SOUND VELOCITY, EQUATION OF STATE
and
STRANGENESS in NEUTRON STAR MATTER

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- Dense Matter in Neutron Stars: Speed of Sound and Equation of State
  - Observational constraints from heavy neutron stars and mergers (GW signals)
  - Bayesian inference and evidence for (or against) phase transitions
- Strangeness and Baryonic Matter
  - Hyperon-nucleon interactions, three-body forces and hypernuclei
  - Hyperons in the core of neutron stars ? Scenarios and the “hyperon puzzle”

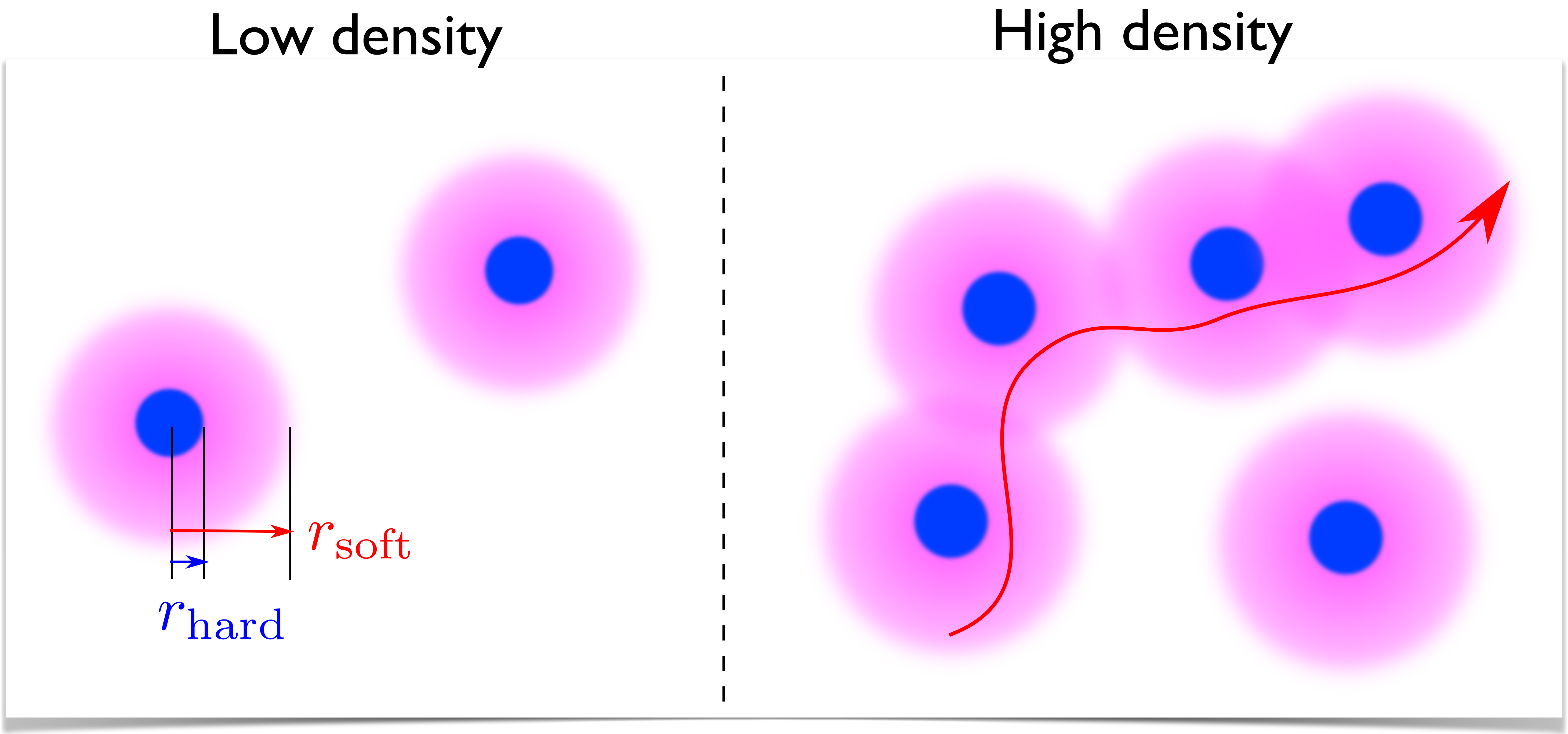
The BASIC QUESTION

- **COLD** and **DENSE** baryonic matter in the core of **NEUTRON STARS**

$$\mathrm{r_{hard}} \sim 0.5\ \mathrm{fm}$$

$$\mathrm{r_{soft}} \sim 1\ \mathrm{fm}$$

$$\left(\frac{\mathrm{r_{hard}}}{\mathrm{r_{soft}}}\right)^{3} << 1$$



K. Fukushima,
T. Kojo, W.W.
Phys. Rev. D 102 (2020)
096017

- **Phase transition ?**
- Continuous **crossover** ?
- **Strangeness** degrees of freedom ?
- Percolation ?
- Many-body forces ?

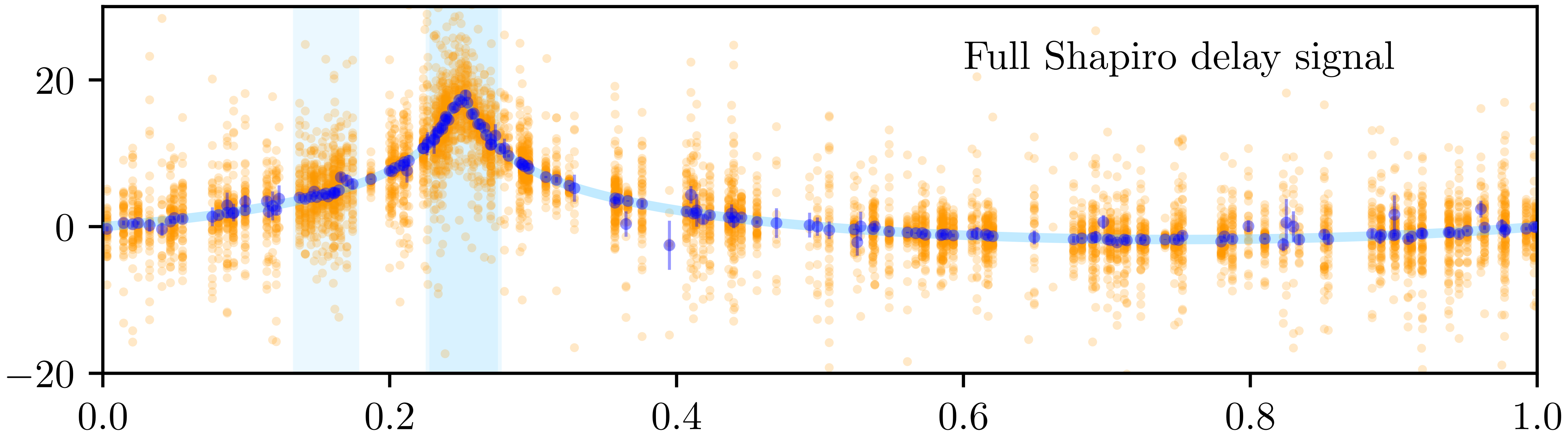
Part One

Equation-of-State of Dense Baryonic Matter :

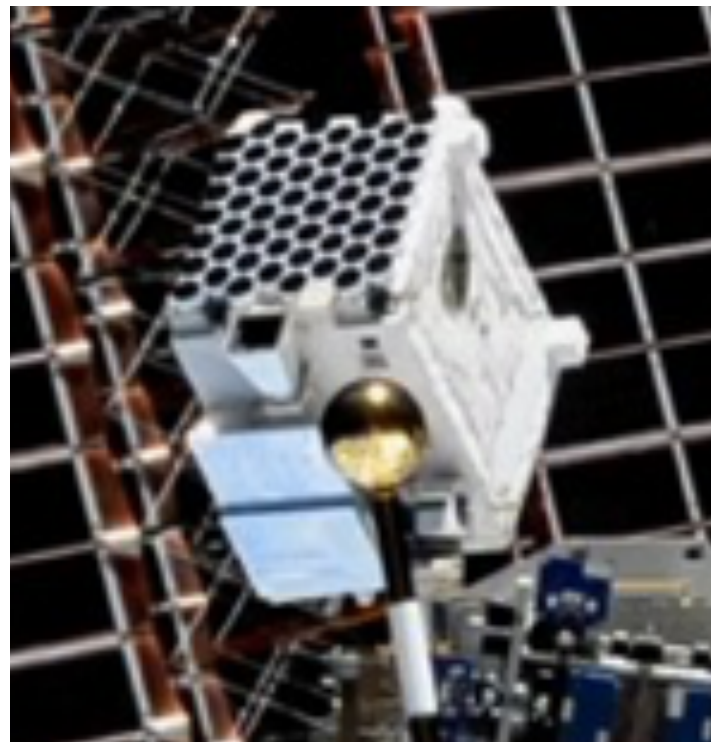
Observational Constraints from Neutron Stars

NEUTRON STARS : DATA

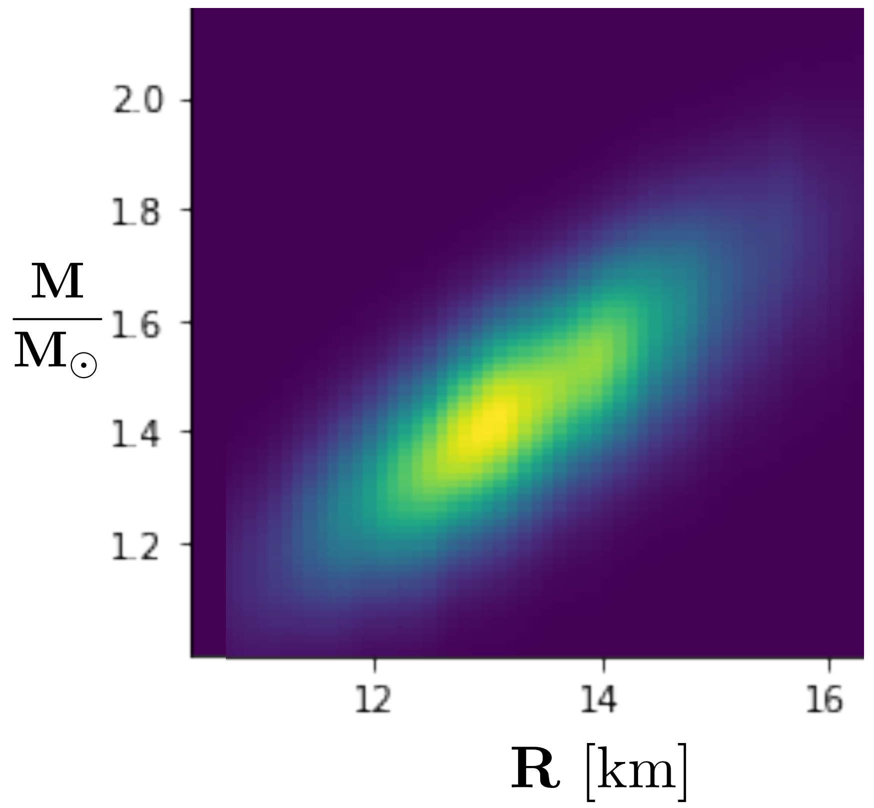
- Database for **inference of Equation-of-State** and other properties of neutron stars

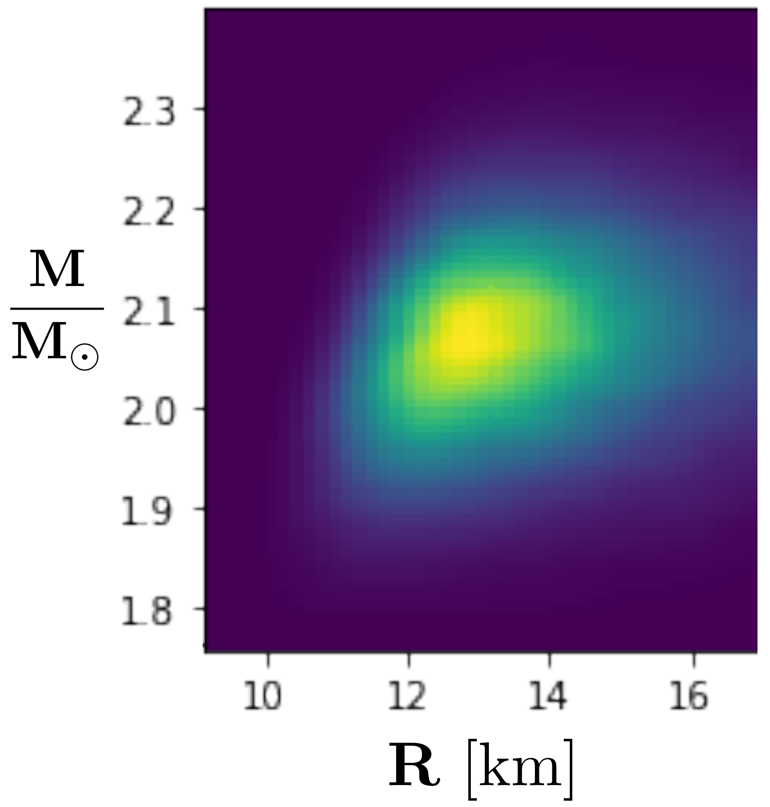


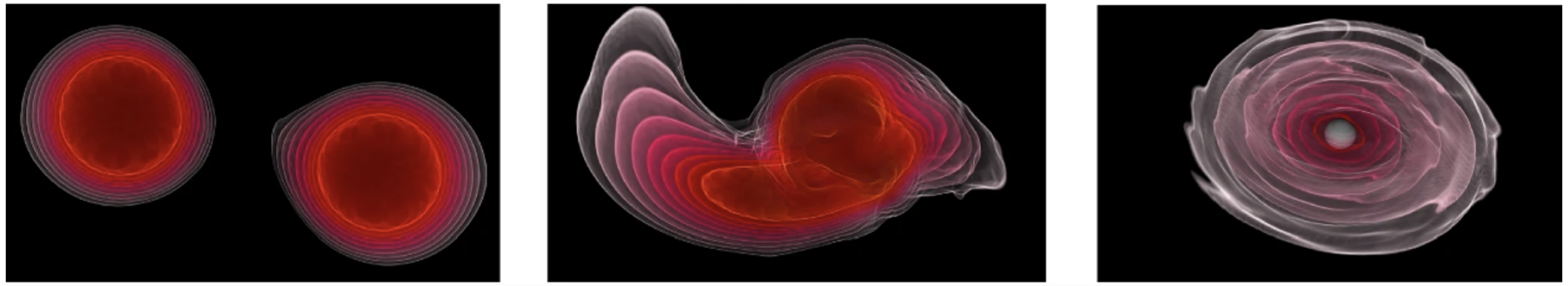
- **Neutron star masses**
  Shapiro delay measurements
  (Green Bank Telescope)



- **Masses and radii**
  X rays from hot spots on the surface of rotating neutron stars
  (NICER telescope @ ISS)







- **Tidal deformabilities**
  Gravitational wave signals of neutron star mergers
  (LIGO and Virgo collab.)

NEUTRON STARS : DATA

- **Masses of** $2\,\mathrm{M}_\odot$ **stars** (Shapiro delay measurements)

PSR J0348+0432

$$\boxed{\mathrm{M} = 2.01 \pm 0.04\ \mathrm{M}_\odot}$$

J. Antoniadis et al.: Science 340 (2013) 1233232

PSR J1614-2230

$$\boxed{\mathrm{M} = 1.908 \pm 0.016\ \mathrm{M}_\odot}$$

Z. Arzoumanian et al., Astrophys. J. Suppl. 235 (2018) 37

PSR J0740+6620

$$\boxed{\mathrm{M} = 2.08 \pm 0.07\ \mathrm{M}_\odot}$$

E. Fonseca et al., Astrophys. J. Lett. 915 (2021) L12

- **Masses and Radii** (NICER)

PSR J0030+0451

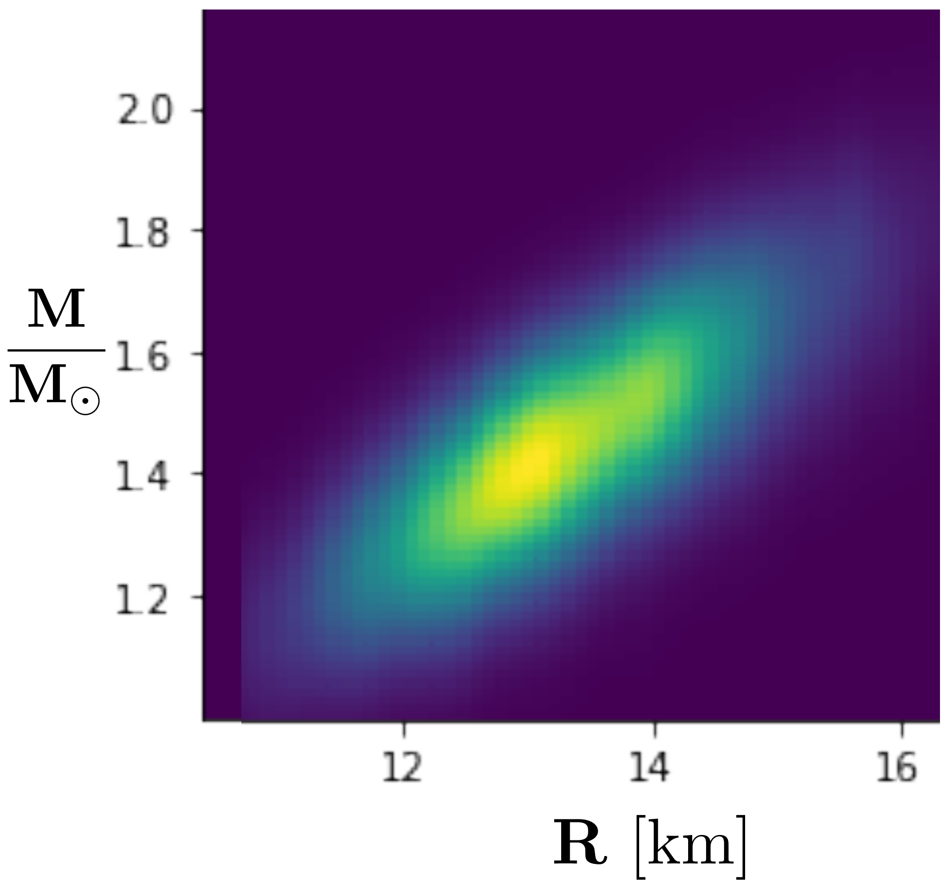
$$\boxed{\mathrm{M} = 1.34 \pm 0.16\,\mathrm{M}_\odot \qquad \mathrm{R} = 12.71^{+1.14}_{-1.19}\ \mathrm{km}}$$

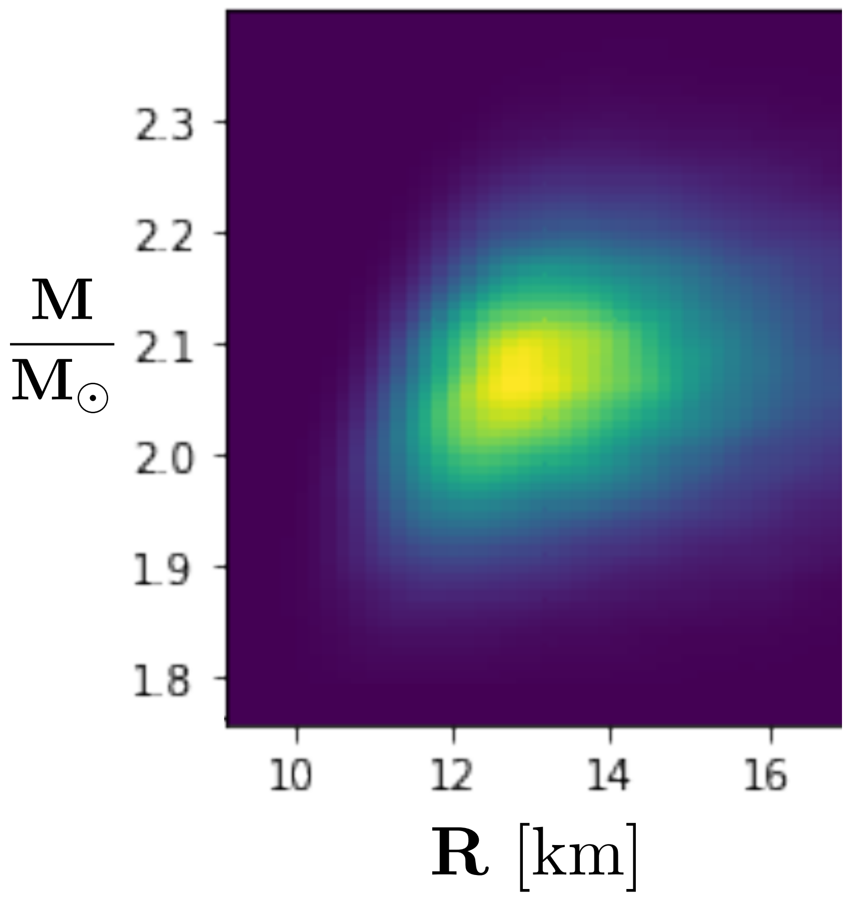
T.E. Riley et al. (NICER), Astroph. J. Lett. 887 (2019) L21

PSR J0740+6620

$$\boxed{\mathrm{M} = 2.07 \pm 0.07\,\mathrm{M}_\odot \qquad \mathrm{R} = 12.39^{+1.30}_{-0.98}\ \mathrm{km}}$$

T.E. Riley et al. (NICER + XMM Newton), Astroph. J. Lett. 918 (2021) L27





NEUTRON STARS : DATA (contd.)

