Hyper(nuclear) $\not\pi$EFT at next-to-leading order: status and perspectives

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ECT* workshop ROCKSTAR: Towards a ROadmap of the Crucial measurements of Key observables in Strangeness reactions for neutron sTARs equation of state

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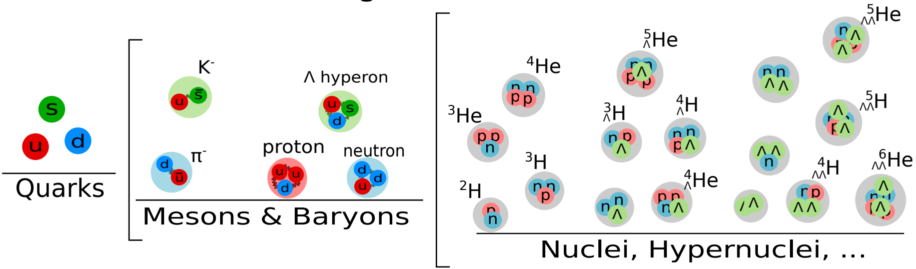
Hypernuclei

Interactions of hadrons :

- currently described by QCD

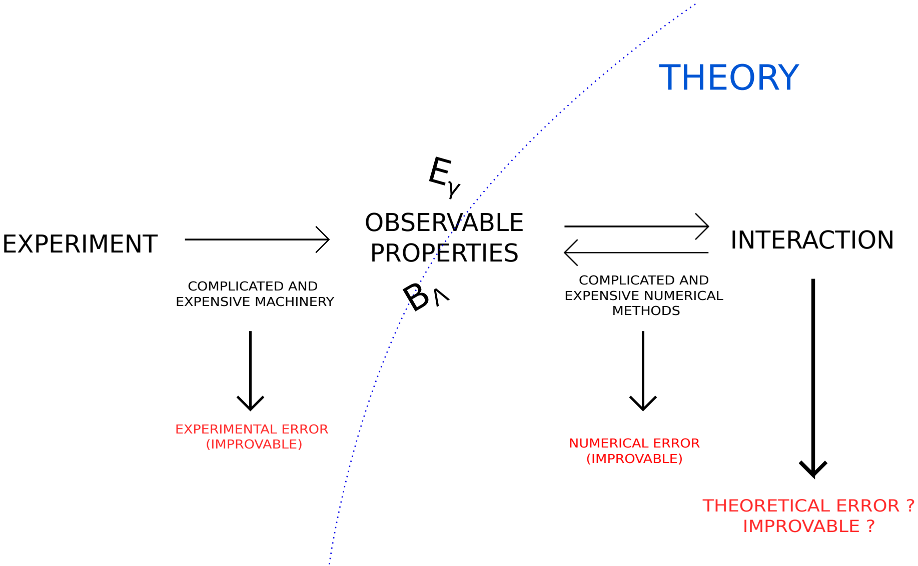
At low and intermediate energies ...



- **QCD is notoriously difficult to solve in this energy regime !**
  $\rightarrow$ lattice QCD and effective field theories (EFTs)

Observed properties of few-body hadronic systems $\longleftrightarrow$ **Precise few-body methods** $\longleftrightarrow$ **Underlying interaction models**

Experiment & Theory



Momentum scales

M_{hi} ... momentum scale of underlying theory

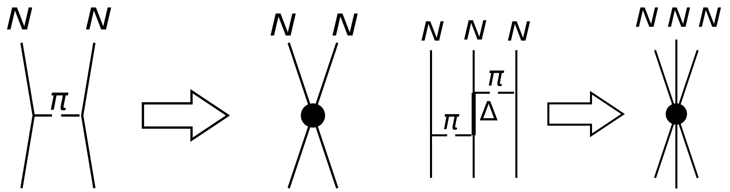
$\rightarrow$ interest in processes at typical momentum Q comparable to lower momentum scale

$$Q \approx M_{\mathrm{lo}} << M_{\mathrm{hi}}$$

Effective Field Theory :

- focus on the low-momentum M_{lo} region in more general case
- most general effective Lagrangian, while keeping the symmetries of the underlying theory
- high-momentum M_{hi} degrees of freedom integrated out
- systematic expansion of an interaction in $(M_{\mathrm{lo}}/M_{\mathrm{hi}})$
- power counting

Nuclear $\not\pi$EFT



Scales :

