Light hypernuclei in the framework of J-NCSM and χEFT

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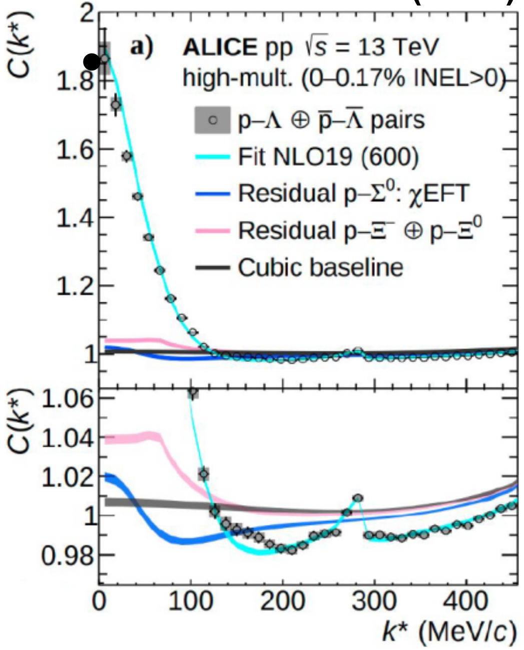
ROCKSTAR: Towards a Roadmap of the Crucial measurements of Key observables in Strangeness reactions for neutron Stars EOS

9th - 13th October 2023, ECT* Trento, Italy

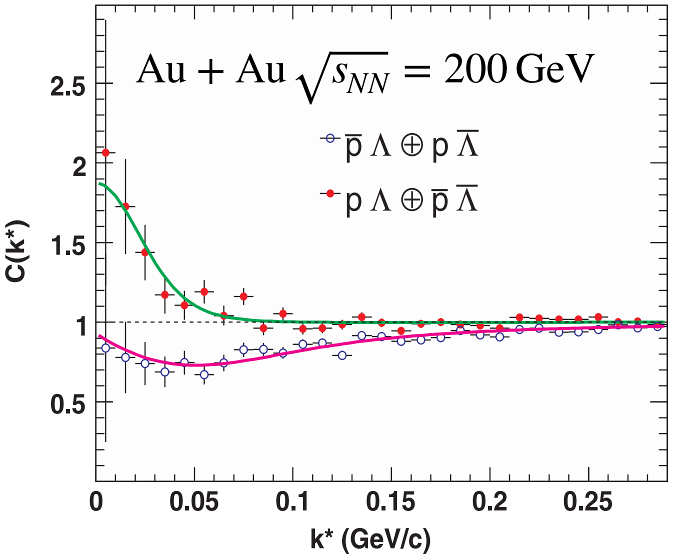
In collaboration with: Johann Haidenbauer, Ulf-G. Meißner, Andreas Nogga

Progress in studying YN interactions

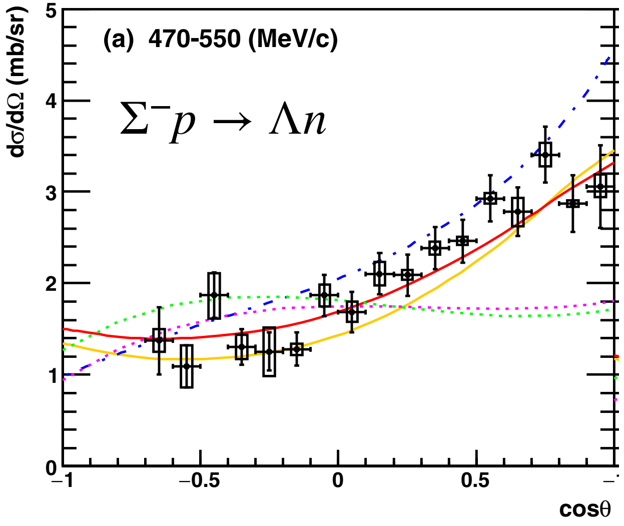
- **ALICE PLB 833 (2022)**



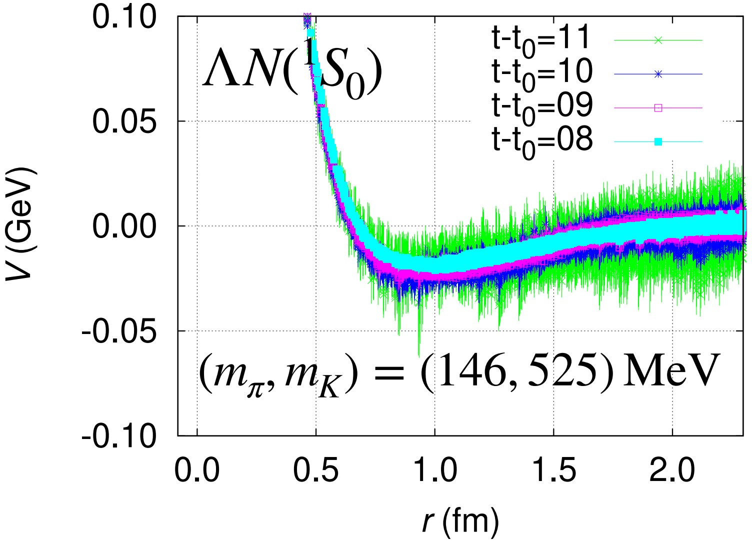
- **STAR collaboration PRC 74(2006)**



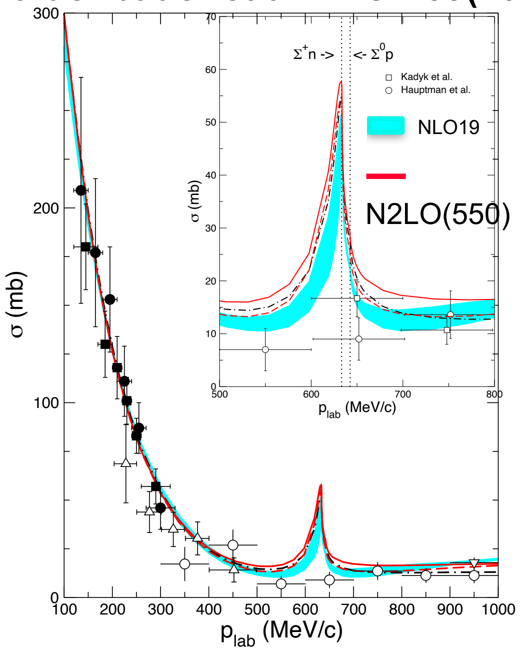
- **K. Miwa et al. PRL 128(2022)**



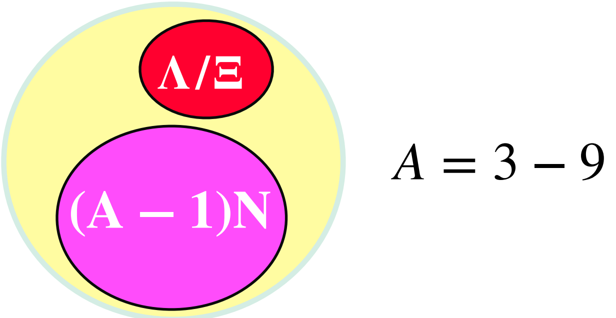
- **H. Nemura et al. PoS LATTICE2021**



- **J. Haidenbauer et al. EPJA 59(2023)**



- **ab initio** treatment of light p-shell $\Lambda(\Xi)$ hypernuclei with the NCSM:



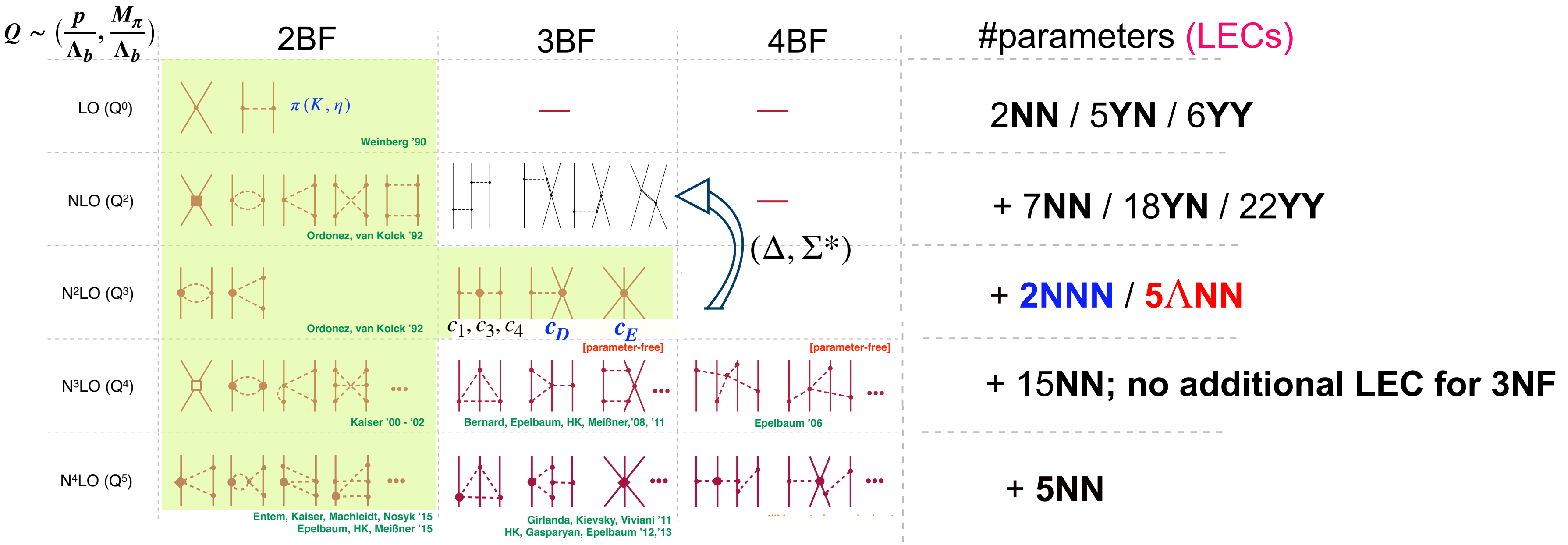
→ **directly** compute B_Λ from the underlying YN interactions

R. Wirth et al. PRL117 (2016), PRC100 (2019)
H. Le et al. EPJA 56 (2020), PRC 117(2023)

BB interactions in χEFT

NN: S. Weinberg, van Kolck, Kaiser, Idaho, Bonn, Bochum, …

YN,YY: H. Polinder et al. NPA 779(2006); J. Haidenbauer et al .NPA 915(2013), EPJA (2019,2020,2023)



(adapted from H. Krebs CD workshop 2021)

- **~5000** NN + ^{2}H → NN forces up to $\mathbf{N^4LO^+}$ $(\chi^2 \sim 1)$ **(P. Reinert et al. EPJA (2018))**

  3NF at $\mathbf{N^2LO}$: $c_{1,3,4}$ from fit to πN data; $c_{E,D}$ from $^3\mathbf{H}$ + **Nd** scattering data **(E. Epelbaum et al. EPJA 56(2020))**

  → good description for energies of light and medium nuclei $(A \leq 40)$ **(LENPIC(2021,2022))**

- **~36 YN** data, no YN bound state → YN forces up to $\mathbf{NLO, N^2LO}$ **(talk by Nogga)**

  YNN forces at $\mathbf{N^2LO}$ with decuplet saturation: 2LECs **(S. Petschauer et al. PRC 93(2016))**

  - fix 2LECs to $B_\Lambda(^4_\Lambda\mathrm{He}(0^+,1^+))$? (**work in progress**)
  - impact to neutron star **(talk by Weise)**

Jacobi-NCSM approach

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga EPJA 56 (2020)

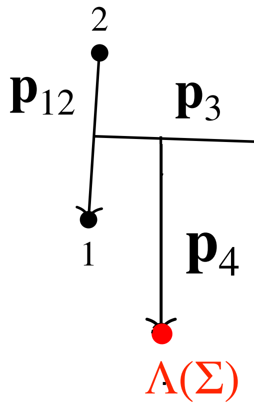
- **Idea:** represent the A-body translationally invariant hypernuclear Hamiltonian:

$$H = T_{rel} + V^{NN} + V^{YN} + V^{NNN} + V^{YNN} + \Delta M + \cdots$$

ΔM: $\Lambda N \leftrightarrow \Sigma N$

in a basis constructed from HO functions

- Jacobi basis: **depends on relative Jacobi coordinates of all particles**

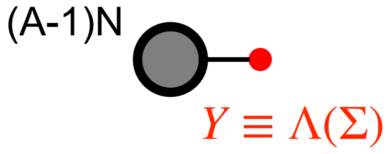


(A-1)N, Λ(Σ)

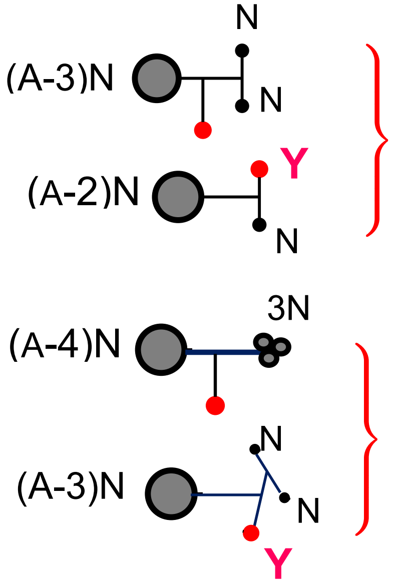
$$| (A-1)N - \Lambda(\Sigma) \rangle = | \mathcal{N} J T, \underbrace{\mathcal{N}_{A-1} J_{A-1} T_{A-1},}_{antisym.(A-1)N} \underbrace{n_Y l_Y I_Y t_Y;}_{\Lambda(\Sigma)\ state} (J_{A-1}(l_Y s_Y) I_Y) J, (T_{A-1} t_Y) T \rangle$$

(independent of ω)

- intermediate bases for evaluating Hamiltonian:



trans. coefficients → **independent of ω**



→ for NN, YN forces

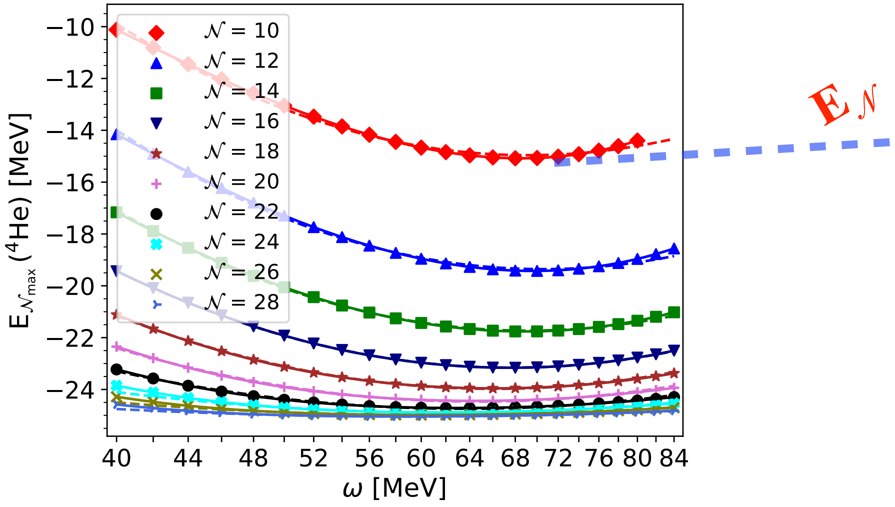
→ for 3N, YNN forces

- basis truncation: $\mathcal{N} = \mathcal{N}_{A-1} + 2n_\lambda + \lambda \leq \mathcal{N}_{max} \Rightarrow E_b = E_b(\omega, \mathcal{N}_{max})$ **extrapolation** → $E_{b,\infty}$

