Λ hypernuclear potentials beyond linear density dependence

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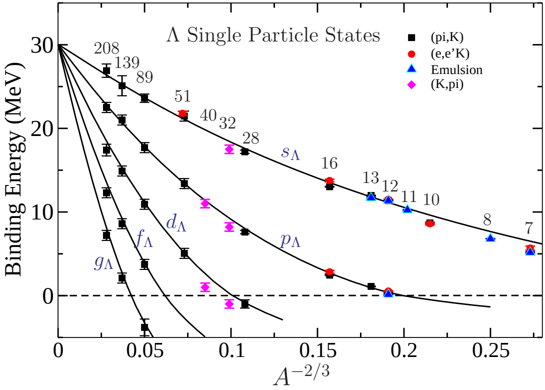
ECT* workshop: **ROCKSTAR**

TOWARDS A ROADMAP OF THE CRUCIAL MEASUREMENTS OF KEY OBSERVABLES IN STRANGENESS REACTIONS FOR NEUTRON STARS EQUATION OF STATE

Trento, October 2023

Update: Millener, Dover, Gal PRC 38, 2700 (1988)



Woods-Saxon $V = 30.05$ MeV, $r = 1.165$ fm, $a = 0.6$ fm

Λ hypernuclei: binding energies.

Figure adapted from A. Gal et al., Rev. Mod. Phys. 88 (2016) 035004

OUTLINE

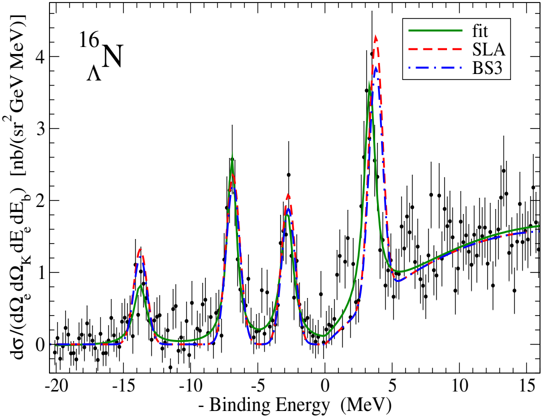
- Experimental binding energies of Λ hypernuclei up to Pb
- PLB 837 (2023) 137669; Fit ${}^{16}_{\Lambda}$N $B_{\Lambda}(1s, 1p)$ and extrapolate up to ${}^{208}_{\Lambda}$Pb (E.F. + A.G.)
- New analysis: least-squares fits to all (18) data points. Focus on the $\rho_{excess} - \rho_{core}$ interaction
  **Just published: NPA 1039 (2023) 122795
- Predictions of $B_{\Lambda}(1s, 1p)$ for ${}^{40}_{\Lambda}$K and ${}^{48}_{\Lambda}$K
- Discussion and summary

Statement of mission

The optical potential employed in this work,
$V_\Lambda^{\rm opt}(\rho) = V_\Lambda^{(2)}(\rho) + V_\Lambda^{(3)}(\rho)$,
consists of terms representing two-body ΛN and three-body ΛNN interactions, respectively.

Our aim in the present phenomenological study is to check to what extent properly chosen Λ hypernuclear binding energy data, with minimal extra assumptions, imply repulsive $V_\Lambda^{(3)}(\rho)$, and how large it is.

Use high-quality data for a single species for calibration.
$^{16}_{\Lambda}$N is not too light, single proton hole in the 1p shell.
1st and 3rd peaks from left are 1s and 1p Λ-nucleus states



