Heavy quarks and quarkonia (Theory)

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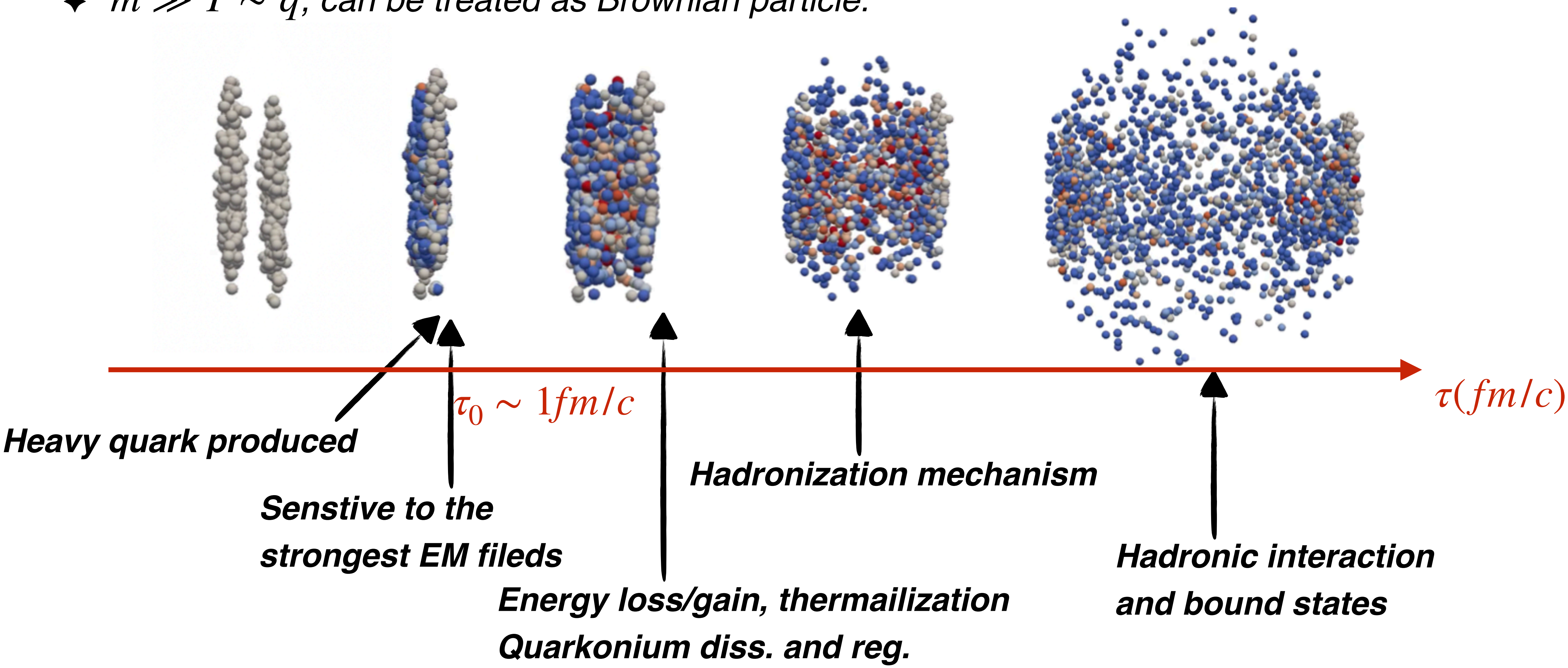
29/05/2023

Heavy quark in Heavy ion collisions

$m_c \sim 1.5GeV, m_b \sim 4.7GeV$

- $\tau_c \sim 1/m_c, \tau_b \sim 1/m_b < \tau_0 \sim 1fm/c$, *"see" full system evolution.*
- $\tau_c, \tau_b < \tau_B \approx R/\gamma \sim 0.1fm/c$, *feel strong electromagnetic fields.*
- $m_c, m_b \gg \Lambda_{QCD}$, *produced by hard scattering, pQCD.*
- $m_c, m_b \gg T$, *Number is conserved during the evolution (thermal production can be neglected).*
- $m \gg T \sim q$, *can be treated as Brownian particle.*



Heavy flavor is a nice probe to each stage of HIC and very useful to study the hot QCD!

