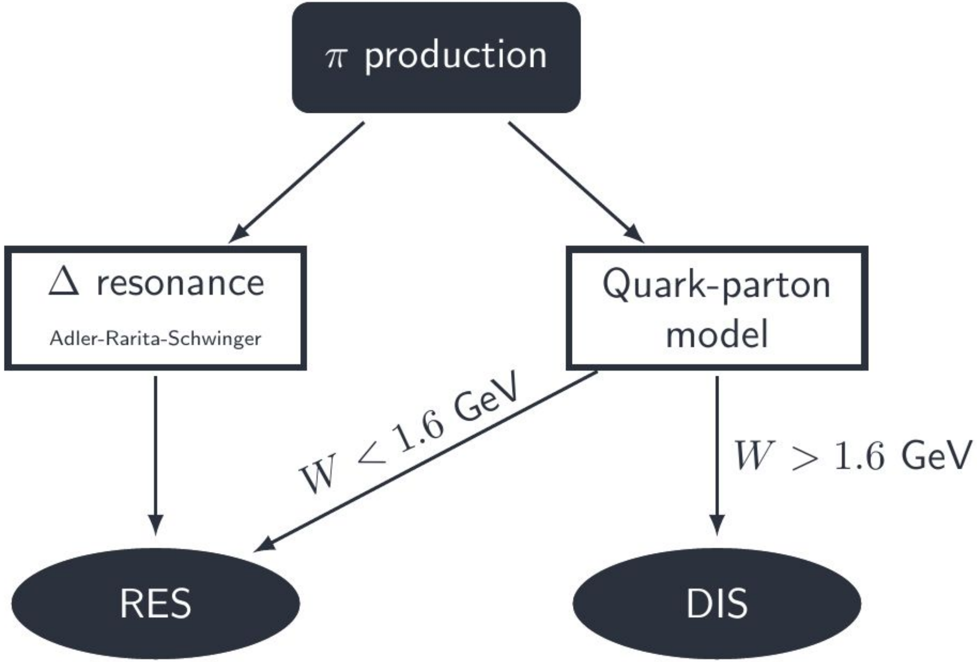
Pion production and absorption in NuWro

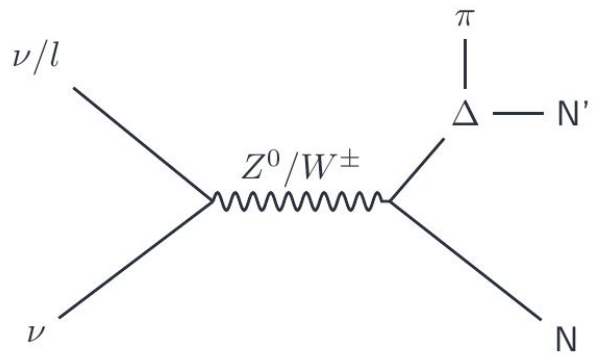
Tomasz Golan (talk given by Jan Sobczyk)
NuWro Collaboration

Modeling neutrino-nucleus interactions
Trento 2018

Pion production in neutrino scattering off nucleon



Single pion production through Delta excitation



The following channels are considered for SPP (labeled as RES):

$$
\begin{aligned}
\nu + p &\rightarrow l^- + \left(\Delta^{++} \rightarrow p + \pi^+\right) \\
\nu + n &\rightarrow l^- + \left(\Delta^{+} \rightarrow p + \pi^0 \text{ or } n + \pi^+\right) \\
\overline{\nu} + p &\rightarrow l^+ + \left(\Delta^{0} \rightarrow p + \pi^- \text{ or } n + \pi^0\right) \\
\overline{\nu} + n &\rightarrow l^+ + \left(\Delta^{-} \rightarrow n + \pi^-\right) \\
\nu(\overline{\nu}) + p &\rightarrow \nu(\overline{\nu}) + \left(\Delta^{+} \rightarrow p + \pi^0 \text{ or } n + \pi^+\right) \\
\nu(\overline{\nu}) + n &\rightarrow \nu(\overline{\nu}) + \left(\Delta^{0} \rightarrow p + \pi^- \text{ or } n + \pi^0\right)
\end{aligned}
$$

Adler-Rarita-Schwinger formalism

$$\frac{\mathrm{d}\sigma}{\mathrm{d}W\,\mathrm{d}Q^2} = G^2\cos^2\theta_C\frac{W\,g(W)}{\pi^2 M E_\nu^2}\left[-(Q^2+m^2)W_1 + \frac{W_2}{M^2}\left(2\,(pq)\,(pk')\,\frac{M^2}{2}\left(Q^2+m^2\right)\right)\right.$$
$$\left.-\frac{W_3}{M^2}\left(Q^2\,(kp) - \frac{1}{2}\left(Q^2+m^2\right)(pq)\right) + \frac{W_4}{M^2}\frac{m^2}{2}\left(Q^2+m^2\right) - 2\frac{W_5}{M^2}m^2\,(kp)\right]$$

with Delta width introduced through Breit-Wigner formula:

$$g(W) = \frac{\Gamma_\Delta/2}{\left(W - M_\Delta\right)^2 + \Gamma_\Delta^2/4}$$

Rarita-Schwinger field Ψ_μ

The final hadronic state is a 3/2-spin resonance described as a Rarita-Schwinger field and the transition from the nucleon to Delta++ state is given as a matrix element of the weak hadronic current:

$$\mathcal{J}_\mu^{\mathrm{CC}} = \mathcal{J}_\mu^V + \mathcal{J}_\mu^A$$

$$\begin{aligned}\langle \Delta^{++}(p') | \mathcal{J}_\mu^V | N(p) \rangle = \sqrt{3}\bar{\Psi}_\lambda(p') \Big[& g^\lambda{}_\mu \Big(\frac{C_3^V}{M}\gamma_\nu + \frac{C_4^V}{M^2} p'_\nu \\ & + \frac{C_5^V}{M^2} p_\nu \Big) q^\nu - q^\lambda \Big(\frac{C_3^V}{M}\gamma_\mu + \frac{C_4^V}{M^2} p'_\mu \\ & + \frac{C_5^V}{M^2} p_\mu \Big) \Big] \gamma_5 u(p), \qquad (5)\end{aligned}$$

$$\begin{aligned}\langle \Delta^{++}(p') | \mathcal{J}_\mu^A | N(p) \rangle = \sqrt{3}\bar{\Psi}_\lambda(p') \Big[& g^\lambda{}_\mu \Big(\gamma_\nu \frac{C_3^A}{M} + \frac{C_4^A}{M^2} p'_\nu \Big) q^\nu \\ & - q^\lambda \Big(\frac{C_3^A}{M}\gamma_\mu + \frac{C_4^A}{M^2} p'_\mu \Big) + g^\lambda{}_\mu C_5^A \\ & + \frac{q^\lambda q_\mu}{M^2} C_6^A \Big] u(p). \qquad (6)\end{aligned}$$