- **NEW** : **very massive** and **fast rotating galactic neutron star**

PSR J0952-0607

$$\boxed{\mathrm{M} = 2.35 \pm 0.17\ \mathrm{M}_\odot}$$

R.W. Romano et al. : Astroph. J. Lett. 935 (2022) L17

(Keck Observatory)

→ equivalent non-rotating mass after rotational correction : $\mathrm{M} = 2.3 \pm 0.2\ \mathrm{M}_\odot$

- **Tidal deformabilities** from binary neutron star mergers (gravitational wave signals)

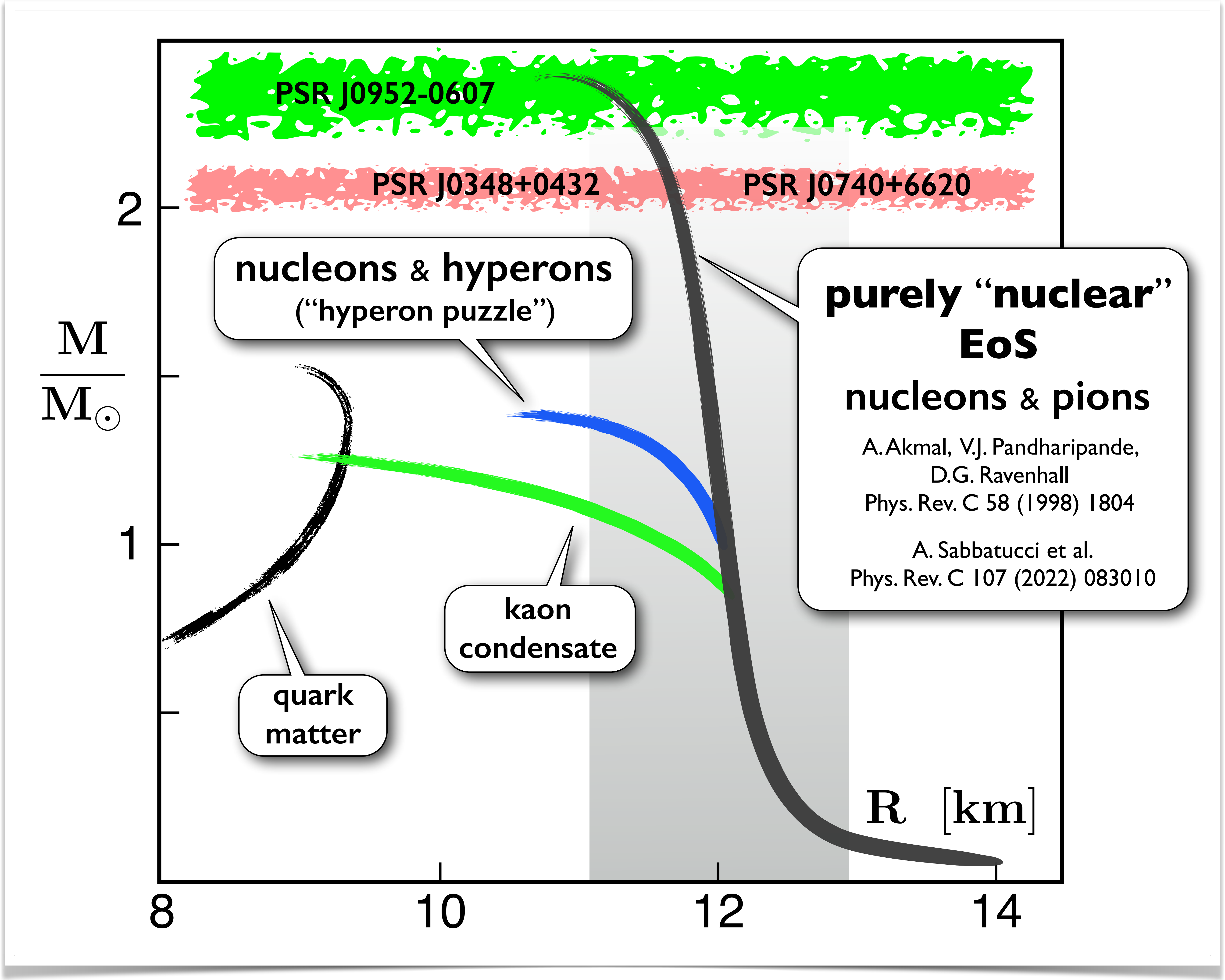
$$\tilde{\Lambda} = \frac{16}{13}\,\frac{(M_1 + 12\,M_2)M_1^4\Lambda_1}{(M_1+M_2)^5} + (1 \leftrightarrow 2)$$

GW170817 $\Lambda_{1.4} = 190^{+390}_{-120}$

B.P. Abbot et al. : Phys. Rev. Lett. 121 (2018) 161101

CONSTRAINTS on EQUATION-of-STATE

- from observations of massive neutron stars



A. Akmal, V.J. Pandharipande, D.G. Ravenhall
Phys. Rev. C 58 (1998) 1804

A. Sabbatucci et al.
Phys. Rev. C 107 (2022) 083010

Tolman - **O**ppenheimer - **V**olkov Equations

$$\frac{dP(r)}{dr} = \frac{G\,[\varepsilon(r) + P(r)]\,\left[m(r) + 4\pi r^3 P(r)\right]}{r\,[r - 2\,G\,m(r)]}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \varepsilon(r)$$

$$M = m(R) = 4\pi \int_0^R dr\, r^2 \varepsilon(r)$$

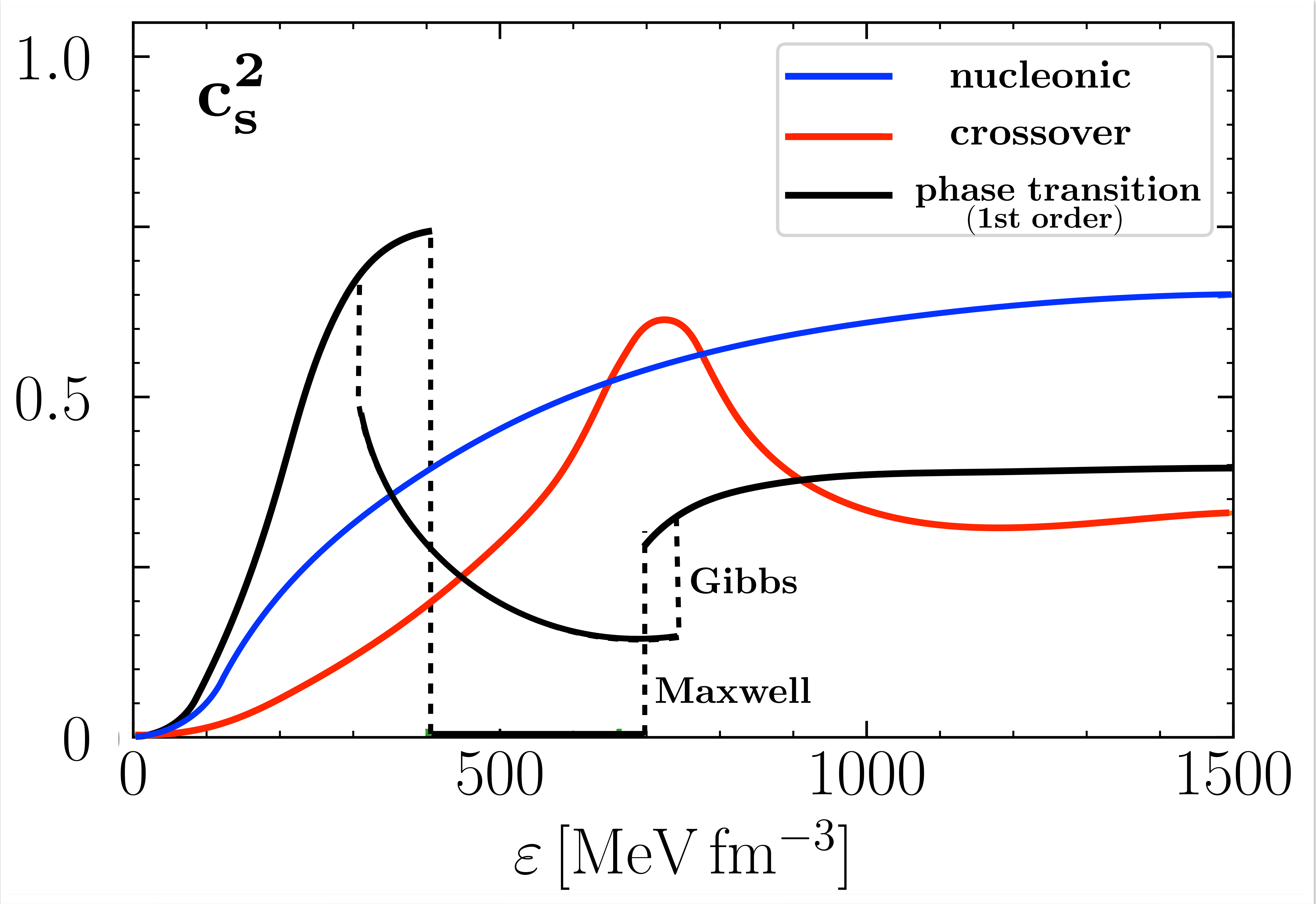
- **Stiff equation-of-state** $\mathrm{P}(\varepsilon)$ **required**
- **Simplest forms of exotic matter**
  (kaon condensate, quark matter, …)
  ruled out

SOUND VELOCITY and EQUATION of STATE

- Key quantity : **Speed of Sound**

$$c_s^2(\varepsilon) = \frac{\partial P(\varepsilon)}{\partial \varepsilon}$$

displays
characteristic signature
of
phase transition
or
crossover



energy density

- **Equation of State :**

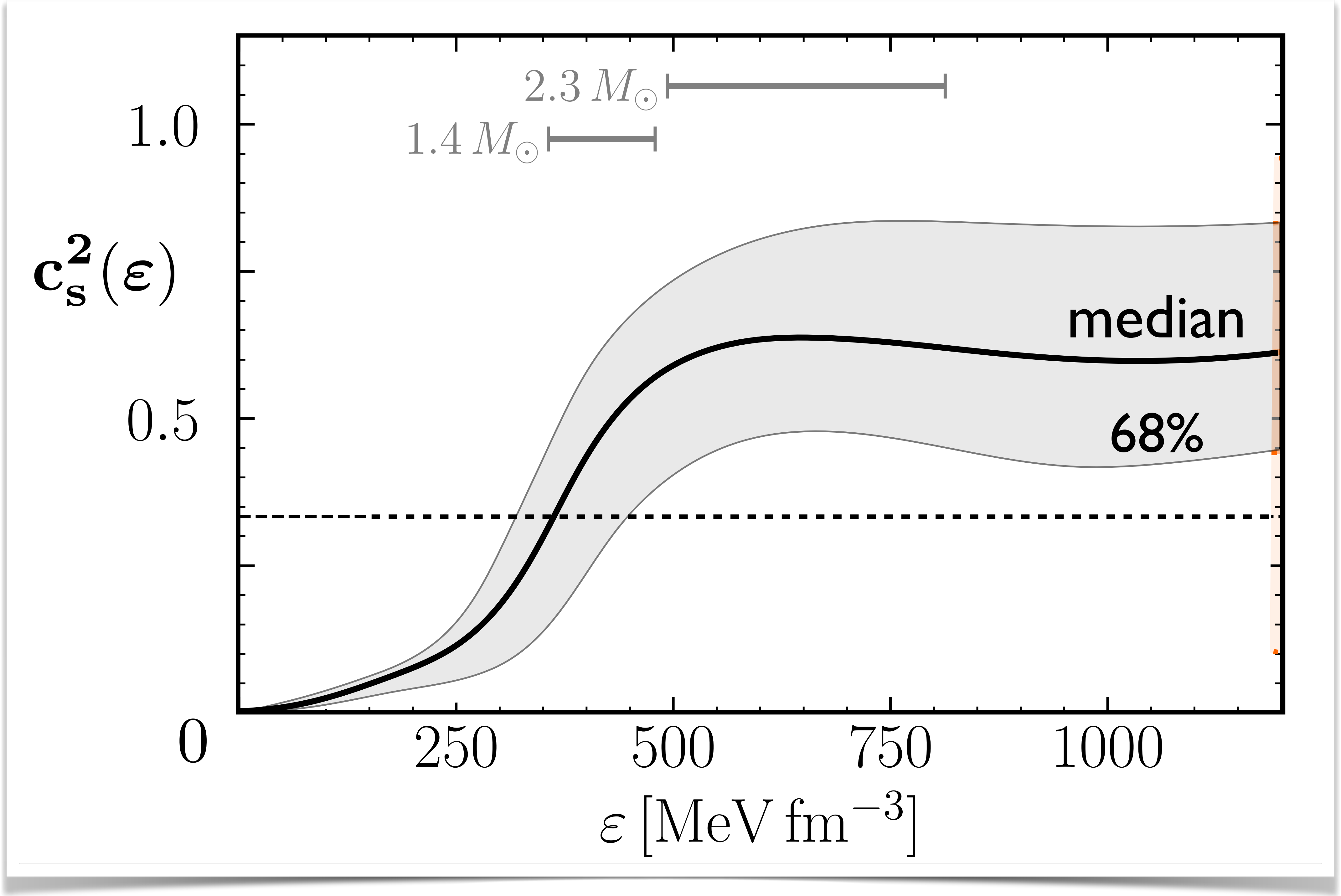
$$P(\varepsilon) = \int_0^{\varepsilon} d\varepsilon'\, c_s^2(\varepsilon')$$

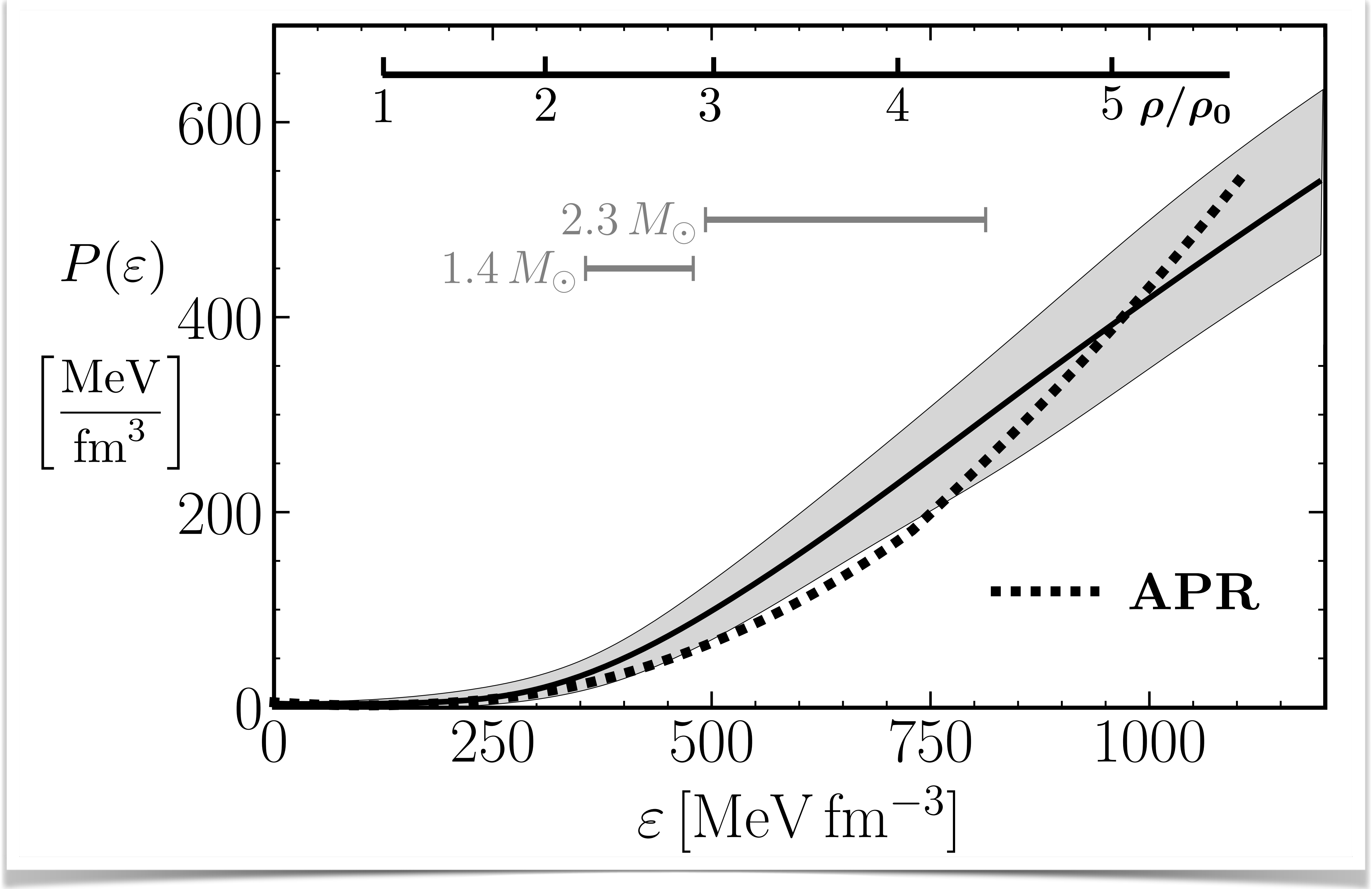
NEUTRON STAR MATTER EQUATION-of-STATE

- Bayesian inference of **sound speed** and **EoS**

PSR masses, NICER & GW data, low-density constraints (ChEFT), **asymptotic constraints** (pQCD)

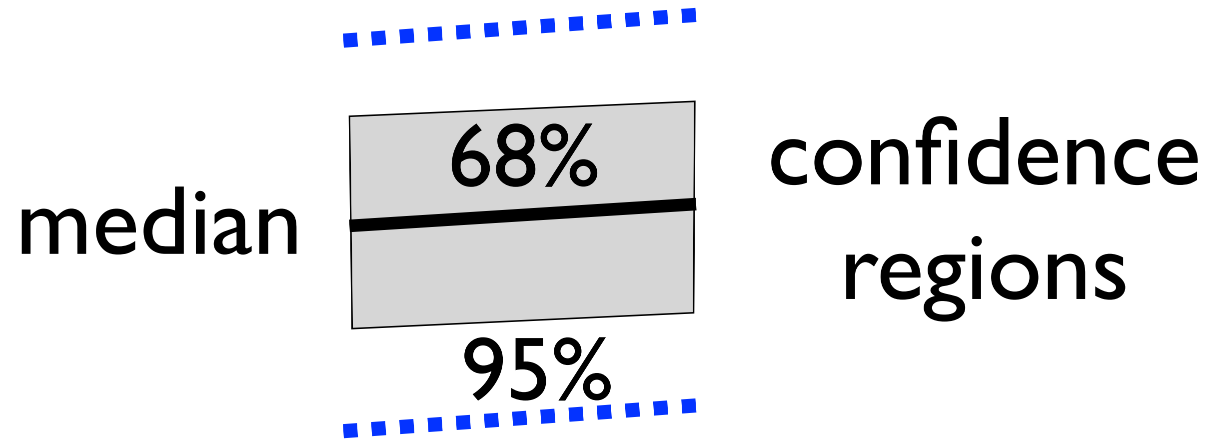
L. Brandes, W. W., N. Kaiser : Phys. Rev. D 107 (2023) 014011 ; arXiv: 2306.06218 [nucl-th] (PRD in print)





