New studies of the K^-pp system

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Interest to antikaon-nucleon systems: quasi-bound state in the $K^- pp$ system
$\rightarrow$ experimental and theoretical efforts with different results

Experimental evidences (FINUDA, DISTO); J-PARK E15 experiment: clear observation of the $K^- pp$ quasi-bound state with $BE = 42 \pm 3$ MeV, $\Gamma = 100 \pm 7$ MeV.

The problem: big difference between theoretical and experimental widths

Our $K^- pp$ calculations: Faddeev-type dynamically exact equations with coupled $\bar{K}NN - \pi\Sigma N$ channels, different $\bar{K}N - \pi\Sigma - \pi\Lambda$ and NN interactions

What could change the theoretical results:

- More accurate model of the $\Sigma N - \Lambda N$ interaction (previous calculations: "ΣN is important to include, but no strong effects")
- Inclusion of the πN interaction (was switched off till now),
- Direct inclusion of the $\pi \Lambda N$ channel

Faddeev-type three-body AGS equations, two channels

$$U_{ij}^{\alpha\beta} = \delta_{\alpha\beta}(1-\delta_{ij})(G_0^{\alpha})^{-1} + \sum_{k=1}^{3}\sum_{\gamma=1}^{3}(1-\delta_{ik})T_k^{\alpha\gamma}{G_0}^{\gamma}U_{kj}^{\gamma\beta}$$

$\bar{K}N$ interaction is stronlgly coupled to $\pi\Sigma$ via $\Lambda(1405)$ resonance $\rightarrow$ $\pi\Sigma$ channel was included directly. Particle channels:

$$\alpha = 1 : |\bar{K}_1 N_2 N_3\rangle, \qquad \alpha = 2 : |\pi_1 \Sigma_2 N_3\rangle \qquad \alpha = 3 : |\pi_1 N_2 \Sigma_3\rangle$$

Separable form of the potentials:

$$V_i^{\alpha\beta} = \lambda_i^{\alpha\beta}|g_i^{\alpha}\rangle\langle g_i^{\beta}| \qquad \rightarrow \qquad T_i^{\alpha\beta} = |g_i^{\alpha}\rangle\tau_i^{\alpha\beta}\langle g_i^{\beta}|$$

the three-body equations are rewritten as

$$X_{ij}^{\alpha\beta}(z) = \delta_{\alpha\beta}Z_{ij}^{\alpha} + \sum_{k=1}^{3}\sum_{\gamma=1}^{3} Z_{ik}^{\alpha}\,\tau_k^{\alpha\gamma}\,X_{kj}^{\gamma\beta}$$

with new four-body transition $X_{ij}^{\alpha\beta}$ and kernel Z_{ij}^{α} operators.

Input for the system: two-body (potentials) T-matrices.

Three antikaon-nucleon interaction models:

- phenomenological $\bar{K}N - \pi\Sigma$ with one-pole $\Lambda(1405)$ resonance
- phenomenological $\bar{K}N - \pi\Sigma$ with two-pole $\Lambda(1405)$ resonance
- chirally motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential, two-pole $\Lambda(1405)$

reproduce (with the same level of accuracy):

- $1s$ level shift and width of kaonic hydrogen *(SIDDHARTA)*
  direct inclusion of Coulomb interaction, no Deser-type formula used
- Cross-sections of $K^- p \to K^- p$ and $K^- p \to MB$ reactions
- Threshold branching ratios γ, R_c and R_n (or γ, $R_{\pi\Sigma}(R_c, R_n)$ for phen.)
- ◦ $\Lambda(1405)$ resonance (*one- or two-pole structure*)
  $M^{PDG}_{\Lambda(1405)} = 1405.1^{+1.3}_{-1.0}$ MeV, $\Gamma^{PDG}_{\Lambda(1405)} = 50.5 \pm 2.0$ MeV [*PDG (2023)*]