Hadronic tensor $W_{\mu\nu}$

Hadronic tensor is defined as:

$$W_{\mu\nu} = \frac{1}{4MM_\Delta} \frac{1}{2} \sum_{\text{spin}} \langle \Delta^{++}, p' | \mathcal{J}_\mu^{\text{CC}} | p \rangle \langle \Delta^{++}, p' | \mathcal{J}_\nu^{\text{CC}} | p \rangle^* \times \frac{\Gamma_\Delta / 2}{((W - M_\Delta)^2 + \Gamma_\Delta^2 / 4)}, \quad (8)$$

$\Gamma_\Delta(W)$ is the Δ width, for which we assume the P-wave ($l = 1$) expression

$$\Gamma_\Delta = \Gamma_0 \left(\frac{q_{\text{cm}}(W)}{q_{\text{cm}}(M_\Delta)} \right)^{2l+1} \frac{M_\Delta}{W} \quad (10)$$

with

$$q_{\text{cm}}(W) = \sqrt{\left(\frac{W^2 + M^2 - m_\pi^2}{2W} \right)^2 - M^2}. \quad (11)$$

$M_\Delta = 1232$ MeV and $m_\pi = 139.57$ MeV is the charged pion mass.

Form factors

- The structure functions depend on vector and axial form factors $C_i^{V, A}$
- There are several parameterisations available in NuWro
- With default taken from

C_5^A axial form factor from bubble chamber experiments

K. M. Graczyk, D. Kiełczewska, P. Przewłocki, and J. T. Sobczyk
Phys. Rev. D **80**, 093001 – Published 2 November 2009

- simultaneus fit to both ANL and BNL
- demonstration that they are consistent!
- C5A = 1.19 and Ma = 0.94

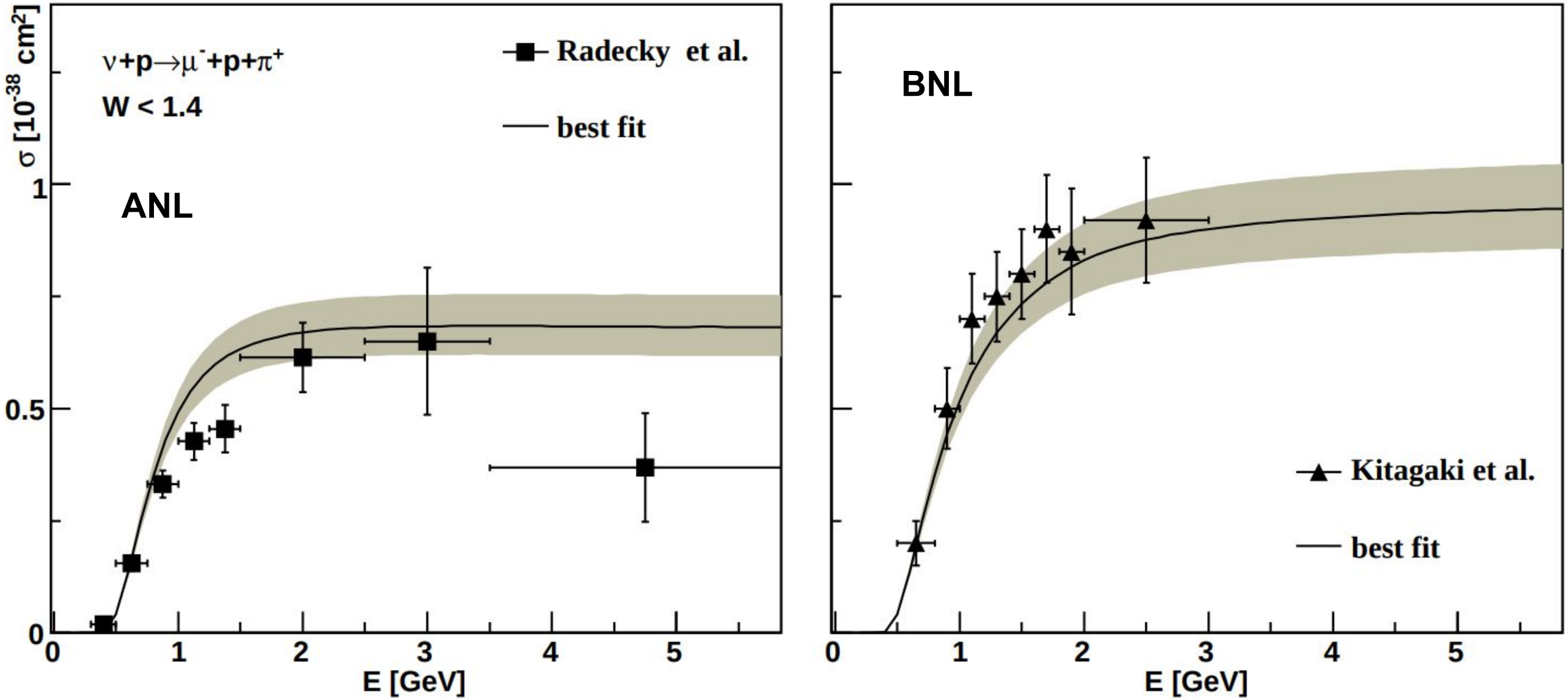
$$C_3^V(Q^2) = 2.13\left(1+\frac{Q^2}{4M_V^2}\right)^{-1} G_D(Q^2),$$

$$C_4^V(Q^2) = -1.51\left(1+\frac{Q^2}{4M_V^2}\right)^{-1} G_D(Q^2),$$

$$C_5^V(Q^2) = 0.48\left(1+\frac{Q^2}{0.776M_V^2}\right)^{-1} G_D(Q^2),$$

$$G_D(Q^2) = \left(1+\frac{Q^2}{M_V^2}\right)^{-2}, \quad \text{and} \quad M_V = 0.84 \text{ GeV}.$$

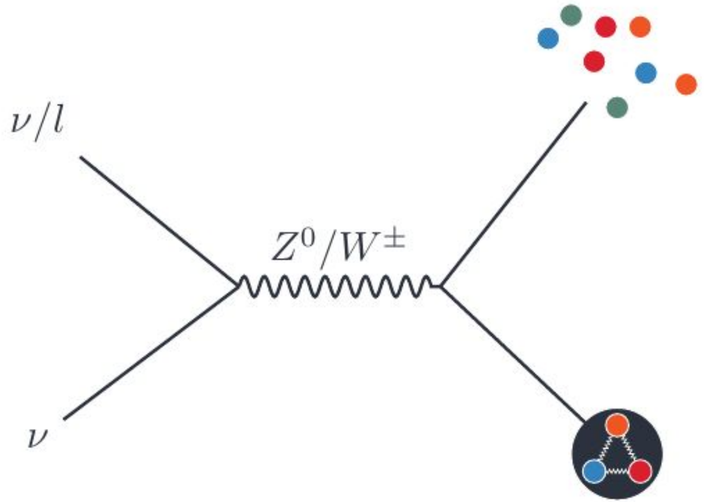
Comparison with ANL / BNL data



Angular distribution

- Δ(1232) resonance decay anisotropy correction based on density matrix measured in ALN/BNL experiments:
  - S.J. Barish et al, Phys. Rev. D19 (1979) 2511
  - G.M. Radecky et al, Phys. Rev. D25 (1982) 1161
  - T. Kitagaki et al., Phys. Rev. D34 (1986) 2554.)
- The cross section is calculated assuming isotropic pion angular distribution
- and reweighted according to ANL/BNL parameterization