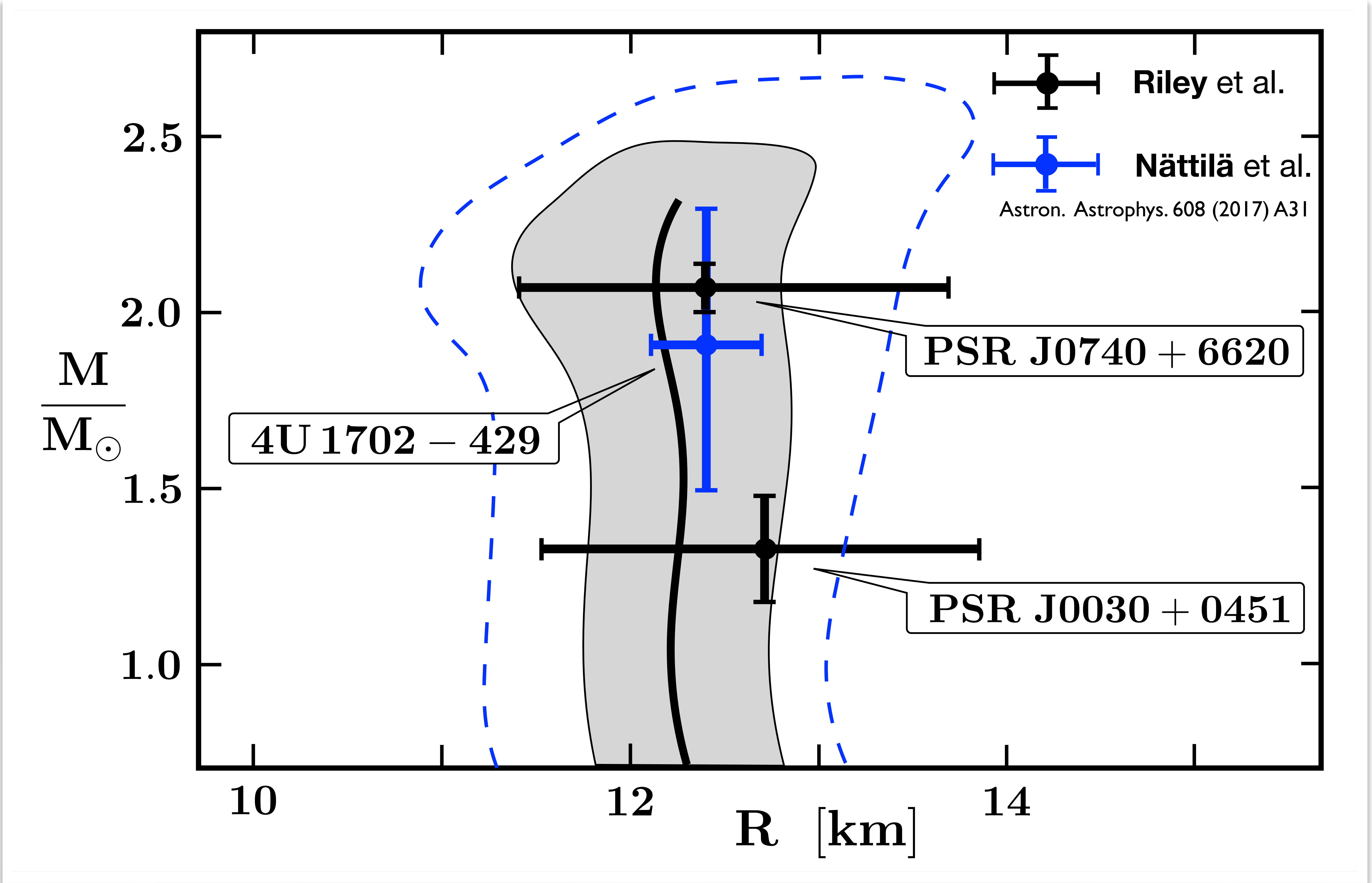
- Squared **speed of sound** exceeds conformal bound $c_s^2 = 1/3$ at densities $\rho > 3\rho_0$
- **Strongly repulsive correlations at high baryon densities**

NEUTRON STAR PROPERTIES

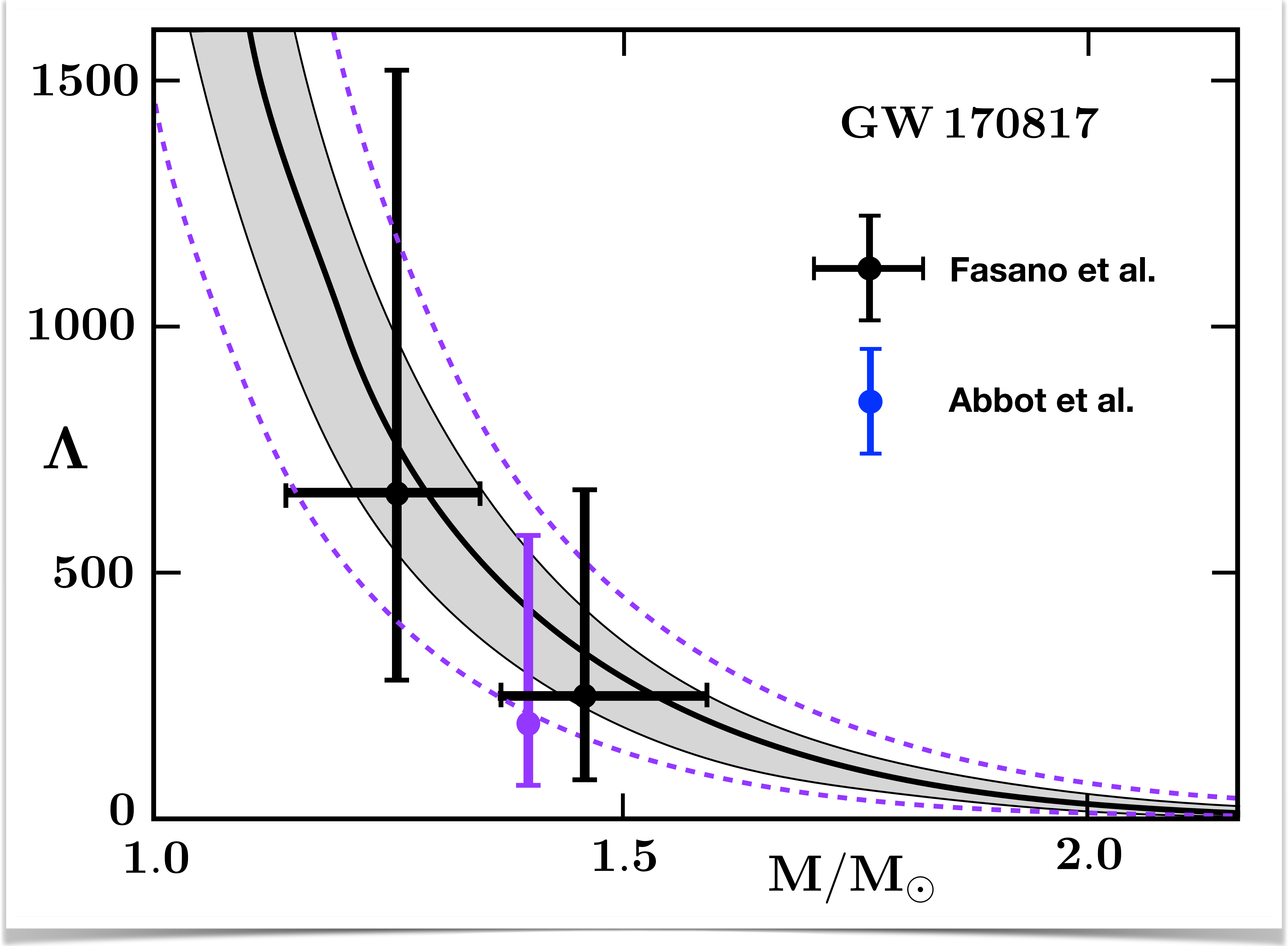


- Bayesian inference posterior credible bands

- Mass - Radius relation (TOV)



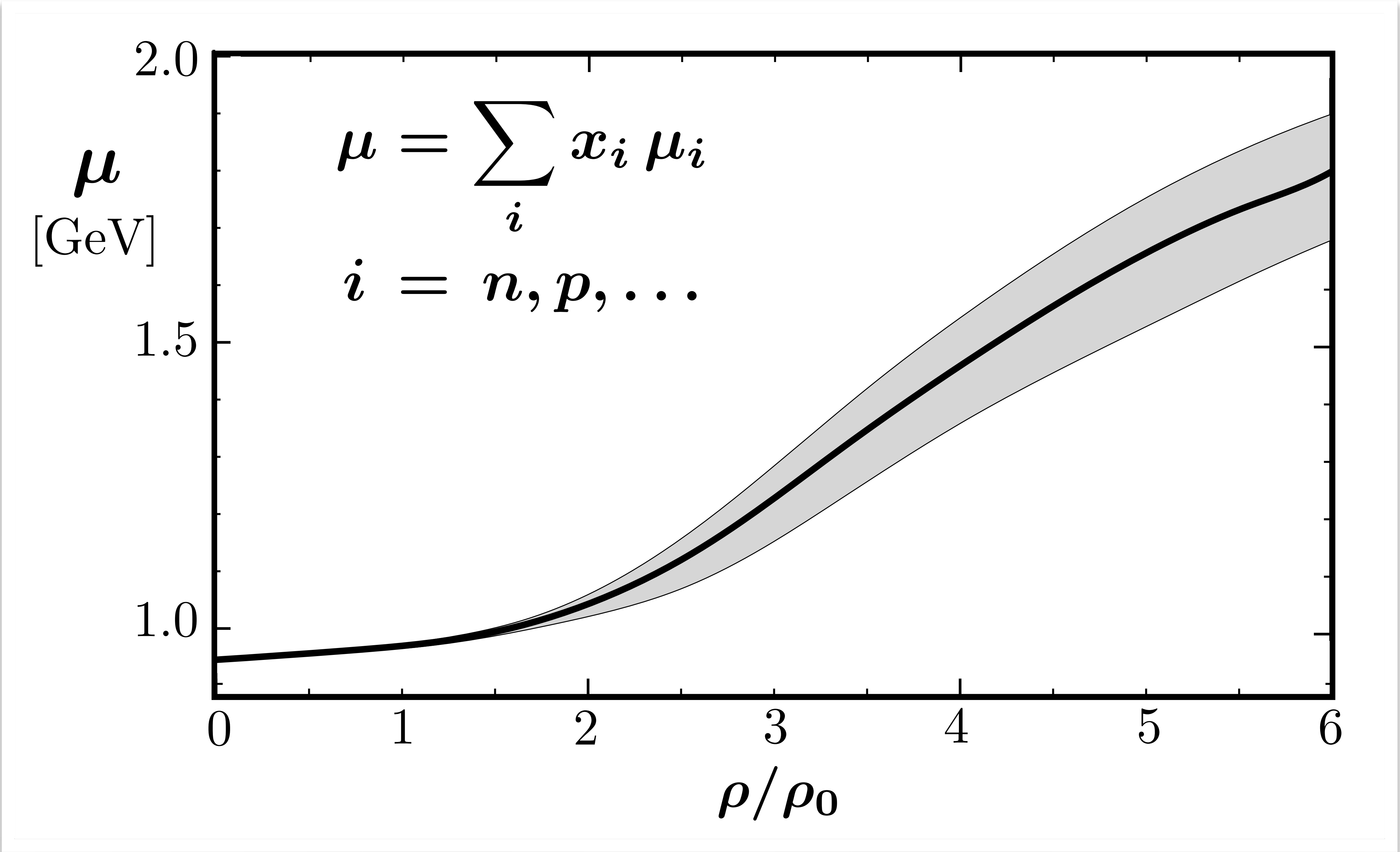
- Tidal deformability



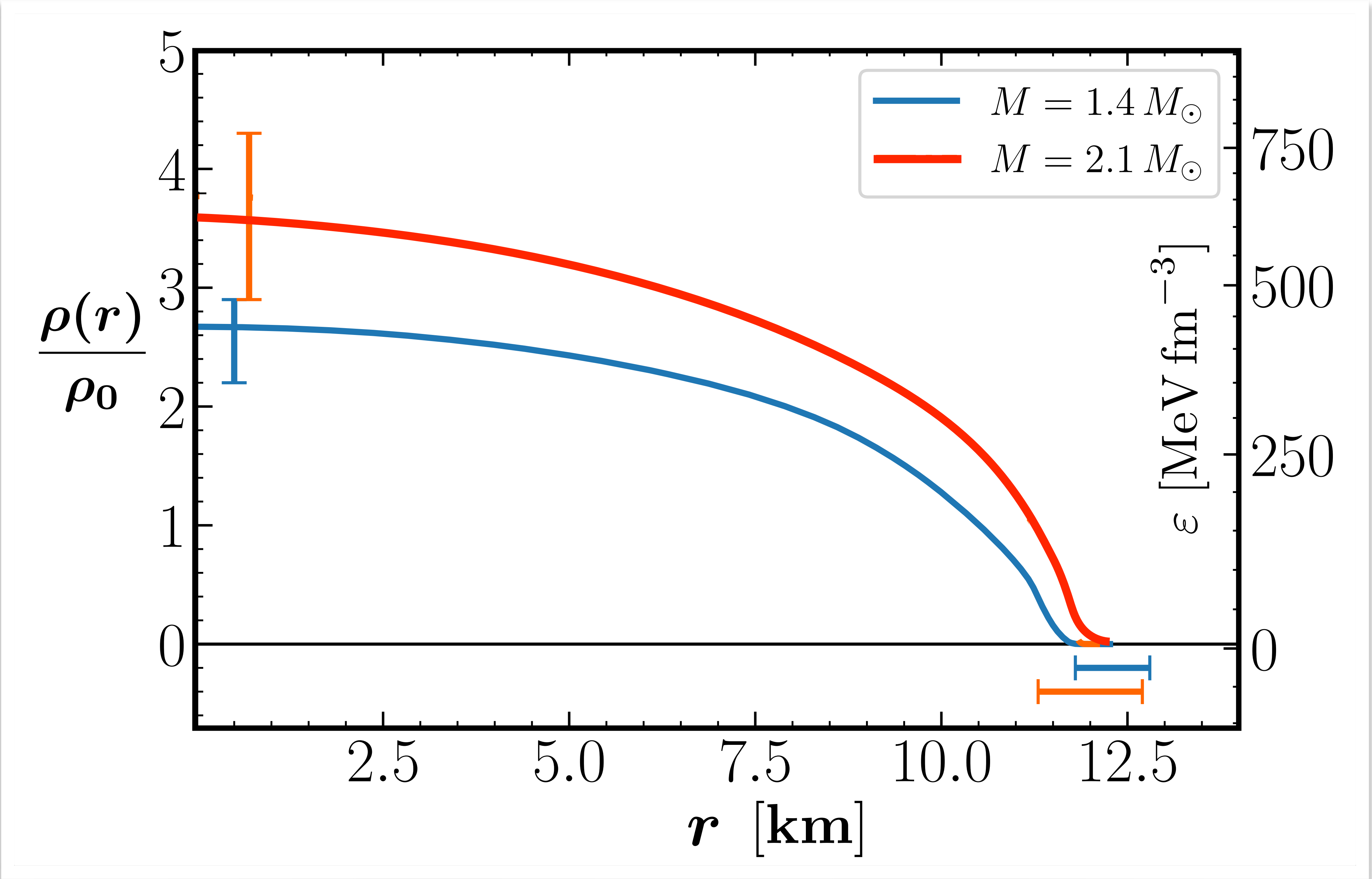
L. Brandes, W. W., N. Kaiser : Phys. Rev. D 107 (2023) 014011 ; arXiv: 2306.06218 [nucl-th]

NEUTRON STAR PROPERTIES (contd.)

- **Baryon chemical potential**



- **Density profiles of neutron stars**



L. Brandes, W. W., N. Kaiser : Phys. Rev. D 107 (2023) 014011 ; arXiv:2306 (Phys. Rev. D in print).

- **Stiff equation-of-state** → **central core densities in neutron stars are NOT extreme :**

$$\rho_c(1.4\,M_\odot) = 2.6^{+0.3}_{-0.4}\,\rho_0 \qquad\qquad \rho_c(2.1\,M_\odot) = 3.6 \pm 0.7\,\rho_0$$

$$(\rho_0 = 0.16\,\mathrm{fm}^{-3})$$

Constraints on
FIRST-ORDER PHASE TRANSITION in NEUTRON STAR MATTER

- **Bayes factor analysis :**

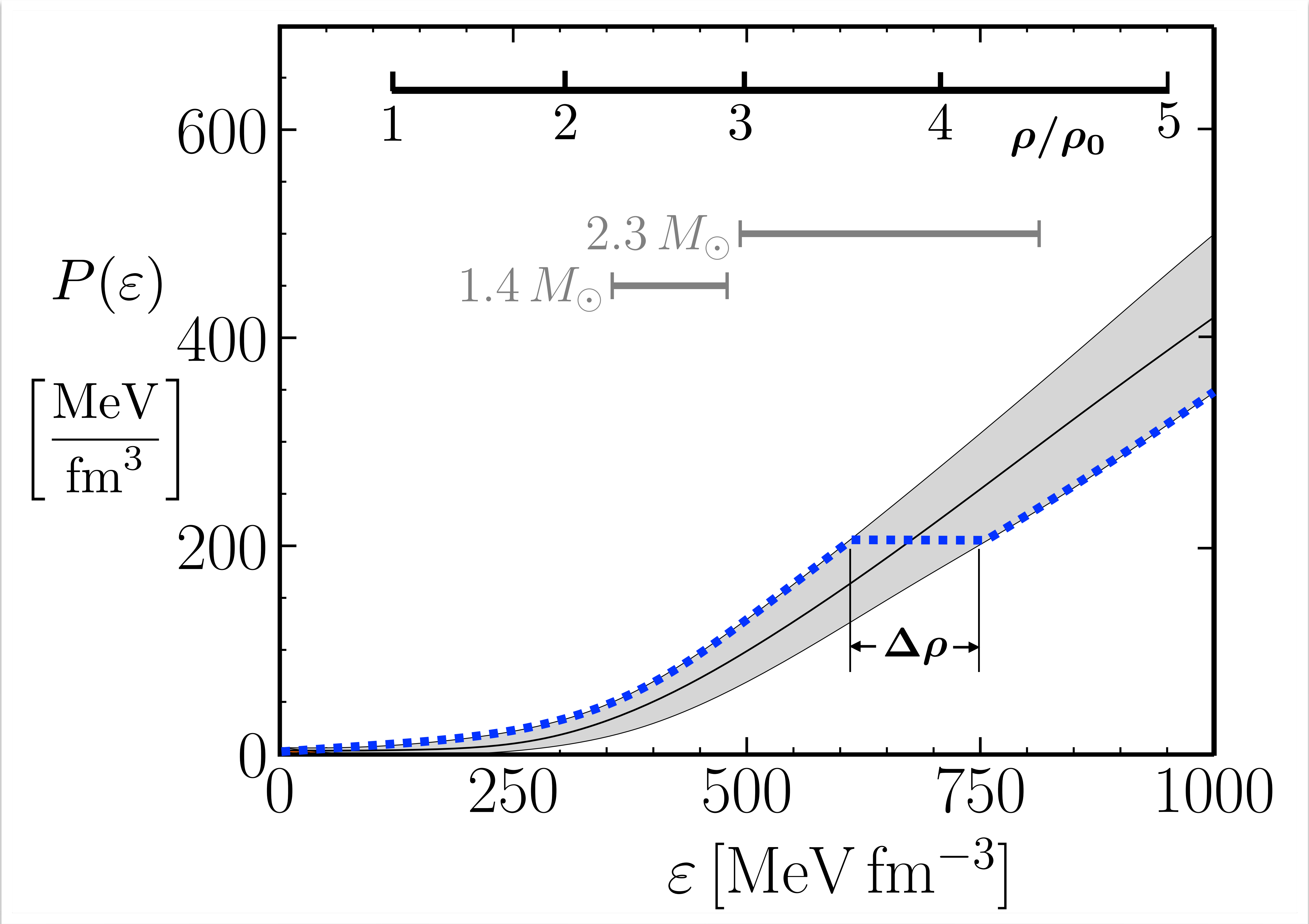
  → **extreme evidence for sound velocities $\mathbf{c_s > 0.5}$ in cores of all neutron stars with**

  $$\mathbf{1.4 \leq M/M_\odot \leq 2.3}$$

- **Evidence against strong 1st order transition :**

  → **maximum possible extension of phase coexistence domain**

  $$\boldsymbol{\Delta\rho/\rho < 0.2}$$



→ **compare with :**
Maxwell construction for nuclear liquid-gas phase transition $(\Delta\rho/\rho > 1)$

