Unified Description of Lepton-Nucleus Scattering within the Spectral Function Formalism

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Modeling Neutrino-Nucleus Interactions
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PREAMBLE

- ⋆ Atomic nuclei are complex many-body systems, whose response to an external probe involves a variety of different reaction mechanism
- ⋆ At large momentum transfer, the formalism based on factorisation provides a unified framework for the interpretation of the *nuclear cross section* in terms of *nucleon cross section* and *nuclear spectral functions*
- ⋆ The spectral function is a fundamental quantity of many-body theory, trivially related to the two-point Green's function. The formalism based on the spectral function allows for a consistent treatment of a variety of reaction mechanisms, and a model independent identification of correlation effects.
- ⋆ Being inherently modular, the formalism is ideally suited for implementation in simulation codes

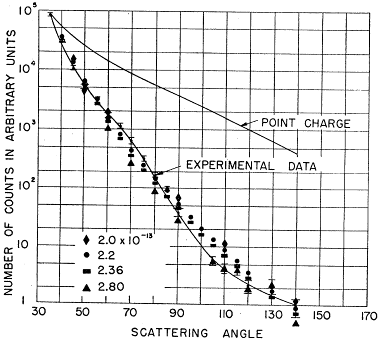
Outline

- ⋆ Elastic electron-nucleus scattering as an archetype example of factorisation : inferring microscopic dynamics from nuclear properties
- ⋆ The lepton-nucleus cross section in the impulse approximation regime: factorisation and the nucleon spectral function
- ⋆ Extended factorisation: Meson-Exchange Currents (MEC) and the extended factorisation scheme
- ⋆ Summary & Prospects

ELASTIC SCATTERING: $e + A \to e' + A$, $\lambda \gg R_A \sim A^{1/3}$

$$\left(\frac{d\sigma}{d\Omega}\right)_{eA} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(\mathbf{q})|^2 \, ,$$

- The Mott x-section described the electromagnetic interaction of a relativistic electron with a point target



Hofstadter et al, A.D. 1953
Gold target, $E_e = 125$ MeV

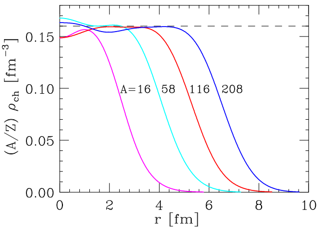
FROM NUCLEAR SYSTEMATICS TO MICROSCOPIC DYNAMICS

- The deviations from the Mott x-section provide information on target size and shape

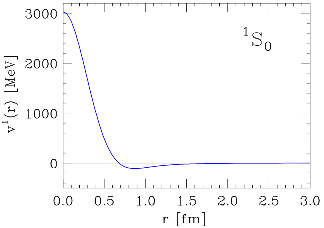
$$F(\mathbf{q}) = \int d^3r \; \rho_{\rm ch}(\mathbf{r})$$

- The observation that the central density of atomic nuclei is largely A-independent for $A > 16$, indicates that nuclear forces are strongly repulsive at short range

Nuclear charge densities



Nucleon-nucleon potential



- The repulsive core is a prominent feature of the nucleon-nucleon potential, giving rise to strong short-range correlations

The Lepton-Nucleus x-Nection

- Consider, for example, the cross section of the process

$$\ell + A \to \ell' + X$$

at fixed beam energy

$$d\sigma_A \propto L_{\mu\nu} W_A^{\mu\nu}$$

  - $L_{\mu\nu}$ is fully specified by the lepton kinematical variables
  - The determination of the nuclear response

$$W_A^{\mu\nu} = \sum_X \langle 0 | J_A^{\mu\,\dagger} | X \rangle \langle X | J_A^{\nu} | 0 \rangle \delta^{(4)}(P_0 + k - P_X - k')$$

    involves

    - the ground state of the target nucleus, $|0\rangle$
    - all relevant hadronic final states, $|X\rangle$
    - the nuclear current operator

$$J_A^{\mu} = \sum_i j_i^{\mu} + \sum_{j>i} j_{ij}^{\mu}$$

THE NON RELATIVISTIC REGIME

- ⋆ In the low-energy regime quasi elastic scattering leading to final states involving nucleons only, i.e.

$$|X\rangle = |(A-1)^\star \, p\rangle \, , |(A-2)^\star \, pp\rangle \ldots$$

  is the dominant reaction mechanism

- ⋆ at low to moderate momentum transfer, typically in the range $|\mathbf{q}| \lesssim 500$ MeV, the non relativistic approximation cal be employed to carry out highly accurate *ab initio* calculations based on realistic nuclear Hamiltonians, strongly constrained by phenomenology

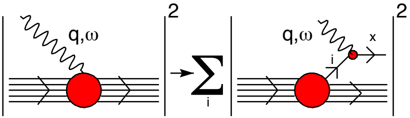
$$H = \sum_i \frac{{\mathbf{p_i}}^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} V_{ijk} \, ,$$

  and consistent nuclear current operators J_A^μ

- ⋆ The non relativistic approach has been widely employed to describe the electromagnetic and weak responses of *isoscalar* nuclei with $A \leq 12$

The Impulse Approximation (IA) Regime

- ⋆ at large momentum transfer, the final state and the current operator can no longer be described within the non relativistic approximation
- ⋆ for $\lambda \ll d_{\rm NN} \sim 1.6$ fm, the average nucleon-nucleon distance in the target nucleus, nuclear scattering reduces to the incoherent sum of scattering processes involving individual nucleons



