Probing hadron-quark mixed phase in twin stars using f-mode

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In collaboration with

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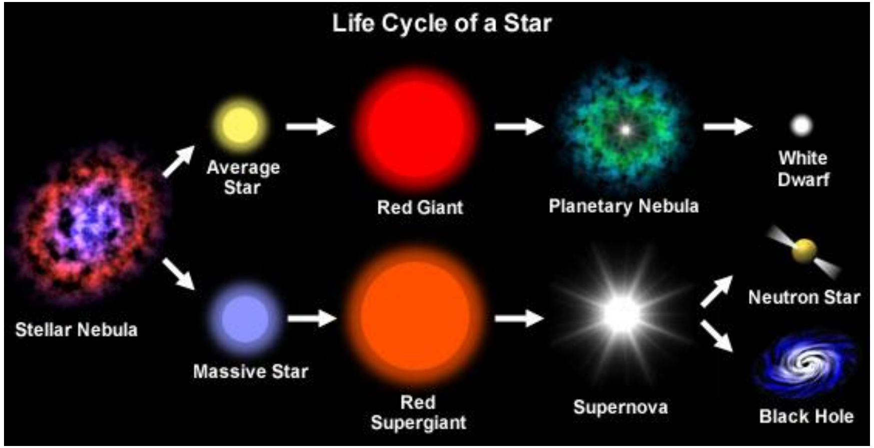
arXiv:2309.08775

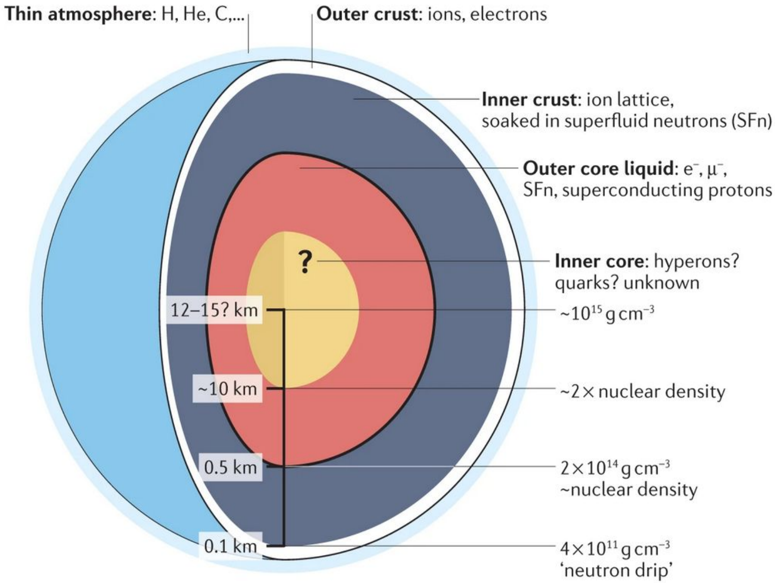
ROCKSTAR: Towards a ROadmap of the Crucial measurements of Key observables in Strangeness reactions for neutron sTARs equation of state

Introduction: Neutron Stars

- Stellar remnant : after the death of a 8 to 20 $M_\odot$ star.
- Core CS density ~ 5-10 x nuclear density .
- Observed with electromagnetic, GW detectors.



Credit: [NASA]



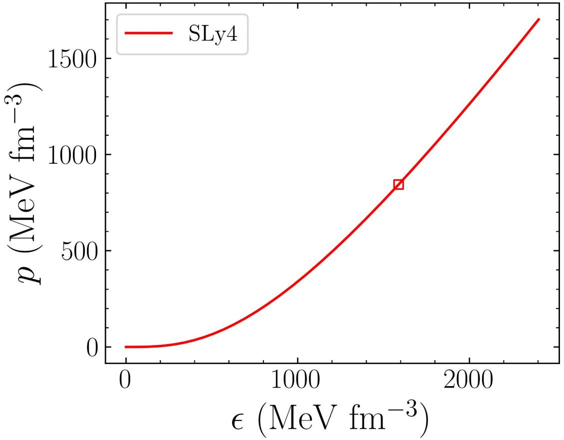
The figure illustrates the thin atmosphere, the outer and inner crust, and the outer and inner core, with the respective densities at different depths. Adapted with permission from NASA, NICER Team.

Pc: *Nature Reviews Physics* **volume 4**, pages 237–246 (2022)

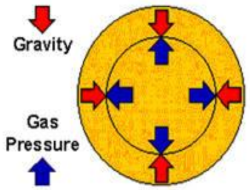
Introduction: Neutron Stars

Theoretical Modelling of Interior

The EoS : $P = P(\epsilon)$



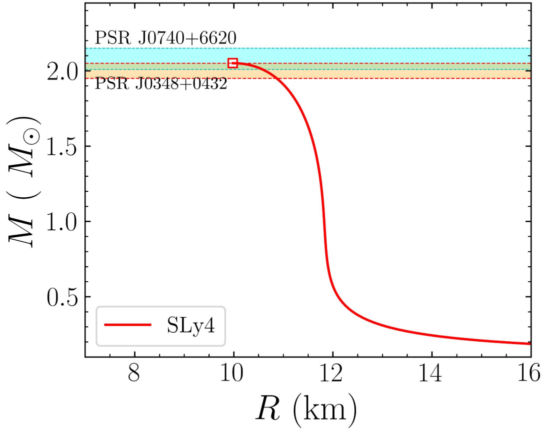
TOV Equations:



$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \epsilon(r)$$

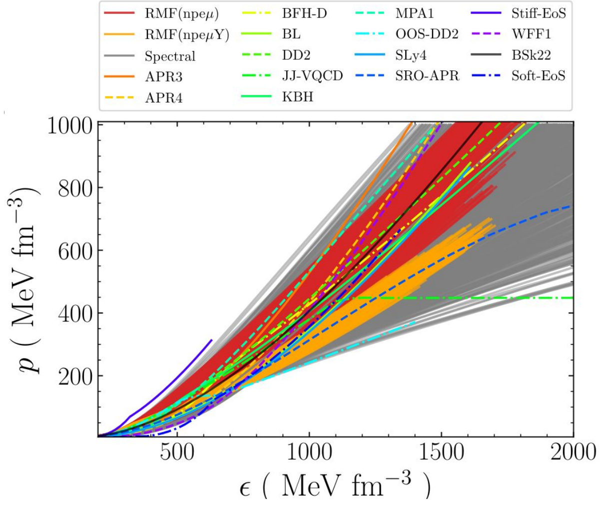
$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\left[P(r) + \epsilon(r)\right] \frac{m(r) + 4\pi r^3 P(r)}{r(r - 2m(r))}$$

NS Configuration:

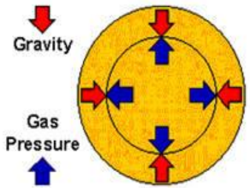


Theoretical Modelling of Interior

The EoS : $P = P(\epsilon)$



TOV Equations:



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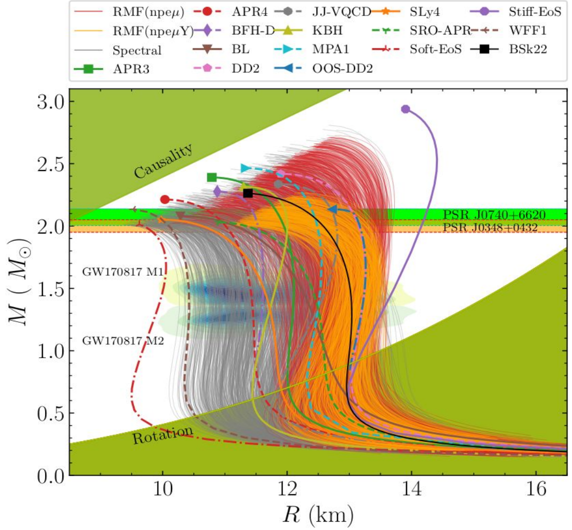
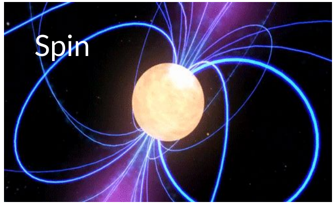
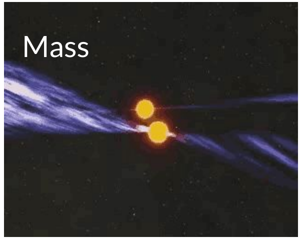
NS Configuration:

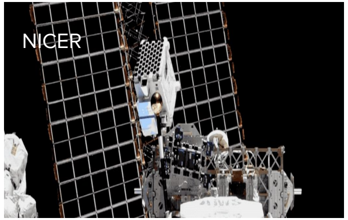


[Pradhan et al, Phys. Rev. D 107, 023010 (2023)](#).

Introduction: Neutron star and Observational constraints

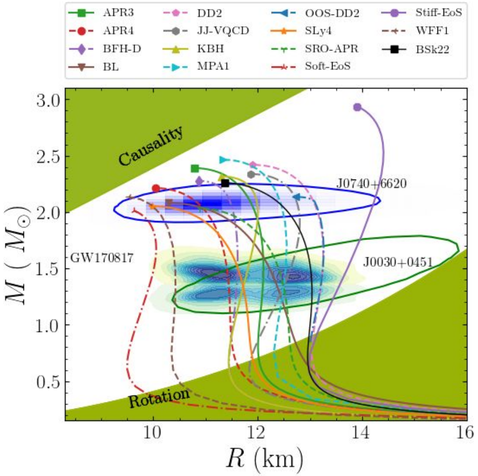


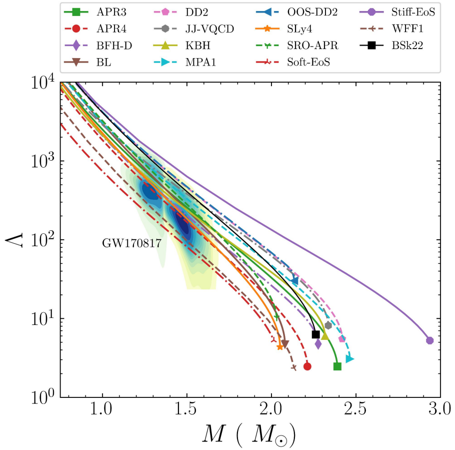


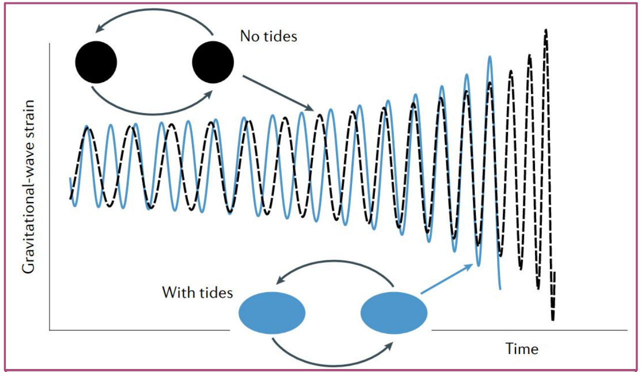


PC: NASA

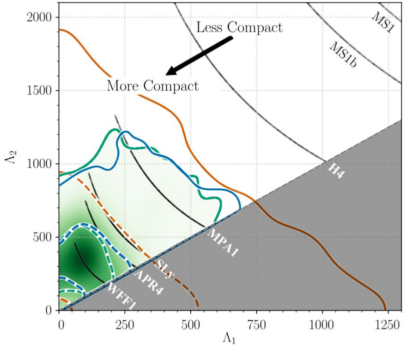
- GW events: GW170817, GW190425
- Post-Merger yet to be detected.







Credit: Zack Carson, Thesis



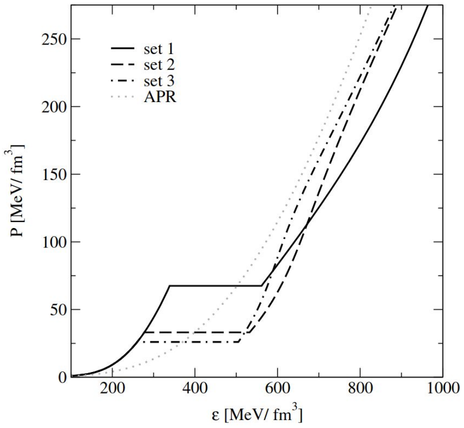
PC: B. P. Abbott *et al.*, Phys. Rev. Lett. 121, 161101 (2018), LVC

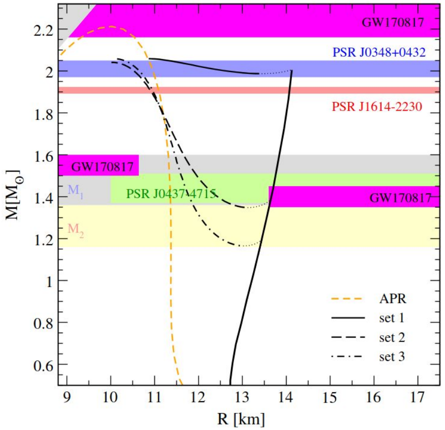
CS EoS and Twin Stars:

- Hadron-Quark phase transition inside the NS core
  - The puzzle: Strong or Crossover ?