INTERMEDIATE SUMMARY

- **Bayesian inference analysis**
  now including heavy $(\mathrm{M} = 2.35 \pm 0.17\,\mathrm{M}_\odot)$ galactic neutron star
  → even **stiffer equation-of-state** required
  → almost **constant neutron star radii** $(\mathrm{R} = 12 \pm 1\,\mathrm{km})$ for all masses

- **Extreme evidence** for sound velocities $\mathrm{c_s} > 1/\sqrt{3}$ in neutron star cores
  → strongly **repulsive correlations** at work

- **Evidence** against **strong 1st order phase transition** in neutron star cores
  → **not** excluded: **baryonic** matter or **hadron-quark** continuous crossover

- **No** extreme **central core densities** even in the heaviest neutron stars:
  $\rho < 4.5\,\rho_0$ for $\mathrm{M} = 2.3\,\mathrm{M}_\odot$ → average baryon-baryon distance : $\mathrm{d} \gtrsim 1\,\mathrm{fm}$

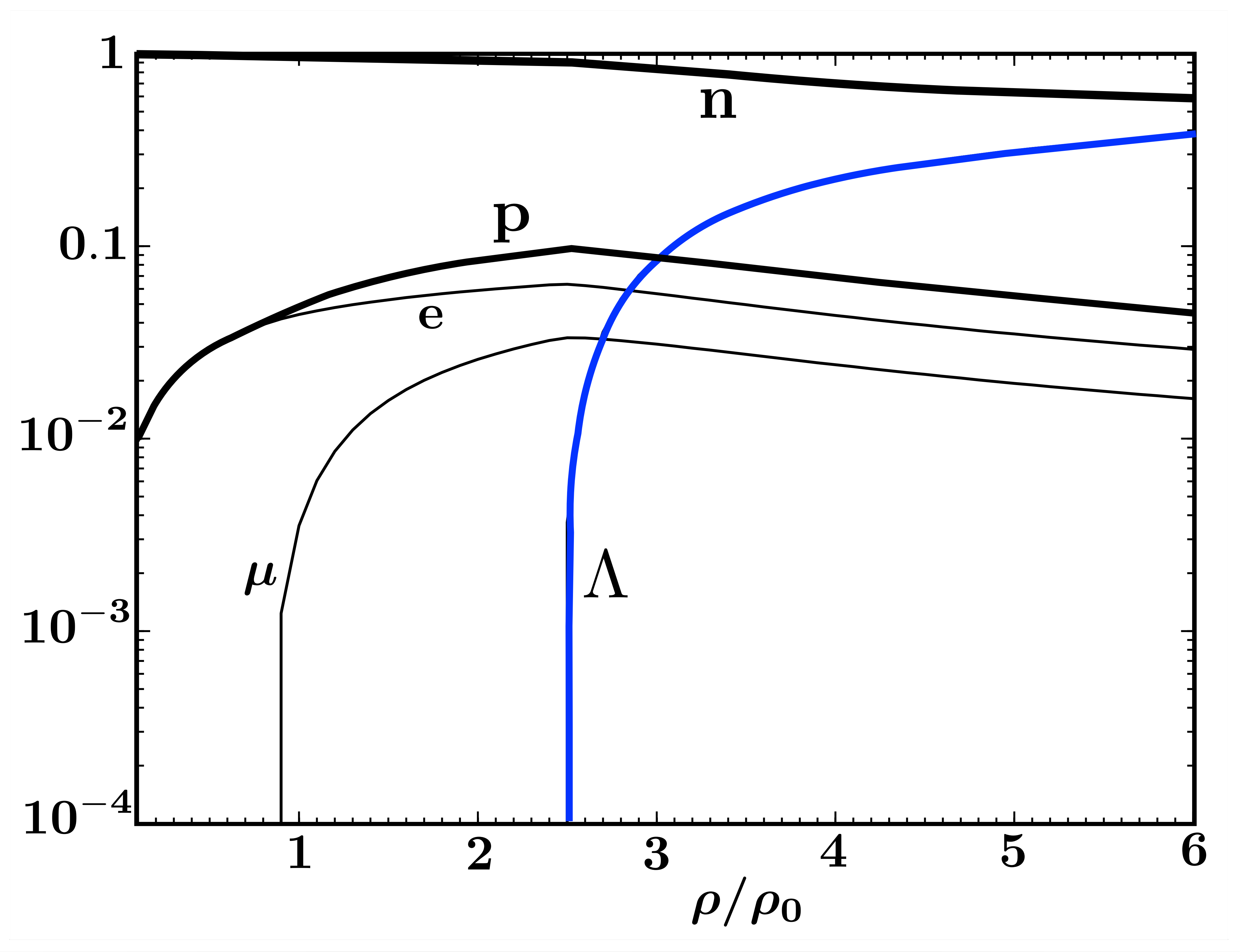
Part Two

Strangeness in Neutron Star Matter ?

Hyperon-Nuclear Interactions
and the
"Hyperon Puzzle"

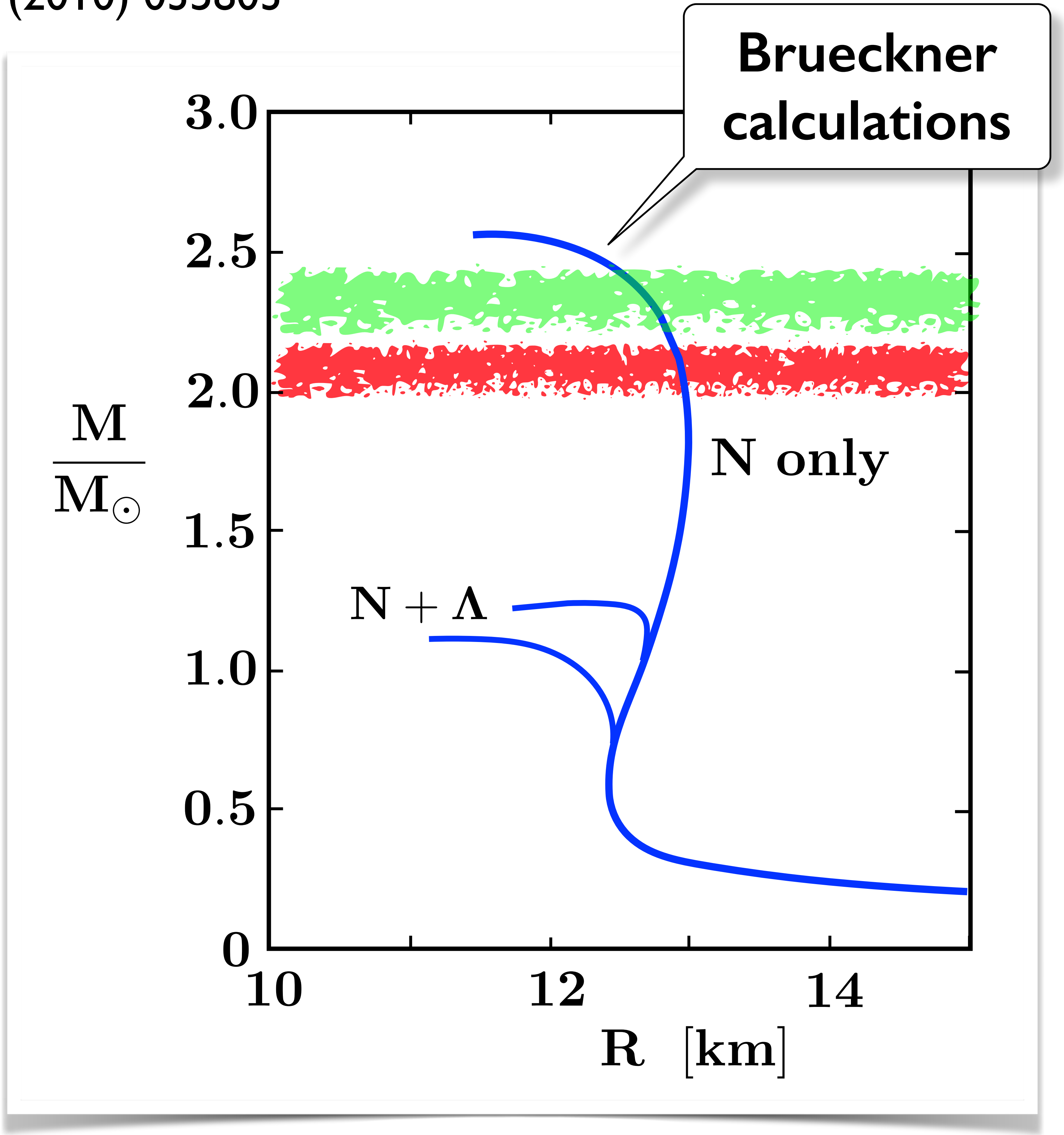
NEUTRON STAR MATTER including HYPERONS

Example: H. Djapo, B.-J. Schaefer, J. Wambach : Phys. Rev. C81 (2010) 035803



“*Hyperon Puzzle*”

D. Lonardoni,
A. Lovato,
S. Gandolfi,
F. Pederiva :
Phys. Rev. Lett.
114 (2015) 092301

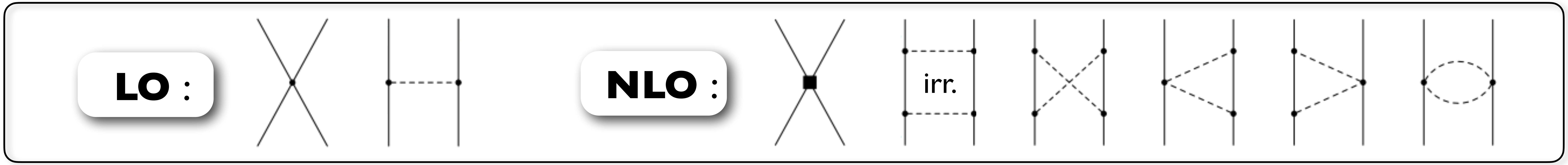

Brueckner calculations

- Inclusion of **hyperons :**

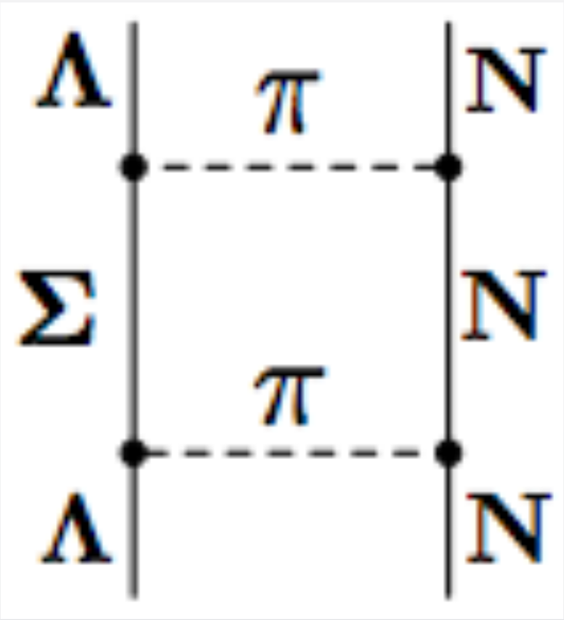
EoS too soft to support two-solar-mass n-stars

strong repulsion in **YN** and/or **YNN interactions** required

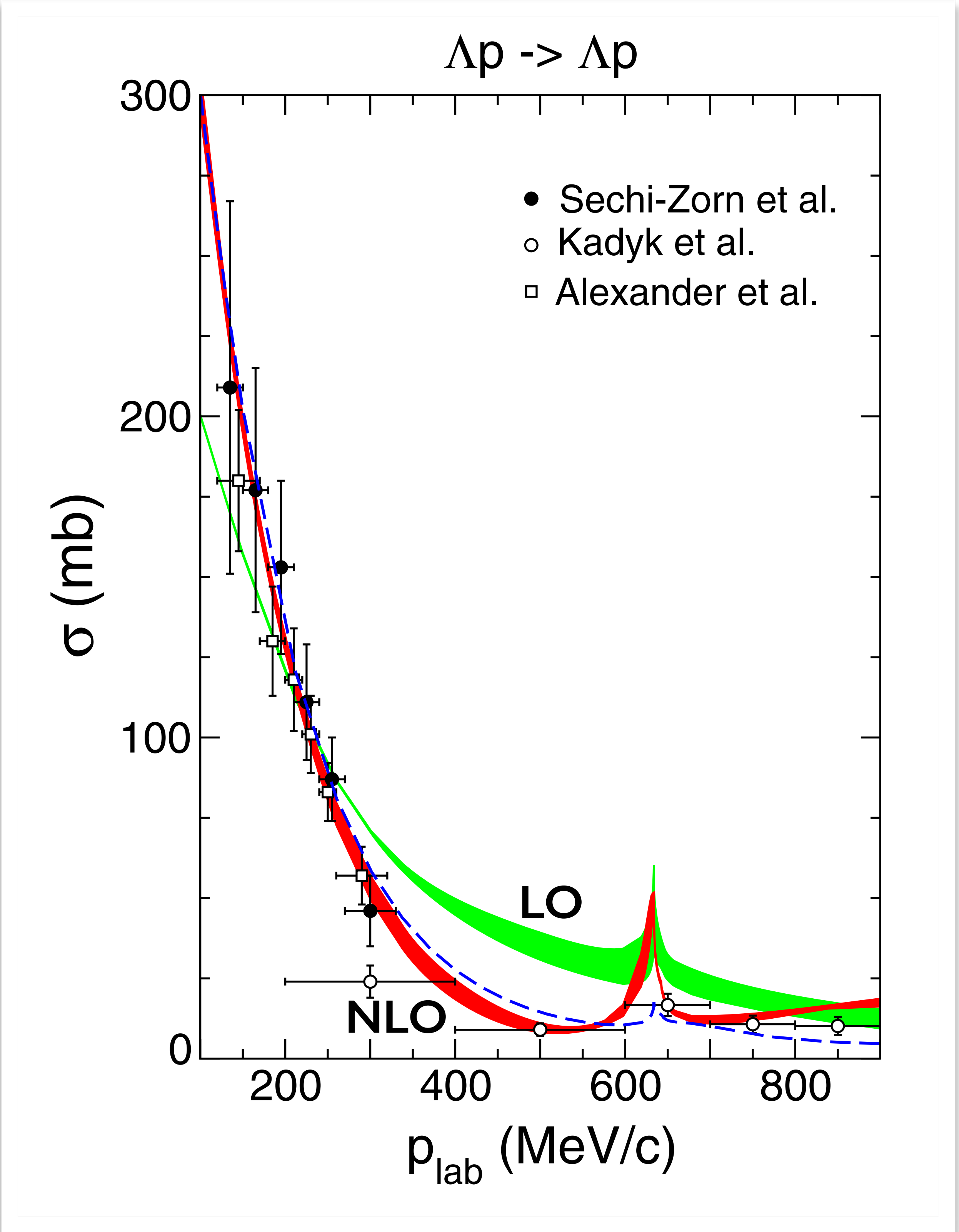
Hyperon - Nucleon Interaction
from CHIRAL SU(3) Effective Field Theory

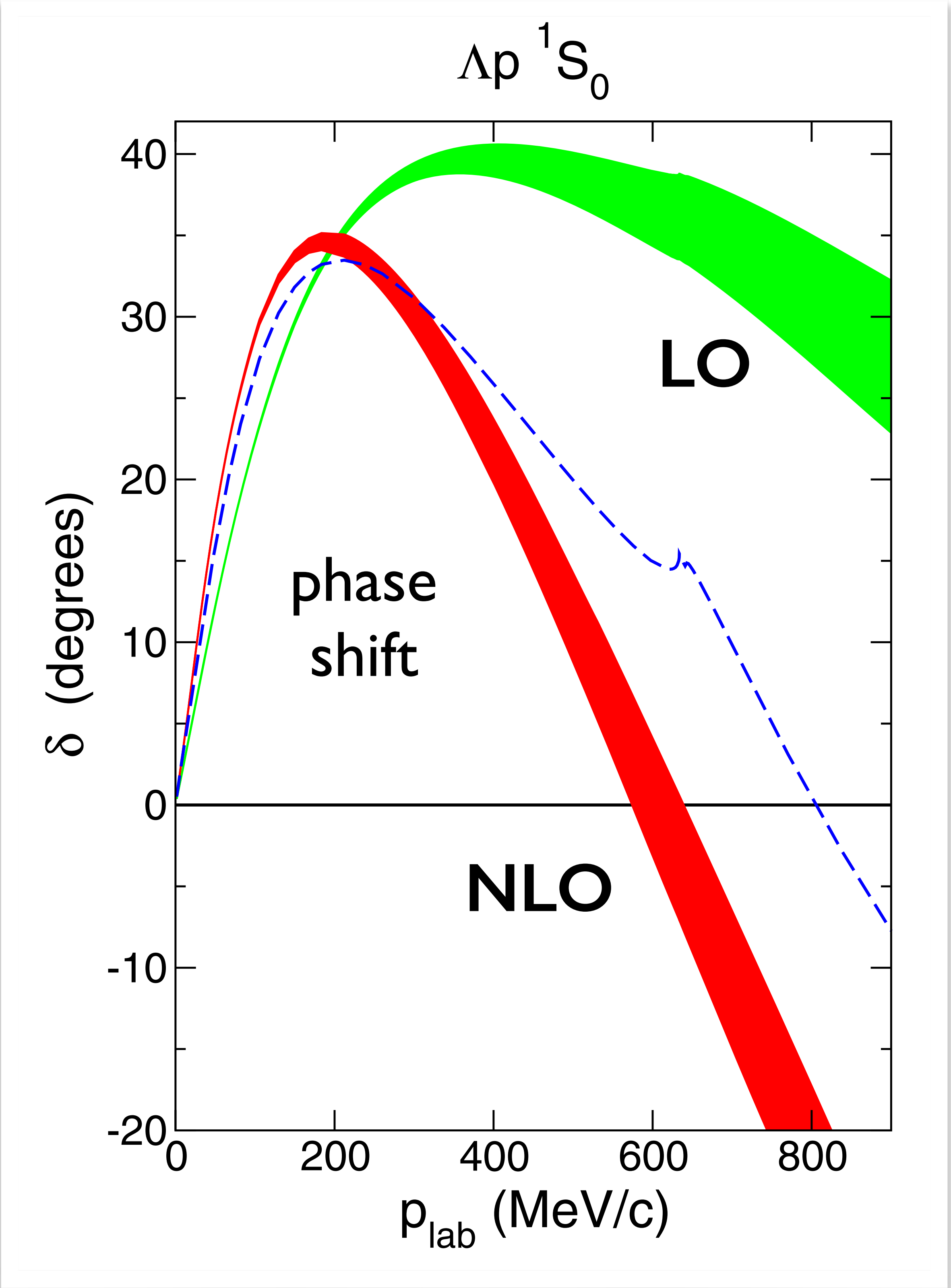


ΛN scattering



Important role of
$\Lambda N \leftrightarrow \Sigma N$
coupled channels





- **moderate attraction** at low momenta
  - → relevant for **hypernuclei**
- **increasing repulsion** at higher momenta
  - → relevant for **dense baryonic matter**