^{16}O$(e, e'K^+)$, F. Garibaldi *et. al.* PRC99 (2019) 054309

The optical potential $V_\Lambda^{\rm OPT}(\rho) = V_\Lambda^{(2)}(\rho) + V_\Lambda^{(3)}(\rho)$, with two-body ΛN and three-body ΛNN terms is

$$V_\Lambda^{(2)}(\rho) = -\frac{4\pi}{2\mu_\Lambda} f_A^{(2)}\, C_{\rm Pauli}(\rho)\, b_0 \rho, \tag{1}$$

$$V_\Lambda^{(3)}(\rho) = +\frac{4\pi}{2\mu_\Lambda} f_A^{(3)}\, B_0\, \frac{\rho^2}{\rho_0}, \tag{2}$$

b_0 and B_0 are strength parameters in units of fm ($\hbar = c = 1$). A is the mass number of the *nuclear core* of the hypernucleus, ρ is a nuclear density normalized to A, $\rho_0 = 0.17$ fm^{-3} is nuclear-matter density, μ_Λ is the Λ-nucleus reduced mass, $f_A^{(2,3)}$ are kinematical factors transforming b_0 and B_0 from the ΛN and ΛNN c.m. systems, respectively, to the Λ-nucleus c.m. system:

$$f_A^{(2)} = 1 + \frac{A-1}{A}\frac{\mu_\Lambda}{m_N}, \quad f_A^{(3)} = 1 + \frac{A-2}{A}\frac{\mu_\Lambda}{2m_N}. \tag{3}$$

$$C_{\mathrm{Pauli}}(\rho) = (1 + \alpha_P \frac{3k_F}{2\pi} f_A^{(2)} b_0)^{-1} \tag{4}$$

with Fermi momentum $k_F = (3\pi^2\rho/2)^{1/3}$.
The parameter α_P in Eq. (4) switches off (α_P=0) or on (α_P=1) the Pauli correlation correction which may be considerable.

The low-density limit of V_Λ^{OPT} requires that b_0 is identified with the c.m. ΛN spin-averaged scattering length (positive here).

Experimental ΛN spin-averaged scattering length=1.7±0.1 fm

In optical model applications it is crucial to ensure that the radial extent of the densities, e.g., their r.m.s. radii, follow closely values derived from experiment. Best known are r.m.s. radii of proton densities throughout the periodic table,

$$\rho = \rho_p + \rho_n.$$

We use charge densities for ρ_p. For ρ_n we use the same radial parameter as for ρ_p in light and medium-weight nuclei, and slightly different parameters for ρ_n in heavy species,

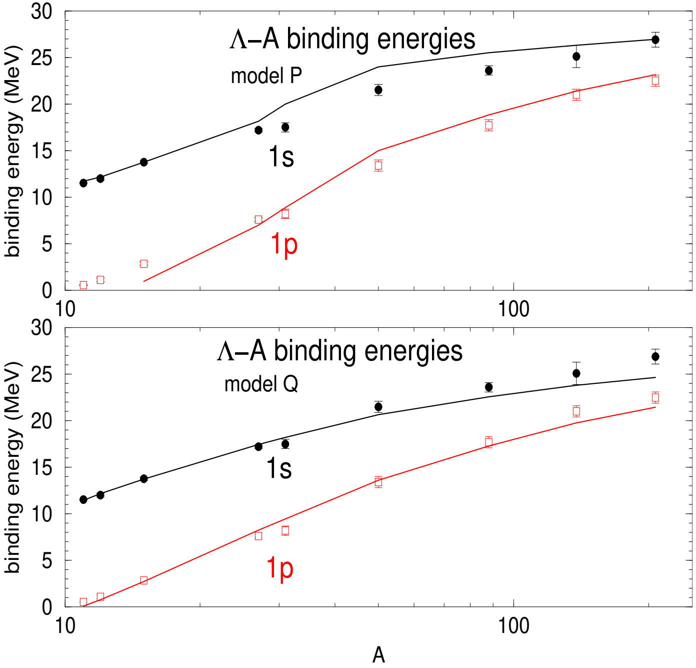
$$r_n - r_p = 1.1\frac{N-Z}{A} - 0.04 \text{ fm, for r.m.s radii}$$

from wide range of strong-interaction probes and model calculations.

The value of $r_n - r_p$ is also known as ‘the neutron skin’.