K^-p cross-sections: theory vs. experiment

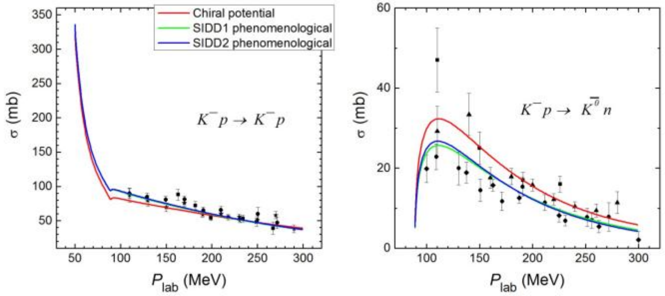
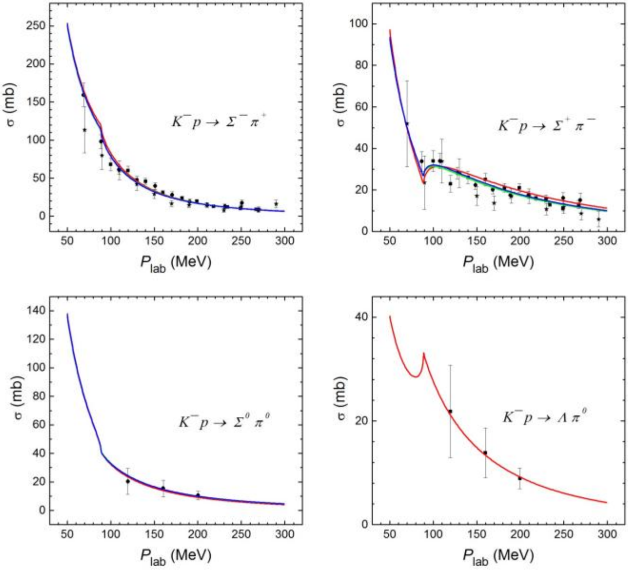


Figure: Comparison with the experimental data on K^-p cross-sections (phenomenological and chirally motivated potentials)

$K^- p$ cross-sections: theory vs. experiment



Physical characteristics of the three $\bar{K}N - \pi\Sigma(-\pi\Lambda)$ potentials: $1s$ level shift and width, strong pole(s), γ, R_c, R_n threshold branching ratios, a_{K^-p} scattering length (physical masses in all channels, Coulomb in K^-p).

	$V^{1,\text{SIDD}}_{\bar{K}N-\pi\Sigma-\pi\Lambda}$	$V^{2,\text{SIDD}}_{\bar{K}N-\pi\Sigma-\pi\Lambda}$	$V^{\text{Chiral}}_{\bar{K}N-\pi\Sigma-\pi\Lambda}$	Exp
ΔE_{1s}	-313	-308	-313	$-283 \pm 36 \pm 6$
Γ_{1s}	597	602	561	$541 \pm 89 \pm 22$
E_1	$1426 - i\,48.0$	$1414 - i\,58$	$1417 - i\,33$	
E_2	–	$1386 - i\,104$	$1406 - i\,89$	
γ	2.36	2.36	2.35	2.36 ± 0.04
R_c	–	–	0.663	0.664 ± 0.011
R_n	–	–	0.191	0.189 ± 0.015
$R_{\pi\Sigma}$	0.709	0.709	–	0.709 ± 0.011
a_{K^-p}	$-0.76 + i\,0.89$	$-0.74 + i\,0.90$	$-0.77 + i\,0.84$	

$$R_{\pi\Sigma} = \frac{R_c}{1 - R_n(1 - R_c)}$$

Two-term Separable New potential (TSN) of nucleon-nucleon interaction

[*N.V.S. Few-body syst 61, 27 (2020)*]

$$V_{NN}^{\rm TSN}(k,k') = \sum_{m=1}^{2} g_m(k)\,\lambda_m\; g_m(k')\,,$$

$$g_m(k) = \sum_{n=1}^{3} \frac{\gamma_{mn}}{(\beta_{mn})^2 + k^2}, \quad \text{for } m = 1,2$$

fitted to Argonne $V18$ potential *[R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, Phys. Rev. C 51, 38 (1995)]* phase shifts