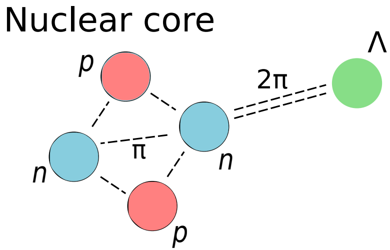
$\rightarrow$ no pionic degrees of freedom

- breakup scale $M_{\rm hi} = m_\pi$
- **rough** typical momentum estimates :

$$Q(^4\mathrm{He}) \approx \sqrt{2M_N B(^4\mathrm{He})/4} \approx 115\ \mathrm{MeV} \longrightarrow \frac{Q}{M_{hi}} \approx 0.8 < 1$$

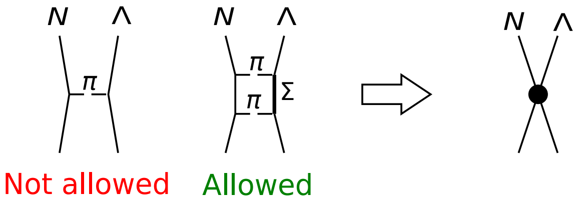
$$Q(^3\mathrm{H}) \approx \sqrt{2M_N B(^3\mathrm{H})/3} \approx 72\ \mathrm{MeV} \longrightarrow \frac{Q}{M_{hi}} \approx 0.5 < 1$$

Hypernuclear $\not\pi$EFT

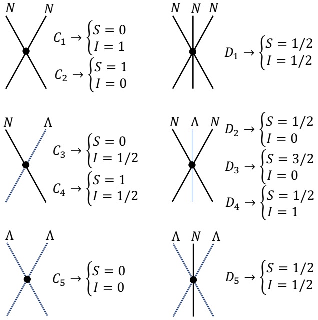


- Λ is weakly bound to the nuclear core (small typical exchange momentum Q)
- long distance forces $\rightarrow 2\pi$ exchange ($M_{\rm hi} = 2m_\pi \approx 280$ MeV)

$$\frac{Q}{M_{hi}} \approx \frac{\sqrt{2M_\Lambda B_\Lambda}}{2m_\pi} \approx 0.3 < 1$$



(Hyper)nuclear $\not{\pi}$EFT at LO



A=2	A=3	A=4	A=5	A=6
$a_{NN}(^1S_0)$	$^3\mathrm{H}$			
$a_{NN}(^3S_1)/^2\mathrm{H}(1^+)$				
$a_{N\Lambda}(^1S_0)$	$^3_\Lambda\mathrm{H}$	$^4_\Lambda\mathrm{H}$		
$a_{N\Lambda}(^3S_1)$		$^4_\Lambda\mathrm{H}^*$		
$a_{\Lambda\Lambda}(^1S_0)$				$^6_{\Lambda\Lambda}\mathrm{He}$

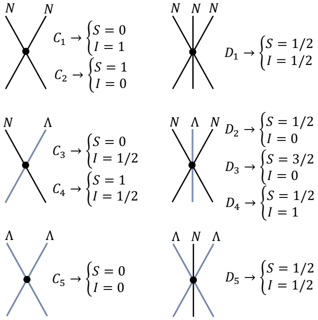
Hamiltonian :

$$H_\lambda^{(\mathrm{LO})} = T_\mathrm{k} + V_2 + V_3$$

$$V_2 = \sum_{I,S} C_\lambda^{I,S} \sum_{i<j} \mathcal{P}_{ij}^{I,S} \delta_\lambda(\mathrm{r}_{ij})$$

$$V_3 = \sum_{I,S,\alpha} D_{\lambda,\alpha}^{I,S} \sum_{i<j<k} \mathcal{Q}_{ijk}^{I,S,\alpha} \sum_{\mathrm{cyc}} \delta_\lambda(\mathrm{r}_{ij})\delta_\lambda(\mathrm{r}_{jk})$$