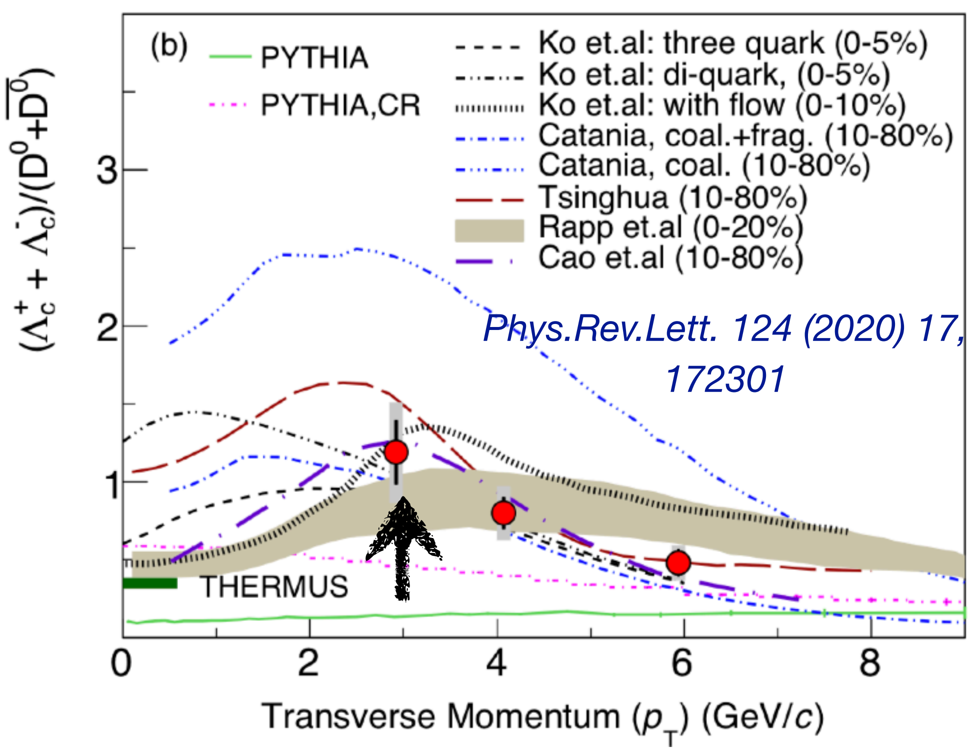
I. *Open Heavy Flavor*

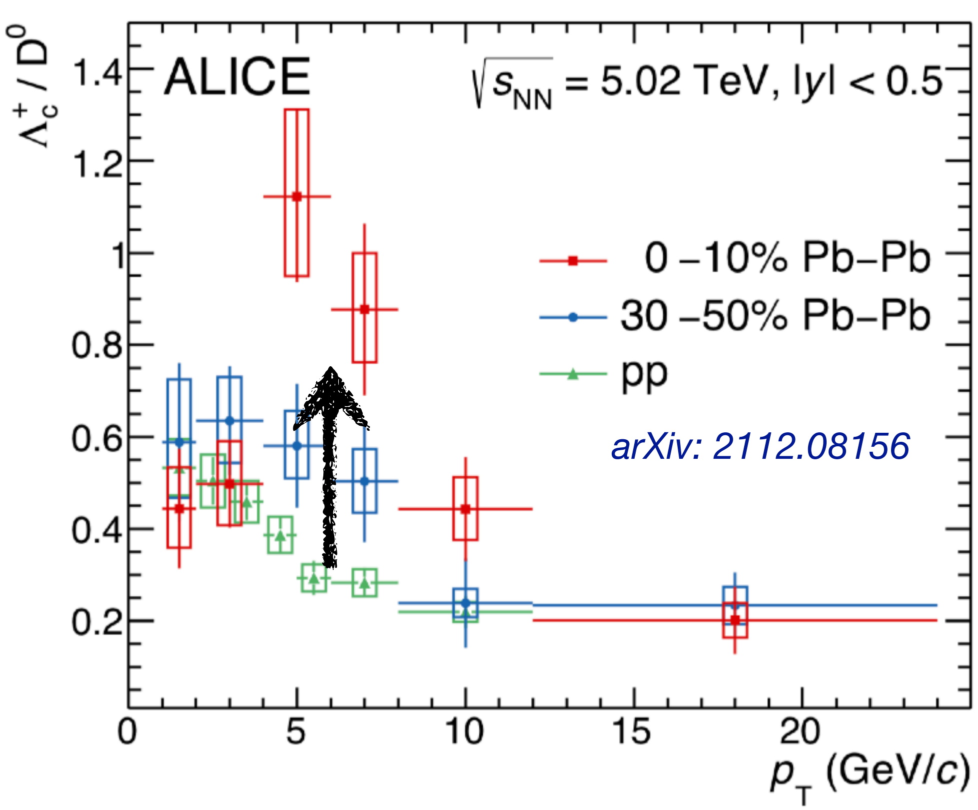
- ✤ *Energy loss (backup slides)*
- ✤ *Hadronization*
- ✤ *EPOS4+HQ framework*

