Recent advances in the calculation of two-point energy correlators in heavy ion collisions

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New jet quenching tools to explore equilibrium and non-equilibrium dynamics in heavy-ion collisions, ECT*, Trento, Italy, February13th, 2024.

C. Andres, FD, R. K. Elayavalli, J. Holguin, C. Marquet, I. Moult, arXiv:<u>2209.11236</u>
C. Andres, FD, J. Holguin, C. Marquet, I. Moult, arXiv:<u>2303.03413</u>
C. Andres, FD, J. Holguin, C. Marquet, I. Moult, arXiv:<u>2307.15110</u>









Energy flux operators



• Correlations of asymptotic energy flux provide valuable information about the underlying theory

$$\mathcal{E}(\vec{n}) = \lim_{r \to \infty} \int_0^\infty dt \, r^2 n^i T_{0i}(t, r\vec{n})$$
$$\mathcal{E}(\vec{n}) |X\rangle = \sum E_a \delta^{(2)} (\Omega_{\vec{p}_a} - \Omega_{\vec{n}}) |X\rangle$$

a



• 2-point function

$$\frac{\langle \mathcal{E}^n(\vec{n}_1)\mathcal{E}^n(\vec{n}_2)\rangle}{Q^{2n}} = \frac{1}{\sigma} \sum_{ij} \int \frac{d\sigma_{ij}}{d\vec{n}_i d\vec{n}_j} \frac{E_i^n E_j^n}{Q^{2n}} \delta^{(2)}(\vec{n}_i - \vec{n}_1) \delta^{(2)}(\vec{n}_j - \vec{n}_2)$$

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• Due to rotational symmetry, the only relevant variable is the opening angle

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Can be expressed as a weighted average of the double-inclusive cross-section

$$\frac{d\Sigma^{(n)}}{d\theta} = \frac{1}{\sigma} \sum_{i,j} \int dE_{i,j} \frac{d\sigma}{d\theta dE_i dE_j} \frac{E_i^n E_j^n}{Q^{2n}}$$

Energy correlators in vacuum

P. T. Komiske, I. Moult, J. Thaler, H. X. Zhu 2201.07800

- In the perturbative region it behaves as a power law
- Confinement scale brakes power law behavior at angles below $\Lambda_{\rm OCD}/E$
 - Small angles correspond to large times, where hadronization is dominant
 - Larger angles correspond to early times



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General advantages:

- Vacuum baseline is understood to a high degree of accuracy in the perturbative regime
- Low sensitivity to soft physics
- Does not require detector information at the particle level
- Alternative to de-clustering observables

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From a theorist perspective:

- Insensitive to large logs from soft divergences
- Collinear resummation needed, but medium contributions do not have collinear divergences
- Calculable in pQCD
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Potential to disentangle effects from different dynamics

- Quark-initiated jet with known initial energy $E(\gamma/Z$ -jet)
 - In this case energy loss effects are subleading and there is no need for resummation of soft emissions

$$\frac{d\Sigma^{(n)}}{d\theta} = \frac{1}{\sigma_{qg}} \int dz \left(\frac{d\sigma_{qg}^{\text{vac}}}{d\theta dz} + \frac{d\sigma_{qg}^{\text{med}}}{d\theta dz} \right) z^n (1-z)^n + \mathcal{O}\left(\frac{\mu_s}{E}\right) \qquad \textcircled{B}_{1-z}$$

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Includes collinear

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- Medium-splittings calculated on a brick
- Soft approximation ($z \rightarrow 0$, keeping zE finite) not suitable
- Two available approximations:
 - Semi-hard approximation: resums multiple scatterings, ignores momentum broadening in an eikonal approach
 FD, Milhano, Salgado, Tywoniuk, Vila <u>1907.03653</u>
 Isaksen, Tywoniuk <u>2107.02542</u>
 - Opacity expansion (GLV): possible unitarity issues, no eikonal assumptions involved