- ⋆ Basic assumptions
  - ▷ $J^\mu_A(q) \approx \sum_i j^\mu_i(q)$ (single-nucleon coupling)
  - ▷ $|X\rangle \to |\mathbf{p}\rangle \otimes |n_{(A-1)}, \mathbf{p_n}\rangle$ (factorization of the final state)
- ⋆ As a zero-th order approximation, Final State Interactions (FSI) and processes involving two-nucleon Meson-Exchange Currents (MEC) are neglected (more on this later)

The IA Cross Section

- ⋆ Factorisation allows to rewrite the nuclear transition amplitude in the form

$$\langle X|J_A^\mu|0\rangle \to \sum_i \int d^3k\ M_n(\mathbf{k})\langle \mathbf{k}+\mathbf{q}|j_i^\mu|\mathbf{k}\rangle$$

  - The nuclear amplitude M_n describes initial sate properties, independent of momentum transfer
  - The matrix element of the current between free-nucleon states can be computed exactly using the fully relativistic expression

- ⋆ Nuclear x-section

$$d\sigma_A = \int d^3k dE\ d\sigma_N\ P(\mathbf{k}, E)$$

- ⋆ The spectral function $P(\mathbf{k}, E)$ describes the probability of removing a nucleon of momentum $\mathbf{p}$ from the nuclear ground state, leaving the residual system with excitation energy E
- ⋆ The lepton-nucleon cross section $d\sigma_N$ can be obtained—at least in principle—from proton and deuteron data

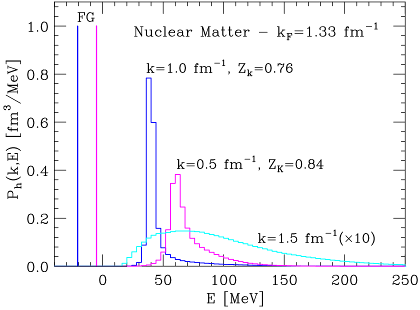
Nuclear Spectral Function

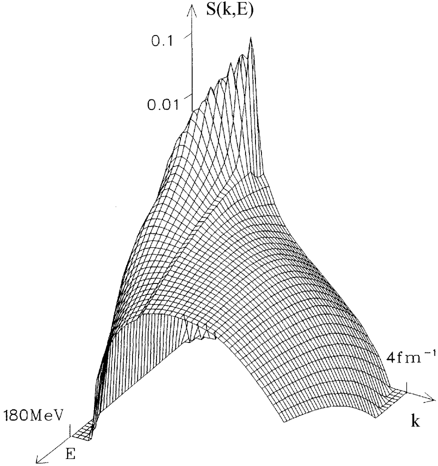
- The analytic structure of the two-point Green's function—dictated by the Källèn-Lehman representations—is reflected by the spectral function

$$P(\mathbf{k}, E) = \sum_{h \in \{F\}} Z_h |M_h(\mathbf{k})|^2 F_h(E - e_h) + P_B(\mathbf{k}, E)$$

- According to the independent particle model (IPM)
  - Spectroscopic factors $Z_h \to 1$
  - Momentum dependence $M_h(\mathbf{k}) = \langle h | a_{\mathbf{k}} | 0 \rangle \to \phi_h(\mathbf{k})$, the momentum-space wave function of the single-particle state h
  - Energy distribution $F_h(E - e_h) \to \delta(E - e_h)$
  - Smooth contribution $P_B(\mathbf{k}, E) \to 0$: pure correlation effect
- The spectral function of uniform nuclear matter can be obtained from accurate non-relativistic calculations

$P(\mathbf{k}, E)$ of Isospin-Symmetric Nuclear Matter



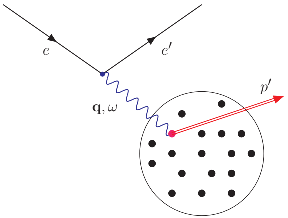


Obtaining $P(\mathbf{k}, E)$ from Electron Scattering Data

- Consider the $(e, e'p)$ Reaction

$$e + A \to e' + p + (A-1)$$

in which both the outgoing electron and the proton, carrying momentum p', are detected in coincidence, and the recoiling nucleus can be left in a any (bound or continuum) state $|n\rangle$ with energy E_n



- In the absence of final state interactions (FSI)—which can be taken into account as corrections—the *measured* missing momentum and missing energy can be identified with the momentum of the knocked out nucleon and the excitation energy of the recoiling nucleus, $E_n - E_0$

$$\mathbf{p}_m = \mathbf{p}' - \mathbf{q} \quad , \quad E_m = \omega - T_{\mathbf{p}'} - T_{A-1} \approx \omega - T_{\mathbf{p}'}$$

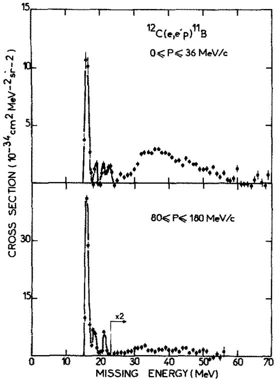
and the differential cross section is given by

$$\frac{d\sigma_A}{dE_{e'}d\Omega_{e'}dE_{p'}d\Omega_{p'}} \propto \sigma_{ep} P(p_m, E_m)$$

