ECT*
EUROPEAN CENTRE
FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

FONDAZIONE
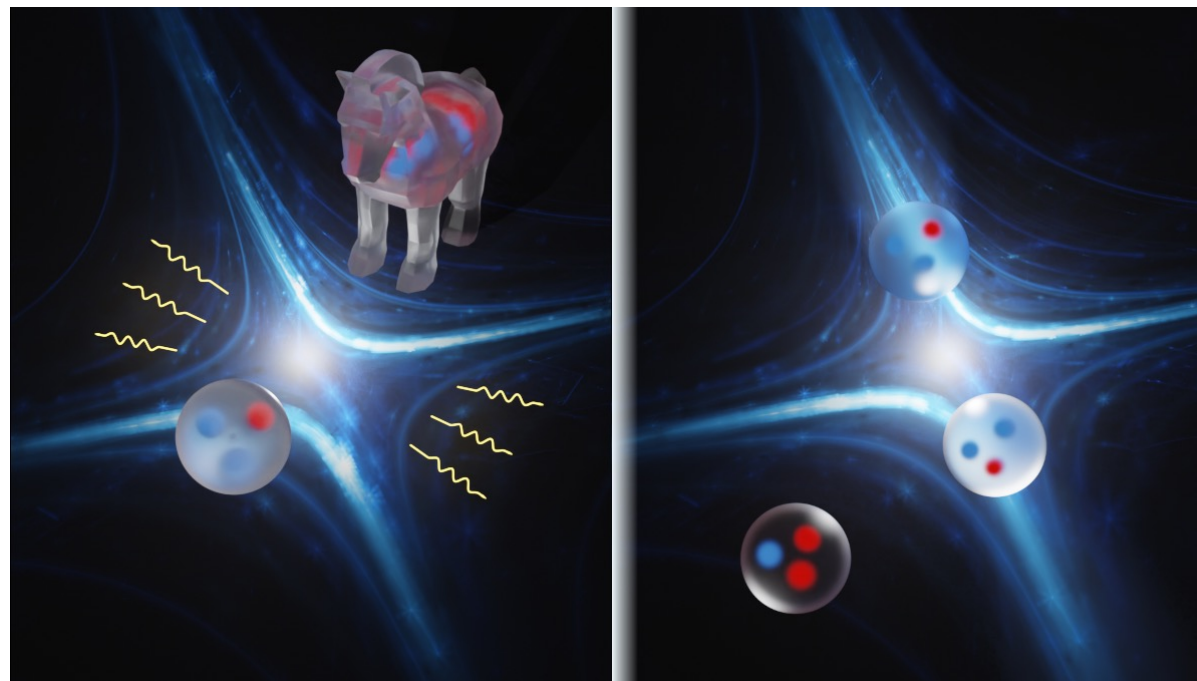BRUNO KESSLER

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Test of the charge symmetry hypothesis of NN interaction from the Coulomb-free p-p scattering cross section and its relation to universality

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Charge independence and charge symmetry

After removing the electromagnetic interactions, the NN force between nn, np, pp are almost the same

Charge independence: equality between pp/nn force and np force

Violation: associated to the mass difference between charged and neutral pions

(identical nucleons exchange a neutral pion, a neutron and a proton may exchange both a neutral and a charged pion)

Charge symmetry: equality between pp and nn forces

Charge symmetry breaking: mainly attributed to the up-down quark mass difference

Its validity is supported to some extent by an approximate equality of binding energies of isobar nuclei.

The charge symmetry breaking of the nucleon-nucleon interaction seems to be manifested in the s-wave scattering lengths, a_{NN} that determine the low-energy behavior of NN scattering.

a_{np} directly determined from experiments

a_{pp} not directly accessible from experiments because of Coulomb effects → need to remove them theoretically to reveal the strong interaction contribution to the scattering length

a_{nn} not directly accessible from experiments because of the absence of neutron targets.

However, the knowledge of these quantities, will help distinguish between the n-n and p-p interactions.

For a_{pp} Coulomb corrections are large: the uncorrected p-p scattering length using available p-p scattering world data at p laboratory energies below 30 MeV is quoted as −7.8063 ± 0.0026 fm (Piarulli et al. PRC 2015).

The removal of electromagnetic interactions is model dependent: the outcome as low as −14.9 ± 0.3 fm (from Ando et al PRC 2007) or up to −17.5 ± 0.3 fm(Walzl et al., PRC 2001).

The scatter of about 2.5 fm can be hazarded as an estimate of the systematic uncertainty coming from theory.

Moreover, by comparing the relative uncertainties before and after the correction, we can conclude that though still very low, the model corrections bring an increase in the relative uncertainty of almost two orders of magnitude.

current accepted values (Machleidt, R. & Slaus, JPG 2001)

$a_{np} = -23.74 \pm 0.02$ fm

$a^{N}_{pp} = -17.3 \pm 0.4$ fm

$a^{N}_{nn} = -18.9 \pm 0.4$ fm

Notice: additional model dependence due to the **arbitrary exclusion of the short-range electromagnetic contributions**

Why don't including all the short-range contributions?