Sievert, Vitev <u>1807.03799</u>

Results in the semi-hard approximation



- No modification at small angles
- Transition towards medium-induced enhancement at larger angles
- Varying \hat{q} has different effects depending on medium resolution

C. Andres, FD, R. K. Elayavalli, J. Holguin, C. Marquet, I. Moult, arXiv:2209.11236 C. Andres, FD, J. Holguin, C. Marquet, I. Moult, arXiv:2303.03413

Results for first order in opacity



- Similar qualitative features compared to the semi-hard approach
- Transition towards medium-enhancement not so well defined
- Enhancement at large angles has a much smaller amplitude

C. Andres, FD, J. Holguin, C. Marquet, I. Moult, arXiv:2303.03413

Model comparison



- Small angle behavior and transition towards medium-enhancement common to all cases shown
- No information can be extracted from amplitude of enhancement at large angles

Moving away from the simplest scenario

- Improve the calculation for in-medium splittings
- Include an evolving medium
 - Brick calculations allow us to build some intuition
 - Any meaningful comparison to data must account for medium evolution effects
- Consider energy-loss effects, which will be important for inclusive jet measurements

Calculation of in-medium splittings

- Collinear (high-energy) limit: All particles have a large longitudinal momentum compared to their transverse momenta (small angles, DGLAP limit)
- Decoupling of transverse and longitudinal dynamics: Effects coming from the transverse (with respect to the direction of the jet) structure of the medium are suppressed by powers of the energy

See talks by Joseph Bahder and Xoán Mayo on Wed morning for advances towards relaxing this approximation

• Medium interactions are resummed through in-medium propagators

$$\begin{array}{ccc} \boldsymbol{p}_1, t_1 & \boldsymbol{\omega} & \boldsymbol{p}_2, t_2 \\ \hline \boldsymbol{p}_1, t_1 & \boldsymbol{\omega} & \boldsymbol{p}_2, t_2 \\ \hline \boldsymbol{p}_1, t_1; \boldsymbol{\omega} \end{array} = \mathcal{G}_R(\boldsymbol{p}_2, t_2; \boldsymbol{p}_1, t_1; \boldsymbol{\omega})$$

 Cross section are expressed in terms of medium averages of products of propagators

FD, Milhano, Salgado, Tywoniuk, Vila <u>1907.03653</u> Isaksen, Tywoniuk <u>2107.02542</u>

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 Take in-medium propagators in the extreme eikonal limit where they can be written as a Wilson line in a straight trajectory in coordinate space

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- The harmonic approximation (HO) is usually employed when using this approach but it is not necessary
- Angular and time scales are easily identifiable from the analytic formulas
- Numerical evaluations are straightforward in the large- N_c limit

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FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653

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$$\rightarrow \underbrace{\mathcal{G}_{0,R}(t_{2},\boldsymbol{\omega},t_{2},\boldsymbol{\sigma}_{1};\omega)}_{Q_{R}(t_{2},\boldsymbol{\omega},t_{2},\boldsymbol{\sigma}_{1};\omega)} \underbrace{V_{L}(t_{2},t_{1};[\boldsymbol{r}])}_{V_{R}(t_{2},t_{1};[\boldsymbol{r}])} \\ \operatorname{Blaizot, FD, Iancu, Mehtar-Tani 1209.4585}_{Apolinario, Armesto, Milhano, Salgado 1407.0599}$$

FD, Milhano, Salgado, Tywoniuk, Vila <u>1907.03653</u>

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$$\to (2\pi)^{2} \delta^{(2)}(\boldsymbol{p}_{2}-\boldsymbol{p}_{1}) e^{-i\frac{p_{2}^{2}}{2\omega}(t_{2}-t_{1})} V_{R}(t_{2}, t_{1}; [\boldsymbol{n}t])$$

