Improved action for contact EFTs

How to simplify our life preserving renormalizability

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Université Paris-Saclay, CNRS/IN2P3, IJCLab, 91405 Orsay, France

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Improved action for contact effective field theory

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(Dated: October 25, 2023)

We present an improved action for renormalizable effective field theories (EFTs) of systems near the two-body unitarity limit. The ordering of EFT interactions is constrained, but not entirely fixed, by the renormalization group. The remaining freedom can be used to improve the theory's convergence, to simplify its applications, and to connect it to phenomenological models. We exemplify the method on a contact theory applied to systems of up to five ^{4}He atoms. We solve the EFT at LO including a subleading interaction that accounts for part of the two-body effective range. We show that the effects of such fake range can be compensated in perturbation theory at NLO, as long as the fake range is smaller or comparable to the experimental effective range. These results open the possibility of using similar improved actions for other many-body systems.

I. INTRODUCTION

https://arxiv.org/abs/2310.15760

M. Schäfer, U. van Kolck

We are talking about: renormalizability…
but first a little context.

Model:

Grasps most of physics from **simple concepts**

Easy to compute

Precise (if it is a good model)

Effective field theory:

Only require knowledge of **fundamental symmetries**

Improvable

Known **theoretical errors**

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Model + Effective field theory:

Only require knowledge of **fundamental symme**

Easy to compute

Precise (if it is a good model)

Improvable

Known **theoretical errors**

A complete theory

$r_{ij} = r_i - r_j$

$$V(r_{ij}) = \delta(r_{ij})$$

Contact theory formally:

$$L = N^\dagger \left(i\partial_0 + \frac{\hbar^2}{2m} \nabla^2 \right) N - C_0 \, N^\dagger N^\dagger N N$$

$$L^{N^{>0}LO} = C_2 \left(N^\dagger \nabla^2 N \, N^\dagger N + h.c. \right) + C_{11} \left(N^\dagger \vec{\nabla} N \, N^\dagger \vec{\nabla} N \right) +$$

$$C_4 \left(N^\dagger \nabla^4 N \, N^\dagger N + h.c. \right) + \ldots$$

$$D_0 \left(N^\dagger N^\dagger N^\dagger N N N \right) +$$

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Including all the derivative/many-body operators one can **express any interaction**

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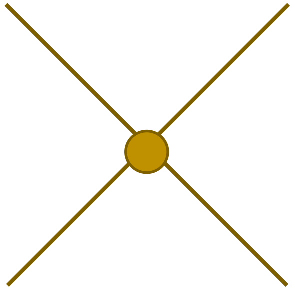

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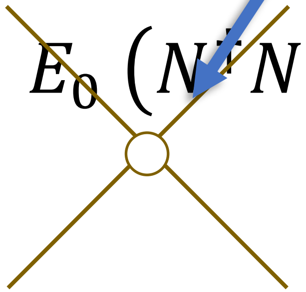
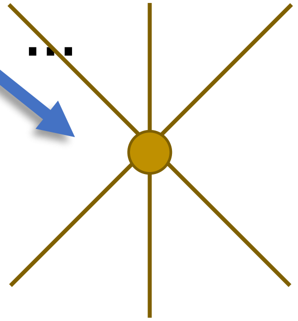
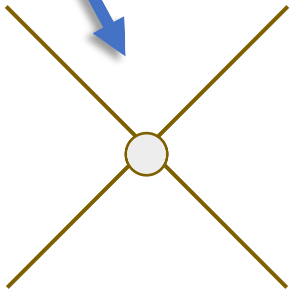
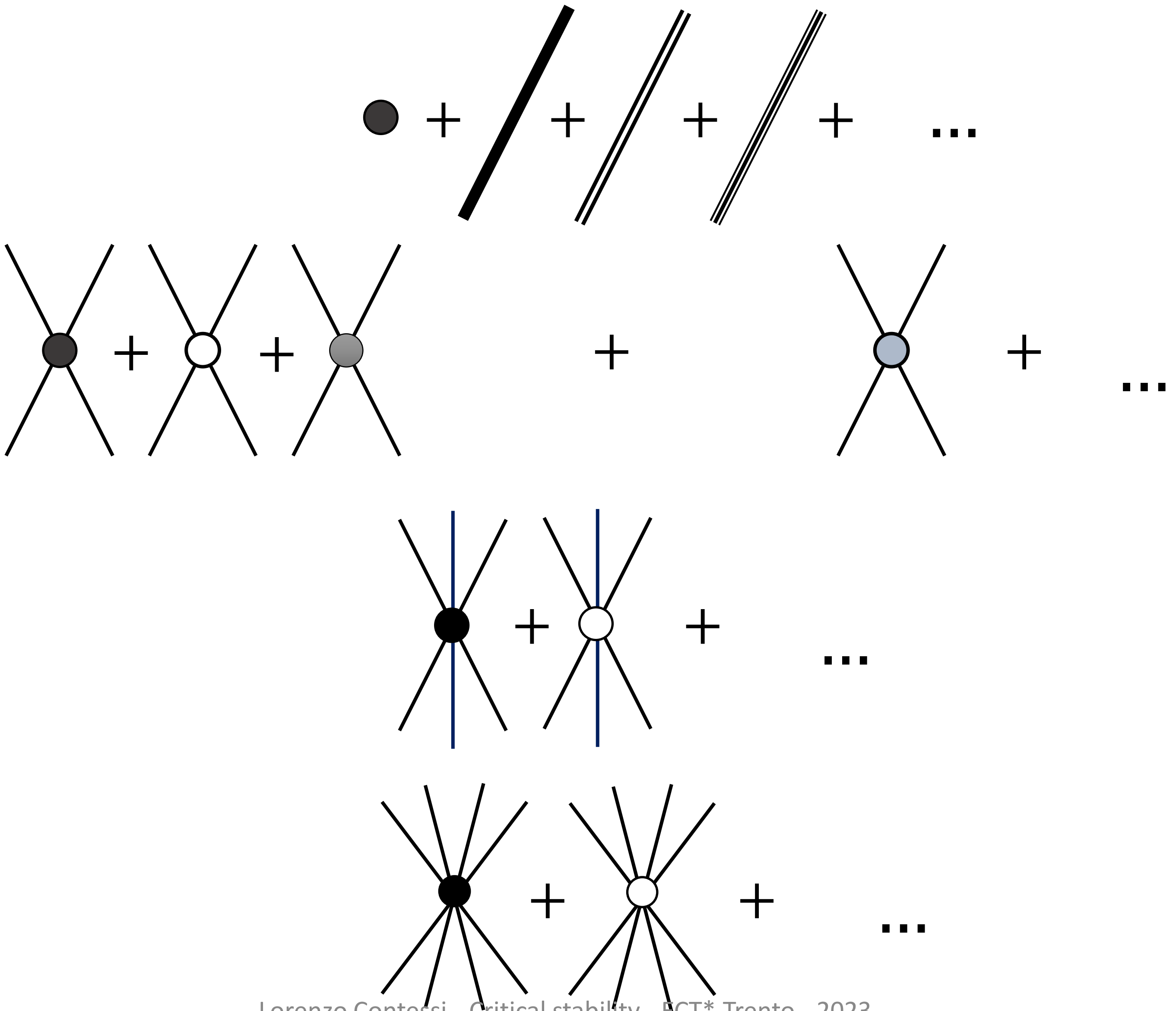






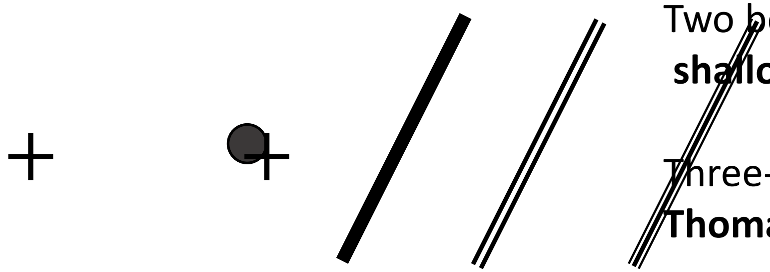
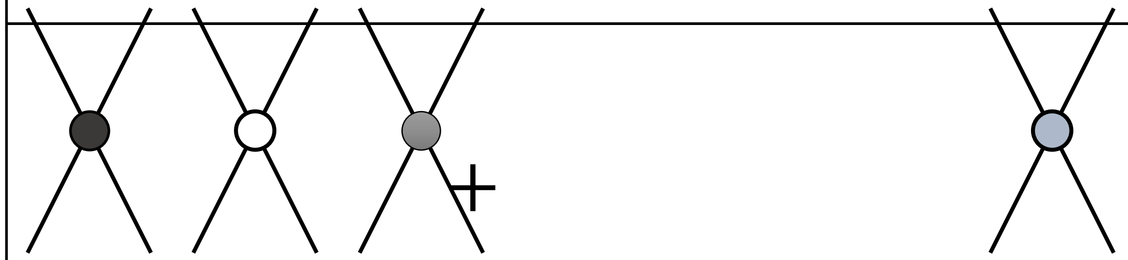
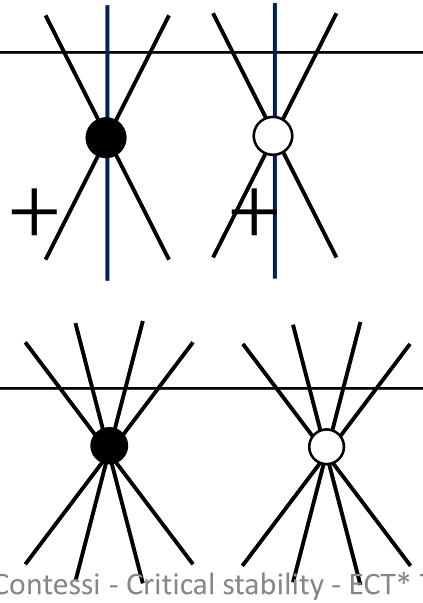
Including all the derivative/many-body operators one can **express any interaction**

