)\right]$$

$$-\frac{dE(b_0\lambda>0)}{dz}\approx\frac{e^2E}{16\pi^2\ell}\left\{\ln\frac{2E}{m}-\frac{1}{3}+\frac{2b_0E}{9m^2}\left(\pi+2\ln\frac{E}{b_0}\right)+2\tau(E-\omega^*)\arctan\frac{2m^2\omega^*\tau}{E(E-\omega^*)}\right\}$$

- the screened limit has a similar behavior except that the effect due to the anomaly is suppressed similarly to the differiential cross section

$$-\frac{dE(b_0\lambda>0)}{dz}\approx\frac{e^2E}{16\pi^2\ell}\left(\ln\frac{2E}{\mu}+\frac{6b_0E}{7\mu^2}\ln\frac{E}{b_0}\ln\frac{4E^2}{\mu^2}+\frac{2b_0\mu\tau}{3E}\right)$$

$$-\frac{dE(b_0\lambda<0)}{dz}\approx\frac{e^2E}{16\pi^2\ell}\left(\ln\frac{2E}{\mu}-\frac{6b_0E}{7\mu^2}\ln\frac{E}{b_0}\ln\frac{4E^2}{\mu^2}\right)$$

Inhomogeneous Media

We focus on homogeneous media for the radiative energy loss, however, implications due to the chiral displacement **b** that lead to the quantum hall effect deserve consideration

The effect can be seen in the photon propagator in the static limit

$$D_{00} = \frac{iq^2}{q^4 + (b \times q)^2},$$
$$D_{0i} = \frac{(b \times q)_i}{q^4 + (b \times q)^2},$$
$$D_{ij} = -\frac{i}{q^4 + (b \times q)^2}\left\{q^2\delta_{ij} + b_i b_j - \frac{[q^4 - (b \cdot q)^2]q_i q_j}{q^4} - \frac{b \cdot q}{q^2}(b_i q_j + b_j q_i)\right\}$$

Though resonance behavior is not as readily apparent as in the case of a homogeneous media, effects due the dispersion relation may have important implications for future study