- In the large- N_c limit, eikonal approximation can also be relaxed, with increased complexity in the formulas and higher computational cost for numerical evaluations

$$\mathcal{G}_{R}(t_{2},\boldsymbol{x}_{2};t_{1},\boldsymbol{x}_{1};\omega) = \int_{\boldsymbol{x}_{1}}^{\boldsymbol{x}_{2}} \mathcal{D}\boldsymbol{r} \exp\left\{\frac{i\omega}{2} \int_{t_{1}}^{t_{2}} d\xi \ \dot{\boldsymbol{r}}^{2}(\xi)\right\} \underbrace{\operatorname{P}\exp\left\{ig \int_{t_{1}}^{t_{2}} d\xi \ A_{R}^{-}(\xi,\boldsymbol{r}(\xi))\right\}}_{V_{R}(t_{2},t_{1};[\boldsymbol{r}])}$$

$$\rightarrow \underbrace{\mathcal{G}_{0,R}(t_{2},\boldsymbol{\omega},\boldsymbol{r},\boldsymbol{t}_{2};\omega)}_{V_{R}(t_{2},t_{2};\boldsymbol{t}_{1};[\boldsymbol{r}])} \underbrace{V_{R}(t_{2},t_{1};[\boldsymbol{r}])}_{V_{R}(t_{2},t_{2};\boldsymbol{t}_{2};\boldsymbol{t}_{2};\omega)} \underbrace{V_{R}(t_{2},t_{2};\boldsymbol{t}_{2};\boldsymbol{t}_{2};\omega)}_{Apolinario, Armesto, Milhano, Salgado 1407.0599}$$

- Going beyond the large- N_c limit requires a much more complex setup where a system of coupled differential equations must be numerically solved

FD, Milhano, Salgado, Tywoniuk, Vila <u>1907.03653</u>

• Corrections due to momentum broadening can be easily included

$$\mathcal{G}_{R}(t_{2}, \boldsymbol{p}_{2}; t_{1}, \boldsymbol{p}_{1}; \omega) \to e^{-i\frac{p_{2}^{2}}{2\omega}(t_{2}-t_{1})} \int_{\boldsymbol{x}} e^{-i(\boldsymbol{p}_{1}-\boldsymbol{p}_{0})\cdot\boldsymbol{x}} V_{R}(t_{2}, t_{1}; [\boldsymbol{x}+\boldsymbol{n}t])$$
$$\to (2\pi)^{2} \delta^{(2)}(\boldsymbol{p}_{2}-\boldsymbol{p}_{1}) e^{-i\frac{p_{2}^{2}}{2\omega}(t_{2}-t_{1})} V_{R}(t_{2}, t_{1}; [\boldsymbol{n}t])$$

- In the large- N_c limit, eikonal approximation can also be relaxed, with increased complexity in the formulas and higher computational cost for numerical evaluations

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$$\rightarrow \underbrace{\mathcal{G}_{0,R}(t_{2},\omega,t_{2},\boldsymbol{x}_{1};\omega)}_{V_{R}(t_{2},t_{1};[\boldsymbol{r}])} \underbrace{V_{R}(t_{2},t_{1};[\boldsymbol{r}])}_{\text{Apolinario, Armesto, Milhano, Salgado 1407.0599}}$$

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FD, Milhano, Salgado, Tywoniuk, Vila <u>1907.03653</u>

Results for a brick with the harmonic approximation



- Large reduction of the enhancement amplitude
- Small dip before transitioning to medium-enhancement, already present when the broadening correction is included
- Overall picture of no modification at small angles followed by a transition to medium-enhancement at large angles still valid

Beyond brick calculations

- Essential if we want to understand anything about how geometry affects our observables
- Repeat the previous calculations with a varying \hat{q}
- It is straightforward when eikonal propagation has been assumed (semi-hard, broadening correction)
- Becomes a lot more involved when eikonal approximation is relaxed (large- N_c , will not be shown in this talk)

Results using a hydro simulation



- We assume $\hat{q} \propto T^3$ and take the temperature from trajectories extracted from a smooth-averaged hydro simulation

Luzum, Romatschke <u>0804.4015</u> <u>0901.4588</u>

- Qualitative behavior similar to brick case
- Dependence on geometry can be explored

