Final State Interactions in GiBUU

With

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Nuclear ground state

- density distribution: Woods-Saxon (or harm. Oscillator)
- particle momenta: ‘Local Thomas-Fermi approximation’

$$f_{(\mathrm{n,p})}(\vec{r},\vec{p}) = \Theta\left[p_{F(\mathrm{n,p})}(\vec{r}) - |\vec{p}|\right]$$

- Fermi-momentum:

$$p_{F(\mathrm{n,p})}(\vec{r}) = \left(3\pi^2\rho_{(\mathrm{n,p})}(\vec{r})\right)^{1/3}$$

- Fermi-energy:

$$E_{F(\mathrm{n,p})} = \sqrt{p^2_{F(\mathrm{n,p})} + m_N^2} + U_{(\mathrm{n,p})}(\vec{r},p_F)$$

potential: see above

Testparticle ansatz

$$\left[\partial_t + (\nabla_p H_i)\nabla_r - (\nabla_r H_i)\nabla_p\right] f_i(\vec{r},t,\vec{p}) = C\left[f_i, f_j, \ldots\right]$$

- *idea:*
approximate full phase-space density distribution by a sum of delta-functions

- $$f(\vec{r},t,\vec{p}) \sim \sum_{i=1}^{N_{\mathrm{test}}} \delta\left(\vec{r}-\vec{r}_i(t)\right)\delta\left(\vec{p}-\vec{p}_i(t)\right)$$

- each delta-function represents one (test-)particle with a sharp position and momentum
- large number of test particles needed

Ensemble techniques

- **“full ensembles” technique**
- every testparticle may interact with every other one
- rescaling of cross section

$$\sigma_{ij} \to \frac{1}{N_{\text{test}}} \sigma_{ij}$$

- Pros:
  - locality of collisions
- Cons:
  - calculational time: collisions scale with $(N_{test})^2$
  - energy not conserved per ensemble, on average only
  - conserved quantum numbers are respected on average only (‘canonical’)

$$\underbrace{K}_{\text{ensemble } i} + \underbrace{\overline{K}}_{\text{ensemble } j} \to \pi\pi$$

Ensemble techniques

- **“parallel ensembles” technique**
- *idea:*
- testparticle index is also ensemble index

- N_{test} independent runs, densities etc. may be averaged

- Pros:
  - calculational time: collisions scale with N_{test}
  - conserved quantum numbers are strictly respected (‘microcanonical’)
- Cons:
  - non-locality of collisions

$$\sigma_{ij} \simeq 30\,\mathrm{mb} \;\rightarrow\; r = 1\,\mathrm{fm}$$

Time evolution

- time axis is discretized
- collisions only happen at discrete time steps,
- between collisions: propagation (through mean fields)
- typical time-step size: $\Delta t = 0.1\text{-}0.2\,\mathrm{fm}/c$
- start at t=0 and run N timesteps until t_{max}
- typically:

$$N\,\Delta t = t_{\max} \approx 20\text{-}50\,\mathrm{fm}/c$$

$$\Longrightarrow \; N \approx 100\text{-}1000$$

- density/potentials: if not analytically, recalc at every step

Collision term

- contains one-, two-, and three-body collisions

$$C = C_{1\to X} + C_{2\to X} + C_{3\to X}$$

(1) resonance decays

(2) two-body collisions

- elastic and inelastic
- any number of particles in final state
- baryon-meson, baryon-baryon, meson-meson

(3) three-body collisions (only relevant at high densities)

- low energies: cross sections based on resonances

$$\text{e.g. } \pi N \to N^*, \; NN \to NN^*$$

- high energies: string fragmentation

Collision term

- 2-to-2 term $\quad (12 \leftrightarrow 1'2')$

-
$$
\begin{aligned}
&C^{(2,2)}(x,p_1)\\
&= C^{(2,2)}_{\text{gain}}(x,p_1) - C^{(2,2)}_{\text{loss}}(x,p_1)\\
&= \frac{\mathcal{S}_{1'2'}}{2p_1^0 g_{1'} g_{2'}} \int \frac{\mathrm{d}^4 p_2}{(2\pi)^4 2p_2^0} \int \frac{\mathrm{d}^4 p_{1'}}{(2\pi)^4 2p_{1'}^0} \int \frac{\mathrm{d}^4 p_{2'}}{(2\pi)^4 2p_{2'}^0}\\
&\quad \times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_{1'} - p_{2'}) \overline{|\mathcal{M}_{12\to 1'2'}|^2}\\
&\quad \times [F_{1'}(x,p_{1'}) F_{2'}(x,p_{2'}) \overline{F}_1(x,p_1) \overline{F}_2(x,p_2)\\
&\qquad - F_1(x,p_1) F_2(x,p_2) \overline{F}_{1'}(x,p_{1'}) \overline{F}_{2'}(x,p_{2'})]
\end{aligned}
$$

$$F(x,p) = 2\pi g\, f(x,p)\, \mathcal{A}(x,p)$$

$$\overline{F}(x,p) = 2\pi g\, [1 - f(x,p)]\, \mathcal{A}(x,p)$$

Pauli-blocking

Cross section: Geometric interpretation

- particle i and particle j collide, if during timestep Δt

$$r_{ij}(t) = |\vec{r}_i(t) - \vec{r}_j(t)| \overset{!}{\leq} \frac{\sqrt{\sigma_{ij}}}{\pi}$$

- problem 1: only for 2-body collisions

- problem 2: not invariant under Lorentz-Trafos

  - different frames may lead to different ordering of collisions
  - specific frame (‘calculational frame’) needed

Relativistically correct collision criterion

Kodama criterion:

Colliding particle‘s coordinates in cm system:

$$\vec{x}_i^{cm}(t^{cm}) = \vec{x}_i^{cm}(t_i^{cm}) + \vec{\beta}_i^{cm}(t^{cm} - t_i^{cm}).$$

Spatial distance of colliding particles as function of time in cm system:

$$d^2(t^{cm}) = \left(\vec{x}_{12}^{cm} + \vec{\beta}_{12}^{cm} t^{cm}\right)^2$$

$$\begin{aligned} \vec{x}_{12}^{cm} &= \vec{x}_1^{cm} - \vec{x}_2^{cm} - \vec{\beta}_1^{cm} t_1^{cm} + \vec{\beta}_2^{cm} t_2^{cm} \\ \vec{\beta}_{12}^{cm} &= \vec{\beta}_1^{cm} - \vec{\beta}_2^{cm} \end{aligned}$$

Impact parameter:

$$b^2 = (\vec{x}_{12}^{cm})^2 - \frac{\left(\vec{x}_{12}^{cm} \cdot \vec{\beta}_{12}^{cm}\right)^2}{\left(\vec{\beta}_{12}^{cm}\right)^2}$$

$$t_{min}^{cm} = -\frac{\vec{x}_{12}^{cm} \cdot \vec{\beta}_{12}^{cm}}{\left(\vec{\beta}_{12}^{cm}\right)^2}$$