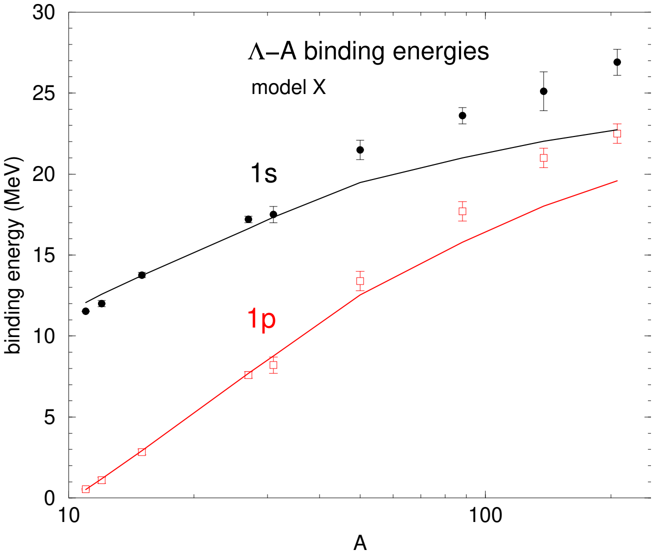
Method

- potential P: fit only b_0 to $B_{1s}({}^{16}_{\Lambda}\mathrm{N}) = 13.76 \pm 0.16$ MeV
- potential P': as P but with inevitable Pauli correlations (not shown)
- potential Q: fit b_0 and B_0 to $B_{1s}({}^{16}_{\Lambda}\mathrm{N}) = 13.76 \pm 0.16$ MeV and $B_{1p}({}^{16}_{\Lambda}\mathrm{N}) = 2.84 \pm 0.18$ MeV. No Pauli correlations.
- potential X: as Q, including Pauli correlations.
- potential Y; as X, including 'core-excess' correction.


Λ−A binding energies
model P
1s
1p
binding energy (MeV)
30
25
20
15
10
5
0
10
100
Λ−A binding energies
model Q
1s
1p
binding energy (MeV)
30
25
20
15
10
5
0
10
100
A

Strength parameters b_0, B_0 (fm) in models P,P',Q plus their respective potential depths $D_\Lambda^{(2)}$, $D_\Lambda^{(3)}$ and sum D_Λ (MeV) at nuclear matter density $\rho_0 = 0.17$ fm^{-3}. Pauli correlations are switched off (on) using $\alpha_P = 0\,(1)$.

Model	α_P	b_0	B_0	$D_\Lambda^{(2)}$	$D_\Lambda^{(3)}$	D_Λ
P	0	0.418	–	-34.1	–	-34.1
P'	1	0.908	–	-32.3	–	-32.3
Q	0	0.706	0.370	-57.6	30.2	-27.4



Fit $B_\Lambda(1s, 1p)$ in ${}^{16}_{\Lambda}N$ by b_0 and B_0, and extrapolate.
Under-binding in medium and heavy hypernuclei.

More on densities

When N > Z define ρ_{nc} by $\rho_n = \rho_{nc} + \rho_{excess}$ where ρ_{nc} referrs to a core of Z neutrons occupying the same orbitals as the protons (ρ_p) and ρ_{excess} is for the excess of N−Z neutrons.

Define $\rho_{core} = \rho_p + \rho_{nc}$ then
$\rho^2 = (\rho_{core} + \rho_{excess})^2 = \rho^2_{core} + \rho^2_{excess} + 2\rho_{core}\rho_{excess}$.

The last term refers to ΛNN interaction where an excess neutron interacts closely with a core nucleon; naively suppressed compared to the other two terms.

More formally, a suppression originates in ΛNN pion-exchange models that couple the isospin $T = 0$ Λ hyperon to the $T = 1$ Σ and $\Sigma^*(1385)$ hyperons, as suggested also in modern χEFT models. Then a $\vec{\tau}_1 \cdot \vec{\tau}_2$ factor vanishes in direct matrix elements when N_1 runs over $T = 0$ closed-shell core nucleons and N_2 is an excess neutron.