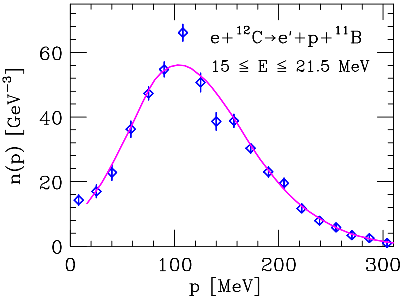
$^{12}C(e, e'p)$ AT MODERATE MISSING ENERGY

- ⋆ At moderate missing energy the recoiling nucleus is left in a bound state, e.g. $|^{11}B(3/2^-),\ p\rangle$, $|^{11}B(1/2^-),\ p\rangle$

- ▶ Missing energy spectrum of ^{12}C measured at Saclay in the 1970s



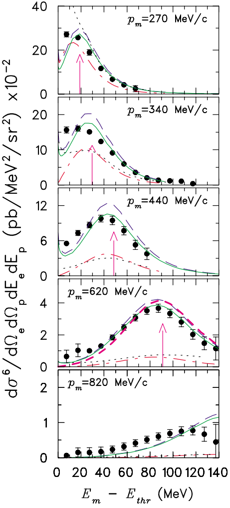
- ▶ P- state momentum distribution.



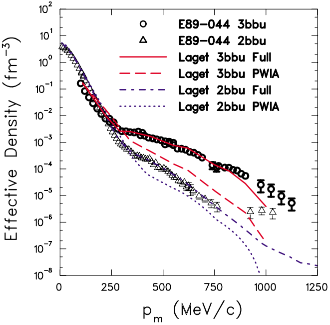
- ⋆ The spectroscopic factors turn out to be significantly less than unity

$(e, e'p)$ Studies of the Correlation Strength

- $^3\text{He}(e, e'p)$ at large $|\mathbf{p}_m|$ and E_m in JLab hall A: strong energy-momentum correlation observed.

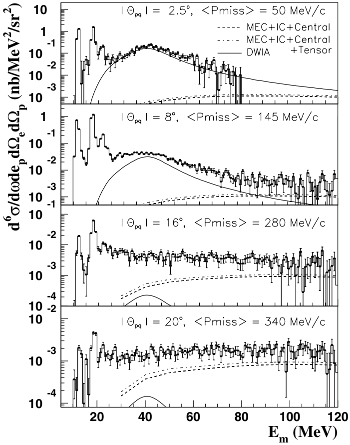


$$n(k) = \int dE\ P(\mathbf{k}, E)$$



LARGE $|\mathbf{p}_m|$ AND E_m STRENGTH IN OXYGEN

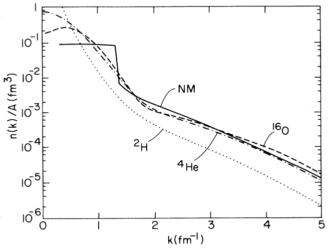
- $|\mathbf{p}_m|$-evolution of missing energy spectrum in Oxygen. Hall A data



- The determination of the spectral function at large missing energy and missing momentum is hindered by significant FSI and MEC effects

THE LOCAL DENSITY APPROXIMATION (LDA)

⋆ Bottom line: accurate theoretical calculations show that the tail of the momentum distribution, arising from the continuum contribution to the spectral function, turns out to be largely A-independent for $A > 2$



⋆ Spectral functions of nuclei can be obtained within the Local Density Approximation (LDA)

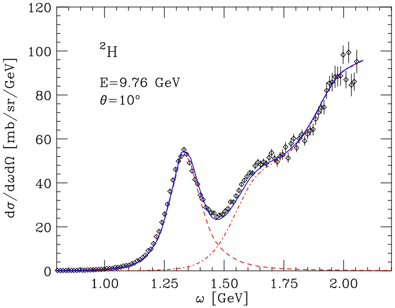
$$P_{\mathrm{LDA}}(\mathbf{k}, E) = P_{\mathrm{MF}}(\mathbf{k}, E) + \int d^3r \, \rho_A(r) \, P^{NM}_{corr}(\mathbf{k}, E; \rho = \rho_A(r))$$

using the Mean Field (MF), or shell model, contributions obtained from $(e, e'p)$ data

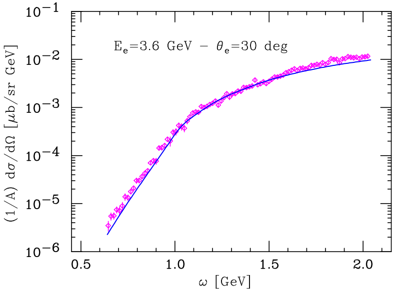
⋆ The continuum contribution $P^{NM}_{corr}(\mathbf{k}, E)$ is computed for uniform nuclear matter at different densities using accurate theoretical approaches

Comparison to $e + A \to e' + X$ Data

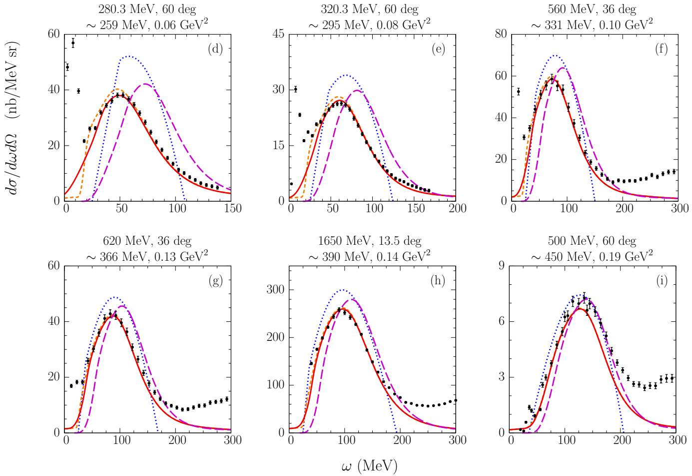
⋆ Deuteron (SLAC data)