Pionless EFT powercounting



U. Van Kolck - Progress in Particle and Nuclear Physics 43, 1999, 337-418

Pionless EFT powercounting

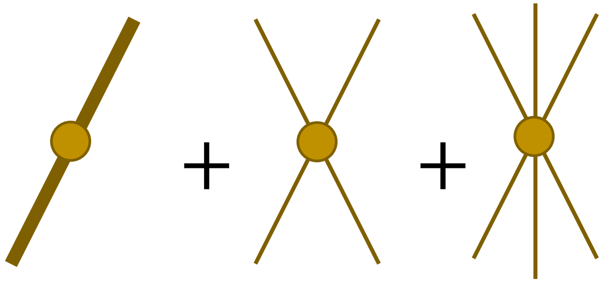
LO		Two body is needed to provide **shallow states**. Three-body needed to avoid **Thomas collapse**.
NLO		
N^2LO		$C_2 \overleftarrow{\nabla} \cdot \overrightarrow{\nabla} \sim c_2 \frac{Q^2}{M_{hi}}$ $\wr$ M_{hi}

Pionless EFT powercounting

In the nuclear case: $\Gamma_{NN} = \frac{Q}{m_\pi} = 0.5 \sim 0.8$

Momentumless 2-3 body

LO

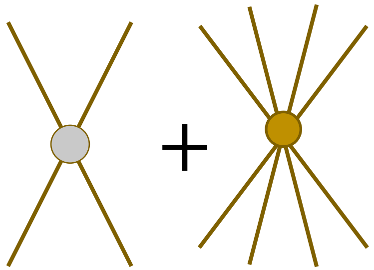


Universality

1

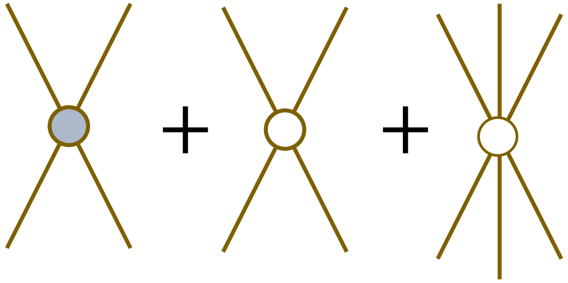
Momentum dependent / 4-body

NLO



$O(\Gamma)$

$\mathrm{N}^2\mathrm{LO}$



$O(\Gamma^2)$

$O(\Gamma^{\geq 3})$

G.P. Lepage, How to renormalize the Schrödinger equation (1997)
U. van Kolck, Nucl.Phys. A645 273-302 (1999)
J.-W. Chen, et al. Nucl.Phys. A653 (1999)
S. König, H. W. Grießhammer, H. W. Hammer, and U. van Kolck, J. Phys. G43, 055106 (2016)
B. Bazak, PRL 122.143001 (2019)

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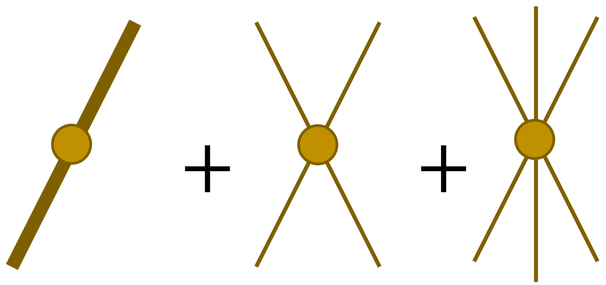

Universality

1

/ 4-body

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0\, \delta\left(\vec{r_i}, \vec{r_j}\right) + \sum_{ijk} D_0\, \delta\left(\vec{r_i}, \vec{r_j}, \vec{r_k}\right)$$

$O(\Gamma)$

Regularize the interaction

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C^\lambda\, e^{-\frac{\lambda^2 r_{ij}^2}{4}} + \sum_{ijk} D^\lambda \sum_{cyc}\left[e^{-\frac{\lambda^2\left(r_{ij}^2 + r_{ik}^2\right)}{4}}\right]$$

$O(\Gamma^2)$

Renormalization

C^λ and D^λ fitted for each cutoff λ.

If $\lambda \to \infty$ any observable becomes λ independent

$O(\Gamma^{\geq 3})$

Aren’t we satisfied with effective field theories?

Nuclear physics is not a dead field that have been completely solved 50 years ago

References:
Renormalizability issue + Energy dependency
van Kolck 2020
Unbound issue
Stetcu 2006
W. G. Dawkins 2020
M. Schafer 2021
Modified chiral
C. J. Yang 2021 (x2)
Halo
HW Hammer 2017

Well… yes and not…

EFTs were **revolutionary** for nuclear physics
But QCD – **Nuclear separation of scales** is not huge (mesons, excitations…)
And this complicates things…

Hard to find a nicely working EFT that is renormalizable

- χEFT has **renormalizability problems**;
- $\not{\pi}$EFT **do not stabilize** nuclei at LO;
- Modified χEFT seems to have the **same stabilization problem**;
- Possible solutions are **non applicable to many-body methods** (e.g. energy dependences);
- Halo-cluster EFTs are **only applicable to specific systems**.

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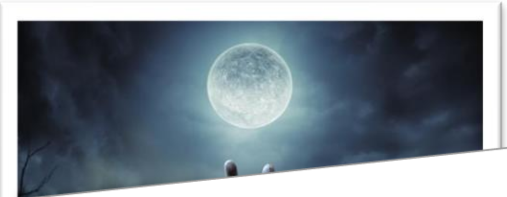

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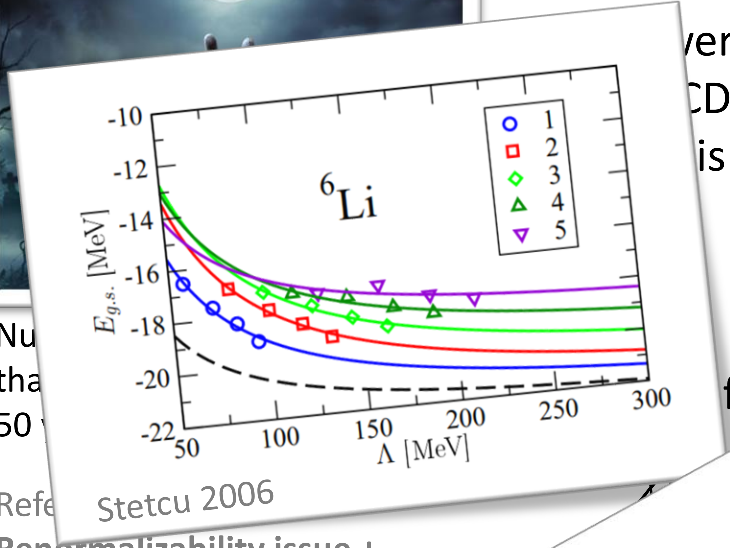
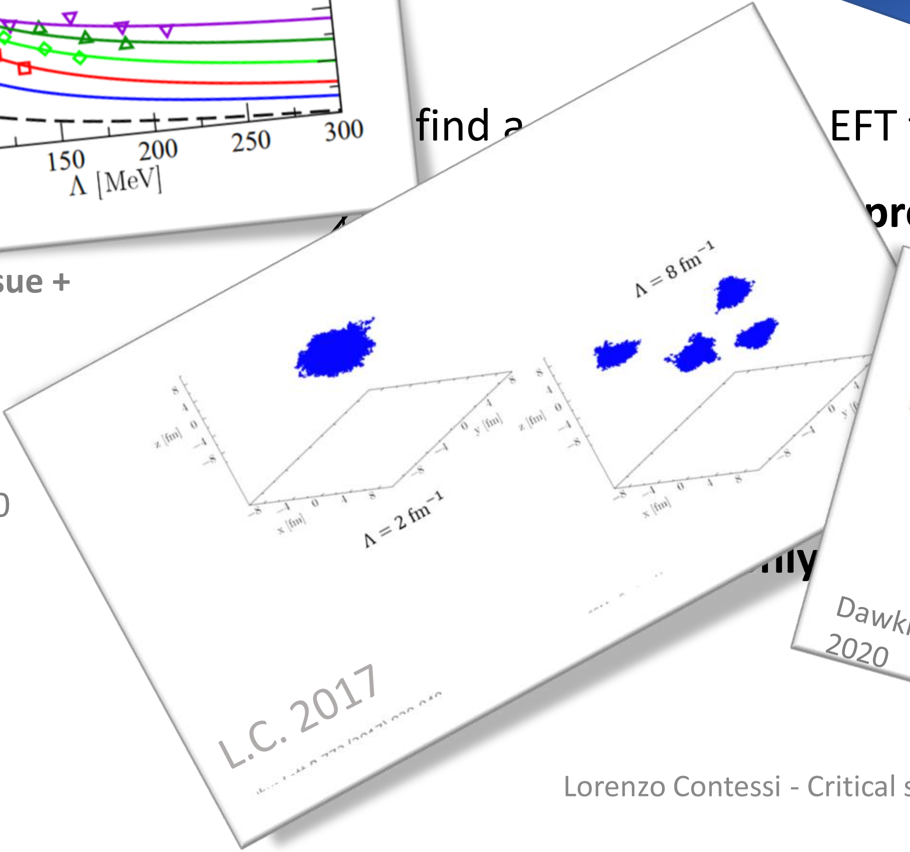


