



# Semi-inclusive Electroweak Reactions

T. W. Donnelly, M.I.T.

Study being done in collaboration with J. W. Van Orden and O. Moreno

... based on the papers

O. Moreno, TWD, J. W. Van Orden and W. P. Ford, *Phys. Rev.* **D90** (2014) 013014 [MDVF]

J. W. Van Orden, TWD and O. Moreno, *Phys. Rev.* **D96** (2017) 113008 [VDM]

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## Outline:

1. General remarks on classes of reactions (inclusive, supra-inclusive, ...)
2. General forms of the inclusive and semi-inclusive cross sections
3. Basics of kinematics for semi-inclusive electroweak reactions
4. General form of the nuclear response; dynamical variables
5. Trajectories in the missing energy – missing momentum plane
6. Discussion of implications



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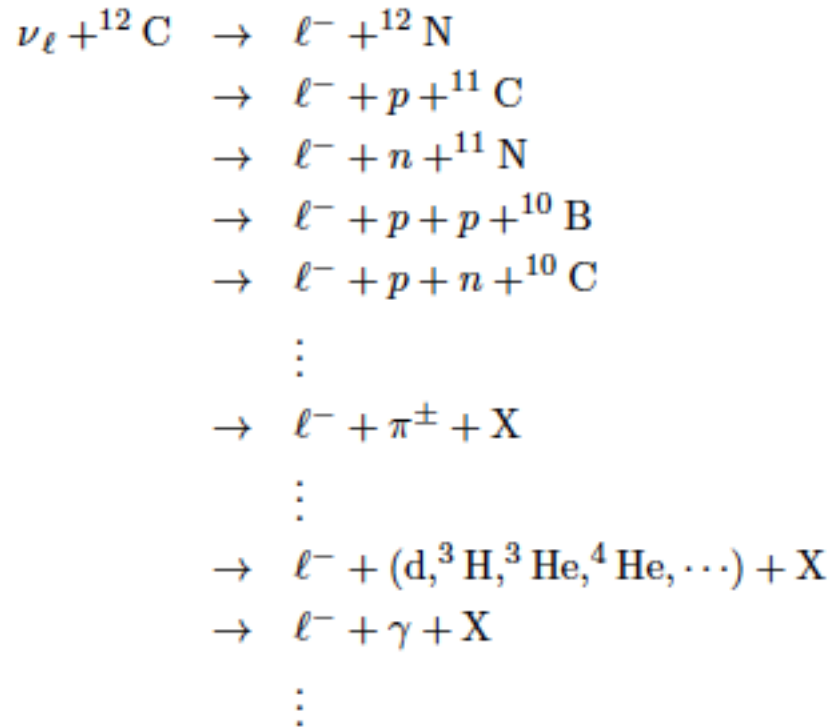
## Note:

Only very basic assumptions here (Lorentz covariance, parity properties, etc.),  
that is, no detailed model assumptions made for most of this talk



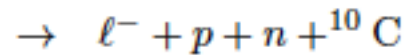
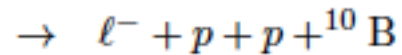
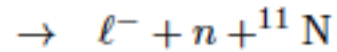
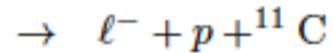
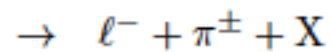
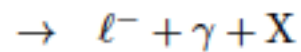
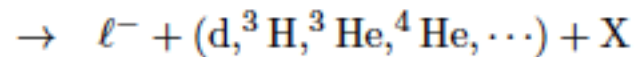
## 1. General remarks on classes of reactions (inclusive, supra-inclusive, ...)

Example of CC neutrino reactions on  $^{12}\text{C}$



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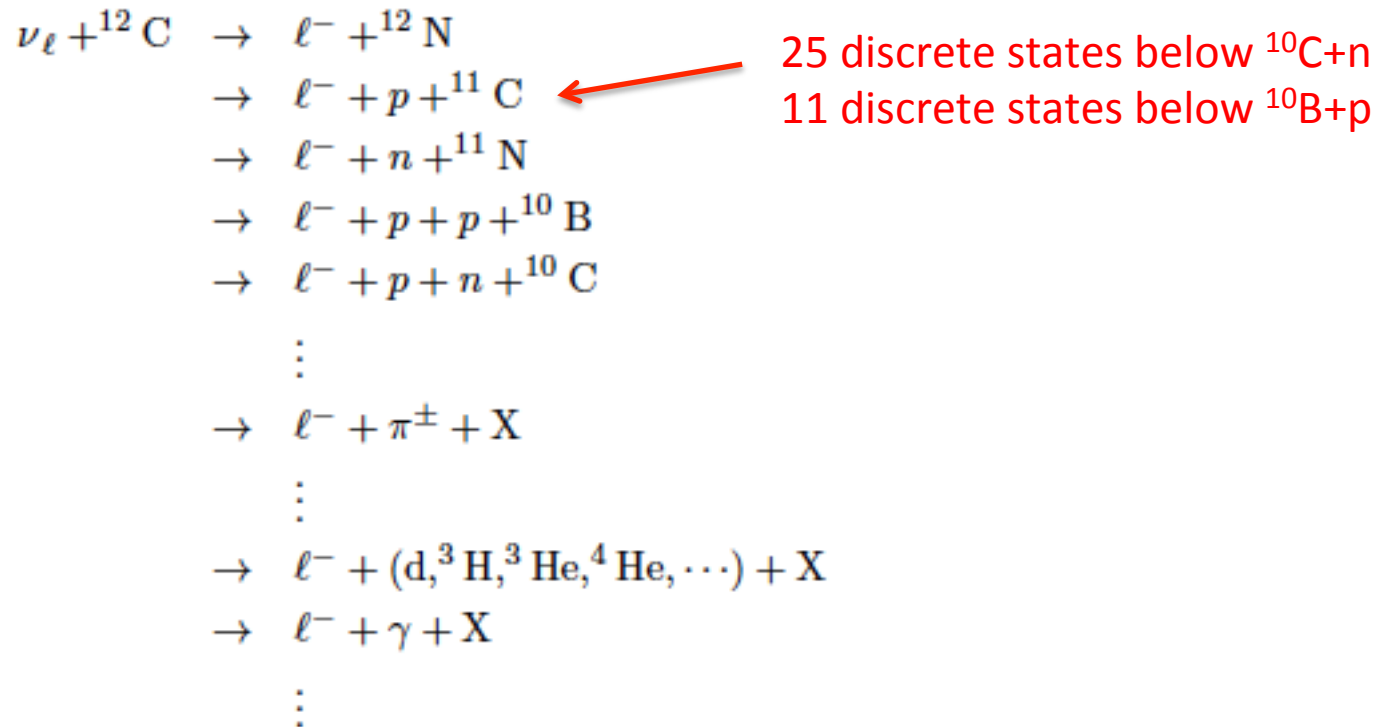
Example of CC neutrino reactions on  $^{12}\text{C}$


$$\vdots$$

$$\vdots$$

$$\vdots$$

1 discrete state that  $\beta$ -decays to  $^{12}\text{C}$

## 1. General remarks on classes of reactions (inclusive, supra-inclusive, ...)

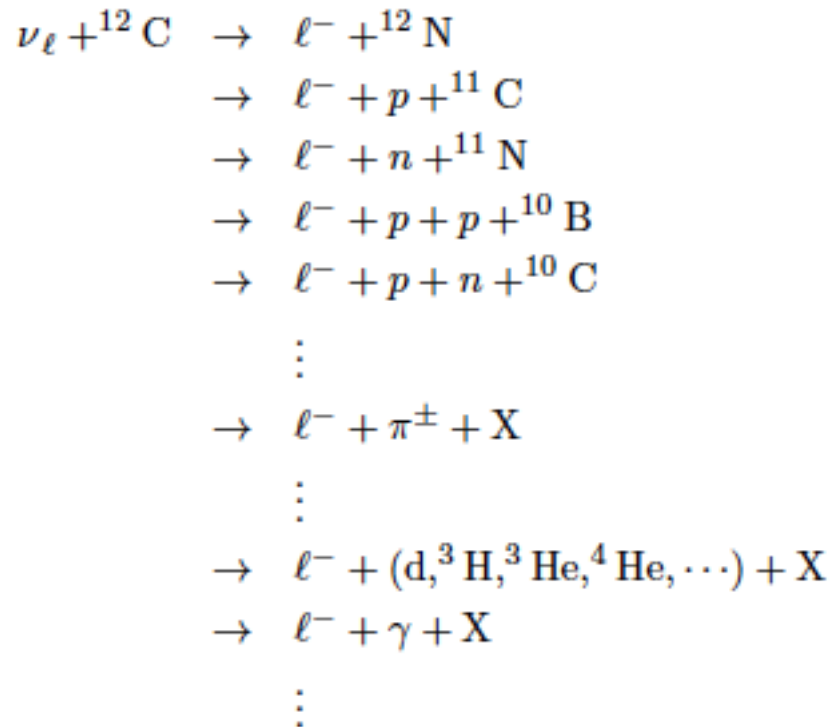
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# 1. General remarks on classes of reactions (inclusive, supra-inclusive, ...)

Example of CC neutrino reactions on  $^{12}\text{C}$



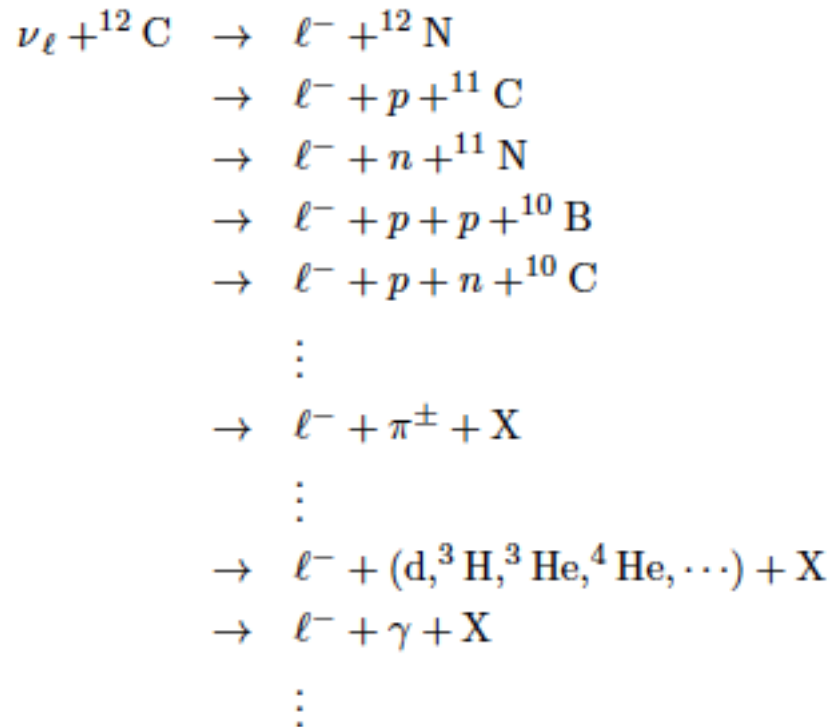
$\Sigma$  All open channels

$= \sigma$  inclusive

... assuming only that  
the muon is detected

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Example of CC neutrino reactions on  $^{12}\text{C}$

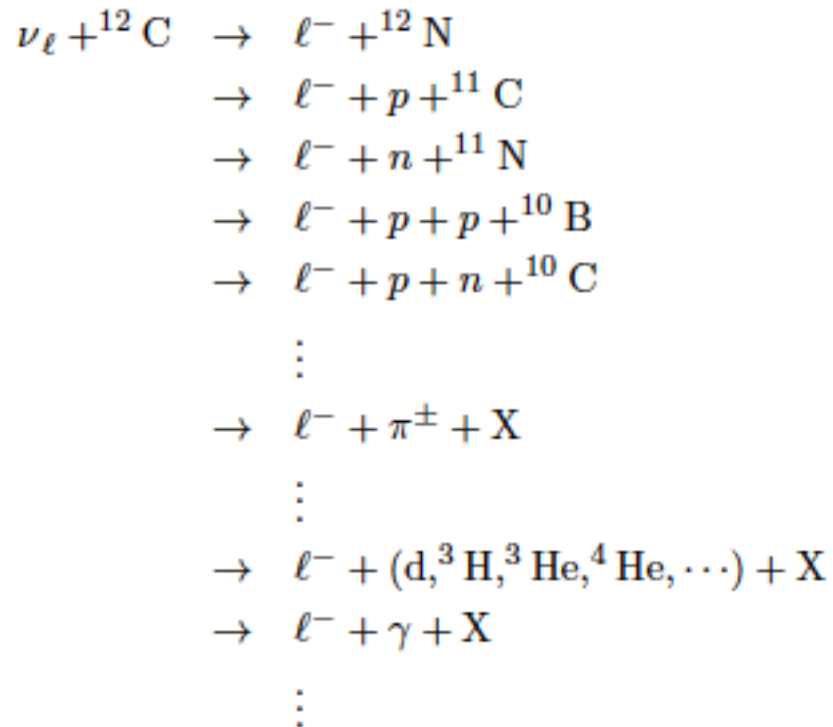


$\Sigma$  All open channels +  
1 or more other particles  
=  $\sigma$  supra-inclusive

... together with detection  
of the muon

# 1. General remarks on classes of reactions (inclusive, supra-inclusive, ...)

Example of CC neutrino reactions on  $^{12}\text{C}$



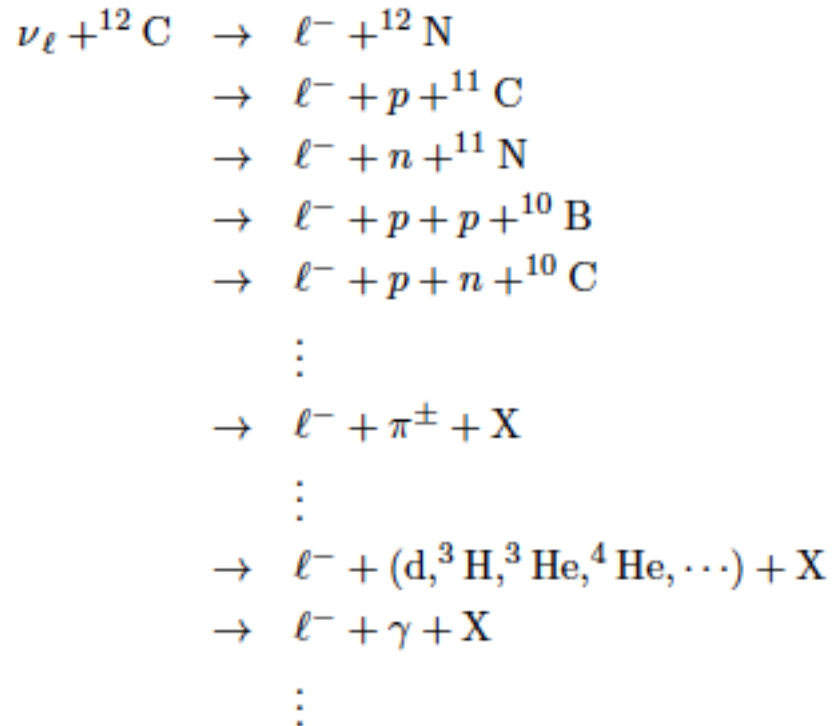
$\Sigma$   
All open channels +  
only 1 other particle  
=  $\sigma$  semi-inclusive

