



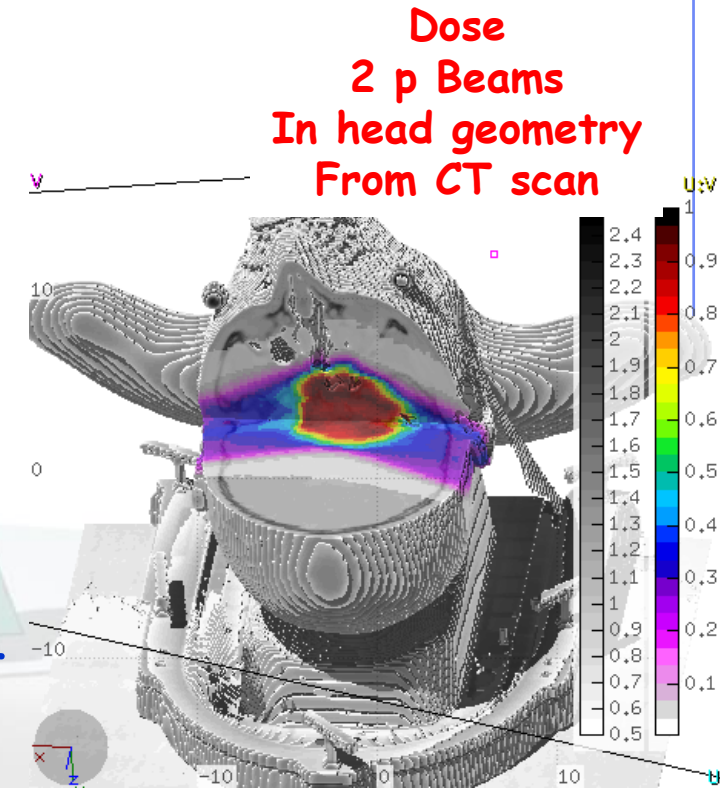
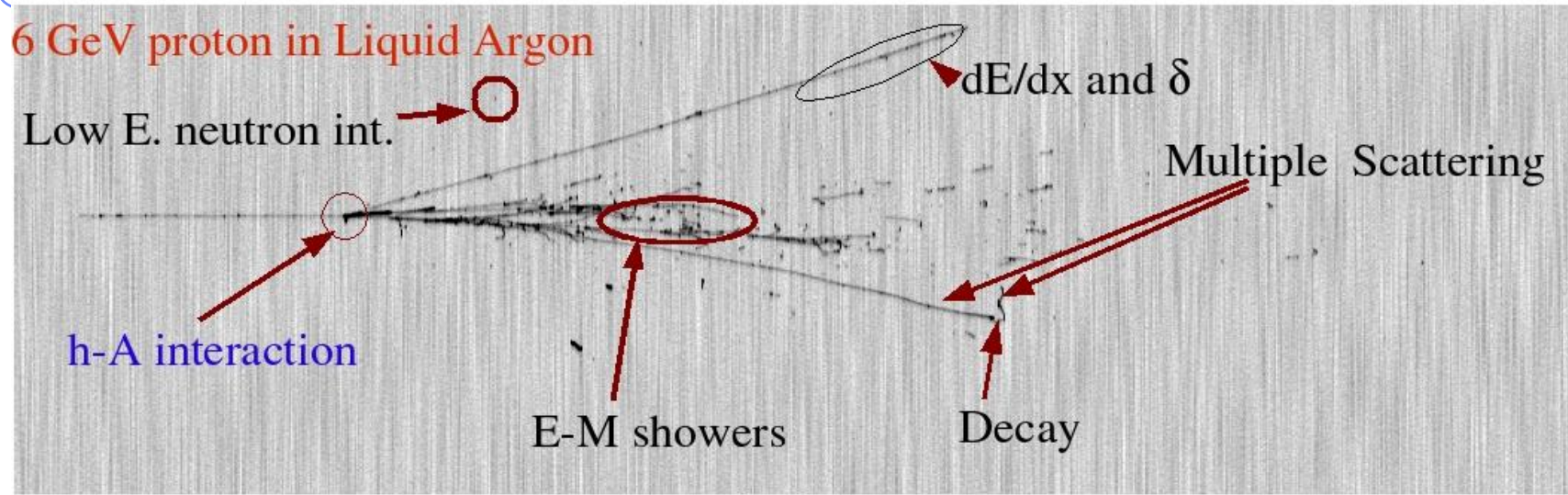
 In memory of  
Hannes Ranft

## Neutrino interactions in FLUKA: NUNDIS

M. Antonello, G. Battistoni, A. Ferrari, M. Lantz, **P. Sala**, G. Smirnov

# FLUKA : a multi-purpose Monte Carlo code

In memory of Hannes Ranft



Developed and maintained under an INFN-CERN agreement  
Copyright 1989-2018 CERN and INFN

Web Site: <http://www.fluka.org>

>10000 registered users

2 user courses /year

july 2018

Trento workshop

# General framework

- Nuclear models in FLUKA have been developed along the years, initially for hadron-nucleus reactions,
- Then extended to treat also
  - Photonuclear reactions
  - Muon-nuclear and electro-nuclear via virtual photon exchange
  - Quasi-elastic electron scattering
  - Muon capture
  - Neutrino interactions
  - Anti-nucleon reactions
- All sharing the same generalized IntraNuclearCascade + Preequilibrium+Evaporation +Gamma deexcitation model
- All DIS share the same fragmentation
- PROs: well tested, valid for all target nuclei, consistent with detector simulations
- CONs : some details of nuclear structure missing, coherent effects have to be inserted ad-hoc

# Neutrinos in FLUKA

- Generators of neutrino-nucleon interactions:
  - QuasiElastic
  - Resonance
  - DIS
- Embedded in FLUKA nuclear models for Initial State and Final State effects
- Only for Argon: absorption of few-MeV (solar) neutrinos on whole nucleus
- Elastic scattering on electrons - to be refreshed
- Products of the neutrino interactions can be directly transported in the detector (or other) materials
- Used for all ICARUS simulations/publications

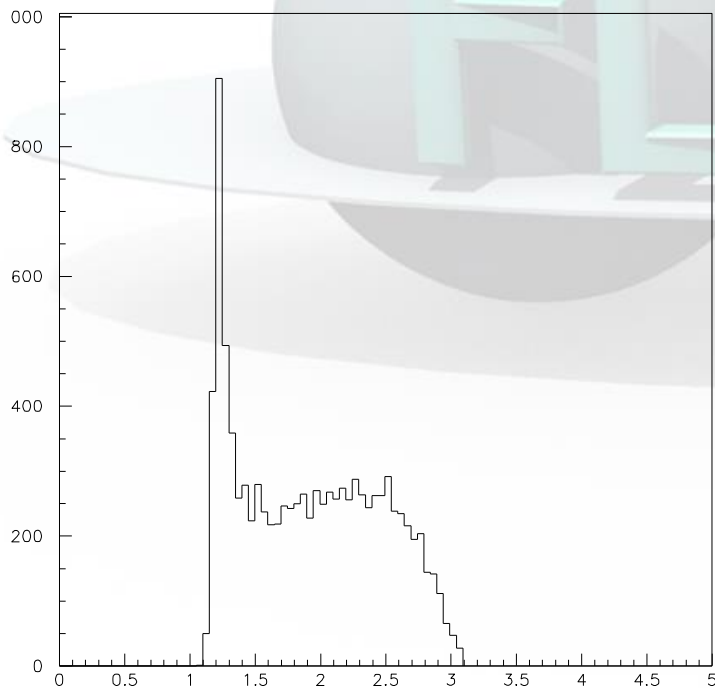
Acta Phys.Polon. B40 (2009) 2491-2505  
CERN-Proceedings-2010-001 pp.387-394.

# Quasi Elastic

- Following Llewellyn Smith formulation
- $M_A = 1.03$ ,  $M_V = 0.84$
- Lepton masses accounted for
- Polarization of the outgoing lepton is calculated (and used in lepton decay) according to Albright-Jarlskog \*
- \* later applied also to leptons produced in RES/DIS

# Resonance production

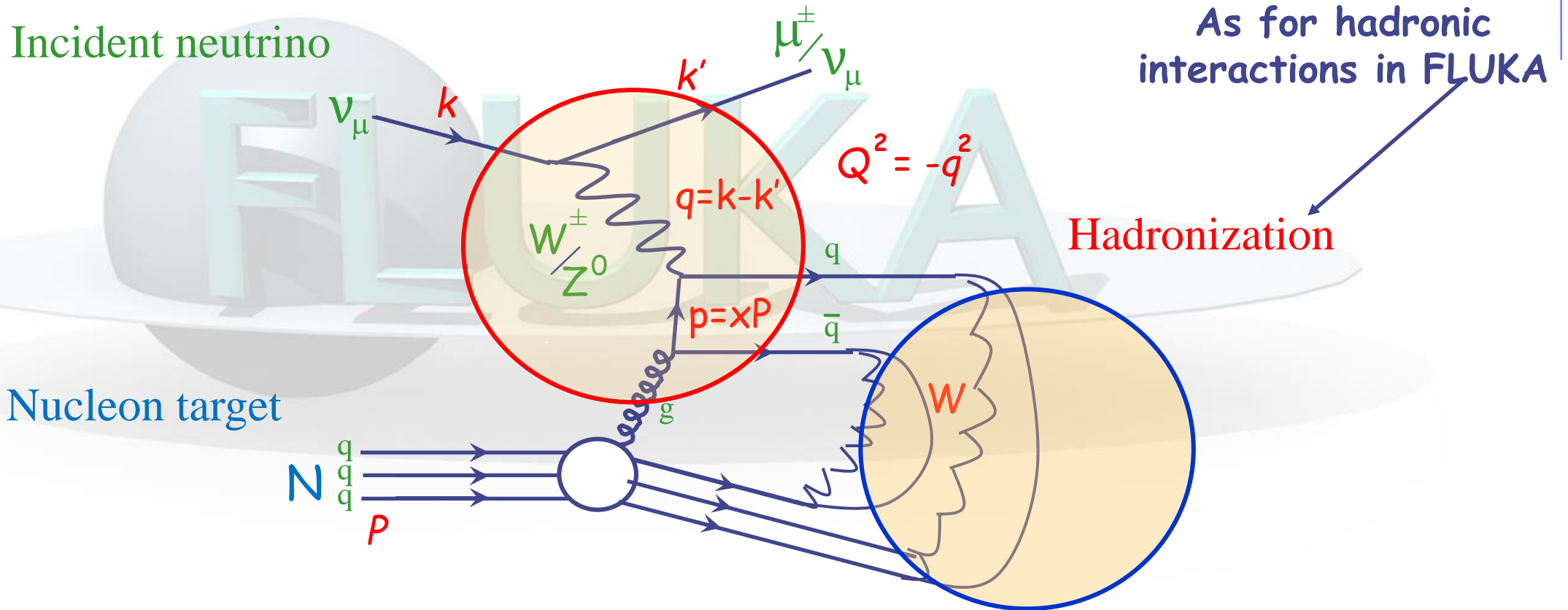
- From Rein-Sehgal formulation
- Keep only  $\Delta$  production
- No non-resonant background term, assuming that the non-resonant contribution comes from NunDIS
- **TRANSITION** from RES to DIS: linear decrease of both  $\sigma$  as a function of  $W$



Hadronic mass distribution for  $\nu_\mu$  CC on p at 5 GeV

# DIS (NUNDIS)

FLUKA hadronization and nuclear interactions work well independently of primary interaction vertex



# Sample $x$ and $Q^2$ from double differential cross sections

$$\frac{d^2\sigma}{dx dQ^2} = \frac{d^2\sigma}{dx dy} \cdot \frac{dy}{dQ^2} = \frac{d^2\sigma}{dx dy} \cdot \frac{1}{2ME_\nu x}$$

$$\frac{d^2\sigma}{dx dy} = \frac{G_F^2 M E_\nu}{\pi(1 + Q^2/M_{W/Z}^2)^2} \sum_{i=1}^5 A_i(x, y, E_\nu) F_i(Q^2, x)$$

## Structure functions $F_i(Q^2, x)$

$$F_2^{\nu p}(Q^2, x) = 2x[d + \bar{u} + s + \bar{c}]$$

$$xF_3^{\nu p}(Q^2, x) = 2x[d - \bar{u} + s - \bar{c}]$$

**Callan-Gross relation:**  $F_1 = \frac{F_2}{2x}$

To be updated to

$$2xF_1(Q^2, x) = F_2(Q^2, x) \frac{1 + 4M^2 x^2 / Q^2}{1 + R(Q^2, x)}$$

**Albright-Jarlskog relations:**

$$F_4 = 0,$$

$$F_5 = \frac{F_2}{x}.$$

$$A_1 = y \left( xy + \frac{m_\ell^2}{2ME_\nu} \right)$$

$$A_2 = 1 - y \left( 1 + \frac{Mx}{2E_\nu} \right) - \frac{m_\ell^2}{4E_\nu^2}$$

$$A_3 = \pm y \left[ x \left( 1 - \frac{y}{2} \right) - \frac{m_\ell^2}{4ME_\nu} \right]$$

$$A_4 = \frac{m_\ell^2}{2ME_\nu} \left( y + \frac{m_\ell^2}{2ME_\nu x} \right)$$

$$A_5 = -\frac{m_\ell^2}{ME_\nu}$$



# Quark dependence $q_i(Q, x)$ determined from Parton Distribution Functions (PDFs)

GRV94	Glück et al., Z. Phys. C67 (1995) 433.
<b>GRV98</b>	<b>Glück et al., Eur. Phys. J. C5 (1998) 461.</b>
BBS	Bourelly et al., Eur. Phys. J. C23 (2003) 487.
CTEQ	J. High Energy Phys. 0207 (2002) 012.
MRST	arXiv:hep-ph/0211080.
Alekhin	Phys. Rev. D68 (2003) 014002.
	...

**DEFAULT OPTION**

**NUNDIS WORKS WITH THESE PDFs**

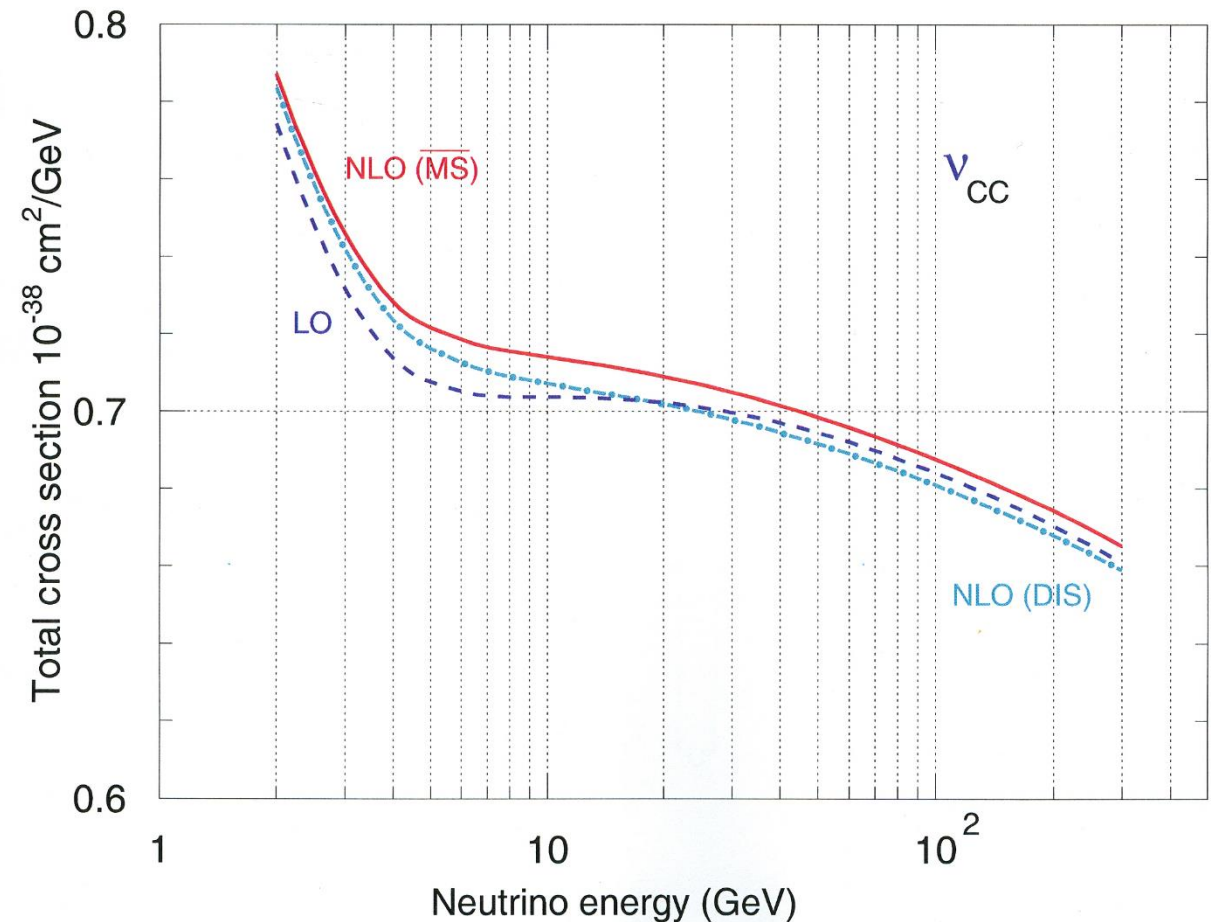
## More on pdfs

Three versions of pdf from the GRV98 analysis are included as options for evaluating nucleon structure functions

1. Leading order analyses (LO)
2. Next to leading order analyses (NLO  $\overline{\text{MS}}$ )
3. Next to leading order analyses (NLO DIS)

An interesting feature of the GRV98 analysis is a low threshold for the transferred, 4-momentum,  $Q^2 = 0.8 \text{ GeV}^2$

**NLO (DIS) is chosen as a default option**



# Extrapolation from $Q^2 = 1.0 \text{ GeV}^2$ to $Q^2 = 0$

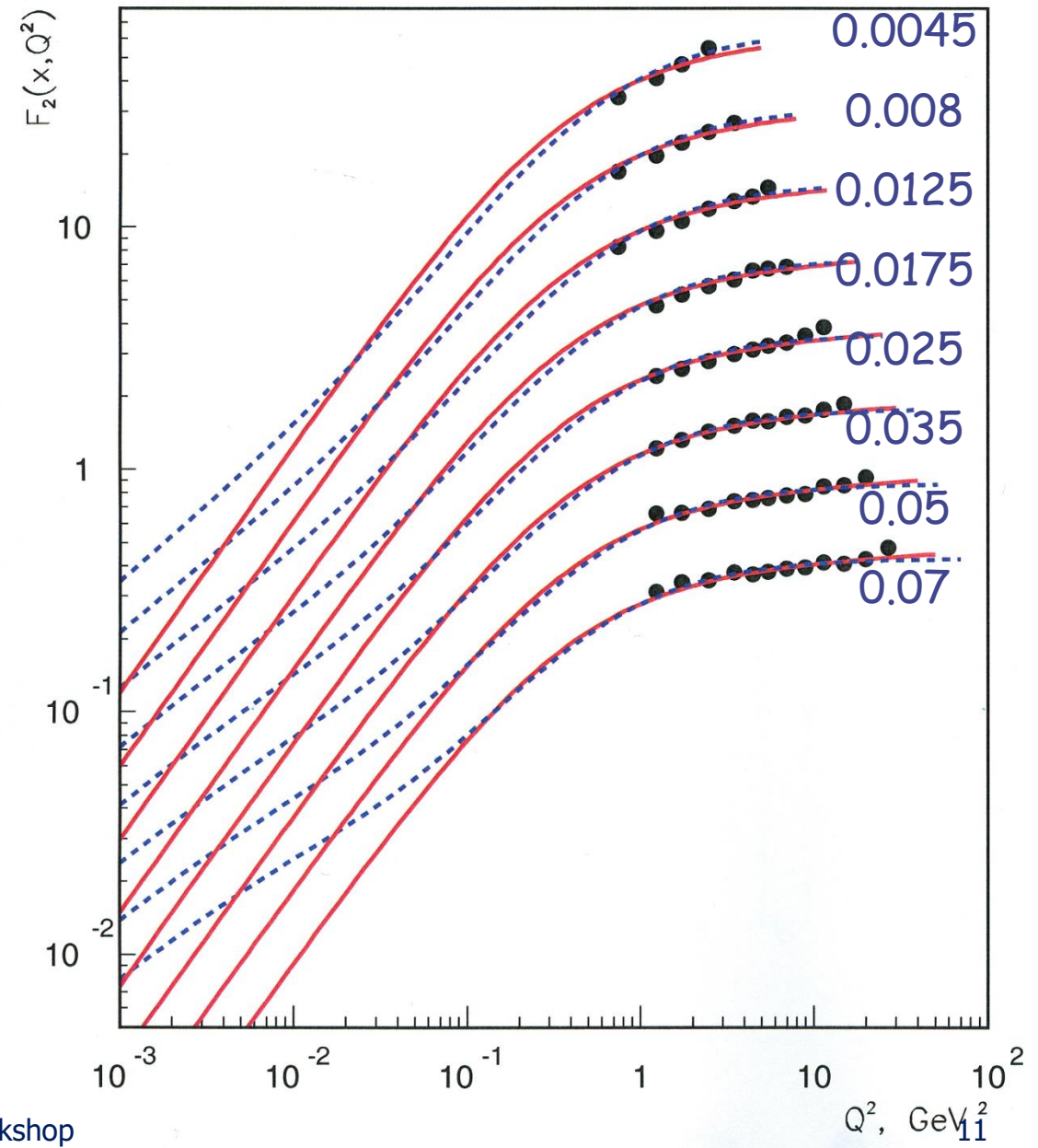
Solid lines: M. Bertini et al. 1996 (Default in NUNDIS)

$$F_2(x, Q^2) = A \left[ 1 + \epsilon \ln(Q^2(1/x - 1) + M^2) \right] \ln(1 + Q^2/(Q^2 + a^2)) .$$