Innovative way:

Coulomb-free $^{1}S_{0}$ $p-p$ scattering length from the quasi-free $p + d \rightarrow p + p + n$ reaction and its relation to universality

Aurora Tumino ✉, Giuseppe G. Rapisarda, Marco La Cognata, Alessandro Oliva, Alejandro Kievsky, Carlos A. Bertulani, Giuseppe D'Agata, Mario Gattobigio, Giovanni L. Guardo, Livio Lamia, Dario Lattuada, Rosario G. Pizzone, Stefano Romano, Maria L. Sergi, Roberta Spartá & Michele Viviani

Proton-proton scattering

The simplest nuclear case where Coulomb + nuclear scattering co-exist and are coherent

→Interference terms are expected to contribute to the cross-section

Nuclear forces are attractive

Coulomb forces are repulsive

Destructive Interference

leading to characteristic minima in the cross section at certain angles

In particular around proton $E_{c.m.}$ = 200 keV and $\vartheta_{c.m.}$= 90°

a deep minimum in the p-p cross-section was predicted and measured

Proton-proton scattering at low energy

COULOMB TERM

COULOMB + NUCLEAR
INTERFERENCE TERM

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{2\mu v^2}\right)^2 \left\{ \frac{1}{\sin^4(\vartheta/2)} + \frac{1}{\cos^4(\vartheta/2)} - \frac{\cos[\eta \ln \tan^2(\vartheta/2)]}{\sin^2(\vartheta/2)\cos^2(\vartheta/2)} \right.$$

Rutherford
scattering

Correction for scattering
between identical particles

Mott scattering

$$-\frac{2}{\eta}\sin\delta_0 \left[\frac{\cos[\delta_0 + \eta \ln \sin^2(\vartheta/2)]}{\sin^2(\vartheta/2)} + \frac{\cos[\delta_0 + \eta \ln \cos^2(\vartheta/2)]}{\cos^2(\vartheta/2)} \right]$$
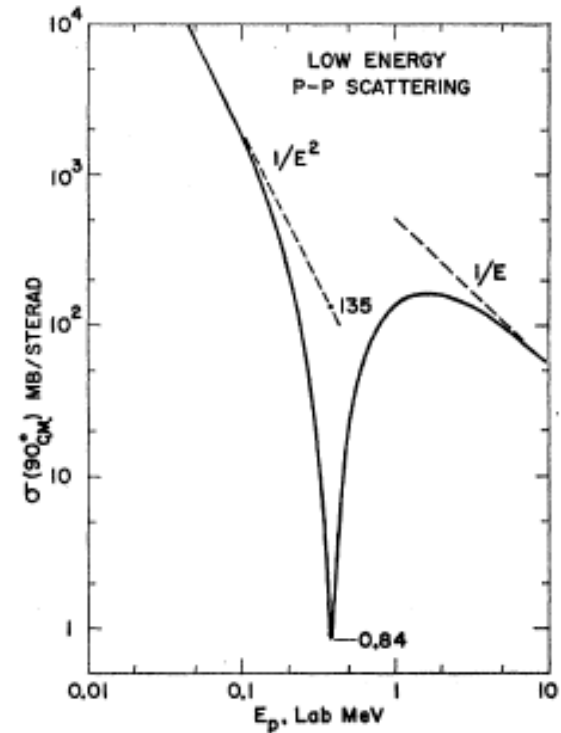
Nuclear + Coulomb Interference term

$$\left. +\frac{4}{\eta^2}\sin^2\delta_0 \right\}$$

NUCLEAR TERM

Scattering from pure
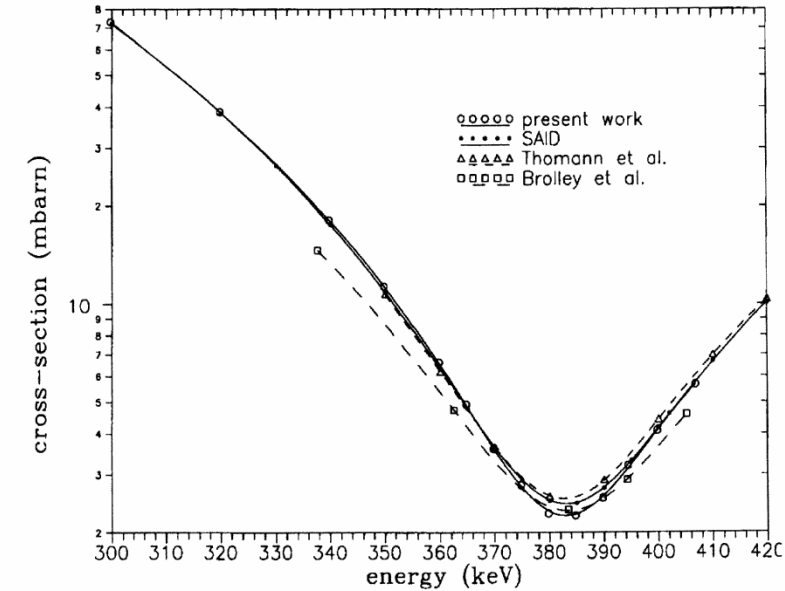nuclear potential

LOW ENERGY
P-P SCATTERING

A number of data sets were recorded in the region of the expected minimum, confirming the theoretical expectations

Such a signature.....

Test for the THM

Idea of the THM is to suppress the Coulomb barrier in binary reactions, due to the off-shell effects.
Also for scattering studied via THM, the off-shell effects are expected to <u>significantly reduce</u> the Coulomb amplitude.
The simplest system to check this hypothesis is p + d → p+p+n, which contains p+p scattering.

Does the THM p-p scattering excitation function exhibit the deep interference minimum?