⋆ Nuclear matter ($A \to \infty$ extrapolation of SLAC data)



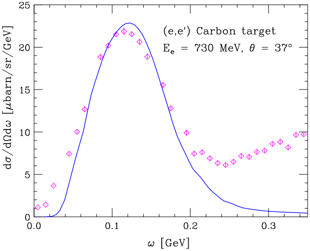
⋆ $e + {}^{12}\text{C} \to e' + X$ quasi elastic cross section computed within the IA including corrections arising from FSI.



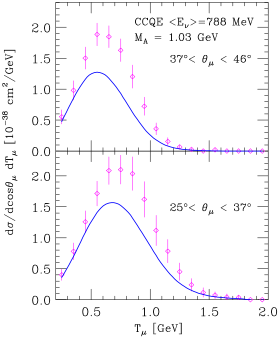
⋆ Recall: no adjustable parameters involved

eA VS νA CROSS SECTION: THE ISSUE OF FLUX AVERAGE

▷ Electron scattering

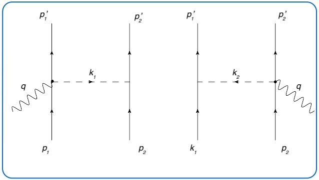


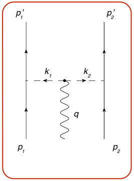
▷ MiniBooNe CCQE cross section



- Theoretical calculations carried out using the same spectral function and vector form factors employed to describe the electron scattering cross section and setting $M_A = 1.03$
- Reaction mechanisms other than single-nucleon knock out contribute to the cross section

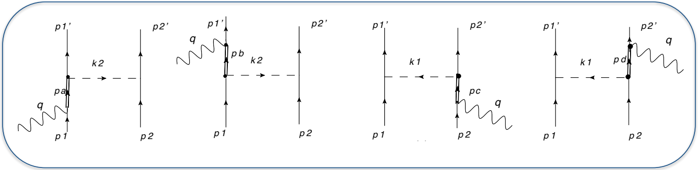
CORRECTIONS TO THE IA: MESON-EXCHANGE CURRENTS





Seagull
or
contact
term

Pion
in
flight
term



The Extended Factorisation *ansatz*

- ⋆ Highly accurate and consistent calculations of processes involving MEC can be carried out in the non relativistic regime
- ⋆ Fully relativistic MEC used within independent particle models, such as the Fermi gas model
- ⋆ Using relativistic MEC and a realistic description of the nuclear ground state requires the extension of the IA scheme to two-nucleon emission amplitudes
    - ▸ Rewrite the hadronic final state $|n\rangle$ in the factorized form

$$|n\rangle \to |\mathbf{p},\mathbf{p}'\rangle \otimes |n_{(A-2)}\rangle = |n_{(A-2)},\mathbf{p},\mathbf{p}'\rangle$$

$$\langle X|j^{\mu}_{ij}|0\rangle \to \int d^3k d^3k' M_n(\mathbf{k},\mathbf{k}') \langle \mathbf{p}\mathbf{p}'|j^{\mu}_{ij}|\mathbf{k}\mathbf{k}'\rangle \, \delta(\mathbf{k}+\mathbf{k}'+\mathbf{q}-\mathbf{p}-\mathbf{p}')$$

The amplitude

$$M_n(\mathbf{k},\mathbf{k}') = \langle n_{(A-2)},\mathbf{k},\mathbf{k}'|0\rangle$$

is independent of q , and can be obtained from non relativistic many-body theory

Two-Nucleon Spectral Function

- ⋆ Calculations have been carried out for uniform isospin-symmetric nuclear matter

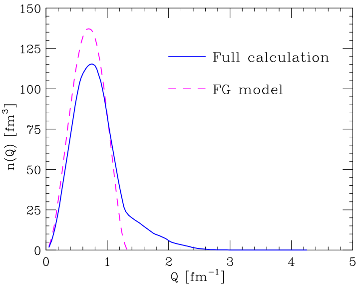
$$P(\mathbf{k}_1, \mathbf{k}_2, E) = \sum_n |M_n(k_1, k_2)|^2 \delta(E + E_0 - E_n)$$