Cross section: Stochastic interpretation

- collision rate per unit phase space

massless, no $(2\pi)^3$

$$\frac{\Delta N_{\rm coll}^{2\to 2}}{\Delta t \Delta^3 x \Delta^3 p_1} = \frac{\Delta^3 p_2}{2E_1\, 2E_2} f_1 f_2 \int \frac{\mathrm{d}^3 p_1'}{2E_1'} \frac{\mathrm{d}^3 p_2'}{2E_2'} |\mathcal{M}| \delta^{(4)}(p_1 + p_2 - p_1' - p_2')$$

$$\sigma_{22} = \frac{1}{2s} \int \frac{\mathrm{d}^3 p_1'}{2E_1'} \frac{\mathrm{d}^3 p_2'}{2E_2'} |\mathcal{M}| \delta^{(4)}(p_1 + p_2 - p_1' - p_2')$$

$$f_i = \frac{\Delta N_i}{\Delta^3 x \Delta^3 p}$$

- collision probability in unit box Δ^3x and unit time Δt

$$P_{22} = \frac{\Delta N_{\rm coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{\rm rel}\sigma_{22} \frac{\Delta t}{\Delta^3 x} \qquad \left(v_{\rm rel} = \frac{s}{2E_1 E_2} \right)$$

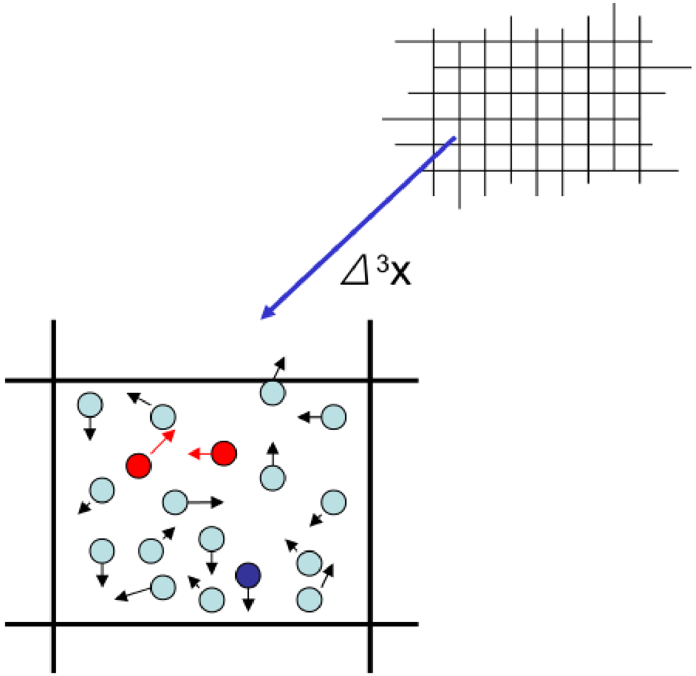
- generalisable to n-body collisions

Cross section: Stochastic interpretation

- discretize time and space

$$\mathrm{P}_{2\to X} = v_{\mathrm{rel}}\sigma_{2\to X}\frac{\Delta t}{\Delta V}$$

$$\mathrm{P}_{3\to X} = \frac{I_{3\to X}}{8E_1E_2E_3}\frac{\Delta t}{(\Delta V)^2}$$



- together with 'full ensemble'
- n particles in cell, randomly select n/2 pairs

$$\mathrm{P}_2 \to \frac{n(n-1)/2}{n/2}\mathrm{P}_2$$

- calculational time: collisions scale approx. with N_{test}
- labeled as "local ensemble method"

Resonance Model

- resonance parameters, decays modes, widths:
  D.Manley, E.Saleski, PRD45 (1992) 4002

PWA of $\pi N \rightarrow \pi N$ and $\pi N \rightarrow \pi\pi N$, consistency!!!

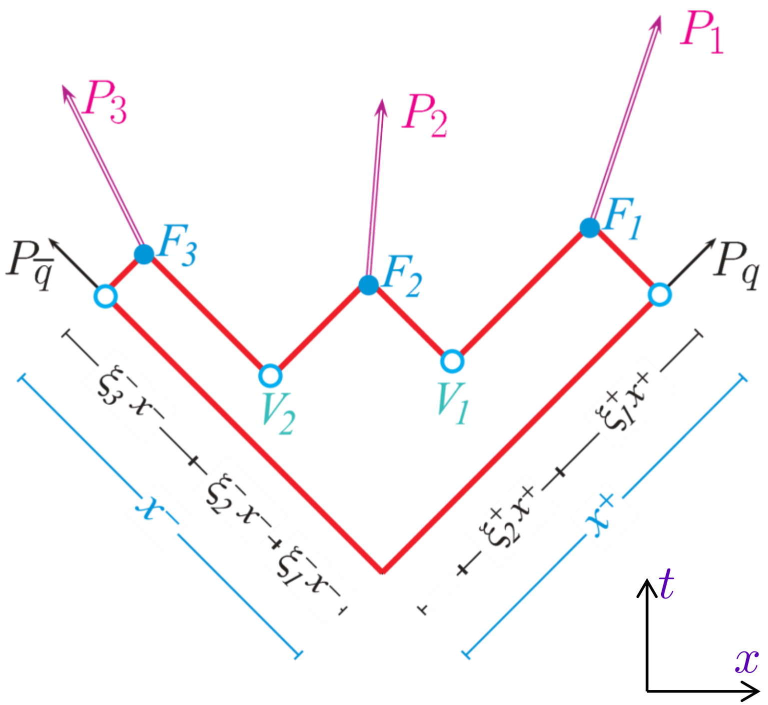
	rating	M_0 [MeV]	Γ_0 [MeV]	$\lvert\mathcal{M}^2\rvert/16\pi$ [mb GeV^2] NR	ΔR	branching ratio in % πN	ηN	$\pi\Delta$	ρN	σN	$\pi N^*(1440)$	$\sigma\Delta$
$P_{11}(1440)$	****	1462	391	70	—	69	—	22_P	—	9	—	—
$S_{11}(1535)$	***	1534	151	8	60	51	43	—	$2_S + 1_D$	1	2	—
$S_{11}(1650)$	****	1659	173	4	12	89	3	2_D	3_D	2	1	—
$D_{13}(1520)$	****	1524	124	4	12	59	—	$5_S + 15_D$	21_S	—	—	—
$D_{15}(1675)$	****	1676	159	17	—	47	—	53_D	—	—	—	—
$P_{13}(1720)$	*	1717	383	4	12	13	—	—	87_P	—	—	—
$F_{15}(1680)$	****	1684	139	4	12	70	—	$10_P + 1_F$	$5_P + 2_F$	12	—	—
$P_{33}(1232)$	****	1232	118	OBE	210	100	—	—	—	—	—	—
$S_{31}(1620)$	**	1672	154	7	21	9	—	62_D	$25_S + 4_D$	—	—	—
$D_{33}(1700)$	*	1762	599	7	21	14	—	$74_S + 4_D$	8_S	—	—	—
$P_{31}(1910)$	****	1882	239	14	—	23	—	—	—	—	67	10_P
$P_{33}(1600)$	***	1706	430	14	—	12	—	68_P	—	—	20	—
$F_{35}(1905)$	***	1881	327	7	21	12	—	1_P	87_P	—	—	—
$F_{37}(1950)$	****	1945	300	14	—	38	—	18_F	—	—	—	44_F

$$\Gamma_{R\rightarrow ab}(m) = \Gamma^0_{R\rightarrow ab}\frac{\rho_{ab}(m)}{\rho_{ab}(M^0)}$$

$$\rho_{ab}(m) = \int p_a^2 p_b^2 \mathcal{A}_a(p_a^2)\mathcal{A}_b(p_b^2)\frac{p_{ab}}{m}B^2_{L_{ab}}(p_{ab}R)\mathcal{F}^2_{ab}(m)$$

