

Constraining the $\pi\Sigma - \bar{K}N$ models with the $\pi\Sigma$ photoproduction data

A. Cieplý

Nuclear Physics Institute, Řež, Czechia

- 1 Chirally motivated $\pi\Sigma - \bar{K}N$ interactions (results overview)
- 2 $\pi\Sigma$ photoproduction mass spectra
 - Formalism
 - Results
 - Current development
- 3 Summary

collaboration with P. C. Bruns, partly with M. Mai (Bonn) and J. Smejkal (Prague)

Chirally motivated $\bar{K}N$ interactions

$\bar{K}N - \pi\Sigma$ system (+ add-ons, mostly more MB)
meson octet - baryon octet coupled channels interactions

involved channels	$\pi\Lambda$	$\pi\Sigma$	$\bar{K}N$	$\eta\Lambda$	$\eta\Sigma$	$K\Xi$
thresholds (MeV)	1250	1330	1435	1660	1740	1810

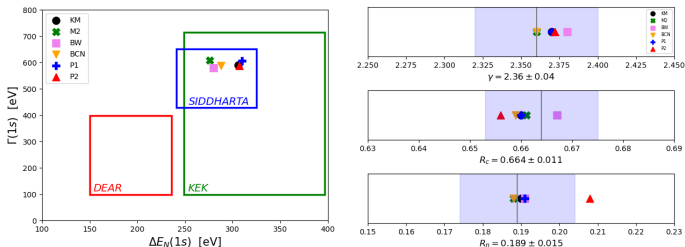
- strongly interacting multichannel system with an s-wave resonance, **the $\Lambda(1405)$** , just below the K^-p threshold
- modern theoretical treatment based on **effective chiral Lagrangians**
- effective potentials constructed to match the chiral meson-baryon amplitudes up to LO or NLO order
- Lippmann-Schwinger (or Bethe-Salpeter) equation to sum the major part of the perturbation series



N. Kaiser, P.B. Siegel, W. Weise - Nucl. Phys. A 594 (1995) 325

K^-p data fits: low energy x-sections, threshold BRs, kaonic hydrogen 1s level

Experimental data reproduction



left - kaonic hydrogen characteristics:

1s level energy shift $\Delta E_N(1s)$ and absorption width $\Gamma(1s)$

right - threshold branching ratios:

$$\gamma = \frac{\Gamma(K^- p \rightarrow \pi^+ \Sigma^-)}{\Gamma(K^- p \rightarrow \pi^- \Sigma^+)},$$

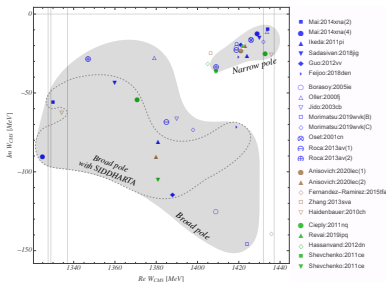
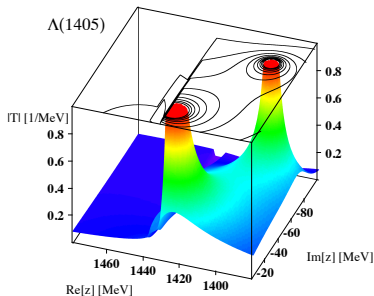
$$R_n = \frac{\Gamma(K^- p \rightarrow \pi^0 \Lambda)}{\Gamma(K^- p \rightarrow \text{neutral states})},$$

$$R_c = \frac{\Gamma(K^- p \rightarrow \pi^+ \Sigma^-, \pi^- \Sigma^+)}{\Gamma(K^- p \rightarrow \text{inelastic channels})}$$