$$n(\mathbf{k}_1, \mathbf{k}_2) = \int dE \, P(\mathbf{k}_1, \mathbf{k}_2, E)$$

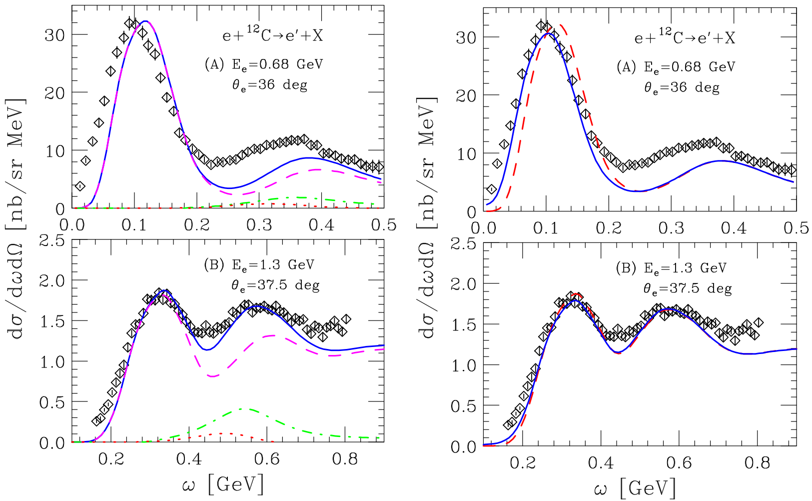
- ⋆ Relative momentum distribution

$$n(\mathbf{Q}) = 4\pi |\mathbf{Q}|^2 \int d^3q \, n\left(\frac{\mathbf{Q}}{2} + \mathbf{q}, \frac{\mathbf{Q}}{2} - \mathbf{q}\right)$$

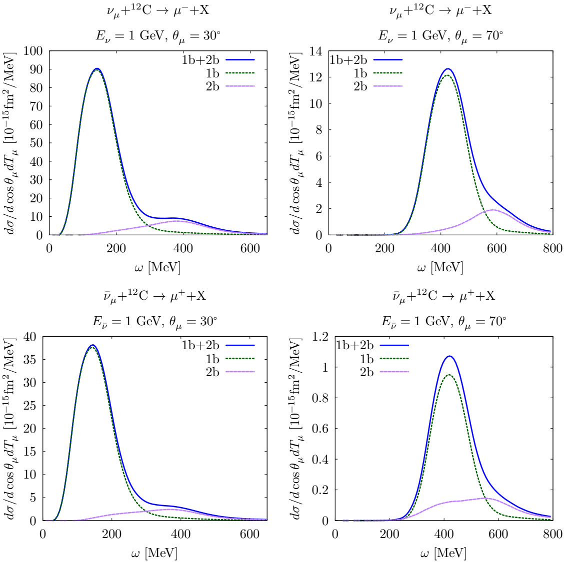
$$\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2 \ , \ \ \mathbf{Q} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}$$



MEC Contribution to $e + {}^{12}\text{C} \rightarrow e' + X$

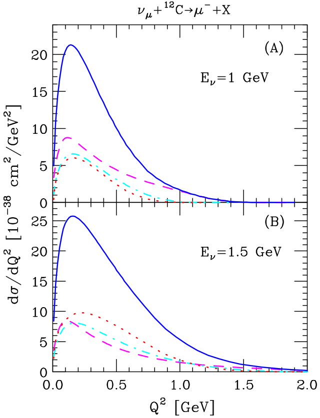


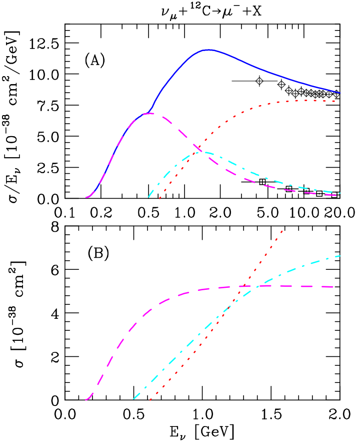
MEC Contribution to $\nu_\mu + {}^{12}\mathrm{C} \to \mu^- + X$ (preliminary!)



INELASTIC CONTRIBUTION TO $\nu_\mu + {}^{12}\mathrm{C} \to \mu^- + X$

- Factorization ansatz + LDA spectral function



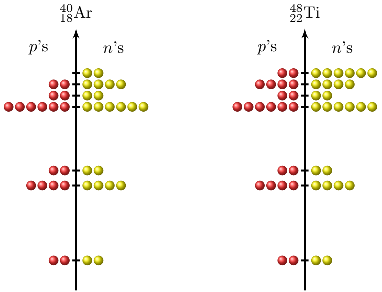


Summary & Prospects

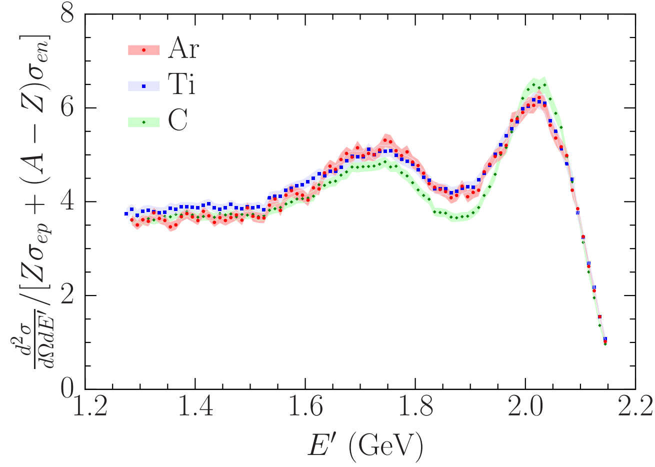
- ⋆ The formalism based on nuclear spectral functions—extensively applied to study electron-nucleus scattering—is approaching the level of maturity needed to perform meaningful calculations of neutrino-nucleus interactions
- ⋆ Being based on intrinsic properties of the target, the formalism can be applied to obtain a *consistent* description of a variety of reaction channels, and appears to be easily implementable in generators
- ⋆ Needed developments include a study of processes leading to the collective excitations of the nuclear target and the treatment of final state interactions in inelastic channels
- ⋆ In the winter of 2017, JLab experiment E12-14-012 has collected $\mathrm{Ar}(e, e'p)$ and $\mathrm{Ti}(e, e'p)$ data, to be used to obtain the argon spectral functions. The first paper, reporting inclusive titanium data has been accepted for publication in PRC

The E12-14-012 Experiment: Why Argon and Titanium?