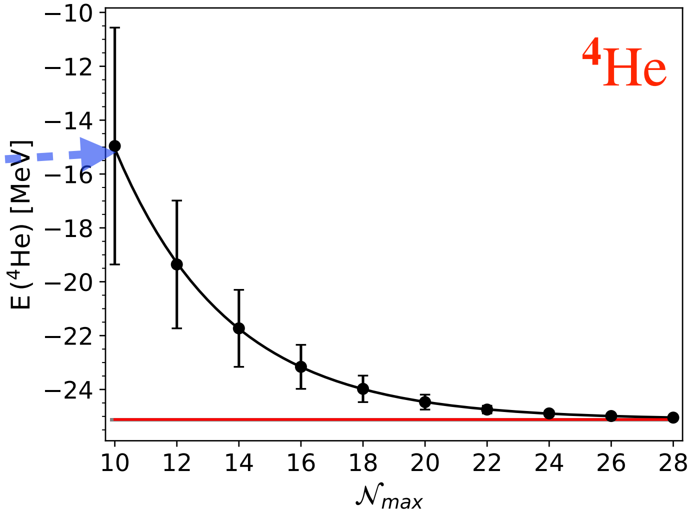
Extrapolation in ω & $\mathcal{N}$ spaces

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga EPJA 56 (2020)

- $\mathbf{E_b}(\omega, \mathcal{N}) = \mathbf{E}_{\mathcal{N}} + \kappa(\log(\omega) - \log(\omega_{\mathbf{opt}}))^2$



- $\mathbf{E}_{\mathcal{N}} = \mathbf{E}_{\infty} + \mathbf{A}e^{-\mathbf{b}\mathcal{N}}$



NN: **bare** SMS N^{2}LO(550)

$\mathbf{E}(^4\mathbf{He}, \mathbf{NCSM}) = -25.14 \pm 0.06$

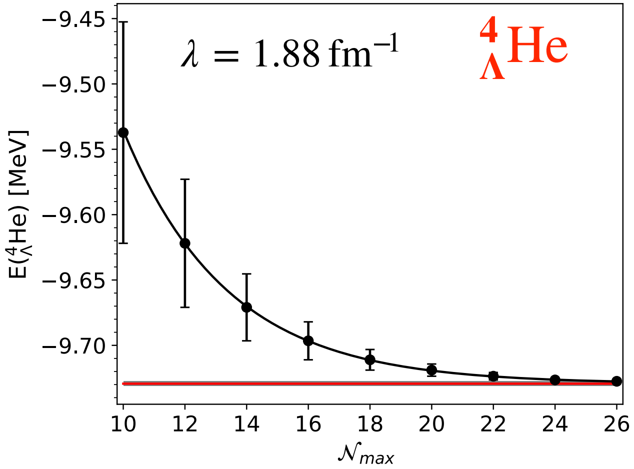
$\mathbf{E}(^4\mathbf{He}, \mathbf{FY}) = -25.15 \pm 0.02$

$\delta E = E_{\infty} - E_{\mathcal{N}_{max}}$

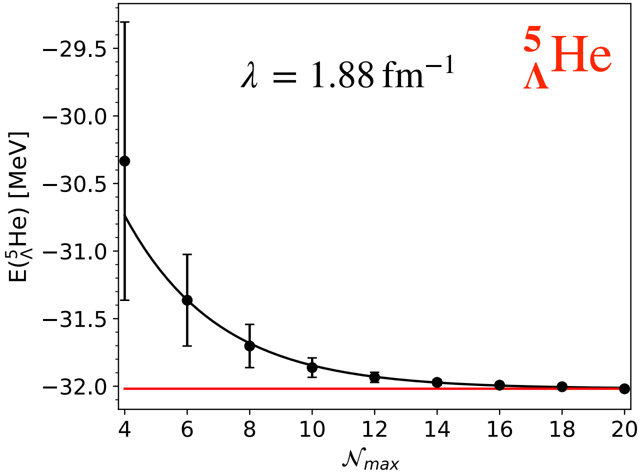
Numerical uncertainties

- NCSM calculations for hypernuclei with **bare SMS** NN (3N) and YN interactions **converge poorly**
- NCSM uncertainties for **SRG-evolved** potentials:
  - **~ several keV for $\mathbf{A \le 5}$**
  - **~ hundred(s) keV for $\mathbf{A = 7\,(8)}$**

$\mathbf{E}(^4_\Lambda\mathbf{He}) = -9.729 \pm 0.002$



$\mathbf{E}(^5_\Lambda\mathbf{He}) = -32.018 \pm 0.001$



Similarity Renormalization Group (SRG)

Idea: **continuously apply unitary transformation to H to suppress off-diagonal matrix elements**

→ speed up the convergence of NCSM calculations (observables e.g. energies are conserved)

F.J. Wegner NPB 90 (2000). S.K. Bogner, R.J. Furnstahl, R.J. Perry PRC 75 (2007)

$$\frac{dV(s)}{ds} = [[T_{rel}, V(s)], H(s)], \qquad H(s) = T_{rel} + V(s) + \Delta M$$

$$s = 0 \to \infty \qquad V(s) = V_{12}(s) + V_{13}(s) + V_{23}(s) + V_{123}(s), \quad V_{123,s=0} \equiv V^{bare}_{NNN}; \quad (V^{bare}_{YNN} = 0)$$

- separate flow equations for 2- and 3-body interactions:

S.K. Bogner et al PRC75 (2007),
K. Hebeler PRC85 (2012)

$$\frac{dV^{NN}(s)}{ds} = [[T^{NN}, V^{NN}], T^{NN} + V^{NN}]$$

$$\frac{dV^{YN}(s)}{ds} = [[T^{YN}, V^{YN}], T^{YN} + V^{YN} + \Delta M]$$

$$\frac{dV_{123}}{ds} = [[T_{12}, V_{12}], V_{31} + V_{23} + V_{123}] + [[T_{31}, V_{31}], V_{12} + V_{23} + V_{123}] + [[T_{23}, V_{23}], V_{12} + V_{31} + V_{123}] + [[T_{rel}, V_{123}], H_s]$$

→ **SRG-induced YNNs are generated even if** $V^{bare}_{YNN} = 0$

- perform evolution in p-space. Evolved potentials **can be directly used in many-body & nuclear matter calculations**