(Hyper)nuclear $\not\pi$EFT at LO



A=2	A=3	A=4	A=5	A=6
$a_{NN}(^1S_0)$	$^3\mathrm{H}$	$^4\mathrm{He}$		
$a_{NN}(^3S_1)/^2\mathrm{H}(1^+)$				
$a_{N\Lambda}(^1S_0)$	$^3_\Lambda\mathrm{H}$	$^4_\Lambda\mathrm{H}$	$^5_\Lambda\mathrm{He}$	
$a_{N\Lambda}(^3S_1)$	$^3_\Lambda\mathrm{H}^*$	$^4_\Lambda\mathrm{H}^*$		
	$\Lambda\mathrm{nn}$			
$a_{\Lambda\Lambda}(^1S_0)$	$\Lambda\Lambda\mathrm{n}$	$^4_{\Lambda\Lambda}\mathrm{H}$	$^5_{\Lambda\Lambda}\mathrm{H}$	$^6_{\Lambda\Lambda}\mathrm{He}$
		$\Lambda\Lambda\mathrm{nn}$		

Hamiltonian :

$$H_\lambda^{(\mathrm{LO})} = T_\mathrm{k} + V_2 + V_3$$

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$$V_3 = \sum_{I,S,\alpha} D_{\lambda,\alpha}^{I,S} \sum_{i<j<k} \mathcal{Q}_{ijk}^{I,S,\alpha} \sum_{\mathrm{cyc}} \delta_\lambda(\mathrm{r}_{ij})\delta_\lambda(\mathrm{r}_{jk})$$

Regularization and renormalization

Regularization :

For singular interaction the solution of the Schrödinger equation requires regularization.

$\rightarrow$ regulator function smears the δ-function over λ^{-1}

$$\delta_\lambda(r_{ij}) = \frac{\lambda^3}{8\pi^{3/2}} \exp\left(-\lambda^2 r^2/4\right)$$

$\rightarrow$ all low-energy constants (LECs) gain specific λ dependencies $C(\lambda)$, $D(\lambda)$, ...

$\rightarrow$ all observables are independent of arbitrary λ value (renormalization)

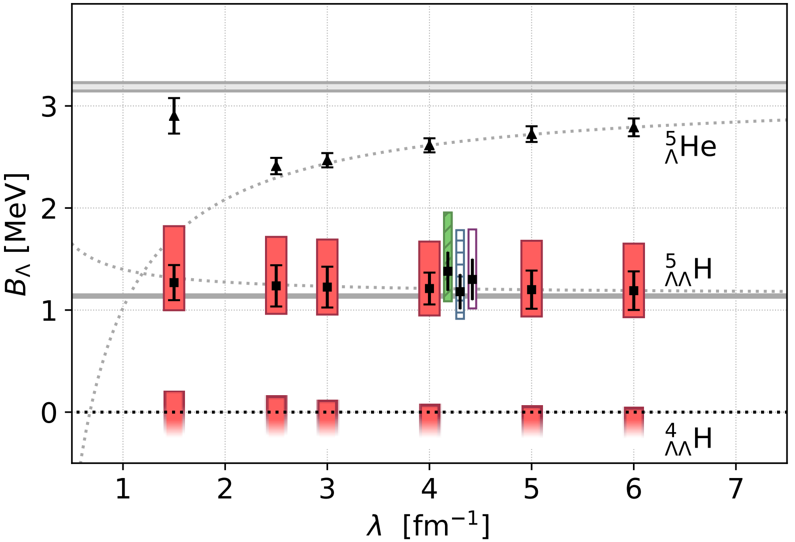
Truncation of the EFT at the selected order :

$\rightarrow$ for $\Lambda >> M_{hi}$ observables acquire residual cutoff dependence $\mathcal{O}(Q/\lambda)$

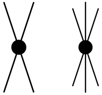
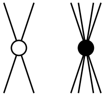
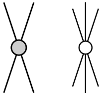
$\rightarrow$ truncation of the Lagrangian at the given order induces relative error of $\mathcal{O}(Q/M_{\rm hi})$

The onset of ΛΛ hypernuclear binding

(L. Contessi, M. Schäfer, N. Barnea, A. Gal, and J. Mareš, Phys. Lett. B **797**, 134893 (2019))



Hierarchy of interaction terms - power counting

LO			$C_0^{(0)}\delta(\mathrm{r}_{12}),\ D_0^{(0)}\delta(\mathrm{r}_{12})\delta(\mathrm{r}_{23})$
NLO			$C_1^{(1)}\nabla^2_{\mathrm{r}_{12}}\delta(\mathrm{r}_{12}),\ E_0^{(1)}\delta(\mathrm{r}_{12})\delta(\mathrm{r}_{23})\delta(\mathrm{r}_{34})$
N^2LO		...	$C_2^{(2)}(\nabla_{\mathrm{r}_1}\cdot\nabla_{\mathrm{r}_2})\delta(\mathrm{r}_{12})$, more 3-body, higher-body
$N^{>2}LO$	...		