- ⋆ The reconstruction of neutrino and antineutrino energy in liquid argon detectors will require the understanding of the spectral functions describing both protons and neutrons
- ⋆ The $\text{Ar}(e, e'p)$ cross section only provides information on proton interactions. The information on neutrons can be obtained from the $\text{Ti}(e, e'p)$, exploiting the pattern of shell model levels



First Results from JLab E12-14-012



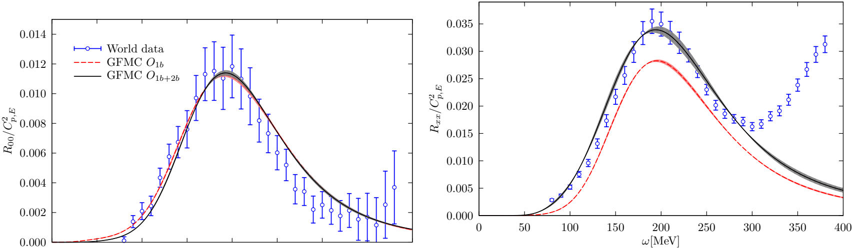
Backup slides

GFMC Results

- ⋆ Electromagnetic responses of ^{12}C at $|\mathbf{q}|$=570 MeV

$$\frac{d^2\sigma}{d\Omega_{e'}dE_{e'}} \propto \left(\frac{Q^2}{|\mathbf{q}|^2}\right)^2 \left[\epsilon R_L(|\mathbf{q}|,\omega) + \frac{1}{2}\frac{|\mathbf{q}|^2}{Q^2} R_T(|\mathbf{q}|,\omega)\right]$$

$$\epsilon = \left(1 + 2\frac{|\mathbf{q}|^2}{Q^2}\tan\frac{\theta_e}{2}\right)^{-1}$$



- ⋆ Note that, even at moderate momentum transfer, the non relativistic approach fails to describe the transverse response in the region of large energy transfer, where the contribution of inelastic processes is large

- ⋆ Within the factorization *ansatz* underlying the IA, the target response reduces to

$$W_A^{\mu\nu} = N \int d^3k \, dE \, \frac{m}{E_k} P(\mathbf{k}, E) w^{\mu\nu}$$

$$w^{\mu\nu} = \sum_x \int d^3p_x \langle \mathbf{k}, n | j^\mu | x, \mathbf{p}_x \rangle \langle \mathbf{p}_x, x | j^\nu | n, \mathbf{k} \rangle \delta^{(4)}(k + \tilde{q} - p_x)$$

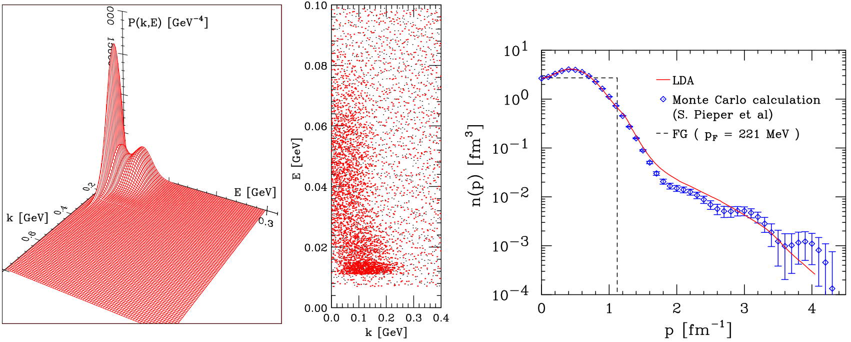
- ⋆ $w^{\mu\nu}$ is the tensor describing the interaction of a free neutron of momentum $\mathbf{k}$ at four momentum transfer

$$\tilde{q} \equiv (\tilde{\omega}, \mathbf{q}) \quad , \quad \tilde{\omega} = \omega + M_A - E_R - E_k$$

- ⋆ The substitution $\omega \to \tilde{\omega} < \omega$ accounts the fact that an amount $\delta\omega = \omega - \tilde{\omega}$ of the energy transfer goes into excitation energy of the residual system.
- ⋆ The spectral function $P(\mathbf{k}, E)$ describes the probability of removing a nucleon of momentum $\mathbf{k}$ from the target nucleus, leaving the residual system with excitation energy E

SPECTRAL FUNCTION AND MOMENTUM DISTRIBUTION OF ^{16}O

- $n(k) = \int dE \; P(k,E)$



- shell model states account for $\sim 80\%$ of the strenght
- the remaining $\sim 20\%$, arising from NN correlations, is located at high momentum and large removal energy

Determination of the Spectroscopic Factor

- The spectroscopic factor of the p-state with $j = 3/2$ is obtained from

$$Z_p = \frac{(2j+1)}{Z} \int_{\Delta k} \frac{d^3k}{(2\pi)^3} \int_{\Delta E} dE \, P_{\rm expt}(|\mathbf{k}|, E) = 0.625$$

with

$$\Delta k \equiv [0\text{–}310] \text{ MeV} \quad , \quad \Delta E \equiv [15\text{–}22.5] \text{ MeV}$$

- Models based on the mean field approximation predict $Z_p = 1$
- The deviation of Z_p from unity implies that dynamical effects not taken into account within the independent particle picture reduce the average number of protons occupying the $j = 3/2$ p-state from 4 to 2.5
- The result obtained from the LDA analysis is within 2% of the experimental value

Early Studies of the Correlation Strength

- The $(e, e'p)$ cross section at large E_m and p_m , tipically $E_m \gtrsim 50$ MeV and $p_m \gtrsim 250$ MeV, gives access to the *correlation strength*. Strong energy–momentum correlation clearly observed.