Triplet and singlet scattering lengths a and effective ranges $r_{\rm eff}$

$$a_{np}^{\rm TSN} = -5.400\,{\rm fm}, \qquad r_{{\rm eff},np}^{\rm TSN} = 1.744\,{\rm fm}$$
$$a_{pp}^{\rm TSN} = 16.325\,{\rm fm}, \qquad r_{{\rm eff},pp}^{\rm TSN} = 2.792\,{\rm fm},$$

deuteron binding energy $E_{\rm deu} = 2.2246$ MeV.

New NN potential

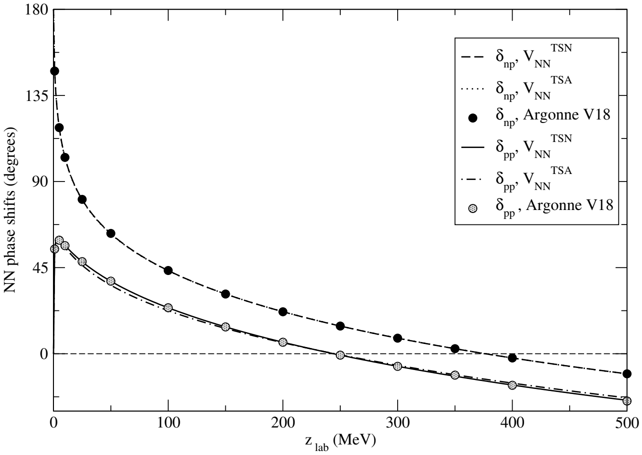


Figure: Phase shifts of np and pp scattering calculated using the new V_{NN}^{TSN} and $V_{NN}^{\mathrm{TSA-B}}$ potentials plus phase shifts of Argonne $V18$

Previously used YN potential: parameters of

$$V_{I,S}^{\Sigma N}(k,k') = \lambda_{I,S}^{\Sigma N}\, g_{I,S}^{\Sigma N}(k)\, g_{I,S}^{\Sigma N}(k'), \quad g_{I,S}^{\Sigma N}(k) = \frac{1}{(k^2 + \beta_{I,S}^{\Sigma N})^2}$$

were fitted to experimental cross-sections

- $I = 3/2$: Real parameters, one-channel case
- $I = 1/2$:
  1. Two-channel $\Sigma N - \Lambda N$ potential, real parameters,
  2. One-channel (exact) optical $\Sigma N (-\Lambda N)$ potential, complex energy-dependent strength

To study dependence of the $K^- pp$ pole position on the $\Sigma N - \Lambda N$ interaction – new fits: spin-dependent (SDep) or spin-independent (SInd) potentials fitted to experimental cross-sections with/without scattering lengths (ScL/noScL) from an "advanced" potential [*J. Haidenbauer et al., nucl-th/2301.00722*]

Dependence on $\Sigma N - \Lambda N$

Dependence of the binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the $K^- pp$ system on $\Sigma N - \Lambda N$ interaction models. Previous results are from [*N.V.S. Few-body syst 61, 27 (2020)*]

	$V_{\bar{K}N}^{1,\mathrm{SIDD}}$		$V_{\bar{K}N}^{2,\mathrm{SIDD}}$		$V_{\bar{K}N}^{\mathrm{Chiral}}$	
	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}
$V_{\Sigma N-\Lambda N}^{\mathrm{Prev}}$	52.2	67.1	46.6	51.2	29.4	46.4
$V_{\Sigma N-\Lambda N}^{\mathrm{noScL,SInd}}$	63.5	96.5	52.2	61.0	28.3	49.5
$V_{\Sigma N-\Lambda N}^{\mathrm{noScL,SDep}}$	52.5	67.0	46.1	49.8	29.6	46.8
$V_{\Sigma N-\Lambda N}^{\mathrm{ScL,SInd}}$	38.1	48.9	35.4	41.1	29.5	39.3
$V_{\Sigma N-\Lambda N}^{\mathrm{ScL,SDep}}$	34.3	65.6	34.7	53.4	27.9	42.8

Strong dependence (phenomenological $\bar{K}N - \pi\Sigma$ potentials)

Previous calculations: "πN interaction is weak" – $V^{\pi N}$ was set to zero.