Nuclear $\not\pi$EFT LO and NLO potential

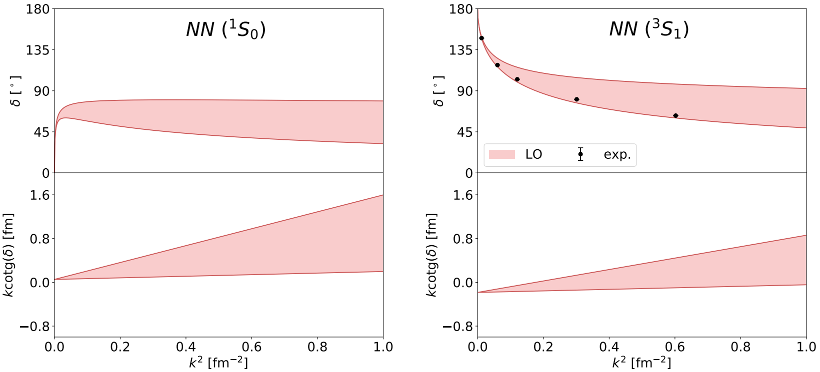
Leading order potential (3 LECs) :

$$V_\lambda^{(\mathrm{LO})} = \sum_{i<j} \left[C_0^{(0)}(\lambda) P_{ij}^{I=1,S=0} + C_1^{(0)}(\lambda) P_{ij}^{I=0,S=1} \right] e^{-\frac{\lambda^2}{4} r_{ij}^2}$$
$$+ D_0^{(0)}(\lambda) \sum_{i<j<k} \mathcal{Q}_{ijk}^{I=1/2,S=1/2} \sum_{\mathrm{cyc}} e^{-\frac{\lambda^2}{4}(r_{ij}^2 + r_{jk}^2)}$$

Next-to-leading order potential (6 LECs) :

$$V_\lambda^{(\mathrm{NLO})} = \sum_{i<j} \left[C_0^{(1)}(\lambda) P_{ij}^{I=1,S=0} + C_1^{(1)}(\lambda) P_{ij}^{I=0,S=1} \right] e^{-\frac{\lambda^2}{4} r_{ij}^2}$$
$$+ \sum_{i<j} \left[C_2^{(1)}(\lambda) P_{ij}^{I=1,S=0} + C_3^{(1)}(\lambda) P_{ij}^{I=0,S=1} \right] \left(e^{-\frac{\lambda^2}{4} r_{ij}^2} \overrightarrow{\nabla}_{ij}^2 + \overleftarrow{\nabla}_{ij}^2 e^{-\frac{\lambda^2}{4} r_{ij}^2} \right)$$
$$+ D_0^{(1)}(\lambda) \sum_{i<j<k} \mathcal{Q}_{ijk}^{I=1/2,S=1/2} \sum_{\mathrm{cyc}} e^{-\frac{\lambda^2}{4}(r_{ij}^2 + r_{jk}^2)}$$
$$+ E_0^{(1)}(\lambda) \sum_{i<j<k<l} \mathcal{Q}_{ijkl}^{I=0,S=0} e^{-\frac{\lambda^2}{4}(r_{ij}^2 + r_{ik}^2 + r_{il}^2 + r_{jk}^2 + r_{jl}^2 + r_{kl}^2)}$$

LO $\not\pi$EFT



Leading order (LO) :

(exp. constraints)

a^0_{NN} (a^0_{nn}) $= -18.95(40)$ fm

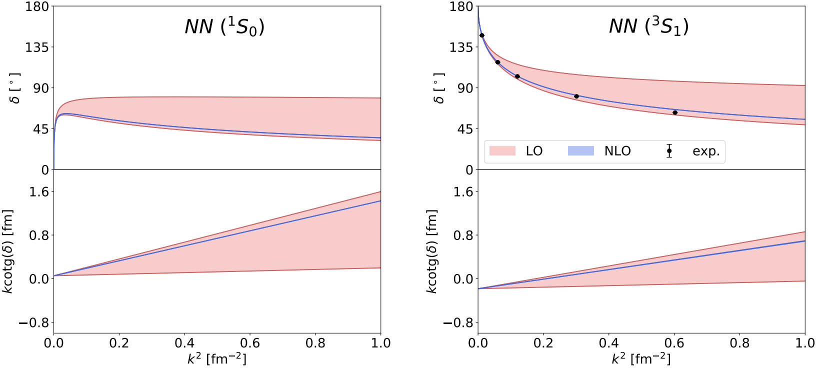
a^1_{NN} (a^1_{np}) $= 5.419(7)$ fm

$B(^3\mathrm{H}) = 8.482$ MeV

Effective range expansion :

$$k\,\mathrm{cotg}(\delta) = -\frac{1}{a} + \frac{1}{2} r k^2 + \ldots$$

NLO $\pi\!\!\!/$EFT



Leading order (LO) :