THM in short

Basic principle: relevant low-energy two-body σ from quasi-free contribution of an appropriate three-body reaction in quasi free kinematics

$$A + a \rightarrow b + B + s \quad \rightarrow \rightarrow \rightarrow \quad A + x \rightarrow b + B$$
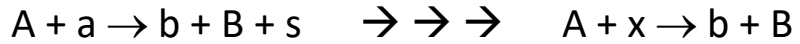
a: x $\oplus$ s clusters

Quasi free mechanism

✓ only *x* - A interaction

✓ *s* = spectator ($p_s \sim 0$)
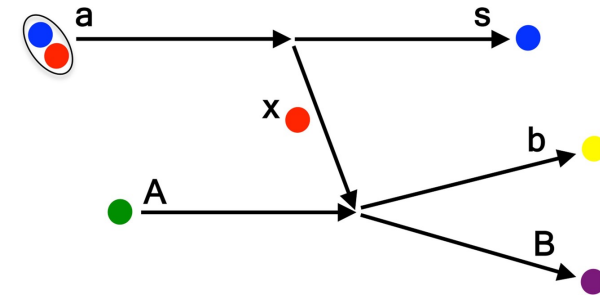
$E_A > E_{Coul} \Longrightarrow$

NO Coulomb suppression

NO electron screening

Repulsion wall

$$\frac{d^3\sigma}{d\Omega_c d\Omega_C dE_c} \propto KF \cdot \left| \Phi(p_s) \right|^2 \frac{d\sigma^{off}}{d\Omega}$$

need to normalize the two-body σ to direct data
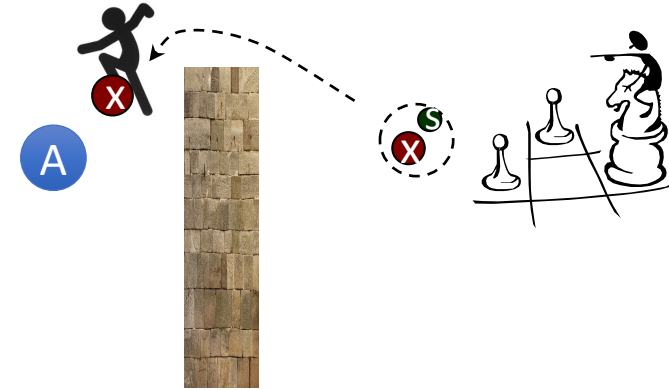
THM applied so far to more than 30 reactions, such as ^{6}Li(p,α)^{3}He, ^{7}Li(p,α)α, ^{2}H(d,p)^{3}H, ^{2}H(d, n)^{3}He, ^{10}B(p,α)^{7}Be, ^{11}B(p,α)^{8}Be, ^{17,18}O(p,α)^{14,15}N, ^{13}C(α, n)^{16}O, ^{7}Be(n,α)^{4}He, ^{18}F(p,α)^{15}O, ^{19}F(p,α)^{16}O, ^{10}B(p,α)^{7}Be, ^{11}B(p,α)^{8}Be, ^{12}C(^{12}C,α)^{20}Ne, ^{12}C(^{12}C,p)^{23}Na ...

See for review:
R.G. Pizzone et al, EpJA, (2020)
A. Tumino et al, Annual Review of Nuclear and Particle Science (2021)

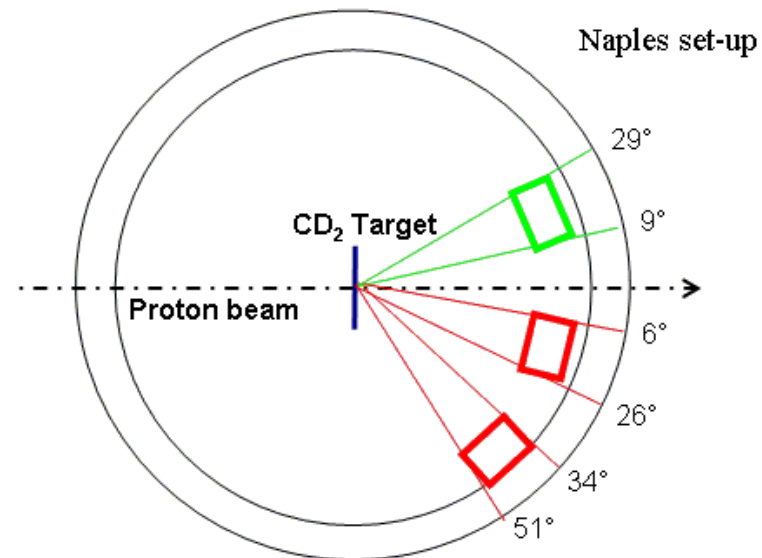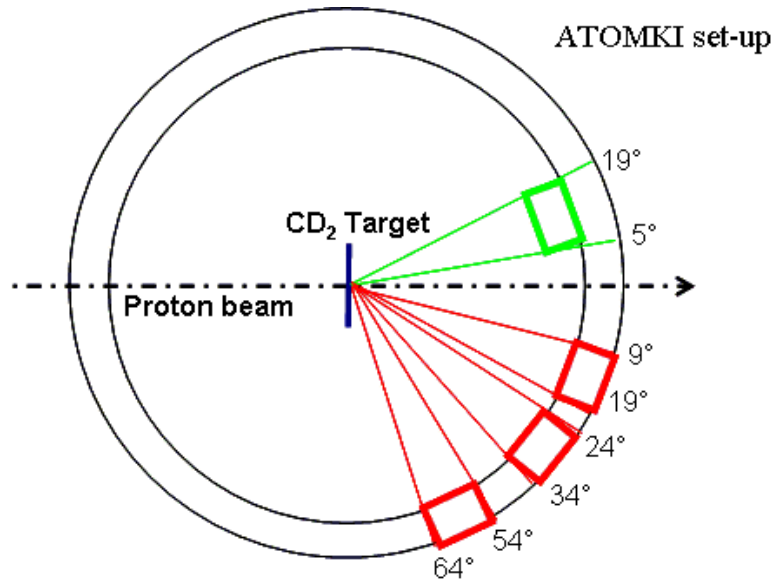# Our measurements on the p+d --> p+p+n above and below the Coulomb barrier