J. Haidenbauer, S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W.W.
Nucl. Phys. A 915 (2013) 24

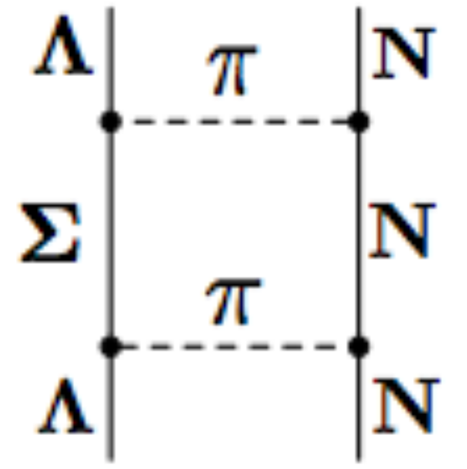
Λ Hyperon - Nucleon Interaction update

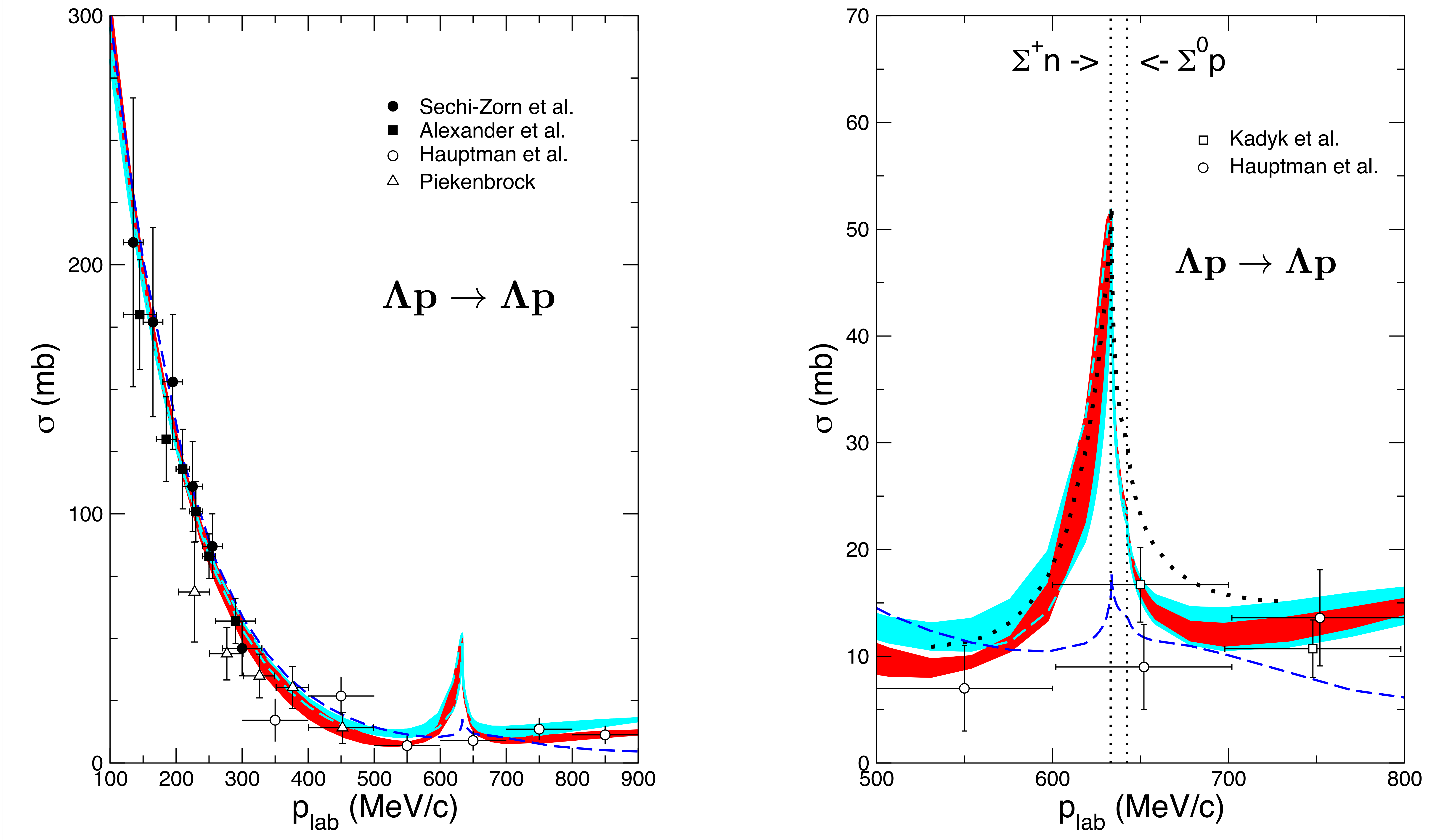
J. Haidenbauer, U.-G. Meißner, A. Nogga Eur. Phys. J. A56 (2020) 91

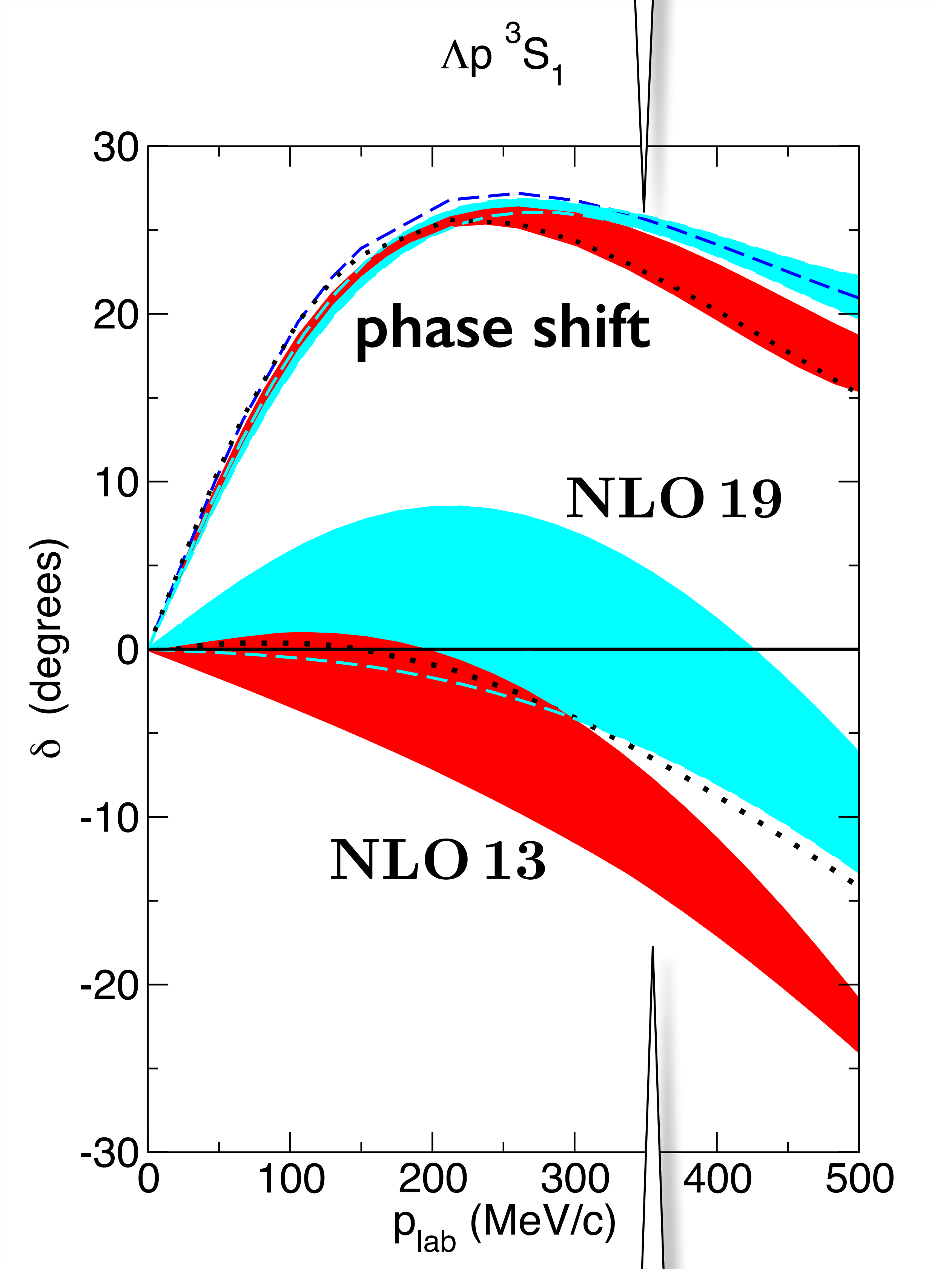
- Reduced no. of independent parameters (contact terms) at NLO by symmetries connecting NN and YN S-waves

blue : NLO 19
red : NLO 13

including $\Lambda N \leftrightarrow \Sigma N$ coupling





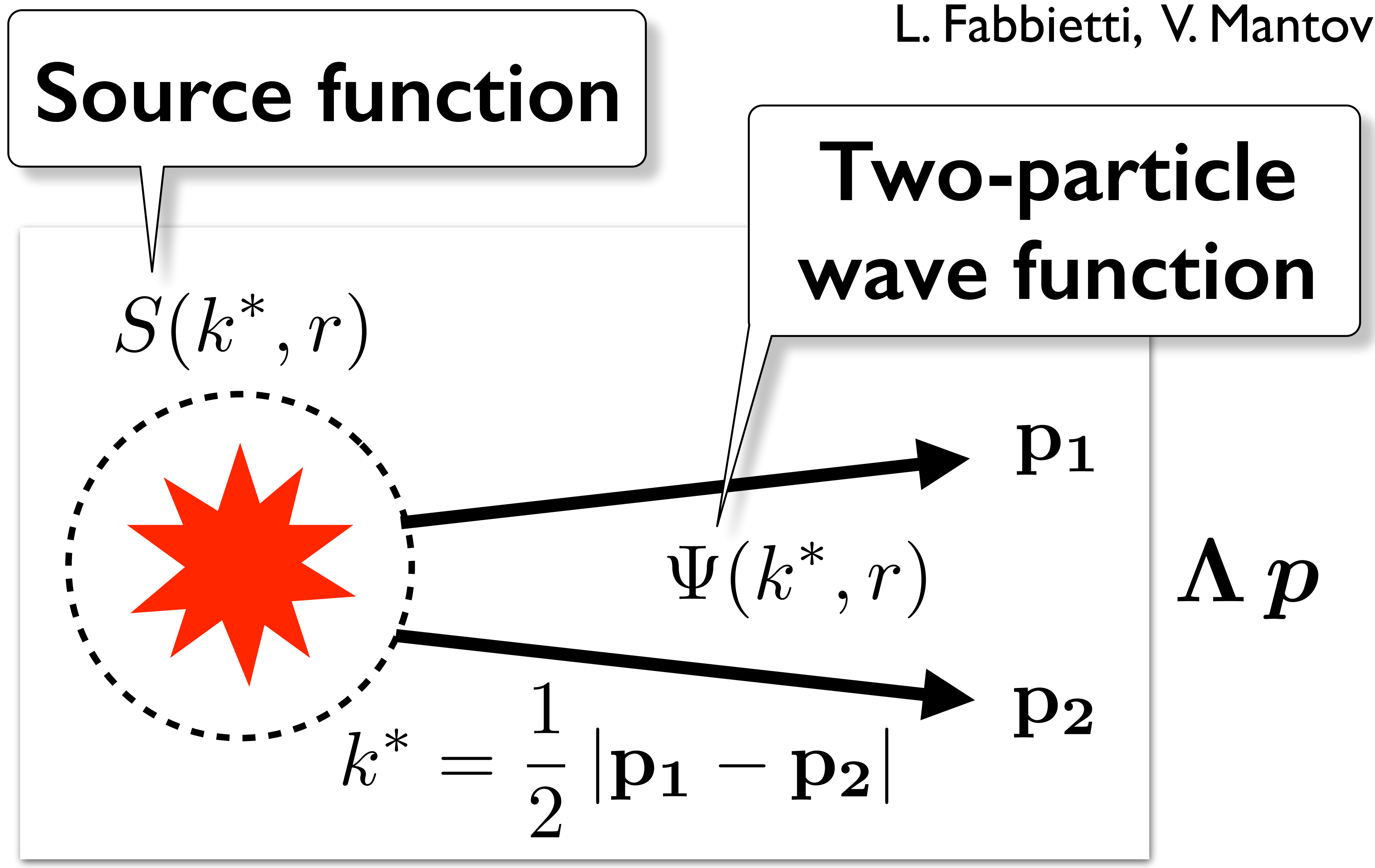


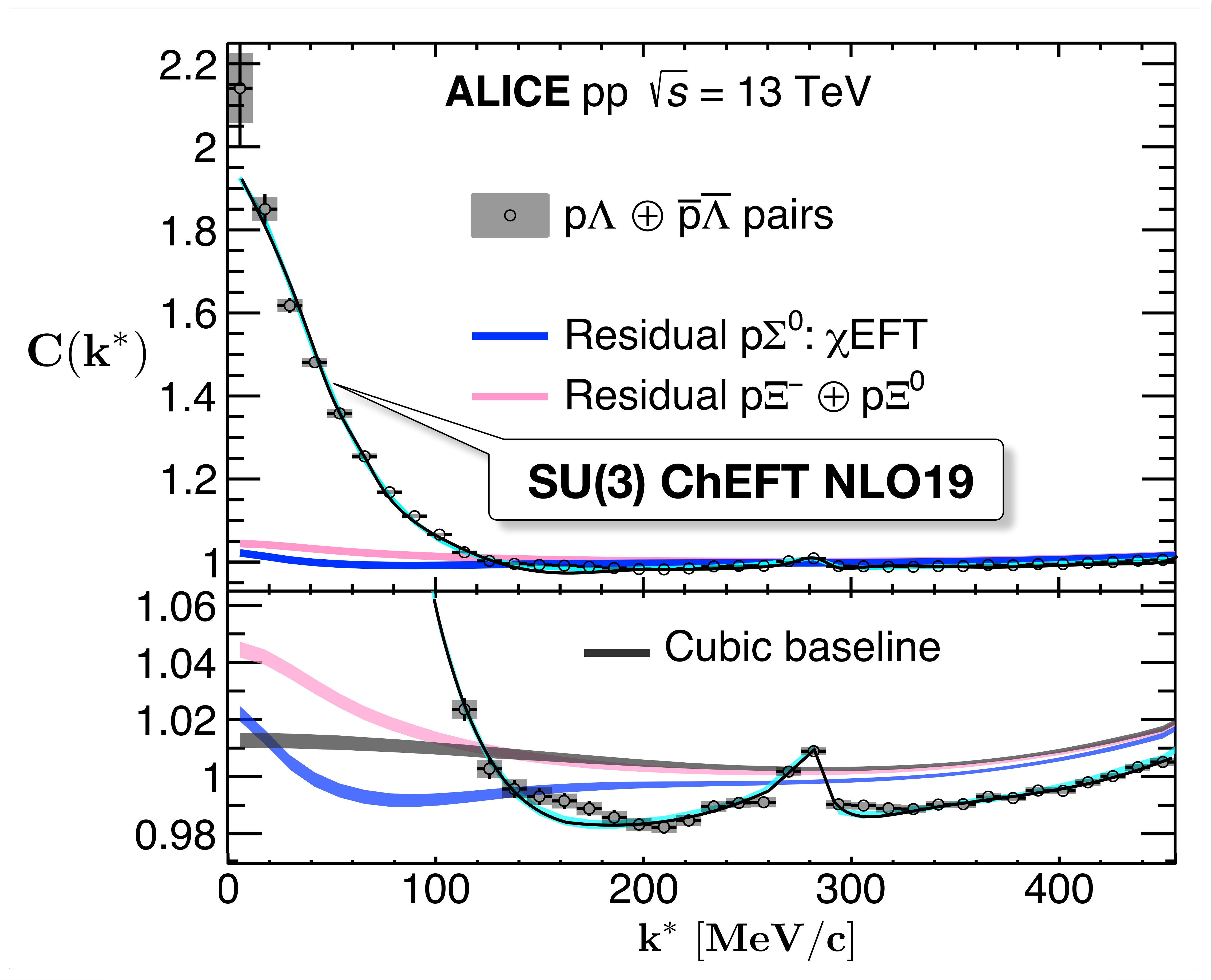
without $\Lambda N \leftrightarrow \Sigma N$ coupling

$\Lambda\, p$ CORRELATION FUNCTION

- Femtoscopy two-particle correlation studies from p p collisions with ALICE @ LHC

L. Fabbietti, V. Mantovani Sarti, O.Vazquez Doce : Ann. Rev. Nucl. Part. Sci. 71 (2021) 377





ALICE Collab. : Phys. Lett. B 833 (2022) 1372782

- Correlation function

$$C(k^*) = \frac{\langle \mathcal{P}_1(k^*)\mathcal{P}_2(k^*)\rangle}{\langle \mathcal{P}_1(k^*)\rangle\langle \mathcal{P}_2(k^*)\rangle} = \int d^3r\, S(k^*, r)|\Psi(k^* r)|^2$$

- Accurate test of
  low-momentum $\Lambda\, p$ interaction

Recent developments :

CHIRAL SU(3) **HYPERON-NUCLEON INTERACTION** at **NNLO**

J. Haidenbauer, U.-G. Meißner, A. Nogga, H. Le : Eur. Phys. J. A 59 (2023) 63

- Updated tests in comparison with YN data
- More **attraction** in the ΛN channel
- Λ - **nuclear single-particle potential** (Brueckner-Hatrtree-Fock calculations)

$U_\Lambda(\rho = \rho_0)$	NLO19	NNLO	empirical (hypernuclei)
[MeV]	-34.8 ± 1.5	-37.4 ± 1.1	$-27 \leftrightarrow -30$

averaged over cutoffs $(\lambda = 550 - 600 \text{ MeV})$

A. Gal, E.V. Hungerford, D. J. Millener
Rev. Mod. Phys. 88 (2016) 035004

- With chiral hyperon-nucleon **two-body interactions** :
too much binding in lower (s and p) shell-model levels of **heavy hypernuclei** ($^{208}\text{Pb}_\Lambda$)

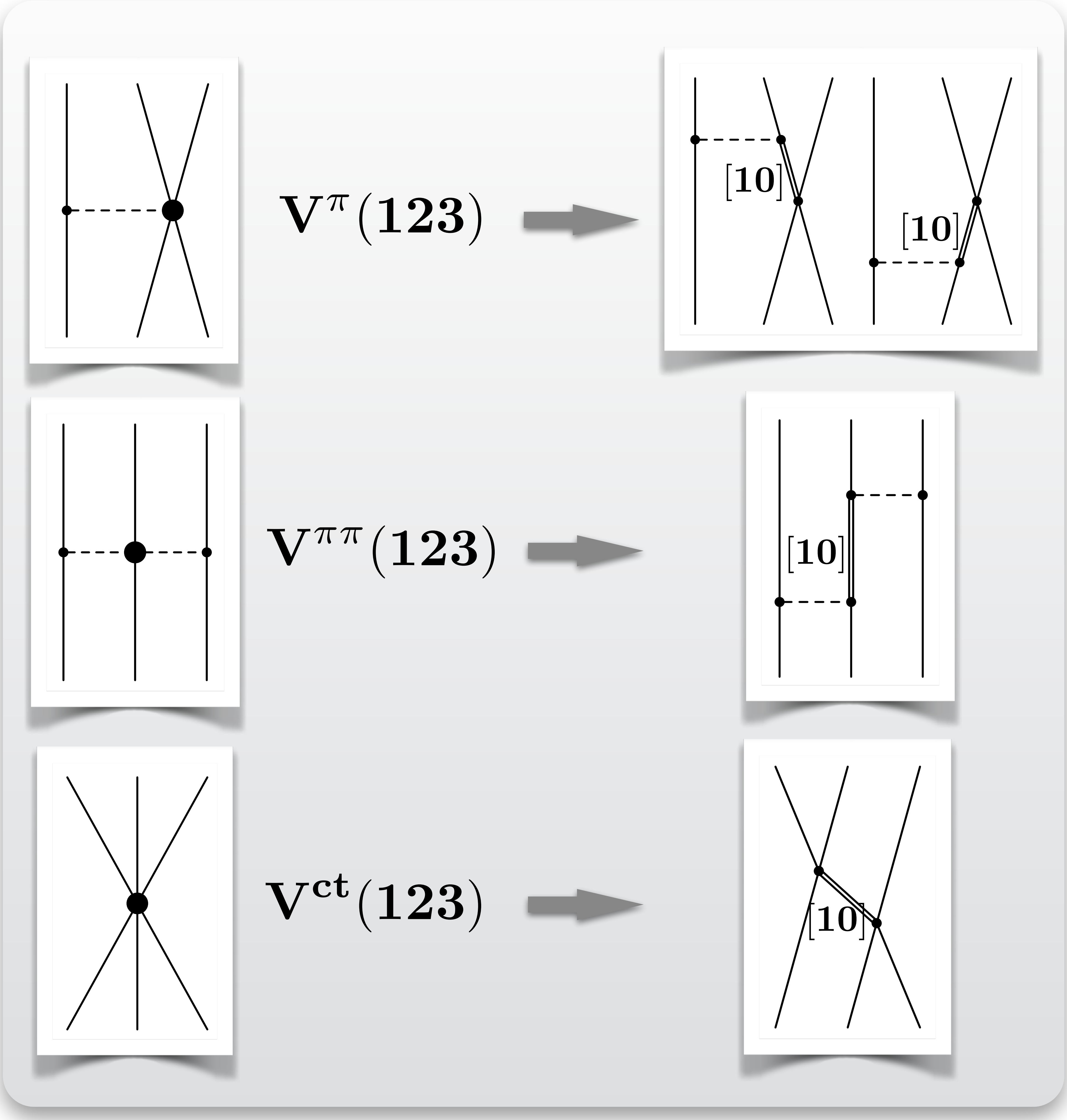
J. Haidenbauer, I. Vidaña : Eur. Phys. J. A 56 (2020) 55