Beyond the harmonic approximation

- Theorists like the harmonic approximation because it allows us to perform analytical calculations when resumming multiple scatterings
- But, the harmonic approximation misses the rare large momentum transfer scatterings characteristic of point-like particles (Molière)
- These rare hard scatterings can be straightforwardly implemented in the semi-hard approximation and the broadening correction, but not clear the approximations are compatible
- When eikonal approximation is relaxed (large- N_c), going beyond the harmonic approximation has not yet been done but feasible to generalize from progress achieved for spectrum in the soft-limit

Andres, Apolinario, FD 2002.01517

Summary of available calculations

	Brick	Evolving media	High-momentum tails
Semi-hard			But
Broadening correction			But
Large-Nc		Feasible	Computationally costly
Isaksen, Tywoniu <u>k</u> $\gamma \rightarrow a\overline{a}$?	Straightforward	Straightforward

Energy loss

- If we don't know the initial energy of the initial hard parton we cannot neglect the effects of energy loss due to multiple soft emissions
- This is the case for inclusive jet measurements, which will be the first heavy ion measurements of energy correlators
- Energy loss effects on the EECs were studied recently in a simplified model with a resolution scale finding very small modifications

Barata, Caucal, Soto-Ontoso, Szafron 2312.12527

• Nevertheless, coherent energy loss from the whole jet shifts the jet energy scale resulting in a shift in the vacuum baseline



Results for inclusive jets

 In order to account for the shift in the energy scale we convolute our results for known initial energies with quenching weights accounting for the energy carried away by an arbitrary number of soft emissions

Salgado, Wiedemann hep-ph/0302184

 We use our calculations done with a expanding medium (modeled by a smooth-averaged hydro simulation) with the semi-hard approximation with the broadening correction



Results for inclusive jets



- All structure seen at small angles due to shift in energy scale, independent of fitting parameter
- Transition to medium-enhancement still present
- Amplitude of medium-enhancement at large angles largely overestimated

Outlook

- Steady progress towards meaningful comparisons with data
- Plenty of room for improvement from the theory side
 - Fully relaxing the eikonal approximations for expanding media
- In order to use this observable to extract information from the QGP we must consider variations in centrality, jet energy, center of mass energy, etc.
- Time to start thinking about more differential observables?

Thank you!

E

-z

• At leading order



E

• At leading order

• For jets we are interested in the collinear (or OPE) limit

E

• At leading order

• For jets we are interested in the collinear (or OPE) limit

$$\mathcal{O}(x)\mathcal{O}(y) \xrightarrow{x \to y} \sum_i |x - y|^{\gamma_i} c_i \mathcal{O}_i$$

• At leading order

• For jets we are interested in the collinear (or OPE) limit

Light-ray OPE

E

• At leading order

• For jets we are interested in the collinear (or OPE) limit

Light-ray OPE

E

$$\frac{d\Sigma^{(1)}}{d\theta} \sim \frac{1}{\theta^{1-\gamma(3)}}$$

 $\gamma(3)$ is the twist-2 spin-3 QCD anomalous dimension

• At leading order

• For jets we are interested in the collinear (or OPE) limit

Light-ray OPE

E

 10^{0}

 10^{-3}

 10^{-2}

 10^{-1}

θ

EECs from massive jets

 Dead-cone effect: radiation from heavy quarks is suppressed at small angles

$$\frac{d\sigma_M^{\text{vac}}}{d\theta dz} \sim \frac{\theta^3}{(\theta^2 + \frac{\theta_0^2}{1-z})^2} \frac{d\sigma^{\text{vac}}}{d\theta dz} \qquad \begin{array}{l} \text{Dead-cone angle!} \\ \theta_0 = \frac{M}{E} \end{array}$$

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- Deviation from power-law behavior occurs at the dead-cone angle in the perturbative regime
- In the massless case, the transition to the non-perturbative regime can be modeled by putting a gluon mass



FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653



FD, Milhano, Salgado, Tywoniuk, Vila <u>1907.03653</u>

E

• For a static medium of length L within the harmonic approximation with jet quenching parameter \hat{q} one can read off the relevant scales directly from the formulas