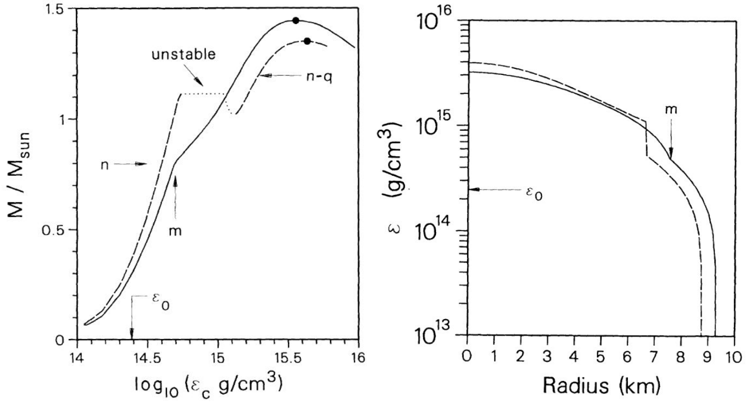
- Maxwell construction with Seidov condition (Z. Seidov, 1971, [Soviet Ast., 15, 347]),

$$\frac{\Delta\varepsilon}{\varepsilon_c} \geq \frac{1}{2} + \frac{3}{2}\frac{P_c}{\varepsilon_c}$$





D. E. Alvarez-Castillo, D. B. Blaschke, A. G. Grunfeld, and V. P. Pagura,PRD,99, 063010 (2019)



N. K. Glendenning, PRD,4,46,(1992)

The EoS Model : Phenomenological Description

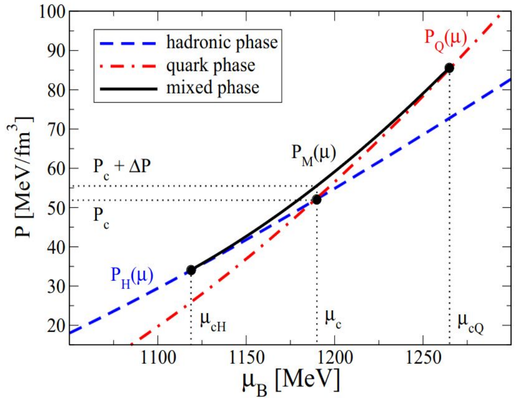
- A. Ayriyan , H. Grigorian, EPJ Web of Conferences. p. 03003,(2018),
- A. Ayriyan, N. Bastian, D. Blaschke, H. Grigorian, K. Maslov, D. N. Voskresensky, PRC 97, 045802 (2018), ,
- V. Abgaryan,D. Alvarez-Castillo, A. Ayriyan, D. Blaschke,H. Grigorian, Universe, 4, 94 (2018).

★ Surface tension effect leads to existence of pasta phases.

★ A parabolic interpolation method used to construct the mix phase.

$$p(\mu) = \begin{cases} p^H(\mu), & \mu \le \mu_{cH}, \\ P^M(\mu) = \alpha_2(\mu-\mu_c)^2 + \alpha_1(\mu-\mu_c) + P_c + \Delta P, & \mu_{cH} \le \mu \le \mu_{cQ}, \\ p^Q(\mu), & \mu \ge \mu_{cQ} \end{cases}$$

$\alpha_1,\ \alpha_2,\ \mu_{cH},\ \mu_{cH}$ → Determined from the continuity of pressure and its derivative.



★ Mix Phase is parametrized by $\Delta p = \Delta P/P_c$.

★ $\Delta p = 0$: Maxwell Construction.

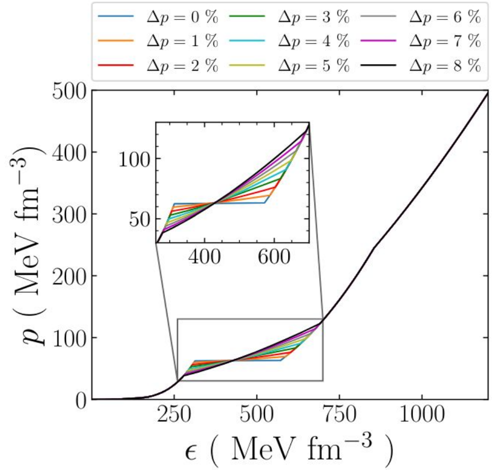
ACB4 Parametrization:

- D. E. Alvarez-Castillo., D. Blaschke, PRC, 96, 045809,(2017) ,
- V. Paschalidis, K. Yagi, D. Alvarez-Castillo, D Blaschke, A Sedrakian, PRD, 97, 084038, (2018).

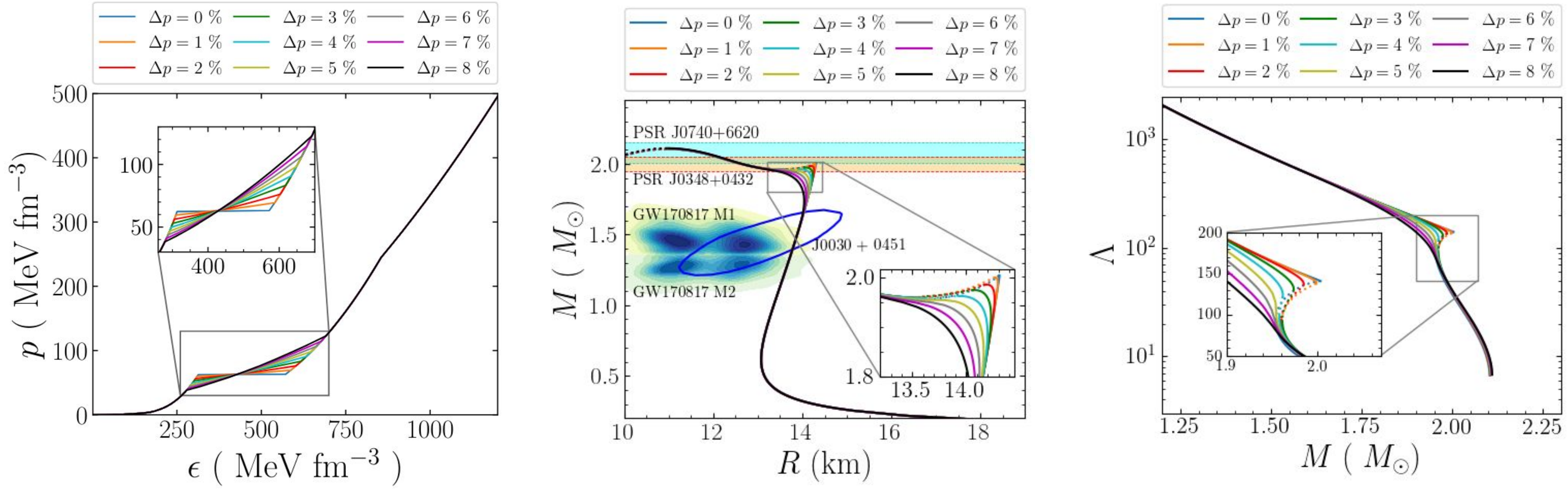
$$P(n) = \kappa_i \left(\frac{n}{n_0}\right)^{\Gamma_i},\ n_i < n < n_{i+1},\ i = 1,..4$$

$$P(\mu) = \kappa_i \left[(\mu - m_{0,i})\frac{\Gamma_i - 1}{\kappa_i \Gamma_i}\right]^{\frac{\Gamma_i}{(\Gamma_i - 1)}}$$

i	Γ_i	κ_i [MeV fm^{-3}]	n_i [fm^{-3}]	$m_{0,i}$ [MeV]
1	4.921	2.1680	0.1650	939.56
2	0.0	63.178	0.3174	939.56
3	4.00	0.5075	0.5344	1031.2
4	2.80	3.2401	0.7500	958.55