... together with detection  
of the muon



## 1. General remarks on classes of reactions (inclusive, supra-inclusive, ...)

Example of CC neutrino reactions on  $^{12}\text{C}$



inclusive

exclusive-1 = semi-inclusive

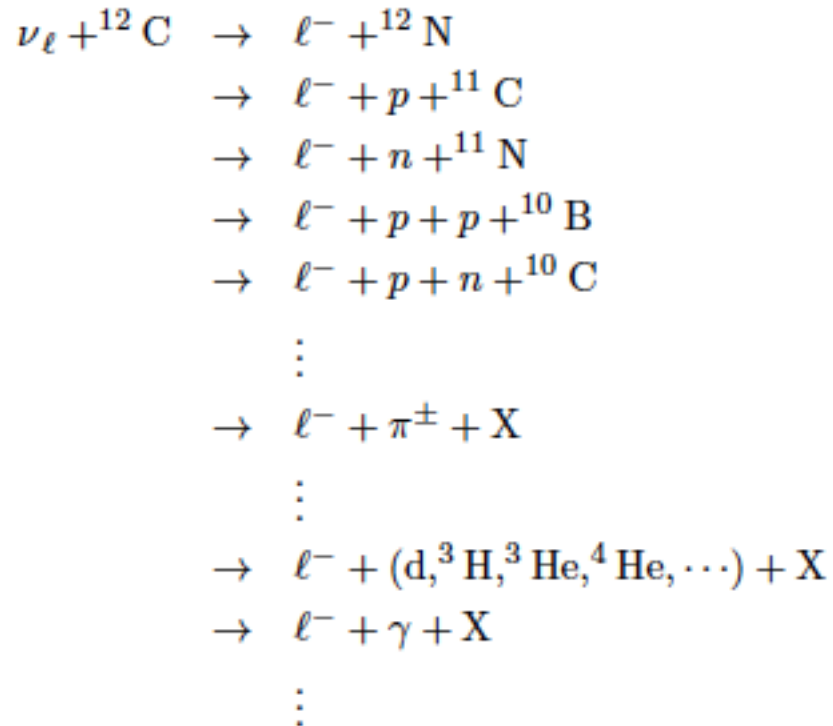
exclusive-2

exclusive-3

...

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Example of CC neutrino reactions on  $^{12}\text{C}$



inclusive

exclusive-1 = semi-inclusive

exclusive-2

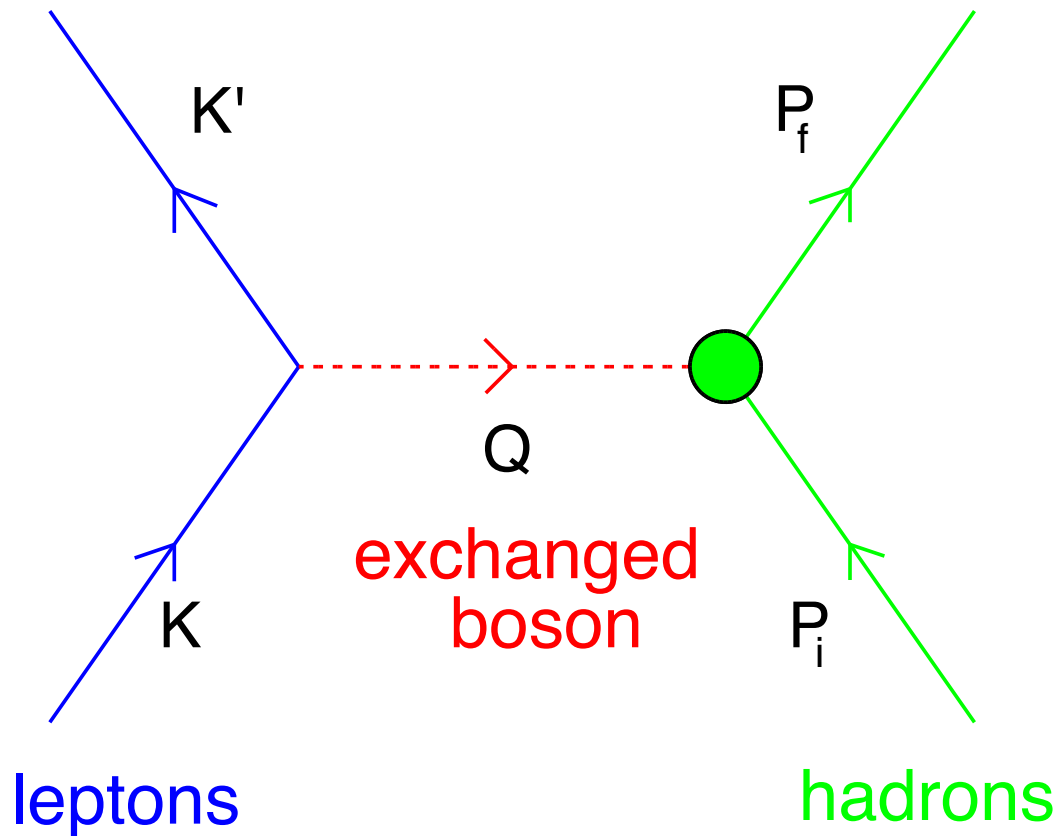
exclusive-3

...

... I will focus on inclusive and semi-inclusive reactions

## 2. General forms of the inclusive and semi-inclusive cross sections

Inclusive: only  $K'$  measured





The general form of the cross section involves the contraction of the leptonic and hadronic tensors:

$$\sigma \sim \eta_{\mu\nu} W^{\mu\nu} = \eta_{\mu\nu}^s W_s^{\mu\nu} + \chi \eta_{\mu\nu}^a W_a^{\mu\nu},$$

where  $\chi = 1$  for incident neutrinos and  $\chi = -1$  for antineutrinos.

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For inclusive scattering one has

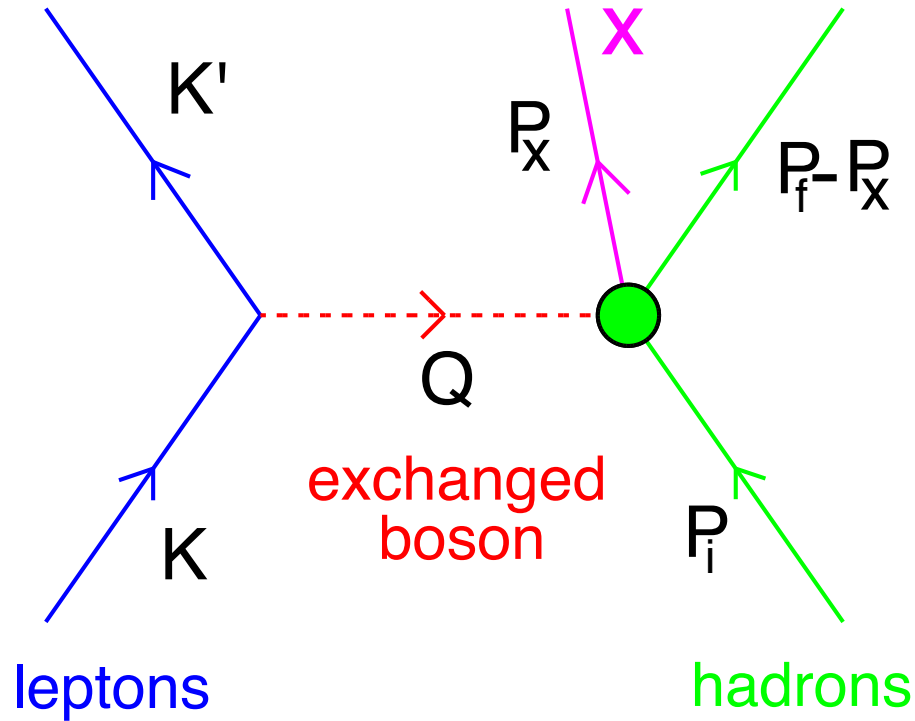
$$\begin{aligned} \eta_{\mu\nu}^s W_s^{\mu\nu} &\sim \hat{V}_{CC} W_{incl}^{CC} + \hat{V}_{CL} W_{incl}^{CL} + \hat{V}_{LL} W_{incl}^{LL} + \hat{V}_T W_{incl}^T \\ \eta_{\mu\nu}^a W_a^{\mu\nu} &\sim \hat{V}_{T'} W_{incl}^{T'}, \end{aligned}$$

where each of the 5 responses is a function of the neutrino momentum  $k$  and 2 other variables, for instance  $(k', \theta)$ , the muon momentum and the lepton scattering angle, or  $(q, \omega)$ , the 3-momentum transfer and energy transfer:

$$\begin{aligned} W_{incl}^K &= W_{incl}^K(k; k', \theta) \\ &= W_{incl}^K(k; q, \omega), \end{aligned}$$

where  $K = CC, CL, LL, T$  and  $T'$ . The factors  $V_K$  are the leptonic kinematic factors (“Rosenbluth factors”) which can be found, for instance, in MDVF.

# Semi-inclusive: $K'$ and $P_X$ measured





In contrast, for semi-inclusive reactions one has more terms. Specifically, for  $CC\nu$  reactions (see MDVF), one has the following completely general structure:

$$\begin{aligned}\eta_{\mu\nu}^s W_s^{\mu\nu} &\sim \hat{V}_{CC} W_{semi}^{CC} + \hat{V}_{CL} W_{semi}^{CL} + \hat{V}_{LL} W_{semi}^{LL} \\ &\quad + \hat{V}_T W_{semi}^T + \hat{V}_{TT} W_{semi}^{TT} + \hat{V}_{TC} W_{semi}^{TC} + \hat{V}_{TL} W_{semi}^{TL} \\ \eta_{\mu\nu}^a W_a^{\mu\nu} &\sim \hat{V}_{T'} W_{semi}^{T'} + \hat{V}_{TC'} W_{semi}^{TC'} + \hat{V}_{TL'} W_{semi}^{TL'}\end{aligned}$$

$$\begin{aligned}W_{semi}^K &= W_{semi}^K(k; k', \theta; p_N, \theta_N^L, \phi_N^L) \\ &= W_{semi}^K(k; q, \omega; p, \mathcal{E}, \phi_N).\end{aligned}$$

Total: 10 response functions, each a function of 6 variables. (Actually, in general there are 16 classes of response for electroweak reactions of all types; for  $CC\nu$  reactions 6 cases do not have the corresponding leptonic factors, labeled  $\underline{TT}, \underline{TC}, \underline{TL}, \underline{CL'}, \underline{TC'}, \underline{TL'}.$ )

The  $\phi_N$  dependence can be made explicit, leaving 6 individual responses, each a function of 5 variables, say  $(k; q, \omega; p, \mathcal{E})$  :

$$\begin{aligned}
W_{semi}^{CC} &= \frac{1}{\rho^2} \{ \rho^2 X_1 + \rho \nu^2 X_2 + X_3 + 2\sqrt{\rho} \nu X_4 \\
&\quad + H^2 X_5 + 2\sqrt{\rho} \nu H X_6 + 2H X_7 \} \\
W_{semi}^{CL} &= \frac{\nu}{\rho^2} \left\{ \rho X_2 + X_3 + \sqrt{\rho} \left( \frac{1}{\nu} + \nu \right) X_4 \right. \\
&\quad \left. + H^2 X_5 + \sqrt{\rho} \left( \frac{1}{\nu} + \nu \right) H X_6 + 2H X_7 \right\} \\
W_{semi}^{LL} &= \frac{1}{\rho^2} \{ -\rho^2 X_1 + \rho X_2 + \nu^2 X_3 + 2\sqrt{\rho} \nu X_4 \\
&\quad + \nu^2 H^2 X_5 + 2\sqrt{\rho} \nu H X_6 + 2\nu^2 H X_7 \} \\
W_{semi}^T &= -2X_1 + X_5 \eta_T^2 \\
W_{semi}^{TT} &= -X_5 \eta_T^2 \cos 2\phi_N \\
W_{semi}^{TC} &= \frac{2\sqrt{2}}{\rho} \eta_T \{ H X_5 + \sqrt{\rho} \nu X_6 + X_7 \} \cos \phi_N \\
W_{semi}^{TL} &= \frac{2\sqrt{2}}{\rho} \eta_T \{ \nu H X_5 + \sqrt{\rho} X_6 + \nu X_7 \} \cos \phi_N \\
W_{semi}^{T'} &= \frac{1}{\sqrt{\rho}} \{ Z_1 + H Z_2 \} \\
W_{semi}^{TC'} &= \frac{2\sqrt{2}}{\rho} \eta_T \{ -(\sqrt{\rho} \nu Y_2 + Y_3) \sin \phi_N + (\sqrt{\rho} Z_2 + \nu Z_3) \cos \phi_N \} \\
W_{semi}^{TL'} &= \frac{2\sqrt{2}}{\rho} \eta_T \{ -(\sqrt{\rho} Y_2 + \nu Y_3) \sin \phi_N + (\sqrt{\rho} \nu Z_2 + Z_3) \cos \phi_N \} ,
\end{aligned}$$

The  $\phi_N$  dependence can be made explicit, leaving 6 individual responses, each a function of 5 variables, say  $(k; q, \omega; p, \mathcal{E})$  :

... for a total of 12 responses

$$W_{semi}^{CC} = \frac{1}{\rho^2} \{ \rho^2 X_1 + \rho \nu^2 X_2 + X_3 + 2\sqrt{\rho}\nu X_4 \\ + H^2 X_5 + 2\sqrt{\rho}\nu H X_6 + 2H X_7 \}$$

$$W_{semi}^{CL} = \frac{\nu}{\rho^2} \left\{ \rho X_2 + X_3 + \sqrt{\rho} \left( \frac{1}{\nu} + \nu \right) X_4 \right. \\ \left. + H^2 X_5 + \sqrt{\rho} \left( \frac{1}{\nu} + \nu \right) H X_6 + 2H X_7 \right\}$$

$$W_{semi}^{LL} = \frac{1}{\rho^2} \{ -\rho^2 X_1 + \rho X_2 + \nu^2 X_3 + 2\sqrt{\rho}\nu X_4 \\ + \nu^2 H^2 X_5 + 2\sqrt{\rho}\nu H X_6 + 2\nu^2 H X_7 \}$$

$$W_{semi}^T = -2X_1 + X_5 \eta_T^2$$

$$W_{semi}^{TT} = -X_5 \eta_T^2 \cos 2\phi_N$$

$$W_{semi}^{TC} = \frac{2\sqrt{2}}{\rho} \eta_T \{ H X_5 + \sqrt{\rho}\nu X_6 + X_7 \} \cos \phi_N$$