Dashed lines: Donnachie-Landshoff 1994

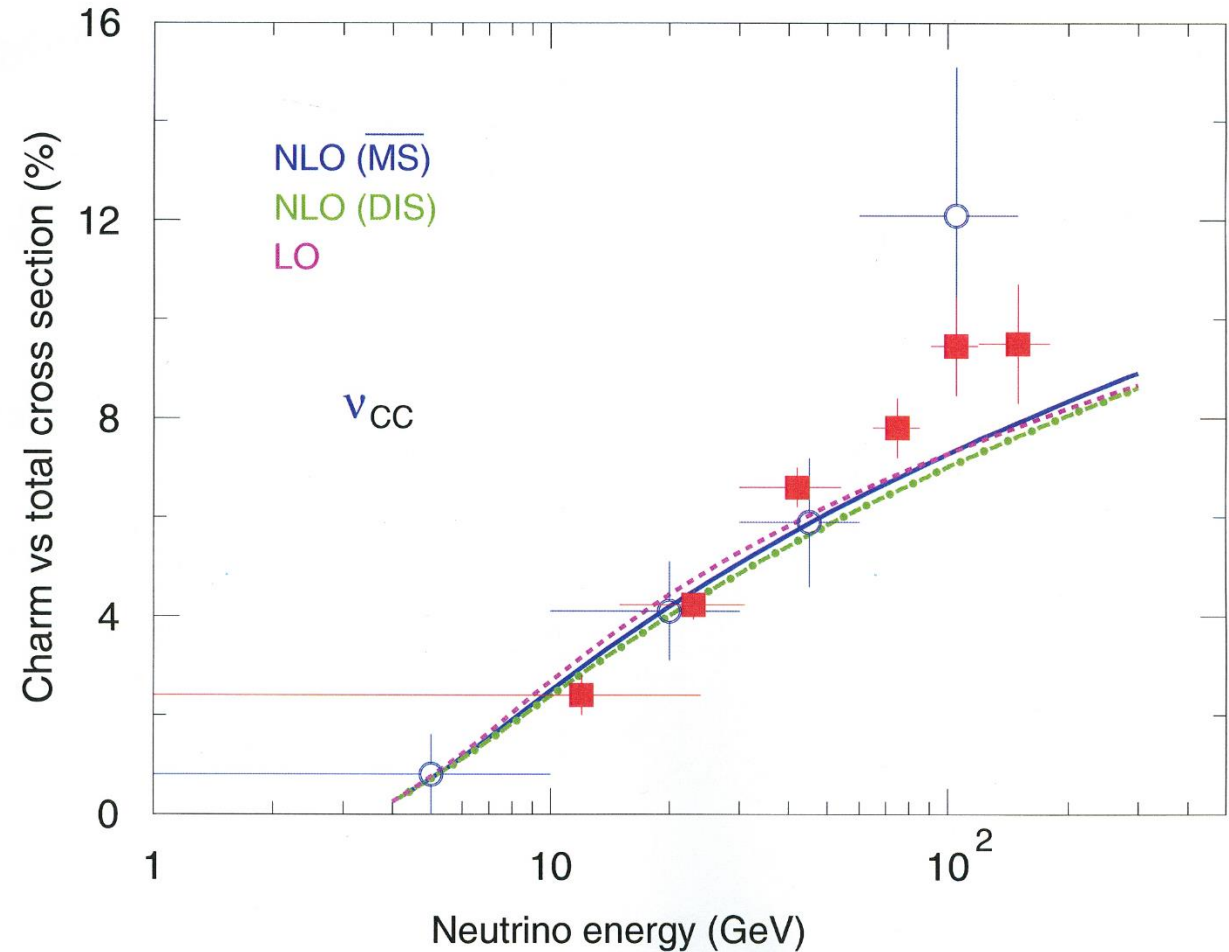
$$F_2(x, Q^2) \sim Ax^{-0.0808} \left( \frac{Q^2}{Q^2 + a} \right)^{1.0808} + Bx^{0.4525} \left( \frac{Q^2}{Q^2 + b} \right)^{0.5475}$$

data points from NMC Collab., M. Arneodo et al., Nucl. Phys. B 483 (1997) 3-43  
Data/cuves scaled for clarity, factors from 1 to 128



# Charm production in neutrino interactions

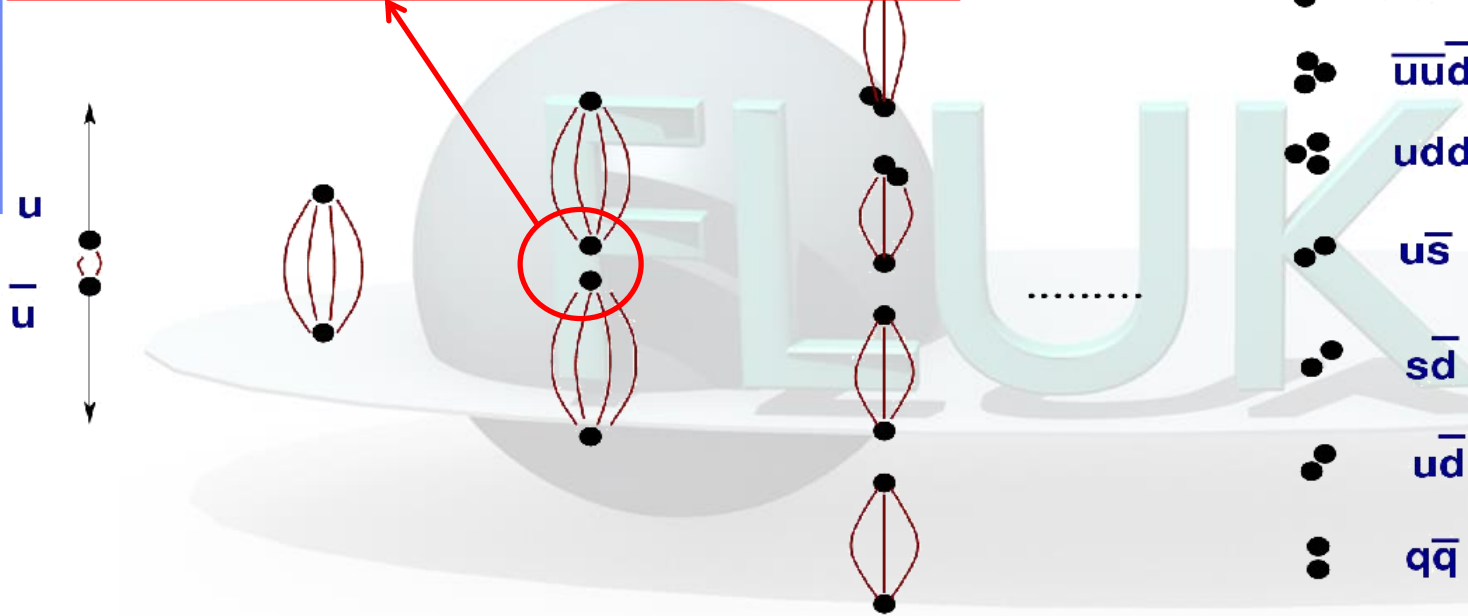
- Ratio of the charm to total cross sections
- Results of NUNDIS simulation with  $M_c = 1.35$  GeV (curves) and experimental data: E531 (open circles) and CHORUS-2011 (filled squares).



# The "hadronization" of chains

An example:

Low mass chain: just 2-3 meson/(anti)baryon resonances



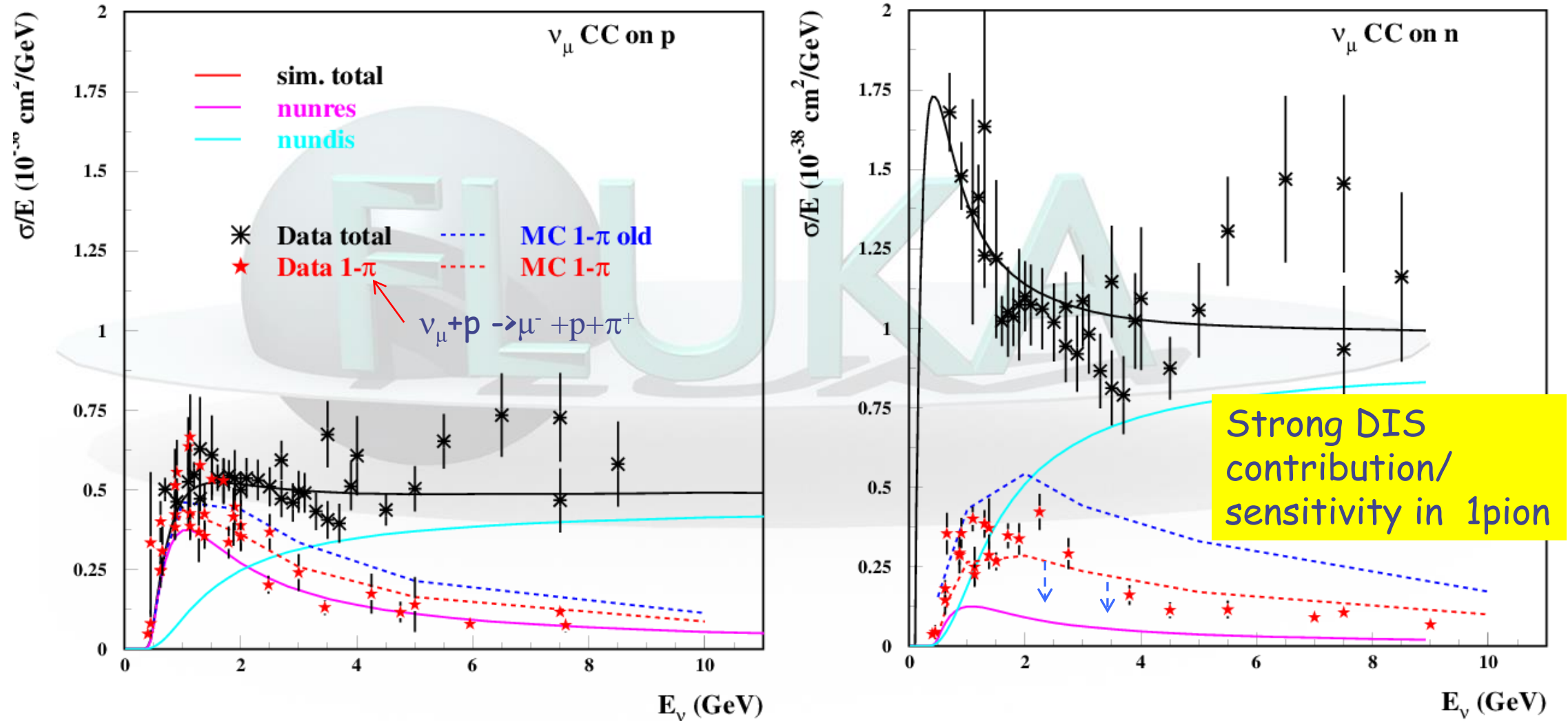
In FLUKA:

- Assumes chain universality
  - Fragmentation functions from hard processes and  $e+e-$  scattering
  - Transverse momentum from uncertainty considerations
  - Mass effects at low energies
- The same functions and (few) parameters for all reactions and energies
- Chains from  $\nu$  DIS :
    - One quark-diquark chain if interaction of valence quark
    - One quark-diquark plus one  $q-qbar$  chain if int on sea quark

gradual transition of **low energies chains** to "phase space explosion" constrained in  $p_T$ , including baryons, mesons, resonances.

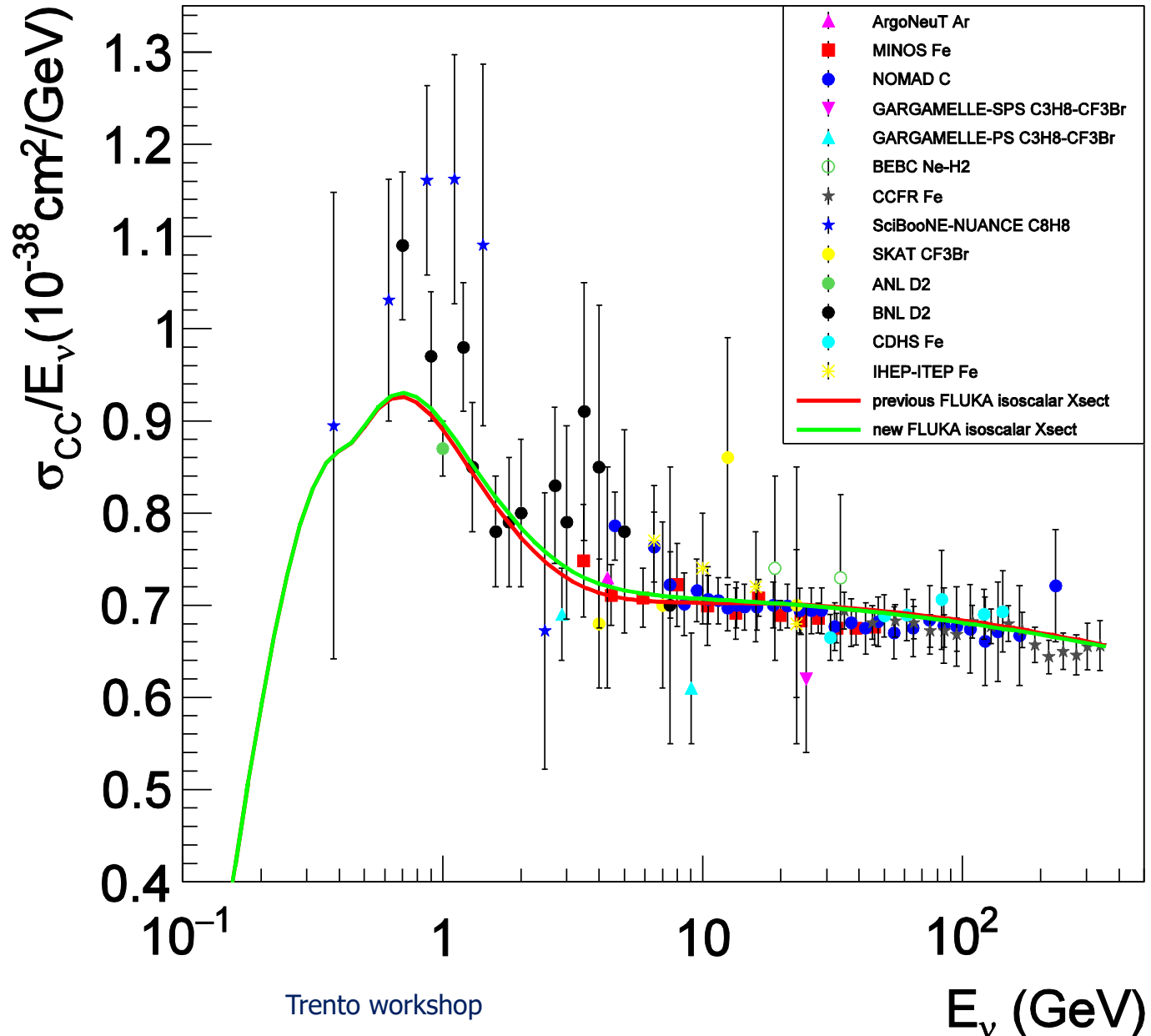
# Single pion production

New *low-mass chain treatment of fragmentation* → improvements in the **RES-DIS** transition

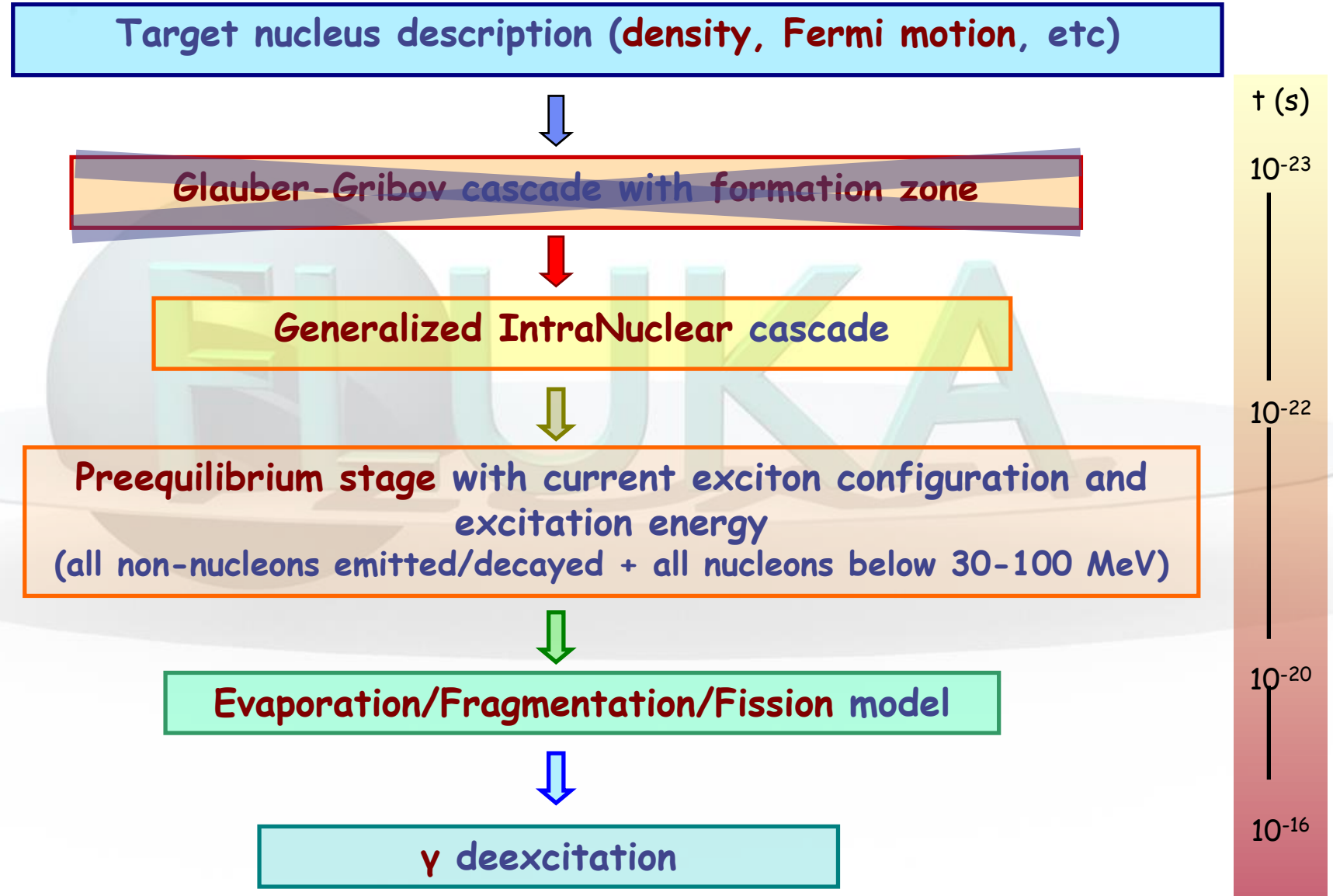


# Comparison with data on total cross section

Isoscalar  
 $\nu_\mu$  - Nucleon total  
 CC cross section  
 Fluka (lines) with  
 two pdf options  
 Vs  
 Experimental data