(Lund) String-fragmentation (Pythia)

- *idea:*
hard qq scattering (pQCD)
creates a color flux tube ('string')
which then fragments into hadrons
(via qq pair production)



- high energy: 10 GeV...
- "Lund string model"
implementation: Pythia (Jetset)
- only low-lying resonances
- phenomenological fragmentation function
(when and how does a string break?)
- parameters fitted to data (different 'tunes‘, e.g. to HERMES data, available)

Init

- in principle:

1) initialize nucleons

2) perform **one** initial elementary event on **one** nucleon

3) propagate nucleons and final state particles

- correct, but ‘waste of time’

- *idea:*

final state particles do not really disturb the nucleus

- 2 particle classes:
  - ‘real particles’
  - ‘perturbative particles’

Particle classes

- ‘real particles’
  - nucleons
  - may interact among each other
  - interaction products are again ‘real particles’

- ‘perturbative particles’
  - final state particles of initial event
  - may only interact with ‘real particles’
  - interaction products are again ‘perturbative particles’

- ‘real particles’ behave as if other particles are not there

- total energy, total baryon number, etc. not conserved!

Init with perturbative particles

- init

1)initialize nucleons

2)perform **one** initial elementary event on **every** nucleon

3)propagate nucleons and final state particles

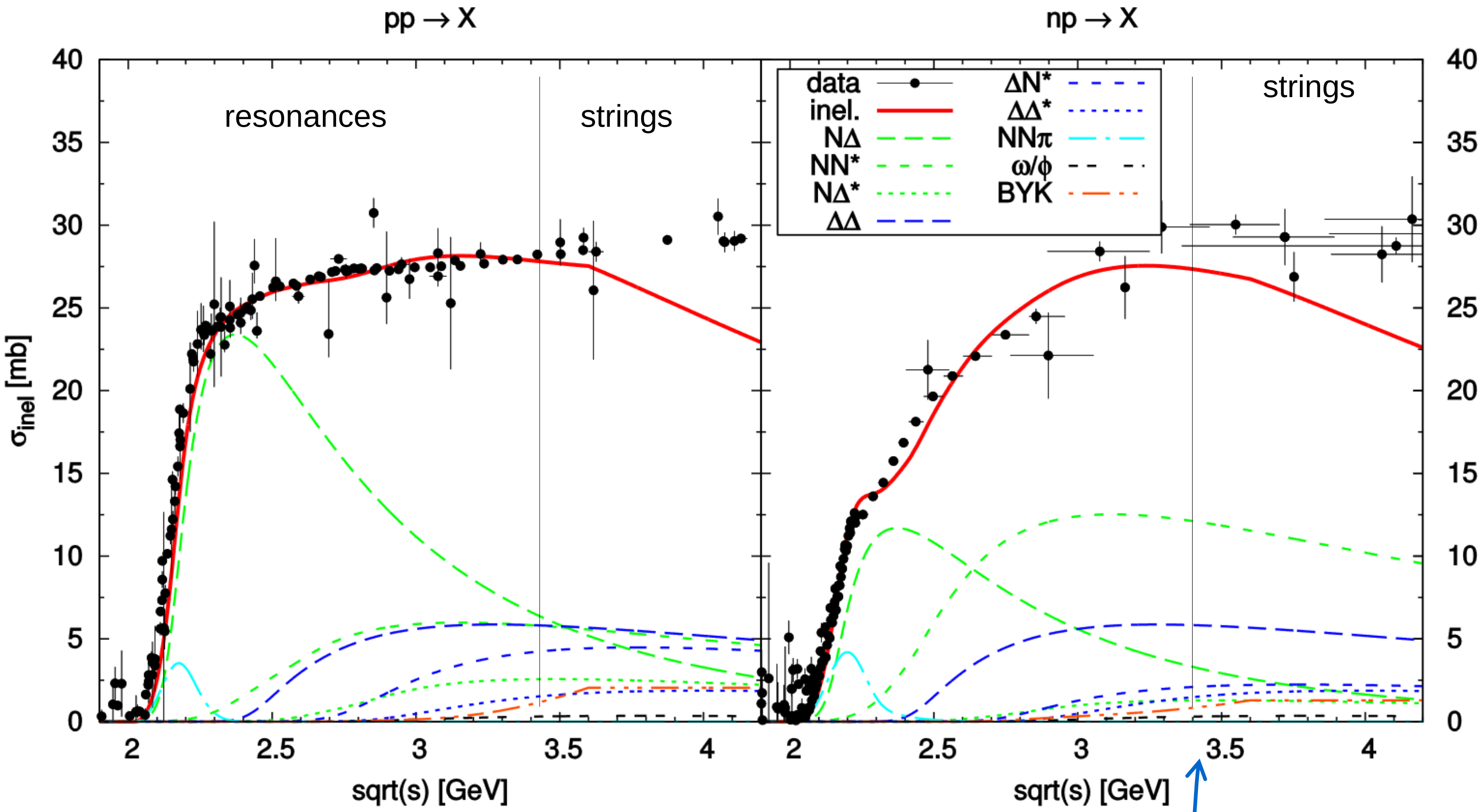
- final states particles are ‘perturbative particles’
- different final states do not interfere

- every final state particle gets a ‘perturbative weight’:
- value: cross section of initial event
- is inherited in every FSI
- *for final spectra the ‘perturbative weights’ have to be added, not only the particle numbers*