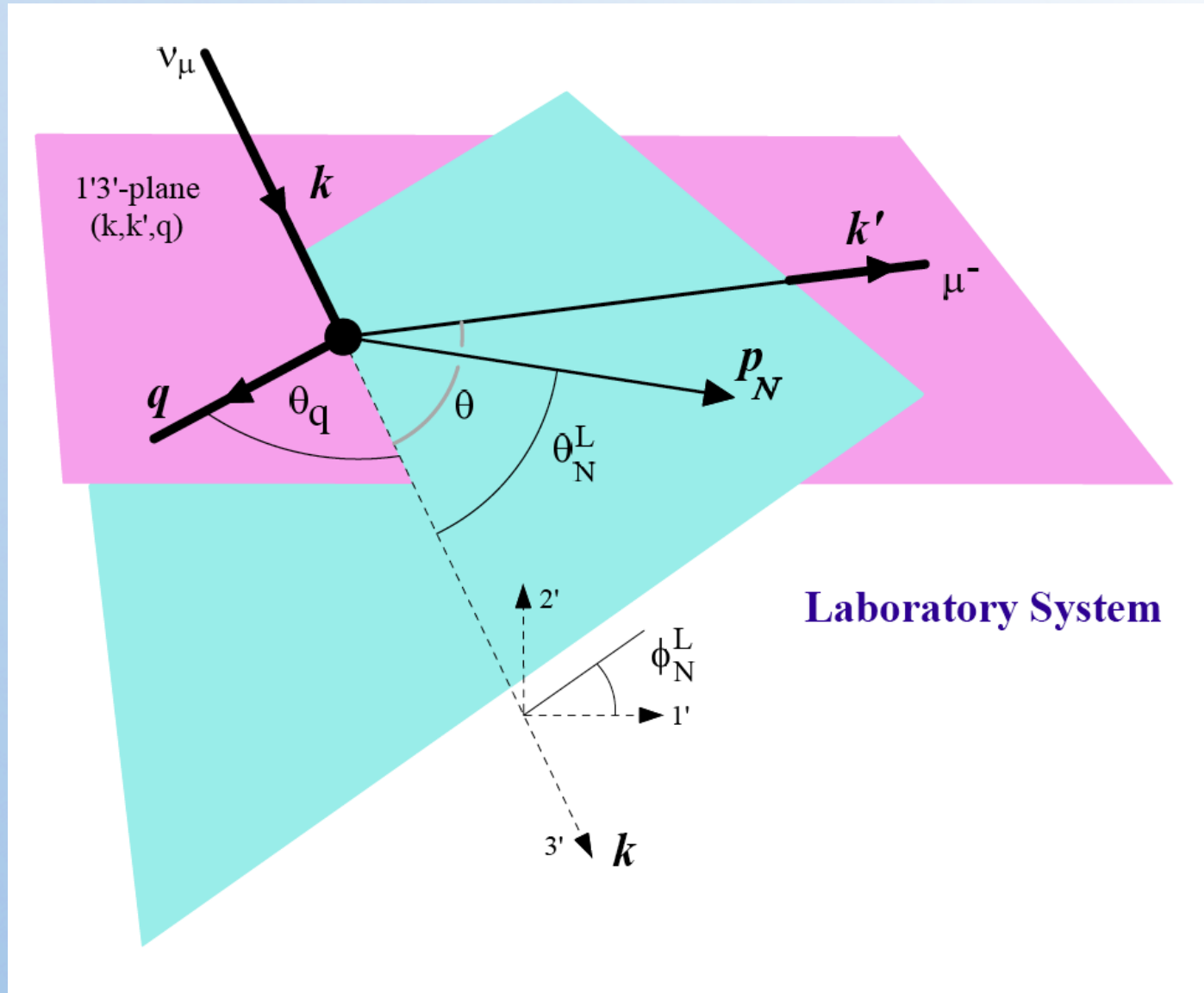
$$W_{semi}^{TL} = \frac{2\sqrt{2}}{\rho} \eta_T \{ \nu H X_5 + \sqrt{\rho} X_6 + \nu X_7 \} \cos \phi_N$$

$$W_{semi}^{T'} = \frac{1}{\sqrt{\rho}} \{ Z_1 + H Z_2 \}$$

$$W_{semi}^{TC'} = \frac{2\sqrt{2}}{\rho} \eta_T \{ -(\sqrt{\rho}\nu Y_2 + Y_3) \sin \phi_N + (\sqrt{\rho} Z_2 + \nu Z_3) \cos \phi_N \}$$

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### 3. Basics of kinematics for semi-inclusive electroweak reactions

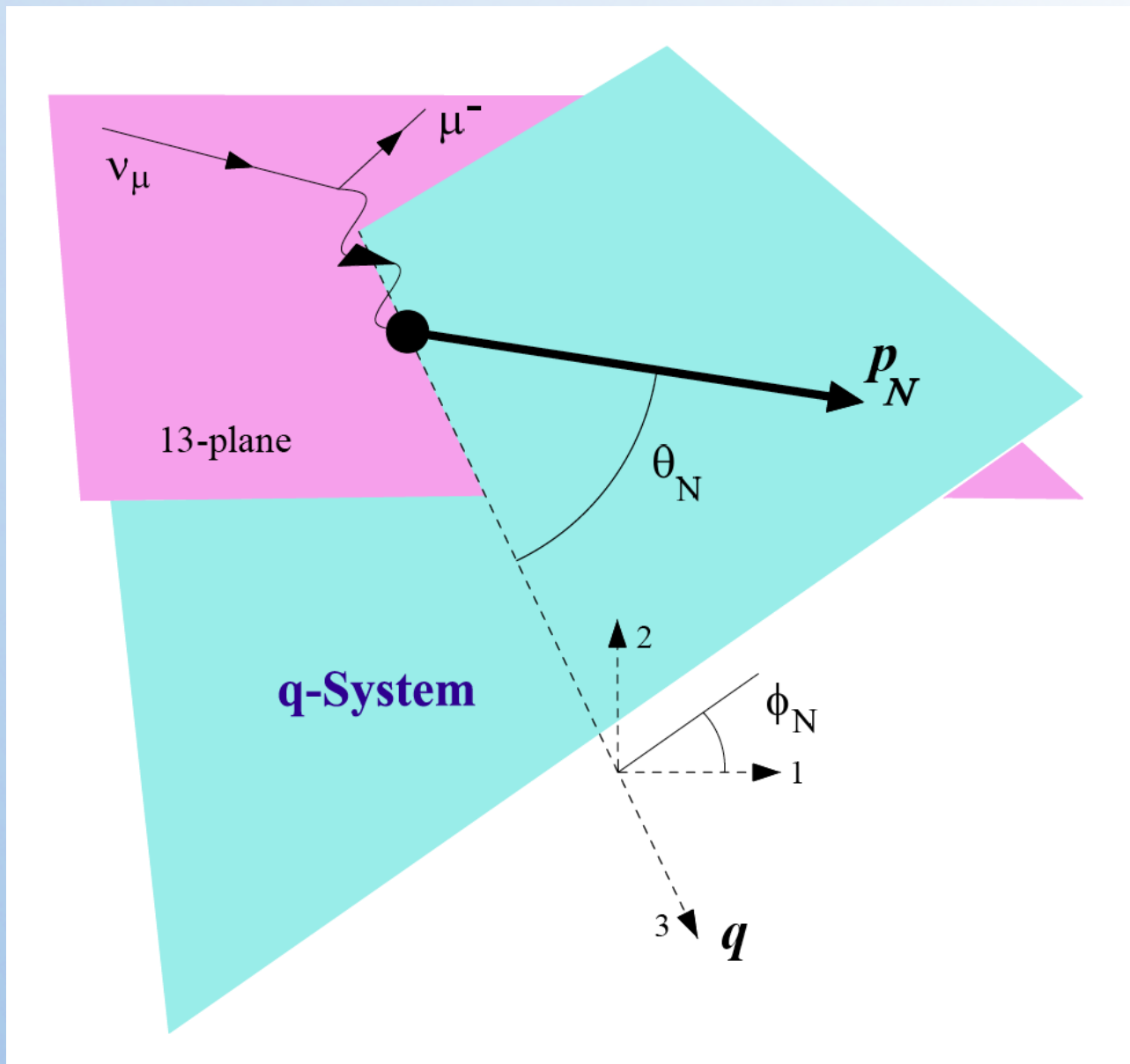


... thus, to begin, we assume that the muon's kinematics are known  $(k', \theta)$  for given neutrino momentum  $(k)$ , and that a nucleon is detected having  $(p_N, \theta_N^L, \phi_N^L)$  in the **laboratory system**

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Next, we change variables to the **q-system** where coordinates are specified with respect to the direction of the 3-momentum transfer:  $(p_N, \theta_N, \phi_N)$





#### 4. General form of the nuclear response; dynamical variables

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Note that as the neutrino momentum  $k$  changes the direction of  $q$  also changes, and thus  $(\theta_N, \phi_N)$  change



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... however, these variables are better suited to treating the general form of the semi-inclusive cross section

All of this is fine; however, it does not capture where the nuclear response is large or small. To do this, it is better to change variables yet again to the missing energy and missing momentum,  $E_m$  and  $p_m$ , as is well known from studies of  $(e,e'p)$  reactions.

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Actually I will use  $\mathcal{E}$  and  $\mathbf{p} = -\mathbf{p}_m$ , as is traditional in scaling analyses ( $\mathcal{E}$  is approximately  $E_m - E_s$  and so is zero when the final daughter nucleus is in its ground state)

Example:  $^{16}\text{O}(\nu_\mu, \mu^- p)$  with

$$k' = 1 \text{ GeV}/c$$

for the muon

$$\theta = 10 \text{ deg.}$$

$$p_N = 50 \text{ MeV}/c$$

for the proton

$$\theta_N^L = 10 \text{ deg.}$$

$$\phi_N^L = 180 \text{ deg.}$$

all of which will be kept **fixed** for this example

Example:  $^{16}\text{O}(\nu_\mu, \mu^- p)$  with

$k' = 1 \text{ GeV}/c$  for the muon  
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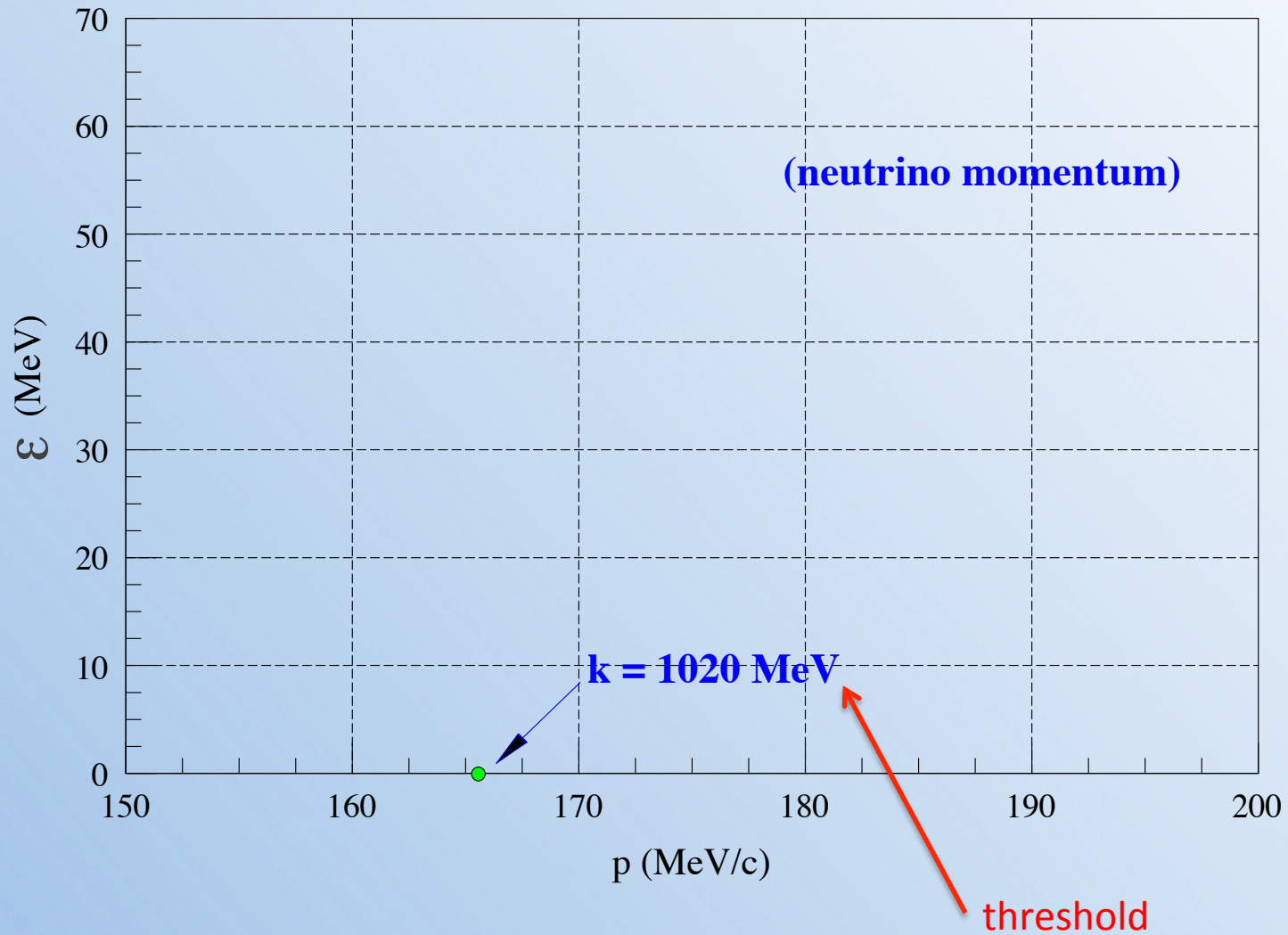
$p_N = 50 \text{ MeV}/c$  for the proton  
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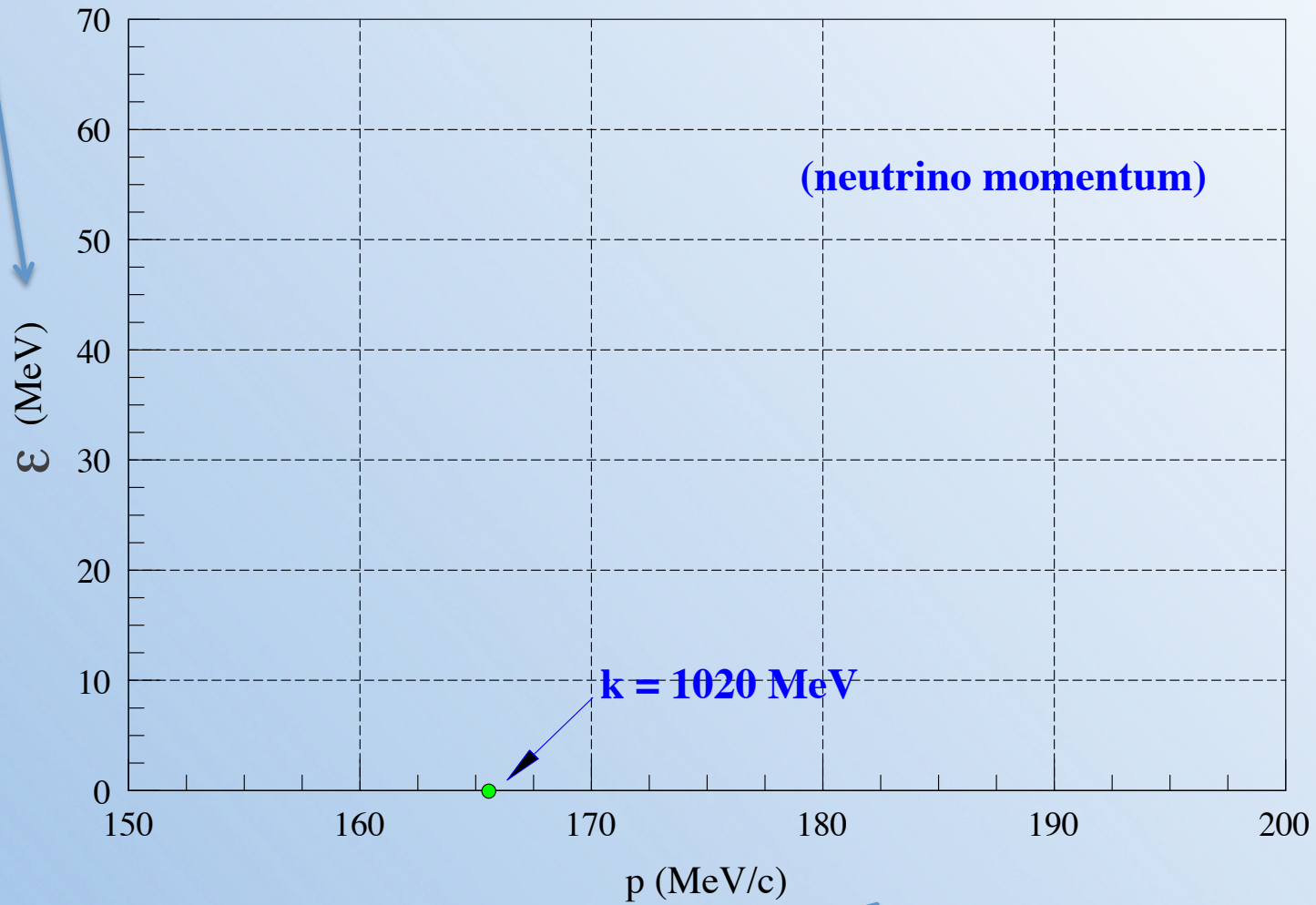
... starting with neutrino momentum  $k = 1020 \text{ MeV}/c$   
= threshold for the chosen kinematics