RUN	Beam energy (MeV)	ϑ_{lab} (deg)	E_{cm} (MeV)	$|p_s|$ (MeV/c)
LNS (2000)	6	16 ÷ 24; 16 ÷ 23 26 ÷34; 16 ÷ 23	0.32 ÷ 0.80 0.49 ÷1.02	5 ÷15 12 ÷30
RBI (2002)	6	11 ÷23; 11 ÷ 23 11 ÷ 23; 39÷ 47	0.156 ÷0.83 0.610 ÷1.34	4.5÷18 10 ÷ 46
ATOMKI (2005)	5	9 ÷19; 9 ÷ 19 9 ÷19; 24 ÷ 34 9 ÷19; 52 ÷ 66	0.09 ÷0.50 0.25 ÷0.70 0.38 ÷1.10	0 ÷ 15 12 ÷ 30 20 ÷ 64
NAPLES (2007)	4.7	9 ÷29; 6 ÷ 26 9 ÷29; 34 ÷ 51	0.1 ÷0.50 0.35 ÷0.90	0 ÷ 15 12 ÷ 35

Interference region

Proton - Proton coincidences

ATOMKI set-up

Naples set-up

Experimental neutron momentum distribution for the ATOMKI (full circles) and Naples runs (open circles). Dashed line: square of the n-p Hulthén bound-state wave function in momentum space.
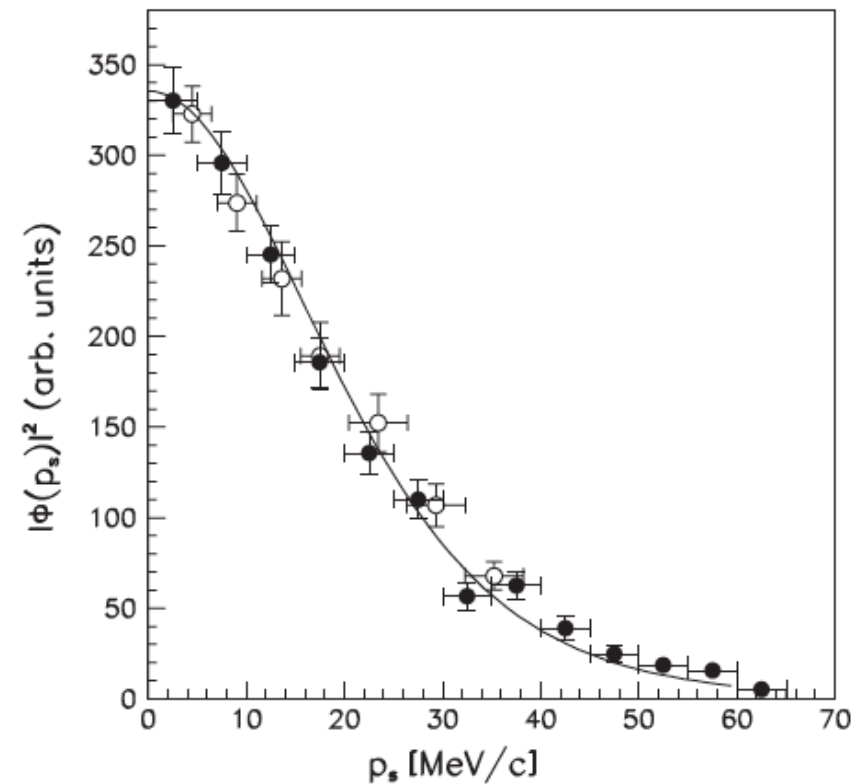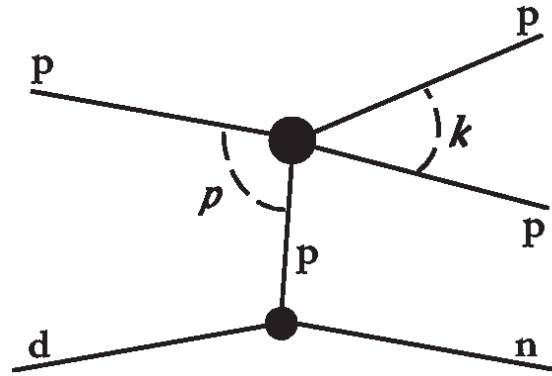
THM p-p cross-section shows the 1/E behaviour also in the region of the expected Coulomb+nuclear interference:
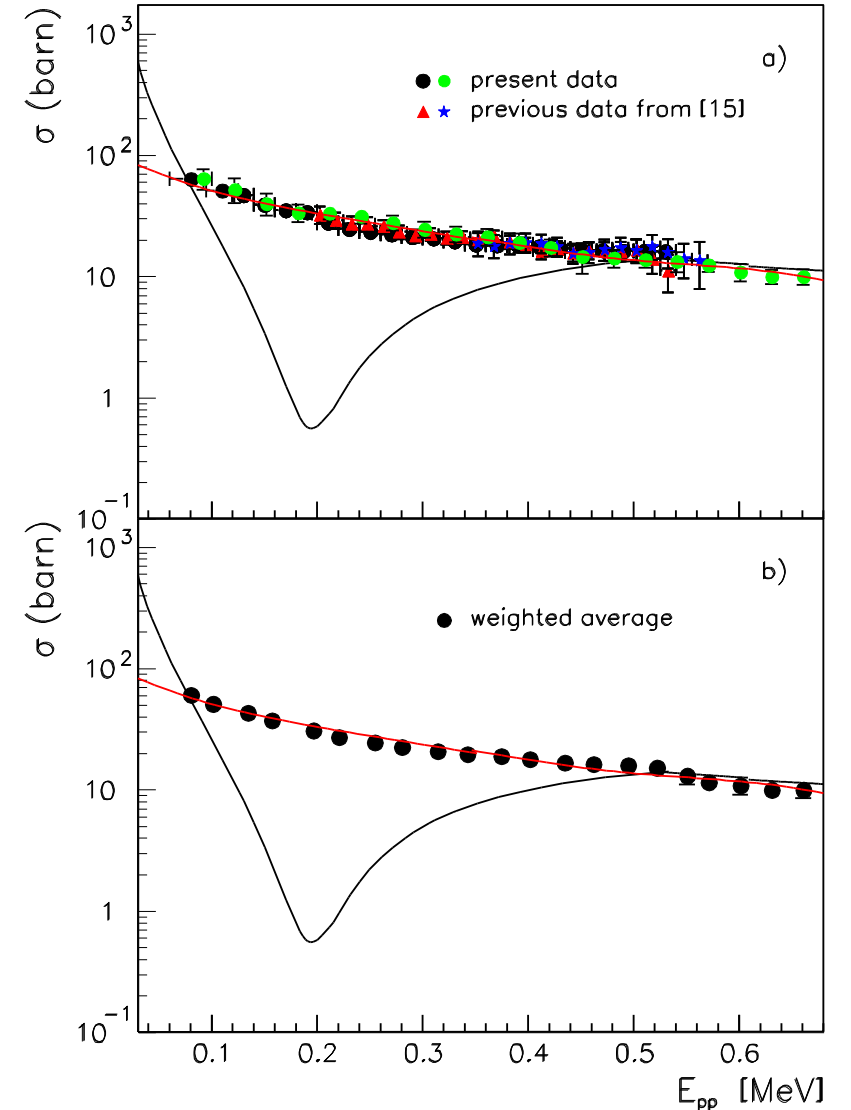
<u>Coulomb effects appear suppressed</u>