# Nuclear interactions in FLUKA: the PEANUT model





# (Generalized) IntraNuclear Cascade

- Primary and secondary particles moving in the nuclear medium
- Target nucleons motion and nuclear potential well according to **local Fermi gas model**
- Interaction probability  
 $\sigma_{\text{free}} + \text{Fermi motion} \times \rho(r) + \text{exceptions (ex. } \pi)$ 
 $\sigma_{\text{free}}$  includes inelastic
- **Glauber cascade at higher energies**
- Classical trajectories (+) nuclear mean potential (**resonant for  $\pi$** )
- Curvature from nuclear potential → **refraction and reflection**
- Interactions are incoherent and uncorrelated
- Interactions in projectile-target nucleon CMS
- Fully relativistic
- **Multibody absorption for  $\pi, \mu^-$**
- **Special for  $K^-$ , antinucleon,  $\pi$**  (phase shifts, annihilation)
- **Quantum effects** (Pauli, formation zone, correlations...)
- **Exact conservation** of energy, momenta and all additive quantum numbers, including nuclear recoil
- First **excited nuclear levels** accounted for (more levels in evaporation/gamma deexc)

# Nucleon Fermi Motion in FLUKA

- Fermi gas model: Nucleons = Non-interacting Constrained Fermions

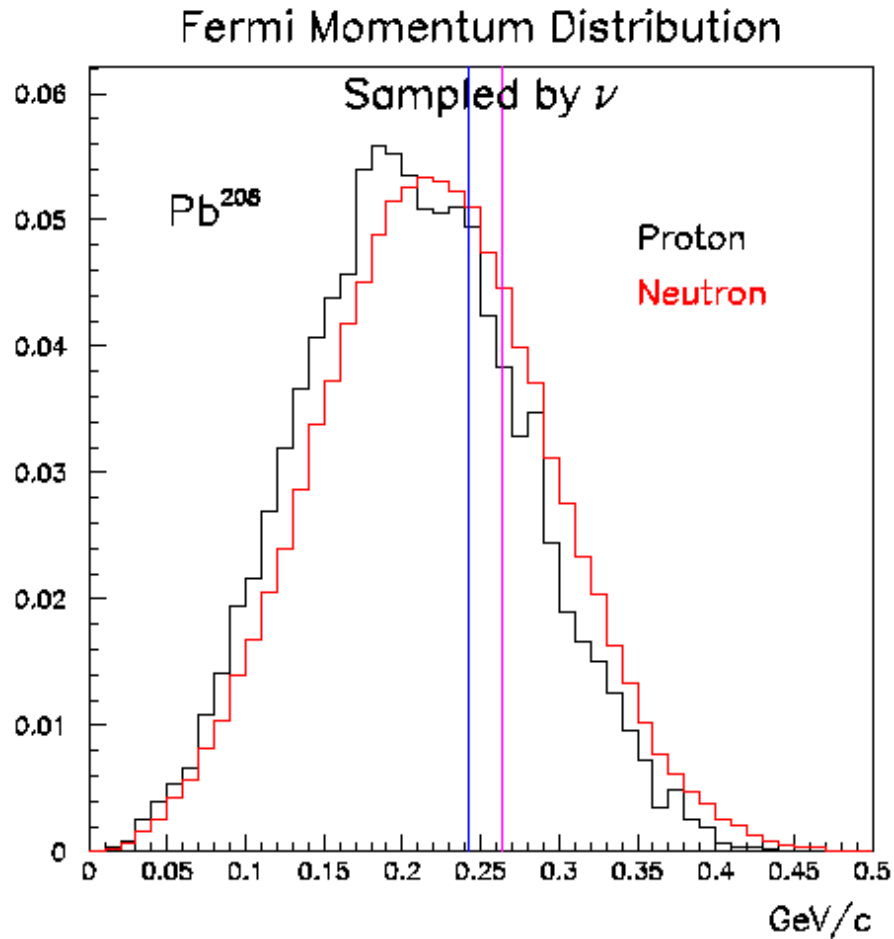
**Momentum distribution**

$$\propto \frac{dN}{dk} = \frac{|k|^2}{2\pi^2}$$

for  $k$  up to a (local) Fermi momentum  $k_F(r)$  given by  $k_F(r) = [3\pi^2 \rho_N(r)]^{1/3}$

- **Momentum smearing** according to uncertainty principle assuming a position uncertainty =  $\sqrt{2}$  fm
- Nuclear density given by symmetrized Woods-Saxon for  $A > 16$  and by a harmonic oscillator shell model for light isotopes
- Proton and neutron densities are different

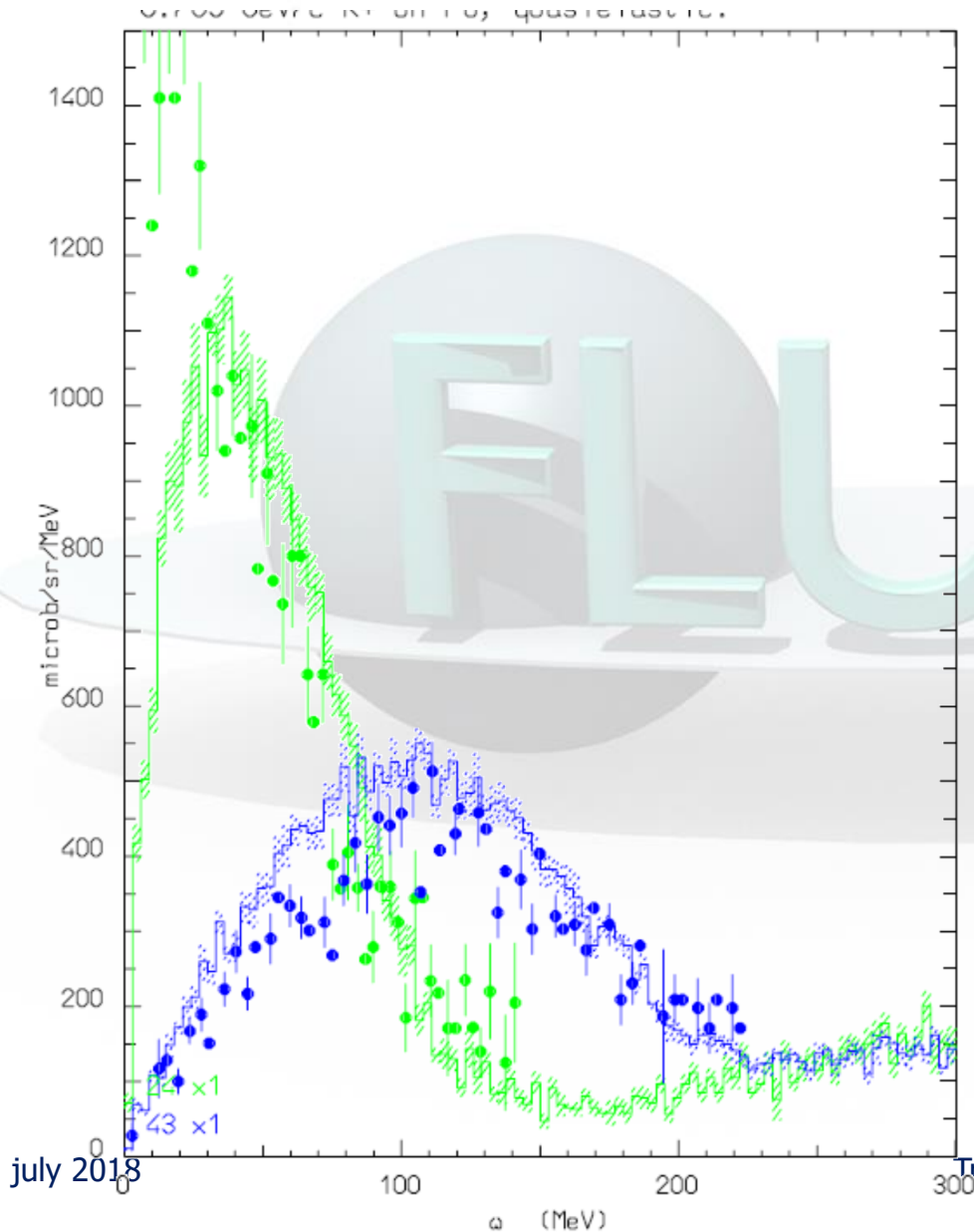
# Example of Fermi distribution



Fermi momentum distribution as "seen" by interacting neutrinos on lead.

Vertical lines: maximum Fermi momentum according to un-smearred distribution

# Positive kaons as a probe of Fermi motion



$K^+$  and  $K^0$

- No low mass  $S=1$  baryons
  - weak  $K^+N$  interaction
  - Only elastic and charge exchange up to  $\approx 800$  MeV/c

$K^+ \text{ Pb} \rightarrow K^+ \text{ Pb}$  705 MeV/c

Residual excitation spectrum

With  $K^+$  at  $24^\circ$  (green)

at  $43^\circ$  (blue)

Histogram : FLUKA

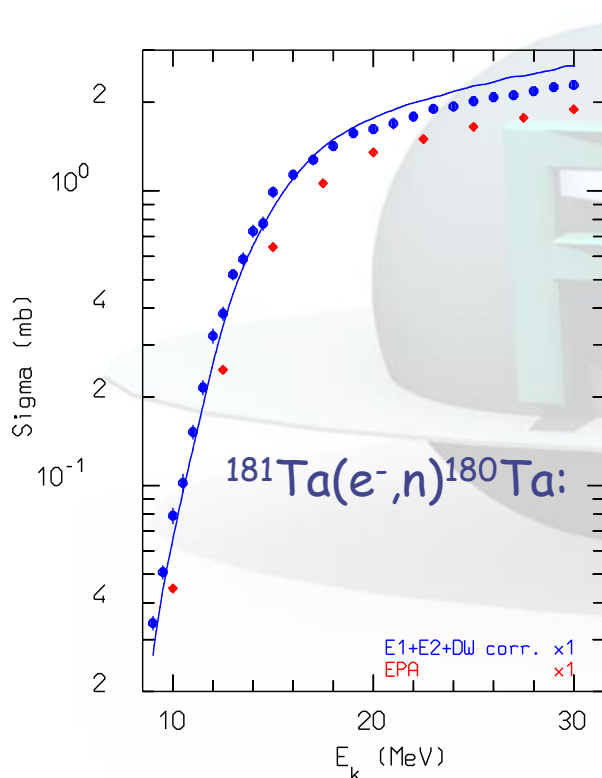
Dots : data (Phys. Rev. C51,669 (1995))

On free nucleon: recoil at 43 MeV or 117 MeV

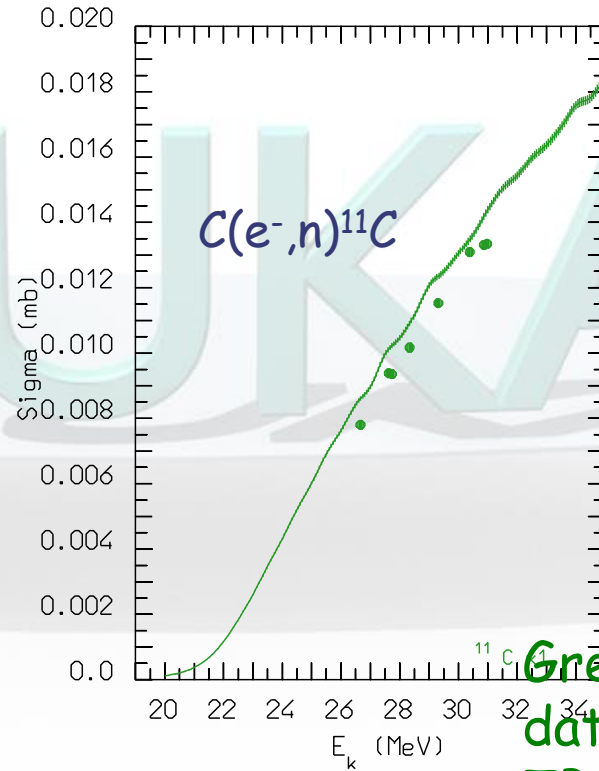
0-deg tail is elastic on nucleus, not included in sim

# Electron scattering

- Quasi-Elastic on nucleon (+ all nuclear)
- Inelastic via virtual photon exchange, recently improved (E1+E2 )



Blue symbols: data from JPG13,515  
Blue line: FLUKA new  
Red symbols: FLUKA with EPA



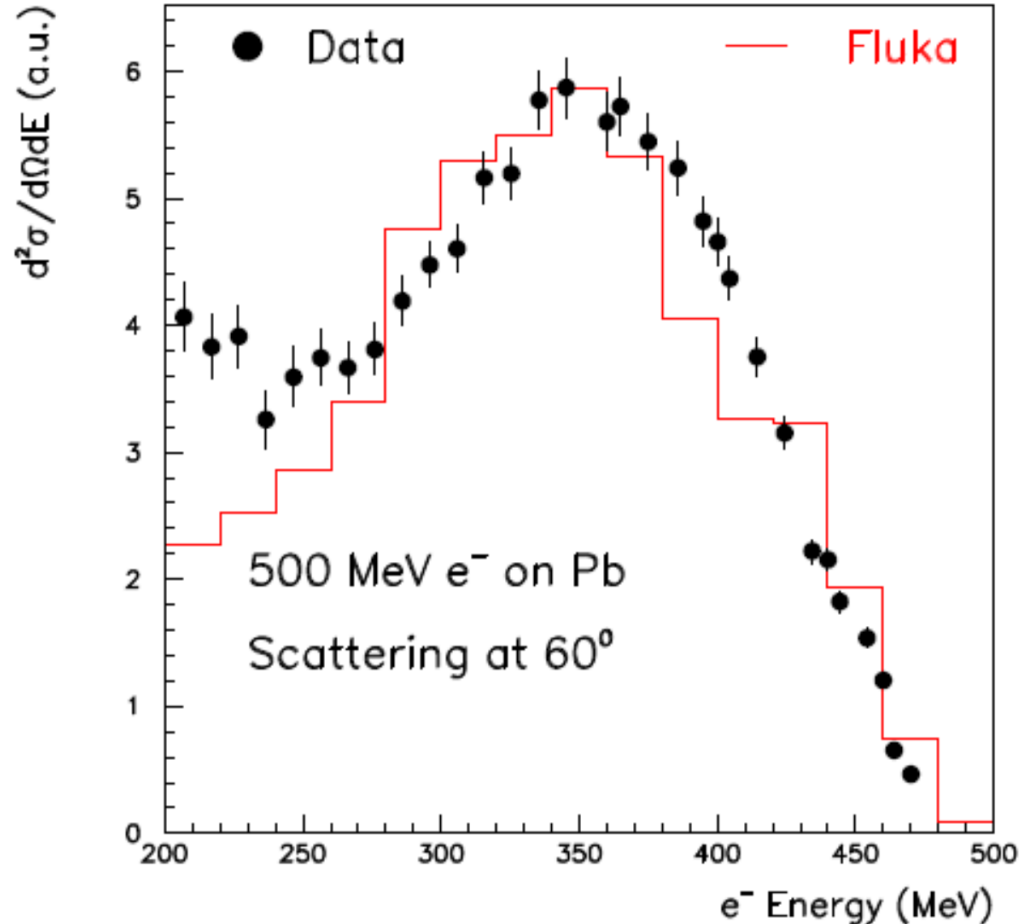
Green symbols:  
data from  
ZPA281,35  
Green curve:  
FLUKA

# First checks with electrons

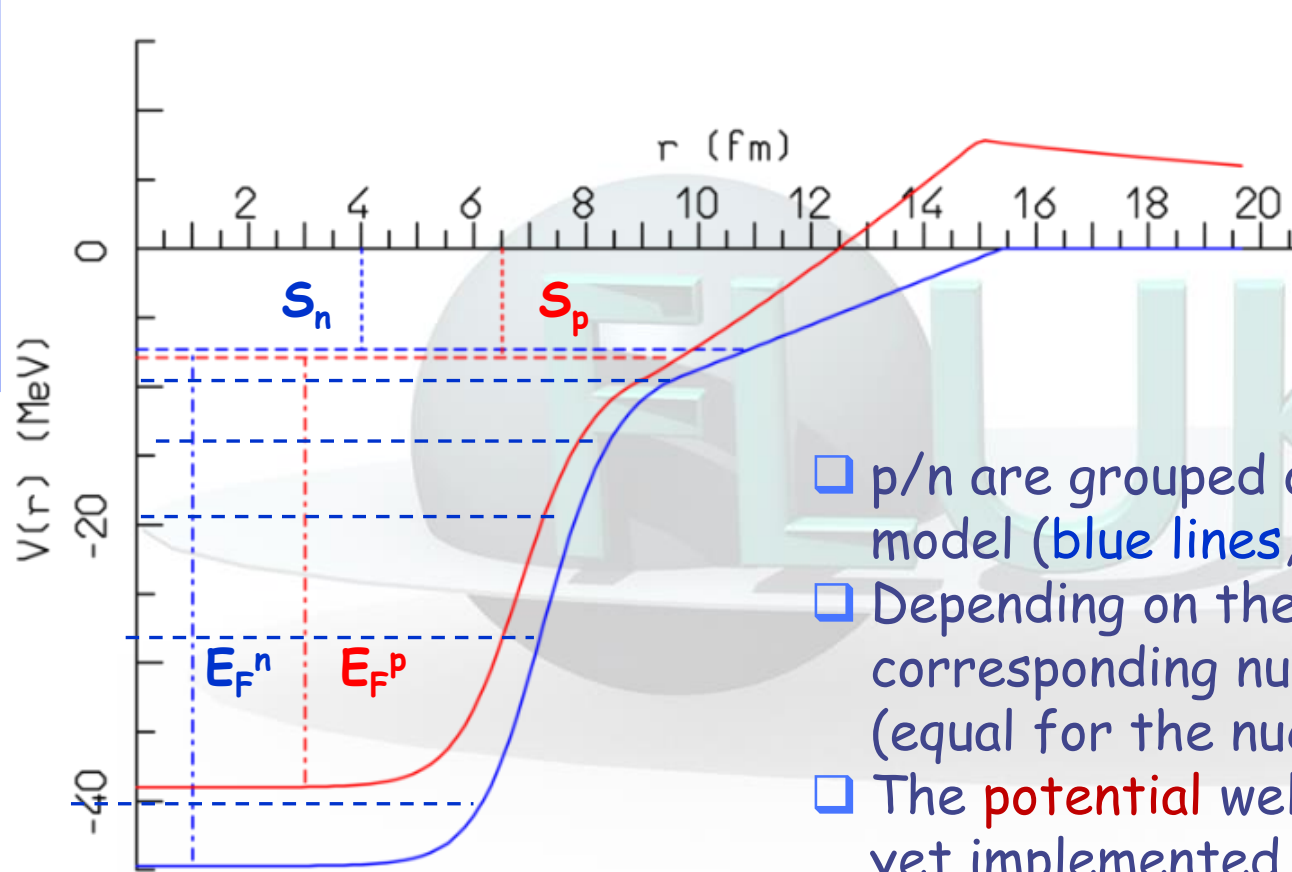
- **Quasi-Elastic scattering** of electrons on Lead, outgoing electron spectrum at  $60^\circ$
- Inelastic tail not included in simulation
- To be improved with the inclusion of **energy-dependent nuclear well**, as already there for nucleon-induced reactions
- **Much more tests needed**

Data:

R.R. Whitney et al., Phys Rev C9,2230 (1974)



# Nucleon levels inside the nuclear potential: schematic drawing



- Blue: neutron
- Red: proton

- ❑ p/n are grouped on energy levels space according to the shell model (blue lines, shown for neutrons)
- ❑ Depending on the level, the maximum radius for the corresponding nucleon is less or equal to the nucleus radius (equal for the nucleons on the Fermi level)
- ❑ The **potential** well depth **depends on the nucleon energy** (not yet implemented for neutrino and electron interactions)
- ❑ Hit nucleon must go above Fermi level, can stay below separation energy.