- Quest for **repulsive** ΛNN **three-body forces**

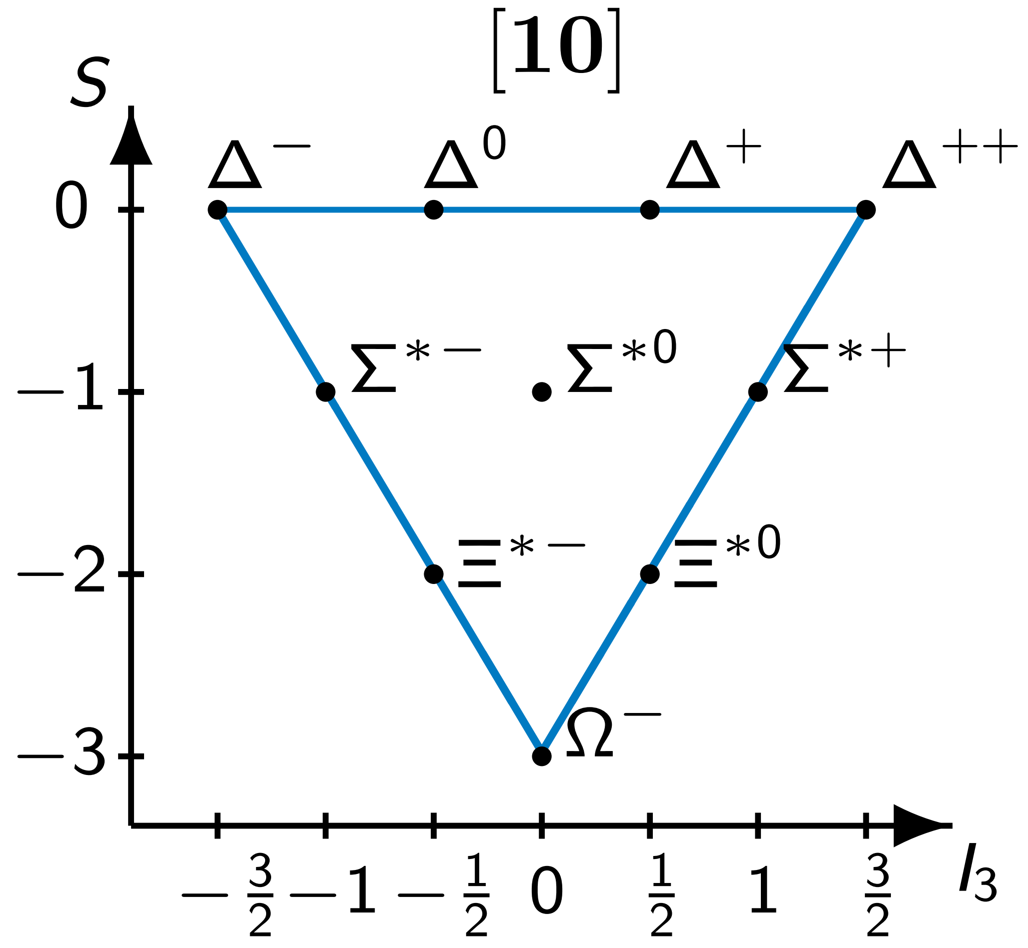
HYPERON-NUCLEON-NUCLEON THREE-BODY FORCES

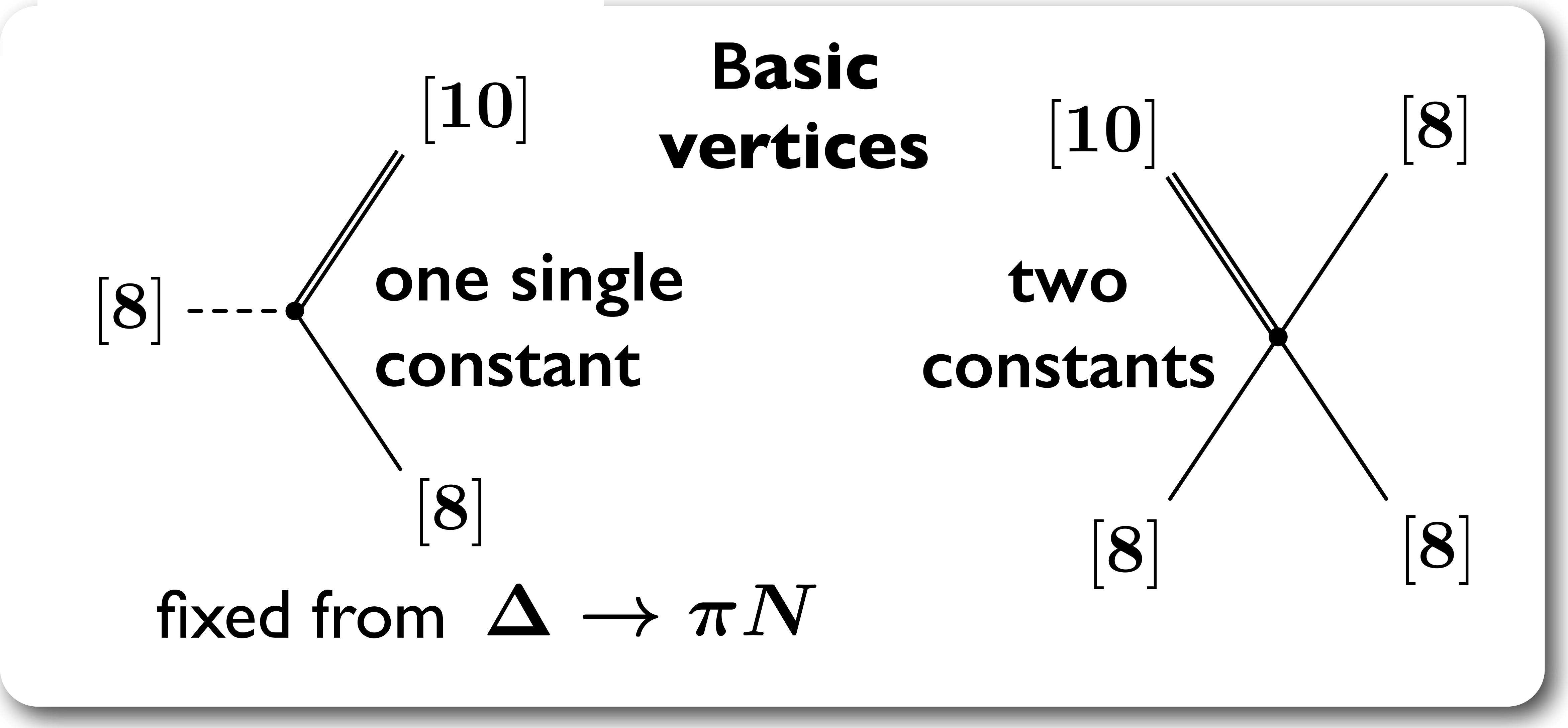
from Chiral $\mathbf{SU(3)_L \times SU(3)_R}$ Effective Field Theory

S. Petschauer, N. Kaiser, J. Haidenbauer, U.-G. Meißner, W.W. : Phys. Rev. C93 (2016) 014001



- **Decuplet Dominance**
  in YNN three-body forces
- Estimates of **YNN** interactions assuming dominant $(\Sigma^*,\ \Delta)$ intermediate states





Basic vertices

one single constant

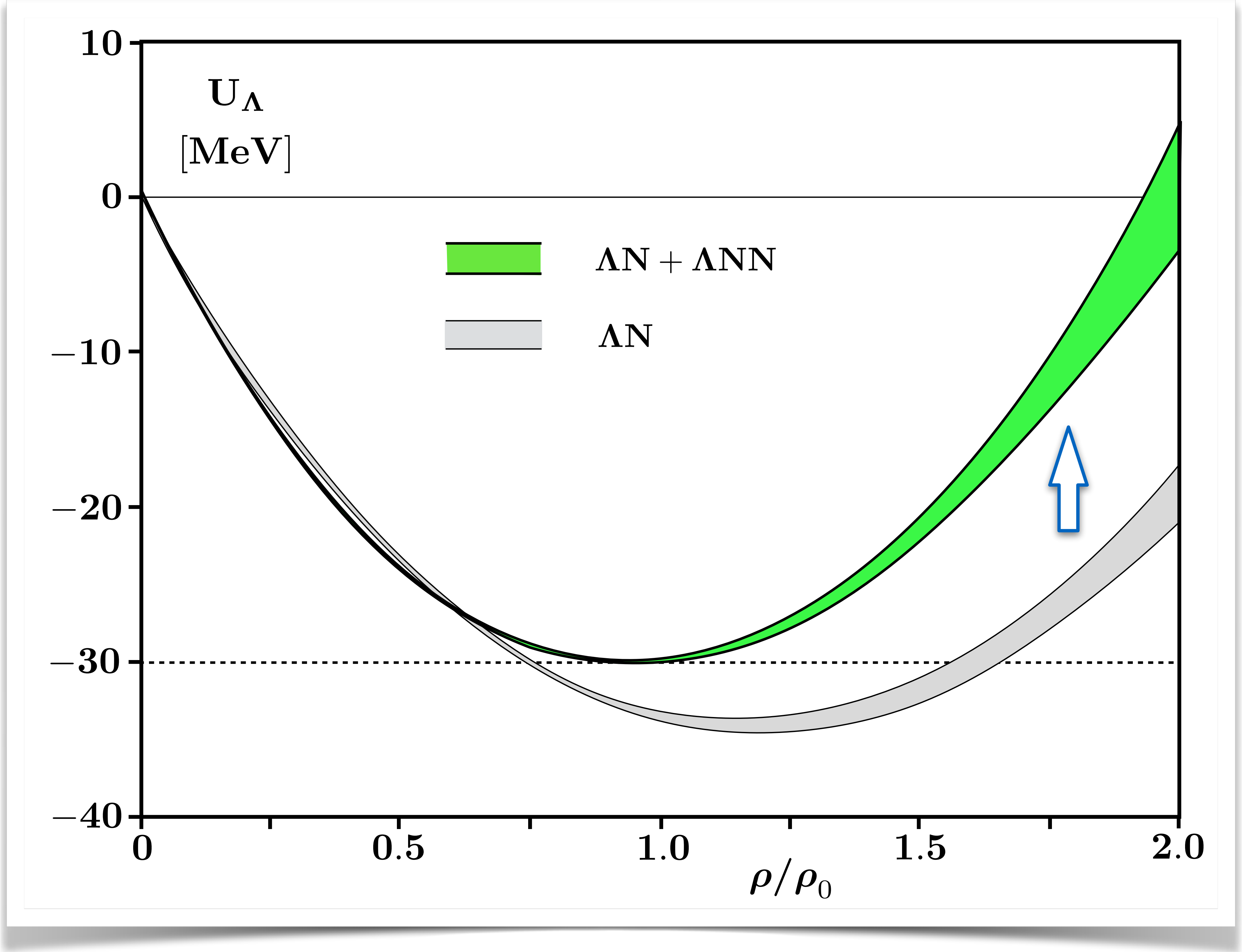
two constants

fixed from $\Delta \to \pi N$

Density dependence of Λ single particle potential

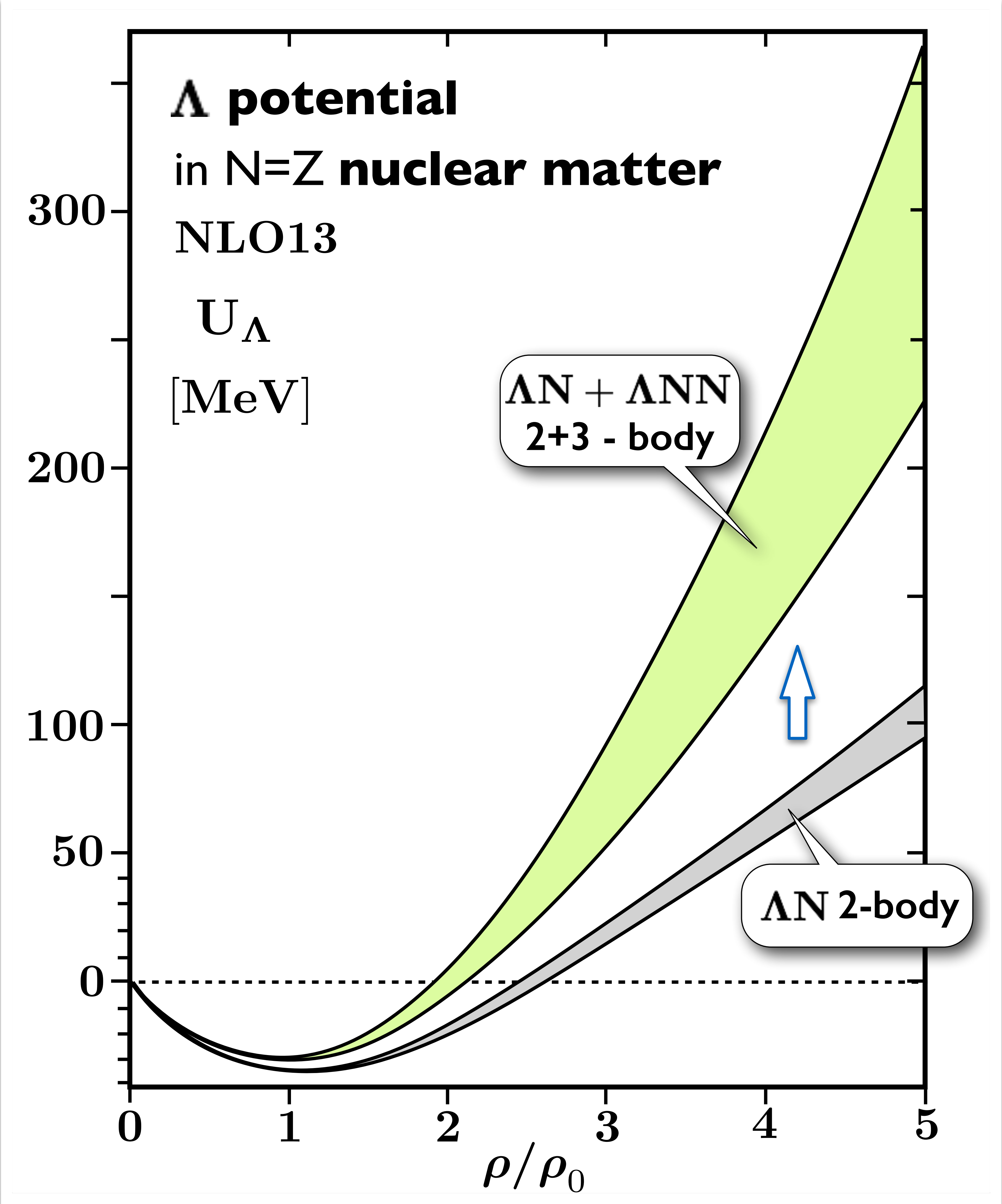
- Coupled-channels G-matrix including $\Lambda NN \leftrightarrow \Sigma NN$
- **Three-body interactions** treated as **density-dependent effective 2-body forces**

D. Gerstung, N. Kaiser, W.W. : Eur. Phys. J. A56 (2020) 175



Chiral
NN (N3LO)
+ YN (NLO)
+NNN+YNN
interactions

Strong additional repulsion from YNN three-body forces

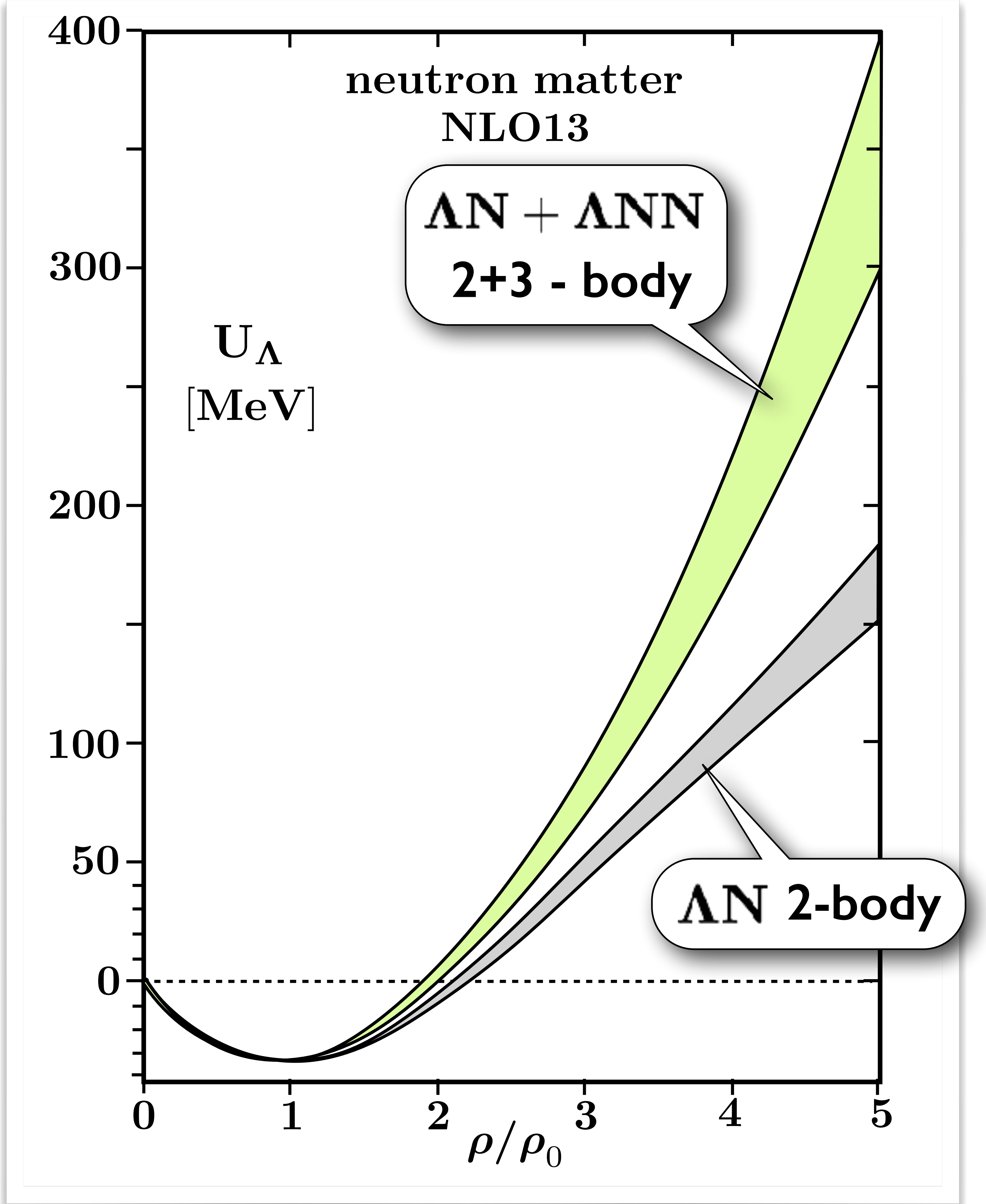


- Constrained by hypernuclear physics : $U_\Lambda(\rho = \rho_0) \simeq -30\,\mathrm{MeV}$

A. Gal, E. Hungerford, D. Millener
Rev. Mod. Phys. 88 (2016) 035004

Λ HYPERONS in NEUTRON STARS ?