Calculated HOES p-p cross section:

$$\left(\frac{d\sigma}{d\Omega_{c.m.}}\right)^{HOES}$$

$$= \frac{1}{k^2}\left(\frac{1}{4}\left[\left|2\mu_{pp}\,e^2\,e^{-\pi\eta}\Gamma(1+i\eta)\right.\right.\right.$$

$$\times \left(\frac{(p^2-k^2)^{i\eta}}{(\mathbf{p}-\mathbf{k})^{2(1+i\eta)}} + \frac{(p^2-k^2)^{i\eta}}{(\mathbf{p}+\mathbf{k})^{2(1+i\eta)}}\right) - 2\,T_{CN}(k,\,p)\left.\right|^2\right]$$

$$+ \frac{3}{4}\left[\left|2\mu_{pp}\,e^2\,e^{-\pi\eta}\Gamma(1+i\eta)\right.\right.$$

$$\times \left(\frac{(p^2-k^2)^{i\eta}}{(\mathbf{p}-\mathbf{k})^{2(1+i\eta)}} - \frac{(p^2-k^2)^{i\eta}}{(\mathbf{p}+\mathbf{k})^{2(1+i\eta)}}\right)\left.\right|^2\right]\right).$$

Red line: HOES p-p cross section
Black line: OES p-p cross section

Comparison between experimental data and nuclear theoretical cross-sections

At low energy THM data follow the p-n trend just accidentally, due to a residual Coulomb interaction. This interaction can be removed taking advantage of the independently calculated HOES behavior by replacing the two proton charges Ze by zero. This gives the HOESn cross section shown as blue solid line.

Red line: HOES p-p cross section
Blue line: HOESn p-p cross section
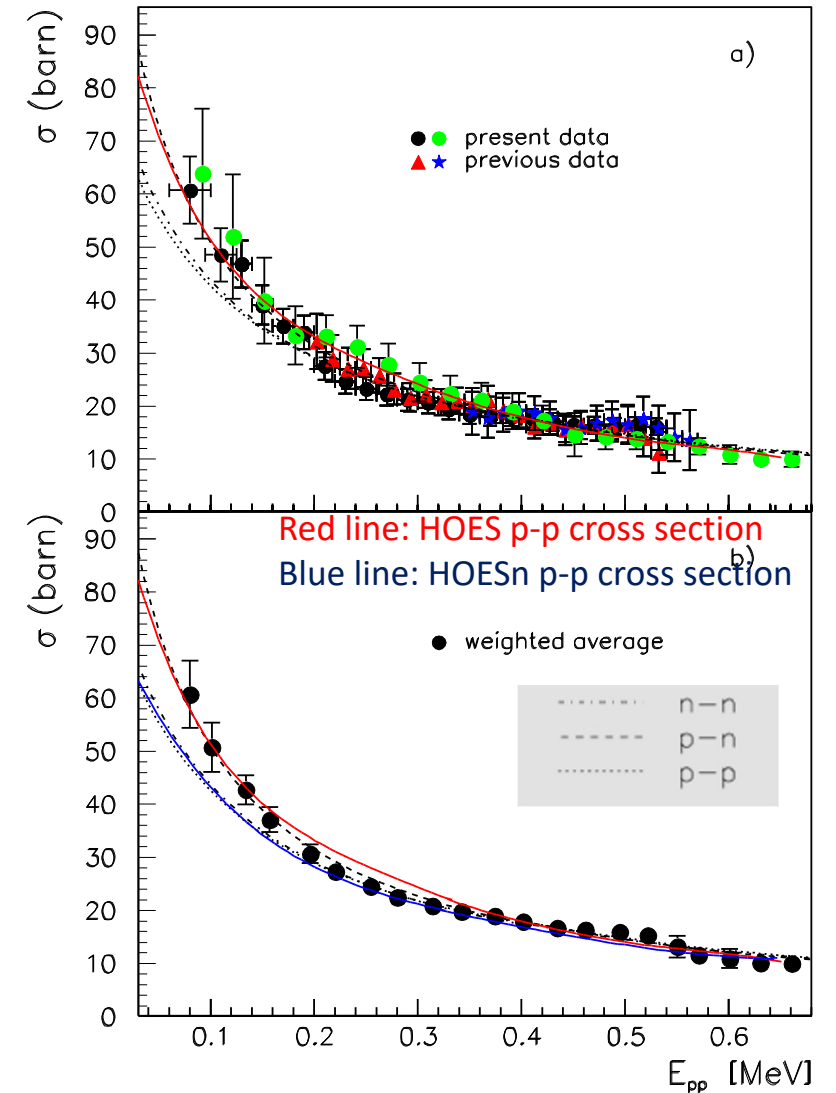
Coulomb-free pp cross section