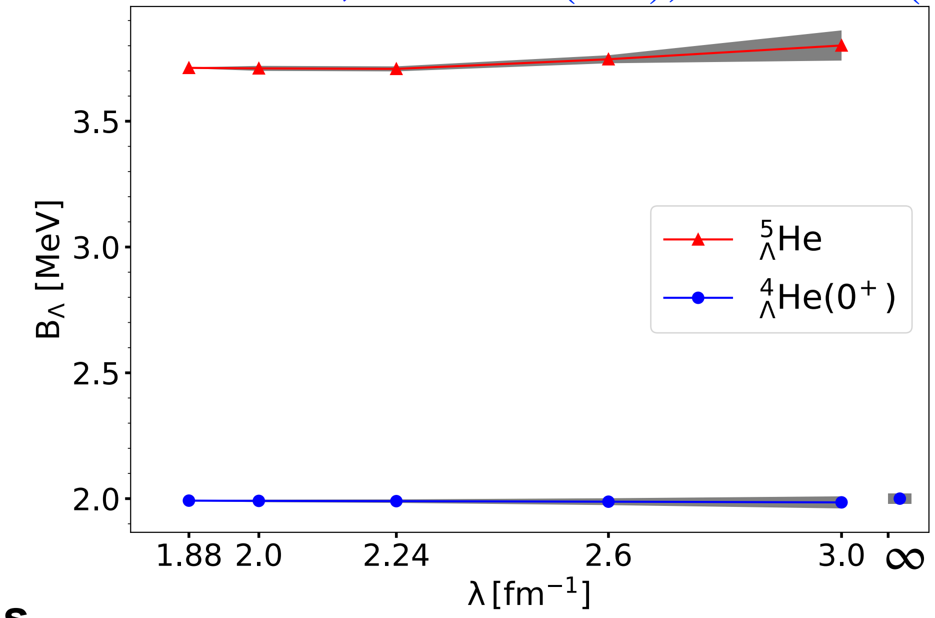
Effect of SRG-induced 4BFs in A=4,5

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga (arXiv:2308.01756)

- induced forces beyond 3BF are not included; **estimate size of omitted forces by varying** $\lambda = (4\mu^2/s)^{1/4}$

λ [fm^{-1}]	$B_\Lambda({}^4_\Lambda\mathrm{He}, 0^+)$	$B_\Lambda({}^5_\Lambda\mathrm{He})$
1.88	1.992 ± 0.002	3.712 ±0.001
2.00	1.991 ± 0.005	3.705± 0.005
2.236	1.990 ± 0.007	3.708 ± 0.006
2.60	1.989 ± 0.014	3.744 ± 0.008
3.00	1.985 ± 0.024	3.806 ± 0.030
∞	2.01 ± 0.02	

NN: N^{4}LO$^+$, **3N**: N^{2}LO(450); **YN**: N^{2}LO(550)



$\lambda = \infty$: FY calculation using **bare NN, 3N & YN potentials**

- variation of B_Λ for $1.88 \le \lambda \le 3.0$ **fm^{-1}**: $\Delta B_\Lambda({}^4_\Lambda\mathrm{He}) = 10 \pm 25$ KeV

  $\Delta B_\Lambda({}^5_\Lambda\mathrm{He}) = 90 \pm 30$ KeV

→ **contributions of SRG-induced 4BFs to** $B_\Lambda({}^4_\Lambda\mathrm{He}, {}^5_\Lambda\mathrm{He})$ **are small**

Results for A=3-8 hypernuclei

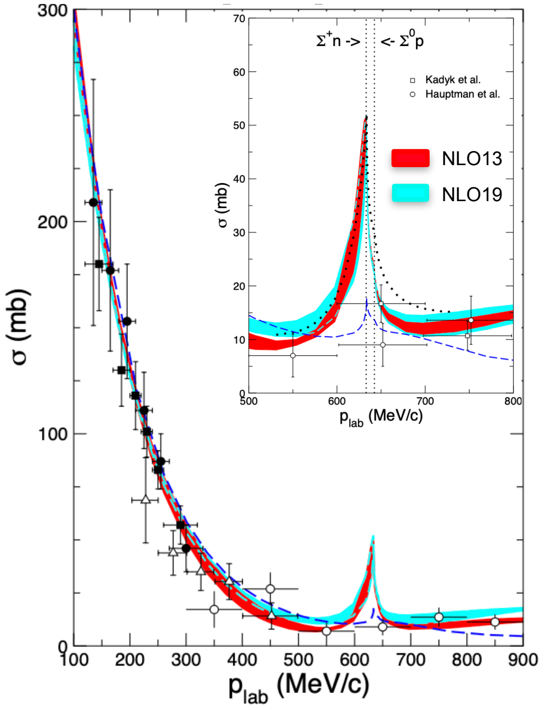
NN: **SMS** **$N^4LO^+(450)$** 3N: **$N^2LO(450)$**

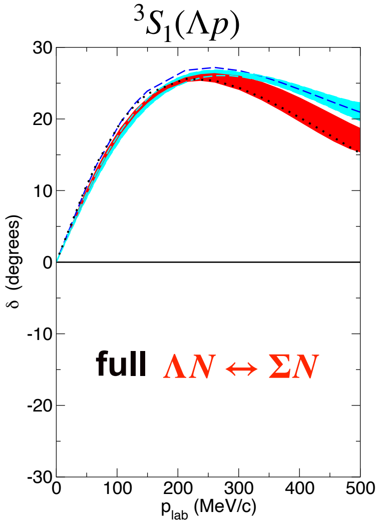
YN: **NLO13**, **NLO19**(500); **+SRG-induced YNN, NNN**

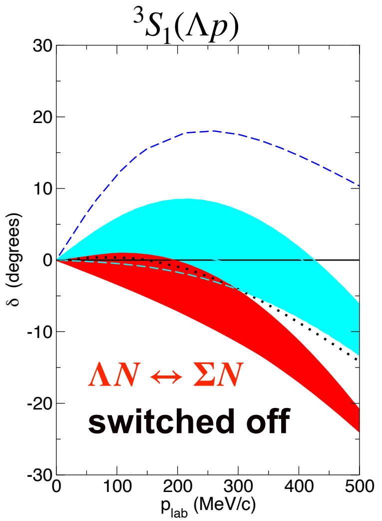
NLO13 & NLO19 YN potentials

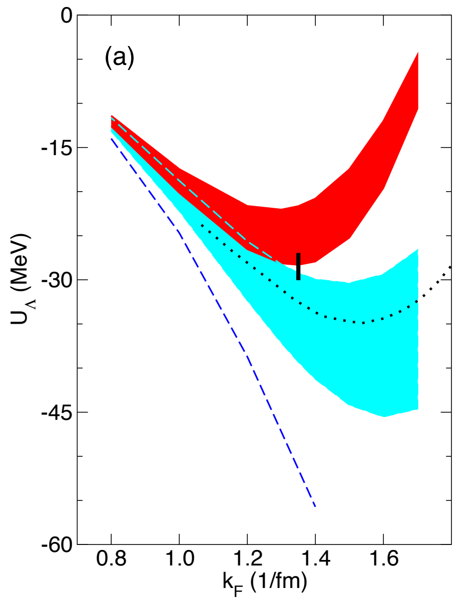
NLO13: J. Haidenbauer et al. NPA 915(2013); NLO19: J. Haidenbauer, U.-G. Meißner, A. Nogga EPJA 56(2020)

- **NLO13:** S-wave LECs are fitted to YN data; **NLO19:** 3 LECs are inferred from NN sector
- almost **phase equivalent** (yield equivalent description of YN scattering data)
- **NLO13 leads to a larger** $V_{\Lambda N \leftrightarrow \Sigma N}$ (especially in $^3S_1$)









$$U_\Lambda(\rho_0, 0) = -28.3, \cdots, -22.3 \quad \textbf{(NLO13)}$$
$$= -39.3, \cdots, -29.2 \quad \textbf{(NLO19)}$$
$$= -33 \quad \textbf{(HAL QCD)}$$