# Nucleon correlation function:

Correlation function: it can be computed within the Fermi-gas model

Due to the anti-symmetrization of the fermion's wave function, given a nucleon in a position  $\vec{r}$  in a nucleus with density  $\rho_0$ , the probability of finding another like nucleon in a position  $\vec{r}'$  is decreased for small values of the distance  $d = |\vec{r} - \vec{r}'|$  by a factor

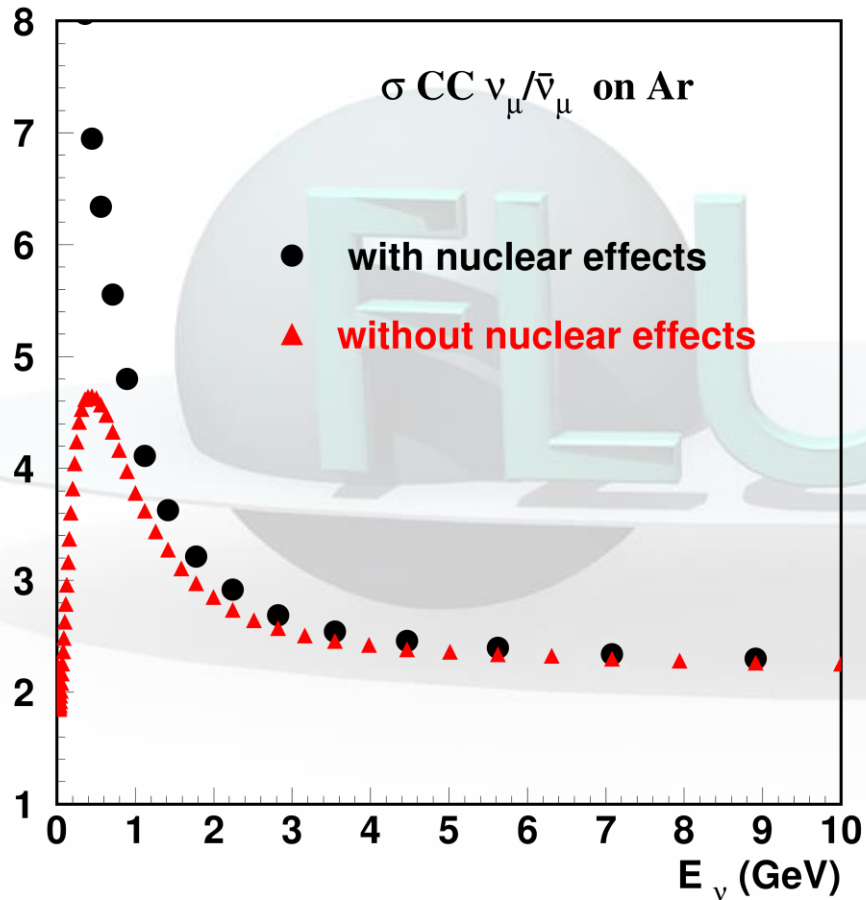
$$g(x) = 1 - \frac{1}{2} \left[ \frac{3}{x^2} \left( \frac{\sin x}{x} - \cos x \right) \right]^2$$

where  $x = K_F d$ , and the factor 1/2 in front of the parenthesis accounts for the two possible spin orientations.

Nucleon hard core effects are also taken into account, forbidding to "find" a nucleon of the same or different type at less than 1-1.5 fm distance. This check is applied at every possible re-interaction, checking against all nucleons already involved in previous interactions



# Effect of Pauli Blocking: example



Ratio of Neutrino/antineutrino  $\sigma_{CC}$  vs  
(a) neutrino energy

For interactions in Ar nuclei,  $\nu_{\mu}$

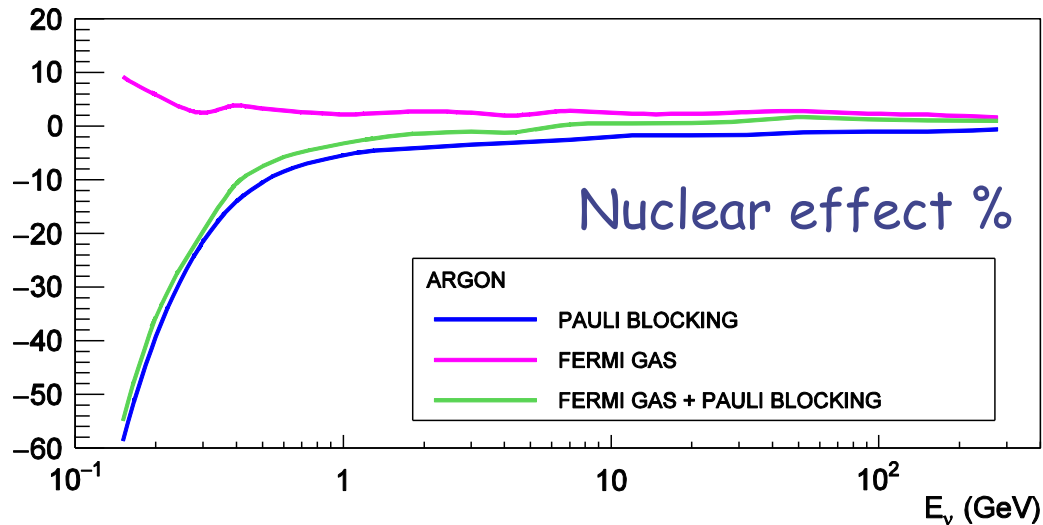
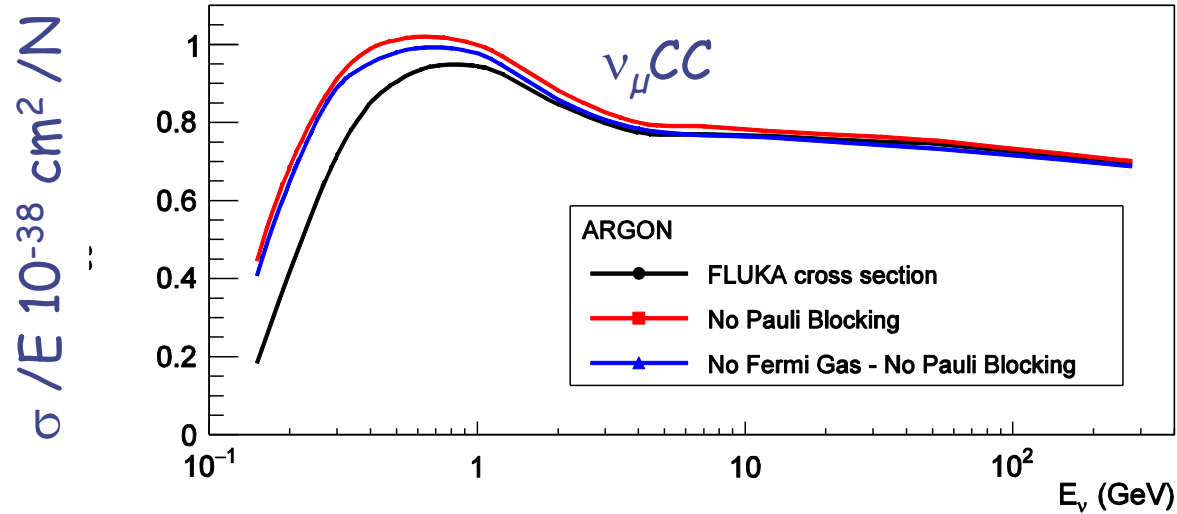
As calculated with FLUKA

Black: full calculation

Red: simple sum of  $\nu$ -N cross section

Smaller  $q^2$  in anti-neutrino results in  
higher Pauli-blocking probability

# Total cross section: nuclear effects in Ar

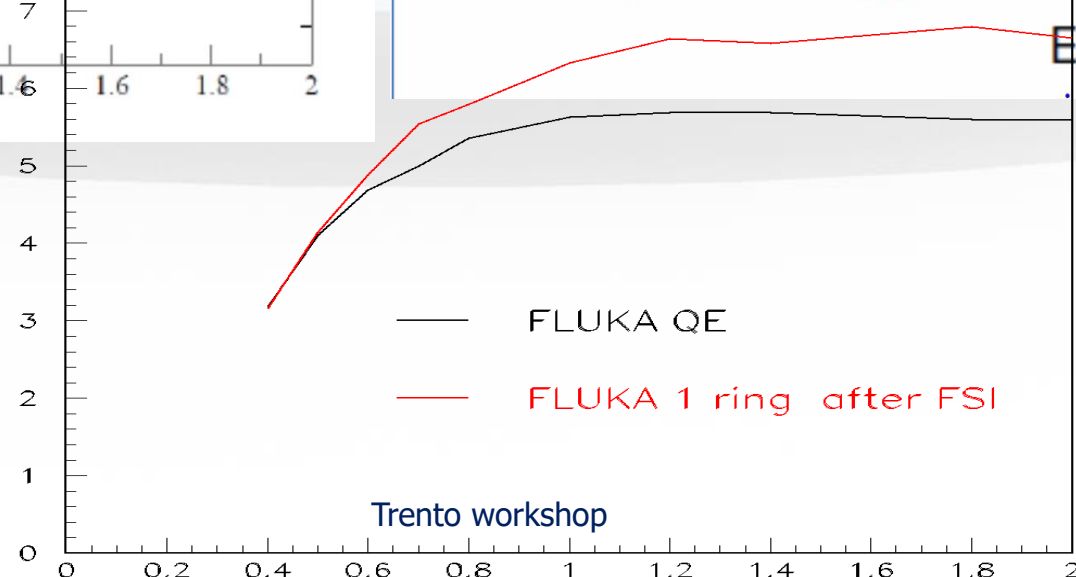
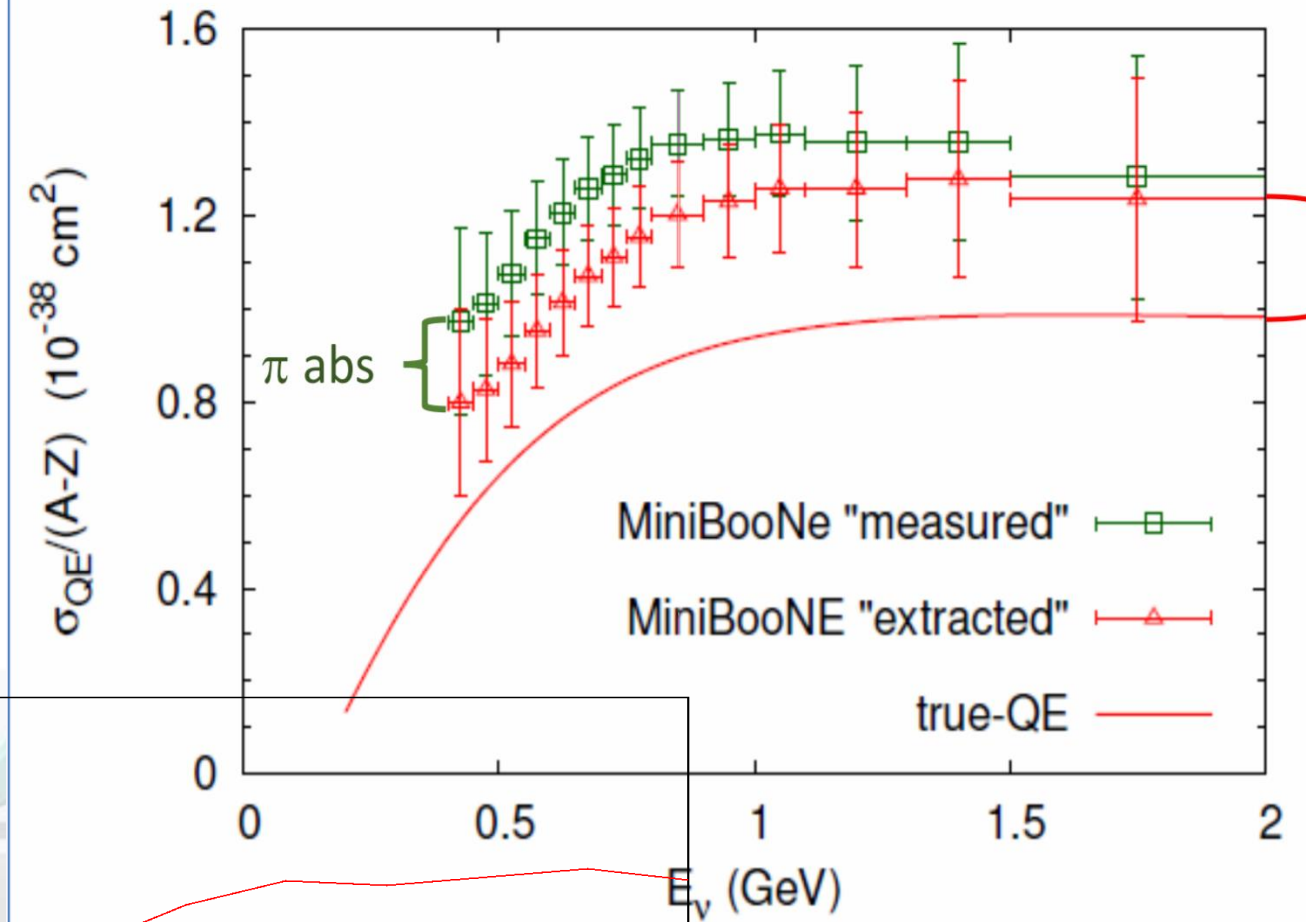
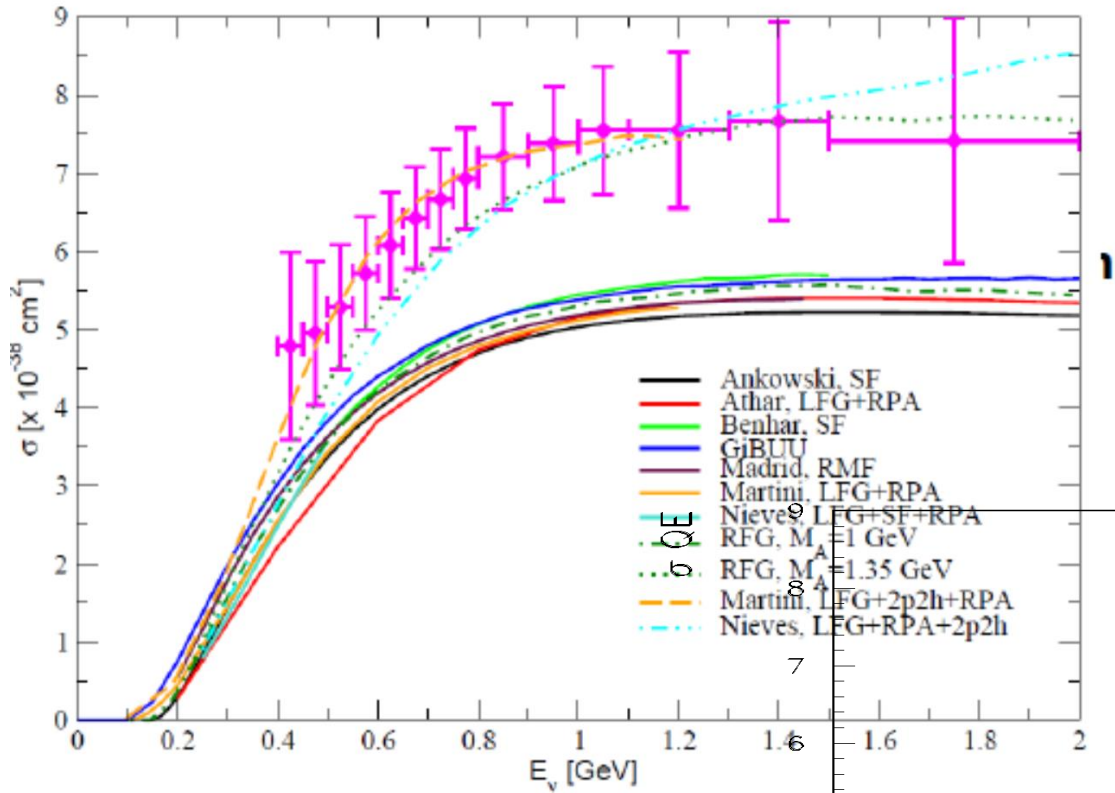


$5 \text{ GeV} < E_{\nu} < 50 \text{ GeV}$   
 Pauli Blocking effect  
 and Fermi Gas effect  
 separately have an  
 impact of  $\sim 2\text{-}3\%$   
 Globally Nuclear  
 effects stay within  
 $\pm 1\%$

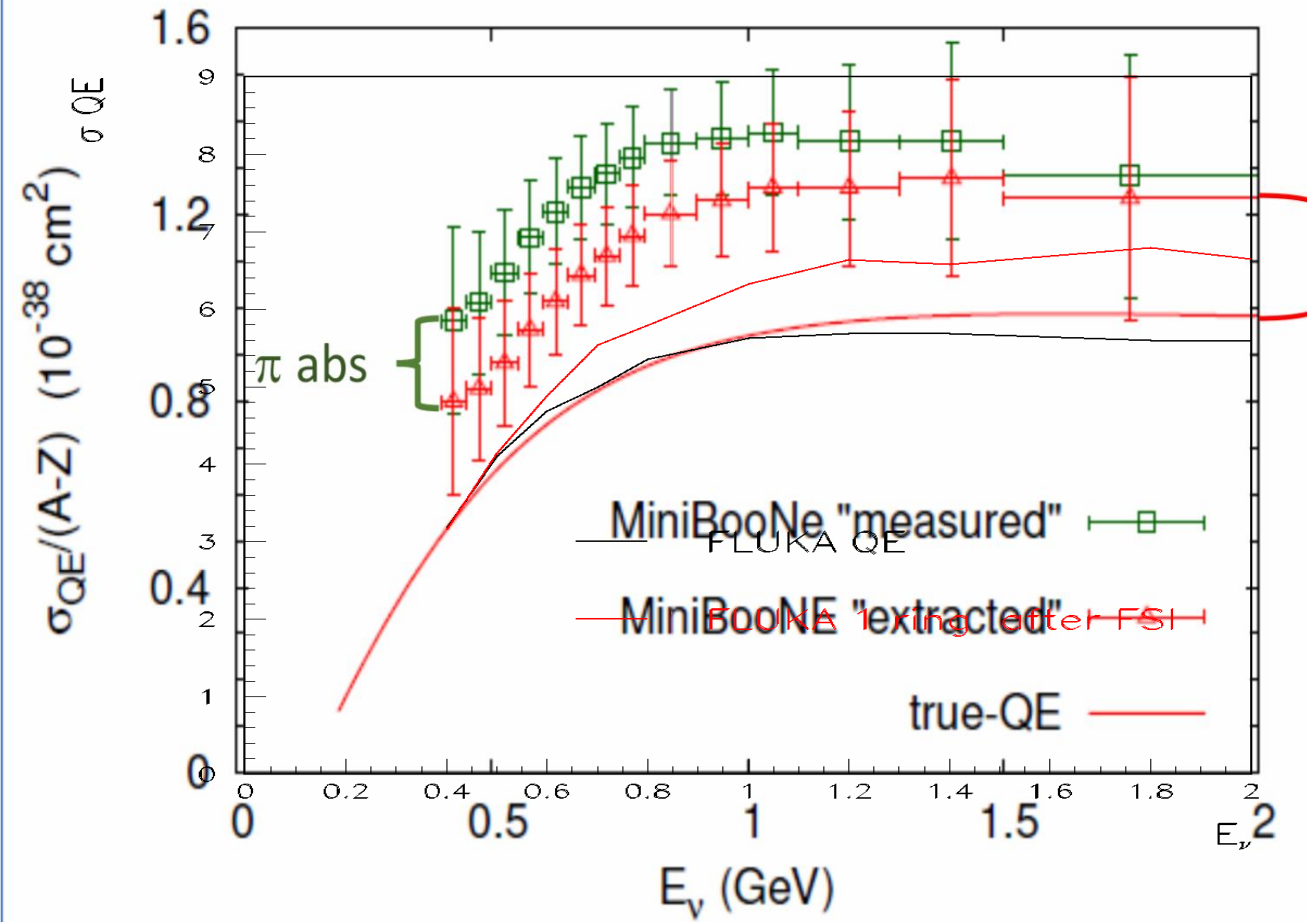
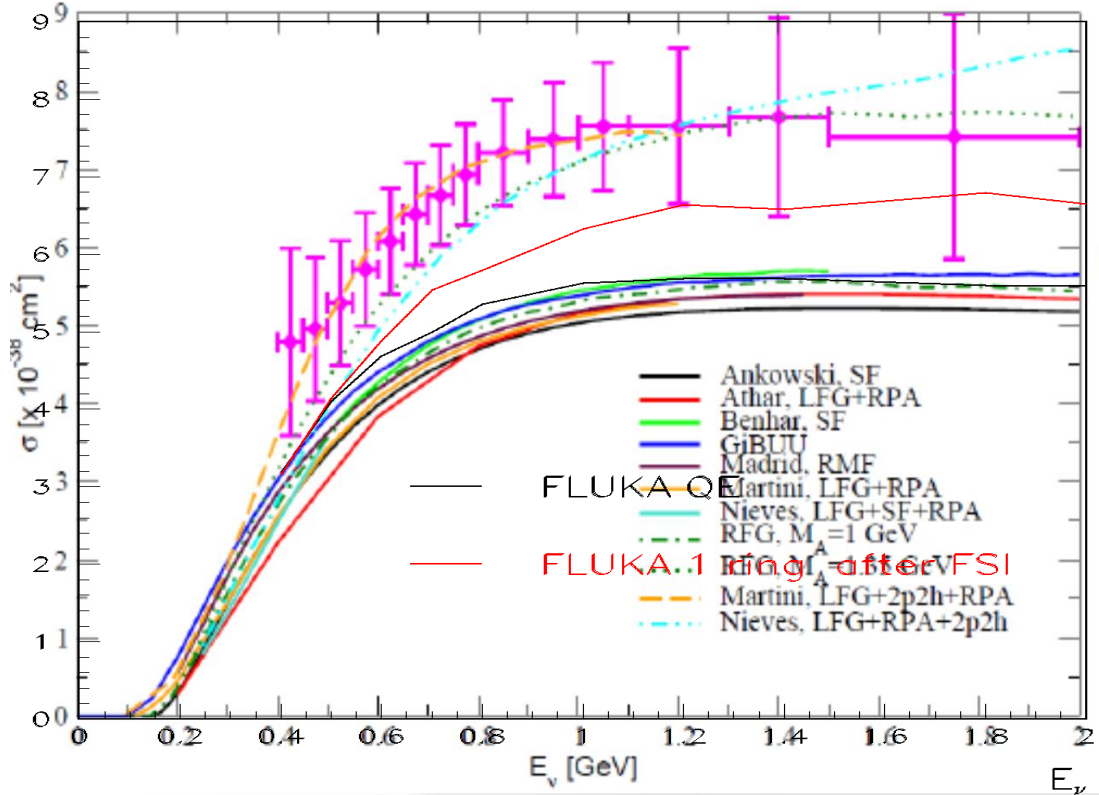
$E_{\nu} < 5 \text{ GeV}$

nuclear effects are  
 dominated by the  
 Pauli Blocking and  
 rapidly increase to  
 the order of  $10\%$  and  
 above