Coulomb free p-p scattering cross section from the 2H(p, pp)n data:
the residual Coulomb interaction has been removed dividing the extracted p-p cross section by a correction term obtained from the ratio between the theoretical HOES p-p (solid black line) and HOESn p-p (solid blue line) cross sections.
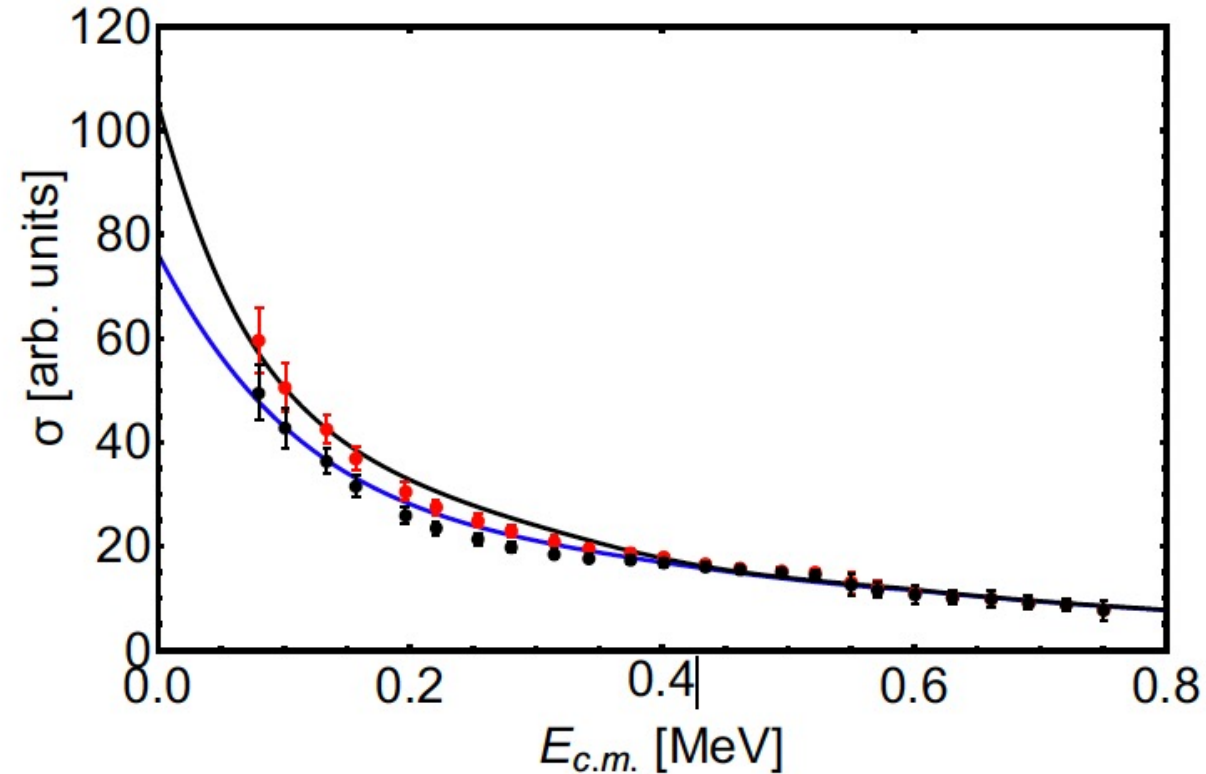
The p-p scattering data free of residual Coulomb interaction were fitted with the s-wave effective-range expansion, the conventional tool to analyze NN scattering data at low energies. For the low energy NN s-wave phase shift, δ:

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r_0 k^2,$$

The s-wave NN scattering cross section is given by

$$\sigma_{tot} = \frac{4\pi}{\left(\frac{1}{a} - \frac{1}{2} r_0 k^2\right)^2 + k^2}.$$

a Bayesian approach was used to fit the data, in order to better sample the parameters of the cross section model according to our data.

The fit was done with three free parameters: a, r0 and a normalization coefficient C.

A flat prior distribution was chosen for a, sampled in the interval (−25 fm, −15 fm) to account for the large historical dispersion of this model parameter. A Gaussian prior distribution was taken for r_0 with centroid at 2.80 fm corresponding to the weighted average of the current accepted values from the three NN combinations and σ = 0.04 fm.