(exp. constraints)

a^0_{NN} (a^0_{nn}) = −18.95(40) fm

a^1_{NN} (a^1_{np}) = 5.419(7) fm

$B(^3\mathrm{H}) = 8.842$ MeV

Next-to-leading order (NLO) :

(exp. constraints)

r^0_{NN} (r^0_{nn}) = 2.75(11) fm

r^1_{NN} (r^1_{np}) = 1.753(8) fm

$B(^4\mathrm{He}) = 28.296$ MeV

Nuclear $\not\pi$EFT up to NLO

Where we stand ?

- NLO $\not\pi$EFT using 6 experimental constraints
  (a, r) of $NN(^1S_0)$ and $NN(^3S_1)$, $B(^3\mathrm{H})$, $B(^4\mathrm{He})$

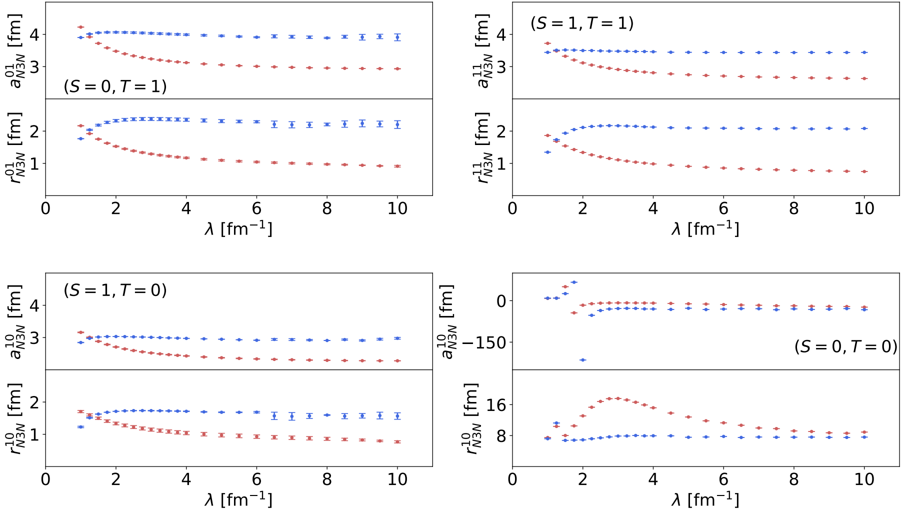
What we want to study ?

- behavior of $\not\pi$EFT predictions with increasing order (LO to NLO)
- comparison with experimental results

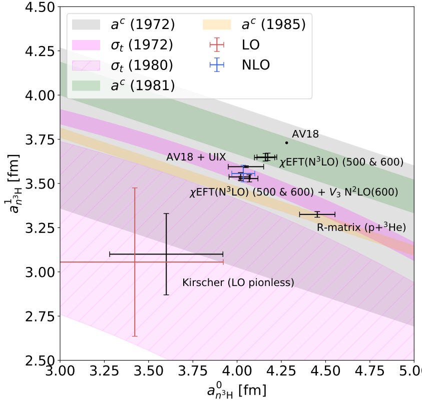
Test of the theory $\longrightarrow$ predictions of few-body nuclear scattering

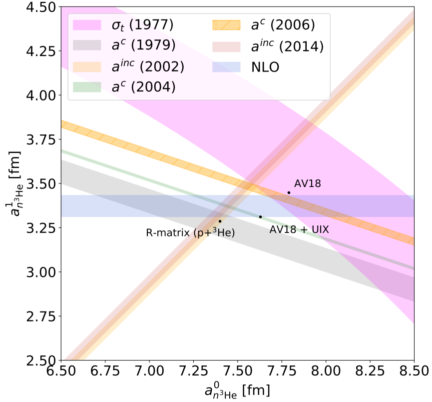
$n + {}^3\mathrm{H}$ and $n + {}^3\mathrm{He}$ scattering (Phys. Rev. C 107 (2023) 064001)