Test of FSI

Baryon-Baryon Collisions

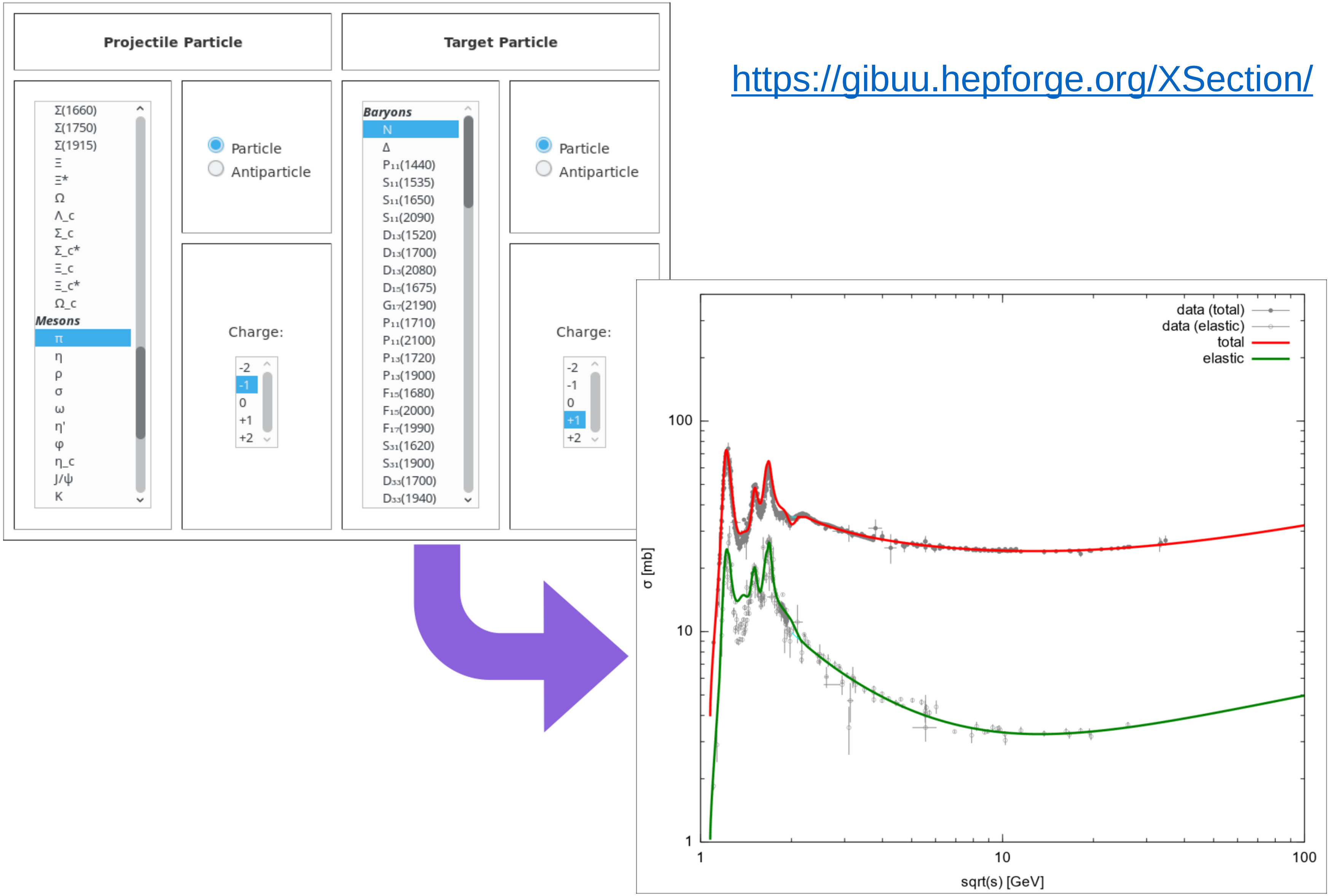
- low energy: resonance model, high energy: string model
- no nice peaks due to two-body kinematics
- *NN*→*NR*,Δ*R* (*R*=Δ,*N**,Δ*)