Nu

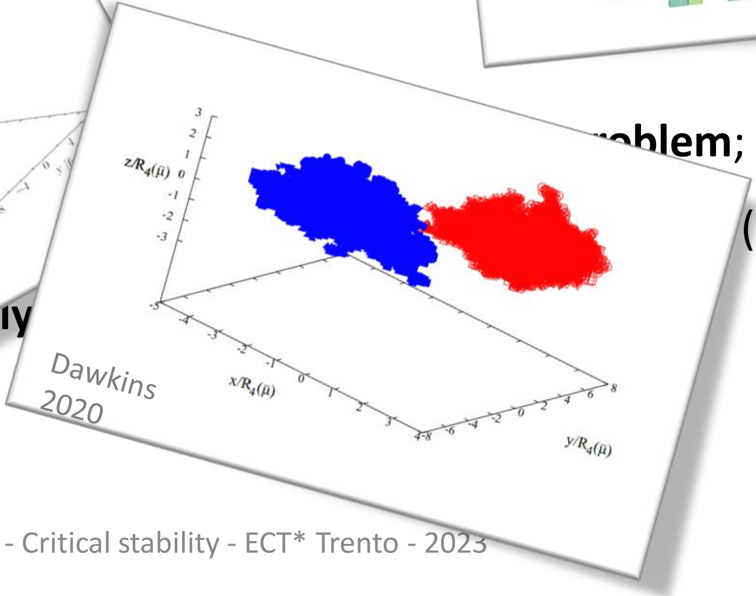
tha

50

find a … EFT that is renormaliz

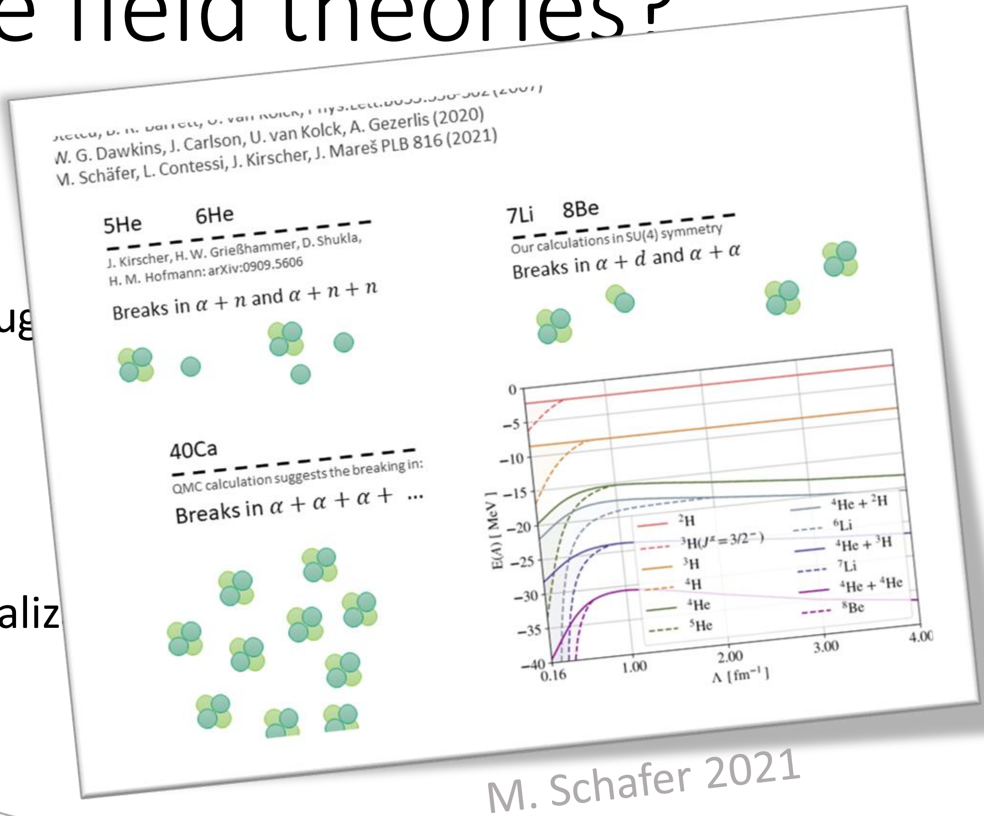


problems;



oblem;

(e.g. energy dependences);



M. Schafer 2021

Refe…

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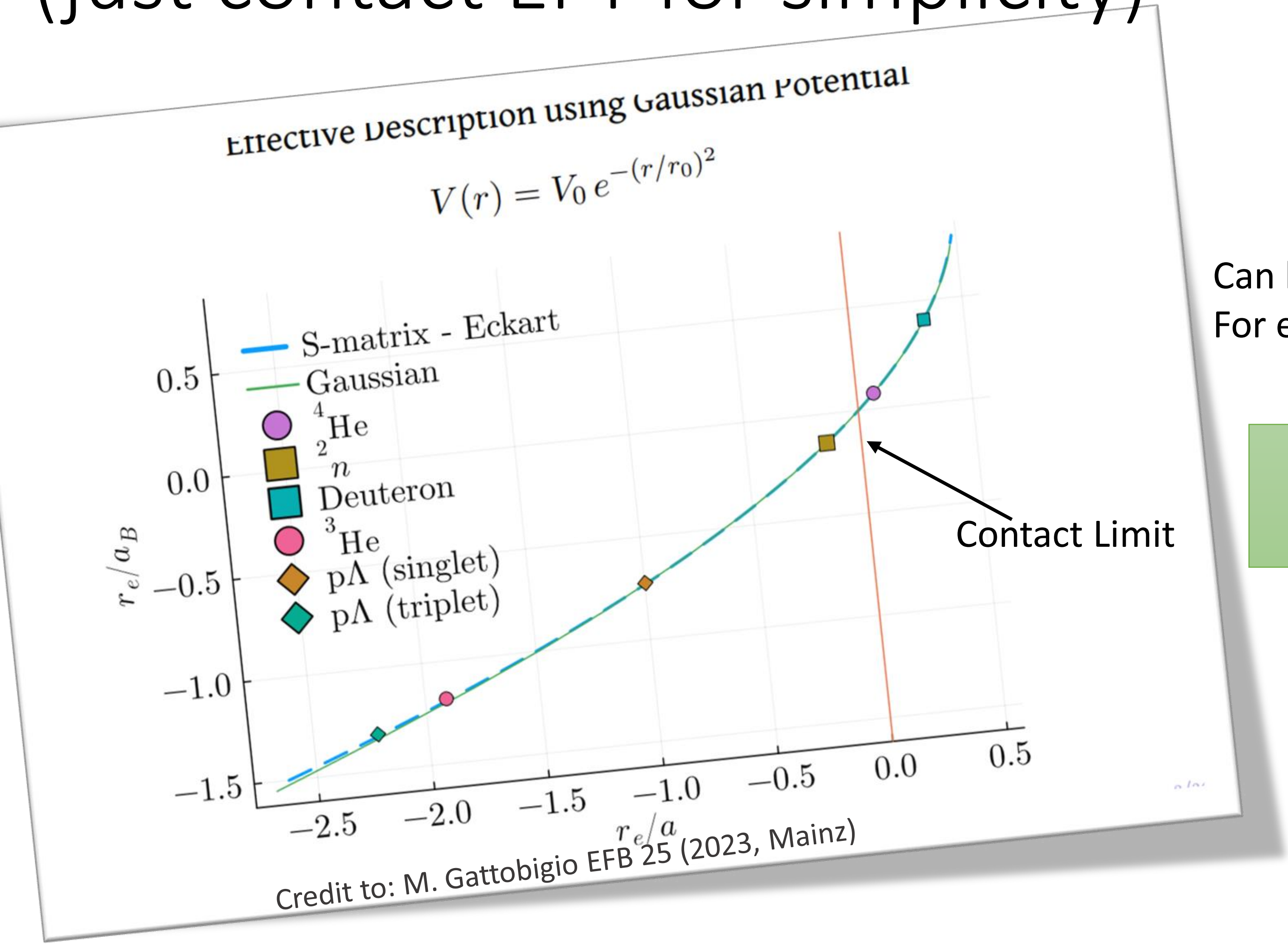
Can a perturbative subleading insertion stabilize a state?

What can we do to avoid cutoff and regulator dependence?

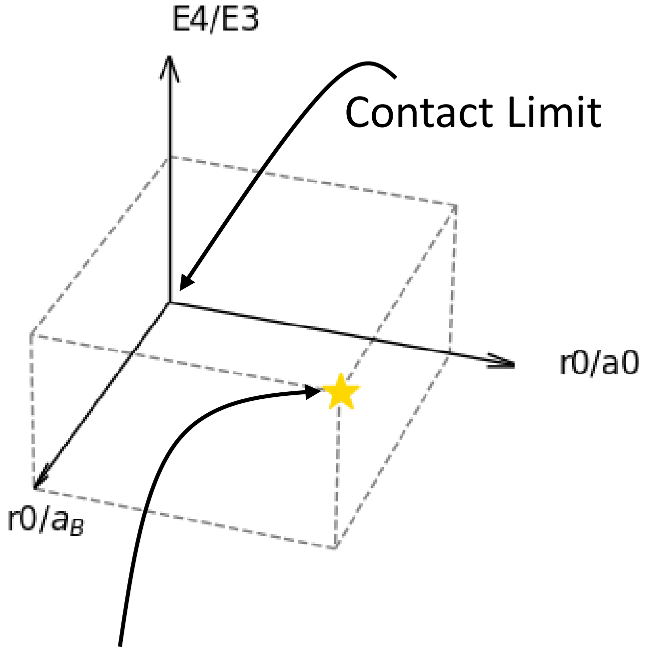
New many-body methods?