Now: parameters of

$$V_I^{\pi N}(k,k') = \lambda_I^{\pi N}\, g_I^{\pi N}(k)\, g_I^{\pi N}(k'), \quad g_I^{\pi N}(k) = \frac{1}{(k^2 + \beta_I^{\pi N})^2}$$

were arbitrary (no bound states) chosen in order to see at the dependences:

- $\beta_{I=1/2}^{\pi N} = \beta_{3/2}^{\pi N} = 3 \text{ fm}^{-1}; \quad \lambda_I^{\pi N} = \lambda_I^{\pi N} = -0.1$ (1a), 0.1 (1b);
- $\beta_{I=1/2}^{\pi N} = \beta_{3/2}^{\pi N} = 1.5 \text{ fm}^{-1}; \quad \lambda_I^{\pi N} = \lambda_I^{\pi N} = -0.03$ (2a), 0.03 (2b)

or fitted to experimental data ("ScL-1" and "ScL-2"):

- $a_{\pi N, I=1/2}^{\text{Exp}} = 0.25$ fm; $a_{\pi N, I=3/2}^{\text{Exp}} = -0.12$ fm; $\quad \beta_{I=1/2}^{\pi N} = \beta_{3/2}^{\pi N} = 3 \text{ fm}^{-1}$
- $a_{\pi N, I=1/2}^{\text{Exp}} = 0.25$ fm; $a_{\pi N, I=3/2}^{\text{Exp}} = -0.12$ fm; $\quad \beta_{I=1/2}^{\pi N} = \beta_{3/2}^{\pi N} = 1.5 \text{ fm}^{-1}$

Dependence of the binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the $K^- pp$ on πN interaction models (with $V^{\mathrm{ScL,SDep}}_{\Sigma N - \Lambda N}$).

	$V^{1,\mathrm{SIDD}}_{\bar{K}N}$		$V^{2,\mathrm{SIDD}}_{\bar{K}N}$		$V^{\mathrm{Chiral}}_{\bar{K}N}$	
	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}
$V^{\mathrm{Prev}}_{\pi N}$	34.3	65.6	34.7	53.4	27.9	42.8
$V^{(1a)}_{\pi N}$	34.6	66.4	34.9	54.2	27.7	43.3
$V^{(1b)}_{\pi N}$	34.1	64.8	34.4	52.7	28.0	42.4
$V^{(2a)}_{\pi N}$	34.6	69.5	35.1	56.6	27.1	43.9
$V^{(2b)}_{\pi N}$	33.9	62.3	34.1	50.8	28.4	41.6
$V^{\mathrm{ScL-1}}_{\pi N}$	37.6	71.4	37.7	58.2	27.2	46.6
$V^{\mathrm{ScL-2}}_{\pi N}$	35.7	70.5	35.9	57.4	27.0	44.6

$$U_{ij}^{\alpha\beta} = \delta_{\alpha\beta}(1-\delta_{ij})(G_0^{\alpha})^{-1} + \sum_{k=1}^{3}\sum_{\gamma=1}^{5}(1-\delta_{ik})T_k^{\alpha\gamma}{G_0}^{\gamma}U_{kj}^{\gamma\beta}$$