Hadronization mechanism in the hot medium

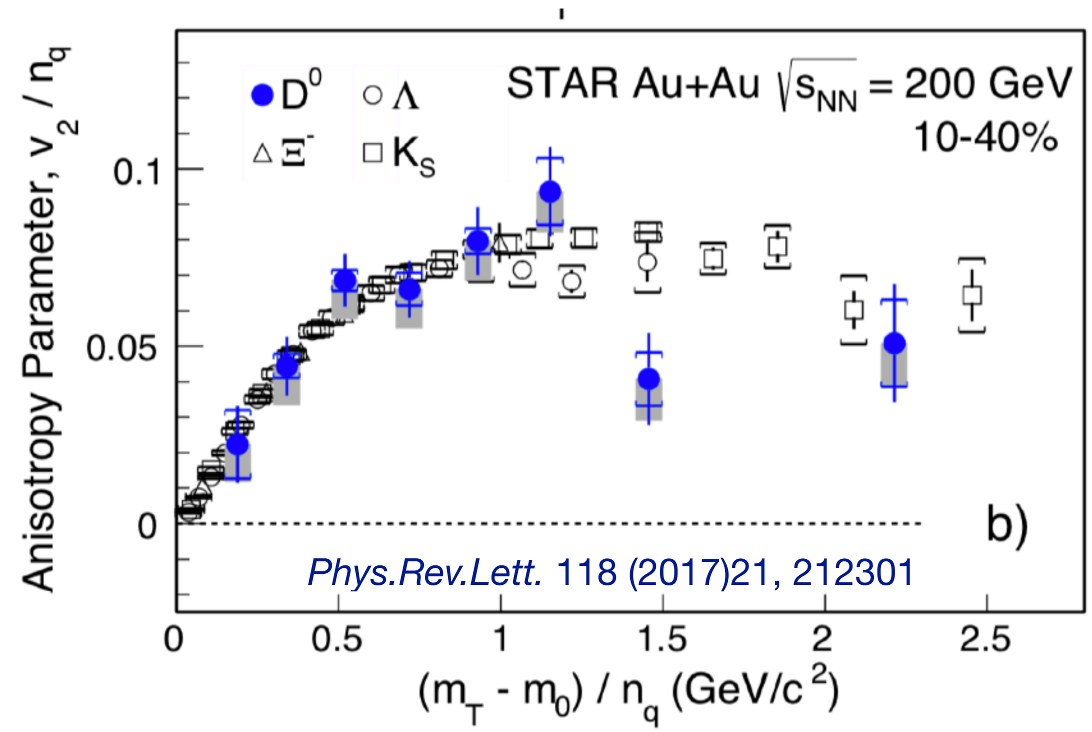
Hadronization in the hot medium shows a huge difference compared to the vacuum case (Fragmentation).