Well… nothing wrong here…

Stability problem in renormalizable theories (just contact EFT for simplicity)



Can be generalized
For each observable



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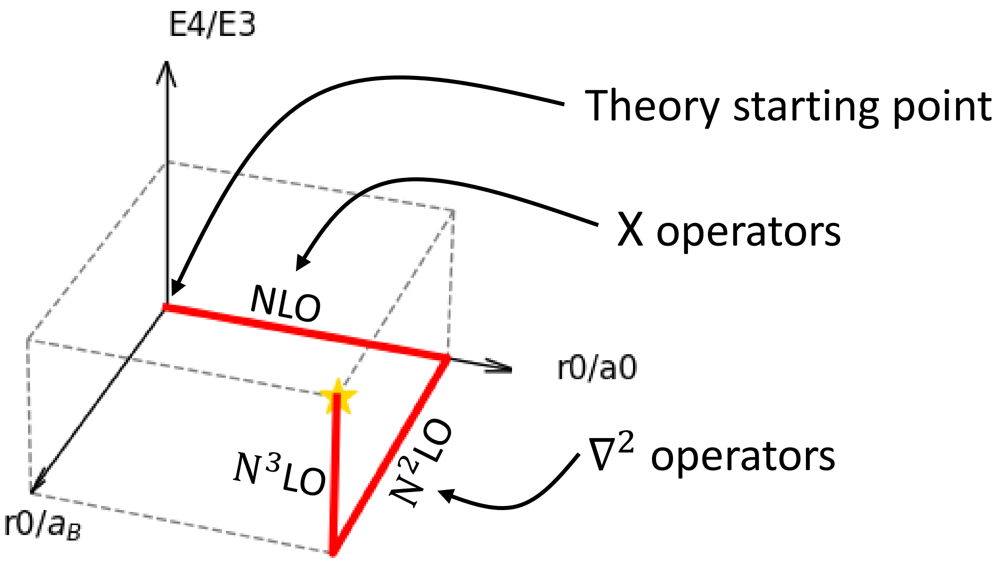
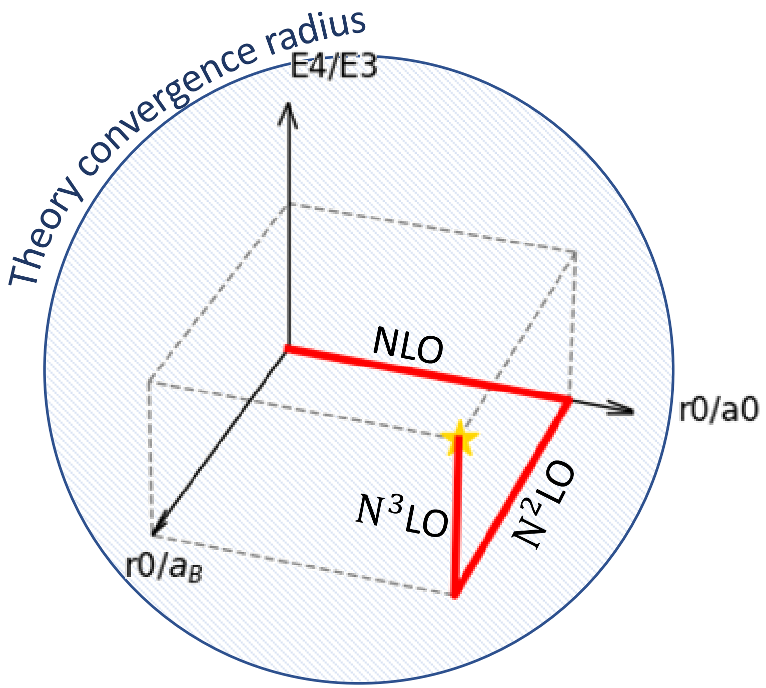


Starting from the **universal point** one can reach the **physical point** with perturbative inclusions.

Contact operators make these lines as perpendicular as possible
(Other expansions are possible)



The radius of convergence of the theory:
points reachable in this way.

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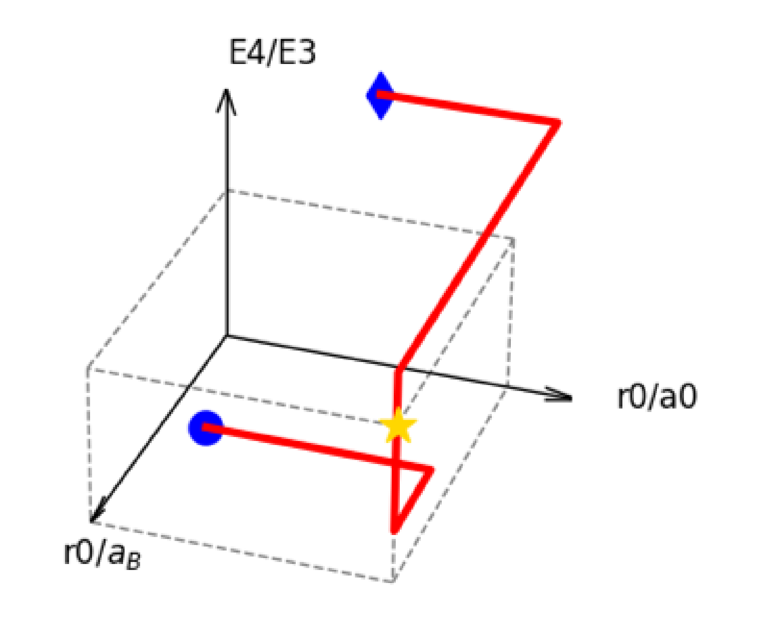
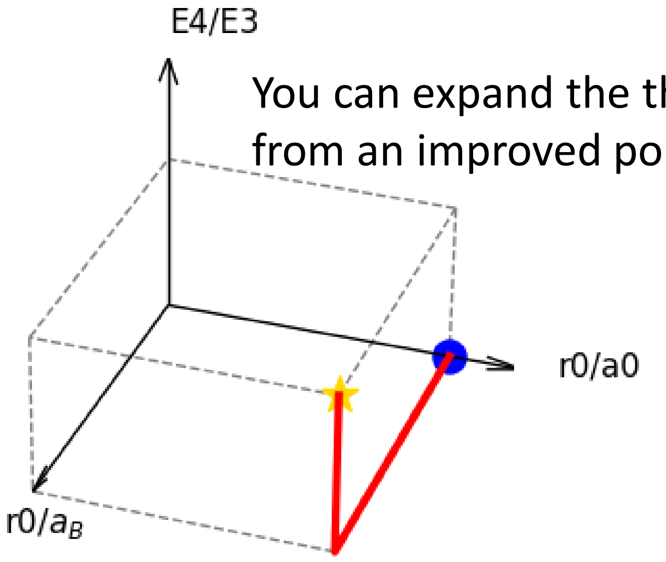




You can expand the theory from an improved point.

LO	$a_0 \to \infty$ **3B**
NLO	$a_0 < \infty$
N2LO	r_0 , **4B**
N3LO	a_1, $\mathbf{3B}p^2$
N4LO	v_2

No needed to start from the universal point.

(the expansion should not necessarily modified)

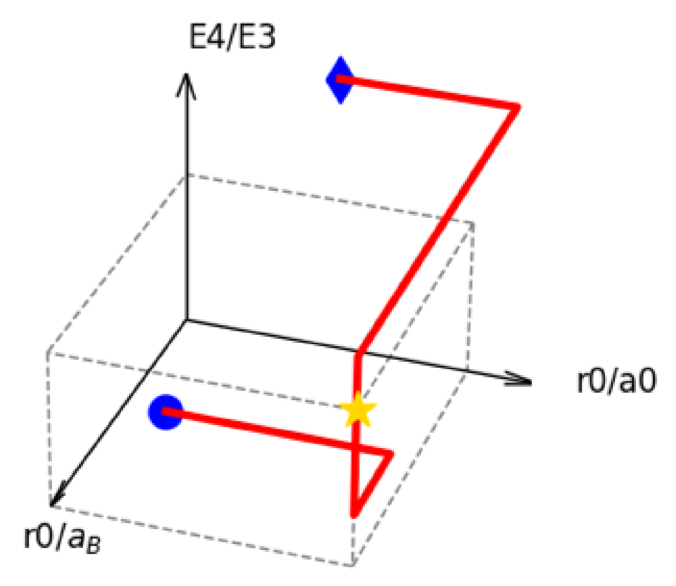
Standard improvement: treat **finite scattering length** as starting point.

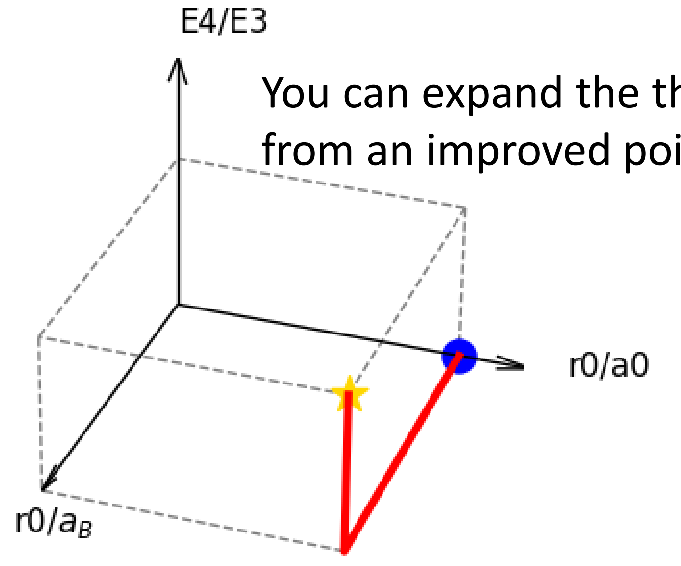
This effectively treat (resumm) **subleading** already **at LO**

doesn't change the power counting:

- The correction remain small
- The rest of the power counting is not perturbed

Stability problem in renormalizable theories (just contact EFT for simplicity)





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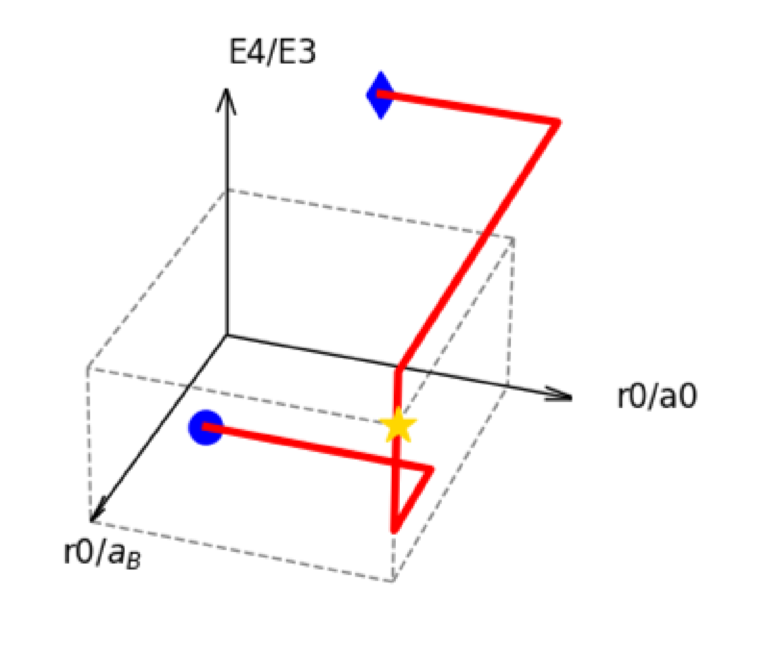