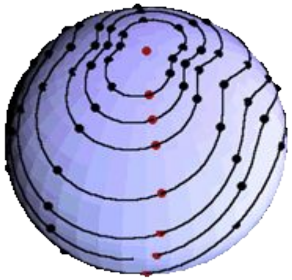


Stellar Properties

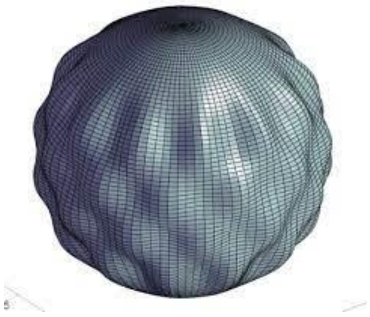


- The second and third family merge to form a single branch for Δp > 4% .
- Precise measurement of *M-R* required for detection of twin star.
- The jump ΔΛ (if any) can be measured ~15 % (< 90% CI) with next-generation GW detectors (P. Landry & K. Chakravarti , arXiv:2212.09733 ,2022).

Neutron Star and Gravitational Wave

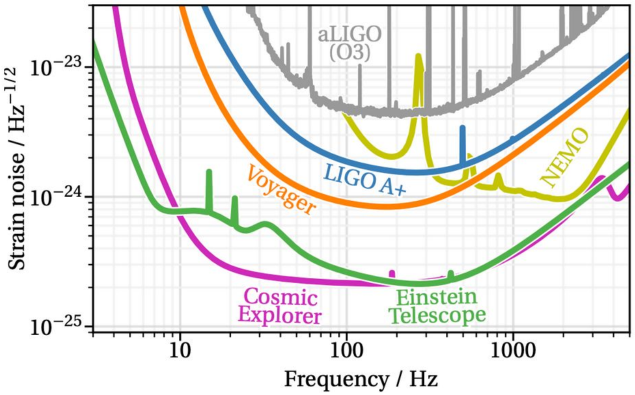


Credit: C. Hanna and B. Owen



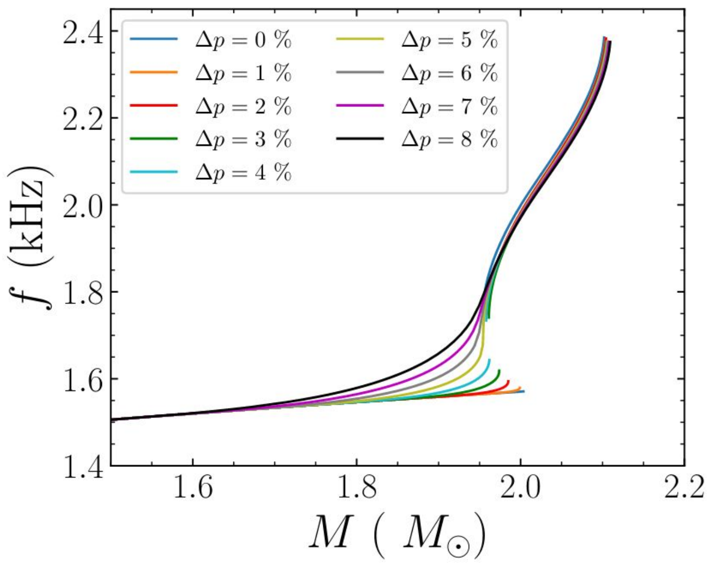
Credit: CERN/Indico

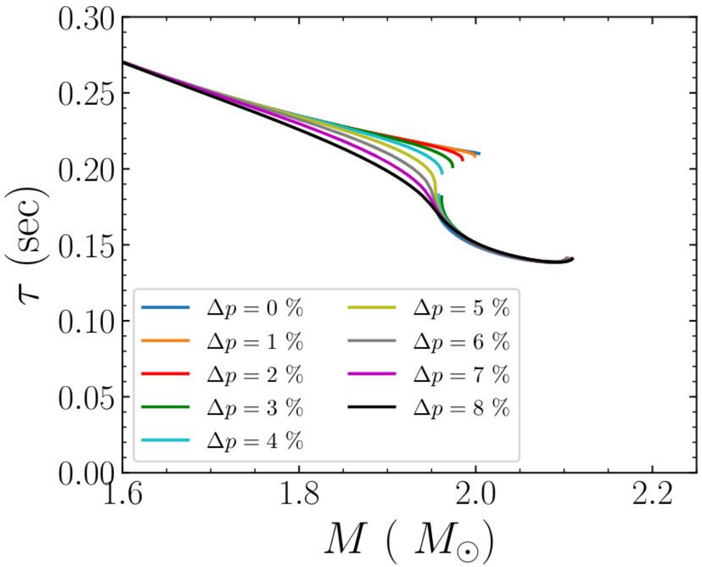
- Non-radial fluid QNMs.
    - fundamental (f) mode,
        - no node, probe for mean density,(1 kHz < f < 3kHz)
    - pressure (p) mode,
        - Sound speed, (5 kHz < f < 10kHz)
    - gravity (g) mode,
        - (50 Hz < f < 500 Hz)
- R-mode, for rotating stars only.
    - Viscosity, (0.5 kHz < f < 2kHz)
- Space-time (w) mode.
    - 5 kHz< f



PC: cosmicexplorer.org/sensitivity

F-mode characteristics





- F-mode characteristics are obtained within ***General relativistic*** formalism.
- Sudden increase (decrease) in the frequency (damping time) observed with appearance of twin star.
- Detections of f-mode GWs from compact stars with known mass may reveal the presence of twin stars.
- Simultaneous measurement of *M-f* (from binary system) can be used to comment on twin stars.

Asteroseismology and Universal Relations (UR): With Twin Stars

- URs among f-mode characteristics (f, τ_f or $\omega=2\pi f+1/\tau_f$) and NS observables.