# CCQE on $^{12}\text{C}$



# CCQE on $^{12}\text{C}$



# Nuclear effects in Minerva

Beam:  $\nu_\mu$  NuMi Low Energy (average 4 GeV)  
Main Target : CH

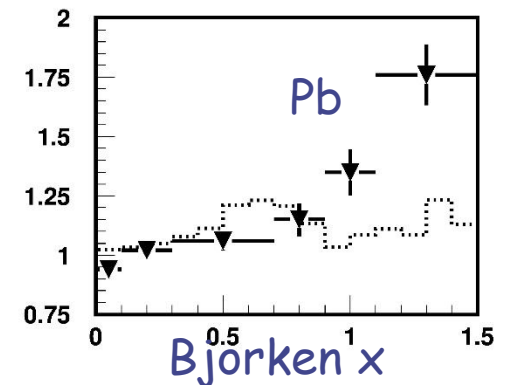
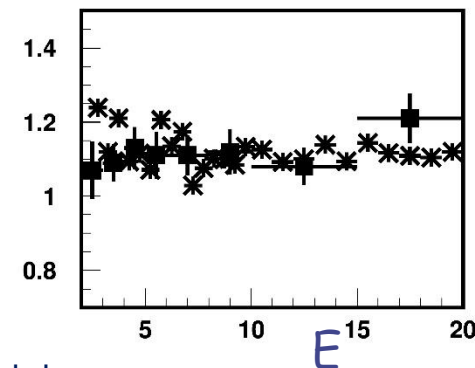
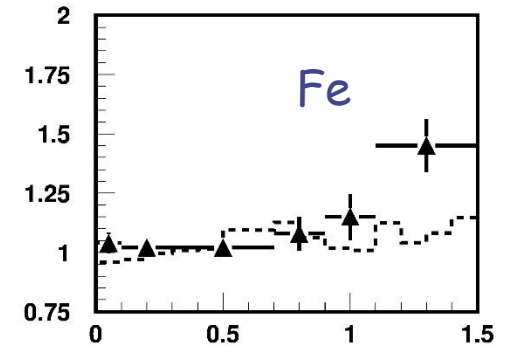
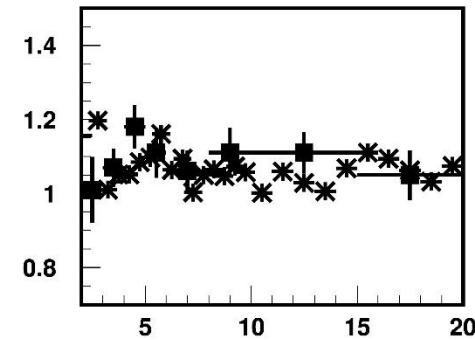
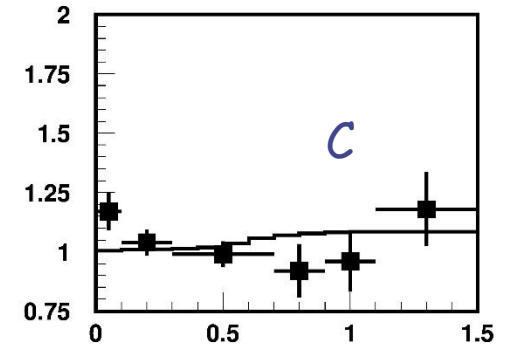
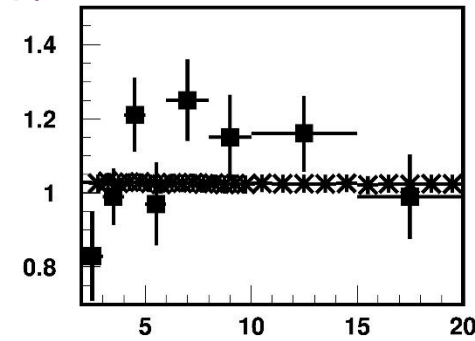
Measured also with C, Fe, Pb targets  
PRL 112, 231801 (2014)

Here: ratio of cross sections / the one in CH

Left: total CC vs neutrino Energy :  
squares: data  
crosses: FLUKA

Right:  $d\sigma/dx$   
symbols: data histos: Fluka  
expt: reduction at low  $x$  and  
enhancement at high  $x$  with incr.  $A$   
Fluka: fails the highest  $x$  (same for  
Genie)

july 2018



# Pions: nuclear medium effects

Free  $\pi$  N interactions  $\Rightarrow$   $\Rightarrow$  Non resonant channel  
 $\Rightarrow$  P-wave resonant  $\Delta$  production

$\Delta$  in nuclear medium  $\Rightarrow$  decay  $\Rightarrow$  elastic scattering, charge exchange  
 $\Rightarrow$  reinteraction  $\Rightarrow$  **Multibody pion absorption**

Assuming for the free resonant  $\sigma$   
 a Breit-Wigner form with width  $\Gamma_F$

$$\sigma_{res}^{Free} = \frac{8\pi}{p_{cms}^2} \frac{M_{\Delta}^2 \Gamma_F^2(p_{cms})}{(s - M_{\Delta}^2)^2 + M_{\Delta}^2 \Gamma_F^2(p_{cms})}$$

An "in medium" resonant  $\sigma$  ( $\sigma_{res}^A$ ) can be obtained adding to  $\Gamma_F$  the imaginary part of the (extra) width arising from nuclear medium

$$\frac{1}{2}\Gamma_T = \frac{1}{2}\Gamma_F - \text{Im}\Sigma_{\Delta} \quad \Sigma_{\Delta} = \Sigma_{qe} + \Sigma_2 + \Sigma_3 \quad (\text{Oset et al., NPA 468, 631})$$

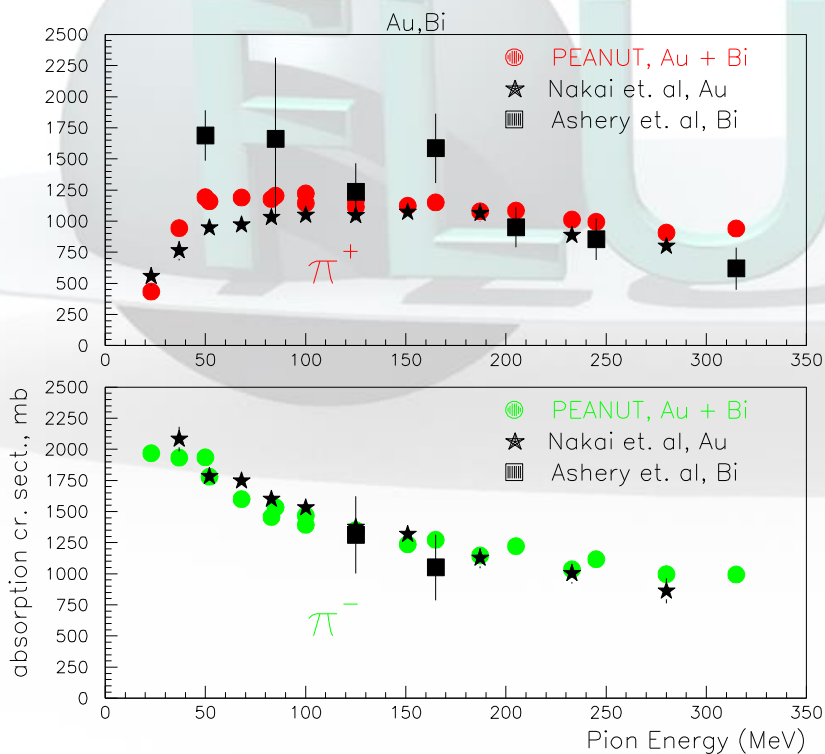
quasielastic scattering, two and three body absorption

The in-nucleus  $\sigma_t^A$  takes also into account a two-body s-wave absorption  $\sigma_s^A$  derived from the optical model

$$\sigma_t^A = \sigma_{res}^A + \sigma_t^{Free} - \sigma_{res}^{Free} + \sigma_s^A \quad \sigma_s^A(\omega) = \frac{4\pi}{p} \left(1 + \frac{\omega}{2m}\right) \text{Im} B_0(\omega) \rho$$

# Pion absorption examples

Pion absorption cross section on Gold and Bismuth in the  $\Delta$  resonance region (multibody absorption in PEANUT)

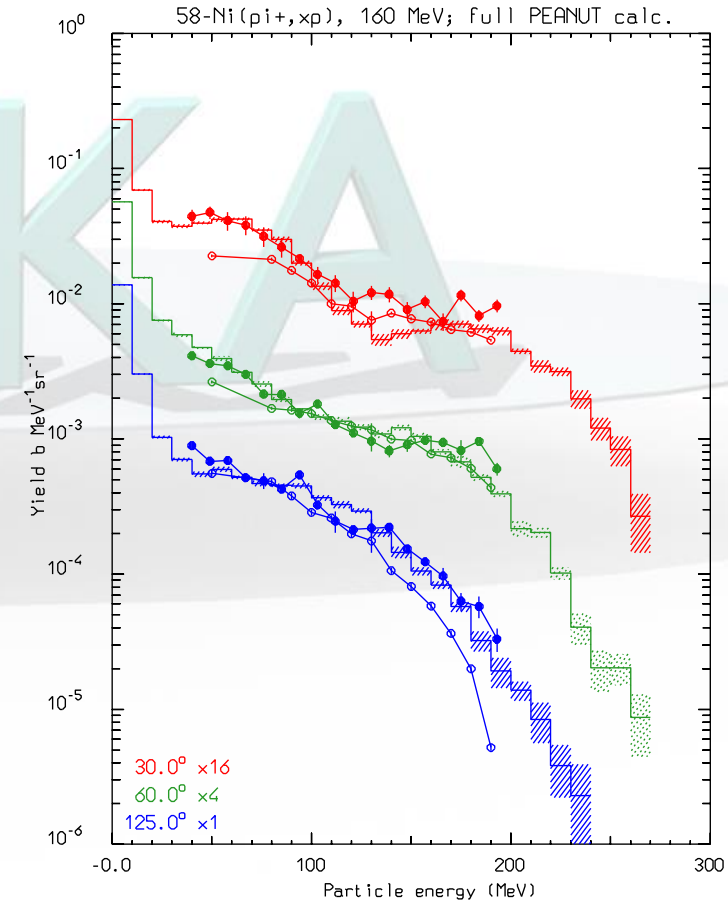


Emitted proton spectra at different angles, 160 MeV  $\pi^+$  on  $^{58}\text{Ni}$

Phys. Rev. C41,2215 (1990)

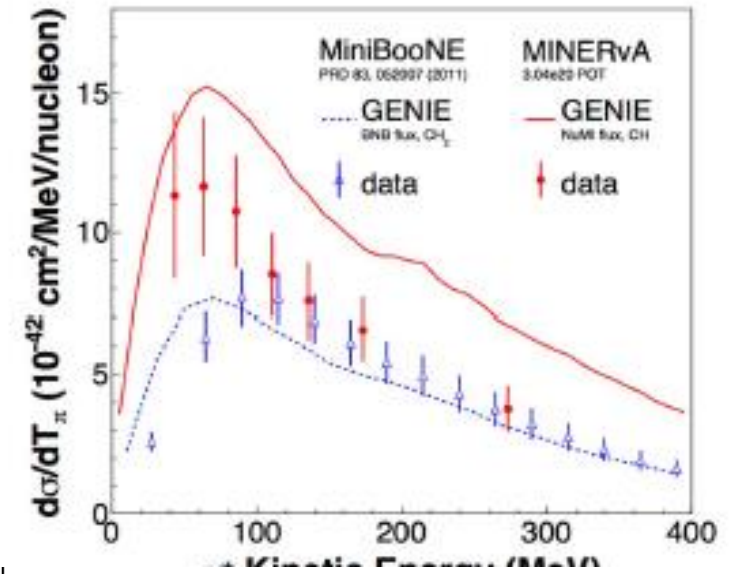
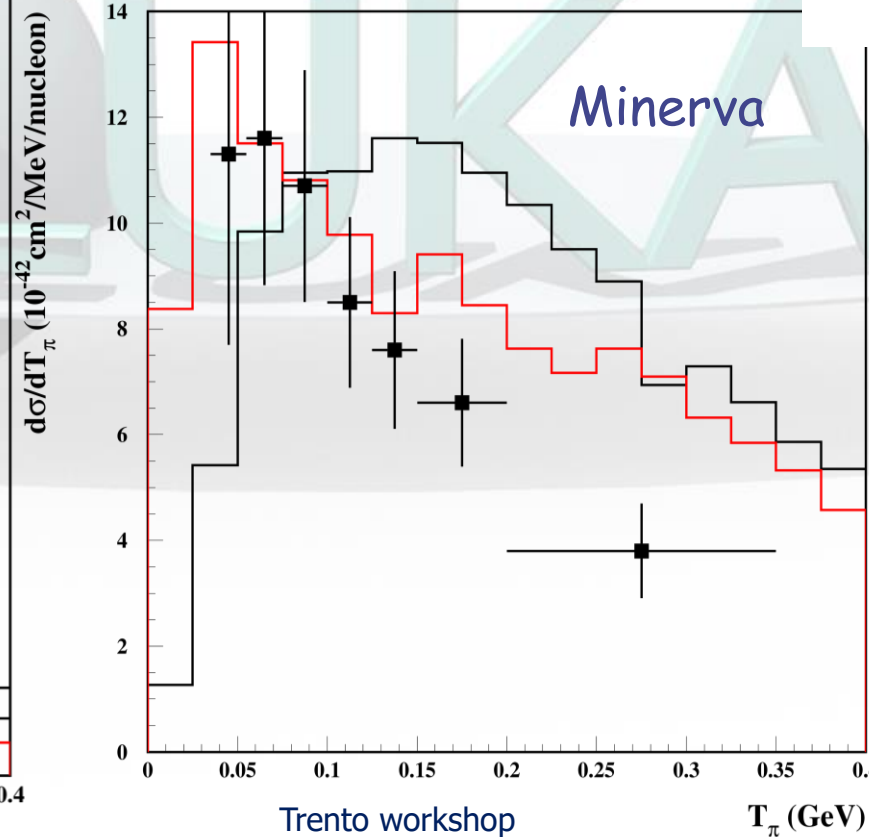
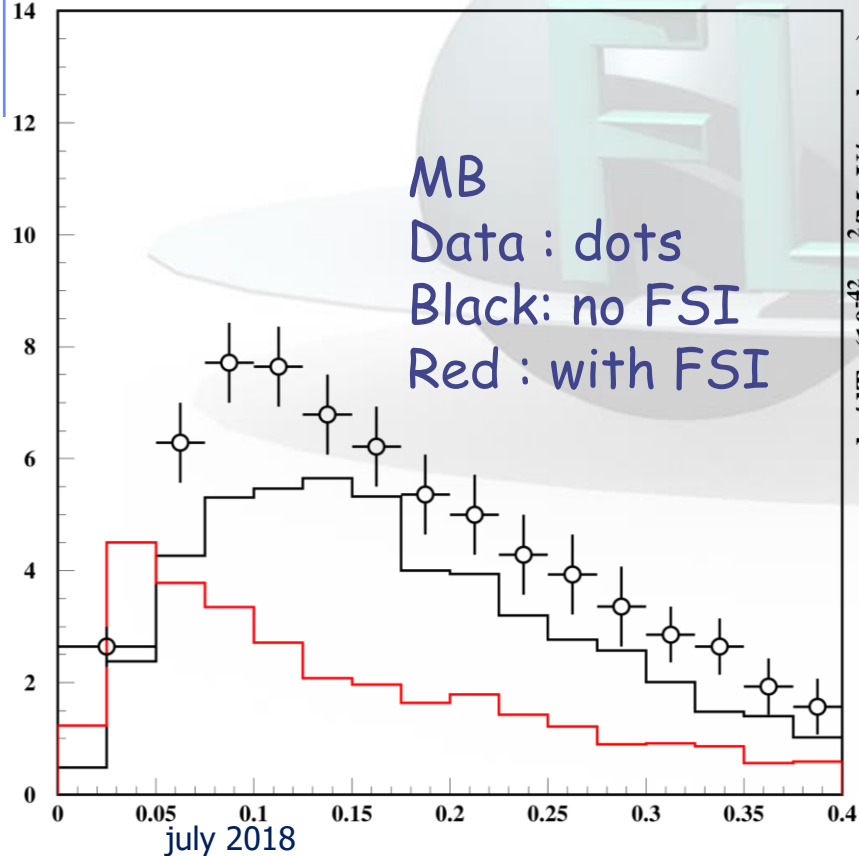
Phys. Rev. C24,211 (1981)

Proton spectra extend up to 300 MeV



# MB and Minerva

Dytman  
NUINT14  
Genie+FSI



MiniBoone : CH<sub>2</sub>, E<sub>v</sub> ≈ 0.8 GeV, cut on single pion, PHYS. REV.D 83, 052007 (2011)

Minerva : CH, E<sub>v</sub> ≈ 4 GeV, cut on W < 1.4 arXiv:1406.6415v3 (2015)

Tension betw the two data sets vs models/ extent of FSI



# another FSI : Formation zone

Naively: "materialization" time (originally proposed by Stodolski).