Deep inelastic scattering



Events with invariant mass W > 1.6 GeV are considered within quark-parton model and labeled as DIS

$$\nu\,(\overline{\nu}) + N \;\rightarrow\; l^-\left(l^+\right) + X$$
$$\nu\,(\overline{\nu}) + N \;\rightarrow\; \nu\,(\overline{\nu}) + X$$

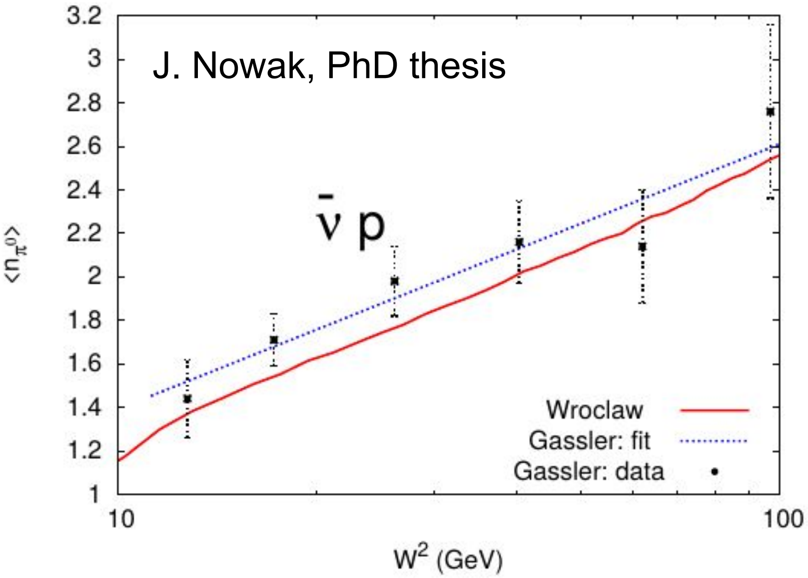
DIS cross section

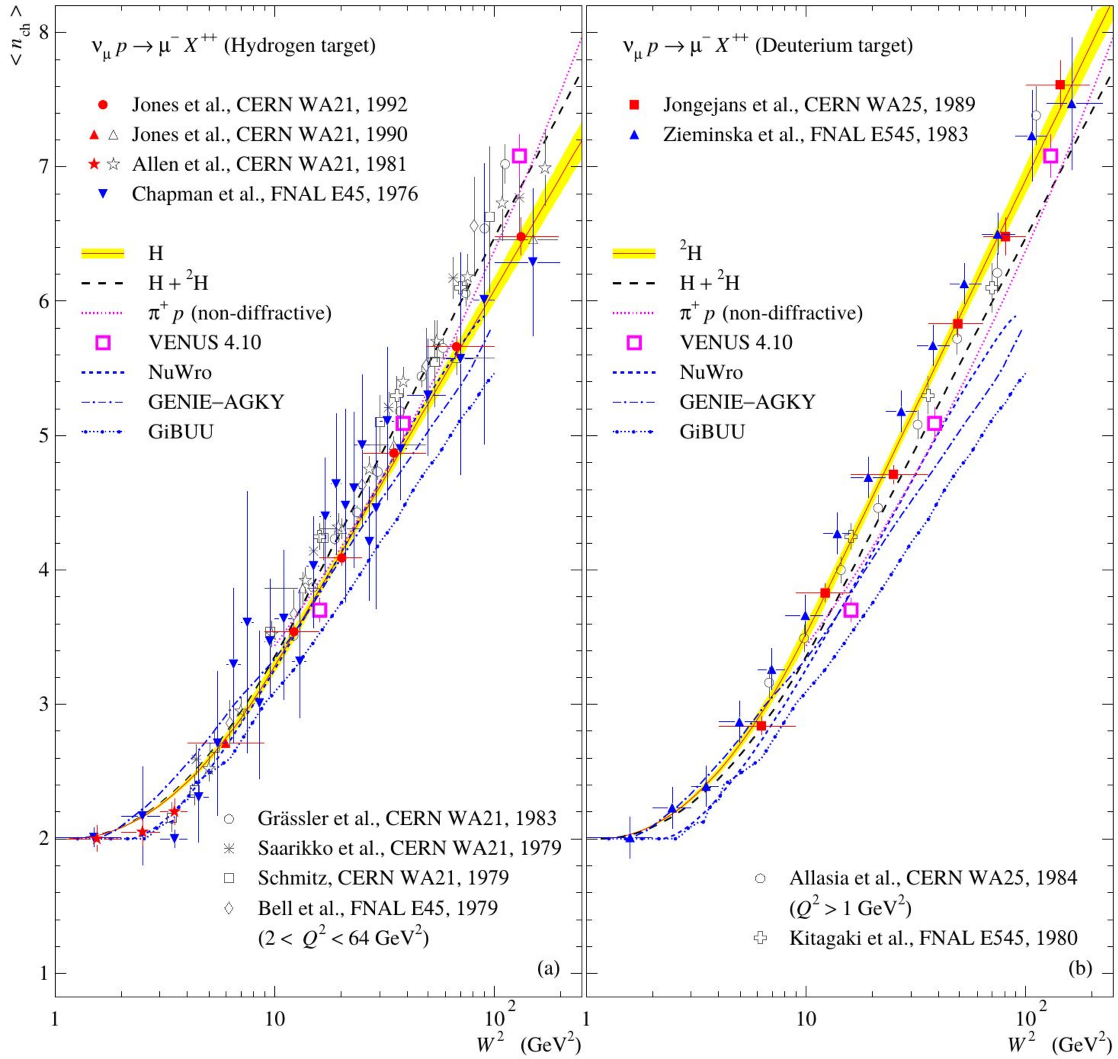
$$\frac{\mathrm{d}^2\sigma^{\nu/\overline{\nu}}}{\mathrm{d}x\mathrm{d}y} = \frac{G^2 M E_\nu}{\pi\left(1+Q^2/M_{W,Z}^2\right)^2}\left[y\left(xy+\frac{m^2}{2E_\nu M}\right)F_1 + \left(1-y-\frac{Mxy}{2E_\nu}-\frac{m^2}{4E_\nu^2}-\frac{m^2}{2ME_\nu x}\right)F_2 \pm \left(xy\left(1-\frac{y}{2}\right)-y\frac{m^2}{4ME_\nu}\right)F_3\right]$$

with Bodek-Young modification to the parton distribution functions for low Q2 included