$\bar{K}N$ interaction is stronlgly coupled to $\pi\Sigma$ via $\Lambda(1405)$ resonance $\rightarrow$ $\pi\Sigma$ and $\pi\Lambda$ channel was included directly. Particle channels:

$$\alpha = 1 : |\bar{K}_1 N_2 N_3\rangle, \qquad \alpha = 2 : |\pi_1 \Sigma_2 N_3\rangle \qquad \alpha = 3 : |\pi_1 N_2 \Sigma_3\rangle$$
$$\alpha = 4 : |\pi_1 \Lambda_2 N_3\rangle \qquad \alpha = 5 : |\pi_1 N_2 \Lambda_3\rangle$$

The three-body equations are rewritten as

$$X_{ij}^{\alpha\beta}(z) = \delta_{\alpha\beta}Z_{ij}^{\alpha} + \sum_{k=1}^{3}\sum_{\gamma=1}^{5}Z_{ik}^{\alpha}\,\tau_k^{\alpha\gamma}\,X_{kj}^{\gamma\beta}$$

Two-body $\bar{K}N - \pi\Sigma - \pi\Lambda$ T-matrices are necessary. The chirally motivated potential was refitted, new phenomenological ones were constructed (before: $\pi\Lambda$ channel was taken in $V^{1,\mathrm{SIDD}}$, $V^{2,\mathrm{SIDD}}$ into account indirectly through complex $\lambda_{I=1}^{11}$).

Three antikaon-nucleon interaction models:

- phenomenological $\bar{K}N - \pi\Sigma - \pi\mathbf{\Lambda}$ with one-pole $\Lambda(1405)$ resonance
- phenomenological $\bar{K}N - \pi\Sigma - \pi\mathbf{\Lambda}$ with two-pole $\Lambda(1405)$ resonance
- chirally motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential, two-pole $\Lambda(1405)$

reproduce (with the same level of accuracy):

- 1s level shift and width of kaonic hydrogen *(SIDDHARTA)*
  direct inclusion of Coulomb interaction, no Deser-type formula used
- Cross-sections of $K^- p \to K^- p$ and $K^- p \to MB$ reactions
- Threshold branching ratios γ, R_c and R_n
- ◦ $\Lambda(1405)$ resonance (*one- or two-pole structure*)
  $M^{PDG}_{\Lambda(1405)} = 1405.1^{+1.3}_{-1.0}$ MeV, $\Gamma^{PDG}_{\Lambda(1405)} = 50.5 \pm 2.0$ MeV [*PDG (2023)*]

New $\bar{K}N$ potentials, K^-p scattering

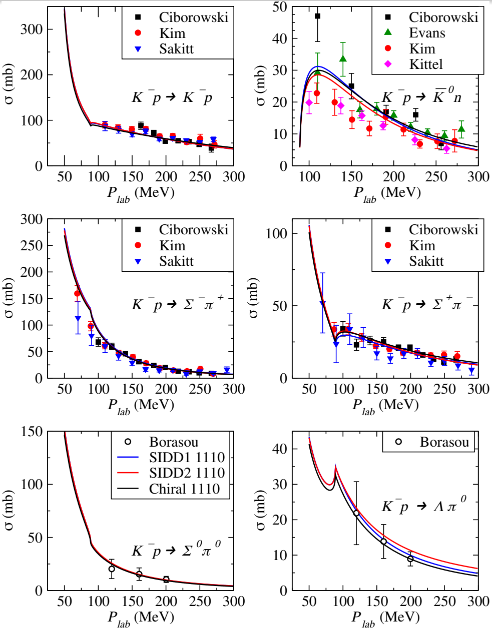


Figure: New $V_{\bar{K}N}$ potentials: one-pole, two-pole phenomenological and chirally motivated

Physical characteristics of the three $\bar{K}N - \pi\Sigma - \pi\Lambda$ potentials: $1s$ level shift and width, strong pole(s), γ, R_c, R_n threshold branching ratios, a_{K^-p} scattering length (physical masses in all channels, Coulomb in K^-p).