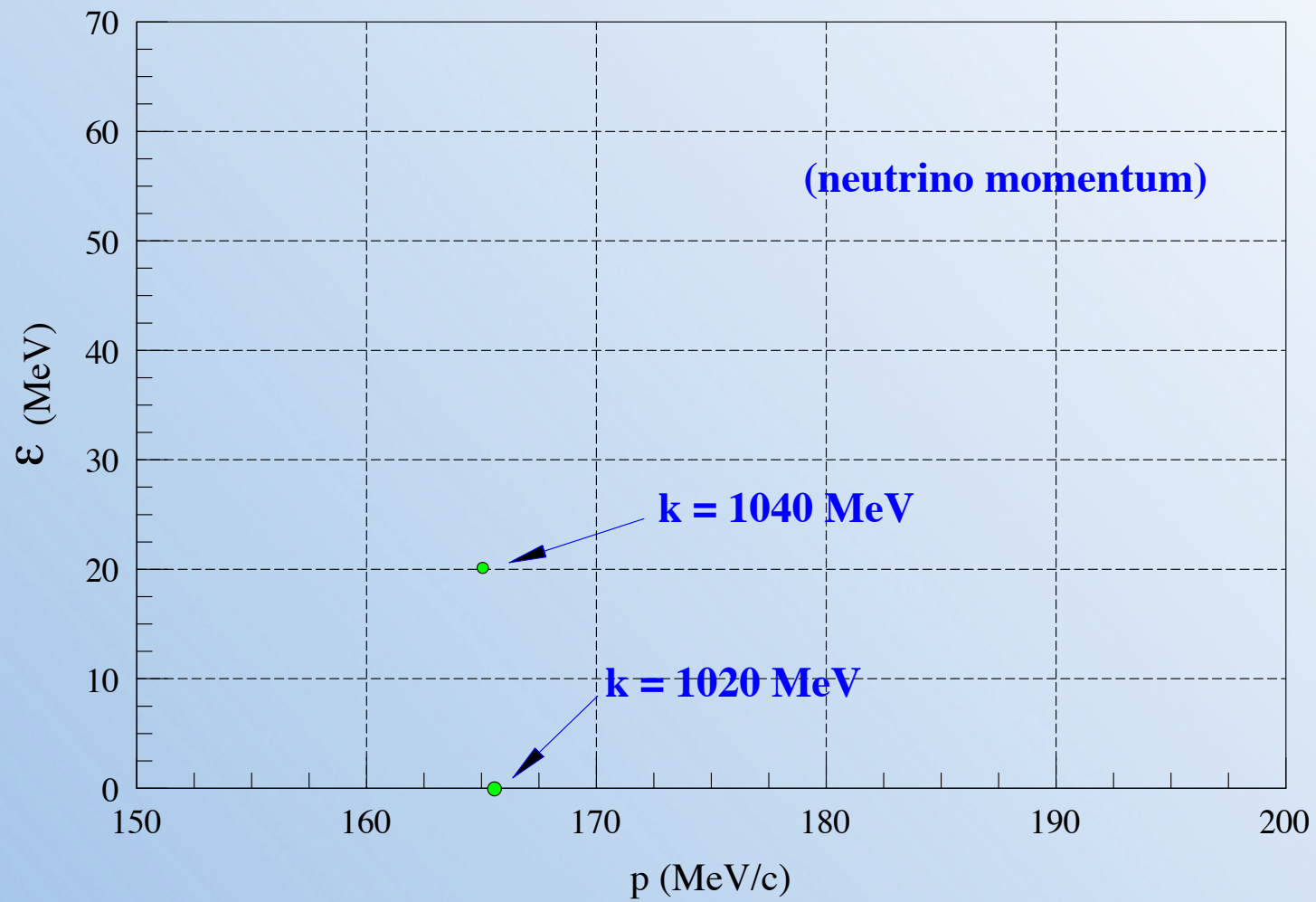


## 5. Trajectories in the missing energy – missing momentum plane

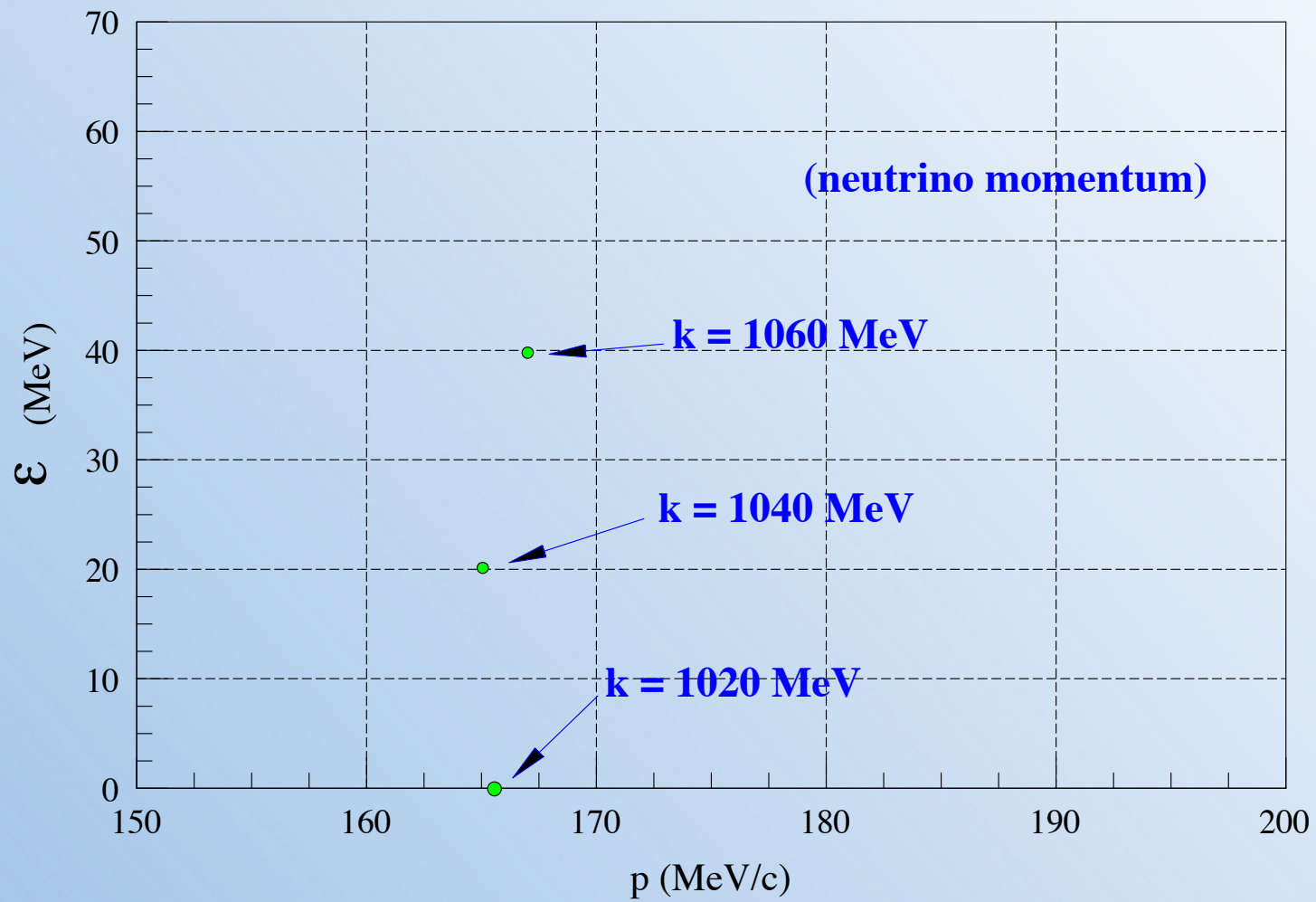


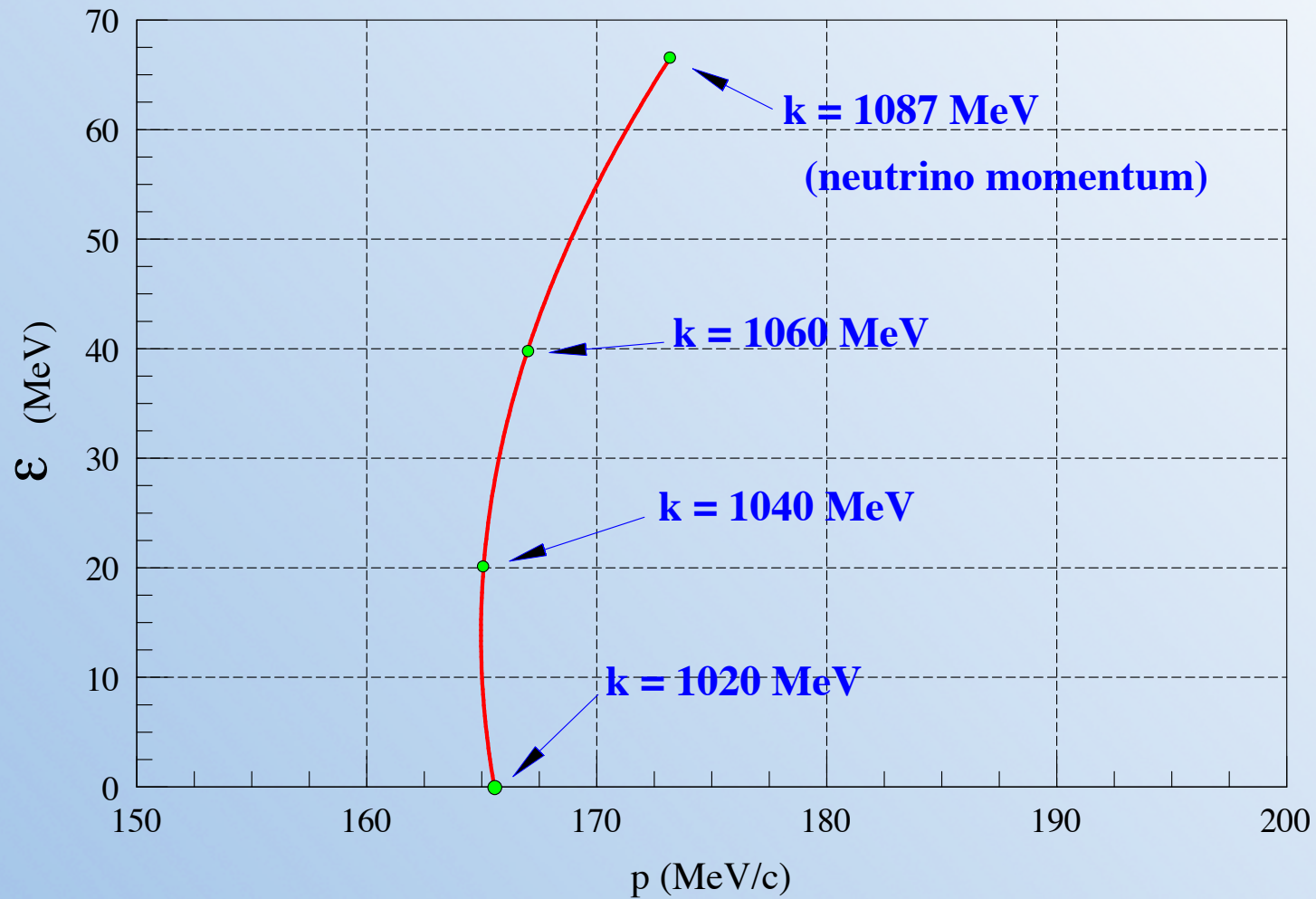
Missing energy – Separation energy





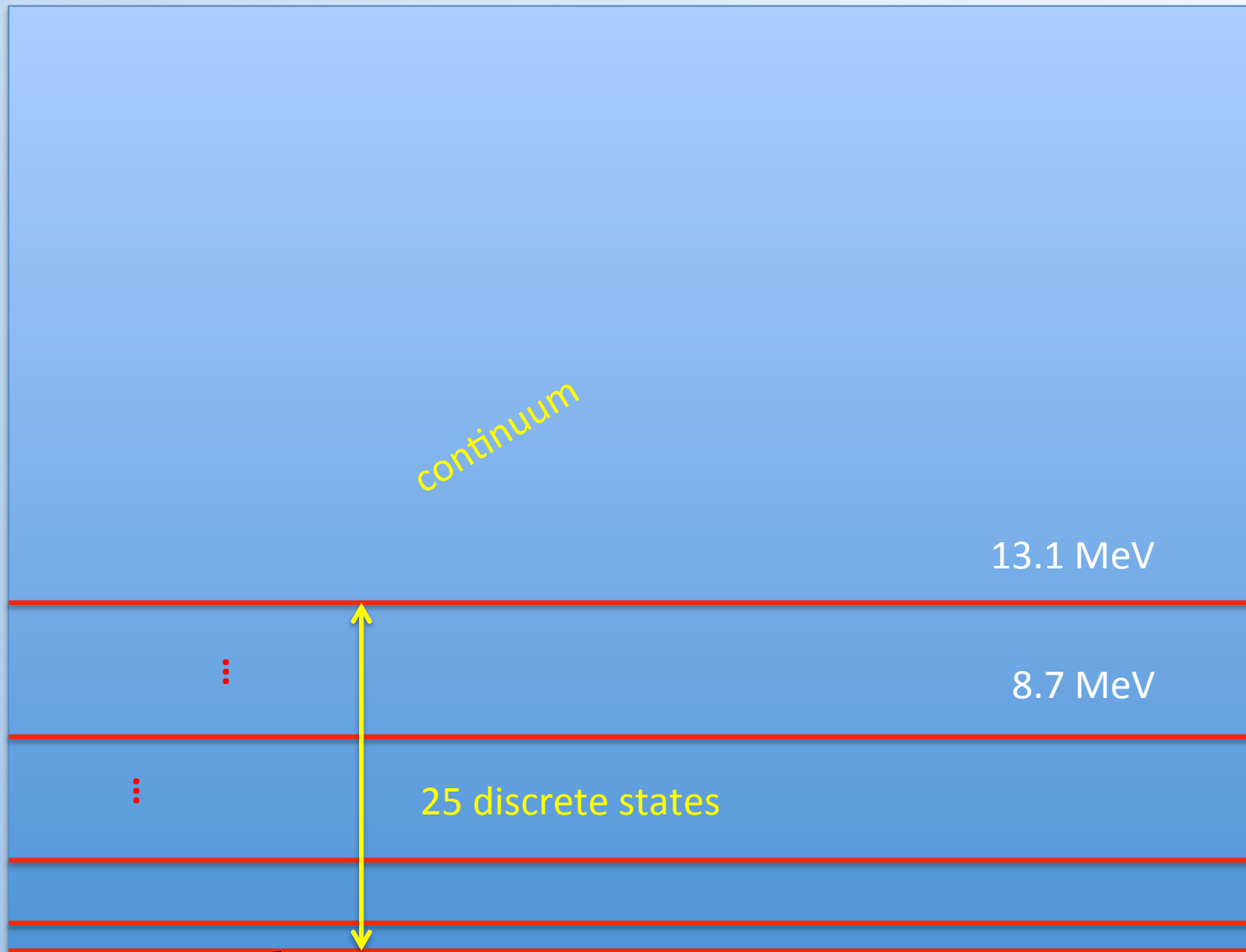






$^{12}\text{C}(\nu_\mu, \mu^- p)$