$\Lambda(1405)$ resonance

- long history since the prediction by Dalitz and Tuan in 1959
- $\Lambda(1405) 1/2^-$ is much lighter than $N^*(1535)$ and a potential spin-orbit partner $\Lambda(1520) 3/2^-$ which is difficult to explain within a standard constituent quark model
- hadronic molecule, a loosely bound $\bar{K}N$ state? a pentaquark?
- most common interpretation - $\bar{K}N$ quasi-bound state submerged in $\pi\Sigma$ continuum, a result of coupled channels $\pi\Sigma - \bar{K}N$ dynamics
- unitary coupled channels approaches based on effective chiral Lagrangian generate two poles related to $\Lambda(1405)$ (Oller, Meißner in 2001)
- topics related to $\Lambda(1405)$ include \bar{K} -nuclei and role of strangeness in dense nuclear matter (e.g. neutron stars)

Model predictions - $\Lambda(1405)$ resonance



Hyodo, Jido - Prog. Part. Nucl. Phys. 67 (2012) 55

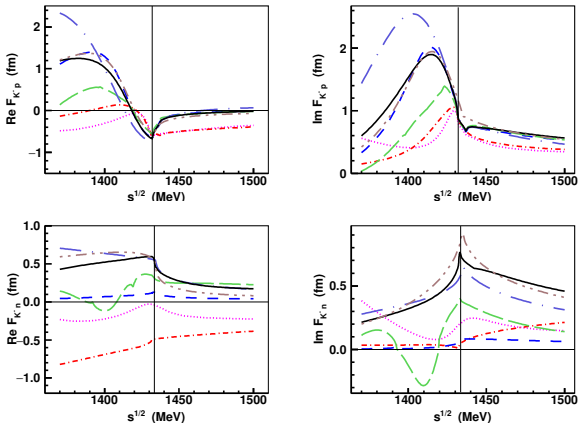
all recent (year ≥ 2000) predictions

M. Mai - Eur. Phys. J. Spec. Top. 230 (2021) 6, 1593

- the *higher* pole around 1425 MeV couples more strongly to $\bar{K}N$, the *lower* pole is much further from the real axis and has larger coupling to $\pi\Sigma$
- all models tend to agree on the position of the $\bar{K}N$ related pole
- the K^-p reactions data are not very sensitive to the position of the $\pi\Sigma$ related pole

Model predictions - K^-N amplitudes

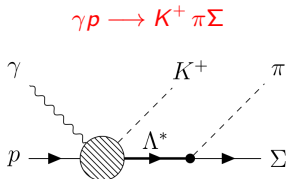
K^-p and K^-n elastic amplitudes



models: Kyoto-Munich, Prague, Bonn, Murcia, Barcelona
updated, original in A.C., M.Mai, U.G.Meißner, J.Smejkal - NPA 954 (2016) 17

Chiral models in need of more data

- Model parameters fitted to experimental data available at energies from K^-p threshold up, provide varied theoretical predictions for subthreshold energies and in the isovector sector.
- In the K^-p reactions data, the $\Lambda(1405)$ is hidden below the threshold. The resonance can be seen in processes, where $\pi\Sigma$ re-scatter in the final state, e.g.



In this **two meson protoproduction reaction** the K^+ meson carries away momentum, enabling a scan in the invariant mass of the $\pi\Sigma$ system down to its production threshold.

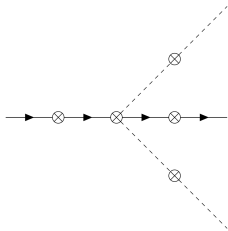
$\pi\Sigma$ photoproduction: Formalism

formalism outlined in: P. C. Bruns - arXiv:2012.11298 [nucl-th] (2020)

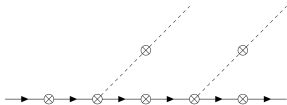
application to $\pi\Sigma$ mass spectra predictions:

P. C. Bruns, A. C., M. Mai - Phys. Rev. D 106 (2022) 7, 074017

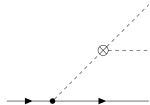
leading-order $B\chi PT$ used to derive expressions for the photoproduction amplitude \mathcal{M} , constructed from tree level graphs:



Weinberg-Tomozawa (WT)



Born term (BT) - B1, B2



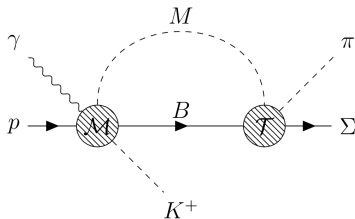
anomalous (AN)

lines: directed - baryons, dashed - pseudoscalar mesons; \otimes symbols - photon insertions