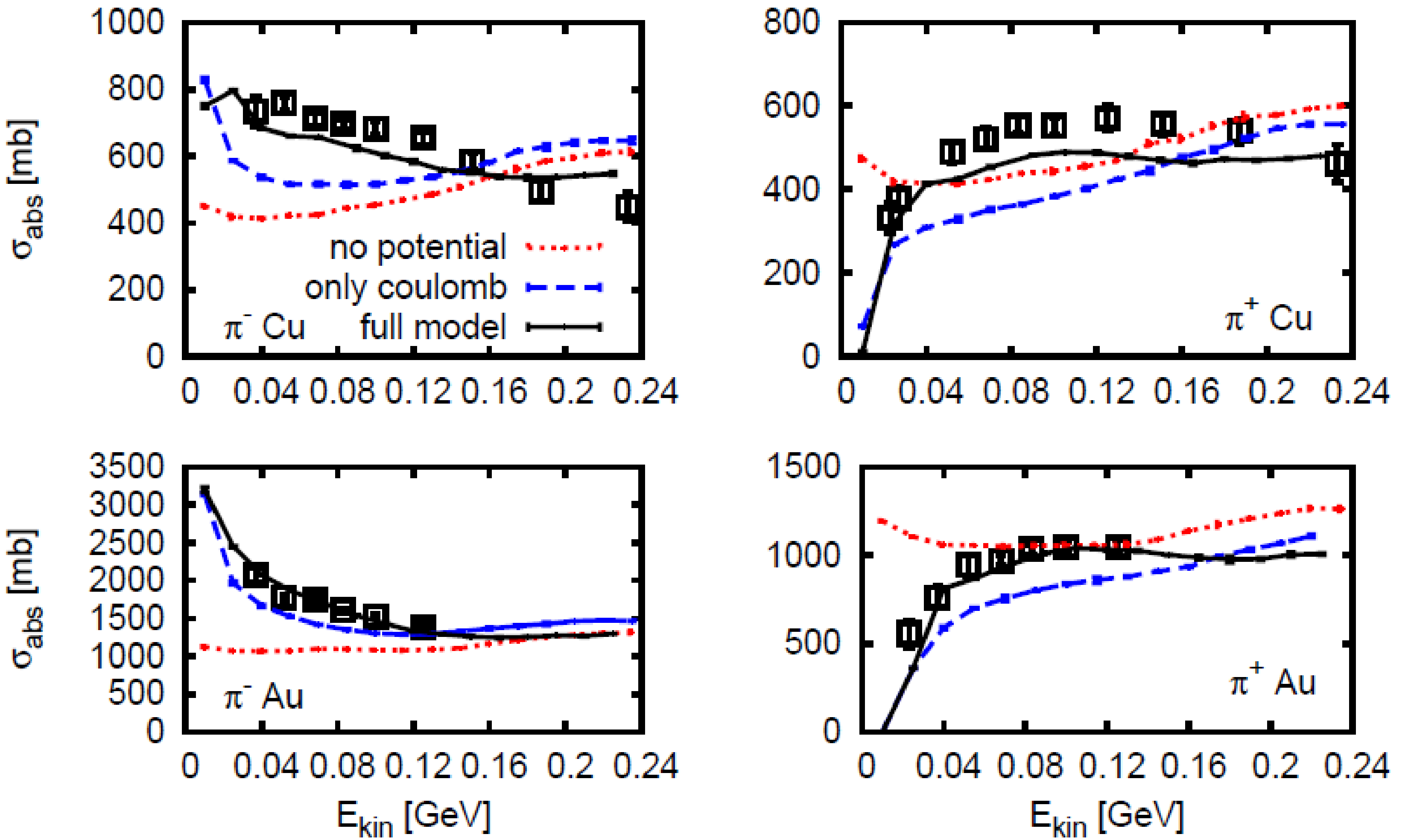
$\sqrt{s} = 3.4 \pm 0.1\,\mathrm{GeV}$

Cross section plotter on GiBUU homepage

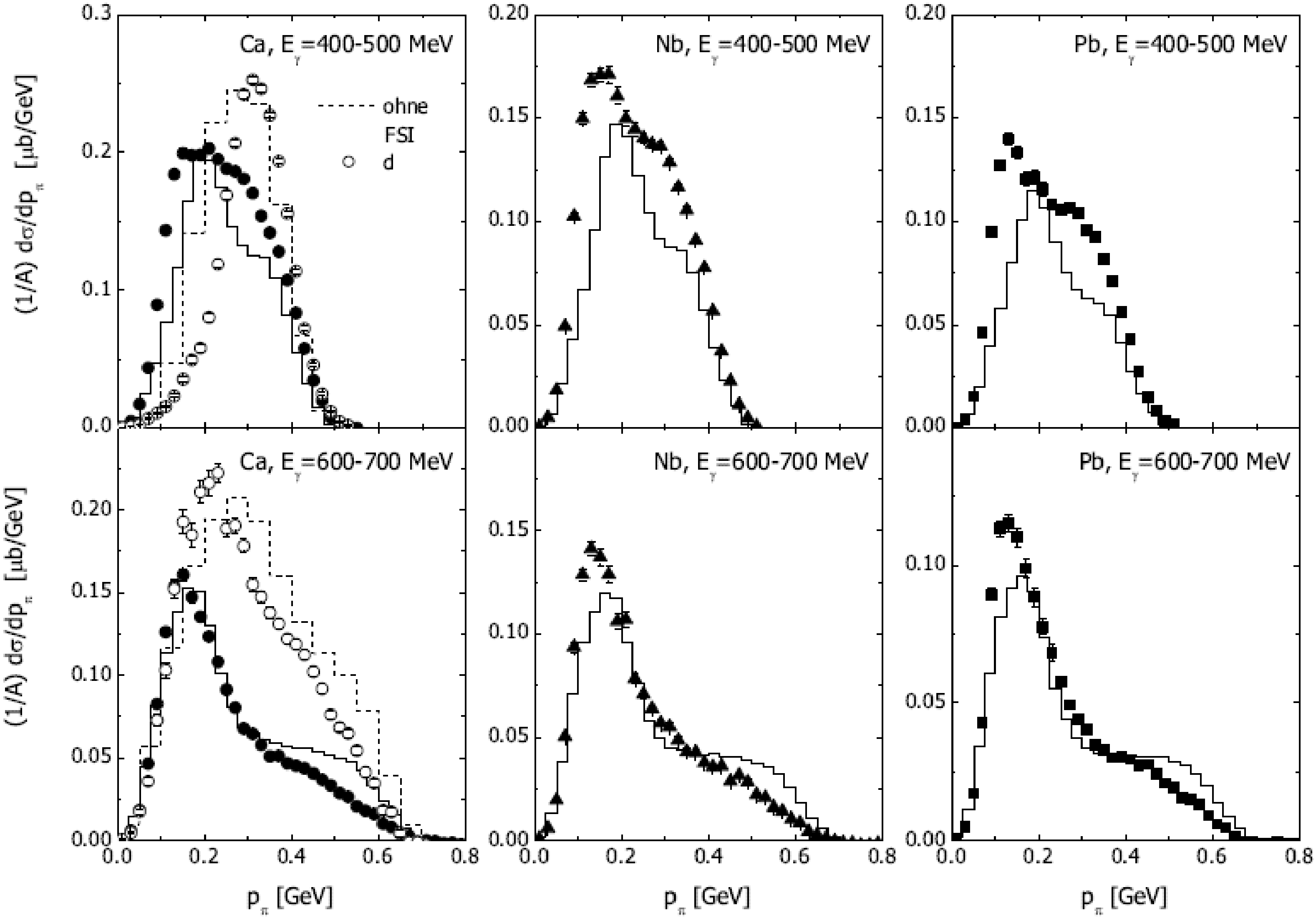
https://gibuu.hepforge.org/XSection/



Pion absorption

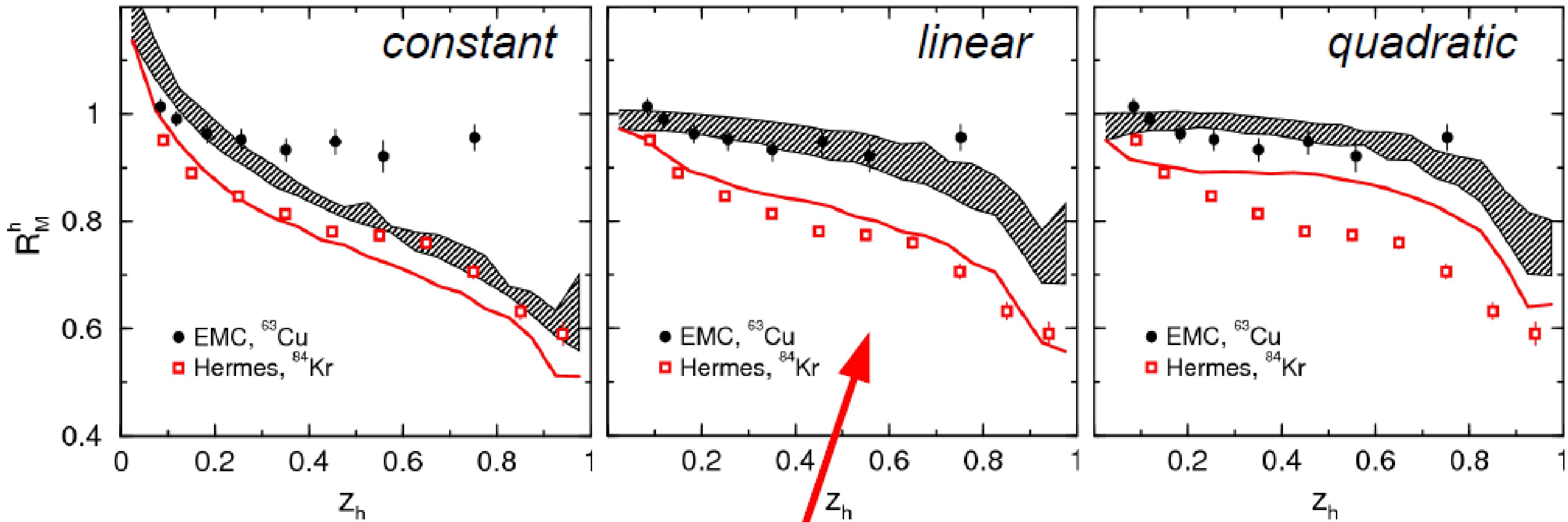


Pi0 Photoproduction



EMC & Hermes

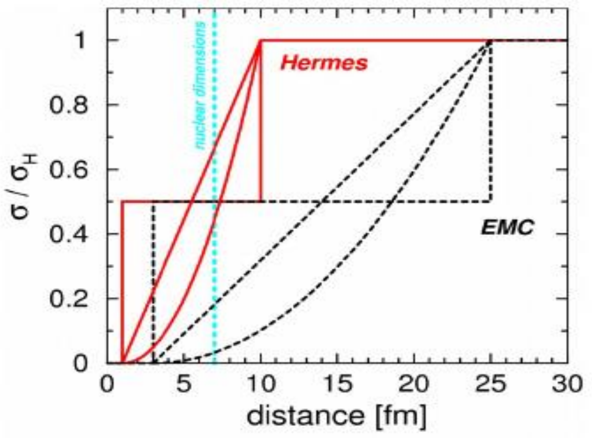
describe simultanously: • EMC@100...280 GeV • Hermes@27 GeV



pre-hadronic cross section:

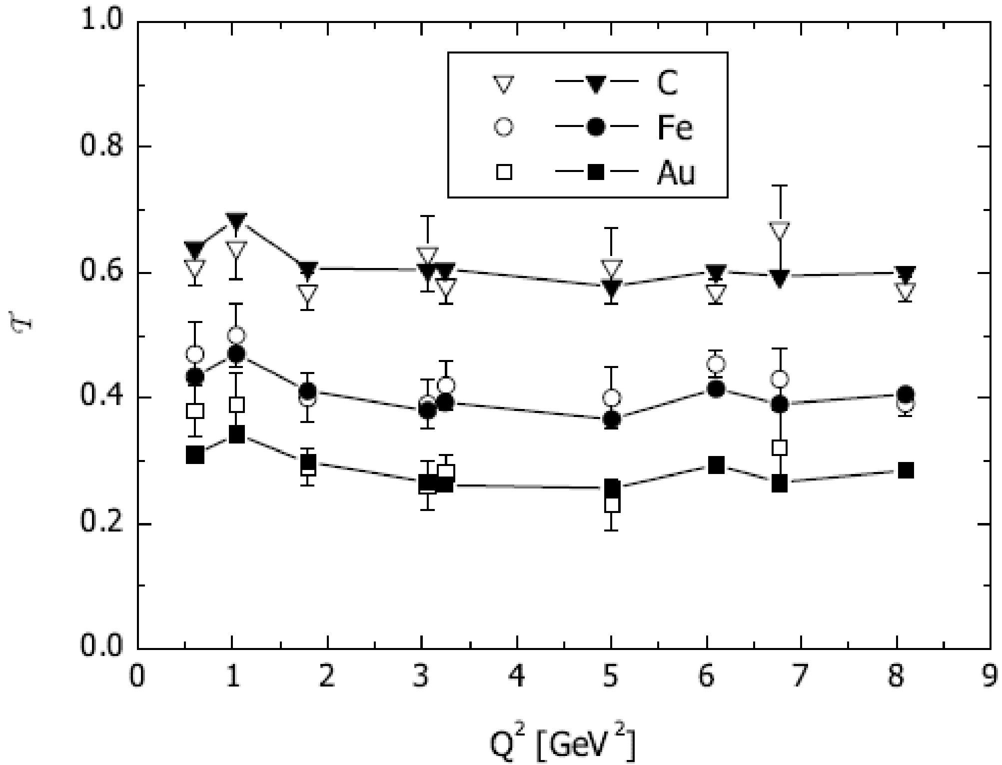
linear increase with time

$$\frac{\sigma^*}{\sigma_H} = \frac{r_{\rm lead}}{Q^2} + \left(1 - \frac{r_{\rm lead}}{Q^2}\right)\left(\frac{t - t_{\rm P}}{t_{\rm F} - t_{\rm P}}\right)$$



cf. also Dokshitzer et al.; Farrar et al.