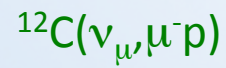
$\mathcal{E}$



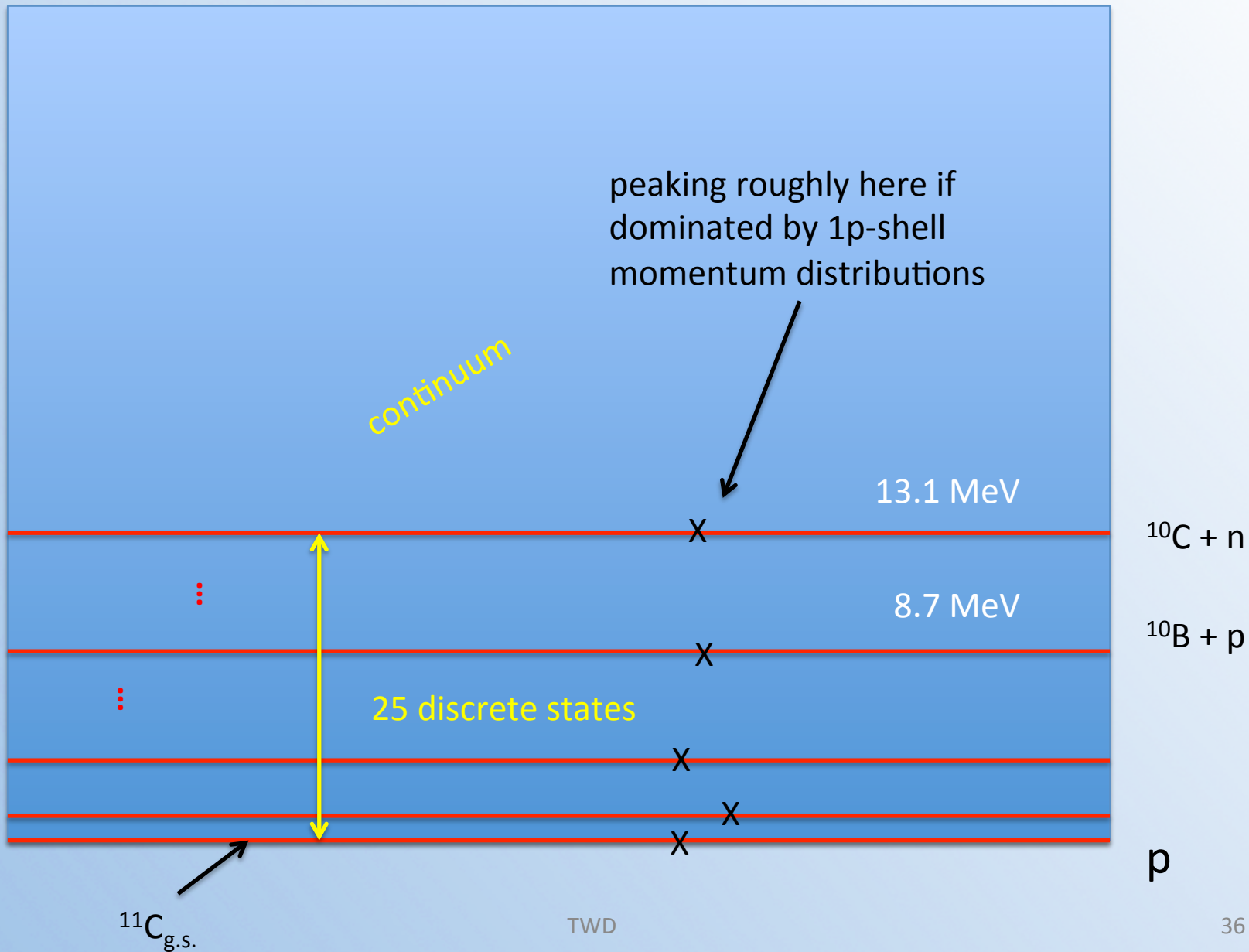
$^{10}\text{C} + n$

$^{10}\text{B} + p$

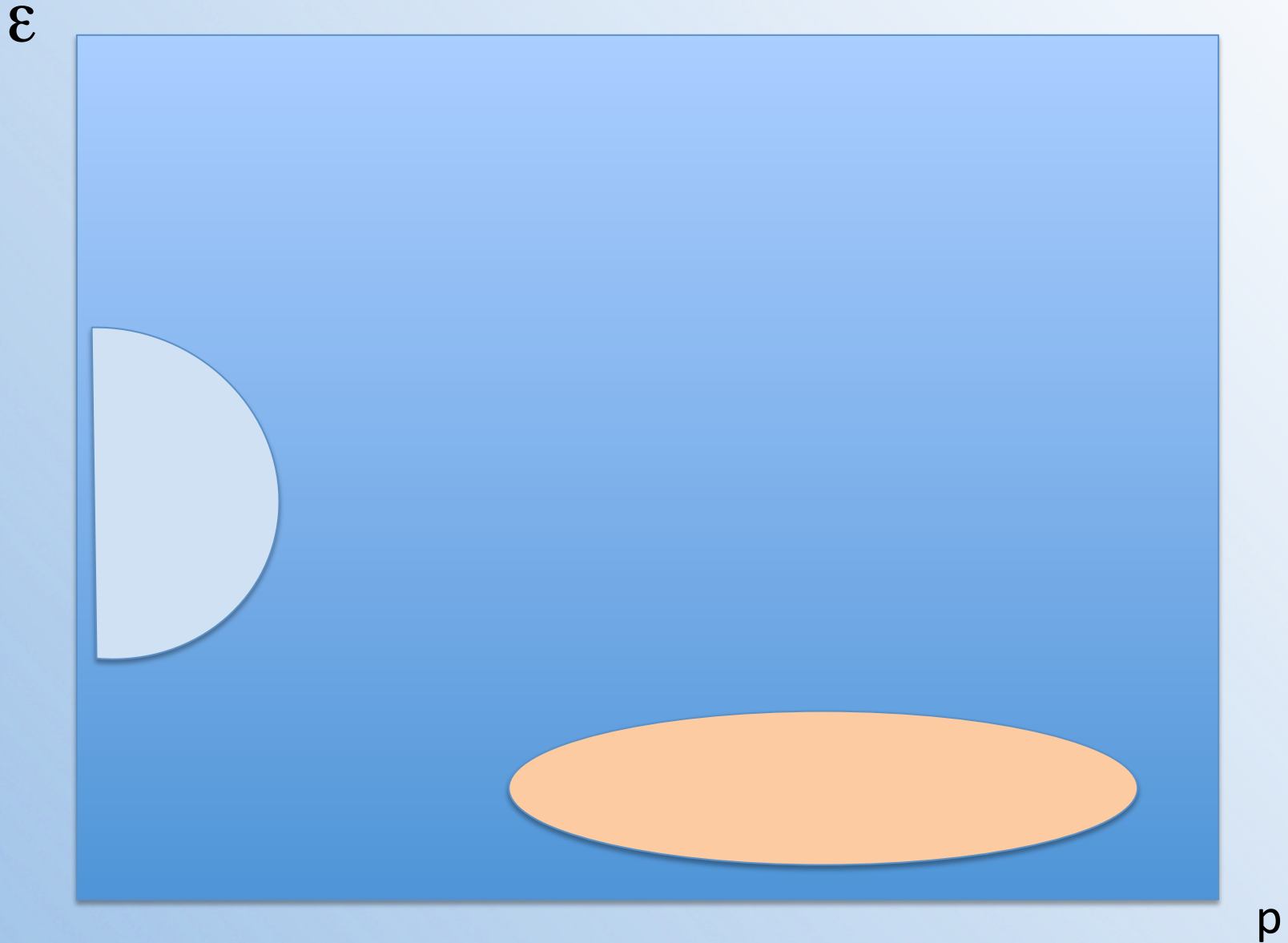
p



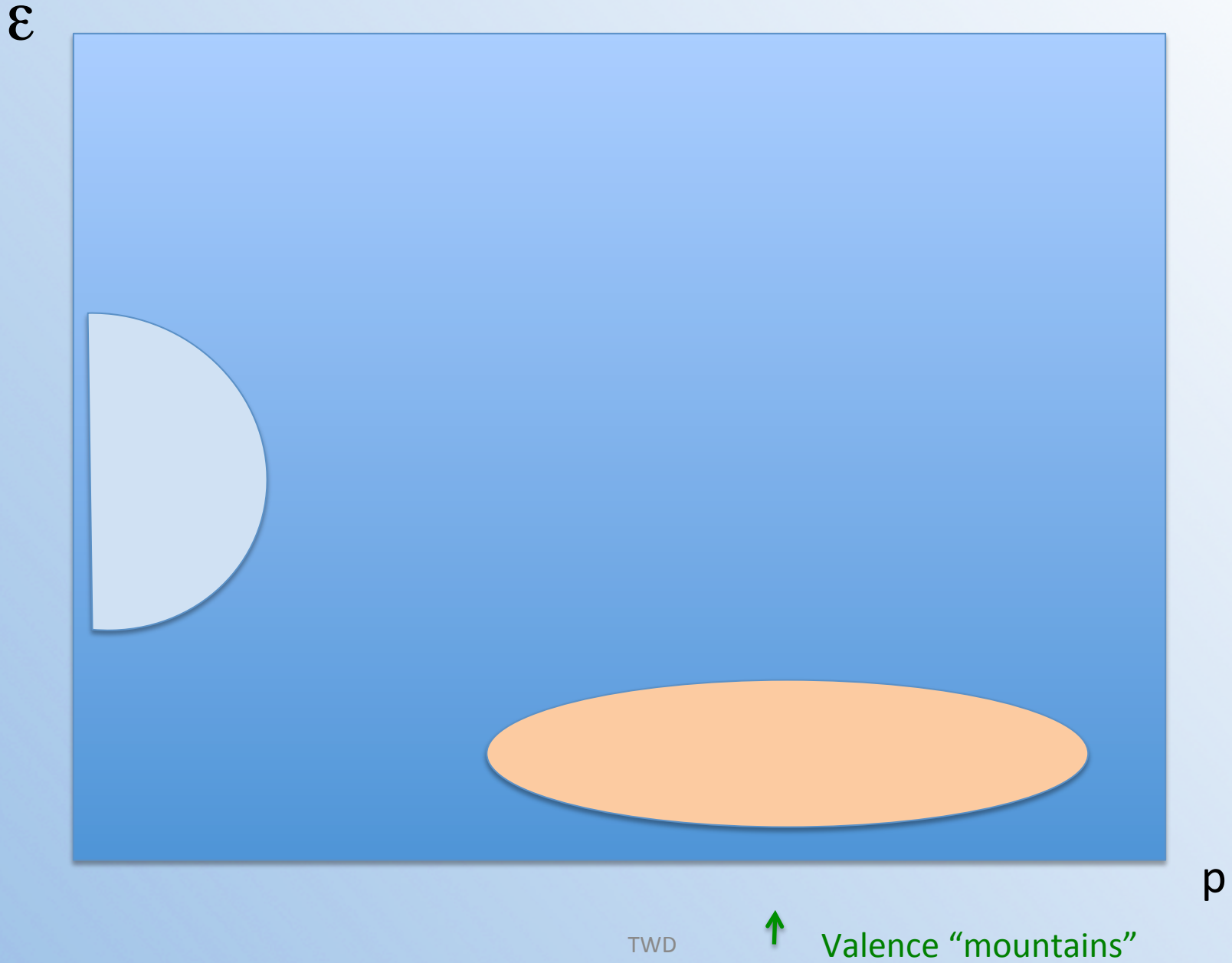
$\mathcal{E}$



## Generic landscape

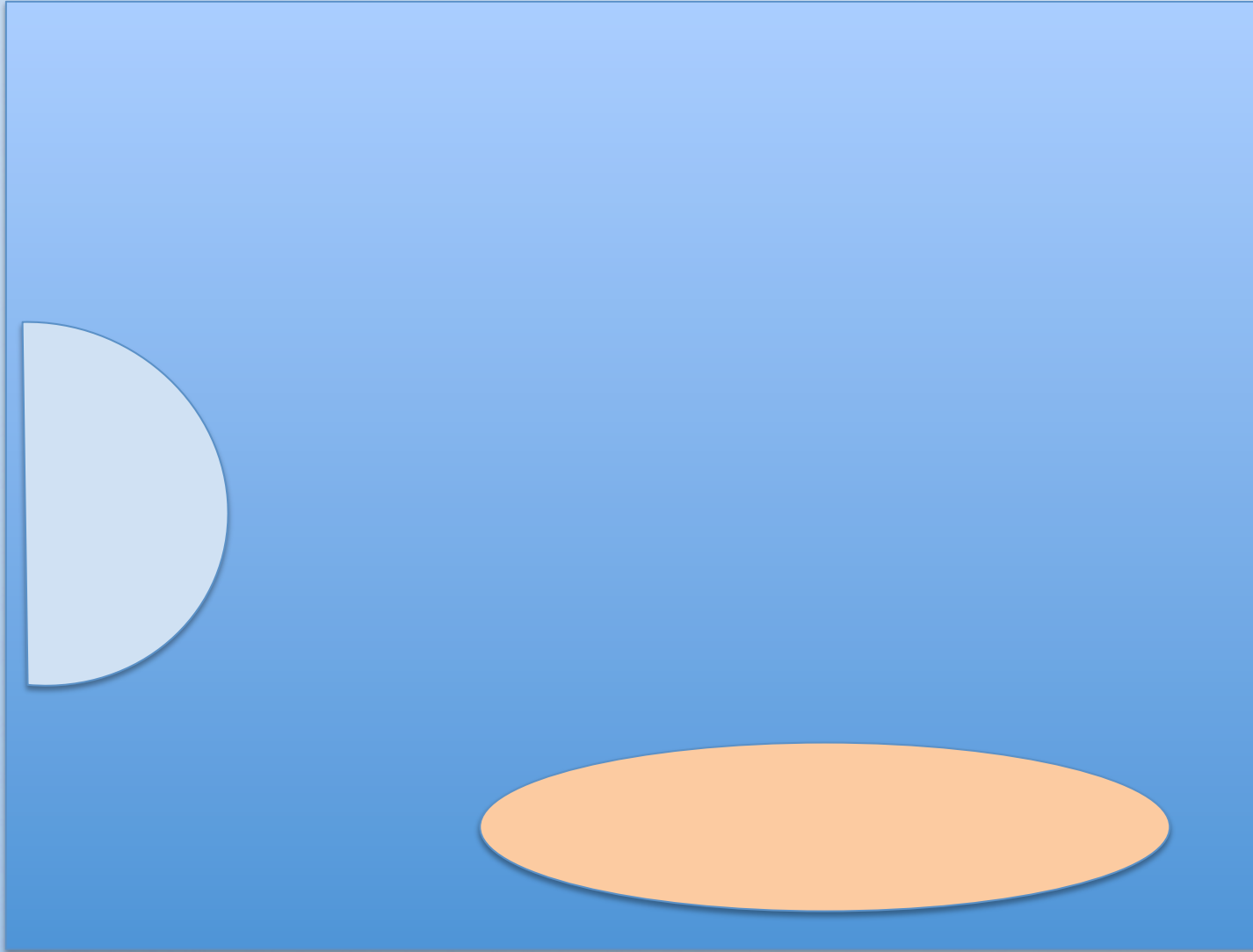


## Generic landscape



## Generic landscape

$\varepsilon$