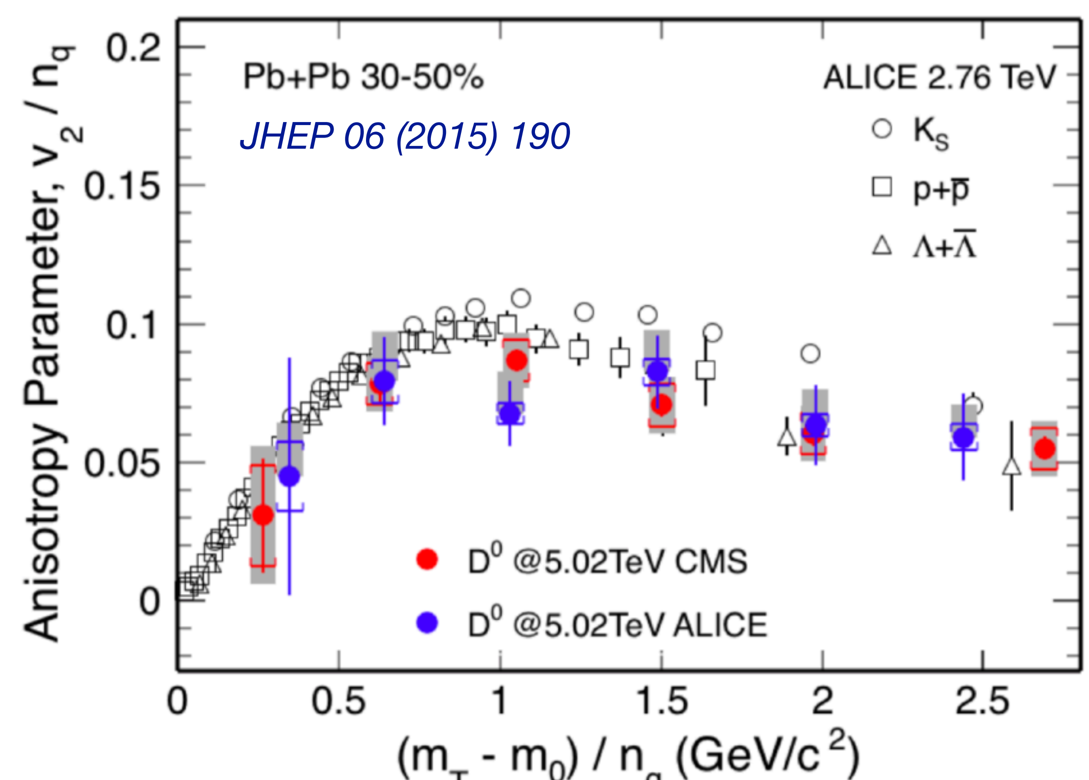
- *Enhancement of Baryon / Meson Ratio*



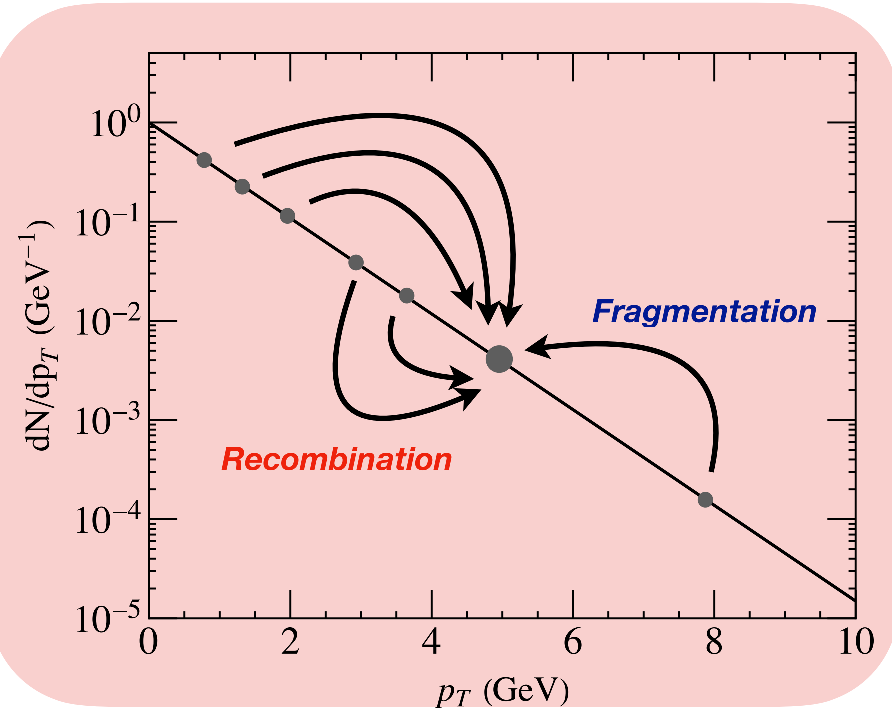


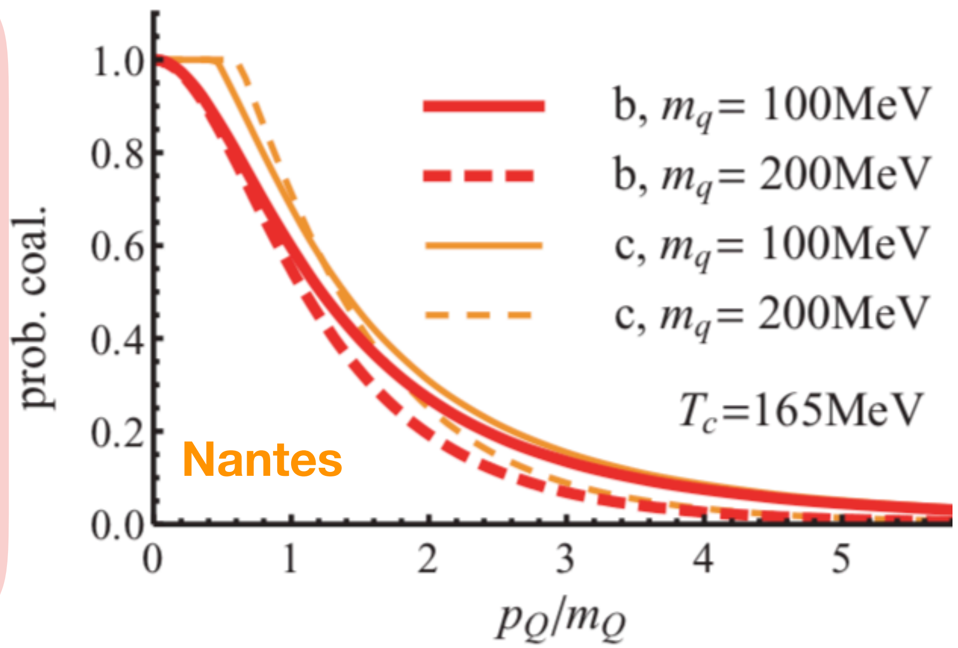
- *Quark Number Scaling of Elliptic flow*