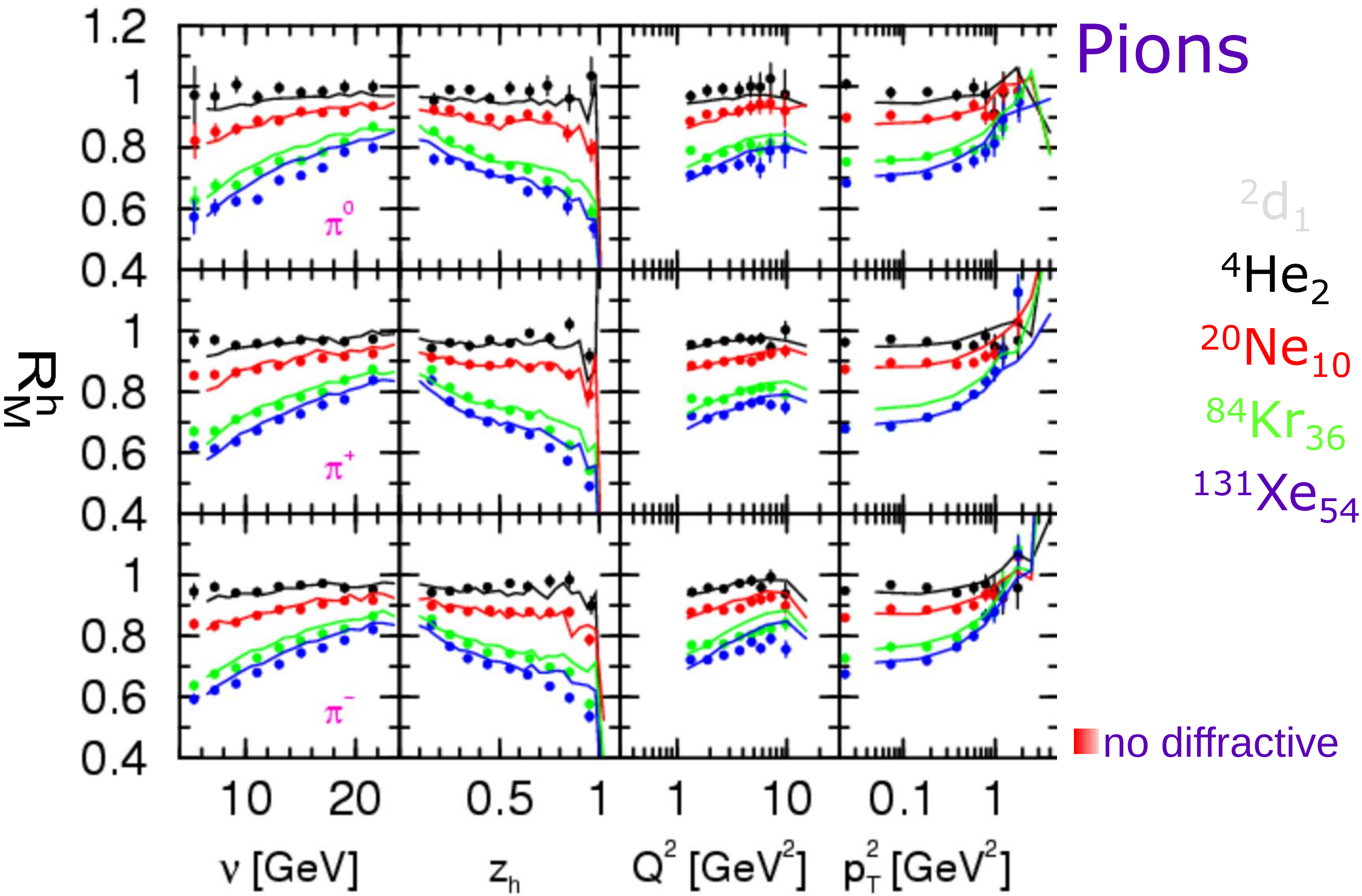
Nucleon transparencies



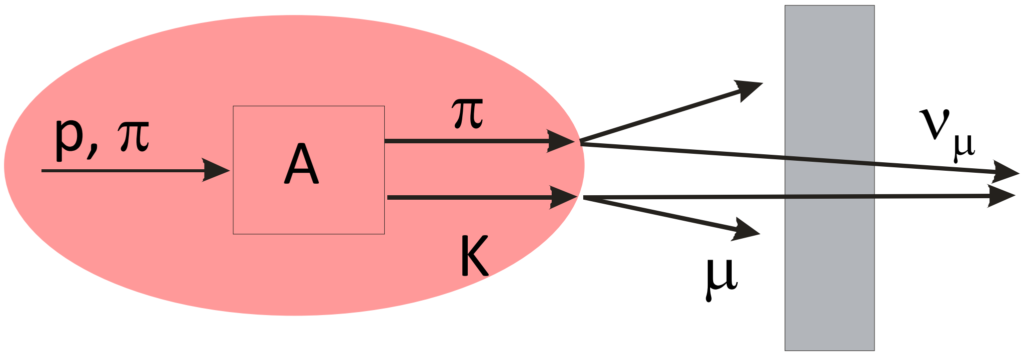
Data from JLAB, SLAC

Excellent agreement over wide kinematical range

Hermes@27: A.Airapetian et al., NPB780(2007)1



HARP, NA61/Shine



aim: adjust flux for …

- DUNE
- T2K
- MINERvA
- …..