$5 + (2 \times 7) + 1 = 20$ tree graphs, 16 independent \mathcal{M}_j structure functions

$\pi\Sigma$ photoproduction: Formalism

Final state interaction of the MB pair needs to be accounted for:



$\pi\Sigma - \bar{K}N$ coupled channels models provide the $f_{\ell\pm}^{c',c}(M_{\pi\Sigma})$ amplitudes, that describe the scattering from channel c to channel c' ($c, c' = \pi\Lambda, \pi\Sigma, \bar{K}N, \eta\Lambda, \dots$)

unitarized amplitudes for $\gamma p \rightarrow K^+ MB$ will be taken as the coupled-channel vector

$$[\mathcal{A}_{0+}^i] = [\mathcal{A}_{0+}^{i(\text{tree})}] + [f_{0+}] [8\pi M_{\pi\Sigma} G(M_{\pi\Sigma})] [\mathcal{A}_{0+}^{i(\text{tree})}]$$

The second term represents the final-state MB rescattering and $G(M_{\pi\Sigma})$ is a diagonal channel-space matrix with entries given by regularized loop integrals

$\pi\Sigma$ photoproduction: Formalism

There are four independent structure functions $\mathcal{A}_{0+}^i(s, M_{\pi\Sigma}^2, t_K)$ constructed from \mathcal{M}_j , projected on s-wave and satisfying the partial-wave unitarity relation

$$\text{Im}(\mathcal{A}_{0+}^i) = (f_{0+})^\dagger(|\vec{p}^*|)(\mathcal{A}_{0+}^i), \quad i = 1, \dots, 4.$$

Neglecting $\ell > 0$ contributions, we get

$$\frac{d^2\sigma}{d\Omega_K dM_{\pi\Sigma}} = \frac{|\vec{q}_K||\vec{p}_\Sigma^*|}{(4\pi)^4 s |\vec{k}|} |\mathcal{A}|^2,$$

$$\begin{aligned} 4|\mathcal{A}|^2 &= (1-z_K) |\mathcal{A}_{0+}^1 + \mathcal{A}_{0+}^2|^2 + (1+z_K) |\mathcal{A}_{0+}^1 - \mathcal{A}_{0+}^2|^2 \\ &+ (1-z_K) \left| \mathcal{A}_{0+}^1 + \mathcal{A}_{0+}^2 + \frac{2|\vec{q}_K|(1+z_K)}{M_K^2 - t_K} ((\sqrt{s} + m_N)\mathcal{A}_{0+}^3 + (\sqrt{s} - m_N)\mathcal{A}_{0+}^4) \right|^2 \\ &+ (1+z_K) \left| \mathcal{A}_{0+}^1 - \mathcal{A}_{0+}^2 - \frac{2|\vec{q}_K|(1-z_K)}{M_K^2 - t_K} ((\sqrt{s} + m_N)\mathcal{A}_{0+}^3 - (\sqrt{s} - m_N)\mathcal{A}_{0+}^4) \right|^2, \end{aligned}$$

with $z_K \equiv \cos\theta_K$, θ_K being the angle between \vec{q}_K and \vec{k} in the overall c.m. frame.

$\pi\Sigma$ photoproduction: Formalism

loop function integral:

$$i G^{c=bj}(M_{\pi\Sigma}) = \int_{\text{reg.}} \frac{d^4 l}{(2\pi)^4} \frac{1}{((p_b + q_j - l)^2 - m_b^2 + i\epsilon)(l^2 - M_j^2 + i\epsilon)}$$

two coupled channels approaches to generate the f_{0+} amplitudes:

Bonn B2, B4 models - M.Mai, U.-G.Meißner, Eur. Phys. J. A 51 (2015) 30

BW model - D.Sadasivan, M.Mai, M.Döring, Phys. Lett. B 789 (2019) 329–335

dimensional regularization used in $G(M_{\pi\Sigma})$, mass scales μ_c

Prague P model - P.C.Bruns, A.C., Nucl. Phys. A 1019 (2022) 122378

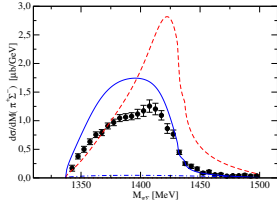
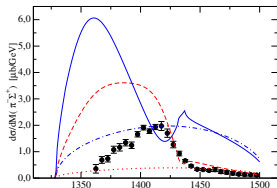
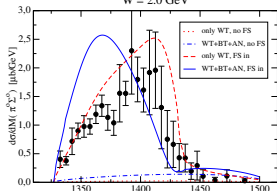
Yamaguchi form factors used in $G(M_{\pi\Sigma})$, inverse ranges α_c

CLAS data: K. Moriya et al. - Phys. Rev. C 87 (2013) 035206

Result: different models provide varied predictions of the $\pi\Sigma$ mass spectra

$\pi\Sigma$ photoproduction: FSI impact

$W = 2.0 \text{ GeV}$



CLAS data (2013) by Moriya et al.

c.m. energy $W = \sqrt{s} = 2.0 \text{ GeV}$

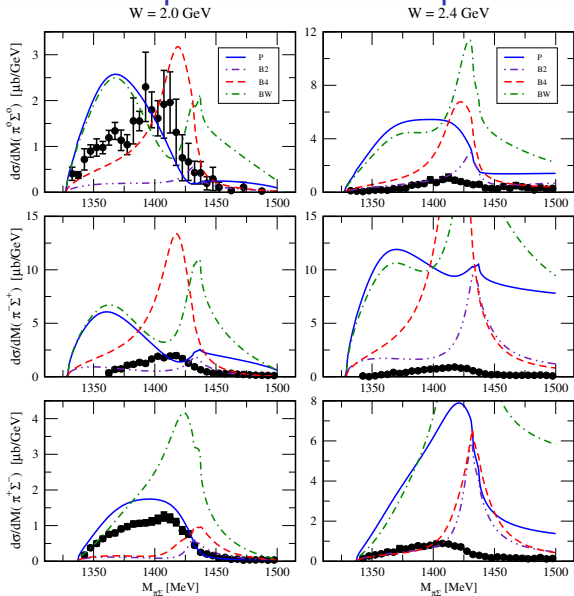
P model used for the MB amplitudes

- **only WT, no FSI:**
small (or zero for $\pi^+\Sigma^-$) cross sections
- **WT+BT+AN, no FSI:**
the cross sections remain flat, the $\pi^-\Sigma^+$ one reaches magnitude comparable with the data
- **addition of FSI:**
 MB rescattering is responsible for the peak structure
 $\pi^0\Sigma^0$ and $\pi^+\Sigma^-$ reproduced rather well
Born terms move the peak to lower energies

parameter-free predictions!

no adjustment to the f_{0+} amplitudes

$\pi\Sigma$ mass spectra - model dependence



New results - FSI not fixed

First time fits to combined K^-p reactions and $\pi\Sigma$ photoproduction data with the FSI no longer fixed at a particular $\pi\Sigma - \bar{K}N$ model parameters setup.

A. C., P. C. Bruns - submitted to NPA, arXiv:2305.06205

- The P model used for the $MB \rightarrow \pi\Sigma$ amplitude represented by the MB rescattering \mathcal{T} bubble:
6 regularization scales α_c , 6 NLO couplings ($b_0, b_F, d_{1...4}$)
- Tree level photoproduction amplitudes representing the \mathcal{M} bubble multiplied by MB and K^+ form factors: 6 regularization scales β_c (3 fixed), and β_K
- Yamaguchi forms adopted for all form factors $g_c(k^*) = 1/[1 + (k^*/\alpha_c)^2]$ etc.
- fitted data: kaonic hydrogen characteristics, K^-p threshold branching ratios, K^-p reaction cross sections, $\pi\Sigma$ photoproduction mass distributions at $\sqrt{s} = 2.1$ GeV