Avoiding explicit models as much as possible, we replace ρ^2 by $\rho^2_{\rm core} + \rho^2_{\rm excess}$, represented by

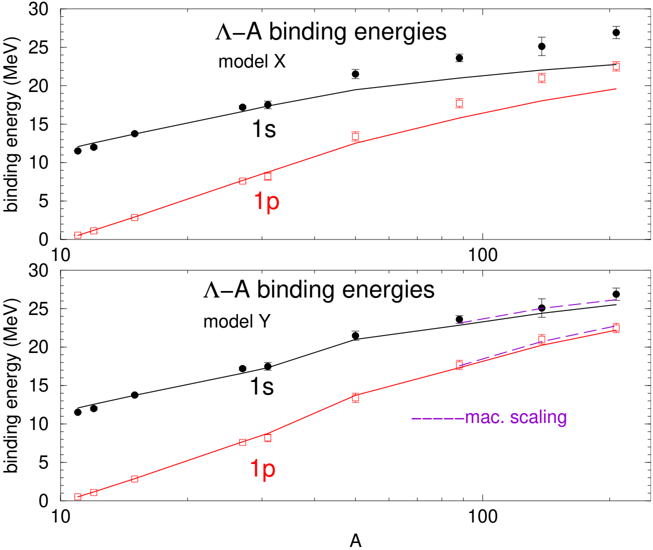
$$\rho^2_{\rm core} + \rho^2_{\rm excess} \rightarrow (2\rho_p)^2 + (\rho_n - \rho_p)^2, \tag{5}$$

in terms of the available densities ρ_p and ρ_n.

It is straightforward to show that the volume integral of $(2\rho_p)^2 + (\rho_n - \rho_p)^2$ is equal to F times the volume integral of ρ^2 where

$$F = \frac{(2Z)^2 + (N-Z)^2}{A^2}. \tag{6}$$

Using $F\rho^2$ in $V_\Lambda^{(3)}(\rho)$ to suppress the bilinear term, instead of using Eq. (5), leads to almost the same calculated binding energies, as shown in the lower part of the following figure.

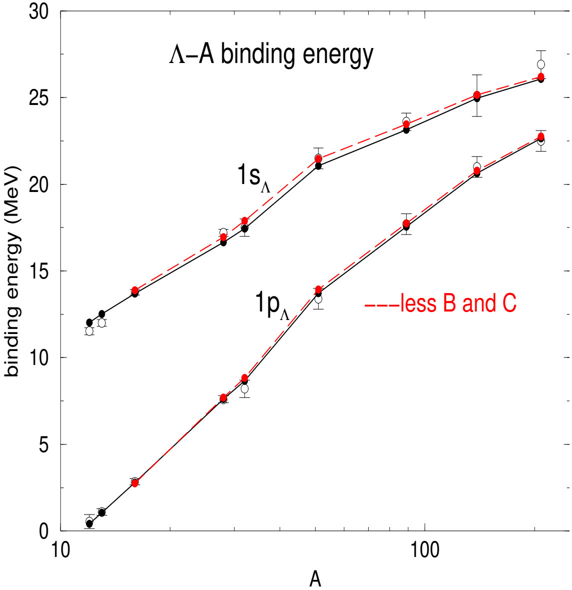


$B_\Lambda(1s, 1p)$ values. Upper part for full ρ^2 term. Lower part for decoupling $N > Z$ excess neutrons from $N = Z$ core nucleons. Dashed line for ρ^2 replaced by $F\rho^2$, with a suppression factor F given by eq.(6) above.

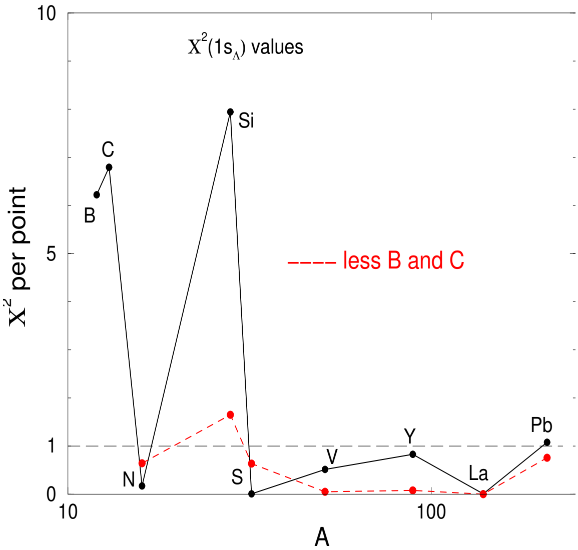
Table: INPUT DATA: $1s_\Lambda$ and $1p_\Lambda$ binding energies (MeV) in hypernuclei ${}^{A}_{\Lambda}Z$, including uncertainties, from several strangeness production reactions (SPR), see Table IV of Gal et.al, Rev. Mod Phys. 88 (2016) 035004.