LO → NLO



Experiment & Theory : $n + {}^3\mathrm{H}$ and $n + {}^3\mathrm{He}$ scattering lengths (Phys. Rev. C 107 (2023) 064001)

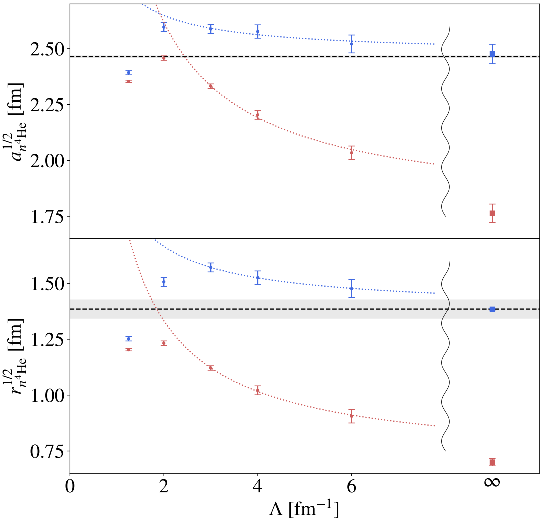




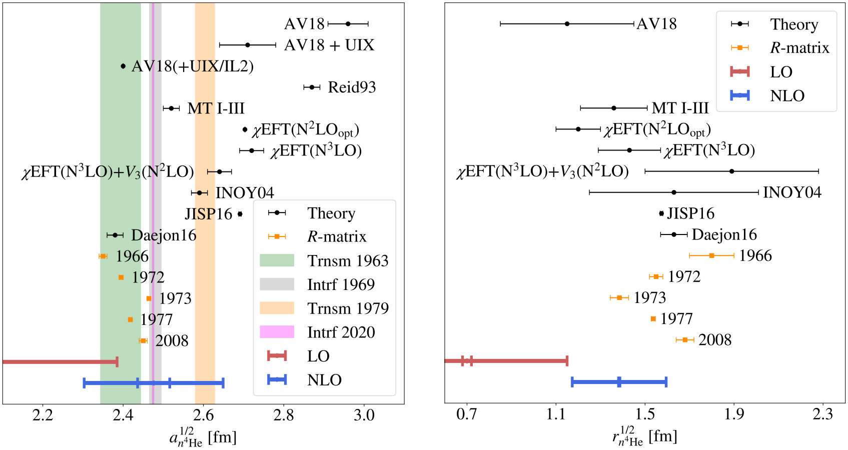
(Phys. Rev. C 42 (1990) 438; Phys. Rev. C 102 (2020) 034007; Few-Body Syst. 34 (2004) 105; Phys. Lett B 721 (2013) 355; Phys. Rev. C 68(R) (2003) 021002)

$n + {}^4\mathrm{He}$ scattering (Phys. Lett. B 844 (2023) 138078)

LO → NLO



$n + {}^4\text{He}$ scattering (Phys. Lett. B 844 (2023) 138078)



For references to all theoretical results see (Phys. Lett. B 844 (2023) 138078).

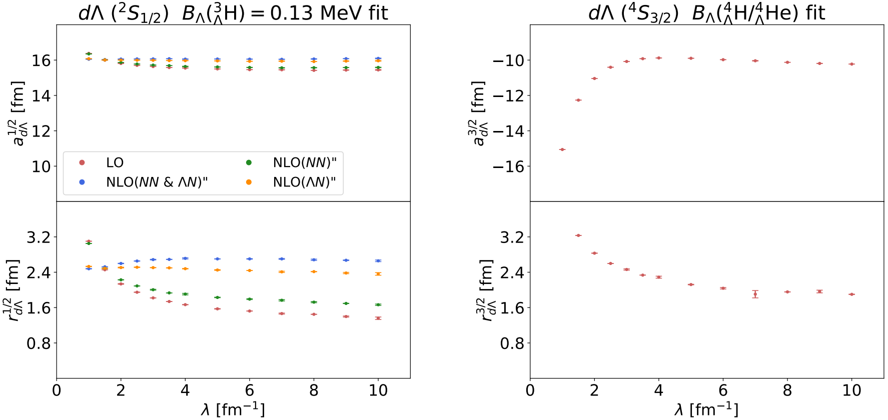
Hypernuclear $\not\pi$EFT up to NLO (preliminary)