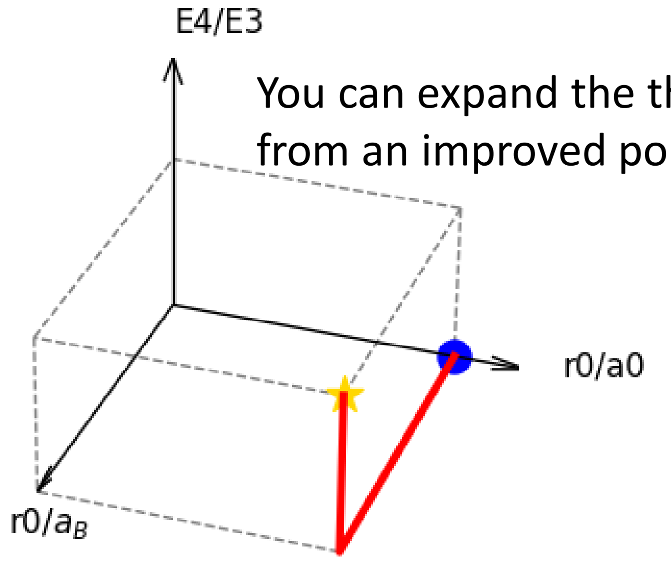

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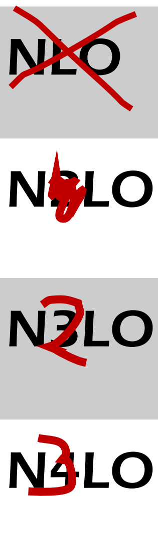

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Designing an improved action (2B)

	Potential	T-matrix	Observable described
Non perturbative	$C_0(a_0 \to \infty)\ \delta(r_{ij})$	$\frac{1}{-ik}$	Universality
Perturbative	$C_1(a_0)\delta(r_{ij})$	$\frac{1}{-ik}\left(1+\frac{\alpha}{a_0}\right)$	a_0
Perturbative	$C_2(a_0, r_0)\ \nabla^2\left(\ \delta(r_{ij})\right)$	$\frac{1}{-ik}\left(1+\frac{\beta}{a_0}-\gamma\ r_0\right)$	r_0

Designing an improved action (2B)

	Potential	T-matrix	Observable described
Non perturbative	$C_0(a_0)\,\delta(r_{ij})$	$\frac{1}{-\frac{1}{a_0}-ik}$	a_0
Perturbative	$C_2(a_0,r_0)\,\nabla^2\left(\delta(r_{ij})\right)$	$\frac{1}{-\frac{1}{a_0}-ik}\left(1+\frac{k^2 r_0}{2\left(k-i\frac{1}{a_0}\right)^2}\right)$	r_0
Perturbative	$C_2(a_0,r_0)\,\nabla^4\left(\delta(r_{ij})\right)$	$\frac{1}{-\frac{1}{a_0}-ik}(1+\alpha_1\,r_0+\beta_1\,\omega)$	ω_0

Designing an improved action (2B)

	Potential	T-matrix	Observable described
Non perturbative	$C_0(a_0)\,\delta(r_{ij})$	$\dfrac{1}{-\frac{1}{a_0}+\frac{1}{2}r_0k^2-ik}$	a_0, r_0
Perturbative	$C_2(a_0,r_0)\,\nabla^2\left(\delta(r_{ij})\right)$	It is not possible to include a contact interaction to correct the effective range (Wigner bound!)	
Perturbative	$C_2(a_0,r_0)\,\nabla^4\left(\delta(r_{ij})\right)$	$\dfrac{1}{-\frac{1}{a_0}-ik}(1+\alpha_1\,r_0+\beta_1\,\omega)$	

Designing an improved action (2B)

	Potential	T-matrix	Observable described
Non perturbative	$C_0(a_0)\,\delta(r_{ij}) + \Delta V$	$\dfrac{1}{-\frac{1}{a_0} + \frac{1}{2} r_0^* k^2 + \Delta\omega\, k^4 + \Delta\omega\, k^6 + \cdots - ik}$	a_0, r_0^*, (+ spurious components)
Perturbative	$C_2(a_0, r_0)\,\nabla^2\left(\delta(r_{ij})\right)$	$T_0\left(1 + \alpha_1(r_0 - r_0^*)\right)$	r_0
Perturbative	$C_2(a_0, r_0)\,\nabla^4\left(\delta(r_{ij})\right)$	$\dfrac{1}{-\frac{1}{a_0} - ik}\left(1 + \alpha_1(r_0 - r_0^*) + \beta_1\,\omega_0\right)$	ω_0

Hamiltonian formulation

$$\delta_\Lambda(\vec{r_i},\vec{r_j}) = e^{-\frac{\lambda^2 r_{ij}^2}{4}}$$

$$\delta\left(\vec{r_i},\vec{r_j},\vec{r_k}\right) = \sum_{cyc}\left[e^{-\frac{\lambda^2 \left(r_{ij}^2+r_{ik}^2\right)}{4}}\right]$$

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0\,\delta_\Lambda(\vec{r_i},\vec{r_j}) + \sum_{ijk} D_0\,\delta_\Lambda\left(\vec{r_i},\vec{r_j},\vec{r_k}\right)$$

Fit to $a_0, r_0 = 0$
And B_3

$$H^{NLO} = \sum_{ij} C_2\,\delta\left(\vec{r_{ij}}\right)\left(\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2\right) + \sum_{ijkl} E_0\,\delta\left(\vec{r_i},\vec{r_j},\vec{r_k},\vec{r_l}\right)$$

Fit to r_0
And B_4

U. Van Kolck (1999)
B. Bazak (2018)

Hamiltonian formulation

Improve action mechanism:
K. Symanzik (1983)

$$\delta_\Lambda(\vec{r_i},\vec{r_j}) = e^{-\frac{\lambda^2 r_{ij}^2}{4}}$$

$$\delta\,(\vec{r_i},\vec{r_j},\vec{r_k}) = \sum_{cyc}\left[e^{-\frac{\lambda^2\left(r_{ij}^2+r_{ik}^2\right)}{4}}\right]$$

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0\,\delta_\Lambda(\vec{r_i},\vec{r_j}) + \sum_{ijk} D_0\,\delta_\Lambda\,(\vec{r_i},\vec{r_j},\vec{r_k}) + \Delta V$$

Small (perturbative) auxiliary interaction

Reproduces $(a_0, r^*, \delta\omega, \delta\omega_2, \dots)$

$$H^{NLO} = \sum_{ij} C_2^*\,\delta\,(\overrightarrow{r_{ij}})\,(\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2) + \sum_{ijkl} E_0^*\,\delta\,(\vec{r_i},\vec{r_j},\vec{r_k},\vec{r_l})$$

Corrects $\mathrm{r}^* \to r_0$ and fit B_4

$$H^{N^{\geq 2}LO}$$

Corrects $\delta\omega, \delta\omega_2$

Hamiltonian formulation

$$\delta_\Lambda(\vec{r_i},\vec{r_j}) = e^{-\frac{\lambda^2 r_{ij}^2}{4}}$$

$$\delta\,(\vec{r_i},\vec{r_j},\vec{r_k}) = \sum_{cyc}\left[e^{-\frac{\lambda^2\left(r_{ij}^2+r_{ik}^2\right)}{4}}\right]$$

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0\,\delta_\Lambda(\vec{r_i},\vec{r_j}) + \sum_{ijk} D_0\,\delta_\Lambda\,(\vec{r_i},\vec{r_j},\vec{r_k}) + \Delta V$$

$$H^{NLO} = \sum_{ij} C_2^*\,\delta\,(\overrightarrow{r_{ij}})\,(\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2) + \sum_{ijkl} E_0^*\,\delta\,(\vec{r_i},\vec{r_j},\vec{r_k},\vec{r_l})$$

$$\Delta V_2 = \sum_{ij}\left(C_0^*(\bar{R}^{-1})\delta_{\bar{R}^{-1}}(\vec{r_i},\vec{r_j}) - C_0(\Lambda)\,\delta_\Lambda(\vec{r_i},\vec{r_j})\right)$$

$$\Delta V_3 = \sum_{ijk}\left(D_0^*(\bar{R}^{-1})\,\delta_{\bar{R}^{-1}}\,(\vec{r_i},\vec{r_j},\vec{r_k}) - D_0(\Lambda)\delta_\Lambda\,(\vec{r_i},\vec{r_j},\vec{r_k})\right)$$

Hamiltonian formulation

$$\delta_\Lambda(\vec{r_i},\vec{r_j}) = e^{-\frac{\lambda^2 r_{ij}^2}{4}}$$

$$\delta\,(\vec{r_i},\vec{r_j},\vec{r_k}) = \sum_{cyc}\left[e^{-\frac{\lambda^2 (r_{ij}^2+r_{ik}^2)}{4}}\right]$$

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$$H^{NLO} = \sum_{ij} C_2^*\,\delta\,(\overrightarrow{r_{ij}})\,(\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2) + \sum_{ijkl} E_0^*\,\delta\,(\vec{r_i},\vec{r_j},\vec{r_k},\vec{r_l})$$

Auxiliary interaction contains a lot of contributions but has no renormalizability problems

Can also be a phenomenological interaction!

$$\Delta V_2 = \sum_{ij}\left(C_0^*(\bar{R}^{-1})\delta_{\bar{R}^{-1}}(\vec{r_i},\vec{r_j}) - C_0(\Lambda)\,\delta_\Lambda(\vec{r_i},\vec{r_j})\right)$$

$$\Delta V_3 = \sum_{ijk}\left(D_0^*(\bar{R}^{-1})\,\delta_{\bar{R}^{-1}}\,(\vec{r_i},\vec{r_j},\vec{r_k}) - D_0(\Lambda)\delta_\Lambda\,(\vec{r_i},\vec{r_j},\vec{r_k})\right)$$

Hamiltonian formulation

$$\delta_\Lambda(\vec{r_i},\vec{r_j}) = e^{-\frac{\lambda^2 r_{ij}^2}{4}}$$

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Option 1:

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0^*(\bar{R}^{-1})\delta_{\bar{R}^{-1}}(\vec{r_i},\vec{r_j}) + \sum_{ijk} D_0\,\delta_\Lambda\,(\vec{r_i},\vec{r_j},\vec{r_k})$$

Option 2:

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0^*(\bar{R}^{-1})\delta_{\bar{R}^{-1}}(\vec{r_i},\vec{r_j}) + \sum_{ijk} D_0^*(\bar{R}^{-1})\,\delta_{\bar{R}^{-1}}\,(\vec{r_i},\vec{r_j},\vec{r_k})$$

See also:
P. Recchia 2022

Subleading orders remain untouched:

$$H^{NLO} = \sum_{ij} C_2^*\,\delta\,(\vec{r_{ij}})\,(\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2) + \sum_{ijkl} E_0^*\,\delta\,(\vec{r_i},\vec{r_j},\vec{r_k},\vec{r_l})$$

Do not forget the four-body force!