p

Deep-lying shells (e.g., “1s foothills”)

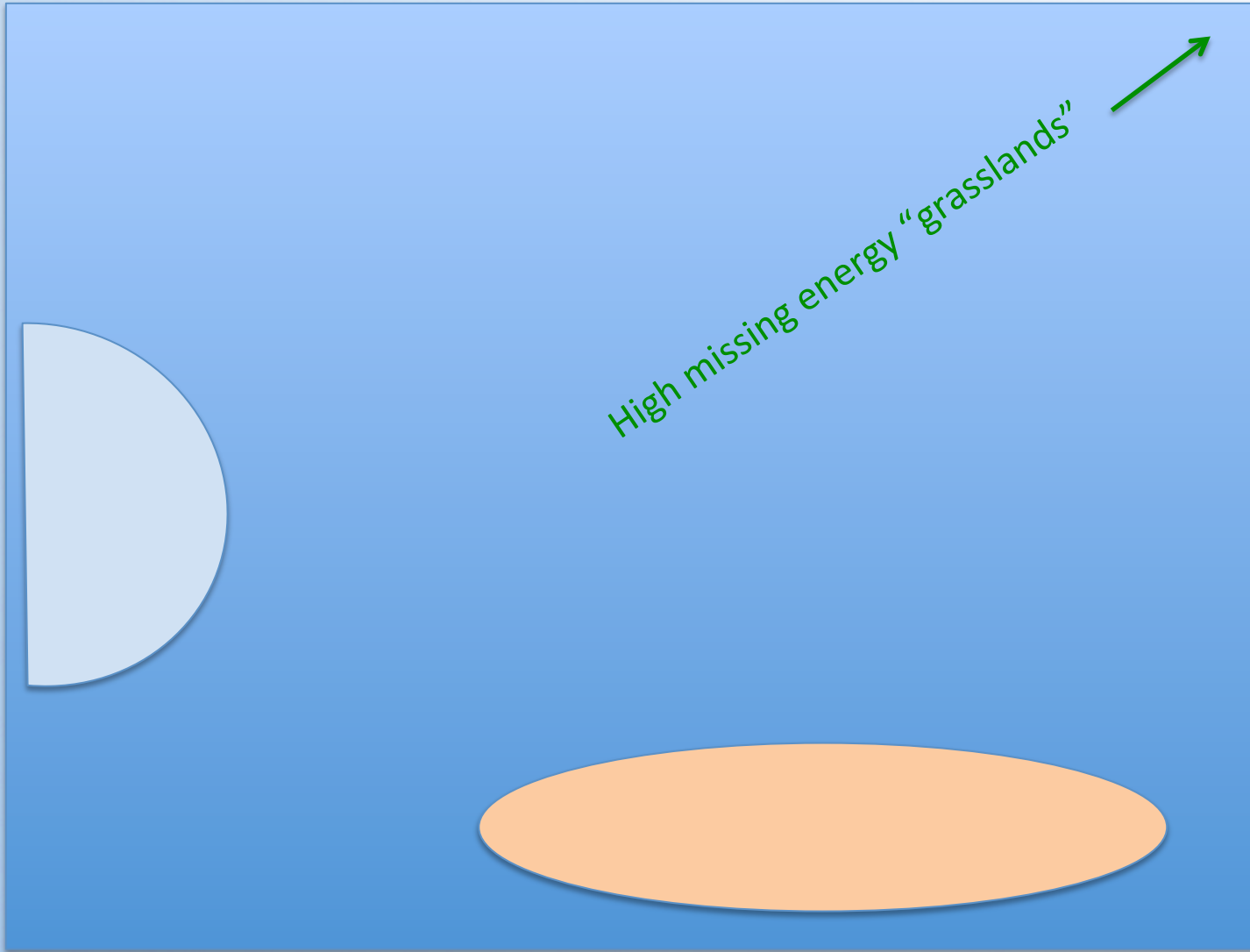
TWD



Valence “mountains”

# Generic landscape

$\epsilon$



p

Deep-lying shells (e.g., “1s foothills”)

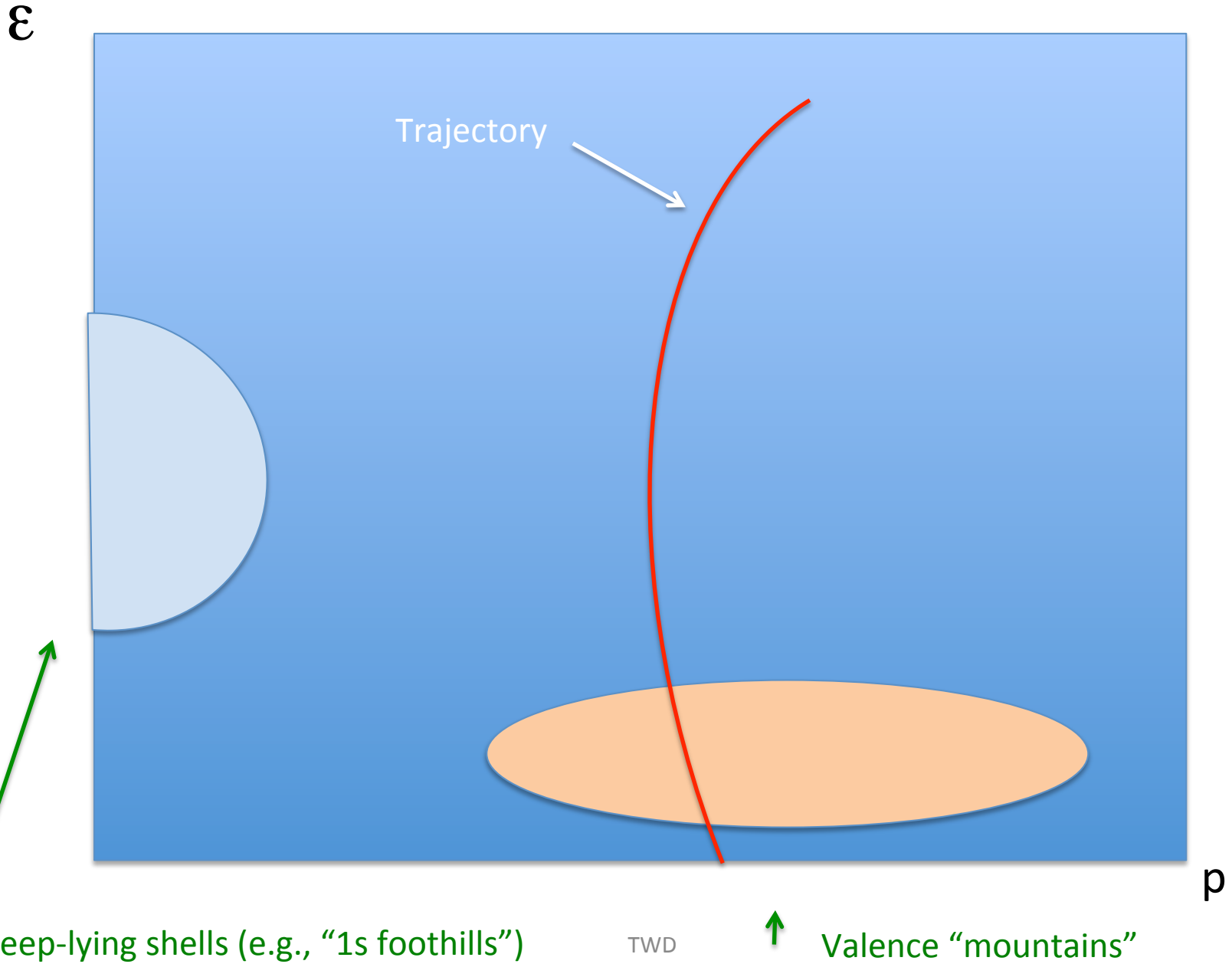
TWD



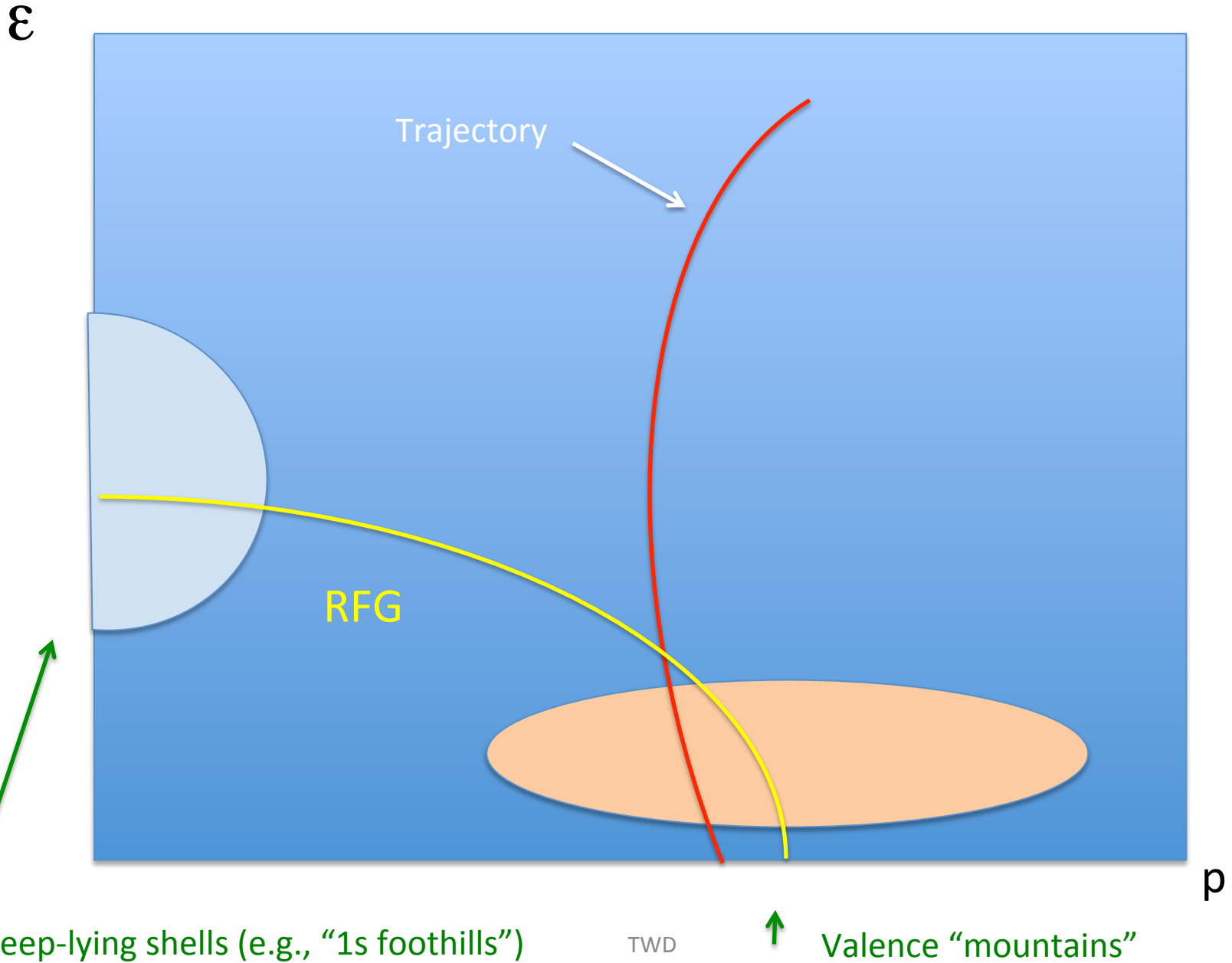
Valence “mountains”