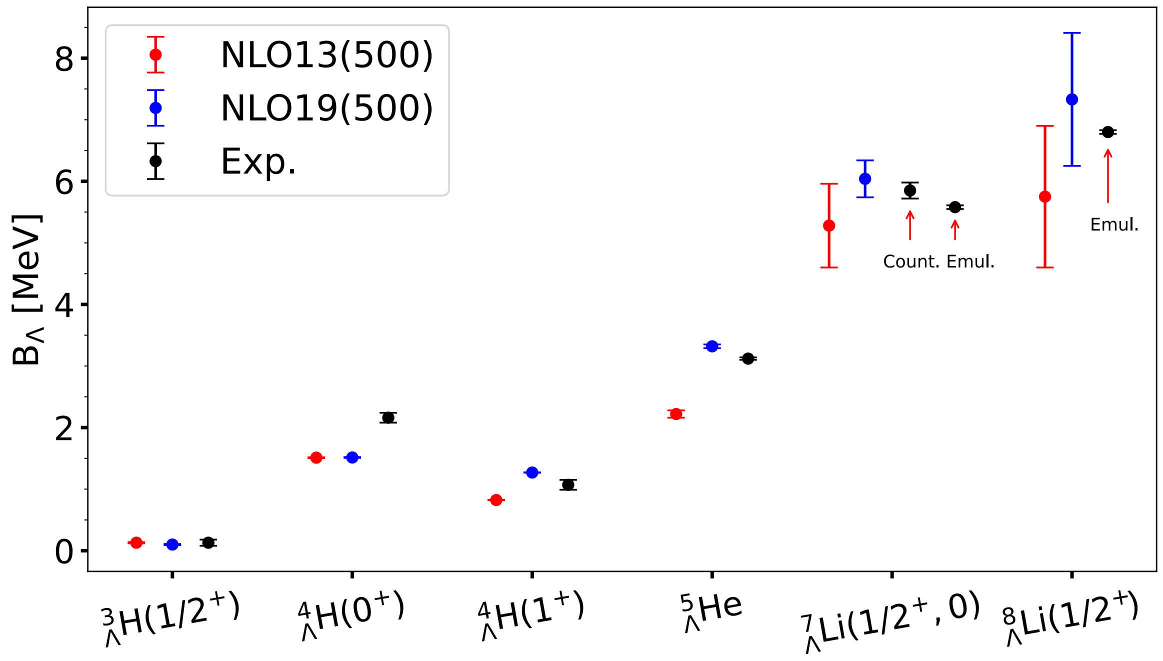
(T. Inoue *PoS* INPC2016 (2016))

→ tool to **assess effect of YNN forces** in many-body systems

(J. Haidenbauer et al EPJA 56(2020))

Results for $B_\Lambda(A \leq 8)$ with NLO13 & NLO19

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga PRC 107(2023)



	E	V_{YN}	SRG-V_{YNN} ΛNN	SRG-V_{YNN} ΛNN-ΣNN	SRG-V_{YNN} total	$\lvert\chi V_{YNN}\rvert$
$^3_\Lambda$H	-2.31	-1.88	0.08	0.04	0.14	~ 0.05
$^4_\Lambda$He(1+)	-9.50	-7.31	0.72	0.05	0.77	~ 0.2 - 0.4
$^4_\Lambda$He(0+)	-10.57	-10.2	0.89	-0.02	0.90	~ 0.2 - 0.3
$^5_\Lambda$He	-32.42	-13.61	2.40	0.15	2.57	~ 0.7 - 1.0

- **NLO13 & NLO19** phase equivalent in 2-body space
- $^4_\Lambda$H(1$^+$), $^5_\Lambda$He, $^7_\Lambda$Li, $^8_\Lambda$Li are **fairly well described** by **NLO19**;

  NLO13 underestimates these B_Λ

→ **signal** of missing **YNN forces;** contribute differently for **NLO13 & NLO19**

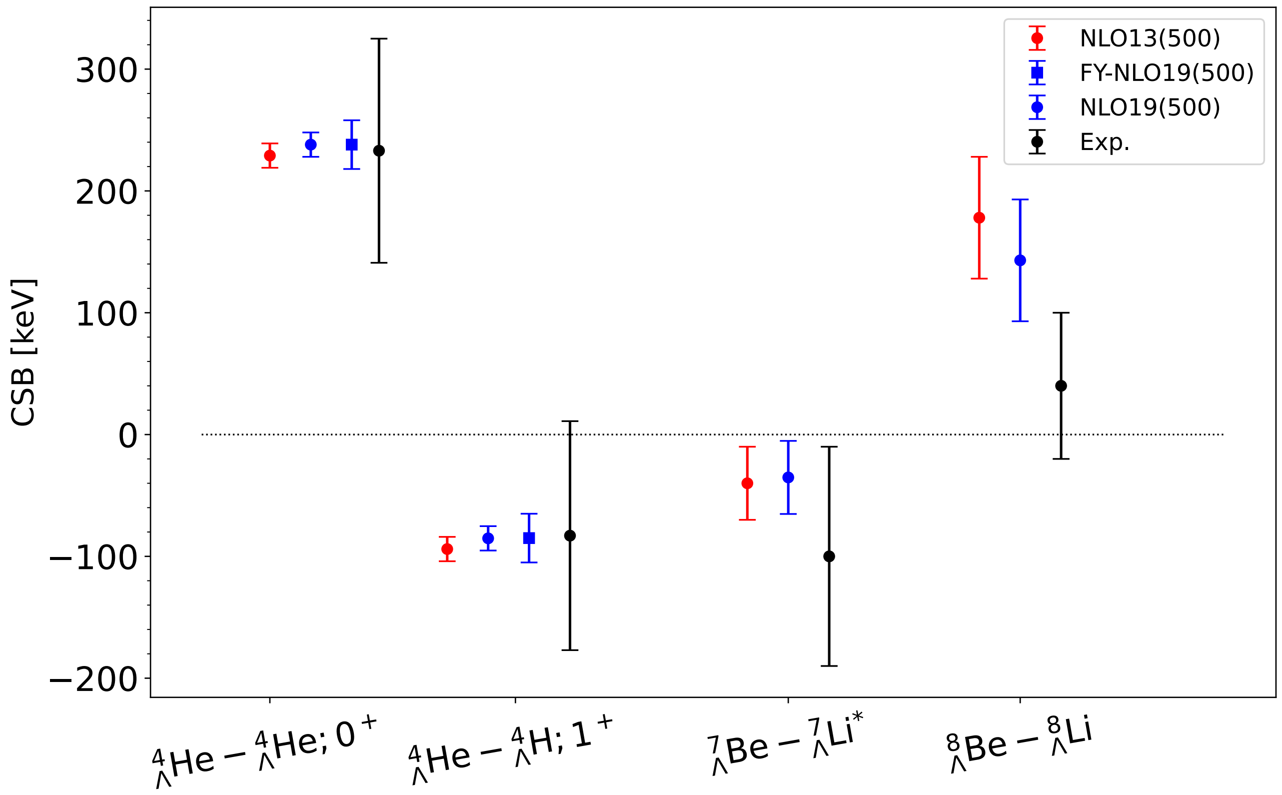
$\lvert\chi V_{YNN}\rvert$ based on **NLO13** & **NLO19** results and cutoff dependence

(J. Haidenbauer et al. EPJA(2019), HL et al. PRC(2023))

→ consistent with estimates based on **chiral truncation** **(see Nogga's talk)**

CSB predictions for A=7-8 multiplets

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga PRC 107(2023)



$$\Delta E(1^+) = B_\Lambda({}^4_\Lambda\mathrm{He}, 1^+) - B_\Lambda({}^4_\Lambda\mathrm{H}, 1^+)$$

$$= -83 \pm 94 \text{ keV} \ (\textbf{up to } 2016)$$

$$\Delta E(0^+) = B_\Lambda({}^4_\Lambda\mathrm{He}, 0^+) - B_\Lambda({}^4_\Lambda\mathrm{H}, 0^+)$$

$$= 223 \pm 92 \text{ keV} \ (\textbf{up to } 2016)$$

(see Nogga's talk)

- CSB predictions for A=7 **are comparable to experiment.**
- yield somewhat **larger CSB in A=8 doublet as compared to experiment**

→ ▸ experimental CSB splitting for A=8 **larger than 40 ± 60 keV**?