Hadronization

- The hadronization is performed using Pythia6 routines
- with hand-crafted parameters tuned to experimental data
- e.g. average pi0 multiplicity:





Charged Hadron Multiplicity

- A lot of effort put into tuning Pythia6 parameters
- Hadronization works very well in the broad range of invariant mass

Mean charged multiplicities in charged-current neutrino scattering on hydrogen and deuterium
K. S. Kuzmin, V. A. Naumov
Phys.Rev. C88 (2013) 065501

Transition region & non-resonant background

The smooth transition from Delta do DIS is made for invariant mass (1.3, 1.6) GeV

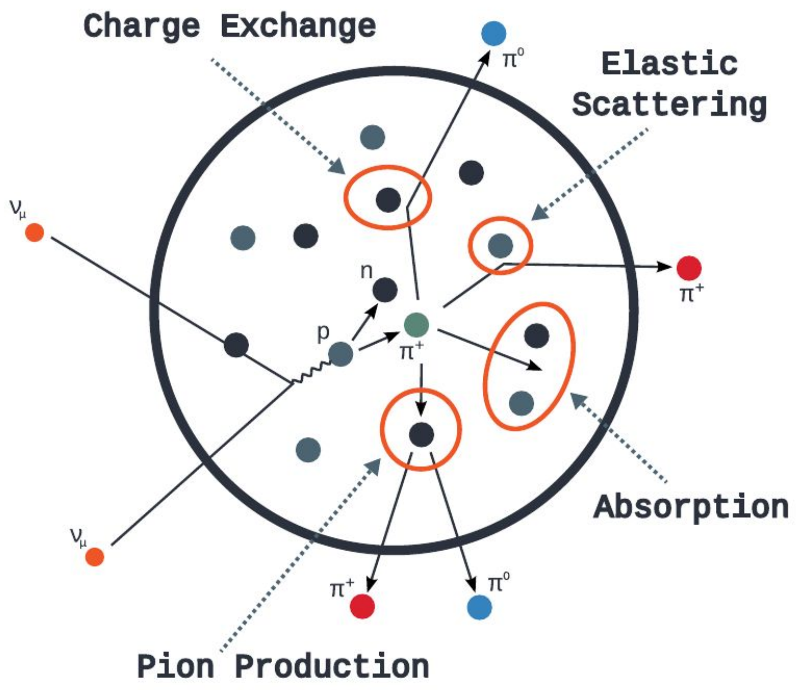
$$\frac{\mathrm{d}\sigma^{SPP}}{\mathrm{d}W} = \frac{\mathrm{d}\sigma^{\Delta}}{\mathrm{d}W}(1-\alpha(W)) + \frac{\mathrm{d}\sigma^{DIS}}{\mathrm{d}W}F^{SPP}(W)\alpha(W) \qquad F^{SPP}(W) = \frac{\mathrm{d}\sigma^{DIS-SPP}/\mathrm{d}W}{\mathrm{d}\sigma^{DIS}/\mathrm{d}W}$$

NRB is different for each SPP channel, so alpha0 is fitted independently for each pion production process

$$\begin{aligned}\alpha(W) &= \Theta\left(W_{min}-W\right)\frac{W-W_{th}}{W_{min}-W_{th}}\alpha_0 \\ &+ \Theta\left(W_{max}-W\right)\Theta\left(W-W_{min}\right)\frac{W-W_{min}+\alpha_0\left(W_{max}-W\right)}{W_{max}-W_{min}} \\ &+ \Theta\left(W-W_{max}\right)\end{aligned}$$

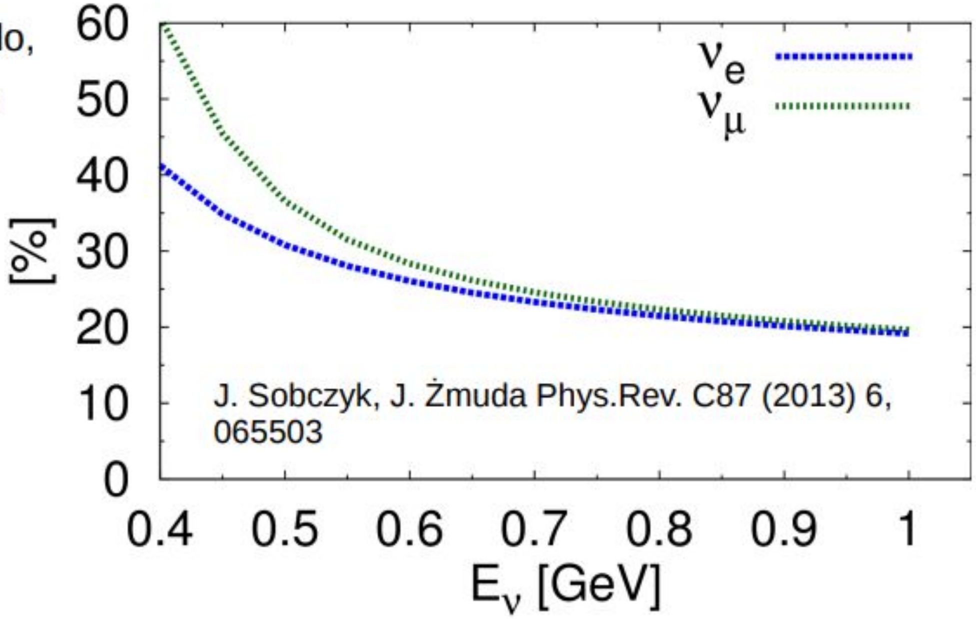
Pion production in neutrino scattering off nucleus

For neutrino-nucleus interactions, pions created in primary scattering off bound nucleon are subject to intranuclear cascade



Nuclear effects for Delta

Δ(1232) self-energy effects (pionless decays, corrections to SPP from E. Oset and L. L. Salcedo, Nucl. Phys. A 468, 631 (1987)) in an approximate manner (as a total cross-section modification)



from J. Żmuda

Intranuclear cascade

- A nucleus is probed in intervals small enough to assume constant density
- So the probability of passing λ without any interactions is given by:

$$P(\lambda) = e^{-\lambda/\tilde{\lambda}}$$

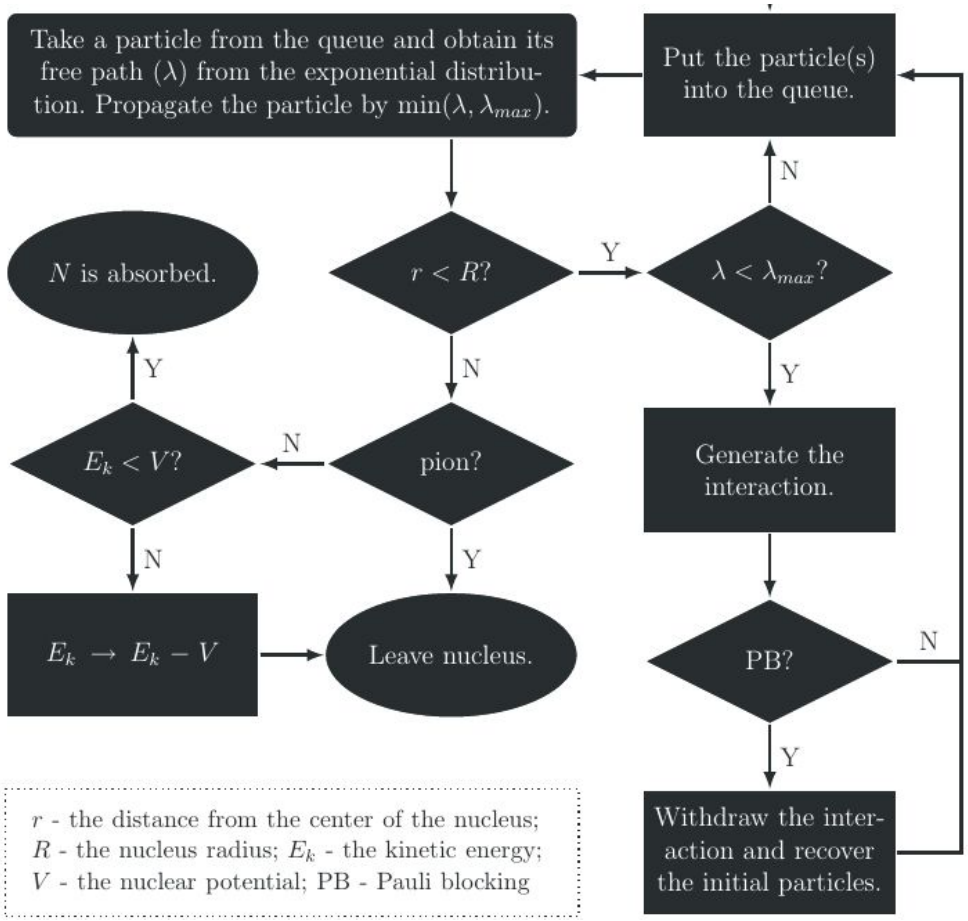
- Where mean free path is calculated assuming ρ being const within λ:

$$\tilde{\lambda} = [\sigma_p \rho_p(r) + \sigma_n \rho_n(r)]^{-1}$$

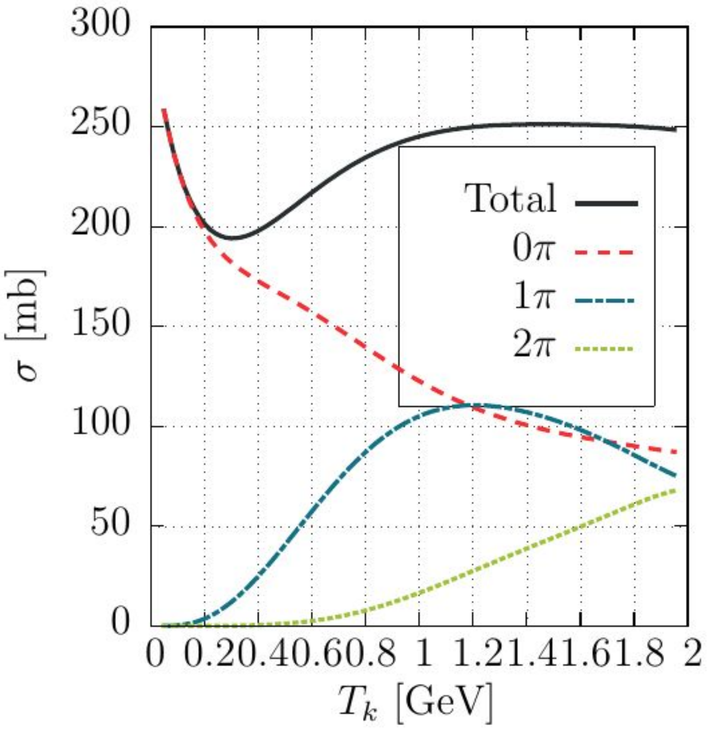
- And free path is chosen randomly according to:

$$\lambda = -\tilde{\lambda} \cdot \ln(\text{rand}[0, 1])$$

INC algorithm



Pion production in nucleon propagation



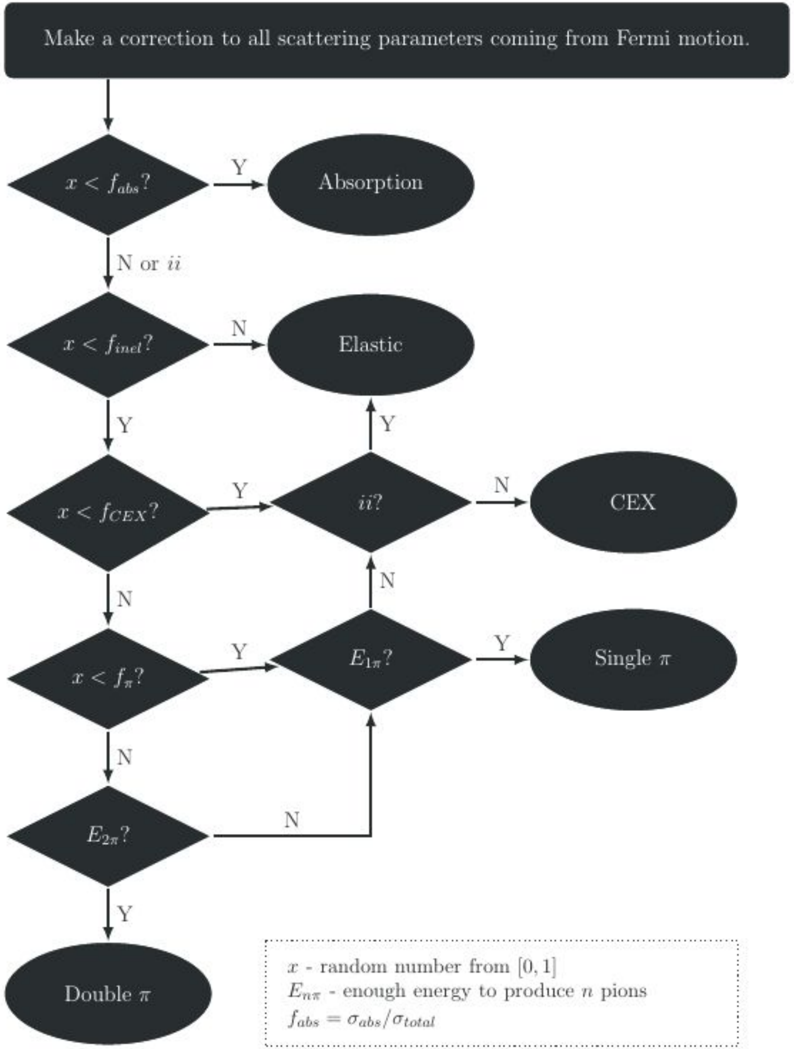
The contribution of pion production channels to total cross section for proton-Carbon scattering.