${}^{A}_{\Lambda}Z$	SPR	B_Λ^{1s}	$\pm$	B_Λ^{1p}	$\pm$
${}^{12}_{\Lambda}B$	$(e, e'K^+)$	11.52	0.02	0.54	0.04
${}^{13}_{\Lambda}C$	(π^+, K^+)	12.0	0.2	1.1	0.2
${}^{16}_{\Lambda}N$	$(e, e'K^+)$	13.76	0.16	2.84	0.18
${}^{28}_{\Lambda}Si$	(π^+, K^+)	17.2	0.2	7.6	0.2
${}^{32}_{\Lambda}S$	(K^-, π^-)	17.5	0.5	8.2	0.5
${}^{51}_{\Lambda}V$	(π^+, K^+)	21.5	0.6	13.4	0.6
${}^{89}_{\Lambda}Y$	(π^+, K^+)	23.6	0.5	17.7	0.6
${}^{139}_{\Lambda}La$	(π^+, K^+)	25.1	1.2	21.0	0.6
${}^{208}_{\Lambda}Pb$	(π^+, K^+)	26.9	0.8	22.5	0.6

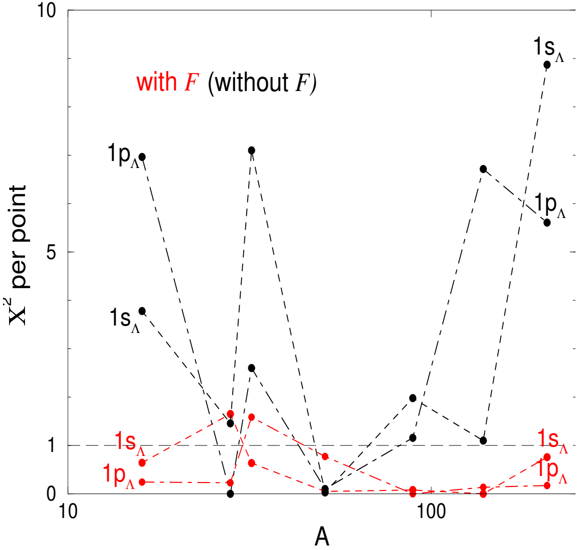
${}^{12}_{\Lambda}B$ is the lightest hypernucleus considered. Its extremely small δB_Λ uncertainty values were increased to ± 0.2 MeV, making the $B_\Lambda^{1s,1p}({}^{12}_{\Lambda}B)$ values consistent with their corresponding values in the charge-symmetric ${}^{12}_{\Lambda}C$ hypernucleus.



Least-squares fits to B_Λ data. Black for the full B_Λ set, red dashed lines excluding ${}^{12}_{\Lambda}$B and ${}^{13}_{\Lambda}$C. Open circles with error bars mark experiment.