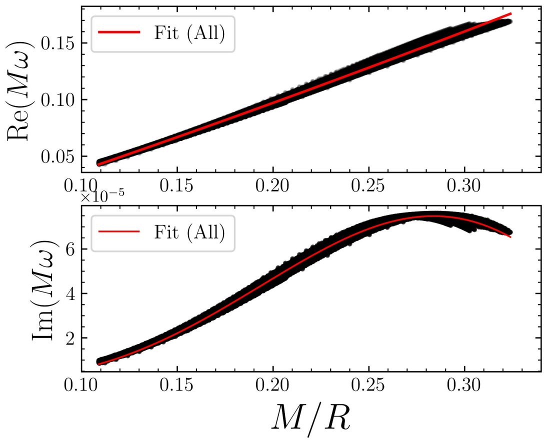
$$f(\mathrm{kHz}) = a_r + b_r\sqrt{\frac{M}{R^3}}$$

→ Empirical relations (EOS dependent)

$$\mathrm{Re}(M\omega) = a_0 + a_1\left(\frac{M}{R}\right) + a_2\left(\frac{M}{R}\right)^2$$

$$\mathrm{Im}(M\omega) = b_0\left(\frac{M}{R}\right)^4 + b_1\left(\frac{M}{R}\right)^5 + b_2\left(\frac{M}{R}\right)^6$$

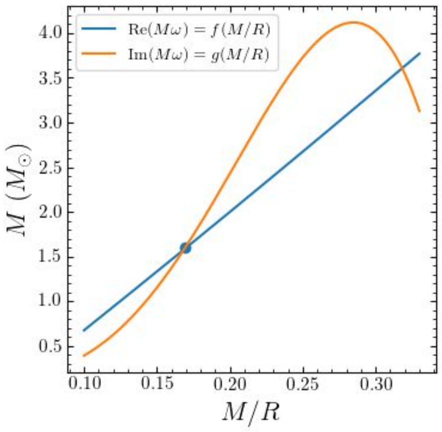
	$\mathrm{Re}(M\omega)$		$\mathrm{Im}(M\omega)$
a_0	-0.027±9 $\times 10^{-5}$	b_0	$(9.81 \pm 0.004) \times 10^{-2}$
a_1	0.610 ± 0.0015	b_1	$(-4.444 \pm 0.003) \times 10^{-1}$
a_2	0.049±0.002	b_2	$(4.91 \pm 0.0045) \times 10^{-1}$

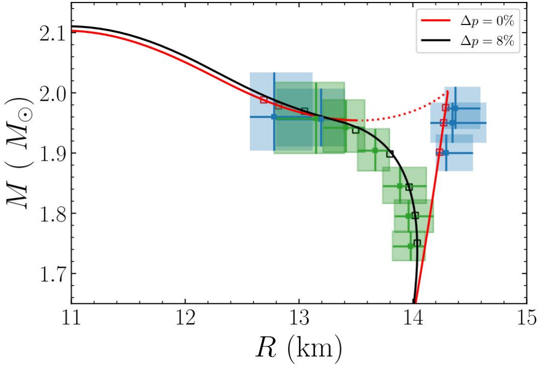


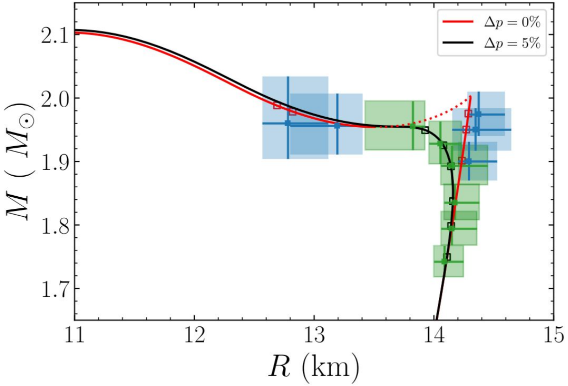
- Scaled Universal relations are more useful.
- Twin stars do not violet the URs. So the URs can be used for EoS inference.
- URs involving tidal deformability have also been examined.

Compact star observables from f-mode observations : Role of UR Uncertainty

- ❖ With the assumption that *f*, *τ* are measured precisely.
- ❖ Errors on UR results uncertainties on *M*-*R*.







★ The presence of the twins maybe confirmed with exact measurement of *f, and τ* .

★ The unstable branch of Δp = 0% can be distinguished from the connecting stable branch of Δp =8%.

★ Differentiating among Δp =0% and Δp =5% is more challenging.

Inclusion of Observational Uncertainties

- F-mode being excited during pulsar glitches. All the energy radiated through GW.
- The burst waveform is modelled as an exponentially damped oscillation.

$$h(t) = h_0 \exp\left(-t/\tau_f\right) \sin\left(2\pi\nu_f t\right), \;\; t > 0$$

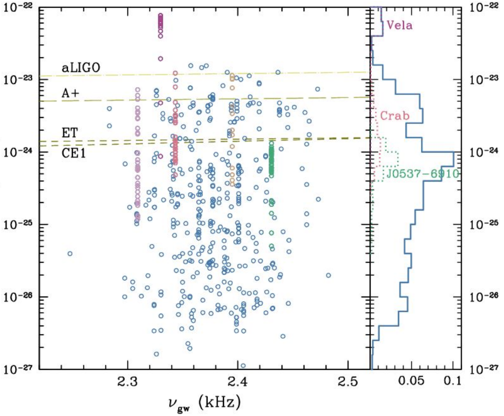
(B.J. Owen,2010, Ho et al. 2020)

$$h_0 = 4.85\times10^{-17}\sqrt{\frac{E_{\rm gw}}{M_\odot c^2}}\sqrt{\frac{0.1{\rm sec}}{\tau_f}\frac{1{\rm kpc}}{d}}\left(\frac{1{\rm kHz}}{\nu_f}\right)$$

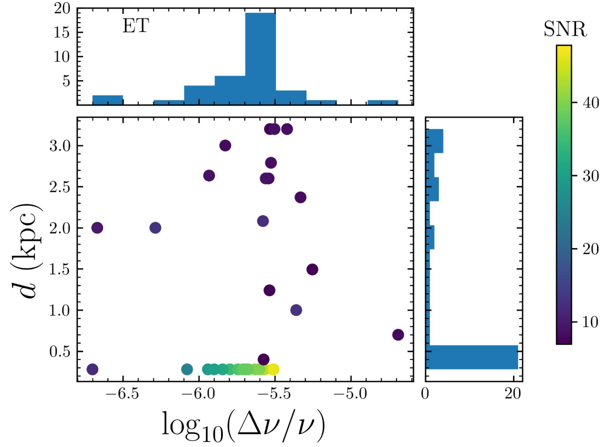
F-mode GW

- B. Abbott et al.,LVC, *ApJ* **874** 163, 2019;
- R. Abbott et al.,LVK, PhRvD, 104, 122004, 2021.
- R. Abbott, et al., LVK, arXiv:2210.10931, 2022.
- R. Abbott, et al., LVK, arXiv:2203.12038, 2022.
- D. Lopez et al., PhRvD, 106, 103037, 2022

$$E_{\rm gw} = E_{\rm glitch} = 4\pi^2 I \nu^2 \left(\frac{\Delta\nu}{\nu}\right)$$



Ho et al ,PRD 101, 103009 (2020)