Pion cascade

- Coefficients: $f_x = \sigma_x / \sigma_{total}$ for channel x
- Cross sections for pion kinetic energy below 350 MeV are taken from Oset et al. (E. Oset et al., Phys.Lett. B165 (1985), pp. 13–18.)
- Data driven xsec are used for higher energies
- For elastic scattering and charge exchange process angular distribution is given by:

$$\frac{d\sigma}{d\Omega} \sim \sum_{i=0}^{7} a_i \cos^i \theta$$

- where a_i are tuned to SAID model



Oset et al model

$$\sigma_{\pi^+ p} = \frac{1}{|\vec{q}|}\frac{2}{3}\left(\frac{f^*}{m_\pi}\right)^2 |G_\Delta|^2 |\vec{q}_{c.m.}|^2 \frac{1}{2}\Gamma$$

$\vec{q}$ - pion momentum in LAB frame

$\vec{q}_{c.m.}$ - pion momentum in center of mass frame

f^* - $\pi N \Delta$ coupling constant $\left(f^{*2}/4\pi = 0.36\right)$

m_π - pion mass

G_Δ - Δ propagator

Γ - Δ width

All other QEL processes (including charge exchange) can be obtained using Clebsch-Gordan coefficients.

- Pion absorption is introduced by a nuclear modification of Δ width:

$$\frac{1}{2}\Gamma \to \frac{1}{2}\Gamma - \mathrm{Im}\,\Sigma_\Delta$$

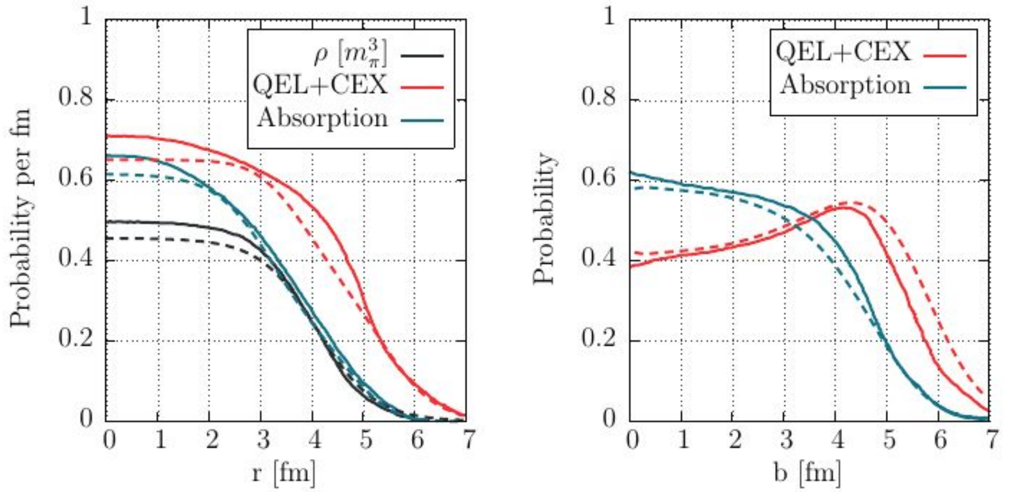
- The Δ self-energy parametrization is taken from *E. Oset and L.L. Salcedo, Nucl. Phys. A468 (1987) 631*:

$$\mathrm{Im}\,\Sigma_\Delta = -\left[C_Q\left(\rho/\rho_0\right)^\alpha + C_{A2}\left(\rho/\rho_0\right)^\beta + C_{A3}\left(\rho/\rho_0\right)^\gamma\right]$$

C_{A2} - contribution coming from two-body absorption

C_{A3} - contribution coming from three-body absorption

Comparison with Oset model



(a) The probability of an interaction per fm as a function of a distance from the center of a nucleus in the case of pion scattering off iron ($T_k = 165$ MeV).

(b) The probability of QEL+CEX and absorption events as a function of the impact parameter of the initial pion in the case of pion scattering off calcium ($T_k = 180$ MeV).

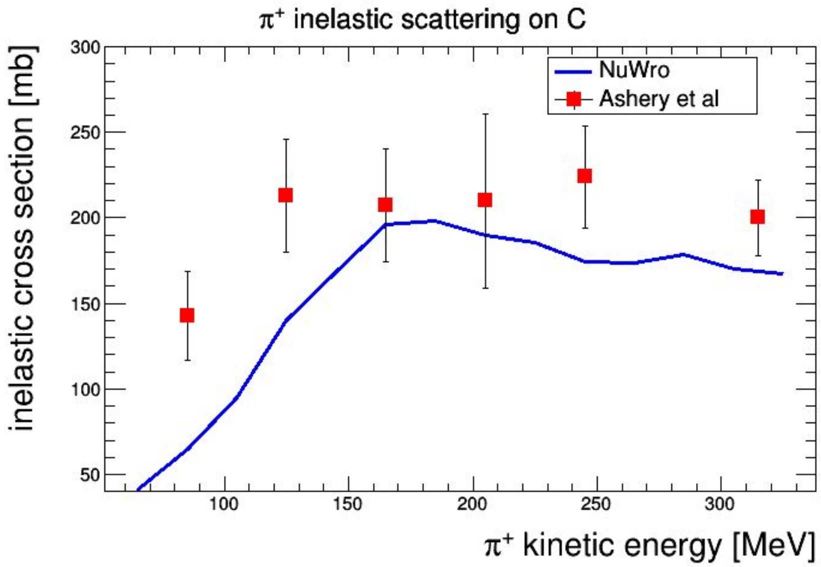
Figure 2.21: The comparison of the original Oset et al. calculations from Ref. [101] (solid lines) and NuWro implementation (dashed lines).

Comparison with Oset model

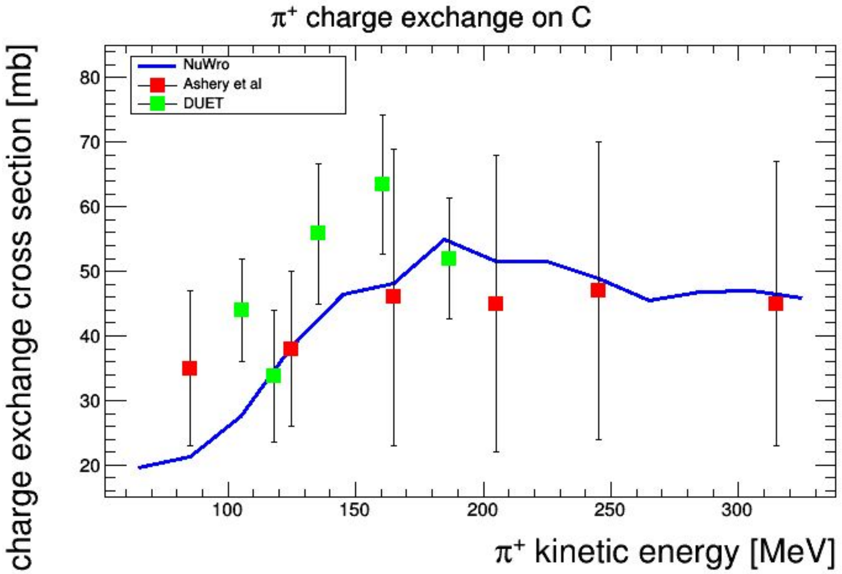
		$T_k = 85$ MeV			$T_k = 245$ MeV			
		$n=1$	$n=2$	$n=3$	$n=1$	$n=2$	$n=3$	$n=4$
$P_n^{(qel)}$	Oset et al.	0.90	0.09	0.01	0.69	0.25	0.05	0.01
	NuWro	0.89	0.10	0.01	0.67	0.24	0.07	0.02
		$n=0$	$n=1$	$n=2$	$n=0$	$n=1$	$n=2$	$n=3$
$P_n^{(abs)}$	Oset et al.	0.81	0.17	0.02	0.37	0.41	0.17	0.04
	NuWro	0.87	0.12	0.01	0.41	0.37	0.16	0.05