Hamiltonian formulation

$$\delta_\Lambda(\vec{r_i},\vec{r_j}) = e^{-\frac{\lambda^2 r_{ij}^2}{4}}$$

$$\delta\left(\vec{r_i},\vec{r_j},\vec{r_k}\right) = \sum_{cyc}\left[e^{-\frac{\lambda^2\left(r_{ij}^2+r_{ik}^2\right)}{4}}\right]$$

Option 1:

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0^*(\bar{R}^{-1})\, e^{-\frac{r_{ij}^2}{4\bar{R}^2}} + \sum_{ijk} D_0\, \delta_\Lambda\left(\vec{r_i},\vec{r_j},\vec{r_k}\right)$$

Option 2:

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0^*(\bar{R}^{-1})\, e^{-\frac{r_{ij}^2}{4\bar{R}^2}} + \sum_{ijk} D_0^*(\bar{R}^{-1}) \sum_{cyc}\left[e^{-\frac{\left(r_{ij}^2+r_{ik}^2\right)}{4\bar{R}^2}}\right]$$

Subleading orders remain untouched:

$$H^{NLO} = \sum_{ij} C_2^*\, \delta\left(\vec{r_{ij}}\right)\left(\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2\right) + \sum_{ijkl} E_0^*\, \delta\left(\vec{r_i},\vec{r_j},\vec{r_k},\vec{r_l}\right)$$

See similarities with: R. Schiavilla (2021)

(but also notice that **the effective range is still subleading**!)

Test:

4He atoms up to 5 particles

D. Blume and C. H. Greene, Monte carlo hyperspherical description of helium cluster excited states, The Journal of Chemical Physics **112**, 8053 (2000), https://doi.org/10.1063/1.481404.

A. R. Janzen and R. A. Aziz, Modern he–he potentials: Another look at binding energy, effective range theory, retardation, and efimov states, The Journal of Chemical Physics **103**, 9626 (1995), https://doi.org/10.1063/1.469978.

E. A. Kolganova, A. K. Motovilov, and W. Sandhas, Scattering length of the helium-atom–helium-dimer collision, Phys. Rev. A **70**, 052711 (2004).

R. Lazauskas and J. Carbonell, Description of ^{4}He tetramer bound and scattering states, Phys. Rev. A **73**, 062717 (2006).

E. Hiyama and M. Kamimura, Variational calculation of 4He tetramer ground and excited states using a realistic pair potential, Phys. Rev. A **85**, 022502 (2012), arXiv:1111.4370 [physics.atom-ph].

	PCKLJS	LM2M2
a_2 [Å]	90.42(92)	100.23
r_2 [Å]	7.27	7.326
B_2 [mK]	1.3094	1.6154
B_3 [mK]	131.84	126.50
B_3^* [mK]	2.6502	2.2779
B_4 [mK]	573.90	559.22
B_5 [mK]	-	1306.7

R. A. Aziz and M. J. Slaman, An examination of ab-initio results for the helium potential energy curve, The Journal of Chemical Physics **94**, 8047 (1991), https://pubs.aip.org/aip/jcp/article-pdf/94/12/8047/9734055/8047_1_online.pdf.

M. Przybytek, W. Cencek, J. Komasa, G. Łach, B. Jeziorski, and K. Szalewicz, Relativistic and quantum electrodynamics effects in the helium pair potential, Phys. Rev. Lett. **104**, 183003 (2010).

R. E. Grisenti, W. Schöllkopf, J. P. Toennies, G. C. Hegerfeldt, T. Kohler, and M. Stoll, Determination of the Bond Length and Binding Energy of the Helium Dimer by Diffraction from a Transmission Grating, Phys. Rev. Lett. **85**, 2284 (2000).

M. Kunitski *et al.*, Observation of the Efimov state of the helium trimer, Science **348**, 551 (2015), arXiv:1512.02036 [physics.atm-clus].

S. Zeller *et al.*, Imaging the He_2 quantum halo state using a free electron laser, Proc. Nat. Acad. Sci. **113**, 4651 (2016), arXiv:1601.03247 [physics.atom-ph].

Few-body sector (LO)

LO	$\widetilde{V}_I = \delta_{\overline{R}^{-1}}(r_{ij}) + \delta_{\Lambda}(r_{ijk})$
LO	$\widetilde{V}_{II} = \delta_{\overline{R}^{-1}}(r_{ij}) + \delta_{\overline{R}^{-1}}(r_{ijk})$
NLO	$\nabla^2 \delta_{\Lambda}(r_{ij})$

$\overline{R}^{-1}$ is the parameter that controls the resummation

Λ is the theory cutoff that should go to "infinity"

3B excited state

Relevant fake ranges $\overline{R}$: $\overline{R} > 6\ Q_3$

$$Q_3 = \sqrt{\frac{2}{3} m\ B_3}$$

Legend:

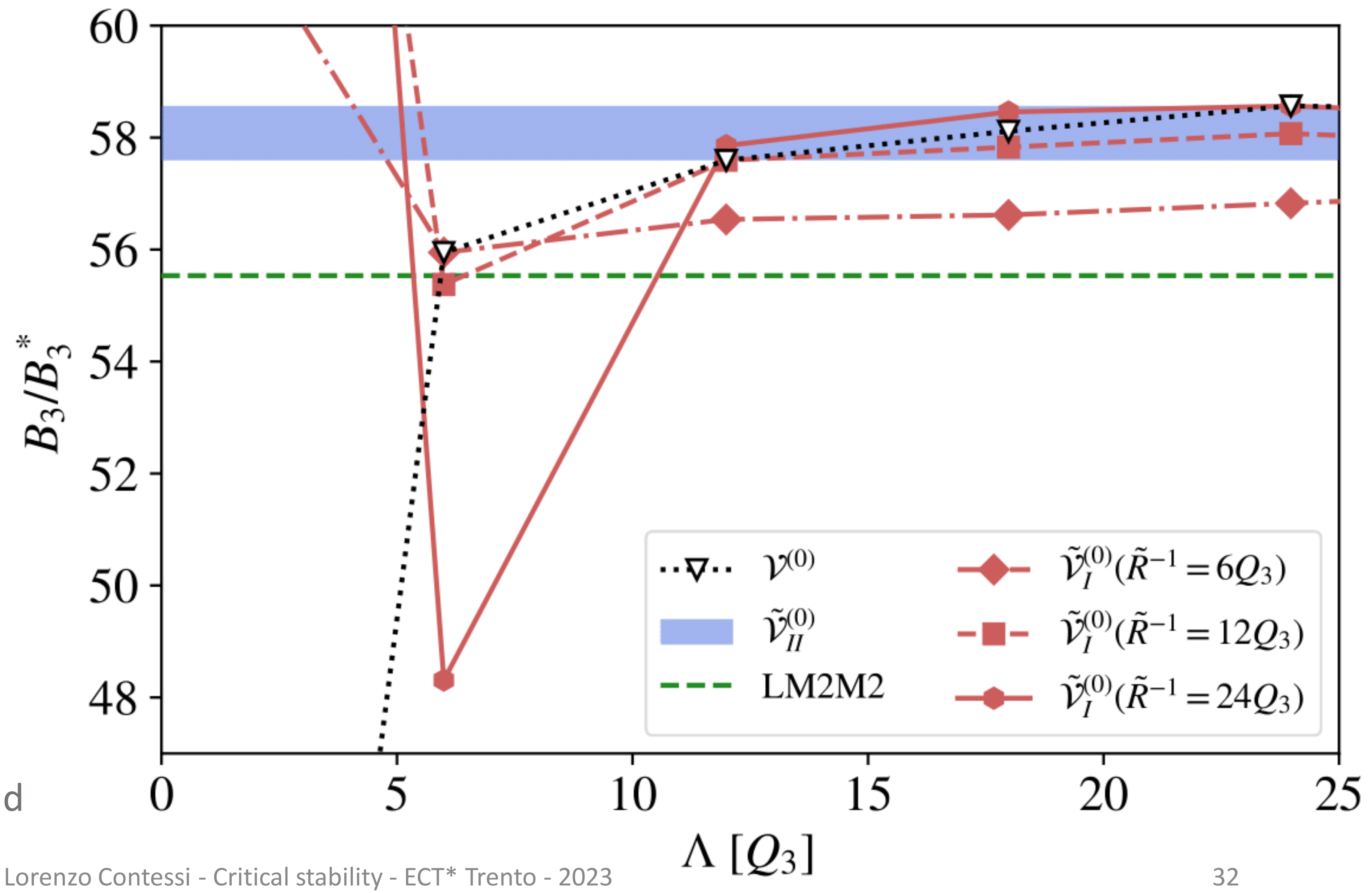
White triangles: the regular LO

Red lines: improved LO w/ 2Body

Blue band: improved LO 2+3 Body

Green Line (white circle): physical value

Vertical dashed line represents the r_0 treshold



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4B ground state

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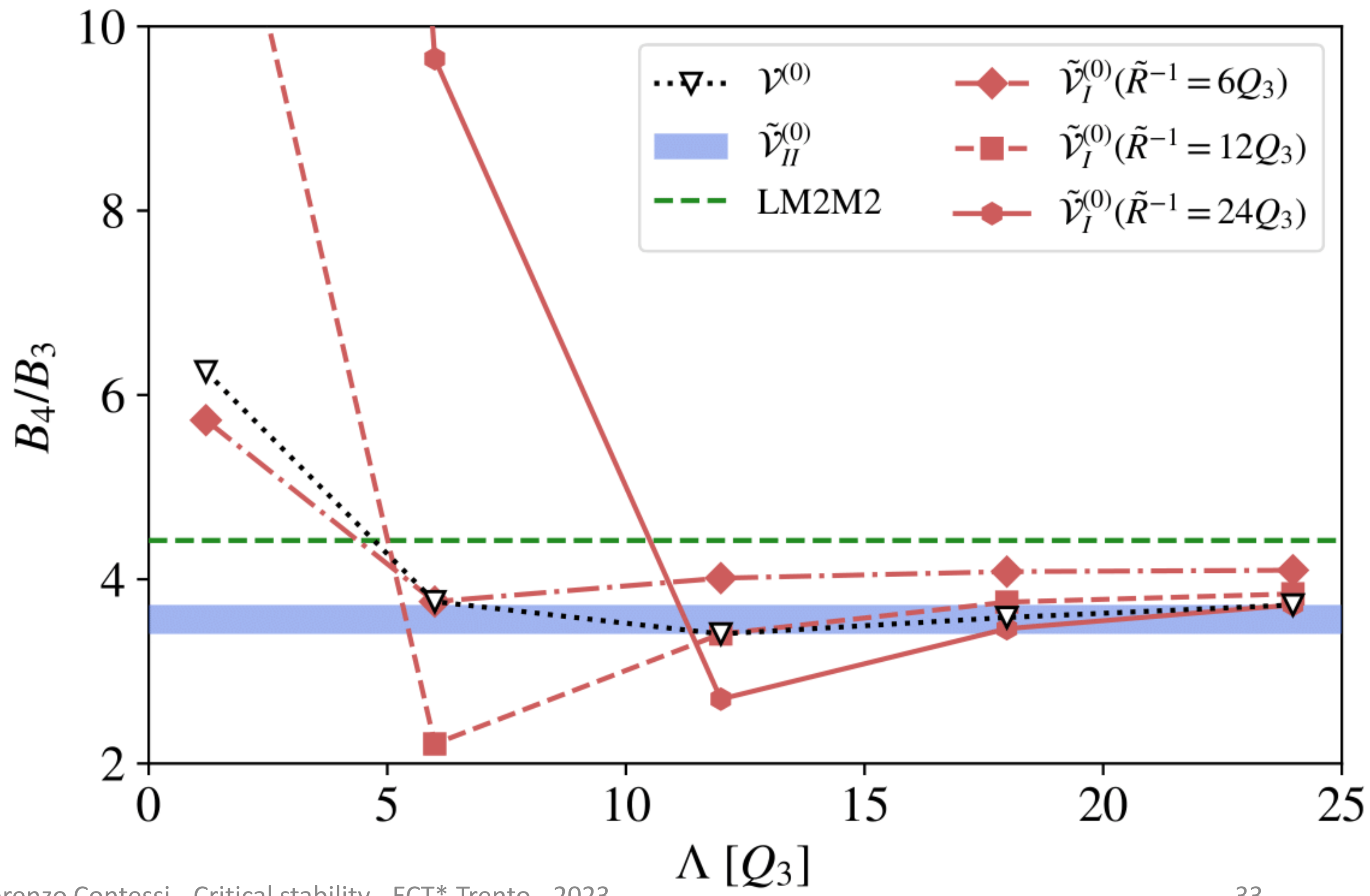

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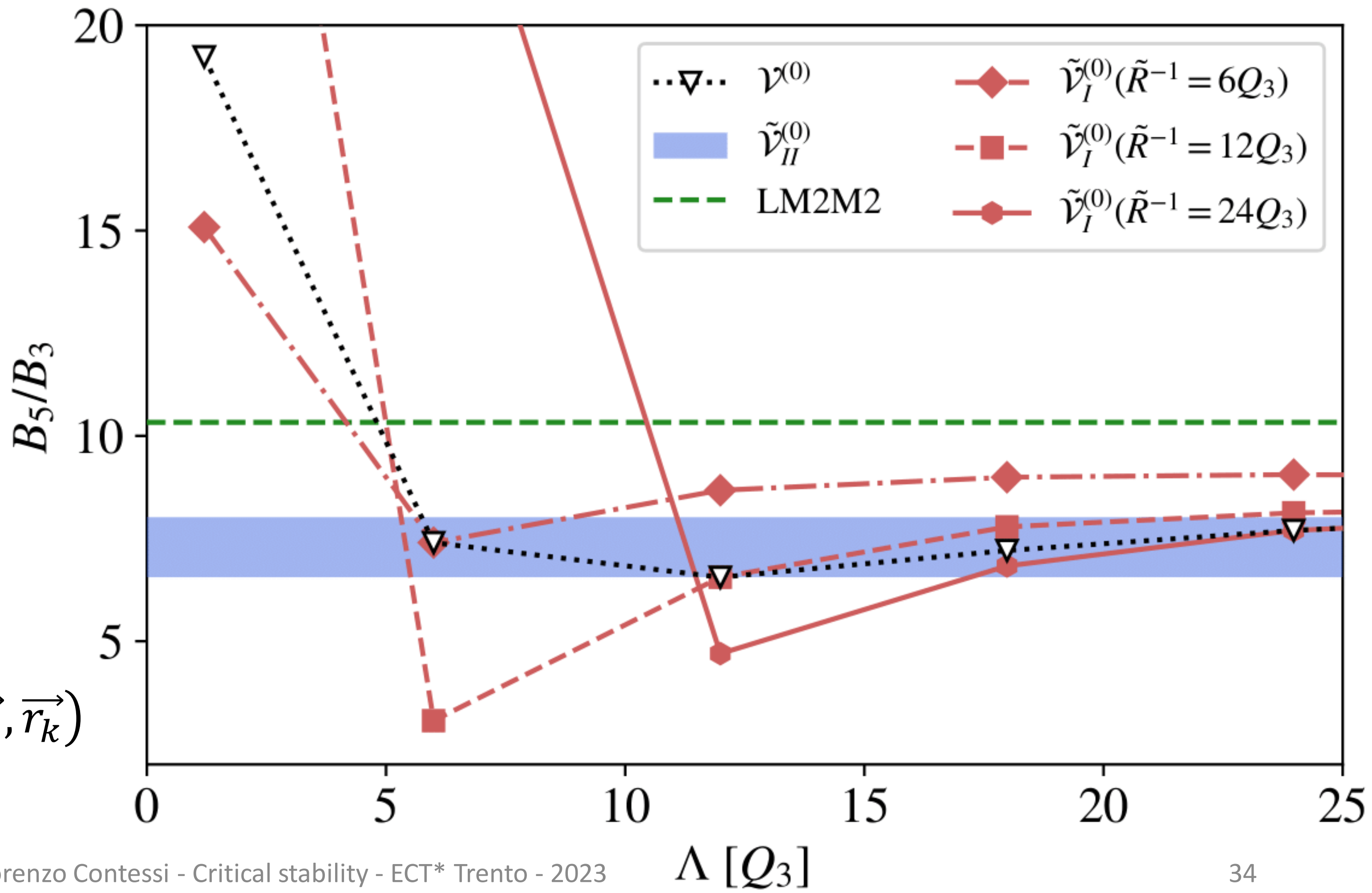
NLO	$\nabla^2\delta_{\Lambda}(r_{ij})$

5B ground state

Relevant fake ranges $\bar{R}$: $\bar{R} > 6\ Q_3$

$$Q_3 = \sqrt{\frac{2}{3} m\ B_3}$$

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0^*(\overline{R}^{-1})\ e^{-\frac{r_{ij}^2}{4\overline{R}^2}} + \sum_{ijk} D_0\ \delta_\Lambda\left(\vec{r_i}, \vec{r_j}, \vec{r_k}\right)$$



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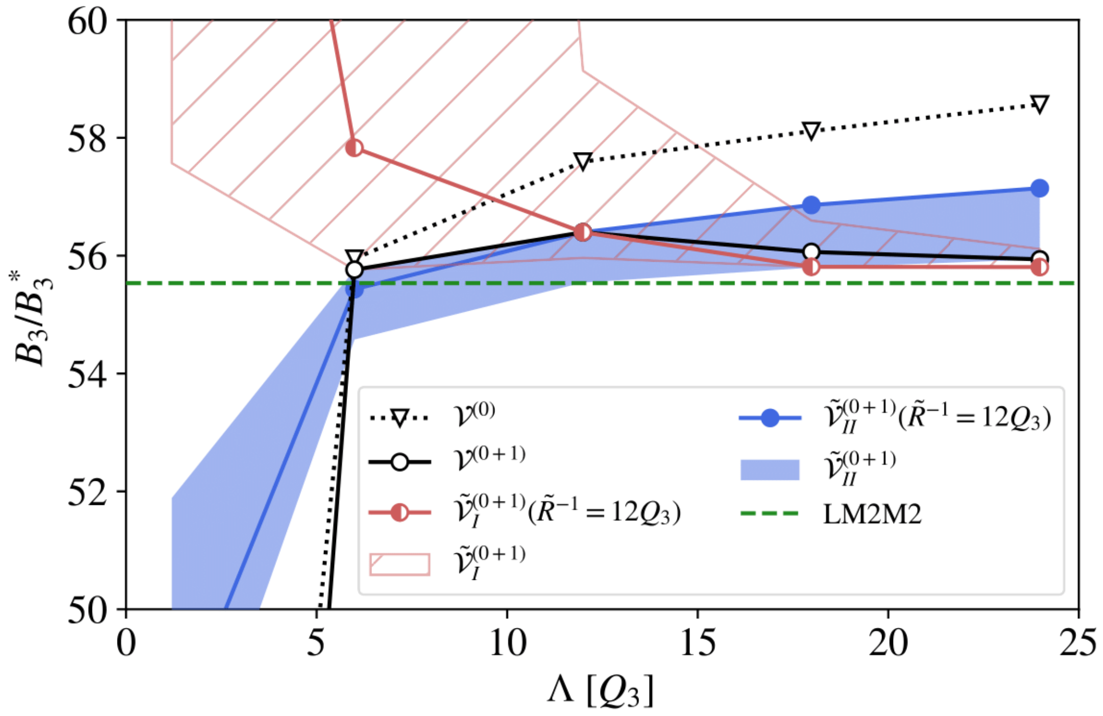