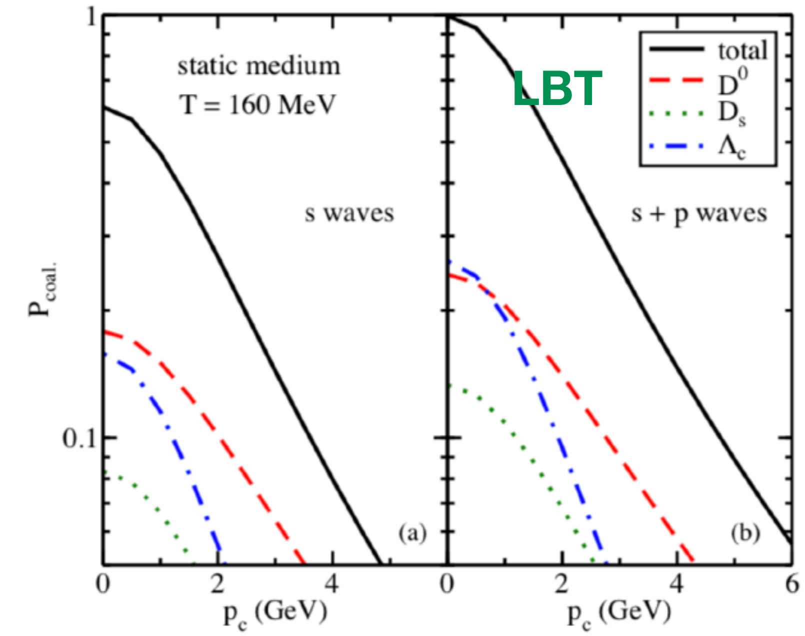


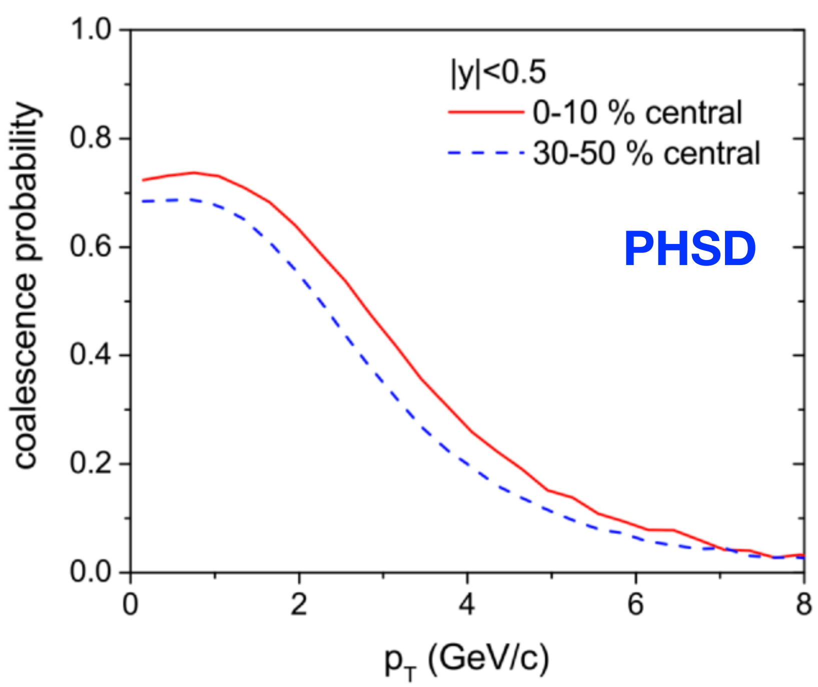


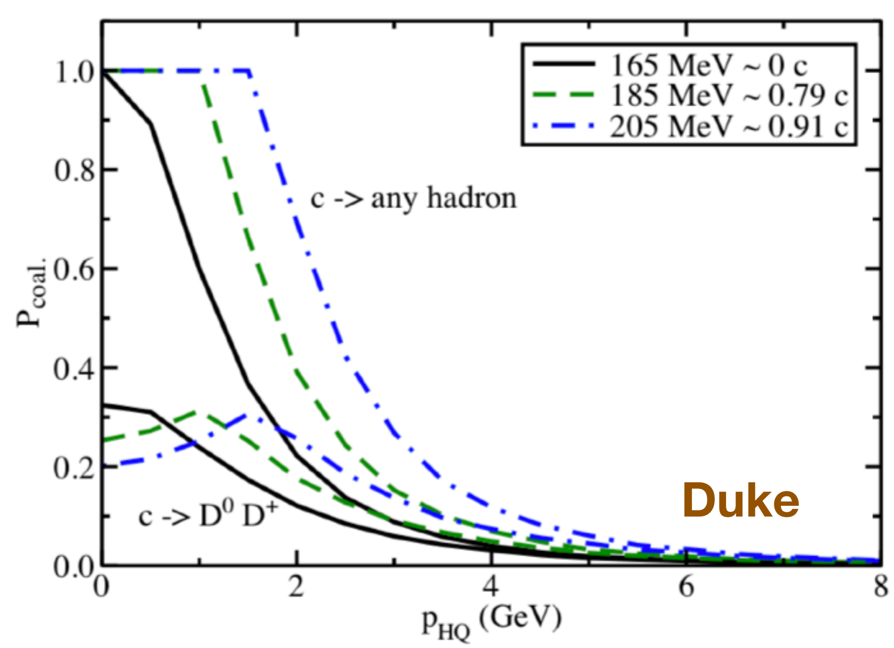
Hadronization mechanism in the hot medium