Qualitative estimate:

In the frame where  $p_{||} = 0$

$$\bar{t} = \Delta t \approx \frac{\hbar}{E_T} = \frac{\hbar}{\sqrt{p_T^2 + M^2}}$$

Particle proper time

$$\tau = \frac{M}{E_T} \bar{t} = \frac{\hbar M}{p_T^2 + M^2}$$

Going to the nucleus system

$$\Delta x_{for} \equiv \beta c \cdot t_{lab} \approx \frac{p_{lab}}{E_T} \bar{t} \approx \frac{p_{lab}}{M} \tau = k_{for} \frac{\hbar p_{lab}}{p_T^2 + M^2}$$

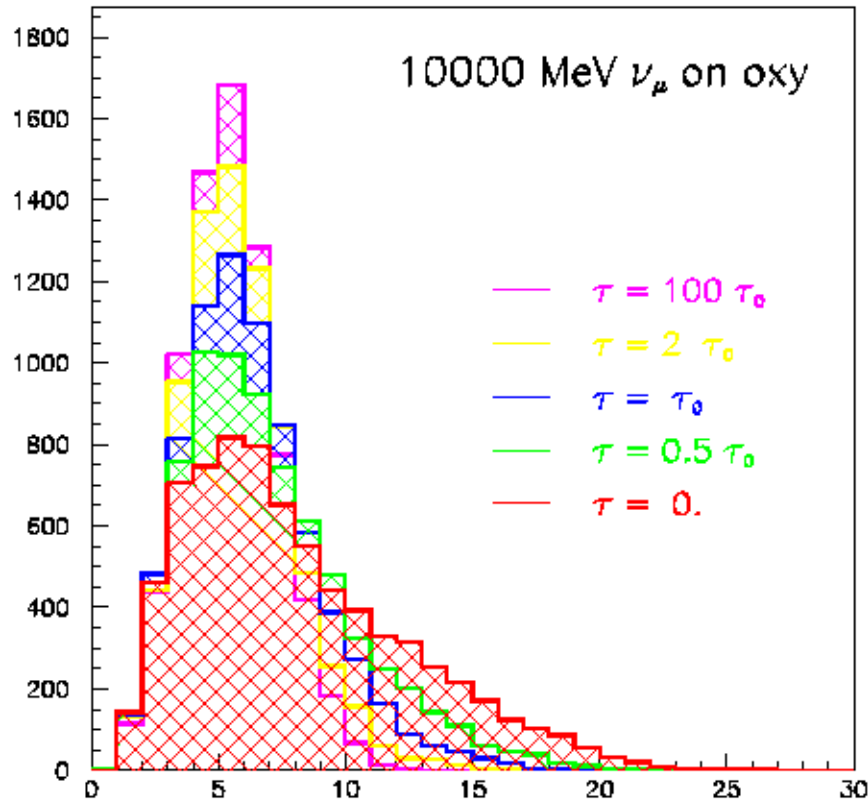
Condition for possible reinteraction inside a nucleus:  $\Delta x_{for} \leq R_A \approx r_0 A^{\frac{1}{3}}$

Decrease of the reinteraction probability

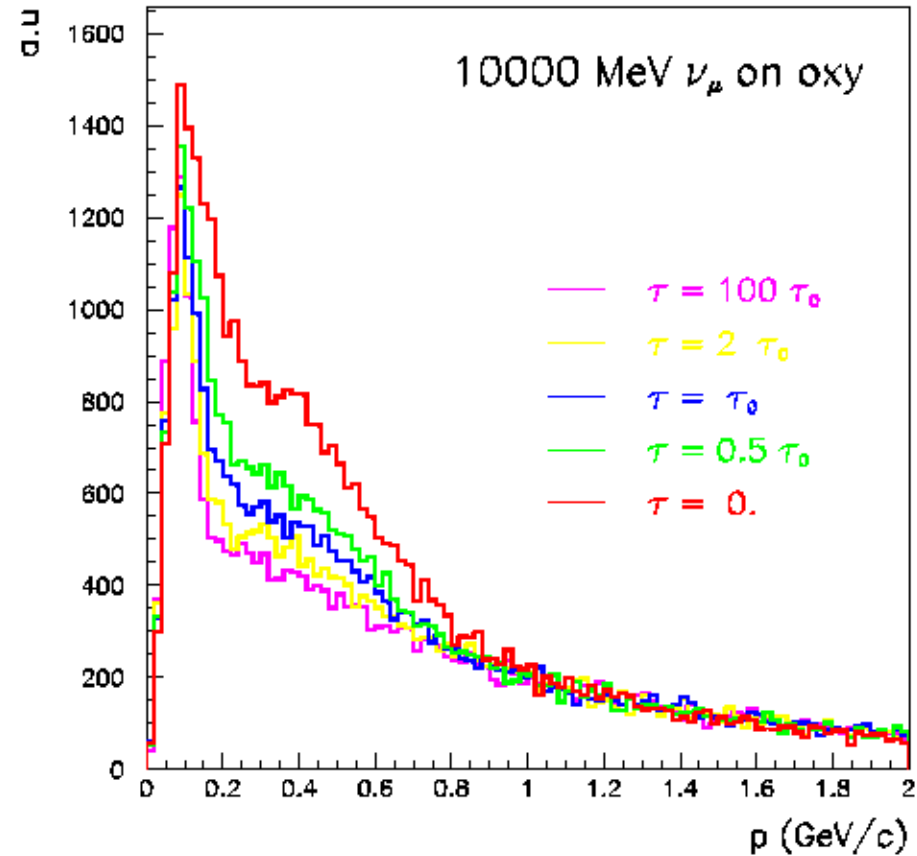
Applied also to DIS neutrino interactions and, in an analogue way, to QE neutrino interactions

# Effect of formation zone

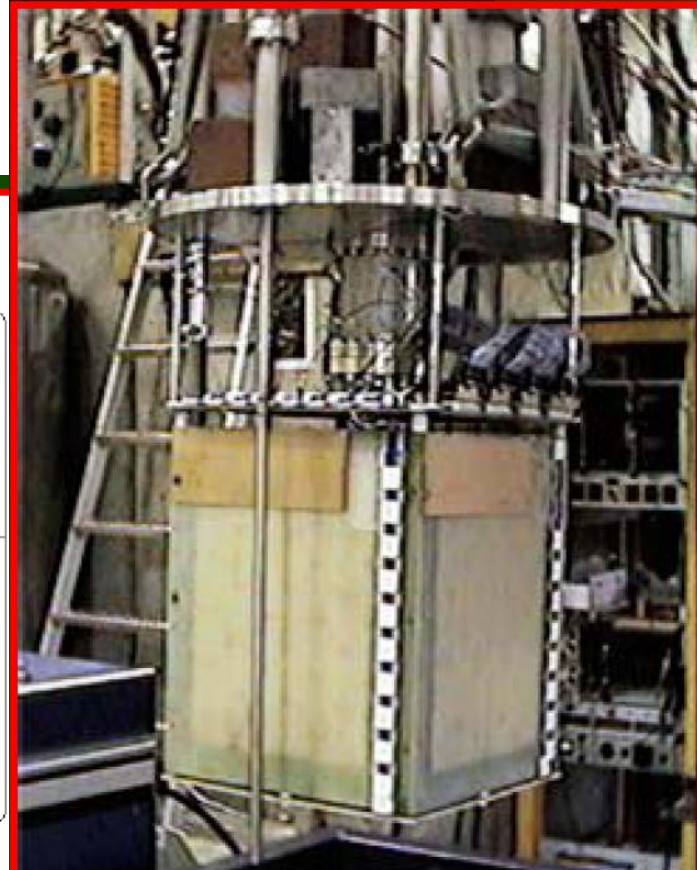
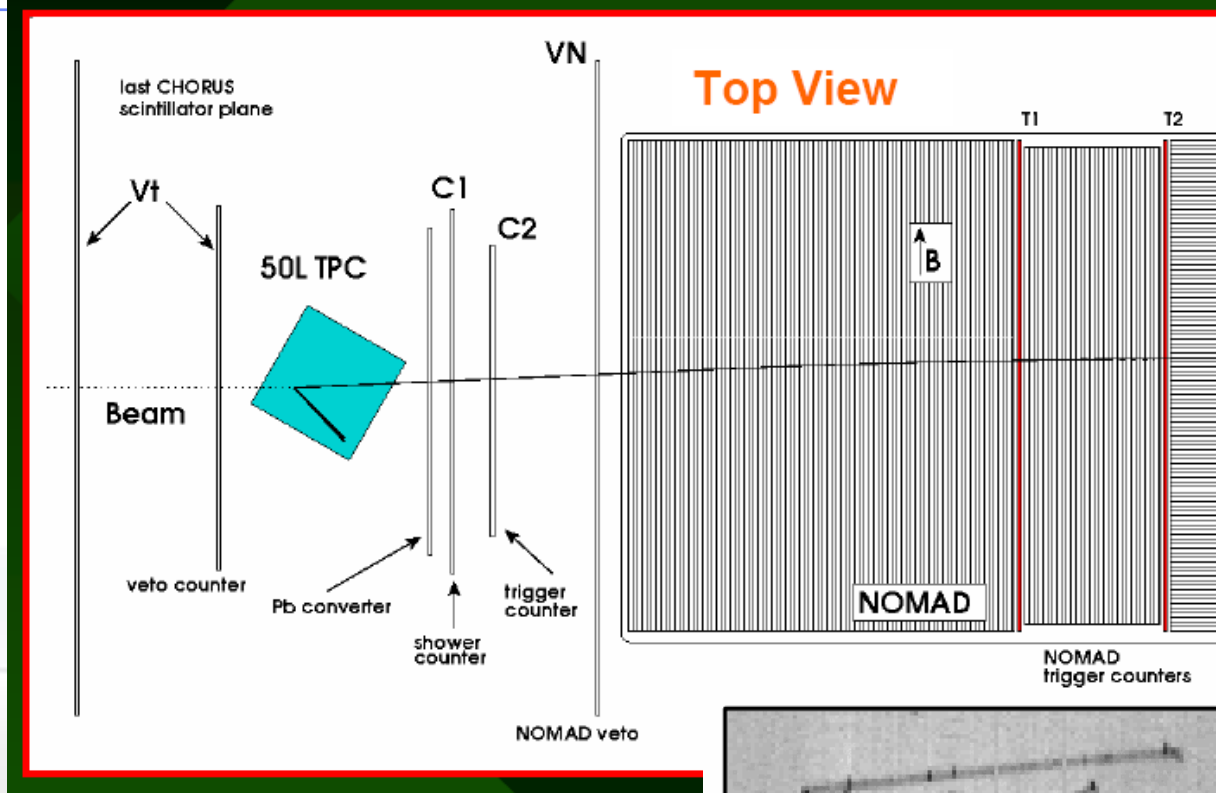
## Total hadron multiplicity



## Charged hadron spectra



# The 50l LAr TPC in the WANF neutrino beam(1997)

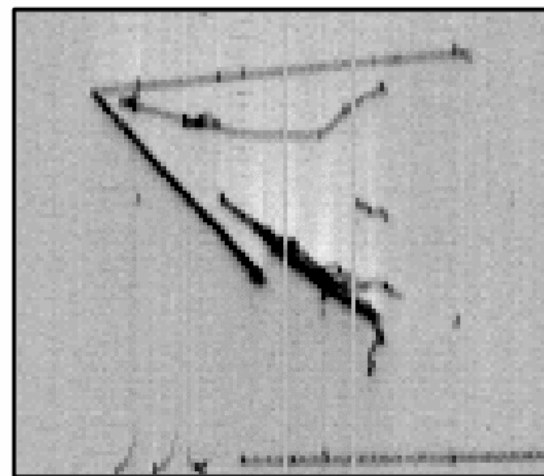


Trigger and  $\mu$   
reconstruction: NOMAD

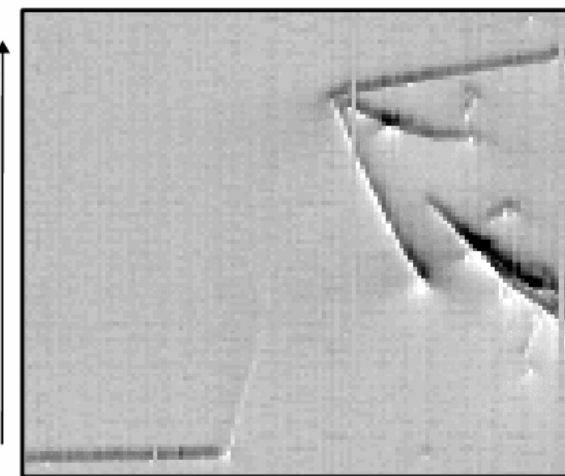
Event selection: "GOLDEN sample"  
= 1  $\mu$  and 1 proton  $>40\text{MeV}$  fully contained

Phys.Rev. D74 (2006) 112001

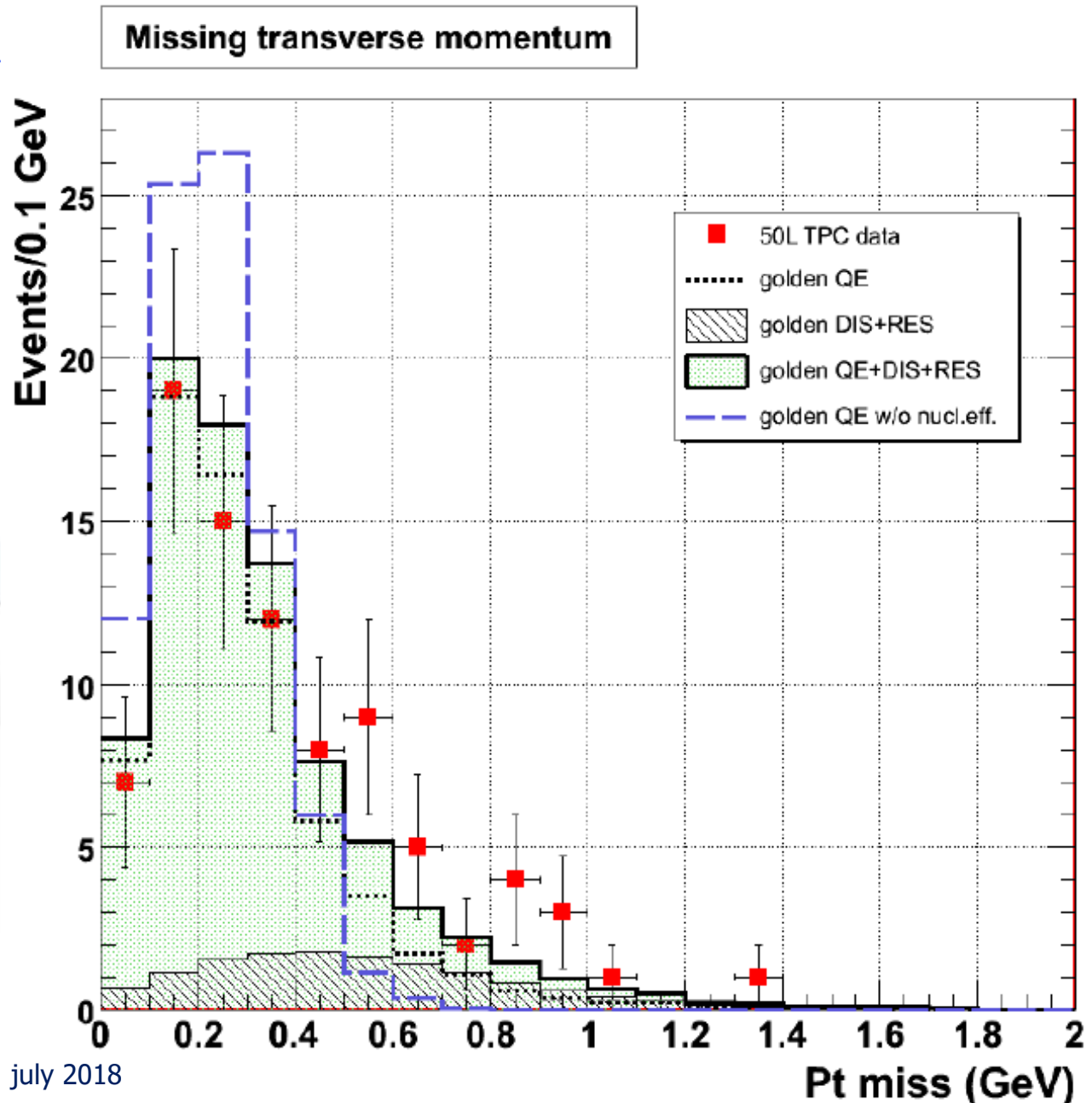
july 2018



Collection wires. (128 wires: 32 cm.)



Induction wires. (128 wires: 32 cm.)



july 2018

- from 400 QE - golden fraction 16%
- background - additional 20% finally expected

$80 \pm 9(\text{stat.}) \pm 13(\text{syst.}) \rightarrow$  mainly QE fraction and beam simul)

to be compared with **86** events observed

Very good consistency with expectations

Note: here DIS and RES from old coupling with the NUX code (A. Rubbia)

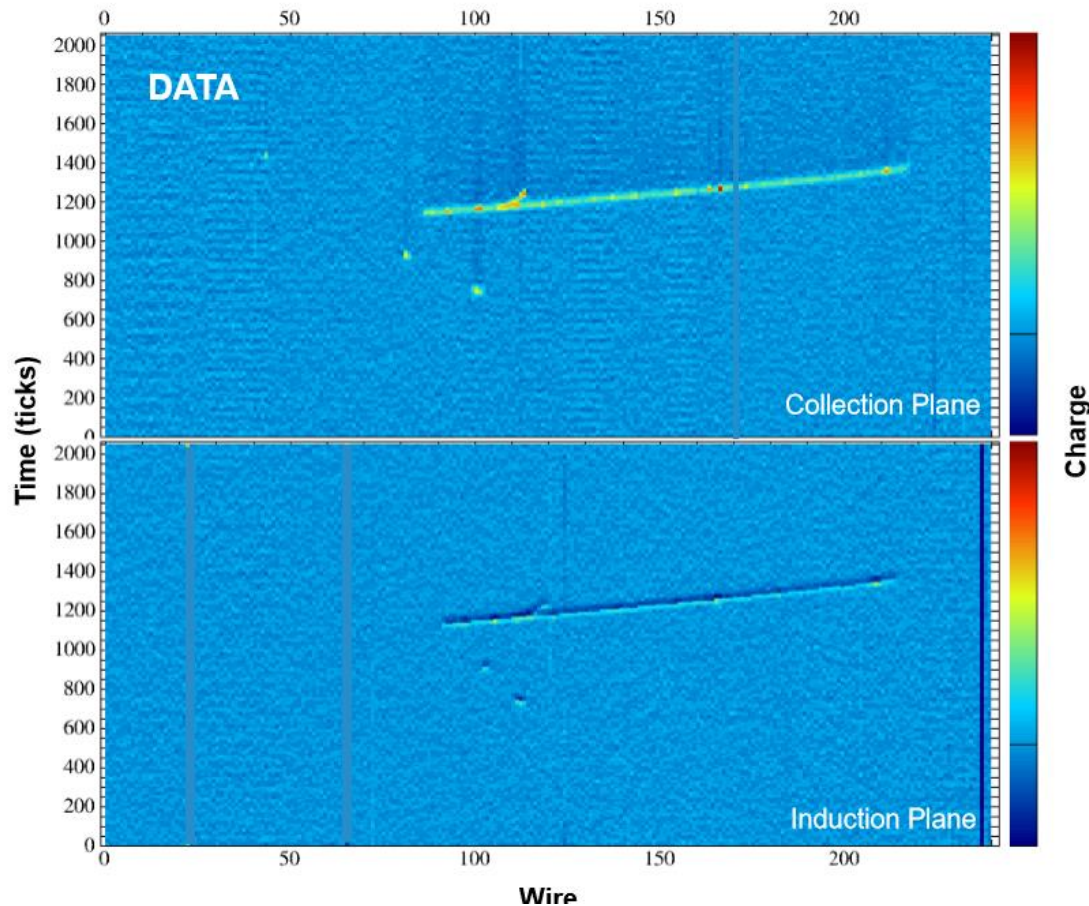
# Last steps of the reaction

- The INC in FLUKA is carried on until all involved nucleons drop below 30-50 MeV kinetic (smooth threshold, depends on number of excited nucleons)
- A **PRE-EQUILIBRIUM** steps comes in : nucleon-hole pairs sharing statistically the residual excitation energy. Exciton number increased by "collisions", particle emission still possible.
- When the exciton number reaches equilibrium, **EVAPORATION / FISSION** comes in. Statistical, includes nucleons and heavy fragments, includes sub-barrier emission, takes into account single excited levels.
- At excitation energies  $<$  separation energy  $\rightarrow$  **emission of gamma rays** (actually also in competition with evaporation) . Uses **atlas of excited levels/transitions** whenever available.