B. K. Pradhan, D. Pathak, and D. Chatterjee, *ApJ* **956** 38, (2023)

Inclusion of Observational Uncertainties

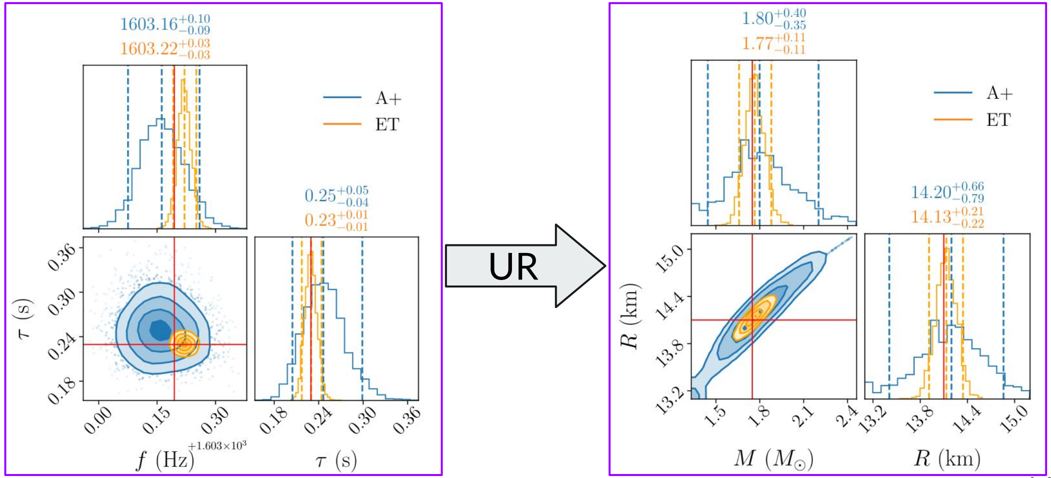
- Parameter Estimation for GW signal parameters are carried out using **Bilby.**

- Priors are kept,
  - logUniform in E_{gw} .
  - $\nu_f \in$ U[800,3500] Hz.
  - $\tau_f \in$ U[0.05,0.7] s.
  - We fix the distance.

$$h(t) = h_0 \exp\left(-t/\tau_f\right) \sin\left(2\pi\nu_f t\right), \ t > 0$$

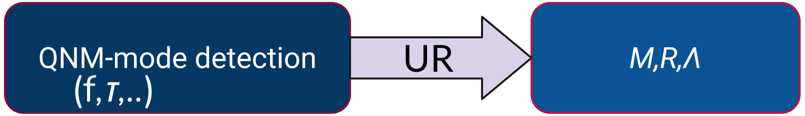
$$h_0 = 4.85\times10^{-17}\sqrt{\frac{E_{gw}}{M_\odot c^2}}\sqrt{\frac{0.1\text{sec}}{\tau_f}}\frac{1\text{kpc}}{d}\left(\frac{1\text{kHz}}{\nu_f}\right)$$

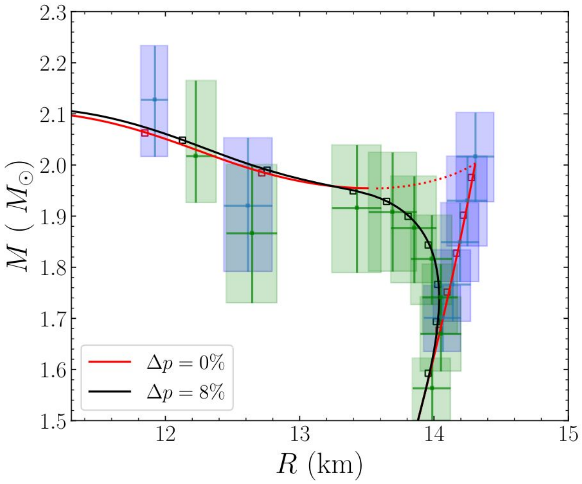
- Frequency can be measured accurately in A+ and ET.
- Damping time can have error ~20-50% in A+ and ~5-15% in ET.
- M-R posterior is obtained using UR.
- With a 90% CI, M can be measured to ~6% in ET.
- With a 90% CI, R can be measured to ~2% in ET.
- Error on M,R are large in A+.



Inclusion of Observational Uncertainties

- Glitching pulsars data taken from the Jodrell Bank Glitch catalogue.
- Spin frequency, distance (d) and sky position to each pulsar are assign from ATNF Pulsar Catalogue.
- Consider few random mass configurations with an assumed EOS model .
- Then f-mode frequency, damping time, moment of inertia to pulsars from the assume EoS model.





- The measurement of *R* from f-mode observation may confirm the presence of twins.
- More challenging for low mass twins. However, we have more observations at low masses.
- Differentiating the nature of Δp is more challenging.

Summary

- F-mode oscillation of hybrid stars and twin stars involving the “pasta phase” is investigated.
- We re-examined the asteroseismology problem considering the twin stars.
- For precise f-mode measurement provides suitable scenario for twin star detection.
- F-mode GW detection with next-generation GW offers a promising scenario for confirming the existence of the twin stars.
- Distinguishing the nature of hadron-quark crossover phase transition is more challenging.
- This work can be improved considering the effect of **rotation** and **magnetic field**.
- A detailed Bayesian study is in progress to constrain the pasta phase parameters from **f-mode/binary** observation.

Thank you

Solving for *f*-mode Characteristics:

Perturbed Metric:

$$ds_p^2 = -e^{2\Phi(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 + h_{\mu\nu}$$

$$h_{\mu\nu} = \begin{pmatrix} r^l H e^{2\Phi} & i\omega r^{l+1} H_1 & 0 & 0 \\ i\omega r^{l+1} H_1 & r^l H e^{2\lambda} & 0 & 0 \\ 0 & 0 & r^{l+2} K & 0 \\ 0 & 0 & 0 & r^{l+2} K sin^2\theta \end{pmatrix} Y_m^l e^{i\omega t},$$

(K. S. Thorne and A. Campolattaro,1967)

- Cowling Approximation: background metric perturbations are neglected $(h_{\mu\nu} = 0)$
- The fluid Lagrange displacement vector (ζ)

$$\zeta^i = \left(\frac{r^l}{r} e^{-\lambda} W(r), \frac{-r^l}{r^2} V(r), 0\right) r^{-2} Y_{lm}(\theta, \phi) \exp(i\omega t)$$

Perturbations inside the star

- Solving differential equations for coupled fluid perturbation and metric functions
- Boundary conditions:
  - Finite near center.
  - Jump condition at phase transition
  - perturbation pressure vanishes at the stellar surface

Perturbations outside the star

- Set fluid variables=0 and integrate the Zerilli equations for perturbations outside the star.
- Search for complex ω, for which one have only outgoing wave solution at infinity.