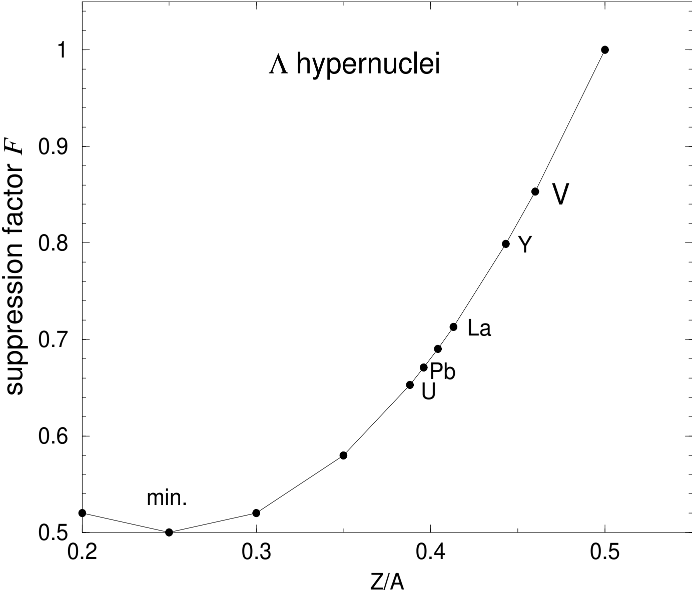
Best-fit χ^2 values for $1s_\Lambda$ states. Black for the full data set, red dashed lines excluding ${}^{12}_{\Lambda}$B and ${}^{13}_{\Lambda}$C.



Best-fit χ^2 values for various $1s_\Lambda$ and $1p_\Lambda$ states, excluding ${}^{12}_{\Lambda}$B and ${}^{13}_{\Lambda}$C. Black without the suppression factor F of Eq. (6), red with the F factor.

The above figure demonstrates the importance of the suppression factor F of Eq. (6) applied to the ρ^2 term of the potential for medium weight and heavy hypernuclei.

The introduction of this factor does not involve any additional parameter beyond b_0 and B_0. Its explicit form, Eq. (6), is based on a simple shell-model picture.


Λ hypernuclei
suppression factor F
Z/A
min.
U
Pb
La
Y
V
1
0.9
0.8
0.7
0.6
0.5
0.2
0.3
0.4
0.5

Results

Results of the least-squares fits are typically $\chi^2 = 7$ for 14 data points, excluding ${}^{12}_{\Lambda}$B and ${}^{13}_{\Lambda}$C.
The two parameters are

$$b_0 = 1.437 \pm 0.095 \text{ fm}, \quad \text{(attraction)}, \tag{7}$$

$$B_0 = 0.190 \pm 0.024 \text{ fm}, \quad \text{(repulsion)} \tag{8}$$

with 100% correlation between the two.

The depths of the partial potentials are (in MeV):

$$D_\Lambda^{(2)} = -38.6 \pm 0.8, \;\; D_\Lambda^{(3)} = 11.3 \pm 1.4, \;\; D_\Lambda = -27.3 \pm 0.6 \tag{9}$$

at nuclear-matter density $\rho_0 = 0.17$ fm^{-3}.

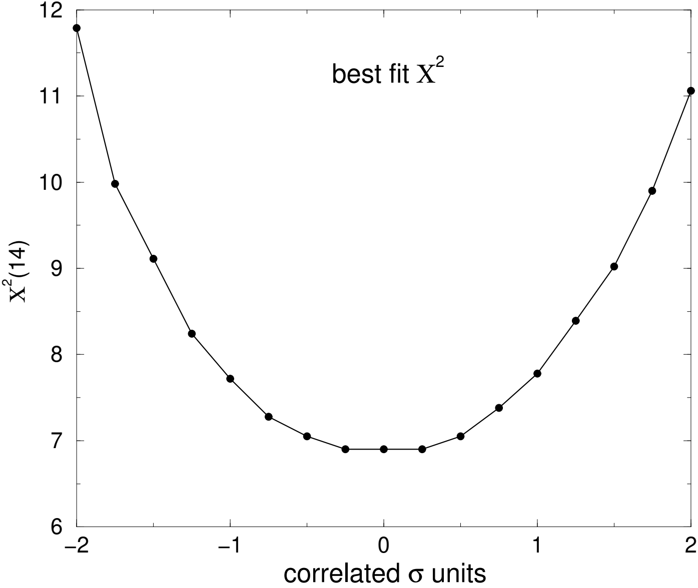
Results

Results of the least-squares fits are typically $\chi^2 = 26$ for 18 data points, INcluding $^{12}_{\Lambda}$B and $^{13}_{\Lambda}$C.
The two parameters are