Table 1 Numbers of low energy parameters.

NN	a^N(fm)	r_0^N(fm)	a^{THM}(fm)	r_0^{THM}(fm)
np	−23.08 ± 0.02	2.77 ± 0.05		
pp	−17.3 ± 0.4	2.85 ± 0.04	$-18.17^{+0.53}_{-0.59}$ fm	2.80 ± 0.05 fm
nn	−18.9 ± 0.4	2.75 ± 0.11		
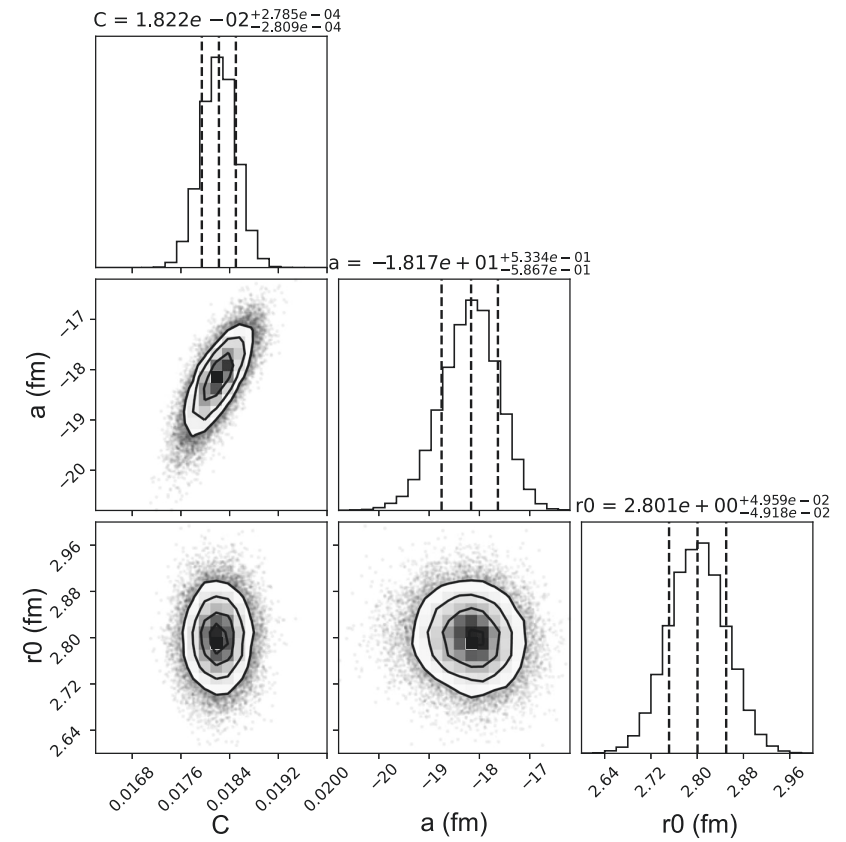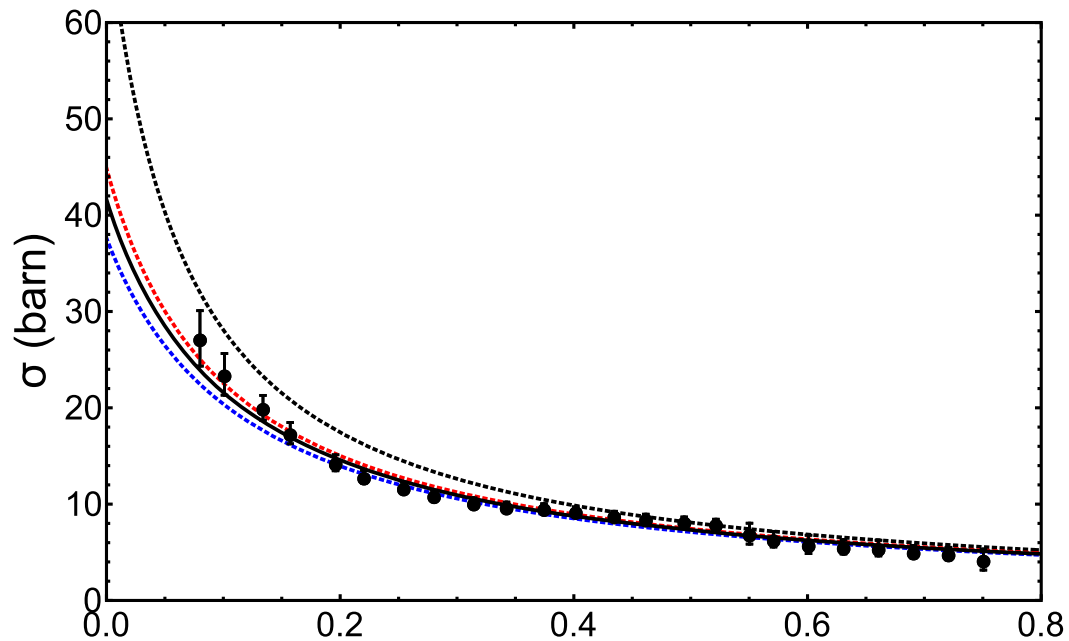
Notice: the NN s-wave phase shift δ contains all short range effects, including the electromagnetic ones. This means that the present analysis of the HOES cross section allows direct access to the short-range p-p interaction as a whole, with its peculiar app and r0 values. A different comparison with the corresponding literature value from the short range physics would therefore be more appropriate

We can exploit universal concepts to better interpret the results, ~~now that Coulomb~~ effects have been removed from the p-p system.