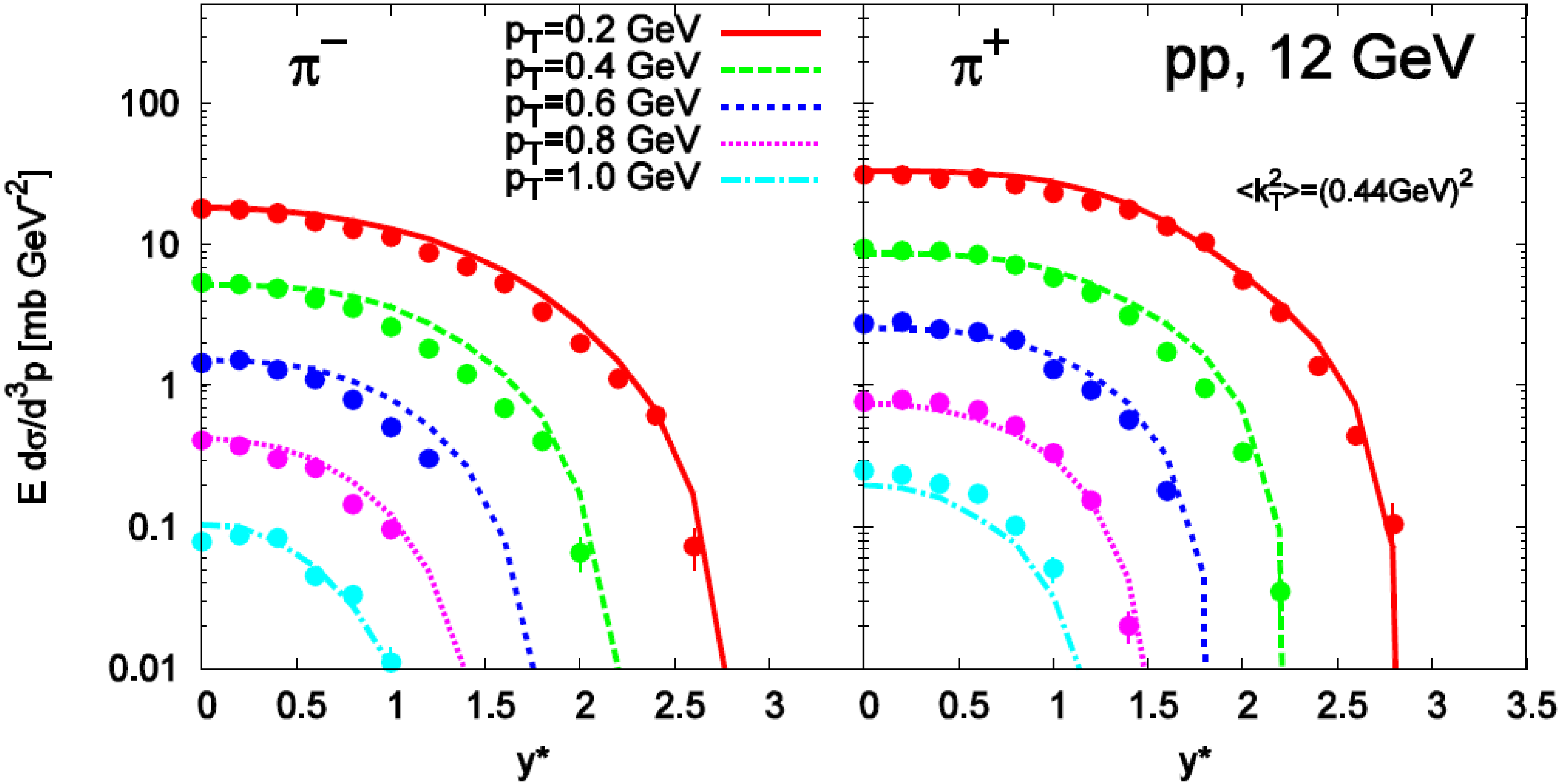
understand hadronic FSI

- proton, pion beam
- beam energies: 3 – 30 GeV/c

- critical test for hadronic fsi

elementary: *pp* → $\pi^{\pm}$ *X*

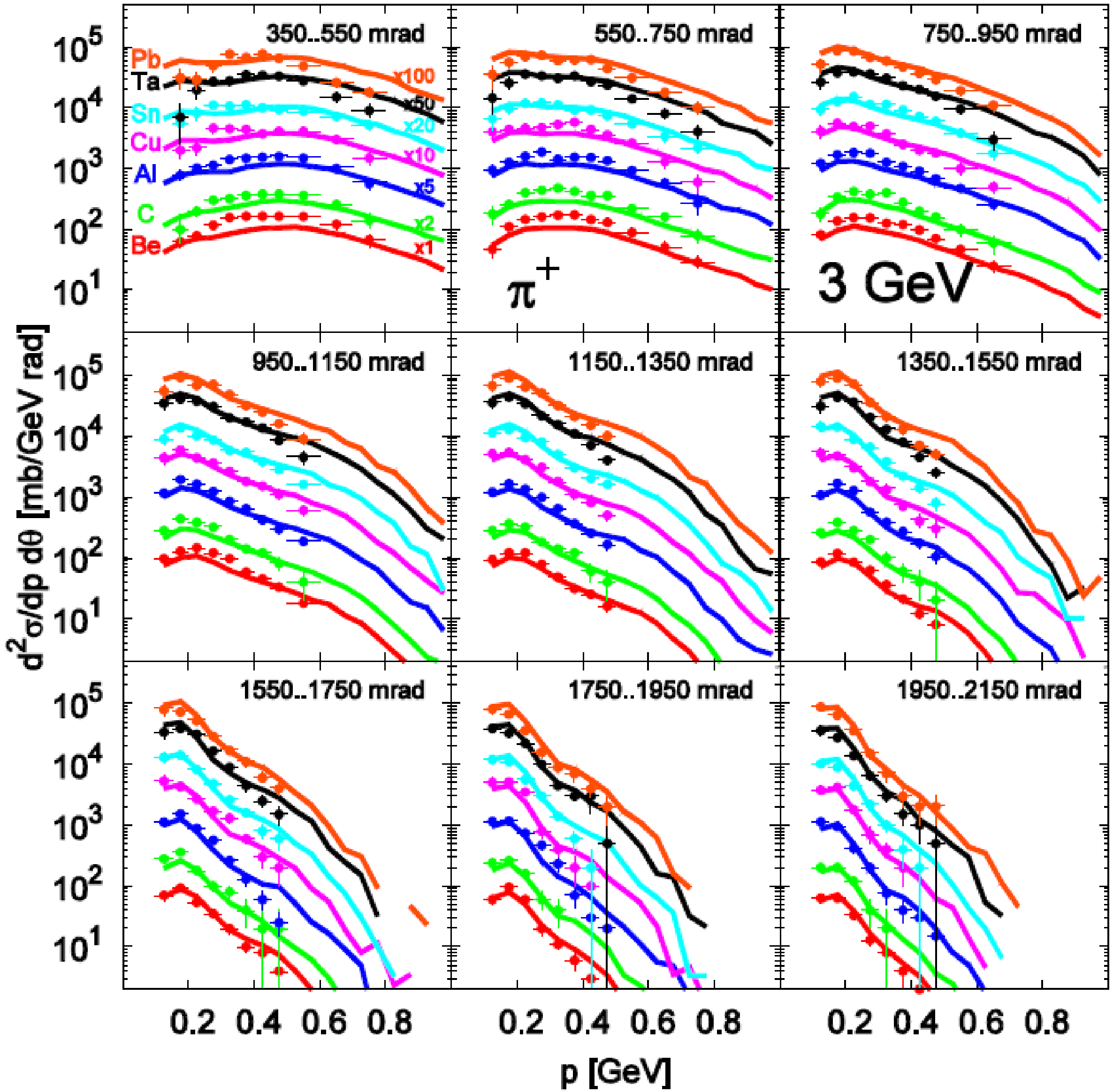
data: V. Blobel et al., Nucl. Phys. B69 (1974) 454



- GiBUU (Pythia v6.4) describes elementary data very well

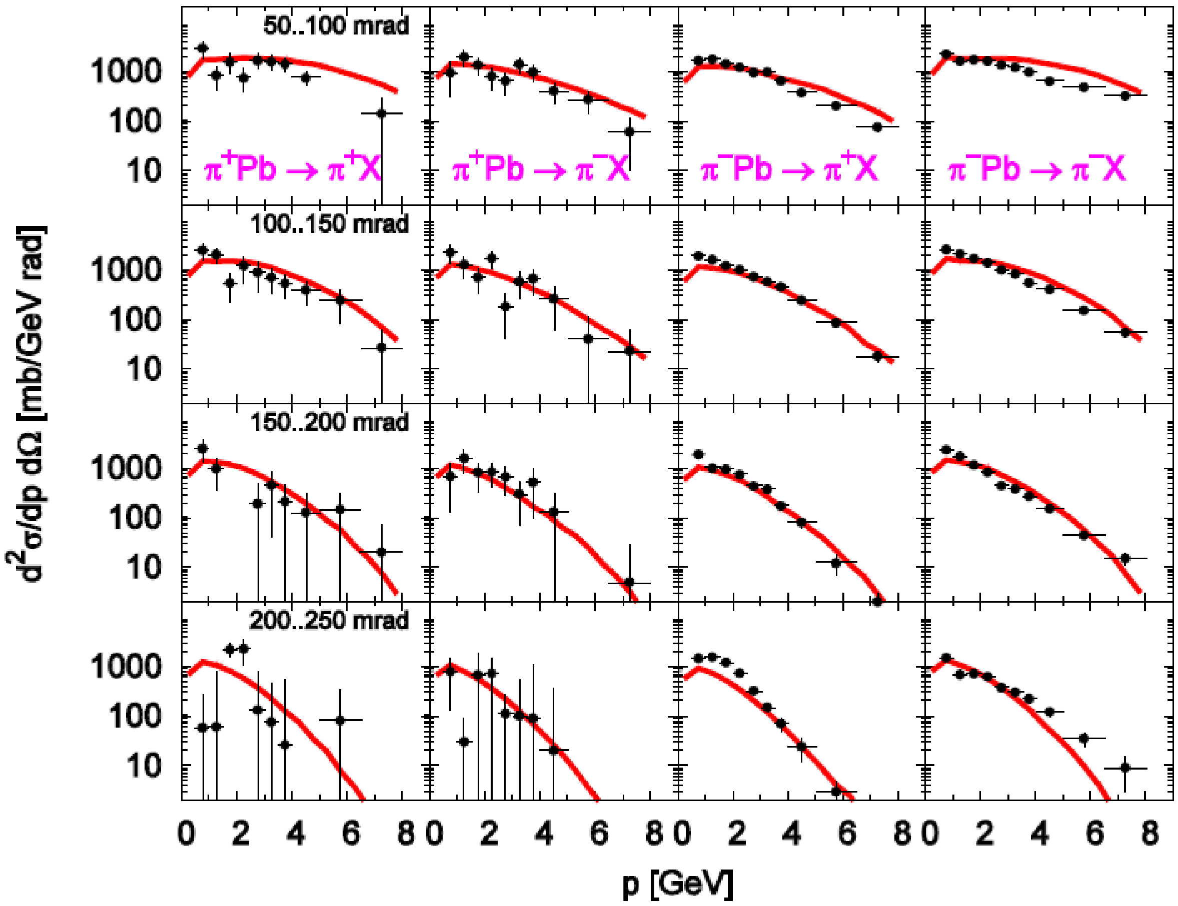
pA → π^+ *X* (backward, 3 GeV/c)

data: M.G. Catanesi et al. (HARP), Phys. Rev. C 77 (2008) 055207



$p^{\pm}$ *Pb* → $\pi^{\pm}$ *X* (forward, 12 GeV/c)

data: M.G. Catanesi et al. (HARP), arXiv:0902.2105 [hep-ex]



- forward production described very well
- pion beam slightly better described than proton beam

Conclusions

- Transport Theory allows to bind nuclei and propagate hadrons in their potentials
- No extra binding energy corrections needed
- Transport will (must) replace simple MCs