S. Detweiler and L. Lindblom,1985, F. J. Zerilli, 1970, Sotani,2001.

Neutron star Asteroseismology

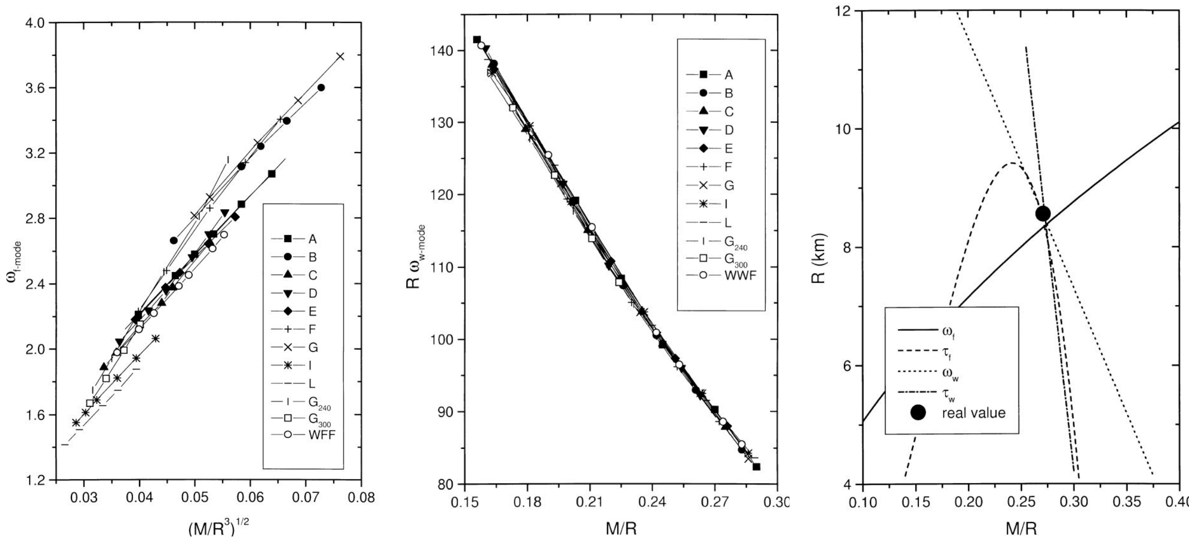
➢ **Nils Andersson and Kostas D. Kokkotas**, Mon. Not. R. Astron. Soc. 299, 1059–1068, (1998).

$$\omega_f(\text{kHz}) \approx 0.78 + 1.635\left(\frac{\bar{M}}{\bar{R}^3}\right)^{1/2}$$

$$\frac{1}{\tau_f(\text{s})} \approx \frac{\bar{M}^3}{\bar{R}^4}\left[22.85 - 14.65\left(\frac{\bar{M}}{\bar{R}}\right)\right]$$

$$\omega_w(\text{kHz}) \approx \frac{1}{\bar{R}}\left[20.92 - 9.14\left(\frac{\bar{M}}{\bar{R}}\right)\right]$$

$$\frac{1}{\tau_w(\text{ms})} \approx \frac{1}{\bar{M}}\left[5.74 + 103\left(\frac{\bar{M}}{\bar{R}}\right) - 67.45\left(\frac{\bar{M}}{\bar{R}}\right)^2\right]$$



Further works improved the **Asteroseismology** problem,

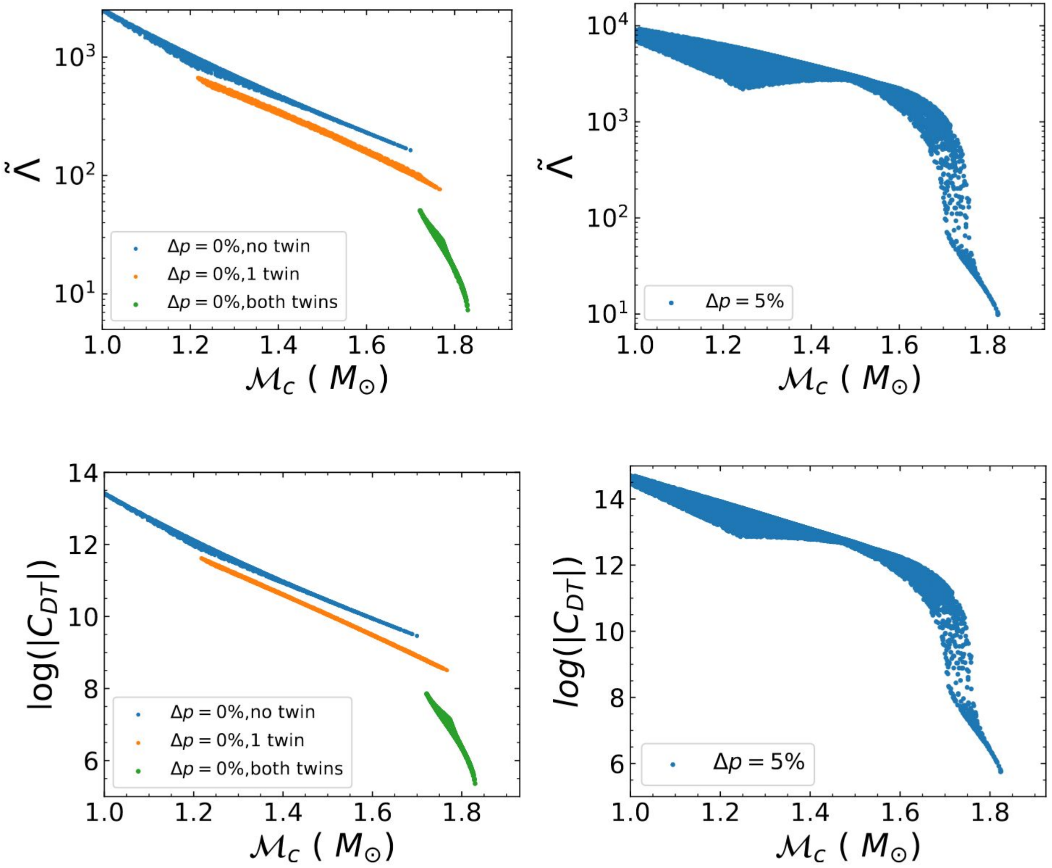
- O. Benhar, V. Ferrari, and L. Gualtieri,Phys. Rev. D 70, 124015 (2004).
- L. K. Tsui and P. T. Leung, Mon. Not. R. Astron. Soc. 357, 1029 (2005).
- J. L. Blázquez-Salcedo, L. M. González-Romero, and F. Navarro-Lérida, Phys. Rev. D,89, 044006 (2014).
- G. Lioutas and N. Stergioulas, Gen. Relativ. Gravit. 50, 12 (2018).
- H. Sotani and B. Kumar, Phys. Rev. D 104, 123002 (2021).
- T. Zhao and J. M. Lattimer, PRD 106, 123002 (2022)

Twin stars in binary :