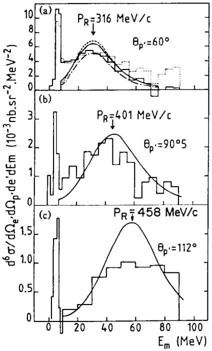


Fig. 5. Missing energy spectra form ^{3}He(e, e′p), showing evidence for an interaction on a two-nucleon correlated pair

CEBAF PROPOSAL COVER SHEET

This Proposal must be mailed to:

CEBAF
Scientific Director's Office
12000 Jefferson Avenue
Newport News, VA 23606

and received on or before OCTOBER 31, 1989

A. TITLE: Selected studies of the ^{3}He and ^{4}He nuclei through electrodisintegration at high momentum transfer

B. CONTACT PERSON: J. MOUGEY

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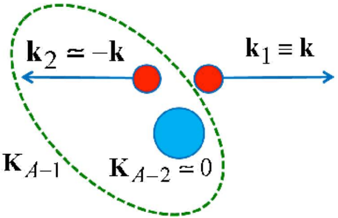
We propose to use the CEBAF Hall A High Resolution Spectrometer pair to study selective aspects of the electromagnetic response of ^{3}He and ^{4}He through (e,e'p) coincidence measurements at Q^2 values from 0.4 to 4.1(GeV/c)^2. In Part I, we propose to study the single nucleon structure of the He isotopes with special emphasis on high momenta (up to ~ 0.6 GeV/c) by the separation of the R_L, R_T and R_{LT} response functions. The Q^2 dependence of the reaction will be examined in Part II by performing longitudinal/transverse (L/T) separations for protons emitted along $\vec{q}$, up to $Q^2 = 4.11$(GeV/c)^2 at quasifree kinematics ($p_m = 0$) and for $Q^2 = 0.5$ and 1.(GeV/c)^2 at $p_m = \pm 0.3$GeV/c. In Part III, we focus on the continuum region to study correlated nucleon pairs. Measurements at $Q^2 = 1.$ (GeV/c)^2 and recoil momenta up to 1 GeV/c are proposed, including separations of the in-plane structure functions for $p_m < 680$ MeV/c.

Energy Dependence of the Correlation Strength

- ⋆ The correlation strength arises from processes involving high momentum nucleons, with $|\mathbf{p}_m| \gtrsim 400$ MeV, in which the residual system is left in a continuum state,
- ⋆ The relevant missing energy scale can be easily understood considering that momentum conservation requires

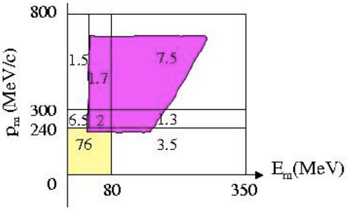


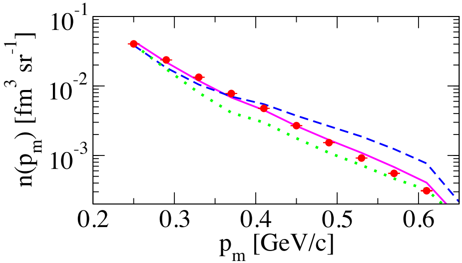
$$E_m = E_{\text{thr}} + \sqrt{|\mathbf{p}_m|^2 + m^2} - m$$

- ⋆ Scattering off a nucleon belonging to a correlated pair entails a strong energy-momentum correlation

Comparison to the Measured Correlation Strength

- ⋆ The correlation strength in carbon has been investigated in JLab Hall C by the E97-006 Collaboration



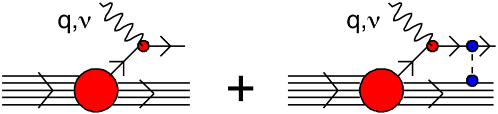


- ⋆ Measured correlation strength (Rohe et al, 2005)

Experiment	0.61 ±0.06
Greens function theory [3]	0.46
CBF theory [2]	0.64
SCGF theory [4]	0.61

Corrections to the IA: Final State Interactions (FSI)

- The measured $(e, e'p)$ x-sections provide overwhelming evidence of the occurrence of significant FSI effects



$$d\sigma_A = \int d^3k dE \, d\sigma_N \, P(\mathbf{k}, E) P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$$

- the particle-state spectral function $P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$ describes the propagation of the struck particle in the final state
- the IA is recovered replacing

$$P_p(|\mathbf{k}+\mathbf{q}|, \omega - E) \to \delta(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2})$$

FSI, CONTINUED

- effects of FSI on the inclusive cross section
  - ⋆ shift in energy transfer due to the mean field of the spectator nucleons
  - ⋆ redistributions of the strength due to the occurrence of rescattering of the knocked out nucleon
- high energy (eikonal) approximation
  - ⋆ the struck nucleon moves along a straight trajectory with constant velocity
  - ⋆ the fast struck nucleon "sees" the spectator system as a collection of fixed scattering centers