▸ A=4 CSB: **too large? different spin-dependence**?

Fitting LECs to new Star measurement

Recent STAR measurement suggests different CSB in A=4: **(STAR collaboration PLB 834 (2022))**

$$\Delta E(1^+) = B_\Lambda({}^4_\Lambda\text{He}, 1^+) - B_\Lambda({}^4_\Lambda\text{H}, 1^+) = -83 \pm 94 \text{ keV (up to 2016)} \Rightarrow \text{(CSB)}$$

$$= \mathbf{-160 \pm 140 \pm 100} \text{ keV} \Rightarrow \text{(CSB*)}$$

$$\Delta E(0^+) = B_\Lambda({}^4_\Lambda\text{He}, 0^+) - B_\Lambda({}^4_\Lambda\text{H}, 0^+) = 233 \pm 92 \text{ keV (up to 2016)} \Rightarrow \text{(CSB)}$$

$$= \mathbf{160 \pm 140 \pm 100} \text{ keV} \Rightarrow \text{(CSB*)}$$

	NLO19(500)	CSB	CSB*
$a_s^{\Lambda p}$	-2.91	-2.65	-2.58
$a_s^{\Lambda n}$	-2.91	-3.20	-3.29
$\boldsymbol{\delta a_s}$	**0**	**0.55**	**0.71**
$a_t^{\Lambda p}$	-1.42	-1.57	-1.52
$a_t^{\Lambda n}$	-1.41	-1.45	-1.49
$\boldsymbol{\delta a_t}$	**-0.01**	**-0.12**	**-0.03**

→ $\delta a(^1S_0)$ **increases;** $\delta a(^3S_1)$ **decreases**

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$$= \mathbf{-160 \pm 140 \pm 100}\ \mathrm{keV} \Rightarrow (\mathrm{CSB}^*)$$

$$\Delta E(0^+) = B_\Lambda({}^4_\Lambda\mathrm{He}, 0^+) - B_\Lambda({}^4_\Lambda\mathrm{H}, 0^+) = 233 \pm 92\ \mathrm{keV}\ (\text{up to } 2016) \Rightarrow (\mathrm{CSB})$$

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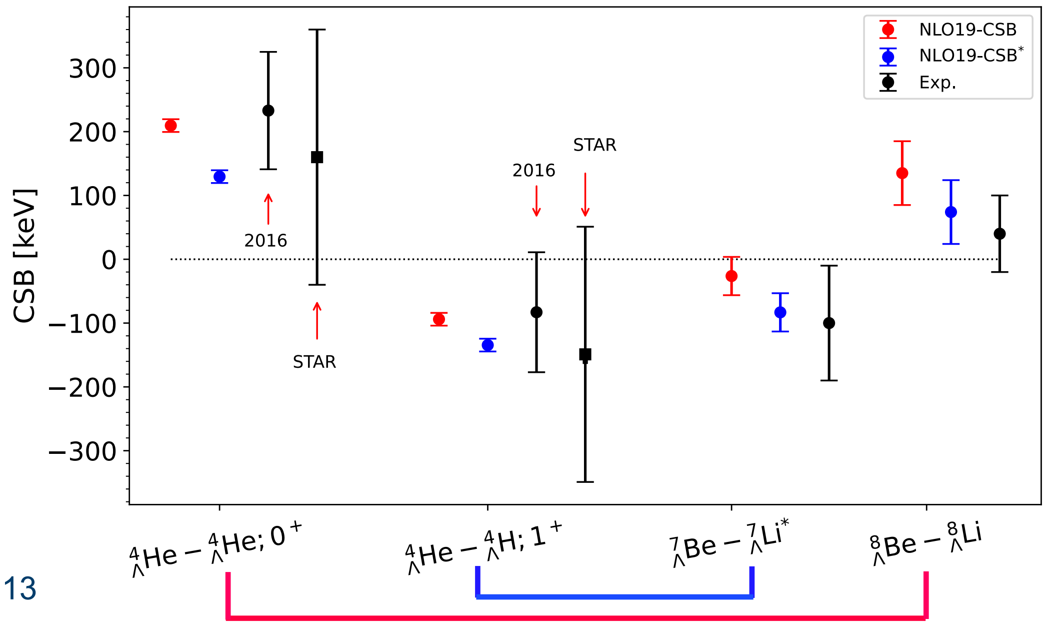
$\boldsymbol{\delta a_t}$	**-0.01**	**-0.12**	**-0.03**

→ $\delta a({}^1S_0)$ **increases;** $\delta a({}^3S_1)$ **decreases**

Impact on CSB in A=7,8 multiplets



- **CSB*** fit yields reasonable CSB in **both A=7 & 8** multiplets
- **correlation** between CSB in A=4(0^+) and A=8, and between $A = 4(1^+)$ and A=7

→ **independent check** for A=4 CSB using A= 7 & 8 results

Results for S=-2 hypernuclei

NN: **SMS** $\mathbf{N^4LO^+(450)}$; $\lambda_{NN} = 1.6\ \text{fm}^{-1}$

YN: **NLO19**(650); $\lambda_{YN} = 0.87\ \text{fm}^{-1}$

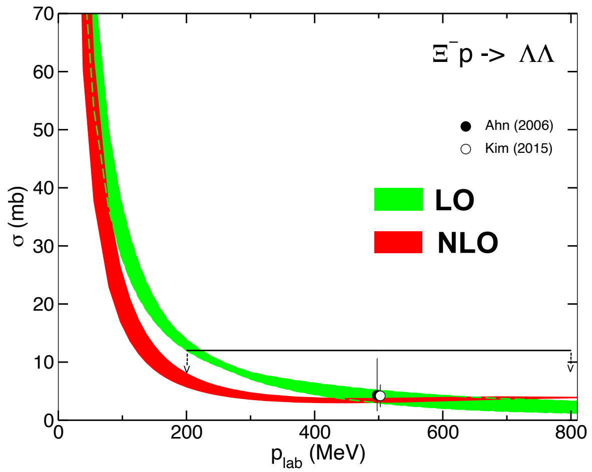
YY: **LO**, **NLO**(600)

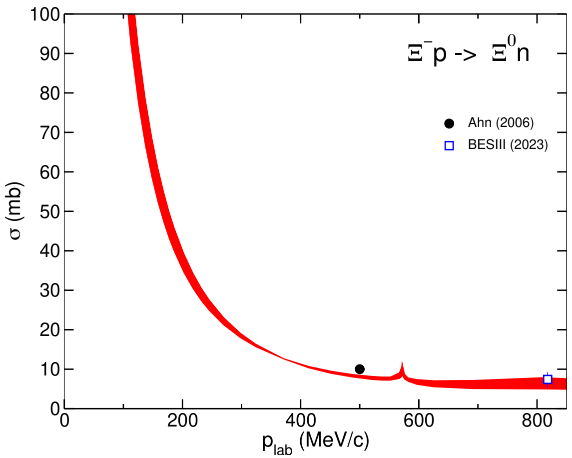
YY (ΞN) interaction at NLO

J. Haidenbauer et al. NPA 954(2016), EPJA 55(2019); J. Haidenbauer EPJ Web Conf. 271(2022)

- additional 2 S-wave LECs are constrained to **a few** ΞN data (weakly bound Ξ states)