Notably, in the universal window the dynamics is largely independent of the details of the interaction. It is dominated by the long-range behavior allowing for a description based on few parameters.

We construct a two-parameter Gaussian NN interaction with fixed range, valid for s-wave in the spin singlet channel

$$V_{NN}(r) = V_0 e^{-r^2/r_G^2} + \frac{e_{NN}^2}{r}$$

with NN ≡ nn, np, pp and $e^2_{pp} = e^2$ and zero otherwise
the Gaussian form selected to represent the short-range interaction is not relevant, other choices are acceptable as well

NN	a^N(fm)	r_0^N(fm)	a^{THM}(fm)	r_0^{THM}(fm)	a^{sr}(fm)	r_0^{sr}(fm)	V_0(MeV)
np	−23.08 ± 0.02	2.77 ± 0.05			−23.74 ± 0.02	2.80 ± 0.08	−29.90
pp	−17.3 ± 0.4	2.85 ± 0.04	$-18.17^{+0.53}_{-0.59}$ fm	2.80 ± 0.05 fm	−17.6 ± 0.4	2.85 ± 0.09	−29.08
nn	−18.9 ± 0.4	2.75 ± 0.11			−18.6 ± 0.4	2.85 ± 0.08	−29.22

Table 1 Numbers of low energy parameters.

Current accepted values of a and r_0 parameters, (N superscript stands for "nuclear") for n-p, p-p and n-n scattering compared with those obtained in this work ("THM" superscript). In the last three columns, the values and the corresponding strength V_0 obtained with the Gaussian characterization are given. The sr superscript stands for "short-range" (nuclear + EM).

The universal window shows the location of the different NN systems using the numbers here obtained: the coordinates are given by
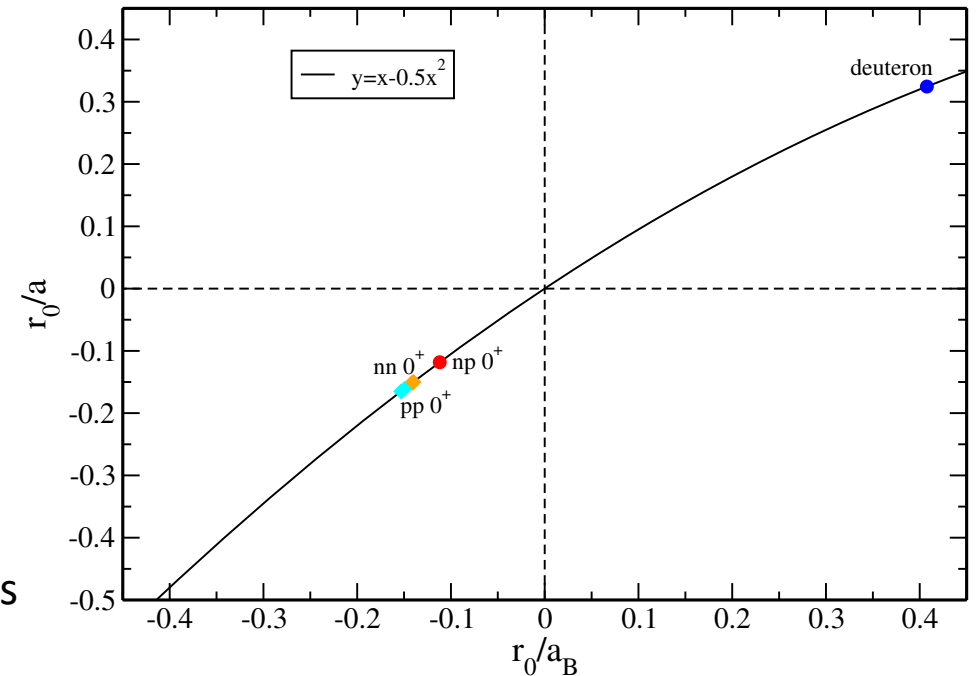
$$[x, y] = [r_0/a_B, r_0/a]$$

With a_B given by

$$\frac{1}{a_B} = \frac{1}{a} + \frac{1}{2}\frac{r_0}{a_B^2}$$

From low-energy effective range plus S-matrix pole equation

Interestingly, they lie on the curve $y = x - 0.5x^2$ verifying the correlation as above.

The NN systems are well determined by the corresponding experimental values, and have a precise position along the y(x) curve.

Using the property highlighted here that the systems move along the universal curve, it is possible to reduce the model dependence in the determination of the scattering parameters as produced by the short-range part of the interaction without discriminating between nuclear and electromagnetic.

Conclusions

First experimental estimate of the Coulomb free $1S_0$ p-p scattering length and effective range

using the following ingredients:
- Coulomb-free p-p scattering cross section below 1 MeV, extracted from the $2H(p, pp)n$ reaction using the THM
- NN s-wave effective range expansion

Universality concepts used to better interpret the results: positions of the NN systems on the universal curve as deduced from the effective description

We propose a new paradigm: to assess the charge symmetry breaking of the short-range interaction as a whole, in line with the current understanding that, at a fundamental level, the charge dependence of nuclear forces is due to a difference between the masses of the up and down quark and to electromagnetic interactions among the quarks.

Our results imply that the whole effect from the different up-down quark masses and residual electromagnetic properties has a smaller impact on the charge symmetry breaking.

Additional experimental and theoretical studies at low energies to better constrain the current existing models of charge symmetry breaking and Coulomb corrections filling up our basic understanding of low-energy NN scattering. In particular, a more precise determination of a_{nn} also using this innovative technique by measuring the quasi-free $n+d \rightarrow n+n+p$ reaction.