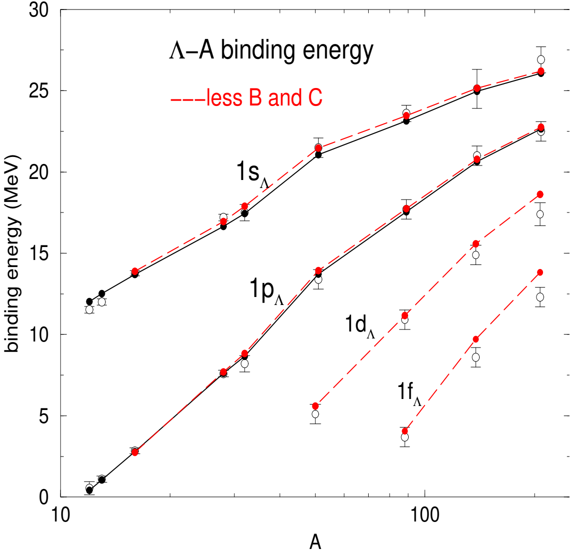
$$b_0 = 1.526 \pm 0.108 \text{ fm}, \quad \text{(attraction)}, \tag{10}$$

$$B_0 = 0.218 \pm 0.026 \text{ fm}, \quad \text{(repulsion)} \tag{11}$$

with 100% correlation between the two.

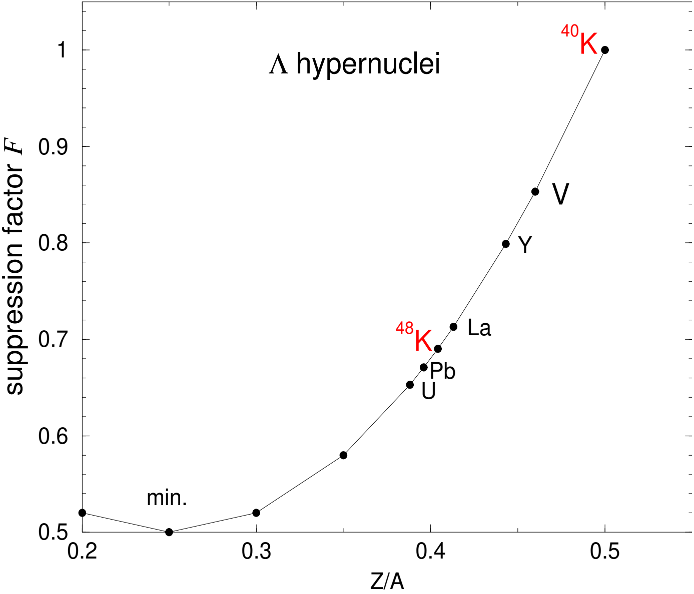

best fit X^2
$X^2(14)$
12
11
10
9
8
7
6
–2
–1
0
1
2
correlated σ units

Higher states



Comparing predictions of binding energies for $1d_\Lambda$ and $1f_\Lambda$ states with experiment, Table IV, Rev. Mod. Phys. 88 (2016) 035004.

Testing the F suppression factor?



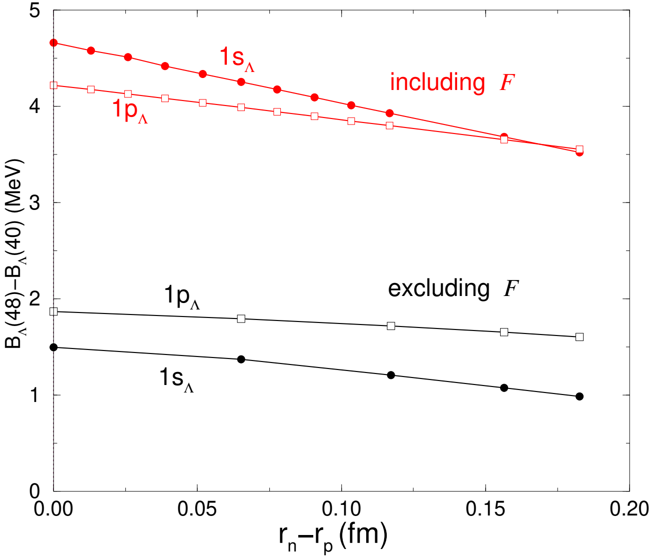
Predictions for $(e, e'K^+)$ experiments on Ca isotopes

Forthcoming ^{40,48}Ca$(e, e'K^+)^{40,48}_{\Lambda}$K experiments at JLab. will study single-particle Λ spectra in $^{40,48}_{\Lambda}$K. With relatively large neutron-excess fraction $(N - Z)/A$ it may be possible to test also the suppression factor F.