  + information on $a_{\Lambda\Lambda}$ (from ${}^{6}_{\Lambda\Lambda}$He calculations; $\Lambda\Lambda$ correlation, ${}^{12}C(K^-, K^+\Lambda\Lambda X)$)



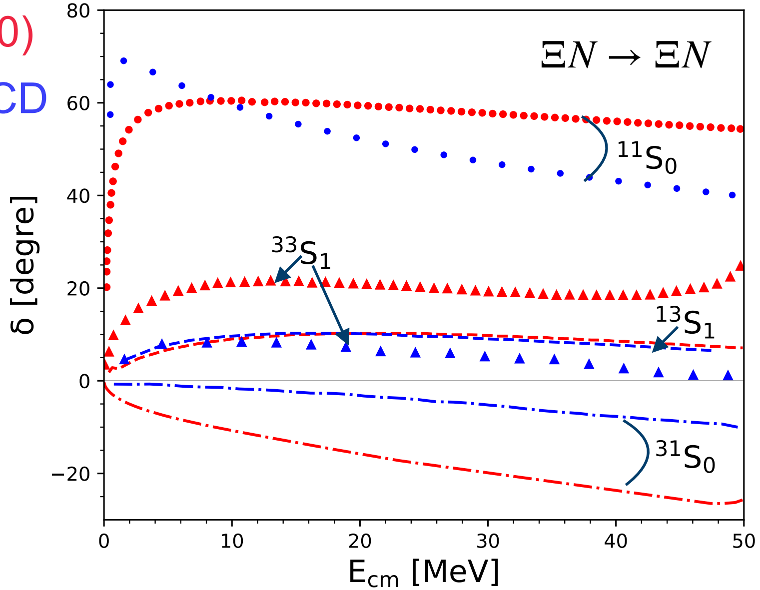


J.K. Ahn et al. PLB 633(2006) (black circle)

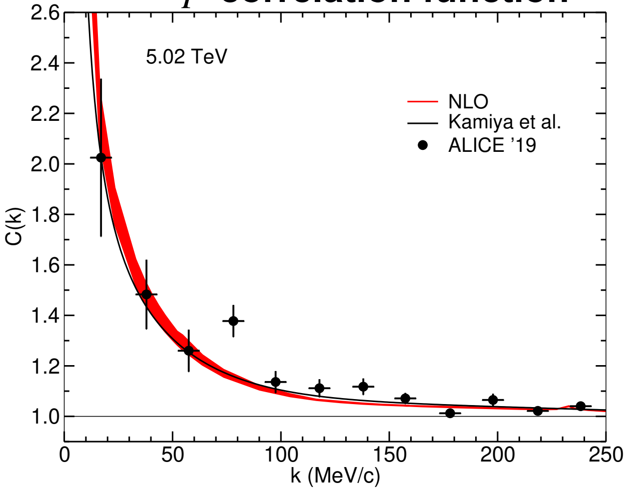
BESIII Collaboration PRL 130(2023) (open square)

red: NLO(500)

blue: HAL QCD (t/a=12)



$\Xi^- p$ correlation function



$U_\Xi(\rho_0,0) \approx -9$ (**NLO(550)**)

$= -6$ (**HAL QCD, t/a=11**)

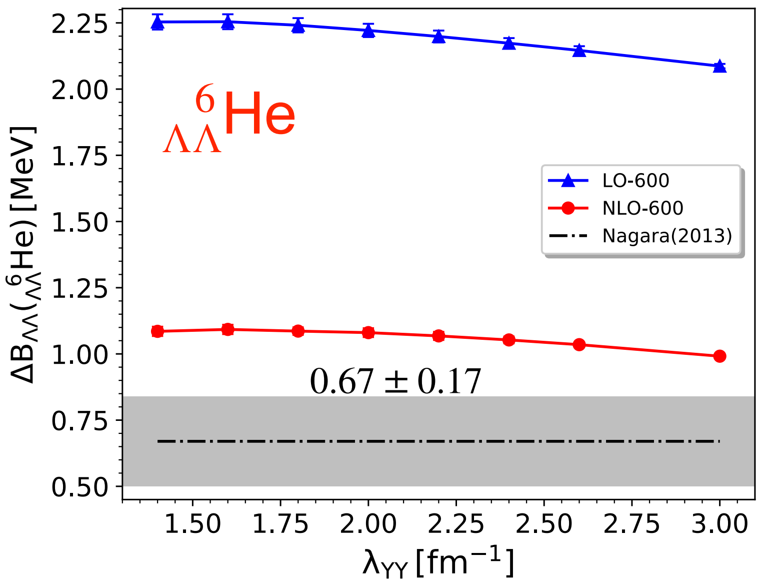
(M. Kohno PRC 100(2019); T. Inoue *PoS* INPC2016 (2016))

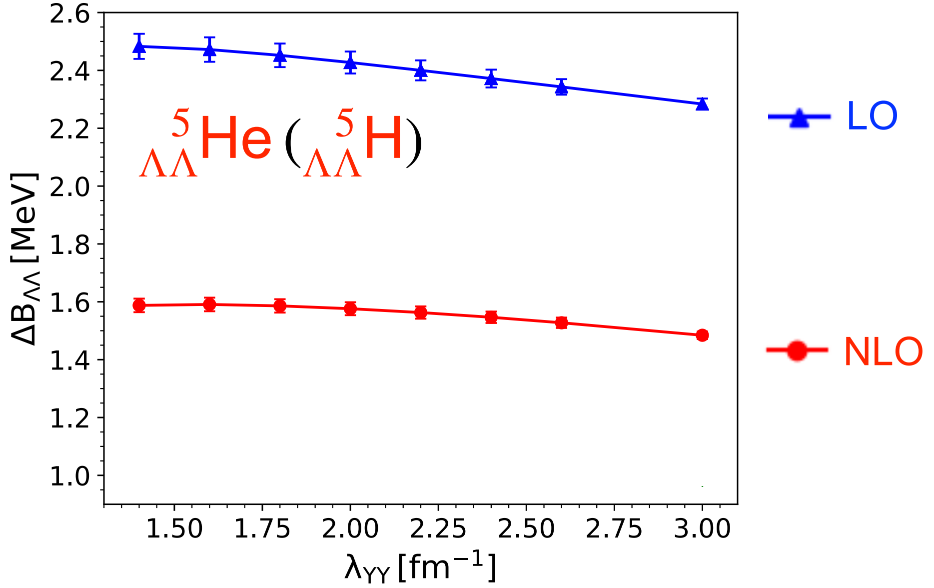
black line: Y. Kamiya et al. PRC 105(2022) (HAL QCD)

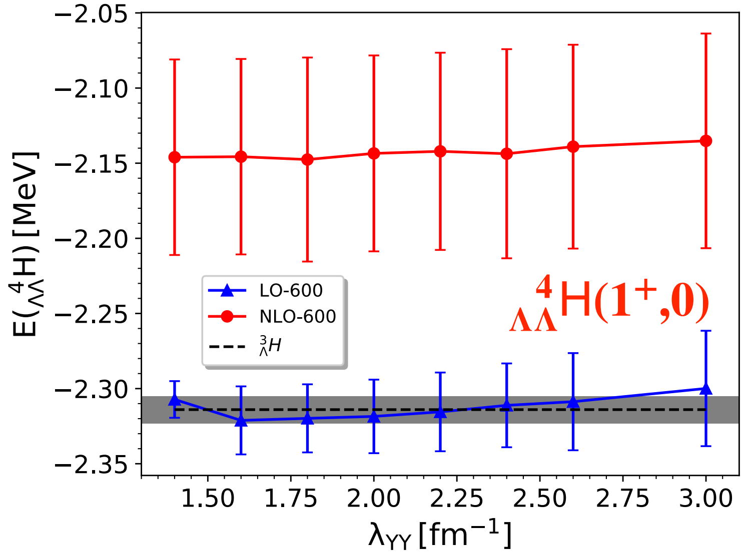
A=4-6 ΛΛ hypernuclei

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga EPJA 57 7(2021)