CLAS data			K^-p threshold		K^-p cross sections							all	
$\pi^0\Sigma^0$	$\pi^+\Sigma^+$	$\pi^+\Sigma^-$	atom	BRs	$\pi^0\Lambda$	$\pi^0\Sigma^0$	$\pi^+\Sigma^+$	$\pi^+\Sigma^-$	K^-p	\bar{K}^0n	$\eta\Lambda$	$\eta\Sigma^0$	total
34	30	32	2	3	4	4	31	32	27	22	24	7	252

$$\chi^2/\text{dof} = \frac{\sum_i N_i}{N_{\text{obs}}(\sum_i N_i - N_{\text{par}})} \sum_i \frac{\chi_i^2}{N_i}, \quad N_{\text{obs}} = 13, \quad N_{\text{par}} = 16$$

New results - FSI not fixed

results for 4 selected solutions (local χ^2 minima):

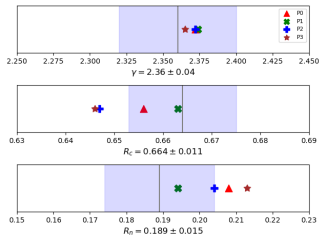
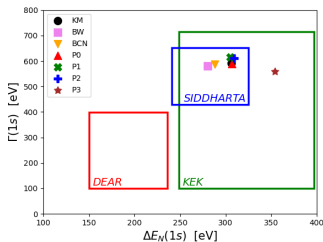
P0 $\chi^2/\text{dof} \approx 5.40$, MB FSI sector fixed to the P model setting, 4 parameters

P1 $\chi^2/\text{dof} \approx 3.34$, both, FSI and tree level photoproduction sectors varied, 16 par.

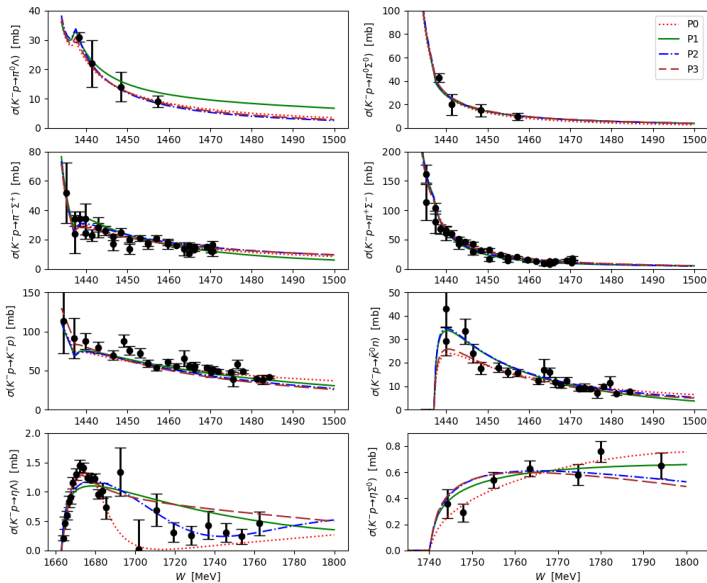
P2 $\chi^2/\text{dof} \approx 4.41$, same as for P1

P3 $\chi^2/\text{dof} \approx 4.72$, same as for P1

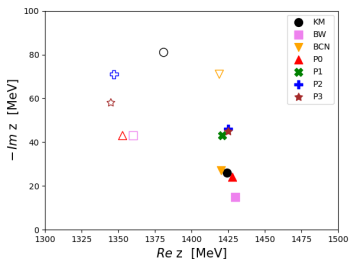
$K^- p$ threshold data:



$K^-p \rightarrow MB$ total cross sections



$\Lambda(1405)$ poles predictions

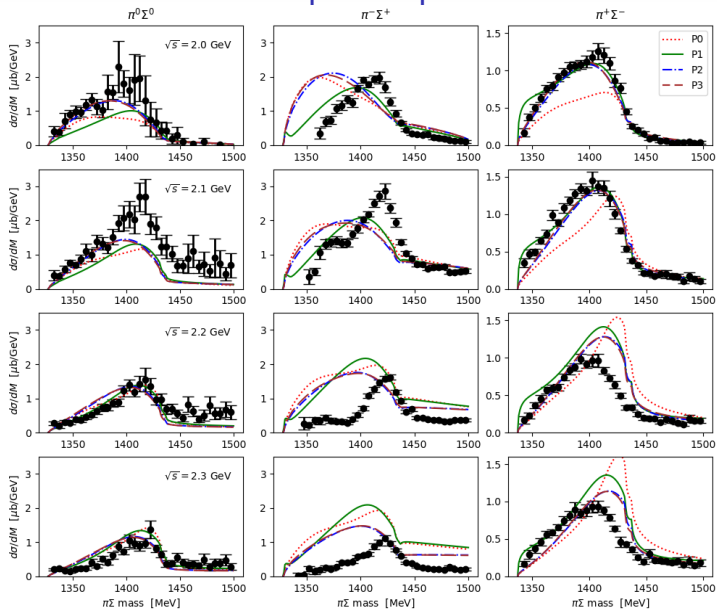


$\Lambda(1405) - z_1$ and z_2 ; $\Lambda(1670) - z_3$

model	z_1 [MeV]	z_2 [MeV]	z_3 [MeV]
P0	(1353,-43)	(1428,-24)	(1677,-14)
P1	—	(1421,-43)	—
P2	(1347,-71)	(1425,-46)	(1725,-57)
P3	(1345,-58)	(1425,-45)	(1665,-7.1)

- The **P1** model provides the best χ^2 but was found **unphysical** due to generating an extremely narrow $I = 1$ resonance close to $\pi\Sigma$ threshold. It is also missing the *lower mass* $\Lambda(1405)$ and the $\Lambda(1670)$ poles.
- Apparently, the K^-p reaction (and to some extent threshold) **data are not very sensitive to the pole positions**. Similar observations by **Revai** (with a non-chiral model), **Shevchenko** (phenomenology), or **Anisovich** (partial wave analysis), all indicating **good reproduction of the K^-p data with one-pole $\Lambda(1405)$ models**.
- All our models agree on the position of the **higher mass $\Lambda(1405)$ pole with $\text{Im } z_2$ much larger than in the fits based merely on the K^-p reactions data**. The *lower mass* $\Lambda(1405)$ pole seems to be constrained around $z_1 \approx (1350, -60)$ MeV. **Model dependence should be checked!**

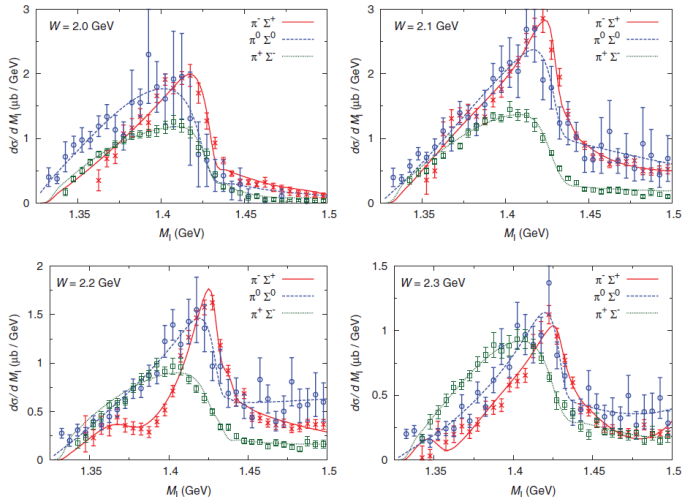
$\pi\Sigma$ mass spectra predictions



$\pi\Sigma$ mass spectra predictions

- Despite being rather simple, the model reproduces reasonably well the $\pi^0\Sigma^0$ and $\pi^+\Sigma^-$ mass distributions.
- The energy dependence seems to be under control thanks to introducing the g_{K^+} form-factor in the tree level photoproduction amplitudes.
- Our model fails to reproduce the $\pi^-\Sigma^+$ mass distributions.
- The applied $\pi\Sigma$ photoproduction formalism may not be realistic: *p-waves, vector meson contributions?*. Thus, the results should be taken with caution and may change once a more advanced photo-kernel is implemented.
- Much better agreement with CLAS photoproduction data can be achieved either by an ad-hoc modelling of the photoproduction dynamics (Roca, Oset, Mai, Meißner) or by employing much more parameters, and keeping some of them energy dependent (Nakamura, Jido).