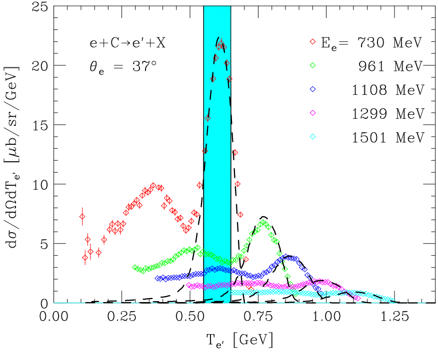
$$\delta(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2}) \to \sqrt{T_{|\mathbf{k}+\mathbf{q}|}}\,\delta(\widetilde{\omega} - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2})$$

$$+(1 - \sqrt{T_{|\mathbf{k}+\mathbf{q}|}})f(\widetilde{\omega} - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2}))$$

- the nuclear transparency T is measured by $(e, e'p)$ experiments, and the folding function f can be computed within nuclear many-body theory using as input nucleon-nucleon scattering data

“Flux-Averaged” Electron Scattering x-Section

- The electron scattering x-section off Carbon at $\theta_e = 37°$ has been measured for a number of beam energies



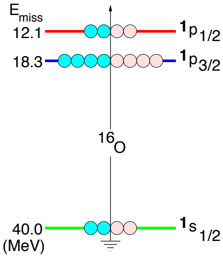
- reaction mechanisms ohter than single-nucleon knock-out contribute to the “flux averaged” cross section

The Mean-Field Approximation

- Nuclear systematics offers ample evidence supporting the assumption that the interaction potentials appearing in the Hamiltonian can be eliminated in favour of a mean field, i.e.

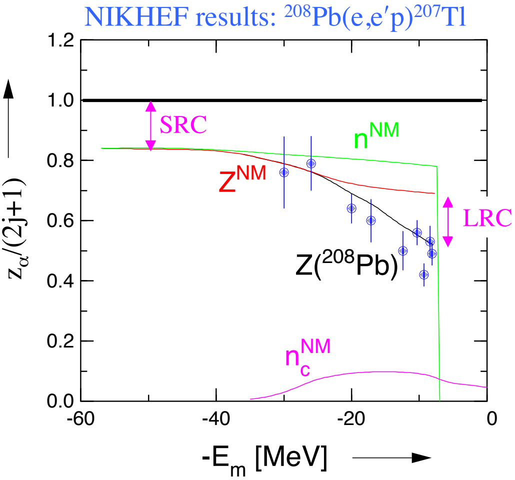
$$\left\{ \sum_{j>i=1}^{A} v_{ij} + \sum_{k>j>i=1}^{A} V_{ijk} \right\} \to \sum_{i=1}^{A} U_i$$

- This assumption lies at the basis of the Independent Particle Model (IPM)



- In the IPM ground-state protons and neutrons occupy the lowest energy levels, comprising the Fermi sea, with unit probability.

Spectroscopic Factors of ^{208}Pb



⋆ Deeply bound states are largely unaffected by finite size and shell effects

Final State Interactions in $(e, e'p)$

- In the presence of FSI, the distorted spectral function describing the mean field region can be written in the form

$$P^D_{MF}(\mathbf{p}_m, \mathbf{p}, E_m) = \sum_\alpha Z_\alpha |\phi^D_\alpha(\mathbf{p}_m, \mathbf{p})|^2 F_\alpha(E_m - E_\alpha)$$

with

$$\sqrt{Z_\alpha}\, \phi^D_\alpha(\mathbf{p}_m, \mathbf{p}) = \int d^3p_i\, \chi^\star_p(\mathbf{p}_i + \mathbf{q})\phi(\mathbf{p}_i)$$

where $\chi^\star_p(\mathbf{p}_i + \mathbf{q})$ describes the distortion arising from FSI effects

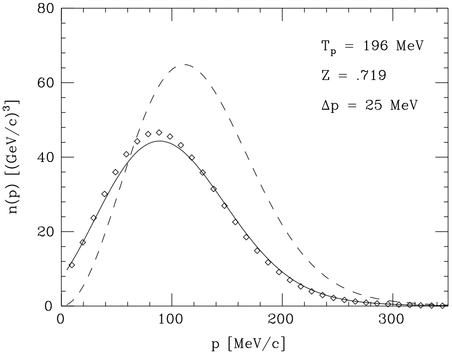
- The large body of existing work on $(e, e'p)$ data suggests that the effects of FSI can be strongly reduced measuring the cross section in *parallel kinematics*, that is with $\mathbf{p} \parallel \mathbf{q}$.
- in parallel kinematics, the distorted momentum distribution at fixed $|\mathbf{p}|$ becomes a function of missing momentum only

$$n^D_\alpha(p_m) = Z_\alpha\, |\phi^D(p_m)|^2 \ ,$$

and the effects FSI can be easily identified.

DISTORTED MOMENTUM DISTRIBUTION

- Knock out of a P-shell protons in oxygen. Proton energy $T_p = 196$ MeV
- Distortion described by a complex optical potential (OP)



- FSI lead to a shift in missing momentum (real part of the OP), and a significant quenching, typically by a factor ~ 0.7 (imaginary part of the OP).