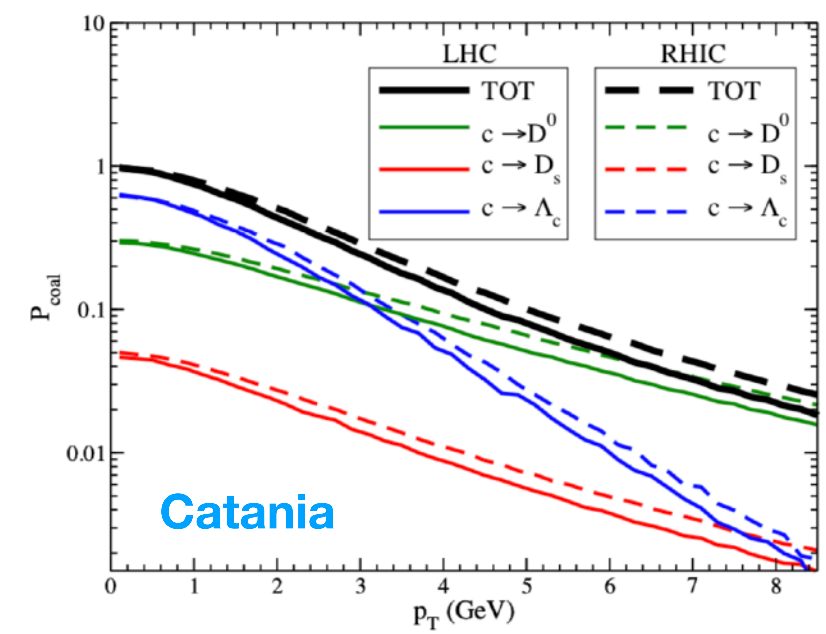












Low pT heavy quark hadronizes via recombination, while high pT through the fragmentation!