Results by Nakamura, Jido (2014)

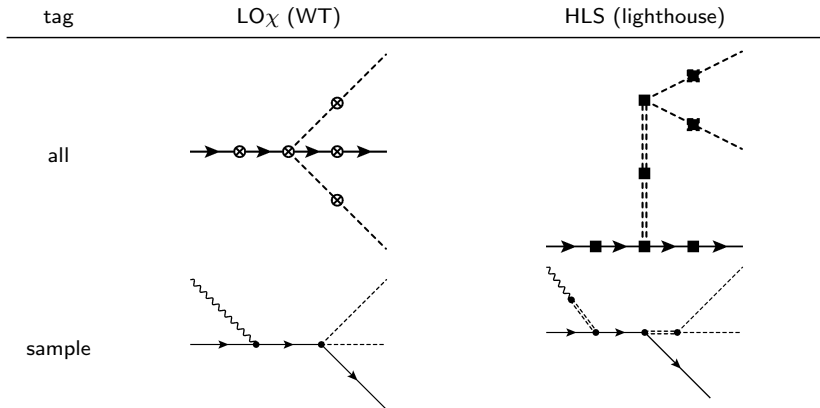


20 subtraction constants, 15 energy dependent complex couplings λ_n^j , common form factor scale $\beta_c = \beta_{K^+} = \Lambda$

Current development

enhanced photo-kernel by including processes involving vector mesons (in particular K^*), adopting the concept of hidden local symmetries (HLS) and VM dominance, Lagrangian from Bando, Kugo, Yamawaki - Phys. Rept. 164 (1988), 217

vector mesons implementation: WT replaced by lighthouse diagrams



Current development

some comments on the WT/lighthouse graphs:

- The lighthouse graphs in which the photon/VM attaches to the baryon or meson line generate the amplitudes that match exactly those obtained in the $\text{LO}\chi$ approach for $M_V \rightarrow \infty$.
- The lighthouse graphs with the photon/VM connecting to the VM propagator are exclusive for the approach based on the HLS and the respective amplitudes vanish in the limit $M_V \rightarrow \infty$.
- Our preliminary results with just one lighthouse amplitude indicate large contributions of the K^* mesons to the $I = 1$ sector, which might be just what we need to reproduce the $\pi^- \Sigma^+$ mass spectra.
- Our evaluation of the lighthouse graphs is almost completed but it is impossible to predict what will be the final picture, especially once we introduce the BT and AN graphs in the HLS approach.

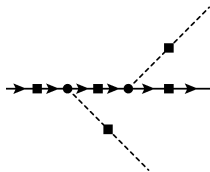
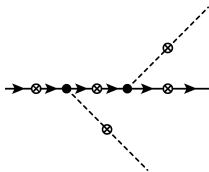
Current development - BT graphs

tag

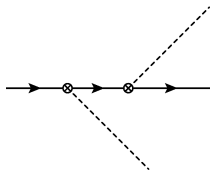
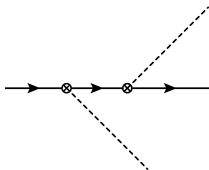
$LO\chi$

HLS

G

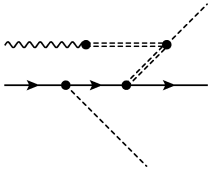
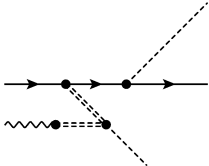


H



- Since we work with real photons and assume the validity of the VMD the G-graphs will provide exactly the same amplitudes in both approaches.
- As there is no BBVM vertex in the HLS Lagrangian but the $BB\gamma M$ one is available (unlike the $BB\gamma$ one), the H-graphs lead to the same amplitudes in both approaches as well.