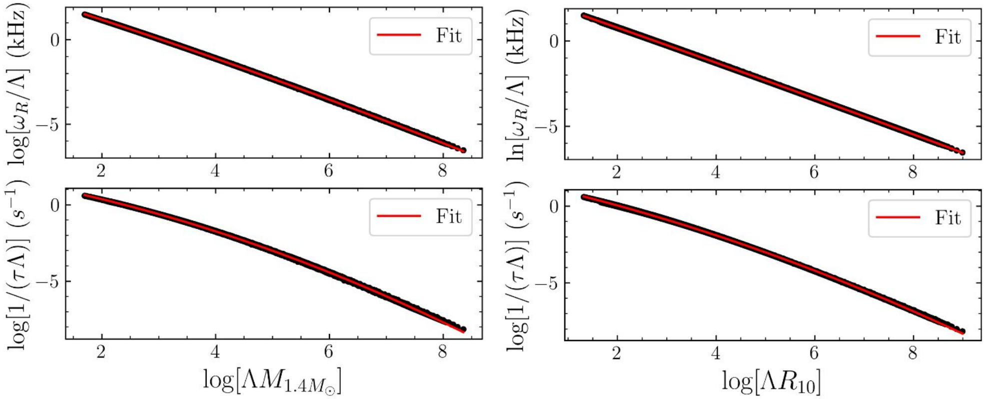
$$\mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

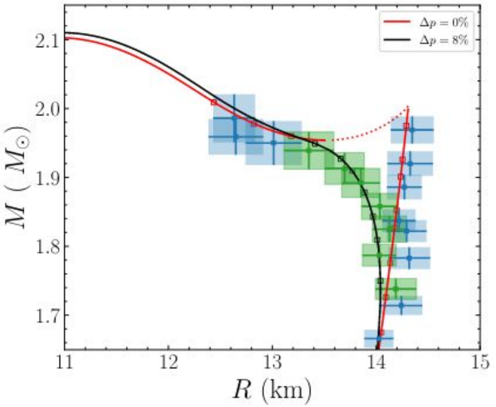
$$\tilde{\Lambda} = \frac{16}{13}\left[\frac{(m_1 + 12 m_2) m_1^4 \Lambda_1}{(m_1 + m_2)^{1/5}} + 1 \longleftrightarrow 2\right]$$

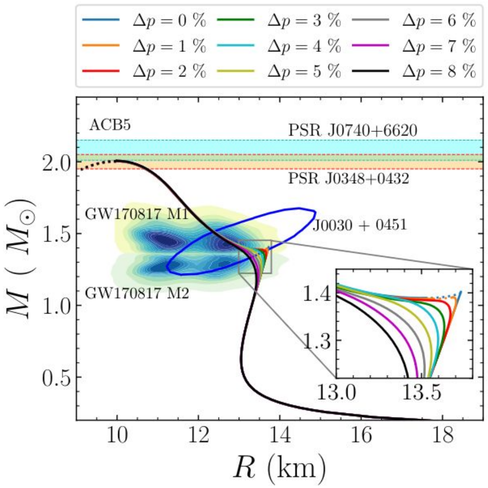
$$C_{DT} = -\frac{1}{X_1 X_2}\left[\frac{\Lambda_1}{(m_1 \omega_1)^2} X_1^6 (155 - 147 X_1) + 1 \longleftrightarrow 2\right]$$

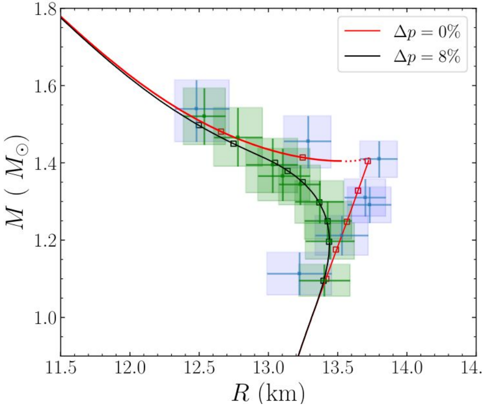


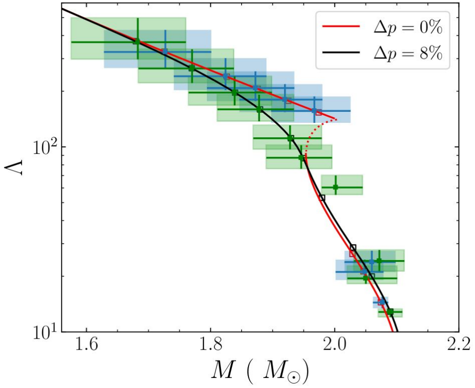
- ❖ There is a jump in the binary parameters in presence of twin stars.
- ❖ Tidal parameter can be more useful.
- ❖ The f-mode parameters can add more information.


Fit
$\log[\omega_R/\Lambda]$ (kHz)
$\ln[\omega_R/\Lambda]$ (kHz)
$\log[1/(\tau\Lambda)]$ (s^{-1})
$\log[\Lambda M_{1.4M_\odot}]$
$\log[\Lambda R_{10}]$


$\Delta p = 0\%$
$\Delta p = 8\%$
M ($M_\odot$)
R (km)


$\Delta p = 0$ %
$\Delta p = 1$ %
$\Delta p = 2$ %
$\Delta p = 3$ %
$\Delta p = 4$ %
$\Delta p = 5$ %
$\Delta p = 6$ %
$\Delta p = 7$ %
$\Delta p = 8$ %
ACB5
PSR J0740+6620
PSR J0348+0432
GW170817 M1
J0030 + 0451
GW170817 M2
M ($M_\odot$)
R (km)


$\Delta p = 0\%$
$\Delta p = 8\%$
M ($M_\odot$)
R (km)


$\Delta p = 0\%$
$\Delta p = 8\%$
Λ
M ($M_\odot$)

Introduction: Neutron star and QNMs

- Non-radial QNMs raised from time varying quadrupole deformations are source of GWs.
- These deformations contain signature of underlying composition of NS and reflect in the form of observed frequencies and damping time.

Fluid displacement vector :

$$\vec{\zeta}(\vec{r},t) = \sum_{lm} \left[\zeta_r(r)\hat{r} + \zeta_h(r) \left(\hat{\theta}\partial_\theta + \hat{\phi}\frac{1}{\sin\theta}\partial_\phi \right) \right] Y_{lm}(\theta,\phi)\; e^{i\omega t}$$

(K. S. Thorne ,1967)

Newtonian limit (T=0, B=0, no rotation)

(Gudrun Kristine Høye,1999)

- Continuity: $\rho' = -\vec{\nabla}.(\rho_0\vec{\zeta})$
- Euler: $\rho_0\vec{\zeta}_{tt} = -\vec{\nabla}p' - \rho_0\vec{\nabla}\phi' - \rho'\vec{\nabla}\phi_0$
- Poisson : ~~$\vec{\nabla}^2\phi' = 4\pi G\rho'$~~ (Cowling Approximation, T. G. Cowling,1941)
- Energy: $p' + \vec{\zeta}.\vec{\nabla}p_0 = \frac{\Gamma_1 p_0}{\rho_0}\left(\rho' + \vec{\zeta}.\vec{\nabla}\rho_0\right)$

- ❑ At center : Regularity
- ❑ At surface:

$$p' + \frac{dp_0}{dr}\zeta_r = 0$$