- $\Lambda\Lambda, \Lambda\Sigma, \Sigma\Sigma, \Xi N$ conversions are explicitly included







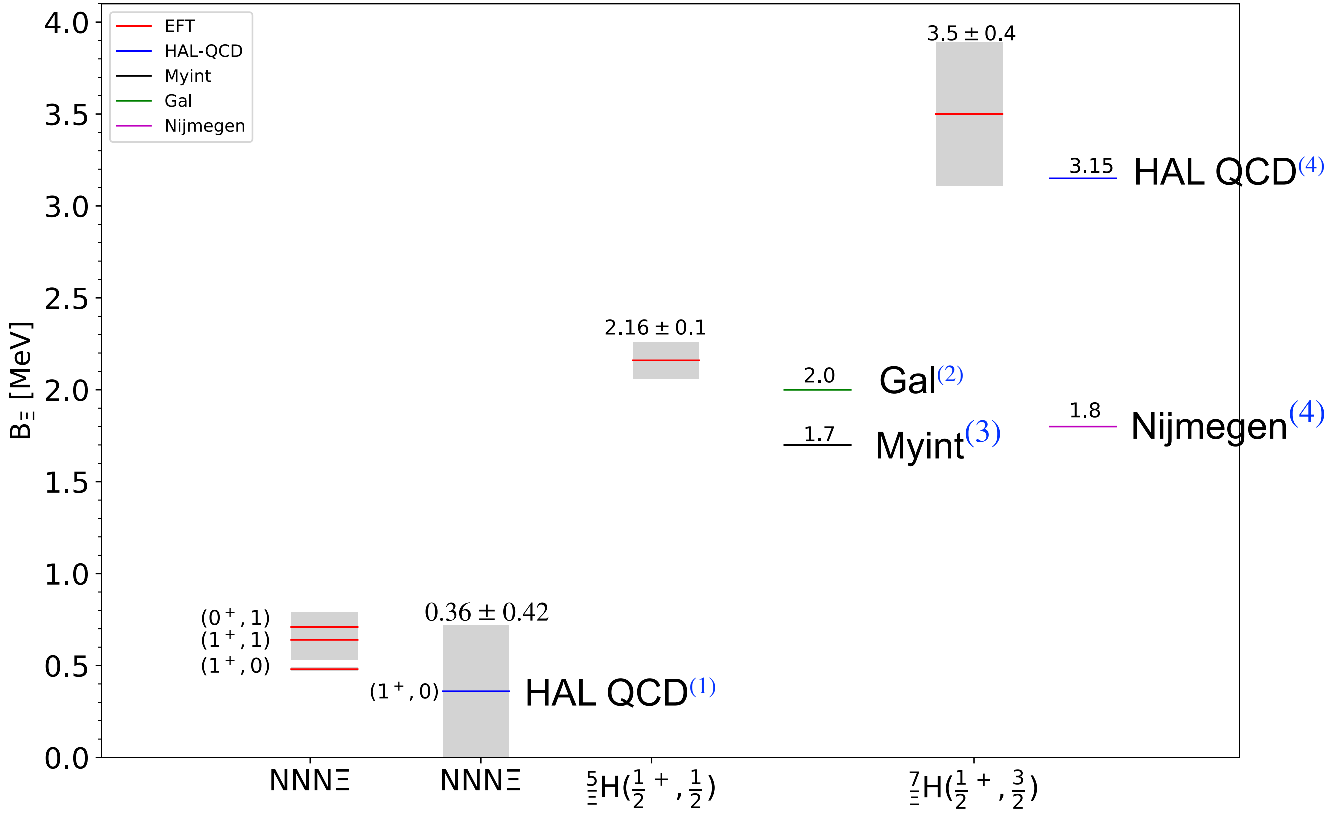
- Effect of SRG-induced YYN forces is negligible
- NLO result is **comparable** to Nagara; LO leads to overbinding
  → use $\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\mathrm{He})$ to **impose constraints on LECs**
- $^{5}_{\Lambda\Lambda}$He ($^{5}_{\Lambda\Lambda}$H) are bound with LO, NLO

  $^{7}\mathrm{Li}(K^{-}, K^{+})\,^{7}_{\Xi}\mathrm{H}; \quad ^{7}_{\Xi}\mathrm{H} \rightarrow {}^{5}_{\Lambda\Lambda}\mathrm{H} + n + n$ (E75 JPARC)
- $^{4}_{\Lambda\Lambda}$H is unbound with NLO. **Existence of** $A = 4\ \Lambda\Lambda$ **hypernucleus is very unlikely**

A=4-7 Ξ hypernuclei

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga, EPJA 57 12(2021)

- $\Xi N - \Lambda\Sigma - \Sigma\Sigma$ transitions are explicitly included; $\Lambda\Lambda - \Xi N(^{11}S_0)$ coupling is incorporated into $V_{\Xi N - \Xi N}$



	B_Ξ [MeV]	Γ [MeV]
${}^{4}_{\Xi}\mathrm{H}(1^+,0)$	0.48 ± 0.01	0.74
${}^{4}_{\Xi}\mathrm{n}(0^+,1)$	0.71 ± 0.08	0.2
${}^{4}_{\Xi}\mathrm{n}(1^+,1)$	0.64 ± 0.11	0.01
${}^{4}_{\Xi}\mathrm{H}(0^+,0)$	-	-
${}^{5}_{\Xi}\mathrm{H}(\frac{1}{2}^+,\frac{1}{2})$	2.16 ± 0.10	0.19
${}^{7}_{\Xi}\mathrm{H}(\frac{1}{2}^+,\frac{3}{2})$	3.50 ± 0.39	0.2

(1) E.Hiyama et al. PRL 124 (2020)
(2) E. Friedman, A. Gal PLB(2021)
(3) K. Myint et al. PTPS 117 (1994)
(4) H. Fujioko APFB2021, 3(2021)

- Coulomb interaction contributes ~ **200, 600 and 400** keV to $NNN\Xi$, ${}^{5}_{\Xi}\mathrm{H}$, ${}^{7}_{\Xi}\mathrm{H}$
- the attraction of chiral ΞN potential in $^{33}S_1$ is **essential for binding of A=4-7 Ξ hypernuclei**
- production: $^{4}\mathrm{He}(K^-,K^+)\,{}^{4}_{\Xi}\mathrm{H}$; $^{7}\mathrm{Li}(K^-,K^+)\,{}^{7}_{\Xi}\mathrm{H}$ (JPARC)

Summary

- At our disposal we have 2 tools to tackle light (hyper)nuclear systems:
  - s-shell (hyper)nuclei: Faddeev-Yakubovsky
  - s-shell & light p-shell: Jacobi NCSM approach; numerical uncertainties (s-shell) ~ few keV

→ **establish a direct link between the interactions and observables ($A \leq 9$)**

- **YN (YY)** at **NLO** yield **reasonable $\mathbf{B}_{\Lambda(\Lambda\Lambda)}$ in A=3-8** hypernuclei; **bound states** in A=4-7 Ξ systems

→ χ**YNN forces** are expected to contribute sizeably to B_Λ (**work in progress**)

use $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\text{He})$ to constrain $\Lambda\Lambda$ scattering length?

- **CSB NLO** interactions **reproduce** experimental CSB for $\boldsymbol{A = 4, 7}$ multiplets;

$\boldsymbol{A = 8}$ CSB prediction **is larger** than experiment

Thank you for the attention!