FD, Milhano, Salgado, Tywoniuk, Vila <u>1907.03653</u>

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- For a static medium of length L within the harmonic approximation with jet quenching parameter \hat{q} one can read off the relevant scales directly from the formulas
 - (Vacuum) formation time:

$$t_f = \frac{2}{z(1-z)E\theta^2}$$

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 $\theta_L \sim (EL)^{-1/2}$

Below θ_L all emissions have a formation time larger than L

FD, Milhano, Salgado, Tywoniuk, Vila <u>1907.03653</u>

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$$\theta_c \sim (\hat{q}L^3)^{-1/2}$$

Below θ_c splittings do not lose color coherence and the medium does not resolve them

FD, Milhano, Salgado, Tywoniuk, Vila <u>1907.03653</u>

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S

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radiation as total charge radiation as independent θ_c then θ_c becomes irrelevant charges 26

Coherence transition



- Extracted the peak angle θ_{peak} for 332 sets of parameters with $E \in [50,700]$ GeV, $L \in [0.2,10]$ fm, $\hat{q} \in [1,3]$ GeV²/fm
- Performed separate fits in the two different regions for the scaling behavior of the peak angle with respect to the 3 parameters

C. Andres, FD, R. K. Elayavalli, J. Holguin, C. Marquet, I. Moult, arXiv:2209.11236 C. Andres, FD, J. Holguin, C. Marquet, I. Moult, arXiv:2303.03413

Massive EEC in HIC

Including mass in medium-induced calculations is straightforward



- Dead-cone is filled for lower energy jets
- When dead-cone is filled the mass changes also the large angle behavior

C. Andres, FD, J. Holguin, C. Marquet, I. Moult, arXiv:2307.15110

Massive EEC in HIC

Experimentally, the cleanest observable would be the b/c ratio



C. Andres, FD, J. Holguin, C. Marquet, I. Moult, arXiv:2307.15110

EECs for inclusive jets

- First measurements will be performed for inclusive jets
- In this case we do not know the energy of the initial parton and therefore energy loss becomes important
- HI-jets have a higher initial energy than pp-jets and therefore the transition to NP regime happens at smaller angles



Results with Yukawa interaction



Results with Yukawa interaction


Results with single scattering (GLV)



Results with single scattering (GLV)



Higher energy power



FD, Milhano, Salgado, Tywoniuk, Vila <u>1907.03653</u> Isaksen, Tywoniuk <u>2107.02542</u>

 Use high-energy limit of propagators: vacuum propagator times a Wilson line in the classical trajectory

 $\mathcal{G}_R(t_2, \boldsymbol{p}_2; t_1, \boldsymbol{p}_1; \omega) \to (2\pi)^2 \delta^{(2)}(\boldsymbol{p}_2 - \boldsymbol{p}_1) e^{-i\frac{\boldsymbol{p}_2^2}{2\omega}(t_2 - t_1)} V_R(t_2, t_1; [\boldsymbol{n}t])$

• Calculate averages of Wilson lines in the large- N_c limit (calculations also available for finite N_c). All averages can be expressed in terms of fundamental dipoles and quadrupoles

$$\frac{1}{N_c} \left\langle \operatorname{Tr} V_1 V_2^{\dagger} \right\rangle = S_{12} \qquad \qquad \frac{1}{N_c} \left\langle \operatorname{Tr} V_1 V_2^{\dagger} V_{\bar{2}} V_{\bar{1}}^{\dagger} \right\rangle = Q$$

FD, Milhano, Salgado, Tywoniuk, Vila <u>1907.03653</u> Isaksen, Tywoniuk <u>2107.02542</u>

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$$\frac{1}{N_c} \left\langle \operatorname{Tr} V_1 V_2^{\dagger} V_2 V_1^{\dagger} \right\rangle = Q$$

$$\underset{\text{emission process}}{\otimes}$$

$$\underset{\text{Broadening}}{\otimes}$$

EECs and color coherence