# First Demonstration of LArTPC MeV-Scale Physics in ArgoNeuT

Ivan Lepetic *APS\_April2018*

ILLINOIS INSTITUTE OF TECHNOLOGY  
For the ArgoNeuT Collaboration



july 2018

Low-energy gammas produced by  
neutrino-nucleus interactions in  
ArgoNeuT

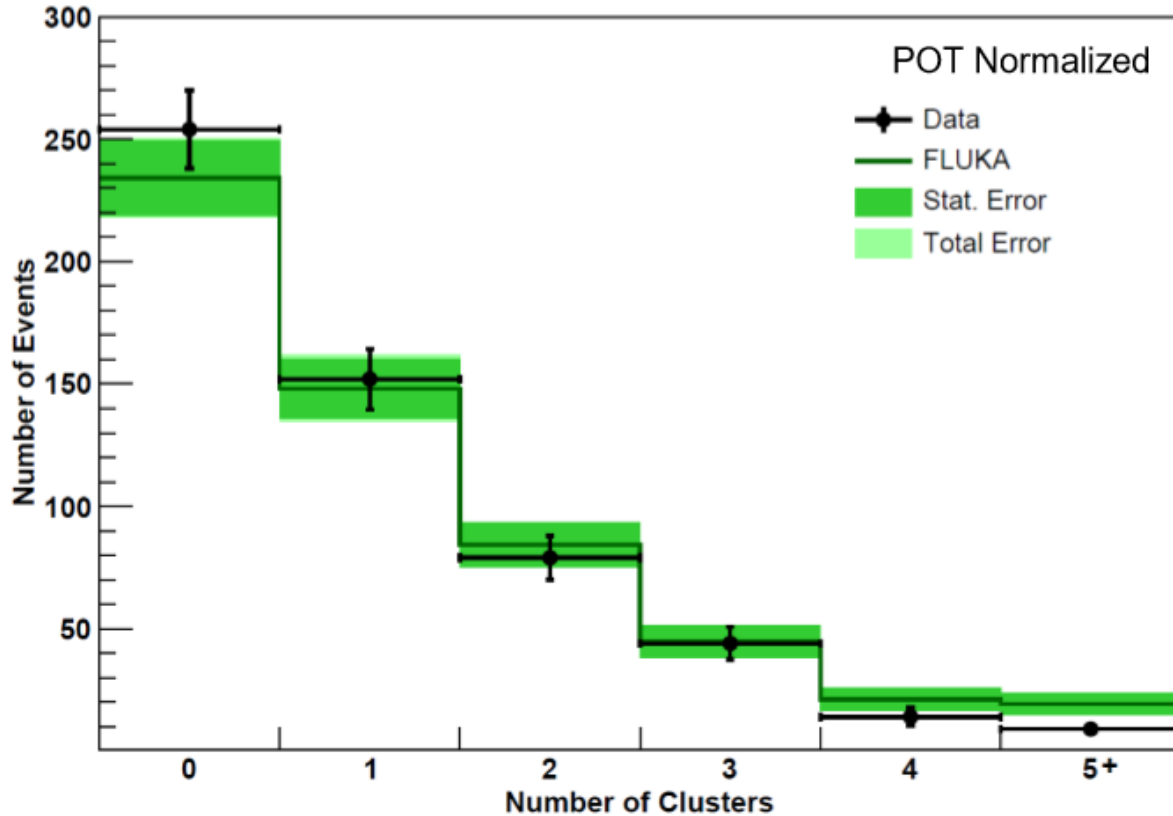
Photons from **neutrino-produced nuclear  
de-excitation** and inelastic neutron  
scattering -



Trento workshop

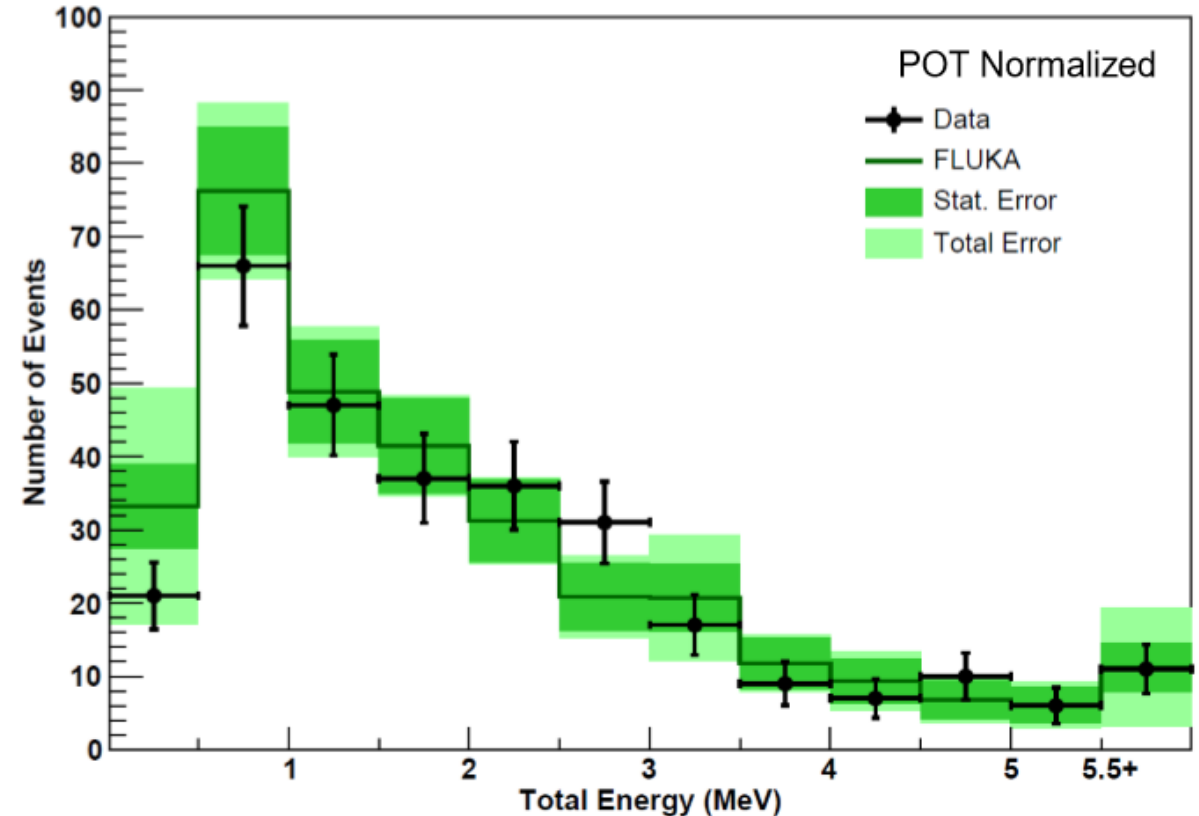
# Data and FLUKA

Number of Clusters in an Event



$$\chi^2/\text{ndf} = 9.62/6. \text{ p-value} = 0.14$$

Total Reconstructed Energy in an Event



$$\chi^2/\text{ndf} = 7.74/12. \text{ p-value} = 0.81$$

Agreement is far worse if either de-excitation or neutron produced gammas are removed.

# Conclusions and perspectives

- A neutrino event generator (NUNDIS) is implemented in FLUKA
- QE, RES, DIS interactions
- Hadronization as for hadronic interactions in FLUKA
- Nuclear effects from the FLUKA nuclear models
- Encouraging comparisons with expt data
  
- More has to be done:
  - Coherent pion production
  - Coherent effects (see high  $x$  in Minerva and proton pairs in Argoneut)
  - More coherent / nuclear structure effects for low energy QE
  - Meson exchange in QE (high  $x$  in Minerva)
  - Radiative corrections in DIS (ongoing)
  - Comparisons against data





**Backup**

R. Augusto, G. Aricò, C. Bahamonde Castro, M.I. Besana, M. Brugger, F. Cerutti, A. Cimmino, L. Esposito, *Alfredo Ferrari*,  
R. Garcia Alia, J. Idarraga Munoz, W. Kozłowska, A. Lechner, M. Magistris, A. Mereghetti, E. Nowak, S. Roesler, F.  
Salvat-Pujol, P. Schoofs, E. Skordis, G. Smirnov, C. Theis, A. Tsinganis, Heinz Vincke, Helmut Vincke, V. Vlachoudis,  
J.Vollaire CERN



G. Battistoni, F. Broggi, M. Campanella, I. Mattei, S. Muraro, P.R. Sala, S.M. Valle INFN. Milano, Italy  
N. Mazziotta INFN Bari, Italy A. Margiotta INFN & Univ. Bologna, Italy M.C. Morone Univ. Roma II, Italy  
F. Ballarini, E. Bellinzona, M. Carante, A. Embriaco, A. Fontana INFN & Univ. Pavia, Italy  
L. Sarchiapone INFN Legnaro, Italy V. Patera, S. Pioli INFN Frascati & Univ. Roma I, Italy  
P. Colleoni, Ospedali Riuniti di Bergamo, Italy G. Magro, M. Pelliccioni CNAO Pavia, Italy  
A. Mairani, CNAO Pavia, Italy & HIT, Germany



P. Degtiarenko, G. Kharashvili, JLab, USA M. Santana, SLAC, USA L. Lari, FNAL USA  
A. Empl, S. Hoang, M. Kroupa, L. Pinsky Univ. of Houston, USA  
K.T. Lee, E. Semones, N. Stoffle, N. Zapp NASA, Houston, USA  
A. Bahadori Kansas Univ. USA M. Trinczec, A. Trudel TRIUMF, Canada



G. Dedes, S. Mayer, K. Parodi, LMU Munich, Germany Anna Ferrari, S. Mueller HZDR Rossendorf, Germany  
S. Brechet, L. Morejon, N. Shetty, S. Stransky, S. Trovati, R. Versaci, ELI-Beamlines, Prague, Czechia  
T.J. Dahle, L. Fjera, A. Rorvik, K. Ytre-Hauge, Bergen Univ., Norway  
F. Belloni INSTN-CEA, France



A. Fedynitch DESY Zeuthen, Germany T. T. Boehlen, Medaustrom, Austria, D.Georg, MedUni, Vienna, Austria  
C. Cuccagna, TERA Switzerland T.V. Miranda Lima Kantonhospital Aarau, Switzerland  
M. Lantz, Uppsala Univ., Sweden F. Fiorini, Oxford Inst. Rad. Oncol., UK  
P. Garcia Ortega IUFFYM, Spain I. Rinaldi, INP Lyon, France  
M. Chin, Malaysia



A. Fassò, M.V. Garzelli, E. Gadioli, J. Ranft



# Nuclear potential for pions

For pions, a complex nuclear potential can be defined out of the  $\pi$ -nucleon scattering amplitude to be used in conjunction with the Klein-Gordon equation

$$\left[ (\omega - V_c)^2 - 2\omega U_{opt} - K^2 \right] \Psi = m_\pi^2 \Psi$$

In coordinate space (the upper/lower signs refer to  $\pi^+ / \pi^-$ ):

$$2\omega U_{opt}(\omega, r) = -\beta(\omega, r) + \frac{\omega}{2M} \nabla^2 \alpha(\omega, r) - \nabla \frac{\alpha}{1 + g\alpha(\omega, r)} \nabla$$

$$\beta = 4\pi \left[ \left( 1 + \frac{\omega}{M} \right) \left( b_0(\omega) \mp b_1(\omega) \frac{N-Z}{A} \right) \rho(r) + \left( 1 + \frac{\omega}{2M} \right) B_0(\omega) \rho^2(r) \right]$$

$$\alpha = 4\pi \left[ \frac{1}{\left( 1 + \frac{\omega}{M} \right)} \left( c_0(\omega) \mp c_1(\omega) \frac{N-Z}{A} \right) \rho(r) + \frac{1}{\left( 1 + \frac{\omega}{M} \right)} C_0(\omega) \rho^2(r) \right]$$

Using standard methods to get rid of the non-locality, in momentum space

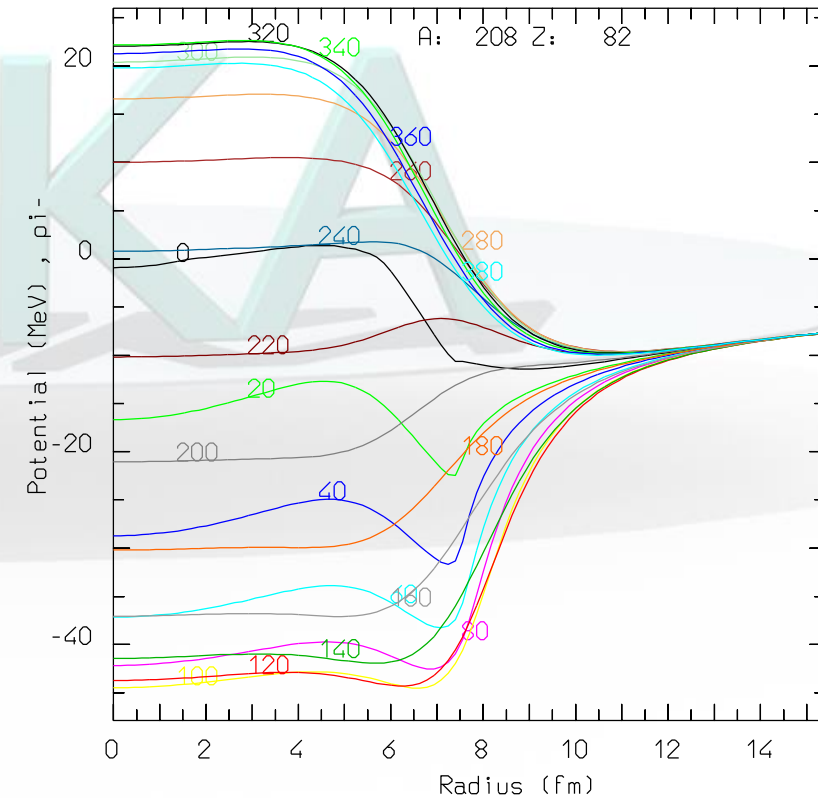
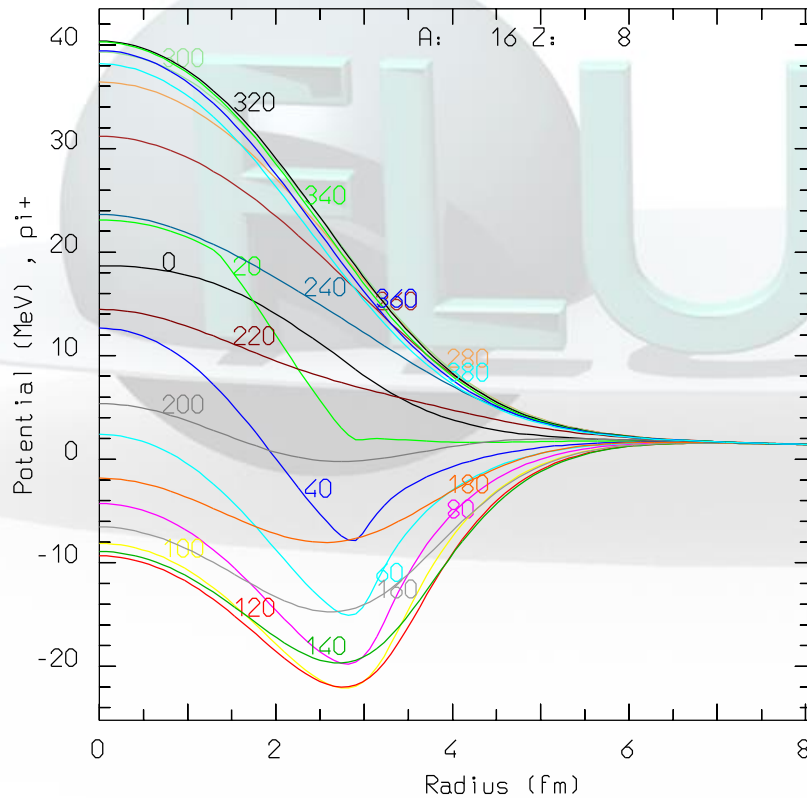
$$2\omega U_{opt}(\omega, r) = -\beta - K^2 \frac{\alpha}{1 + g\alpha} + \frac{\omega}{2M} \nabla^2 \alpha$$

$$K^2 = k_0^2 + V_c^2 - 2\omega V_c^2 - 2\omega U_{opt}(\omega, r) = \frac{k_0^2 + V_c^2 - 2\omega V_c^2 + \beta - \frac{\omega}{2M} \nabla^2 \alpha}{1 - \bar{\alpha}}$$

$$\bar{\alpha} = \frac{\alpha}{1 + g\alpha}$$

# Nuclear potential for pions: examples

The real part of the pion optical potential for  $\pi^-$  on  $^{16}\text{O}$  (left) and  $\pi^+$  on  $^{208}\text{Pb}$  (right) as a function of radius for various pion energies (MeV)

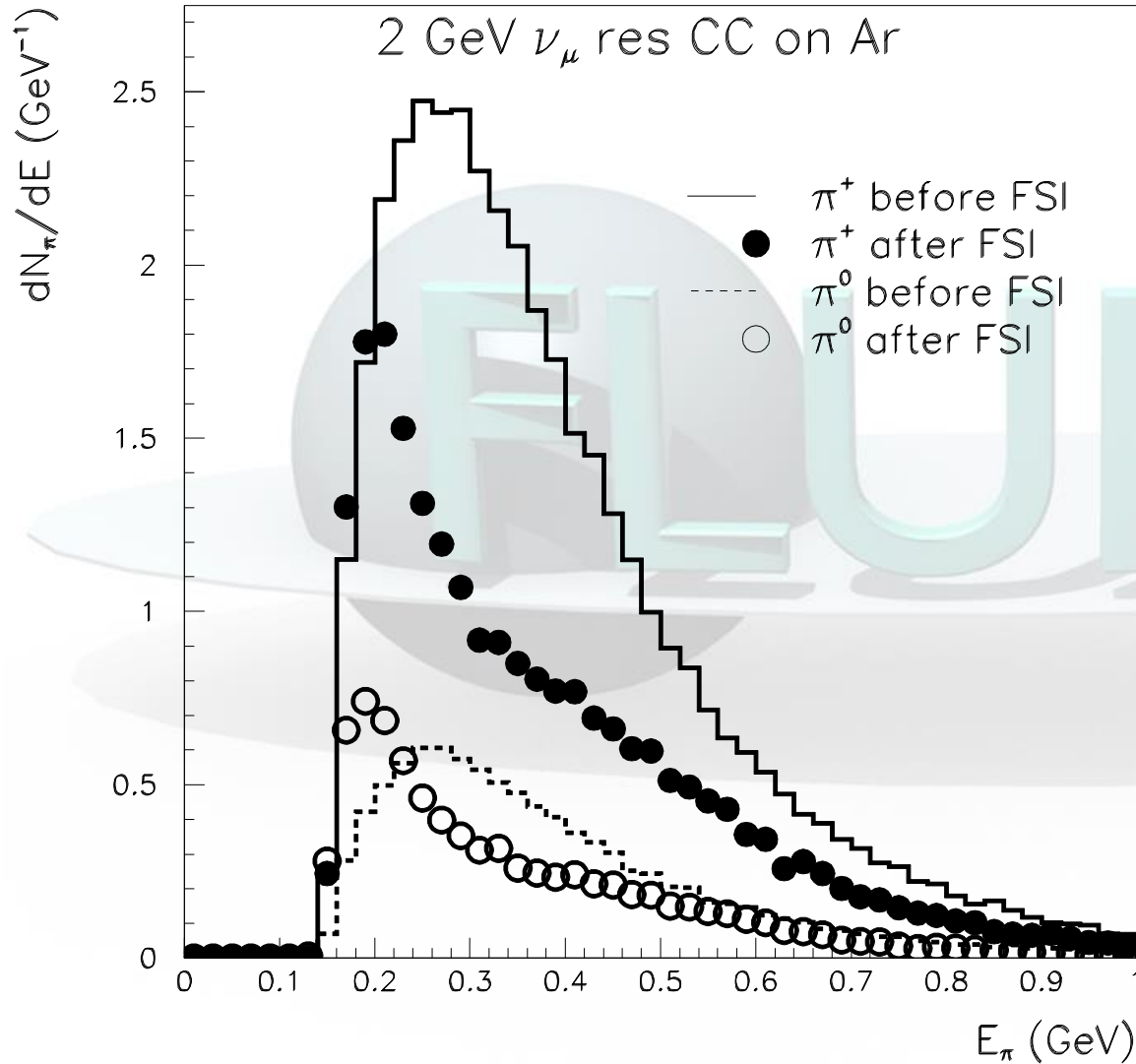


# NUNDIS 2015: kinematics

- Considered kinematical limits for the **PDF** available from GRV94, GRV98, and BBS analyses.