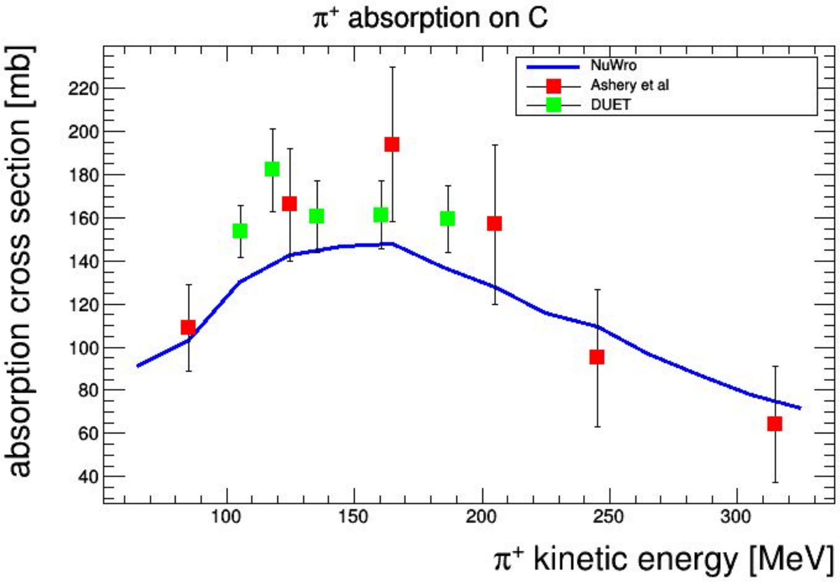
Table 2.6: The probability that the QEL/CEX scatterings proceeds through n collisions ($P_n^{(qel)}$) and the probability that pion absorption occurs after nth QEL/CEX scatterings ($P_n^{(abs)}$) in the case of pion scattering off calcium.