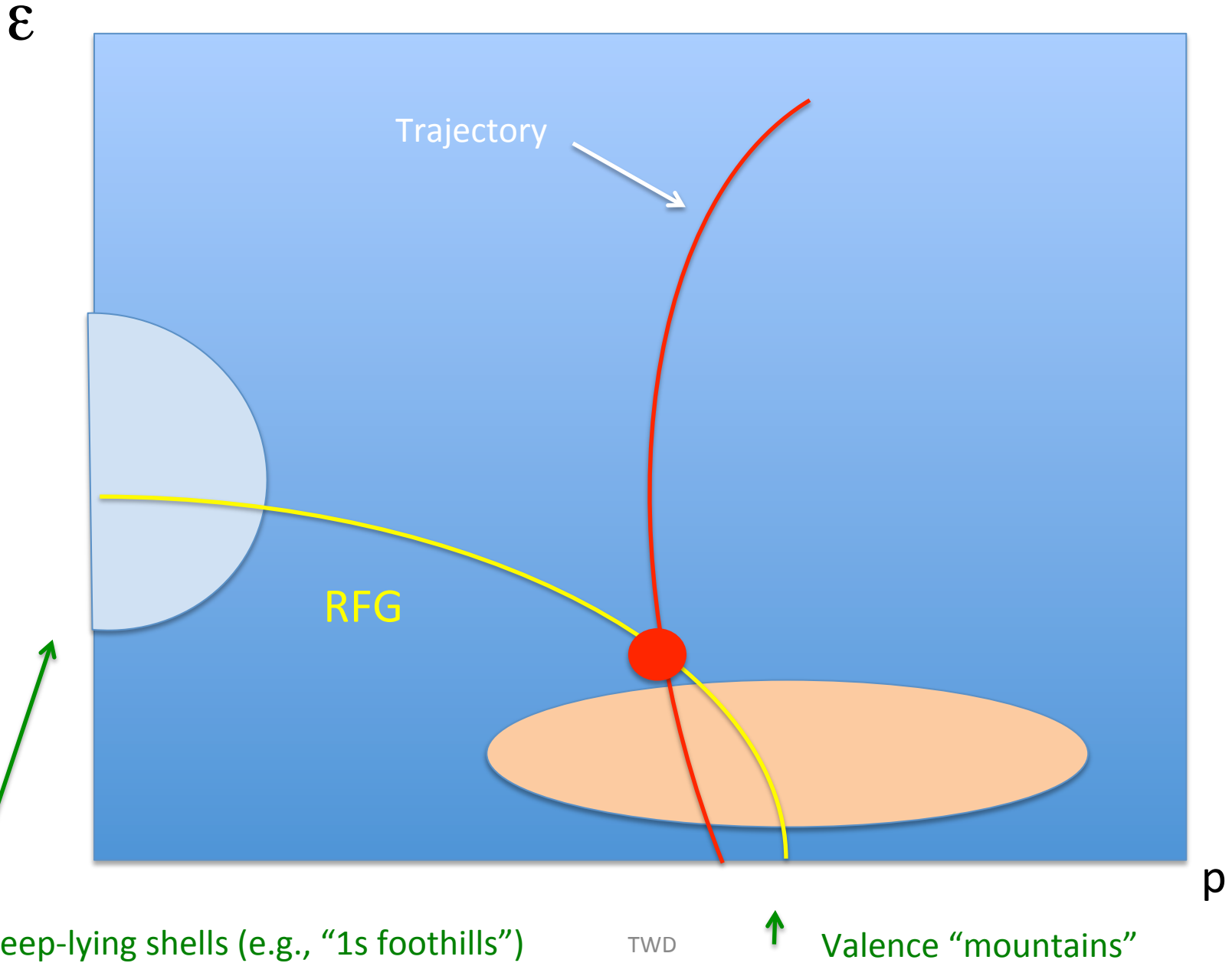
## Generic landscape



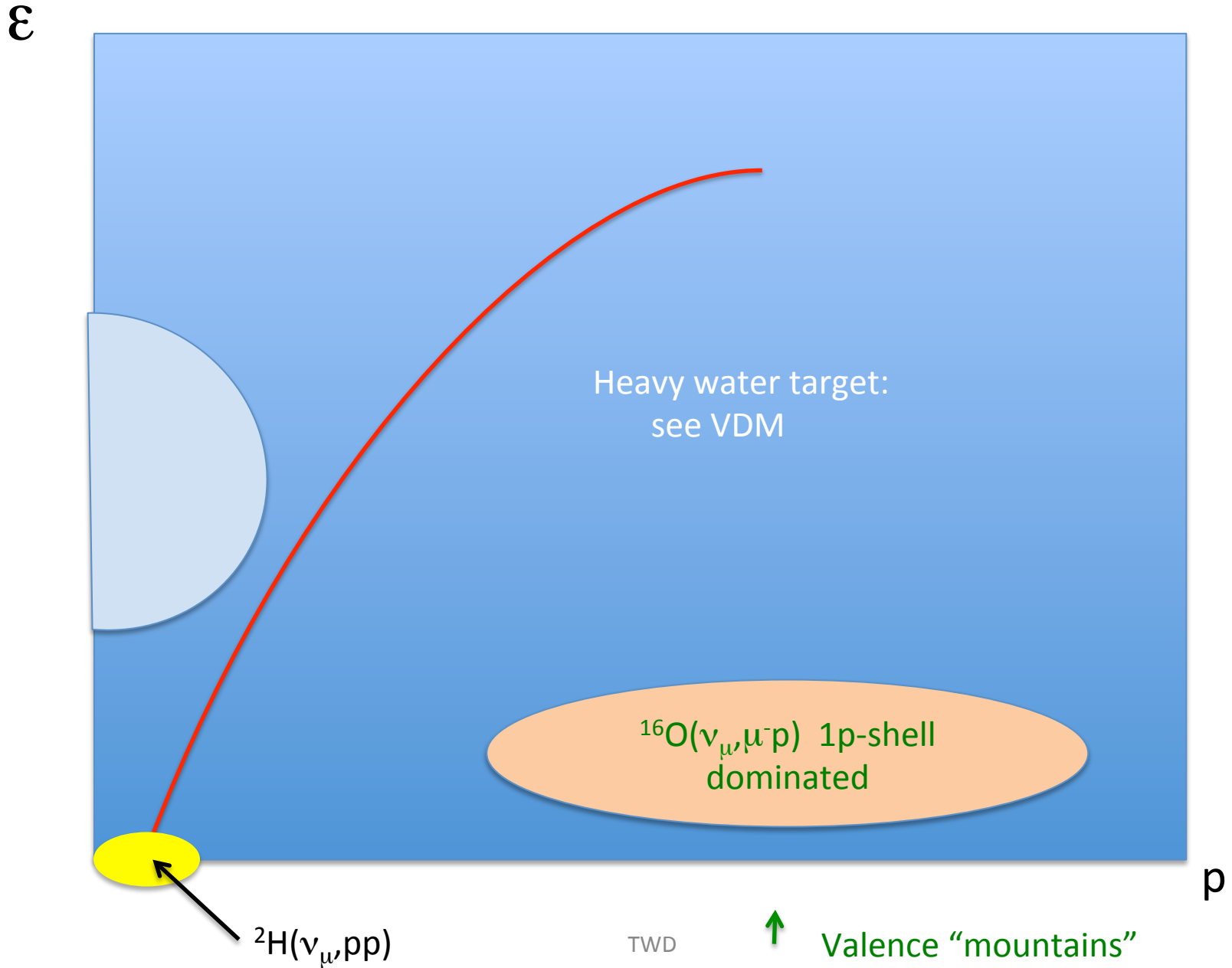
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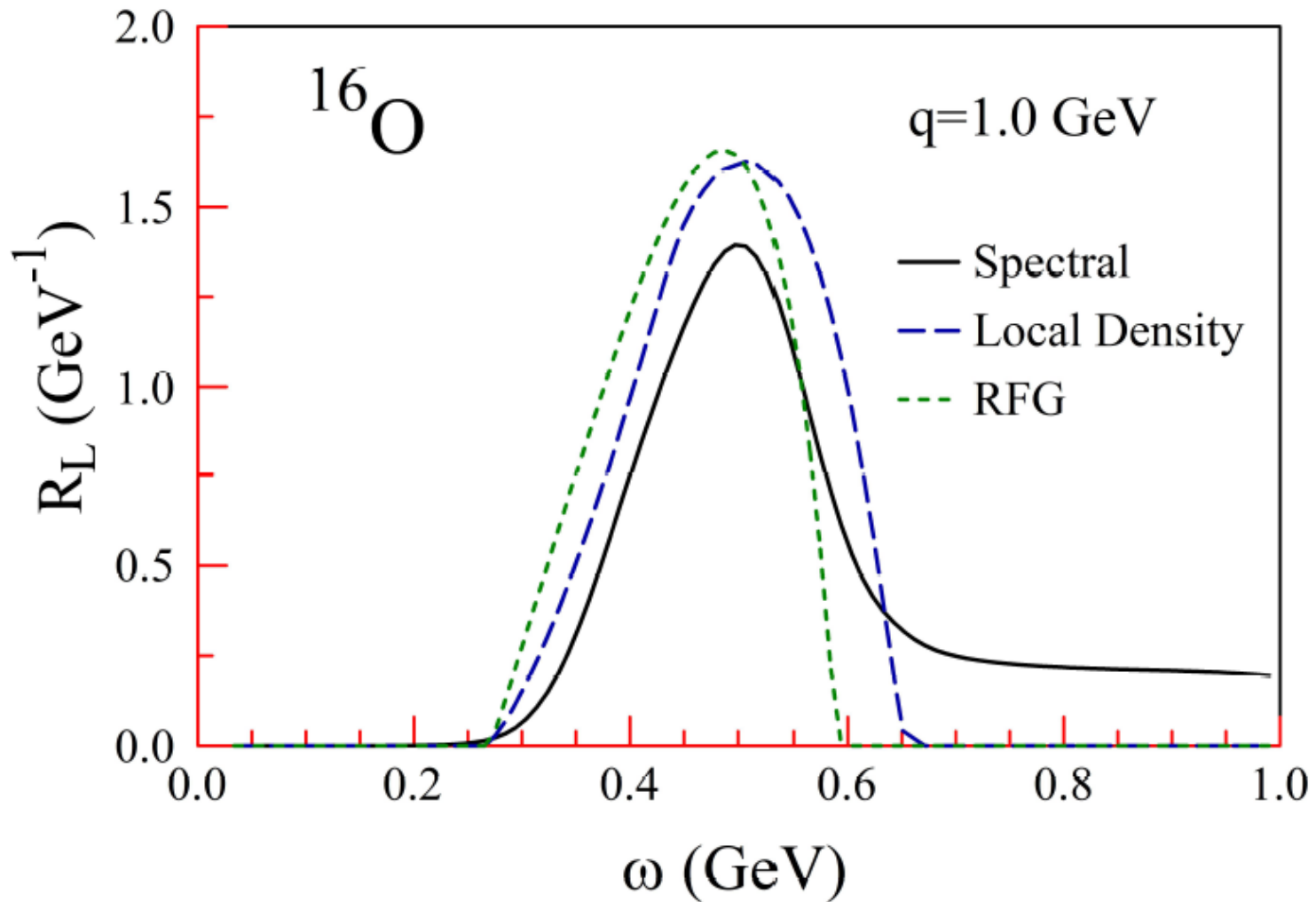