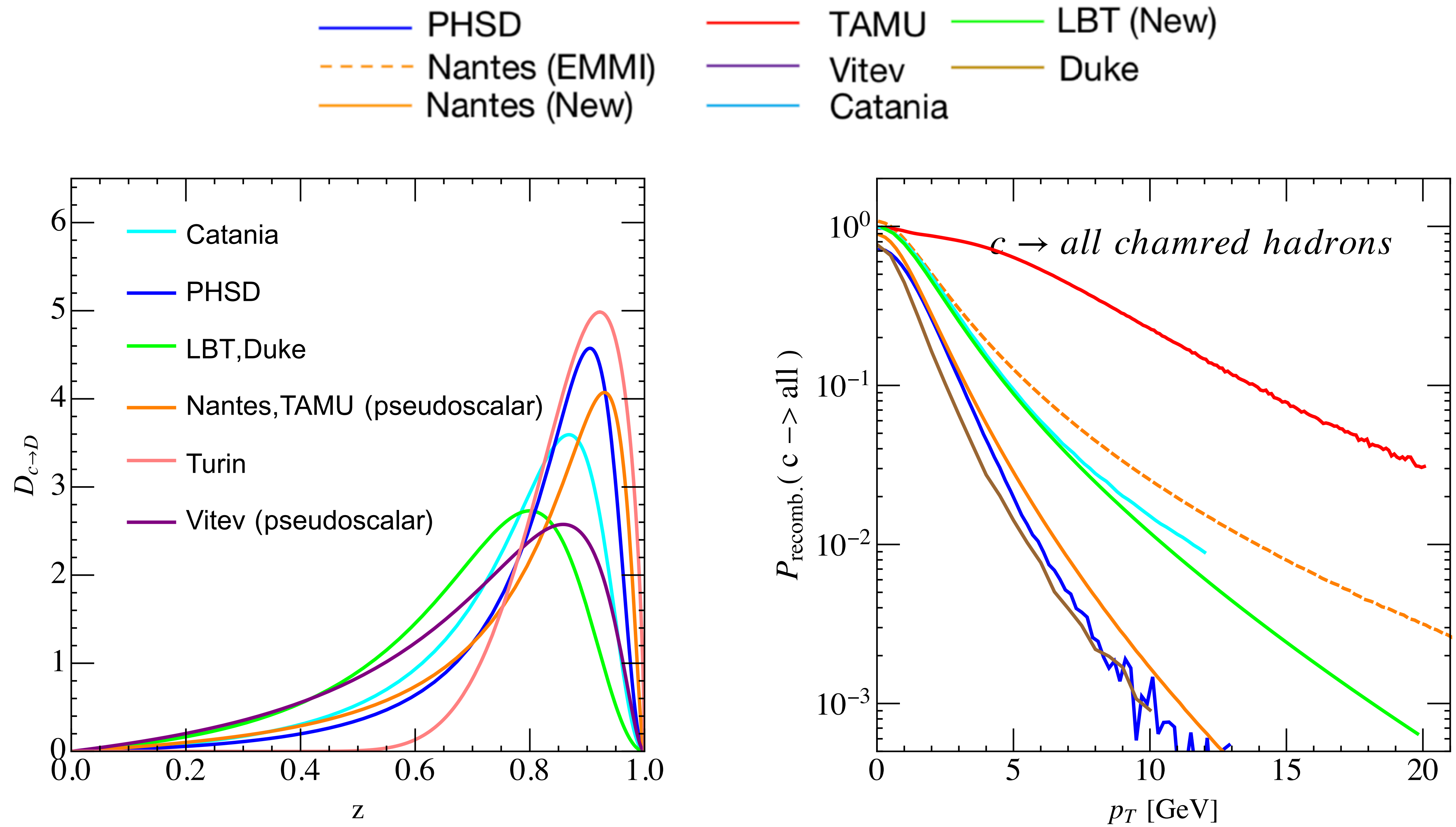
$$P_{frag.}(p_T) = 1 - P_{recomb.}(p_T)$$