- Onset condition for appearance of Λ hyperons in neutron stars : Equality of **chemical potentials**



- **Hyperon chemical potential**

in neutron star matter
from
Chiral SU(3)
EFT interactions

D. Gerstung, N. Kaiser, W.W.
Eur. Phys. J. A56 (2020) 175

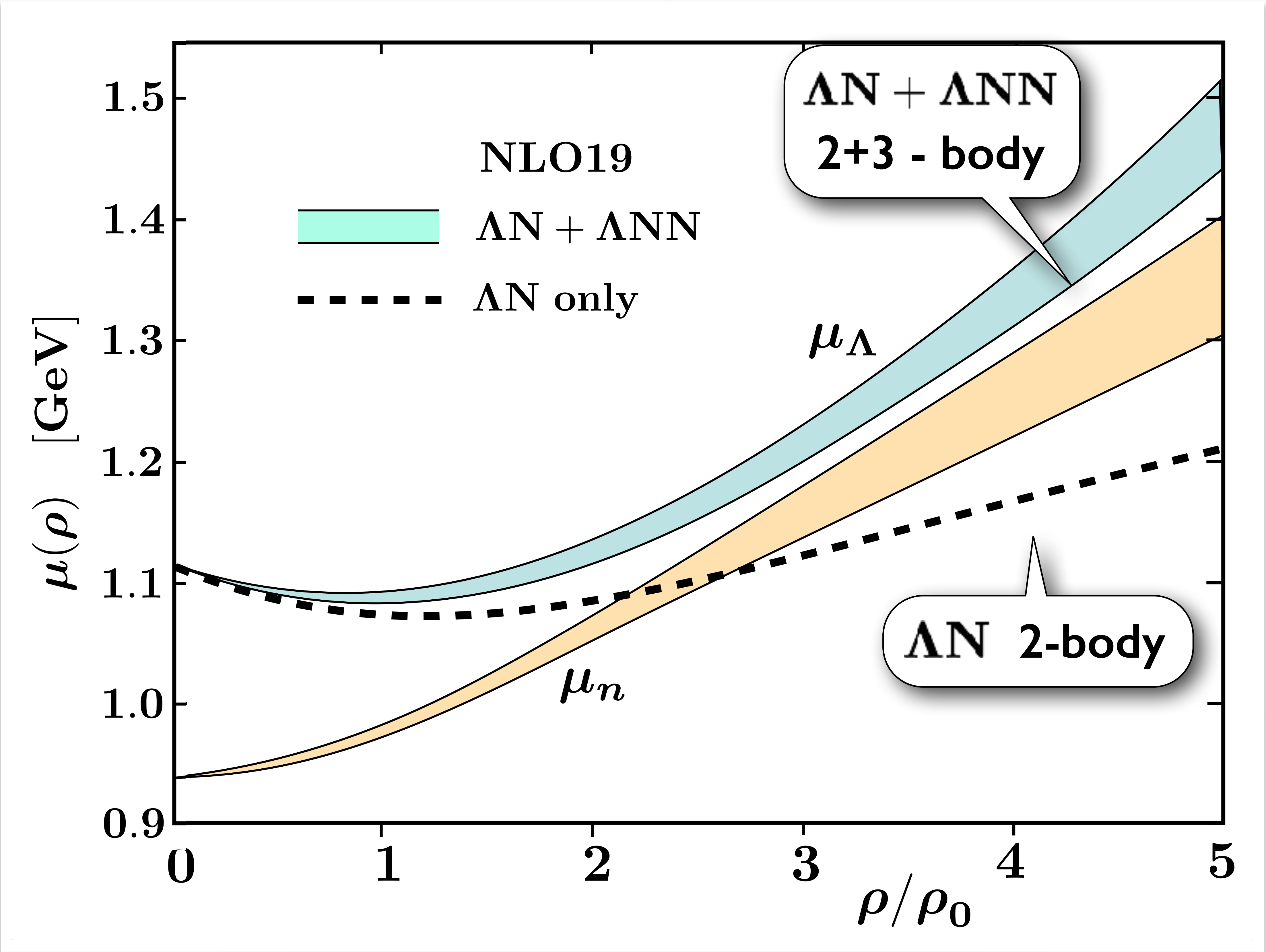
- **Neutron chemical potential**

in neutron star matter
from
Chiral EFT
+ FRG EoS

M. Drews, W.W.
Prog. Part. Nucl. Phys. 93 (2017) 69

(consistent with APR EoS)

$$\mu_\Lambda = \mu_n \qquad \mu_i = \frac{\partial \mathcal{E}}{\partial \rho_i}$$



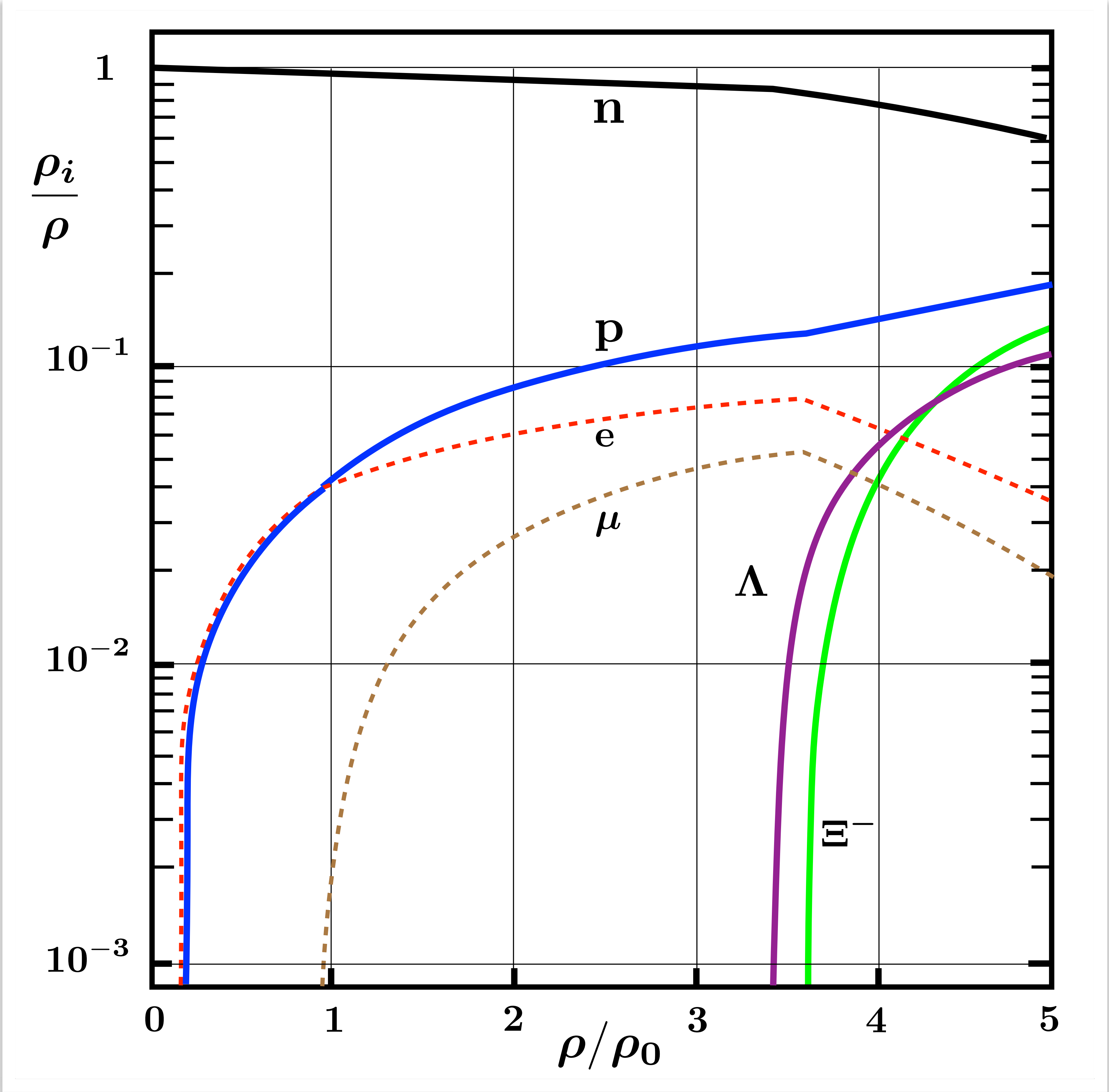
NEUTRON STAR MATTER including HYPERONS

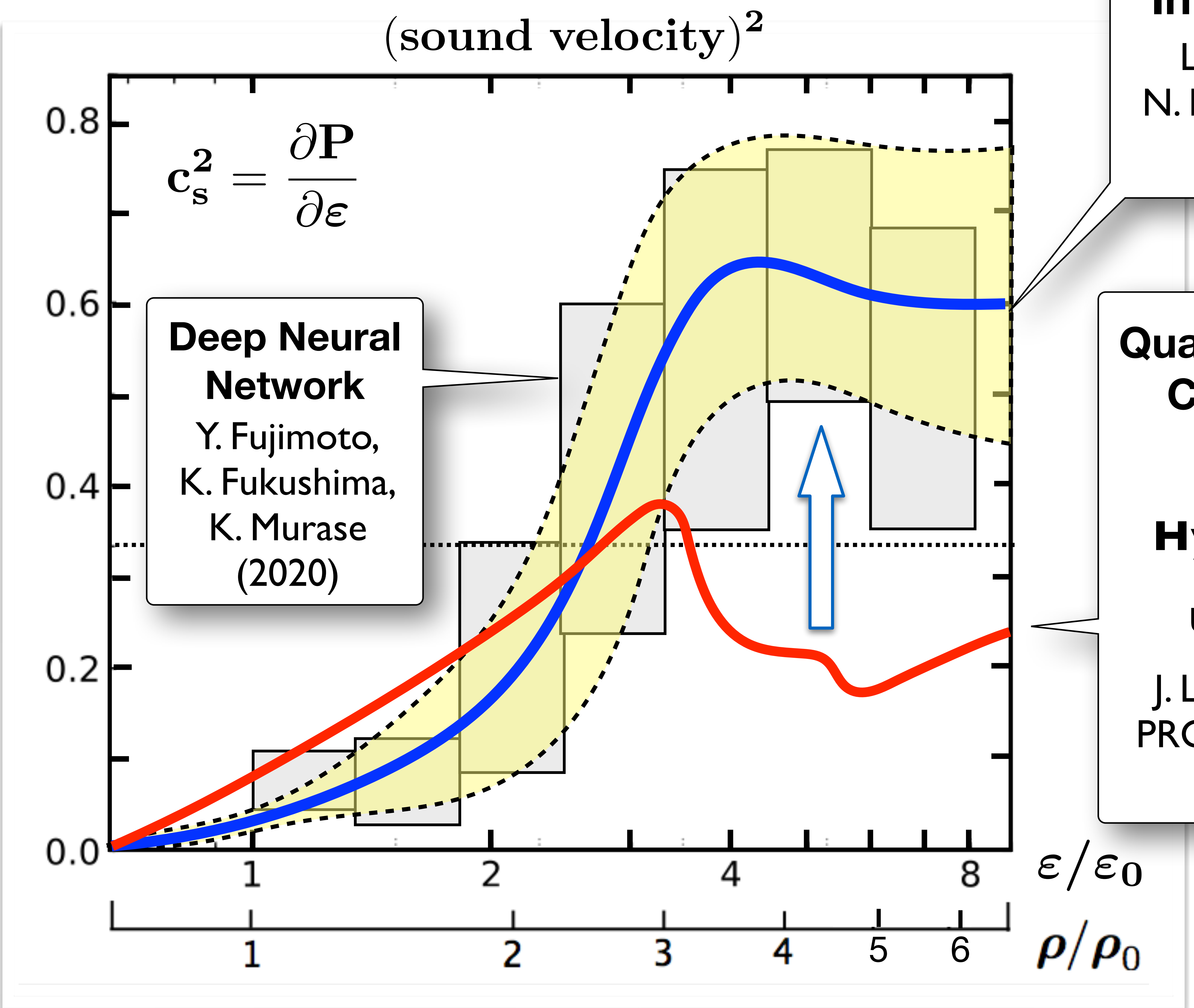
NO "Hyperon Puzzle" ?

- **Quark - Meson Coupling model** $\mathrm{M}_\mathrm{B}^* = \mathrm{M}_\mathrm{B}^{(0)} - \mathrm{g}_\mathrm{B}\,\bar{\sigma} + \frac{\mathrm{d}_\mathrm{B}}{2}\,(\mathrm{g}_\mathrm{B}\,\bar{\sigma})^2$

T.F. Motta, P.A.M. Guichon, A.W. Thomas
Nucl. Phys. A1009 (2021) 122157

- **Effective in-medium baryon masses including non-linear dependence on σ field, with scalar polarizability d_B representing e.g. effects of three-body forces**





Deep Neural Network
Y. Fujimoto,
K. Fukushima,
K. Murase
(2020)

Bayesian Inference
L. Brandes,
N. Kaiser, W.W.
(2023)

Quark-Meson Coupling model
(incl. **Hyperons**)
update:
J. Leong et al.
PRC108 (2023)
015804

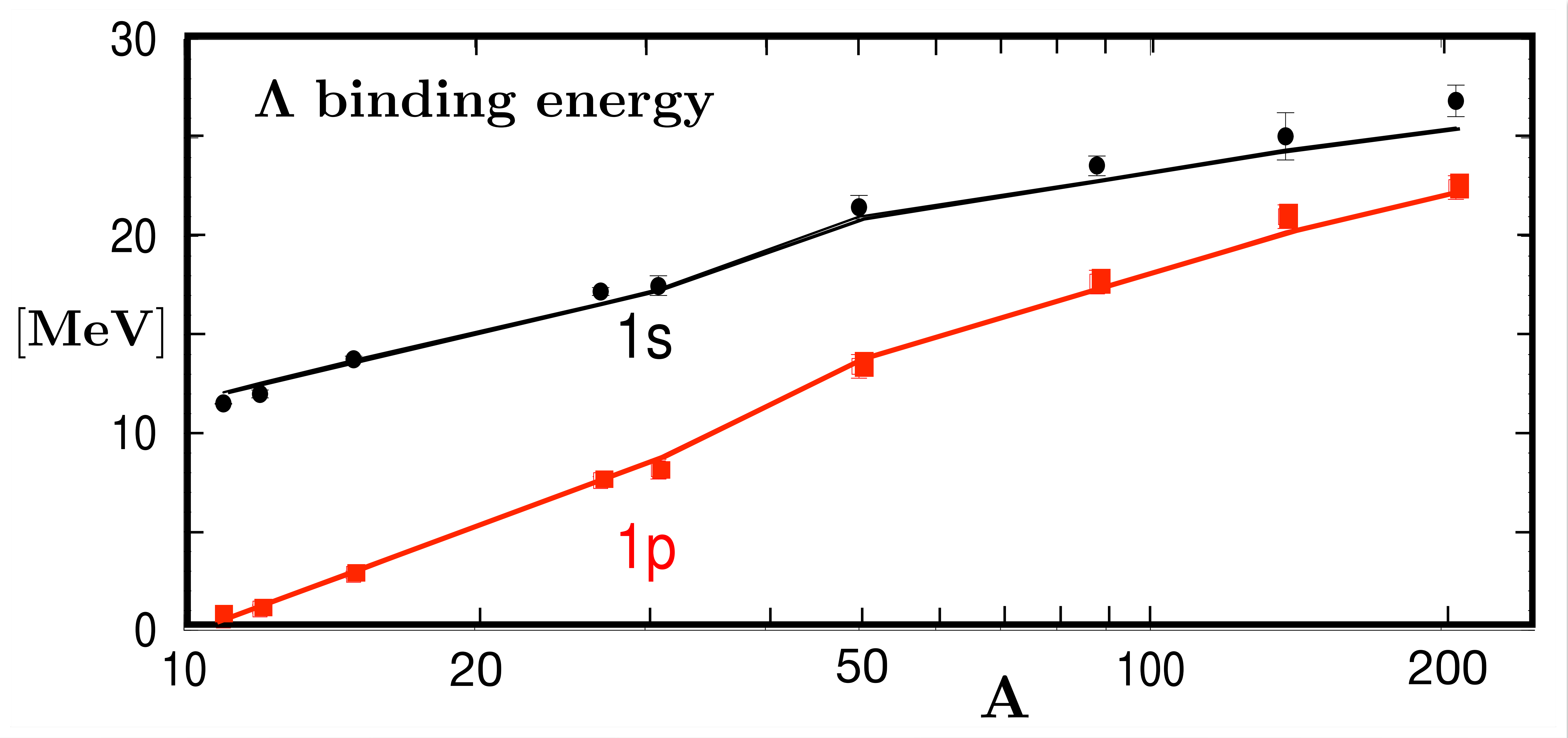
HYPERNUCLEAR PHENOMENOLOGY

- Recent update on Λ- **nuclear binding energies** including ΛNN **three-body forces**
- **2**- and **3-body hyperon-nucleus potentials** using **realistic** (empirically constrained) **density distributions**

$$U_\Lambda(\rho) = U_0^{(2)}(k_F)\,\frac{\rho(r)}{\rho_0} + U_0^{(3)}\left(\frac{\rho(r)}{\rho_0}\right)^2$$

$$(\rho_0 = 0.17\,\mathrm{fm}^{-3})$$

E. Friedman, A. Gal : Phys. Lett. B 837 (2023) 137669 ; Nucl. Phys. A 1039 (2023) 122725



- **Pauli correlations** important
- **Best fit** :

$$U_\Lambda(\rho_0) = -(27.3 \pm 0.6)\,\mathrm{MeV}$$

$$U_0^{(2)} = -(38.6 \pm 0.8)\,\mathrm{MeV}$$

$$U_0^{(3)} = (11.3 \pm 1.4)\,\mathrm{MeV}$$

- Comparison with **SU(3) Chiral EFT**- based 3-body term (NLO19): $U^{(3)}_{0,\mathrm{ChEFT}} = (10 \pm 2)\,\mathrm{MeV}$
  - **Three-body** ΛNN **forces** of such **repulsive magnitude** can solve the hyperon puzzle

CONCLUSIONS and OUTLOOK

- Key to **strangeness** in **neutron stars** :
  - **balance** between **hyperon-nuclear 2-** and **3-body forces**
  - **overbinding** in **hypernuclei** by two-body interactions compensated by **repulsive** hyperon-nuclear three-body forces
- **Equation-of-state** of **neutron star matter** :
  - even **stiffer** than previously expected $(\mathrm{M_{max}} \simeq 2.3\,\mathrm{M_\odot})$
  - increasingly repulsive hyperon-nuclear many-body forces render hyperons in neutron star cores **unlikely**
- Further insights and constraints expected :
  - expanded high-statistics **YN two-body data base**
  - improved **high-resolution hypernuclear spectroscopy**
  - growing quantity and quality of **astrophysical data** focus on EoS and **speed of sound** in **neutron stars**

Supplementary Materials

Density-dependent EFFECTIVE HYPERON - NUCLEON INTERACTION from CHIRAL THREE-BARYON FORCES

S. Petschauer, J. Haidenbauer, N. Kaiser, U.-G. Meißner, W.W.

Nucl. Phys. A957 (2017) 347

$$\mathbf{U}_{\mathbf{eff}}(12) = \sum_i \mathrm{tr}_{\sigma_3} \int_{|\vec{p}|\le p_F^i} \frac{d^3p}{(2\pi)^3} \mathbf{V}(123)$$

- Example: Λ**-neutron** density-dependent effective interaction in a nuclear medium (protons + neutrons)

$$\mathbf{U}^{\pi}_{\mathbf{eff}}(\Lambda n) = \frac{C\,H\,g_A^2}{2f_\pi^4\,\Delta}\,[\rho_n + 2\rho_p] + \mathcal{F}(p_F^p, p_F^n)$$