Current development - more BT graphs

tag	LO_χ	HLS
I	—	
J	—	

- The I and J graphs, that include an anomalous VVM vertex, do not have counterparts in the LO_χ approach.

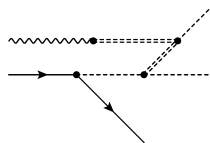
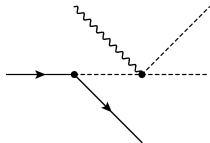
Current development - AN graphs

tag

$LO\chi$

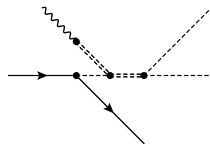
HLS

K



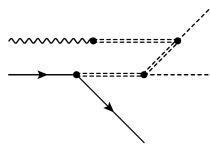
L

—



M

—



Summary

- The up-to-date (NLO) chirally motivated $\pi\Sigma - \bar{K}N$ models provide very different predictions for the MB amplitudes at energies below $\bar{K}N$ threshold.
- The $\Lambda(1405)$ energy region can be accessed by studying processes involving $\pi\Sigma$ rescattering in the final state. The $\pi\Sigma$ photoproduction on protons represents such a process where the **MB rescattering plays a crucial role** as our results demonstrate.
- Our approach to the two-meson photoproduction implements **coupled-channel unitarity, low-energy theorems from ChPT and gauge invariance**. We have revealed **large variations when different models for the MB amplitudes are adopted**.
- Our new results of **fits that combine the K^-p reactions data with those on the $\pi\Sigma$ photoproduction** can achieve a reasonable χ^2/dof but are not satisfactory especially for the $\pi^-\Sigma^+$ mass distributions.
- The presented models tend to limit the mass of the **lower mass $\Lambda(1405)$ pole** and yield a **larger width of the pole that couples more strongly to $\bar{K}N$** . A possible relation to a large absorption width found for the $\bar{K}NN$ bound state at J-PARC?
- **We need a more precise experimental data on low energy K^-p reactions** as the current ones are not too restrictive on the theoretical models. **Our model** (the photoproduction kernel) **should also be improved by implementing the vector mesons** (work in progress).

EXTRA: Comparison with other approaches

our photoproduction amplitude constructed from four \mathcal{A}_{0+}^i amplitudes

$$[\mathcal{A}_{0+}^i(s, M_{\pi\Sigma})] = [\mathcal{A}_{0+}^{i(\text{tree})}(s, M_{\pi\Sigma})] + [f_{0+}(M_{\pi\Sigma})] [8\pi M_{\pi\Sigma} G(M_{\pi\Sigma})] [\mathcal{A}_{0+}^{i(\text{tree})}(s, M_{\pi\Sigma})]$$

L. Roca, E. Oset - Phys. Rev. C 87 (2013) 055201

M. Mai, U.-G. Meißner - Eur. Phys. J. A 51 (2015) 30

makeshift photoproduction amplitude $[\mathcal{A}] = [f_{0+}] [8\pi M_{\pi\Sigma} G(M_{\pi\Sigma})] [C(\sqrt{s})]$

S.X. Nakamura, D. Jido - PTEP 2014 (2014) 023D01

similar to our approach with some non-relativistic simplifications, additional contributions from K^* exchange, phenomenological energy dependent contact terms, and adjustments to the first loop function and to the photoproduction vertex

E. Wang et al. - Phys. Rev. C 95 (2017) 015205

focus on triangle singularity contribution $\gamma p \rightarrow N^*(2030) \rightarrow K^* \Sigma \rightarrow K^+ \Lambda(1405)$
combined with K , K^* meson exchanges and a contact term

$$\mathcal{A} = \mathcal{A}^{\text{tree}} = a t_{\text{triangle}} + b t_{K \text{ exchange}} + c t_{K^* \text{ exchange}} + d t_{\text{contact}}$$

All these fit a good number of model parameters to reproduce the CLAS data.

In contrast, we just demonstrated what can be achieved with **just four new parameters** (originally parameter-free!) approach based on (unitarized) ChPT.