$\mathcal{S}$	LO		NLO		
	A=2	A=3	A=2	A=3	A=4
0	$a_{NN}(^1S_0)$ $a_{NN}(^3S_1)$	^{3}H	$r_{NN}(^1S_0)$ $r_{NN}(^3S_1)$	^{3}H	^{4}He
-1	$a_{N\Lambda}(^1S_0)$ $a_{N\Lambda}(^3S_1)$	$^3_\Lambda$H $^3_\Lambda$H* Λnn	$r_{N\Lambda}(^1S_0)$ $r_{N\Lambda}(^3S_1)$	$^3_\Lambda$H $^3_\Lambda$H* Λnn	$^4_\Lambda$H $^4_\Lambda$H*
-2	$a_{\Lambda\Lambda}(^1S_0)$	$\Lambda\Lambda$n	$r_{\Lambda\Lambda}(^1S_0)$	$\Lambda\Lambda$n	$^4_{\Lambda\Lambda}$H $\Lambda\Lambda$nn

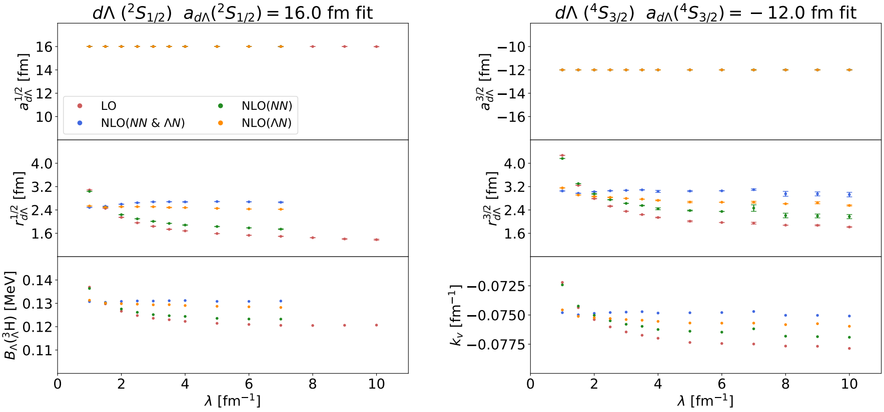
Hypernuclear $\not\pi$EFT up to NLO (preliminary)

$\mathcal{S}$	LO		NLO		
	A=2	A=3	A=2	A=3	A=4
0	$a_{NN}(^1S_0)$ $a_{NN}(^3S_1)$	$^3\mathrm{H}$	$r_{NN}(^1S_0)$ $r_{NN}(^3S_1)$	$^3\mathrm{H}$	$^4\mathrm{He}$
-1	$a_{N\Lambda}(^1S_0)$ $a_{N\Lambda}(^3S_1)$	$^3_\Lambda\mathrm{H}$ $^3_\Lambda\mathrm{H}^*$ Λnn	$r_{N\Lambda}(^1S_0)$ $r_{N\Lambda}(^3S_1)$	$^3_\Lambda\mathrm{H}$ $^3_\Lambda\mathrm{H}^*$ Λnn	$^4_\Lambda\mathrm{H}$ $^4_\Lambda\mathrm{H}^*$
-2	$a_{\Lambda\Lambda}(^1S_0)$	$\Lambda\Lambda n$	$r_{\Lambda\Lambda}(^1S_0)$	$\Lambda\Lambda n$	$^4_{\Lambda\Lambda}\mathrm{H}$ $\Lambda\Lambda nn$

Hypertriton channels (preliminary)



Hypertriton channels (preliminary)



Summary

Nuclear NLO $\not\pi$EFT

- no problem to fit all LECs (6 input data up to NLO)
- works very well in studies of n-$^3\mathrm{H}$, n-$^3\mathrm{He}$, n-$^4\mathrm{He}$ scattering

Λ-hypernuclear NLO $\not\pi$EFT

- difficulties in constraining all LECs (9 input data up to NLO)
    1. three-body $S = 3/2, I = 0$ ΛNN channel $\longrightarrow$ Λd femtoscopy ?
    2. three-body $S = 1/2, I = 1$ ΛNN channel $\longrightarrow$ Λpp femtoscopy ?

ΛΛ-hypernuclear NLO $\not\pi$EFT

- currently not enough data (5 input data required up to NLO)