Variable	Required	GRV94		GRV98		BBS	
		Default	Tested	Default	Tested	Default	Tested
$E_{min}$ (GeV)	—	0.050					
$E_{max}$ (GeV)	$\geq 10^4$	$70 \cdot 10^3$			$10^5$		
$Q_{min}^2$ (GeV <sup>2</sup> )	$\leq 5.5 \cdot 10^{-12}$	0.4	0.4	0.8	0.8	2	0.8
$Q_{max}^2$ (GeV <sup>2</sup> )	$\geq 1.9 \cdot 10^4$	$10^6$	$10^9$	$10^6$	$10^9$	$10^4$	$2 \cdot 10^4$
$x_{min}$	$\leq 1.4 \cdot 10^{-11}$	$10^{-5}$	$10^{-30}$	$10^{-9}$	$10^{-30}$	$10^{-4}$	$10^{-30}$
$x_{max}$	1	0.99999	0.99999	1	1	1	1

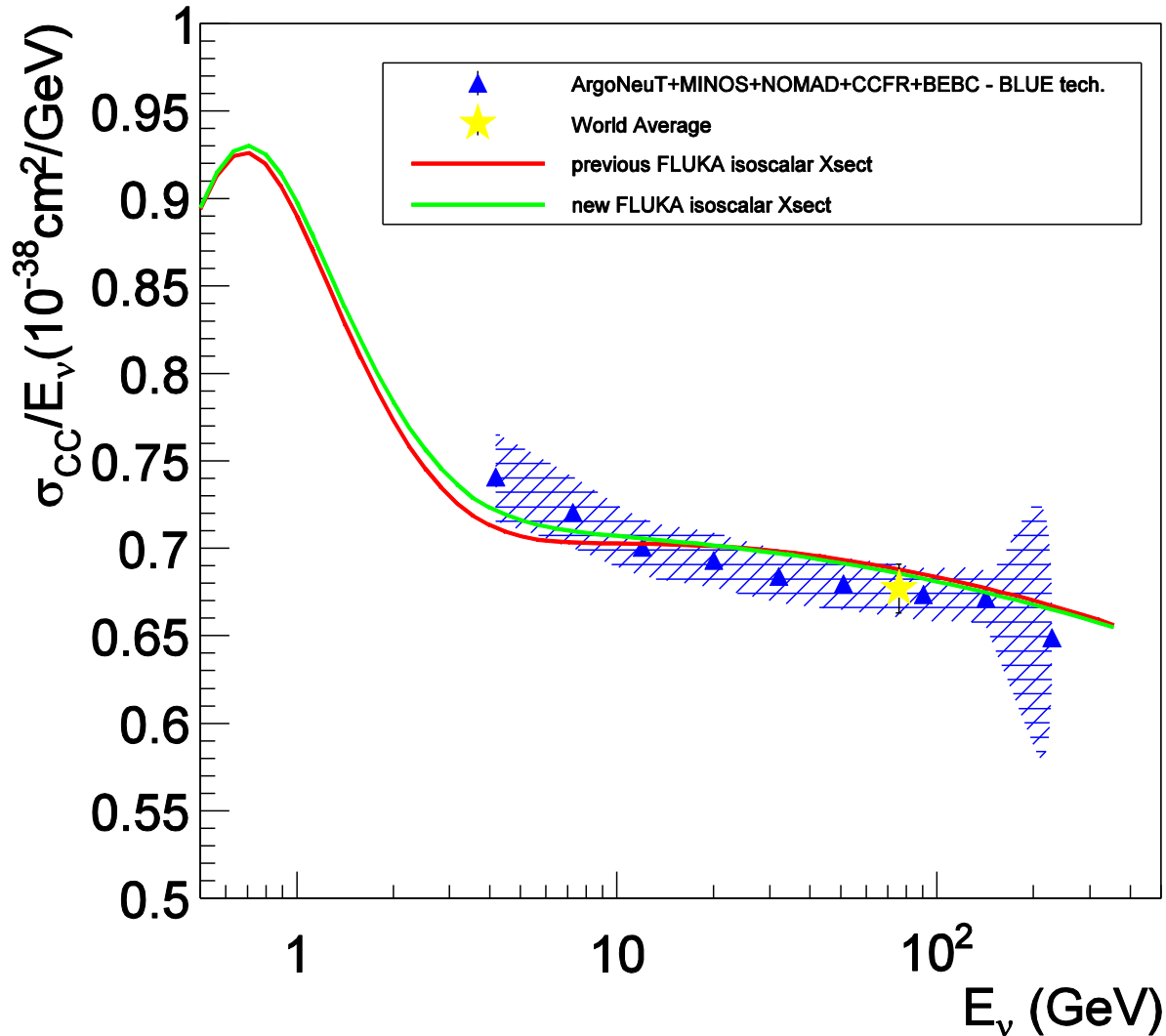
# Expected effect in Ar



Example of expected effect:  
2 GeV  $\nu_\mu$  CC RES interaction in Ar:  
Pion production vs pion total E  
Lines: before FSI  
Symbols: after FSI

Solid and filled symbols: positive pions  
Dashed and open symbols: pizero

# Same, with evaluation of data systematics



Work in progress: Attempt to compare with a combined estimate from available data and relative systematic error, properly accounting for correlations

Focus on the CNGS energy range (5-30 GeV)

**Recent experiments** (like MINOS, NOMAD, CCFR 1997): measure the **shape** of neutrino flux, and get the **Absolute normalization** from **Old measurements** at high energy, performed using Narrow Band Beams (CCFR-E701 / CCFRR-E616 / CDHS) or Wide Band Beams (GARGAMELLE / BEBC)

→ Common systematic errors

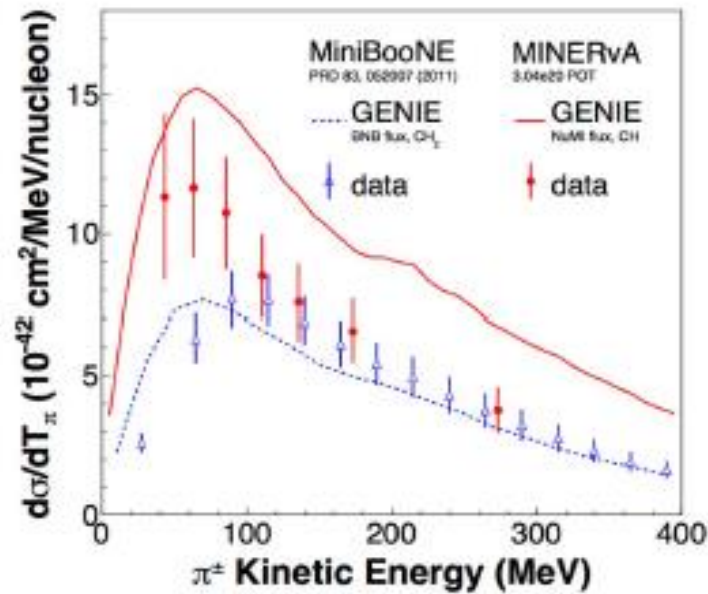
# Data on pion production

## Thoughts on MINERvA vs. MiniBooNE

- ▶ Shapes very similar, no significant dip in either!
- ▶ Small difference in slope (Kinematics, FF, nonres differences).
- ▶ Biggest difference is at low energy.

MiniBoone : CH<sub>2</sub>, E<sub>ν</sub> ≈ 0.8 GeV, cut on single pion, PHYS. REV.D 83, 052007 (2011)

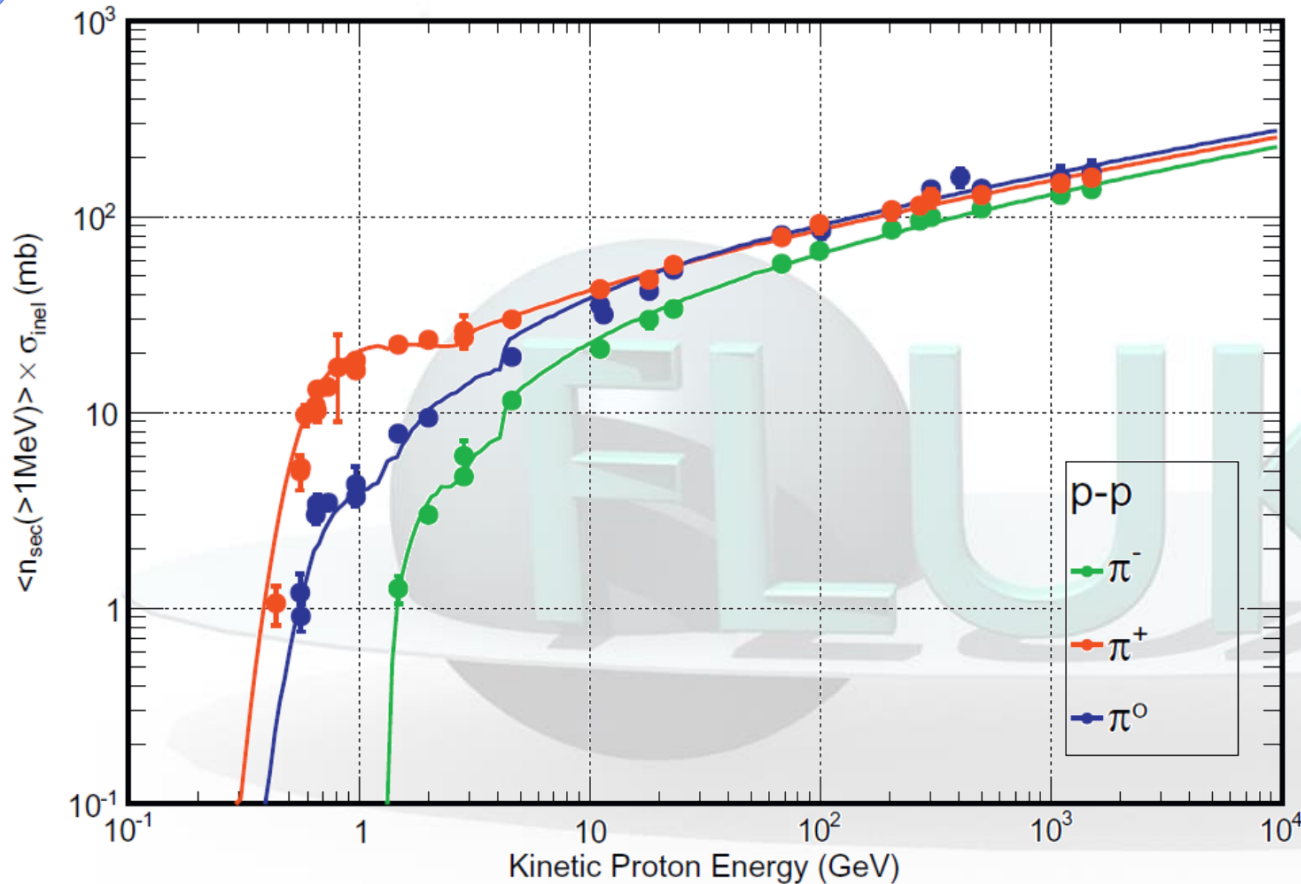
Minerva : CH, E<sub>ν</sub> ≈ 4 GeV, cut on W < 1.4 arXiv:1406.6415v3 (2015)



Tension betw the two data sets vs models/ extent of FSI



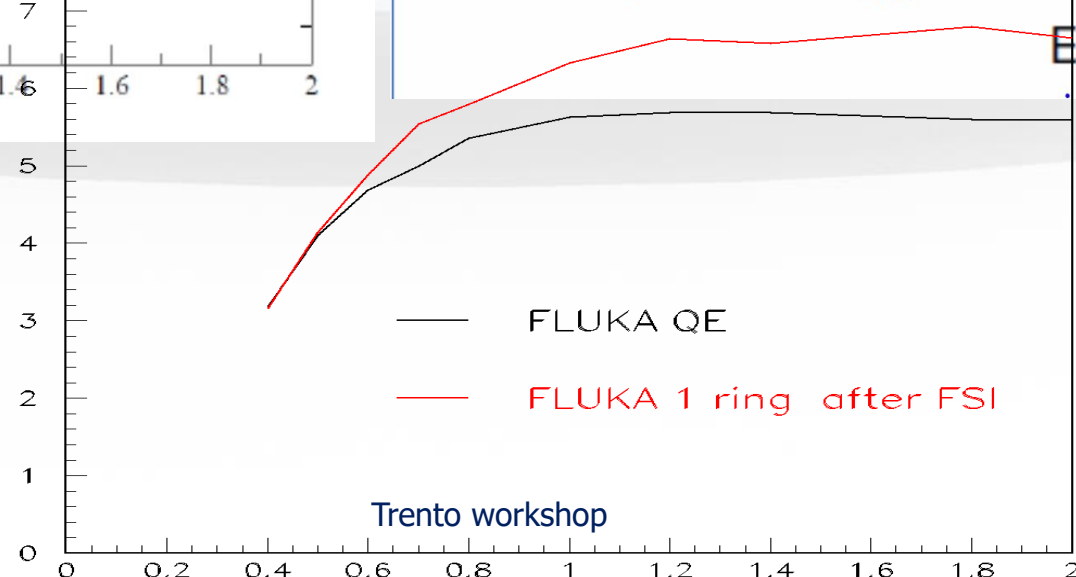
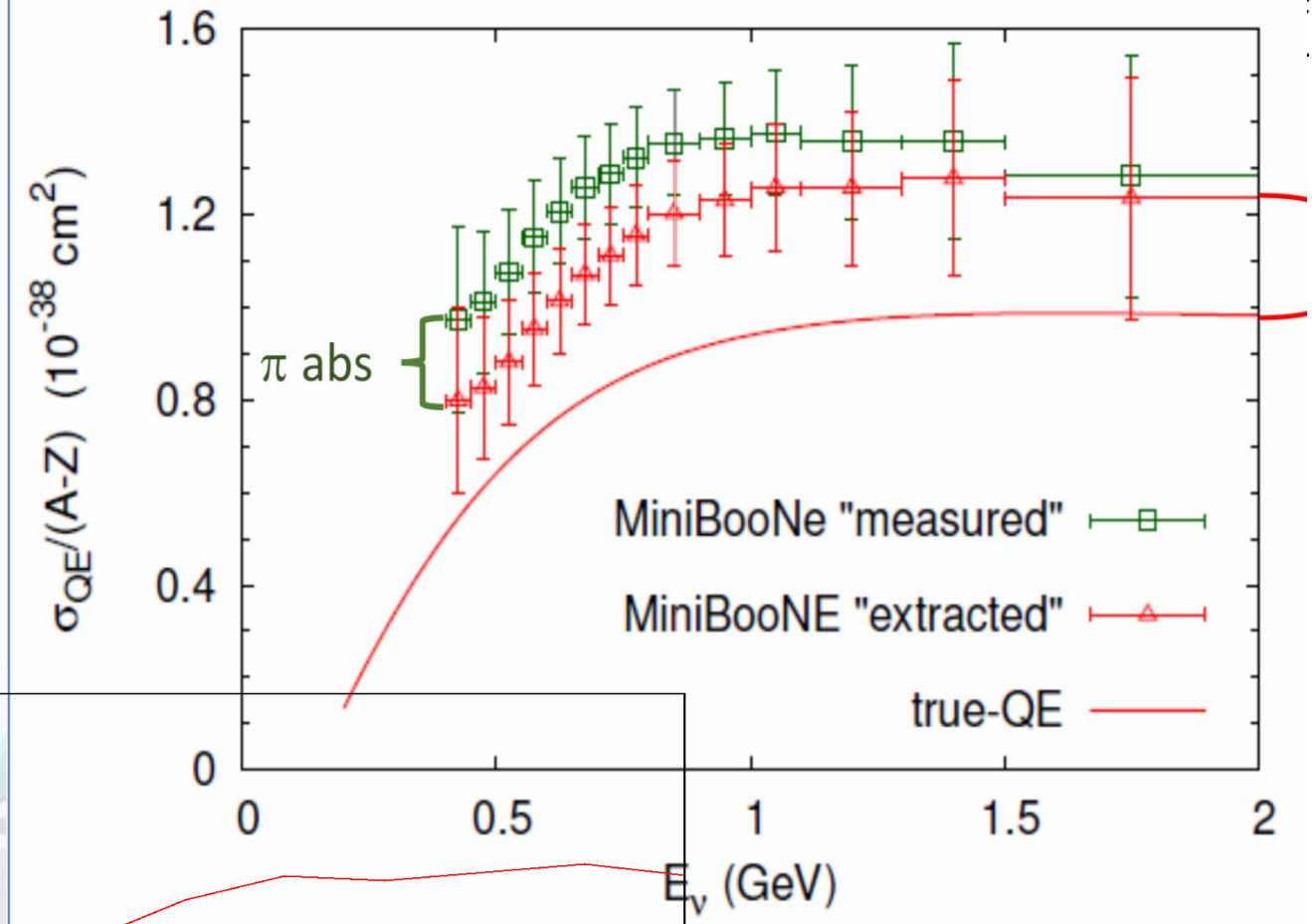
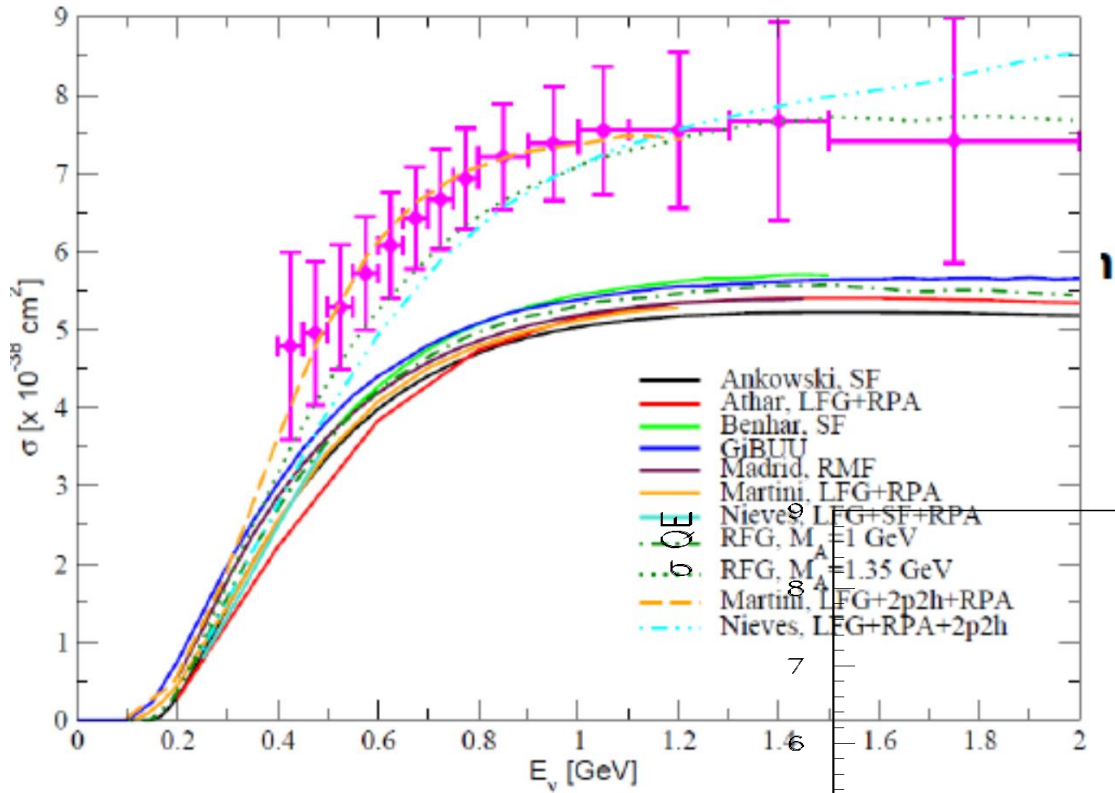
# Pion production in p-p collisions:



Inclusive cross section for the production of  $\pi^0$  (blue),  $\pi^+$  (red), and  $\pi^-$  (green) in p-p collisions as a function of the proton kinetic energy. Lines: simulations, symbols exp. Data. (figure from AstrPhys81, 21 (2016))

**Fig. 2.** Inclusive cross sections for the production of  $\pi^0$  (blue),  $\pi^+$  (red) and  $\pi^-$  (green) in  $p$ - $p$  collision as function of the incoming proton kinetic energy. Lines: FLUKA simulation; points: data from Ref. [28]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

# CCQE on $^{12}\text{C}$



# CCQE on $^{12}\text{C}$