Comparison with pi-nucleus scattering data

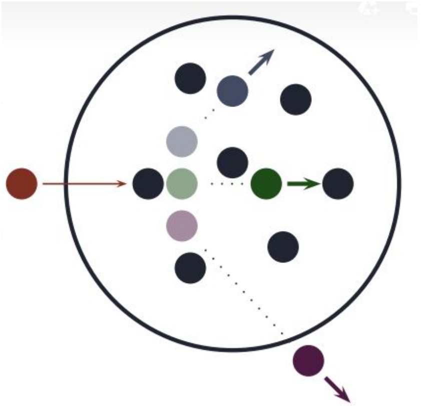


Comparison with piN data





Delta lifetime and formation time



- Pion reinteractions are preceded by:
  - Delta propagation for RES

$$t_f = \gamma \tau_\Delta = \gamma \Gamma^{-1}.$$

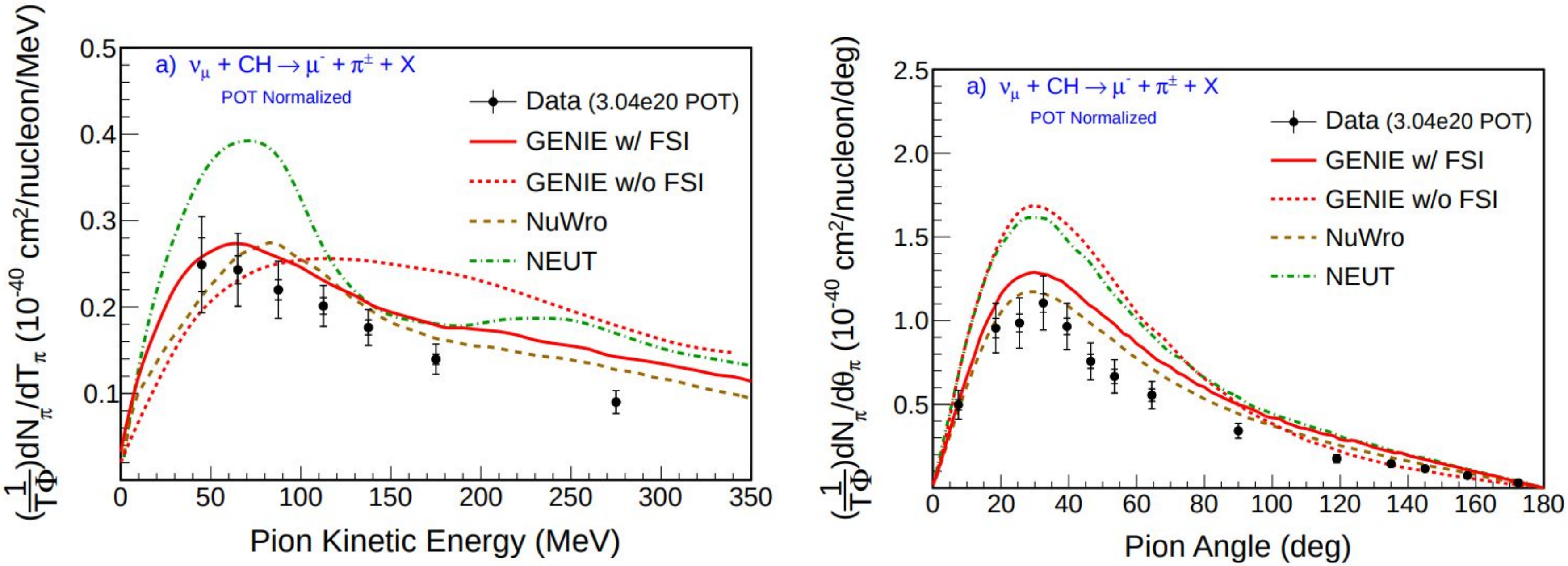
  - Formation zone for DIS

$$t_f = \tau_0 \frac{E \cdot M}{\mu_T^2}$$

E, M - nucleon energy and mass,

$\mu_T^2 = M^2 + p_T^2$ - transverse mass, $\tau_0 = 8fm.$

Comparison with MINERvA nu CC 1pi+



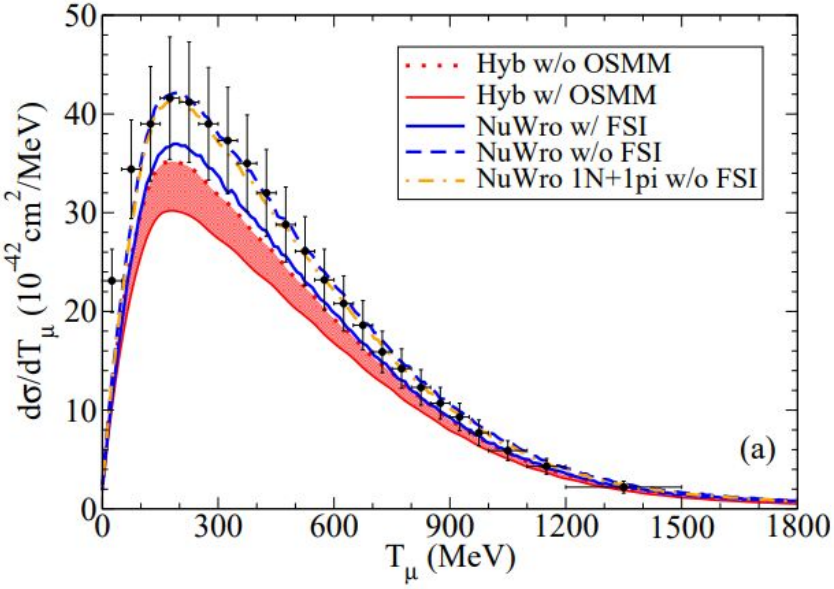
C.L. McGivern *et al.* (MINERvA Collaboration) Phys. Rev. D **94**, 052005

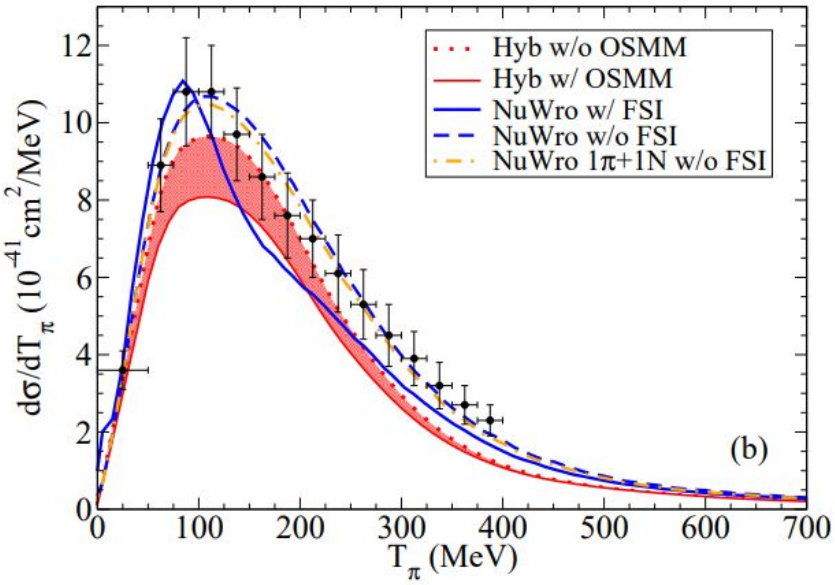
Summary

- NuWro pion production model includes:
  - single resonance pion production through Delta excitation
  - more inelastic processes described by quark-parton model with Pythia6 used for hadronization
  - hadronization parameters are tuned to get a good agreement with experimental data
  - smooth transition region between “RES” and “DIS”
  - final state interactions modeled via intranuclear cascade based on Oset et al model
- The agreement with neutrino pion production data is outstanding
- The NuWro Reweighting Framework allows to reweight pion production model parameters (thanks to L. Pickering and P. Stowell!)
- There is ongoing work on reweighting FSI

Backup slides

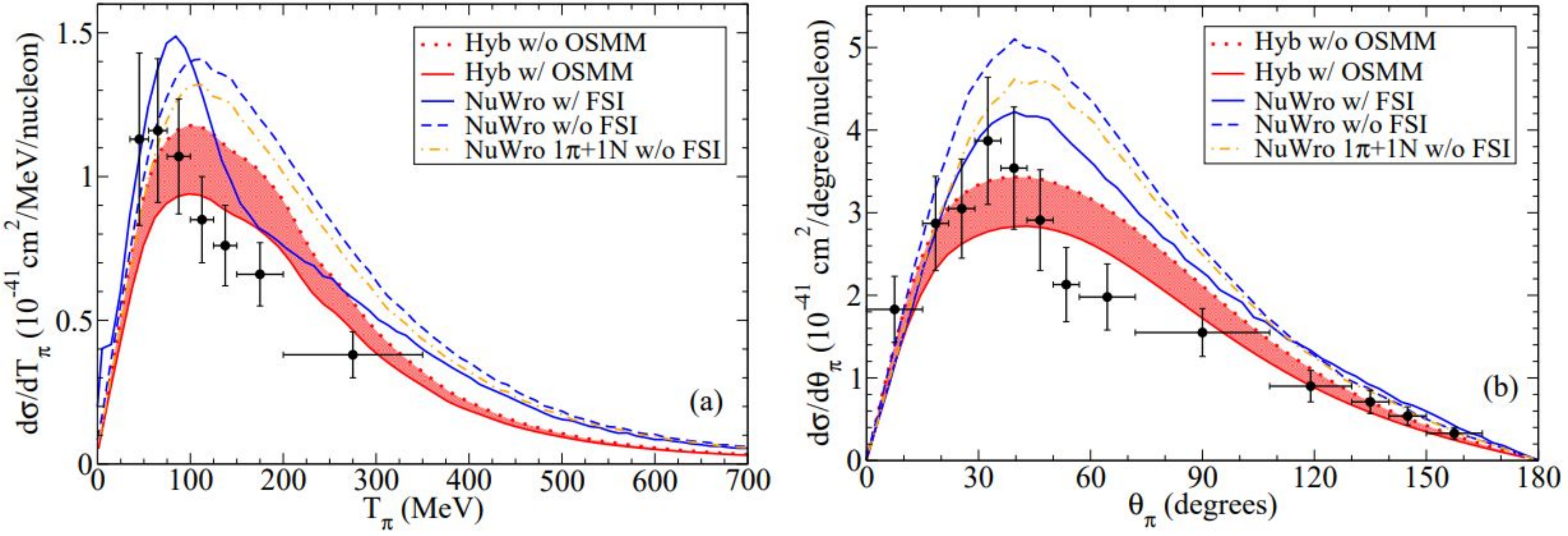
Comparison with MiniBooNE nu CC 1pi+





R. González-Jiménez, K. Niewczas, and N. Jachowicz, Phys. Rev. D **97**, 013004

Comparison with MINERvA nu CC 1pi+



R. González-Jiménez, K. Niewczas, and N. Jachowicz, Phys. Rev. D **97**, 013004