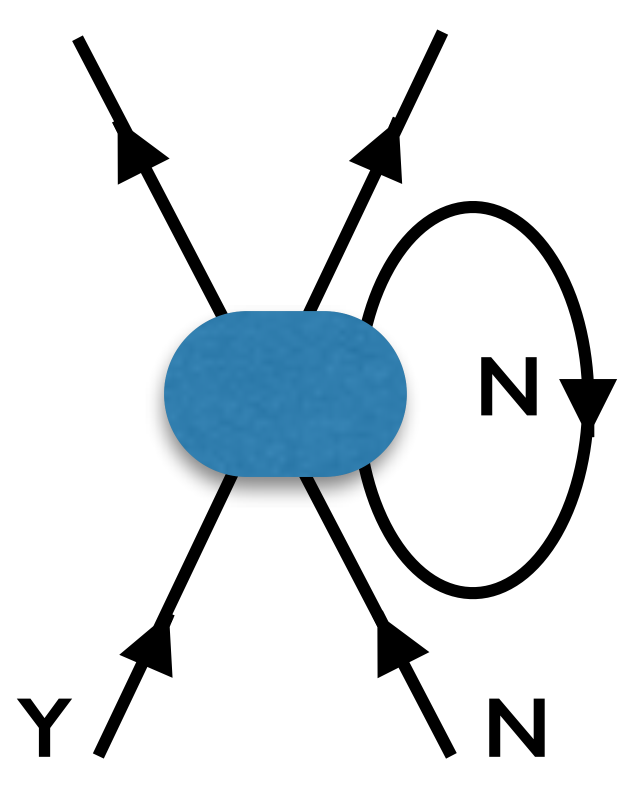
+/-

$$\mathbf{U}^{\pi\pi}_{\mathbf{eff}}(\Lambda n) = \frac{C^2\,g_A^2}{9f_\pi^4\,\Delta}\,[\rho_n + 2\rho_p] + \mathcal{G}(p_F^p, p_F^n)$$

repulsive

$$\mathbf{U}^{ct}_{\mathbf{eff}}(\Lambda n) = \frac{H^2}{18\,\Delta}\,[\rho_n + 2\rho_p]$$

repulsive



- **Decuplet-octet mass difference**

$$\Delta = \mathrm{M}_{[10]} - \mathrm{M}_{[8]} \simeq 270\,\mathrm{MeV}$$

$$C = \frac{3}{4} g_A \simeq 1$$

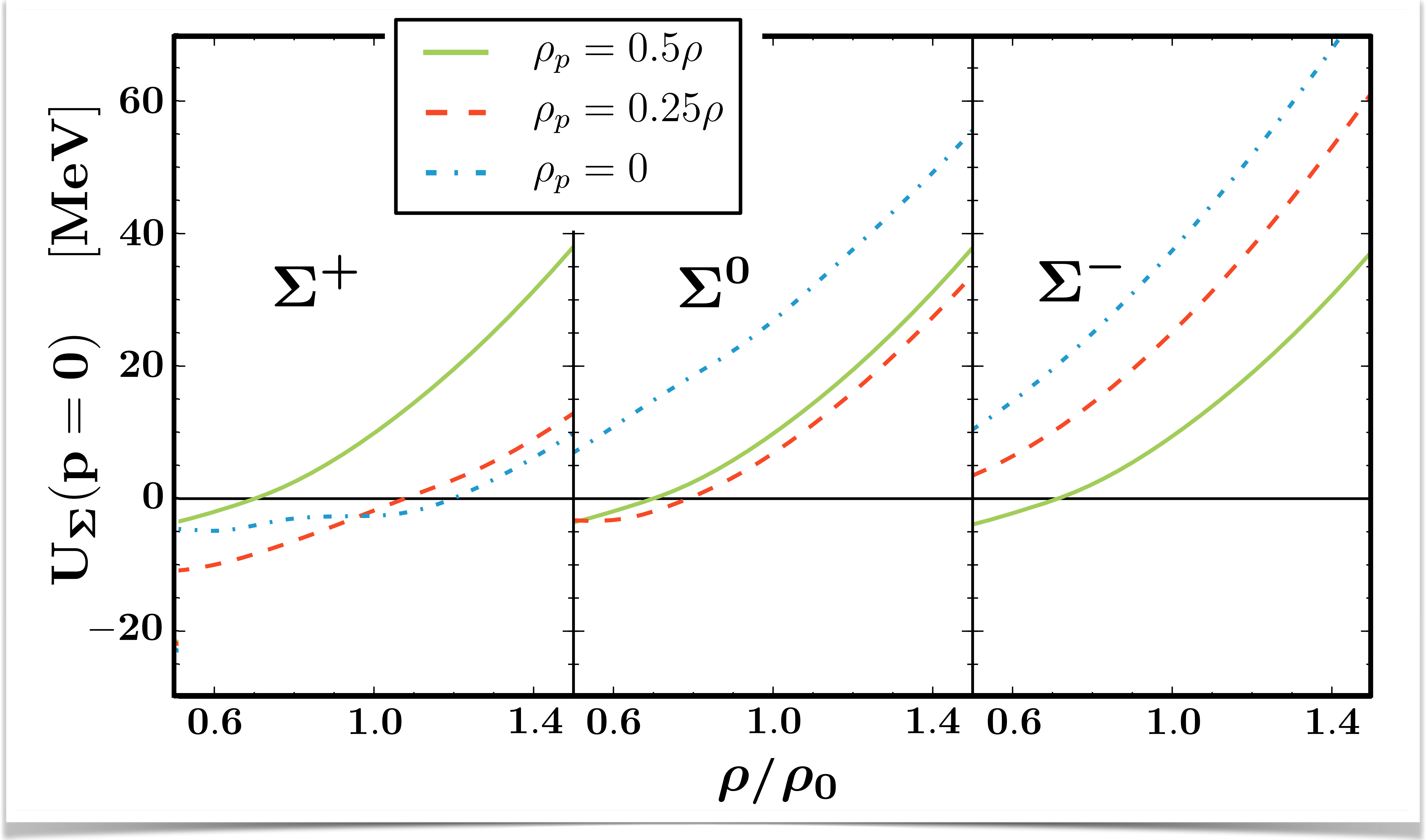
- **Coupling parameters :**

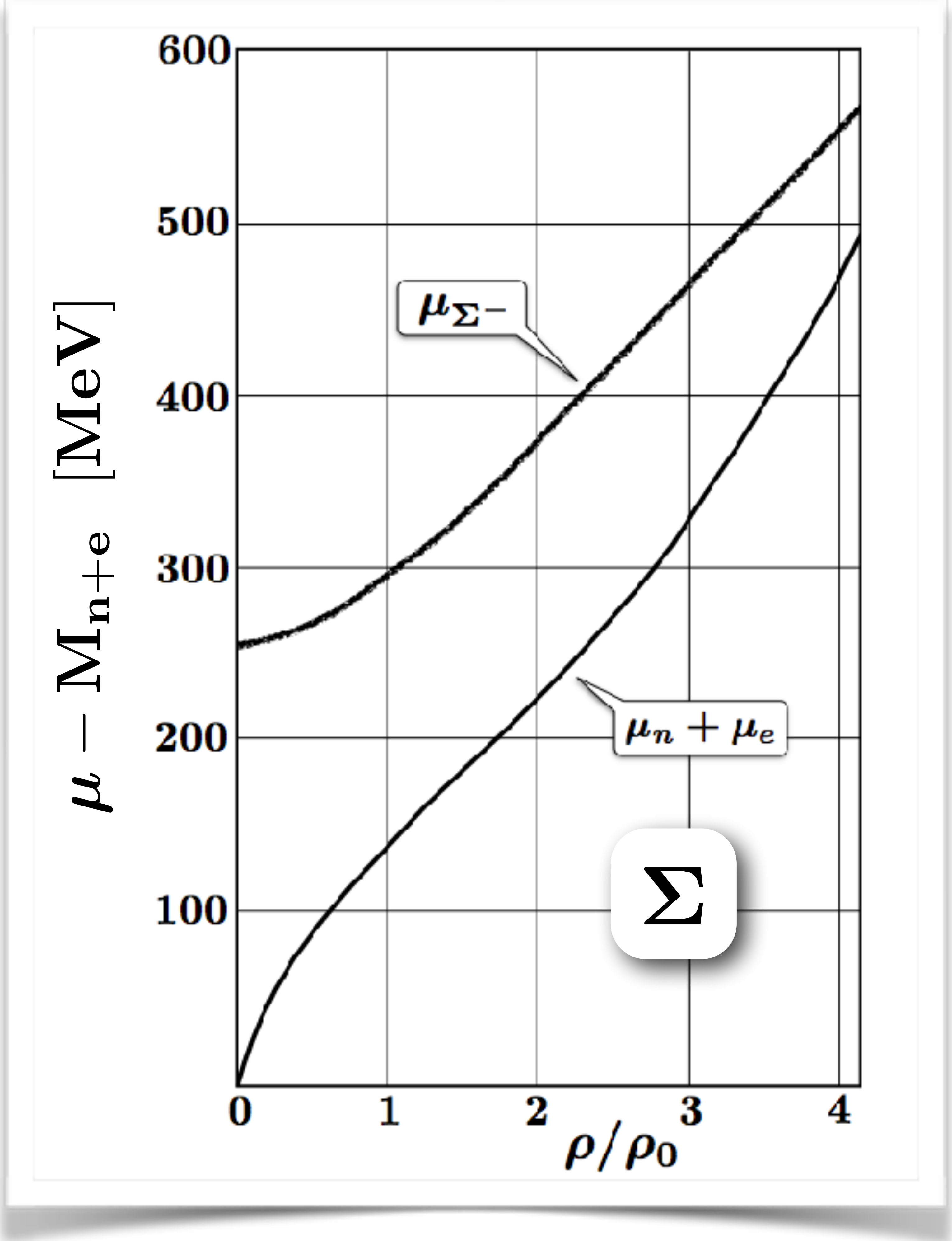
$$H = H_1 + 3H_2 \quad |H| \lesssim f_\pi^{-2}$$

Σ Hyperons in Neutron Stars ?

- Σ **hyperon potentials** from **Chiral SU(3) EFT** interactions

S. Petschauer, J.Haidenbauer, N. Kaiser, U.-G. Meißner, W.W. : EPJ A52 (2016) 15





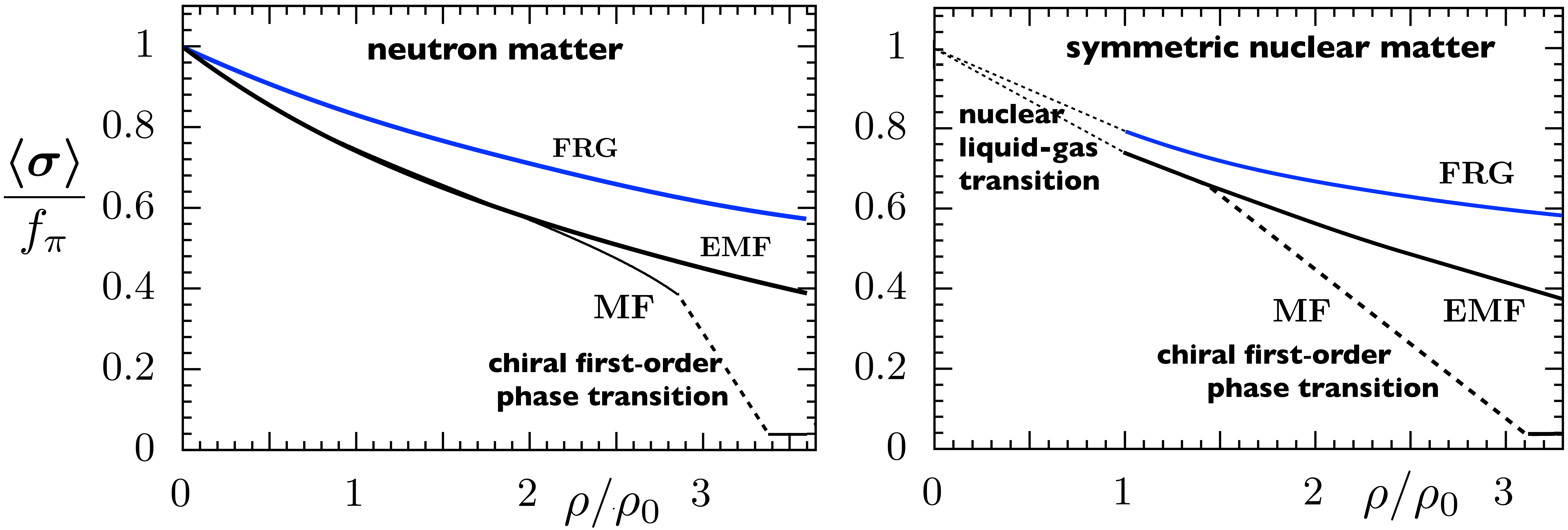
- Σ - nuclear potentials are **repulsive**
- Condition for appearance of Σ^- in neutron star matter :

$$\mu_{\Sigma^-} = \mu_n + \mu_e = 2\mu_n - \mu_p \qquad \mu_i = \frac{\partial \mathcal{E}}{\partial \rho_i}$$

CHIRAL ORDER PARAMETER in **NUCLEAR** and **NEUTRON MATTER** (T = 0)

- Chiral phase transition in dense baryonic matter **?** Studies in chiral meson-nucleon field theory

L. Brandes, N.. Kaiser, W.W. : Eur. Phys. J. A57 (2021) 243



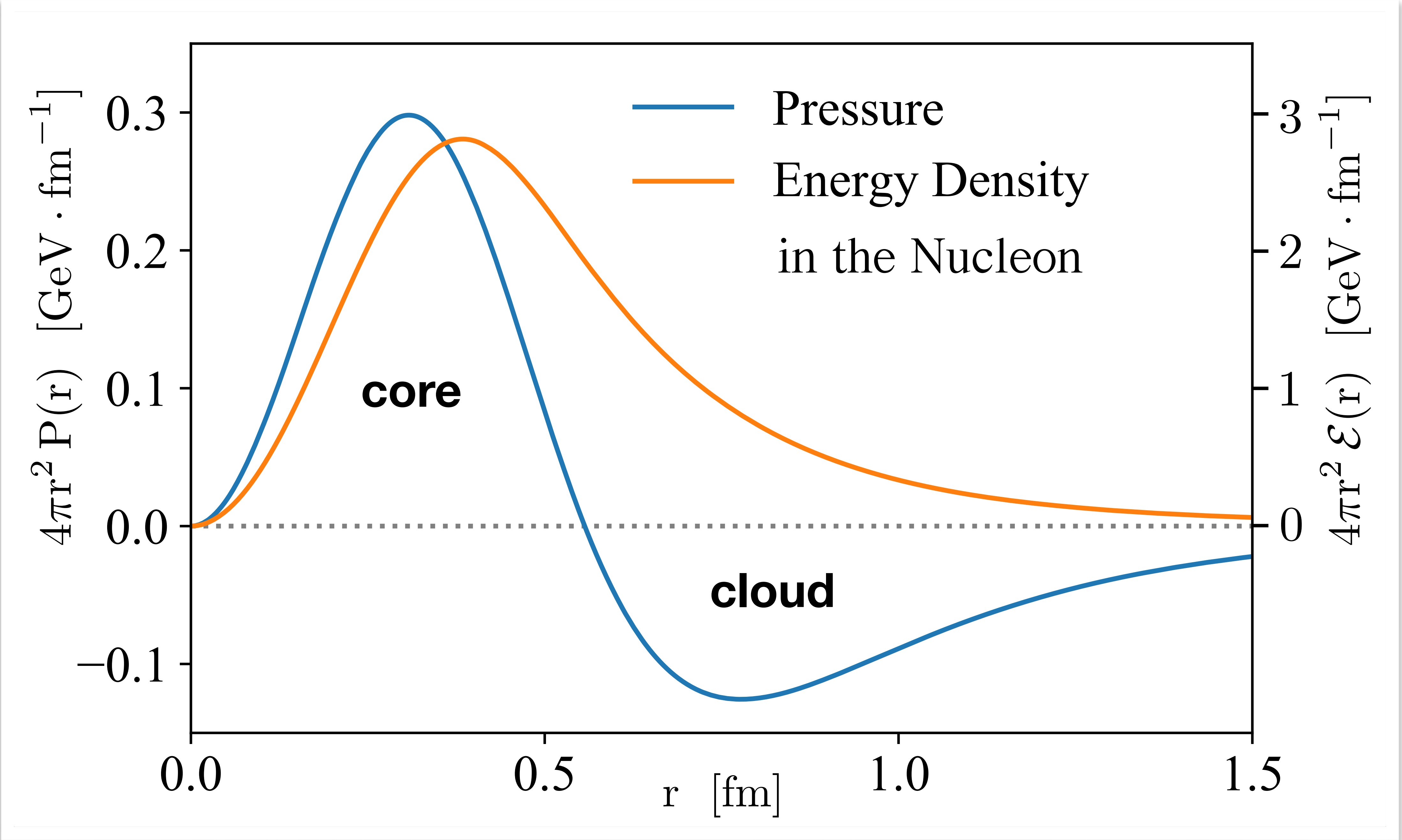
chiral
crossover
transition
at
baryon
densities
$\rho > 6\,\rho_0$

- **Mean-field** (MF) approximation (wrongly) features chiral 1st order transition at $\rho \sim 2-3\,\rho_0$
- **Vacuum fluctuations** (EMF) shift chiral transition to high density ➡ **smooth crossover**
- **FRG** (with non-perturbative **loop corrections** involving **pions** , **nucleons**) enhances this effect

Further **KEYWORDS** on **COLD MATTER** at **EXTREME DENSITIES**
Hadron - Quark Continuity

K. Fukushima, T. Kojo, W.W. : Phys. Rev. D102 (2020) 096017

- Nucleonic scales : **HARD-CORE** deconfinement + **SOFT SURFACE** delocalisation

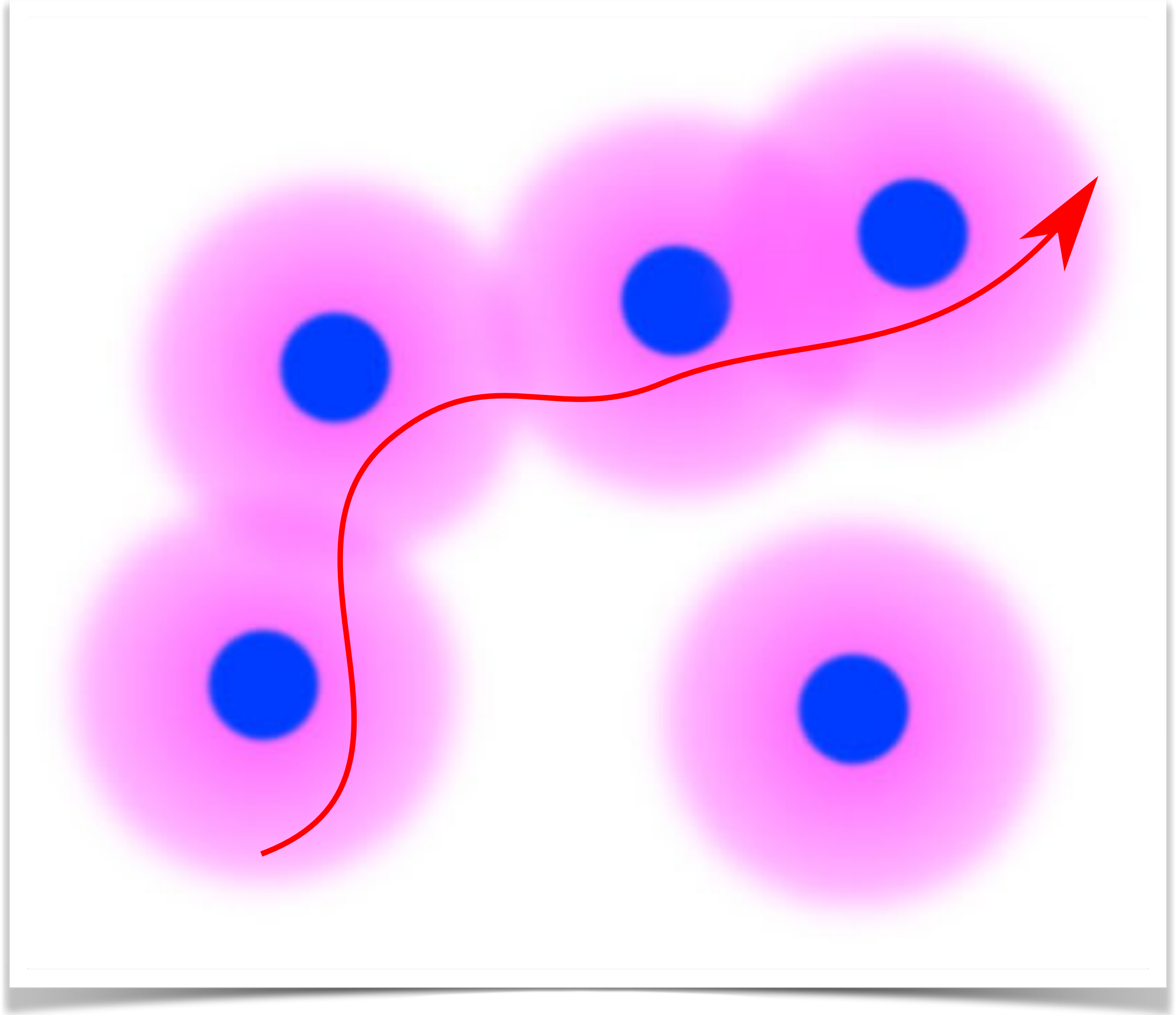


- Nucleon cores touch at baryon densities

$$\rho_B \sim 6\,\rho_0$$

(well above neutron star core densities)

- **Percolation** of mesonic clouds at lower densities induce many-body correlations



F. Karsch, H. Satz
Phys. Rev.
D21 (1980) 1168

- Soft delocalisation and collective mobility of quark-antiquark pairs over larger distances
- No (first-order) phase transition expected at densities relevant to neutron stars