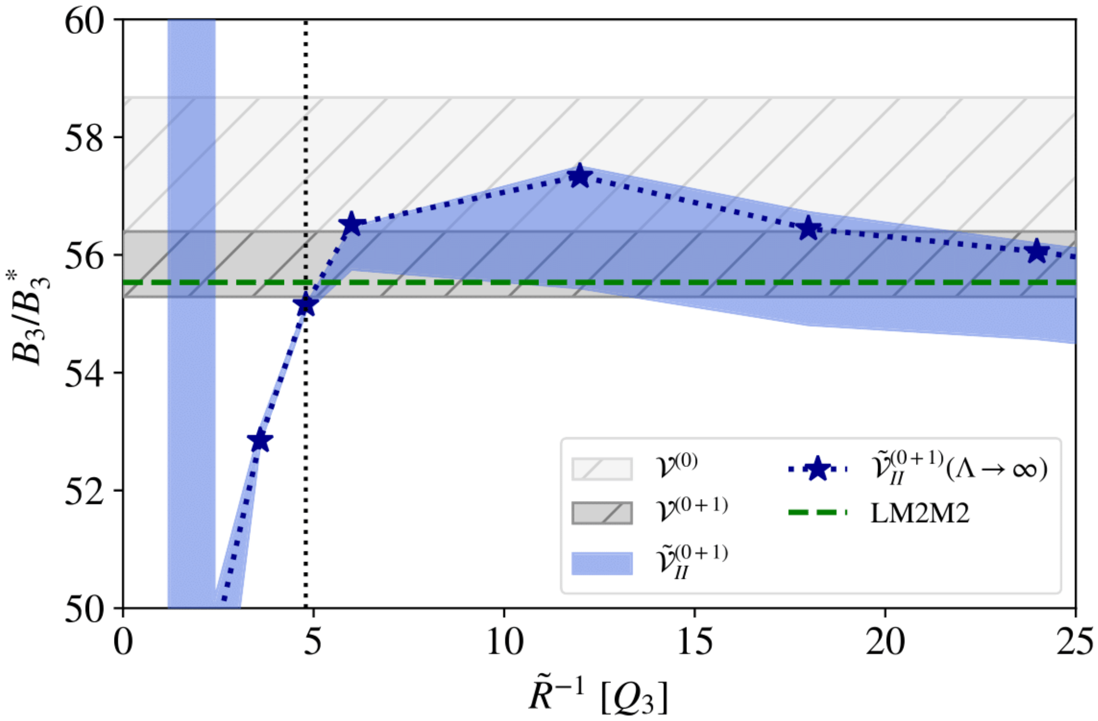

3B excited state

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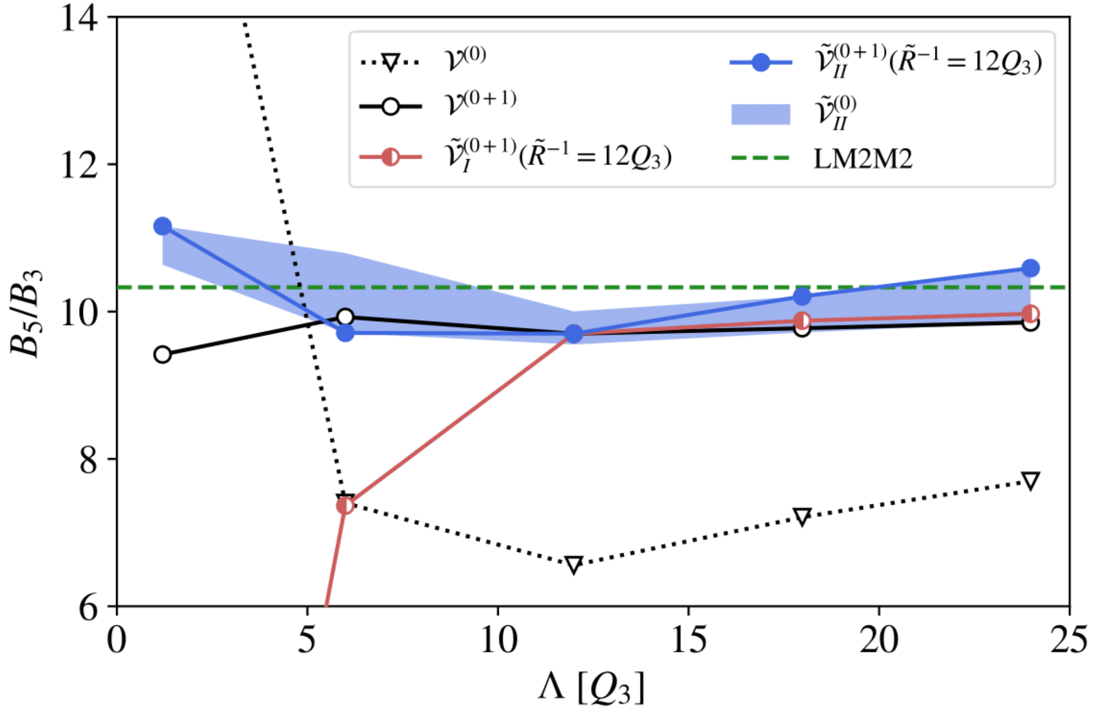

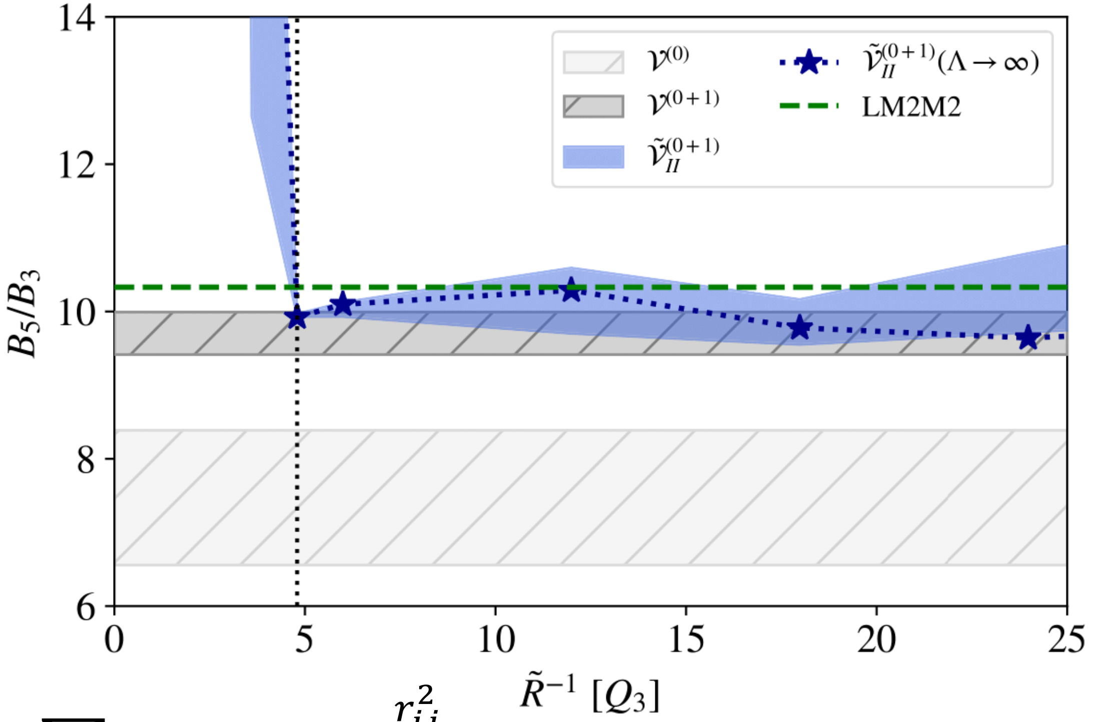

5B ground state

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0^*(\bar{R}^{-1})\, e^{-\frac{r_{ij}^2}{4\bar{R}^2}} + \sum_{ijk} D_0^*(\bar{R}^{-1}) \sum_{cyc}\left[e^{-\frac{(r_{ij}^2+r_{ik}^2)}{4\bar{R}^2}}\right]$$

Relevant fake ranges $\bar{R}$: $\bar{R} > 6\ Q_3$

$Q_3 = \sqrt{\frac{2}{3} m B_3}$

Conclusions (Just two slides left)

- **Resum and treat exactly subleading contributions**
  without Wigner bound/renormalizability issues.

- **Powercounting not affected** and the auxiliary interaction
  it is “reabsorbed” order by order.

- Renormalization is preserved
  check it by **perturbative insertion of the next order** (do it!).

- **NLO results in ^{4}He are promising** (and expandible).

- **Convergence of the powercounting** is the only limit
  for what can be resumed (check it!).

Discussion, applications, perspectives

Model to EFT
improvement

Simplify Many-body calculations
Without compromise renormalizability
(error control and renormalizability)

→

Minimize **problematic operators**:
Many-body forces, stiff potentials
?SRG – ladder resummation?

Add **entire orders** non-perturbatively?
(It might be problematic to “improve” N-body forces)

Thank you for the endurance and attention