Calculated B^{1s}_{Λ} and B^{1p}_{Λ} in $^{40}_{\Lambda}$K and in $^{48}_{\Lambda}$K assuming neutron-skin values of $r_n - r_p = -0.04$ fm in $^{40}_{\Lambda}$K and 0.16 fm in $^{48}_{\Lambda}$K.

B_{Λ} (MeV)	$^{40}_{\Lambda}$K $(F = 1)$	$^{48}_{\Lambda}$K $(F = 1)$	$^{48}_{\Lambda}$K $(F = 0.69)$
$1s_{\Lambda}$	18.70	19.78	22.39
$1p_{\Lambda}$	10.70	12.35	14.35

The effect of the suppression factor F is 2-2.5 MeV.



Scan over calculated B_Λ values in ${}^{48}_{\Lambda}$K and in ${}^{40}_{\Lambda}$K with and without applying the suppression factor F, for neutron densities of ${}^{48}_{\Lambda}$K characterized by variable neutron-skin $r_n - r_p$ values.

Summary

Least-squares fits of two parameters to experimental $1s$ and $1p$ Λ-nuclear binding energies from ${}^{16}_{\Lambda}$N to ${}^{208}_{\Lambda}$Pb lead to well-defined ρ and ρ^2-depended optical potential, (at $\rho_0 = 0.17$ fm^{-3}):
$D_\Lambda^{(2)} = -38.6 \pm 0.8$, $D_\Lambda^{(3)} = 11.3 \pm 1.4$, $D_\Lambda = -27.3 \pm 0.6$ MeV.

For the first time predictions are made of isospin-dependence that could be tested by forthcoming ^{40,48}Ca$(e, e'K^+)^{40,48}_{\Lambda}$K experiments at JLab.

The repulsive ρ^2 term is larger by a few MeV than the one leading to the Λ chemical potential to be larger than the chemical potential for neutrons in pure neutron matter. (Gerstung, Kaiser and Weise, Eur. Phys. J. **A 56**,175 (2020)).

A direction for solving the Hyperon Puzzle?

Thanks for your attention!

Suppressing ρ^2 in medium-weight and heavy species

$\rho = \rho_{core} + \rho_{ex}$.

By definition:

$\int \rho^2 d\vec{r} = A \int \rho \frac{\rho}{A} d\vec{r} = A\bar{\rho}$

$\int \rho_{core}^2 d\vec{r} = 2Z \int \rho_{core} \frac{\rho_{core}}{2Z} d\vec{r} = 2Z\bar{\rho}_{core}$

$\int \rho_{ex}^2 d\vec{r} = (N - Z) \int \rho_{ex} \frac{\rho_{ex}}{N-Z} d\vec{r} = (N - Z)\bar{\rho}_{ex}$

$\rho^2 = (\rho_{core} + \rho_{ex})^2 = \rho_{core}^2 + \rho_{ex}^2 + 2\rho_{core}\rho_{ex}$.

Ignoring the crossed term $2\rho_{core}\rho_{ex}$ and approximating
$\bar{\rho}_{core} = \frac{2Z}{A}\bar{\rho}$, $\bar{\rho}_{ex} = \frac{N-Z}{A}\bar{\rho}$, we get

$\int \rho^2 d\vec{r} \to \int (\rho_{core} + \rho_{ex})^2 d\vec{r} = \frac{(2Z)^2+(N-Z)^2}{A^2} \int \rho^2 d\vec{r}$.

Hence we apply a suppression factor $F = \frac{(2Z)^2+(N-Z)^2}{A^2}$ to the ρ^2 term in the potential.