## Generic landscape

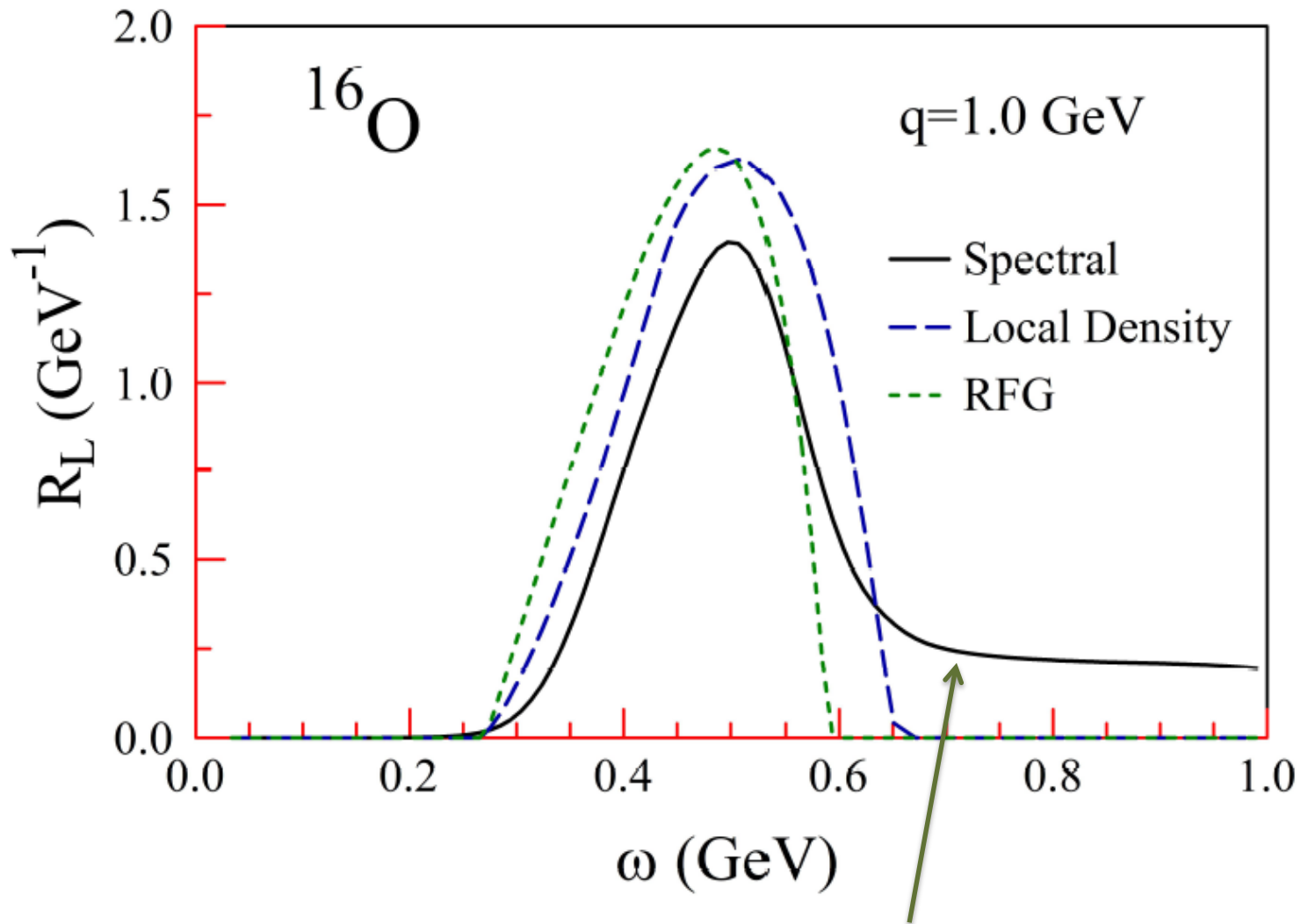


**A few specific results using various models:**

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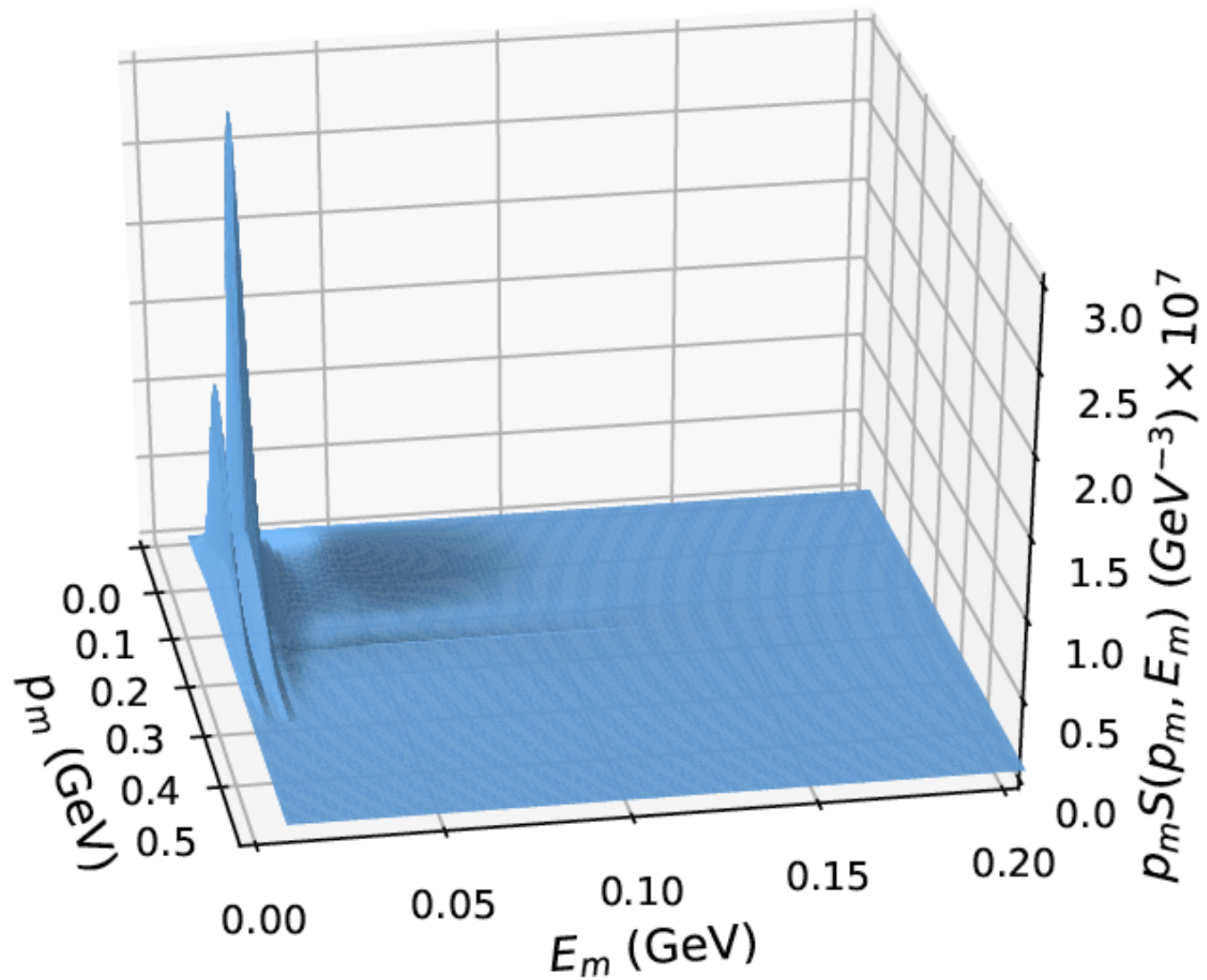


A few specific results using various models:



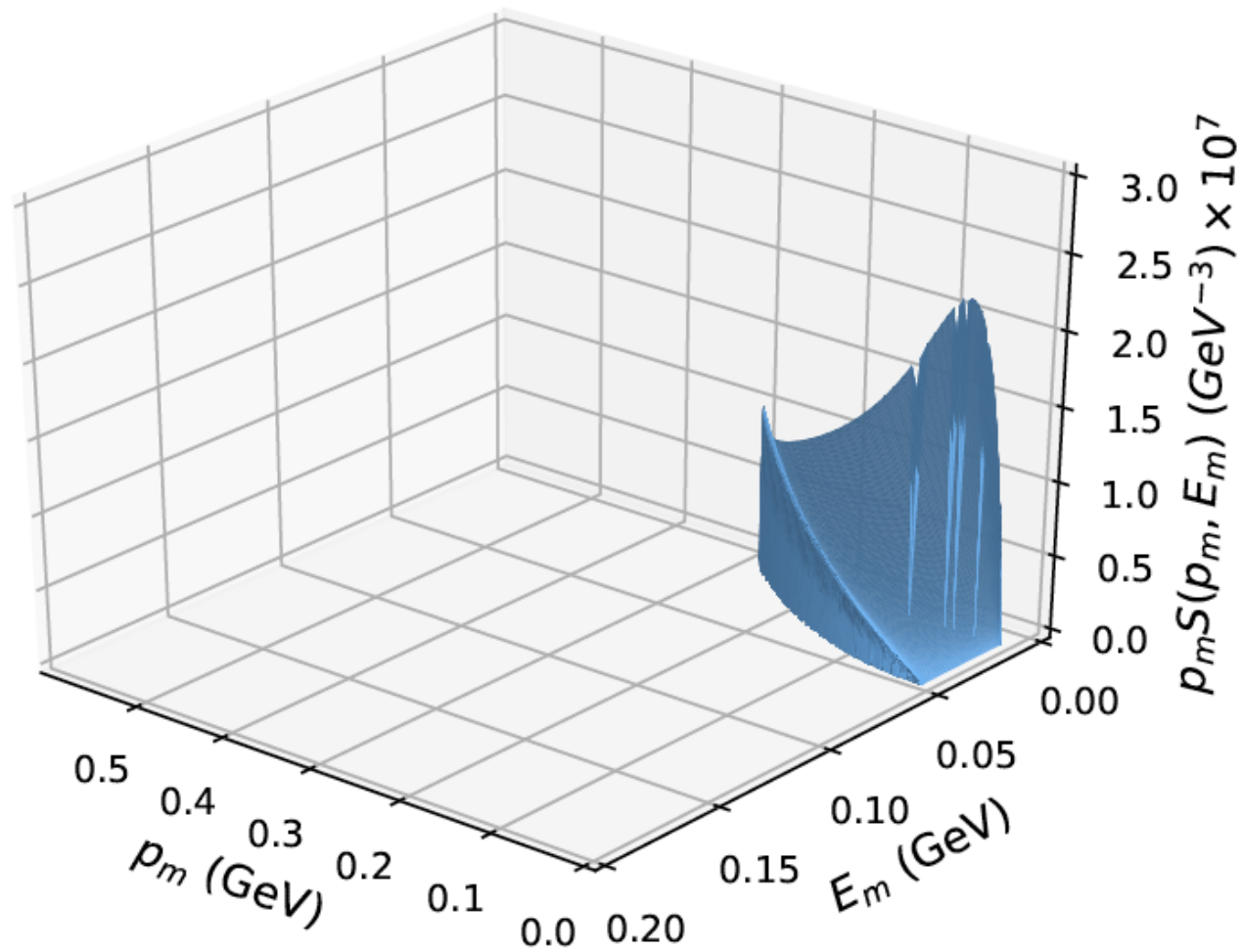
“Spectral” is basically the same as from scaling analyses, from RMFT and from experiment

$^{16}\text{O}$ :  $p_m \times S(p_m, E_m)$  from Omar



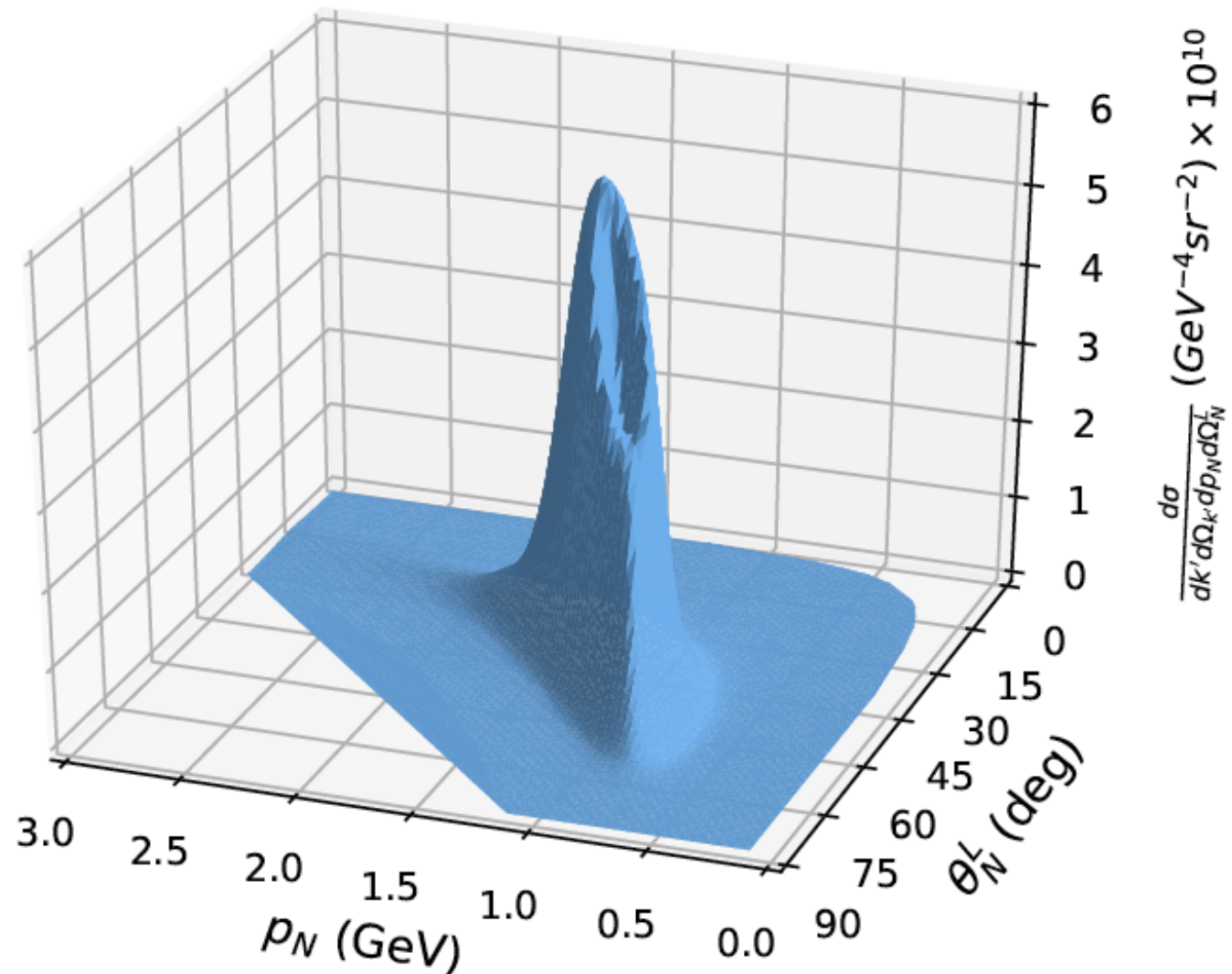


$^{16}\text{O}$ :  $p_m \times S(p_m, E_m)$  based on the LDA (local Fermi gas) approximation



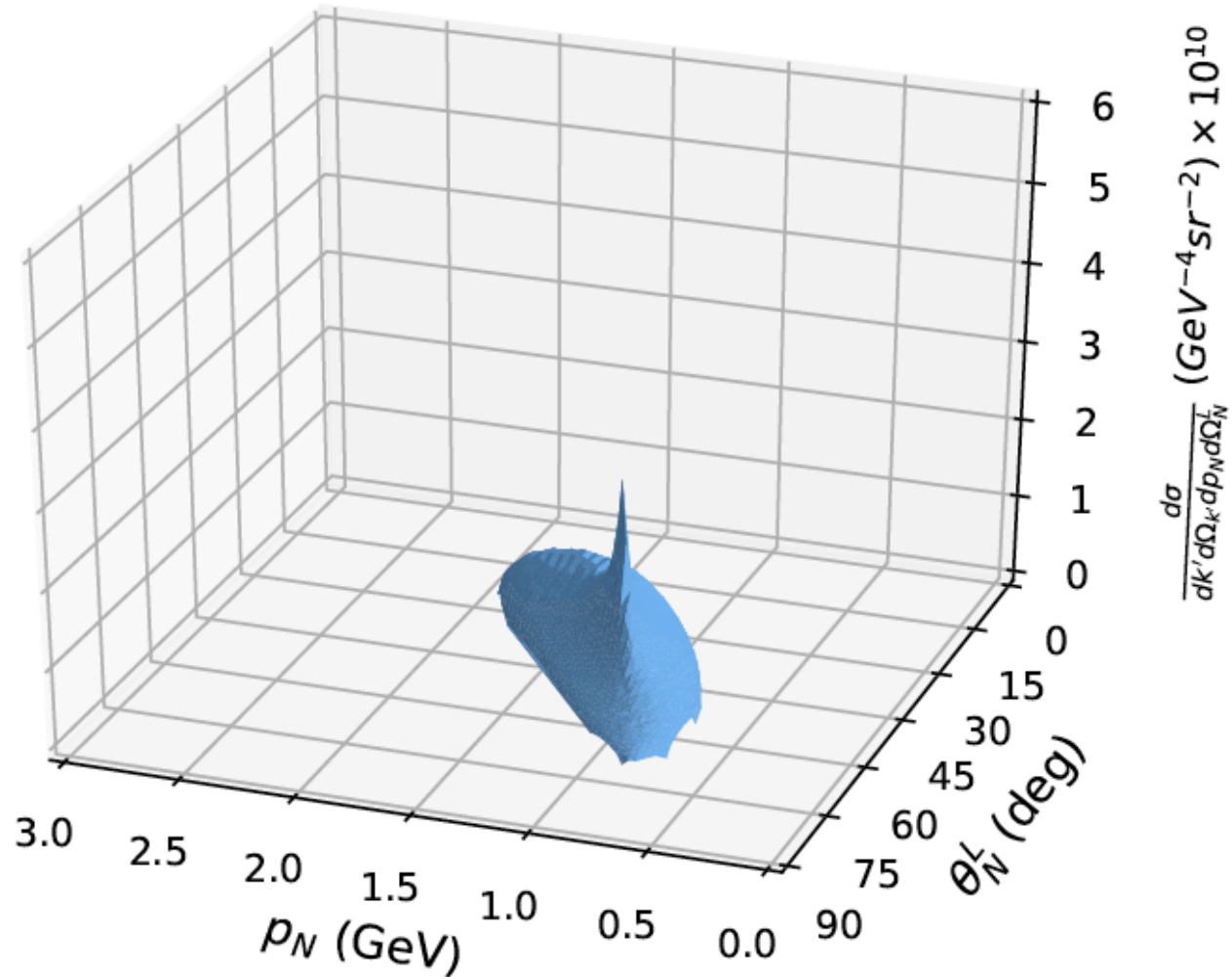
CC neutrino semi-inclusive cross section based on the spectral function from Omar

$$k' = 2.0 \text{ GeV} \quad \theta_l = 25^\circ \quad \phi_N^L = 180^\circ$$



CC neutrino semi-inclusive cross section based on the LDA (local Fermi gas) approximation

$$k' = 2.0 \text{ GeV} \quad \theta_l = 25^\circ \quad \phi_N^L = 180^\circ$$



## 6. Discussion of implications

- For inclusive scattering integrals must be done over the “landscape” whose boundaries are determined by the lepton kinematics
- Most modeling is done for inclusive reactions and there one finds that, as long as the models used are relativistic, the results are not dramatically different (see our recent analysis of T2K oxygen results)
- However, inevitably experimental studies must rely on semi-inclusive simulations, or, indeed, may want the extra hadronic information as in measurements using TPCs
- Despite the fact that inclusive relativistic modeling is reasonably robust, semi-inclusive modeling using naïve models which have been designed for studies of inclusive scattering are at best suspect
- One special case exists: the deuteron. There a delta-function line is all that is found in the “landscape” and so detecting (say) a muon and a proton yields a unique value for the neutrino energy (see a recent study by J. W. Van Orden and TWD of  $CC\nu$  reactions on heavy water)



*thank you...*