	$V^{1,\mathrm{SIDD}}_{\bar{K}N-\pi\Sigma-\pi\Lambda}$	$V^{2,\mathrm{SIDD}}_{\bar{K}N-\pi\Sigma-\pi\Lambda}$	$V^{\mathrm{Chiral}}_{\bar{K}N-\pi\Sigma-\pi\Lambda}$	Exp
ΔE_{1s}	-322.6	-323.5	-311.6	$-283 \pm 36 \pm 6$
Γ_{1s}	645.4	633.8	605.8	$541 \pm 89 \pm 22$
E_1	$1429.5 - i\,35.0$	$1430.9 - i\,41.6$	$1429.6 - i\,33.2$	
E_2	–	$1380.4 - i\,79.9$	$1367.8 - i\,66.5$	
γ	2.35	2.36	2.36	2.36 ± 0.04
R_c	0.666	0.664	0.664	0.664 ± 0.011
R_n	0.190	0.189	0.190	0.189 ± 0.015
a_{K^-p}	$-0.77 + i\,0.97$	$-0.78 + i\,0.95$	$-0.75 + i\,0.90$	

Three-channel $\bar{K}NN - \pi\Sigma N - \pi\Sigma\Lambda$ calculations: dependence of the binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the K^-pp on πN interaction models (with $V^{\mathrm{ScL,SDep}}_{\Sigma N-\Lambda N}$).

	$V^{1,\mathrm{SIDD}}_{\bar{K}N}$		$V^{2,\mathrm{SIDD}}_{\bar{K}N}$		$V^{\mathrm{Chiral}}_{\bar{K}N}$	
	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}
$V^{\mathrm{ScL-1,Prev}}_{\pi N}$	37.6	71.4	37.7	58.2	27.2	46.6
$V^{\mathrm{ScL-1,3ch}}_{\pi N}$	34.5	52.0	42.9	60.4	26.8	59.6
$V^{\mathrm{ScL-2,Prev}}_{\pi N}$	35.7	70.5	35.9	57.4	27.0	44.6
$V^{\mathrm{ScL-2,3ch}}_{\pi N}$	34.3	52.1	40.2	57.7	27.2	56.3

Fine tuning of the K^-pp quasi-bound state:

- different models of $\Sigma N - \Lambda N$ interaction could change the three-body K^-pp pole position quite strongly (two-channel calculations)
- variation of the πN interaction parameters lead to smaller differences
- the results evaluated with chirally motivated model of antikaon-nucleon interaction are less sensitive to the $\Sigma N - \Lambda N$ and πN interactions than those obtained with the phenomenological potentials (two-channel calc.)

	$V_{\bar{K}N}^{1,\mathrm{SIDD}}$		$V_{\bar{K}N}^{2,\mathrm{SIDD}}$		$V_{\bar{K}N}^{\mathrm{Chiral}}$	
	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}
$V^{2\mathrm{ch}}_{\mathrm{Prev}\Sigma\mathrm{N},\pi\mathrm{N}}$	52.2	67.1	46.6	51.2	29.4	46.4
$V^{3\mathrm{ch}}_{\mathrm{New}\Sigma\mathrm{N},\pi\mathrm{N}}$	34.5	52.0	42.9	60.4	26.8	59.6

- Influence of $\Sigma N - \Lambda N$ and πN potentials and the direct inclusion of the $\pi \Lambda N$ channel on the $K^- pp$ quasi-bound state was studied
- The tuned widths calculated width the tree-channel two-pole phenomenological and chiral potentials are close each to another and larger than the previous ones; the three-channel phenom. potential lead to smaller width
- The calculated widths are still much smaller than the experimental ones; why?
- To do: calculate $K^- pp$ with "an advanced" chiral potential (Prague model); perform fine-tuning of the $\bar{K}NNN$ system (with exact optical version of the three-channel $\bar{K}N - \pi\Sigma\ - \pi\Lambda$ potentials)