Hadronization models

	Frag.	Recom.	Recom. Form	Charmed hadrons involved
Catania	Peterson	Phase space Wigner function	$W(x,p) = \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_i^2}{\sigma_{ri}^2} - p_i^2\sigma_{ri}^2\right)$	S-wave, D0,Ds, D*+,D*0,D*s,several excited states of \Lambda_c,\Sigma_c
Duke	Pythia 6.4/ Peterson	Momentum space Wigner function	$W(p) = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 p^2},$	S-wave,D,D*
LBT	Pythia 6.4/ Peterson	Momentum space Wigner function	$W_s(p) = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 p^2},$ $W_p(p) = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} \frac{2}{3}\sigma^2 p^2 e^{-\sigma^2 p^2}.$	S-wave,P-wave,D,Ds,D*, \Lambda_c,\Sigma_c,\Xi_c. \Omega_c
Nantes	HQET	Phase space Wigner function	$W(x_Q, x_q, p_Q, p_q) = \exp\left(\frac{(x_q - x_Q)^2 - [(x_q - x_Q)\cdot u_Q]^2}{2R_c^2} - \alpha_d^2(u_Q \cdot u_q - 1)\right)$	S-wave, D0
PHSD	Peterson	Phase space Wigner function	$W_s(r,p) = \frac{8(2S+1)}{36} e^{-\frac{r^2}{\sigma^2} - \sigma^2 p^2},$ $W_p(r,p) = \frac{2S+1}{36}\left(\frac{16}{3}\frac{r^2}{\sigma^2} + \frac{16}{3}\sigma^2 p^2 - 8\right) e^{-\frac{r^2}{\sigma^2} - \sigma^2 p^2},$	S-wave, P-wave D+,D0,Ds, D*+,D*0,D*s
TAMU	thermal density correlated	Resonance amplitude	$\frac{\gamma_M}{\Gamma} v_{rel} g_\sigma \frac{4\pi}{k^2} \frac{(\Gamma m)^2}{(s - m^2)^2 + (\Gamma m)^2}$	D+,D0,Ds and few excited states. Charm baryons+missing baryons
Turin	Pythia 6.4/ String fragmentation	Invariant mass criterion	$M_D < M_{Cluster} < M_{max}.$	(prompt) D+,D0,Ds,\Lambda_c, \Xi_c,\Omega_c
Los Alamos	HQET	–	–	S-wave, D+,D0,Ds, charm-baryons

··· *Each model with a recombination part can give a nice explanation of the experimental data!*

Hadronization Model comparison



Phase-space vs. momentum space criterion; energy conservation; space-momentum correlation;…

Aiming to compare the hadronization model itself and put the understanding forward.

We prepared several tasks for different groups with the same hadronization hypersurface and charm distribution functions at hadronization hypersurface. (2022.04-now)

More results, analysis, and draft are coming soon!

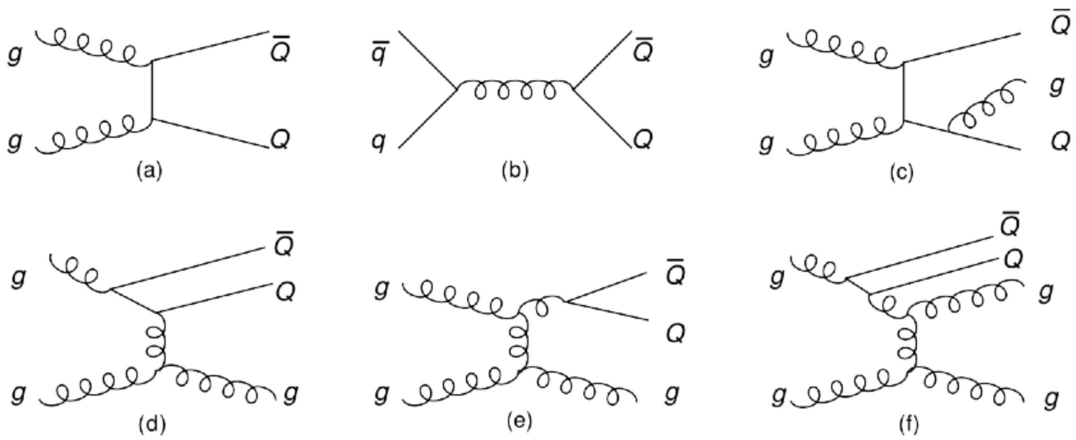
J. Zhao, P.B. Gossiaux, J. Aichelin,…

See detail: J. Zhao, HP2023, Tue. 16:00

EPOS4+HQ

EPOS4: multiple (nucleonic or partonic) scatterings are treated parallelly based on S-matrix theory

https://klaus.pages.in2p3.fr/epos4 ***K. Werner. arXiv: 2301.12517***



- *Heavy quarks are produced perturbatively.*

- *Heavy quark loss energy in hot medium via both collisional and raditive energy loss.*

P.B. Gossiaux, R. Bierkandt and J. Aichelin. PRC79 (2009) 044906

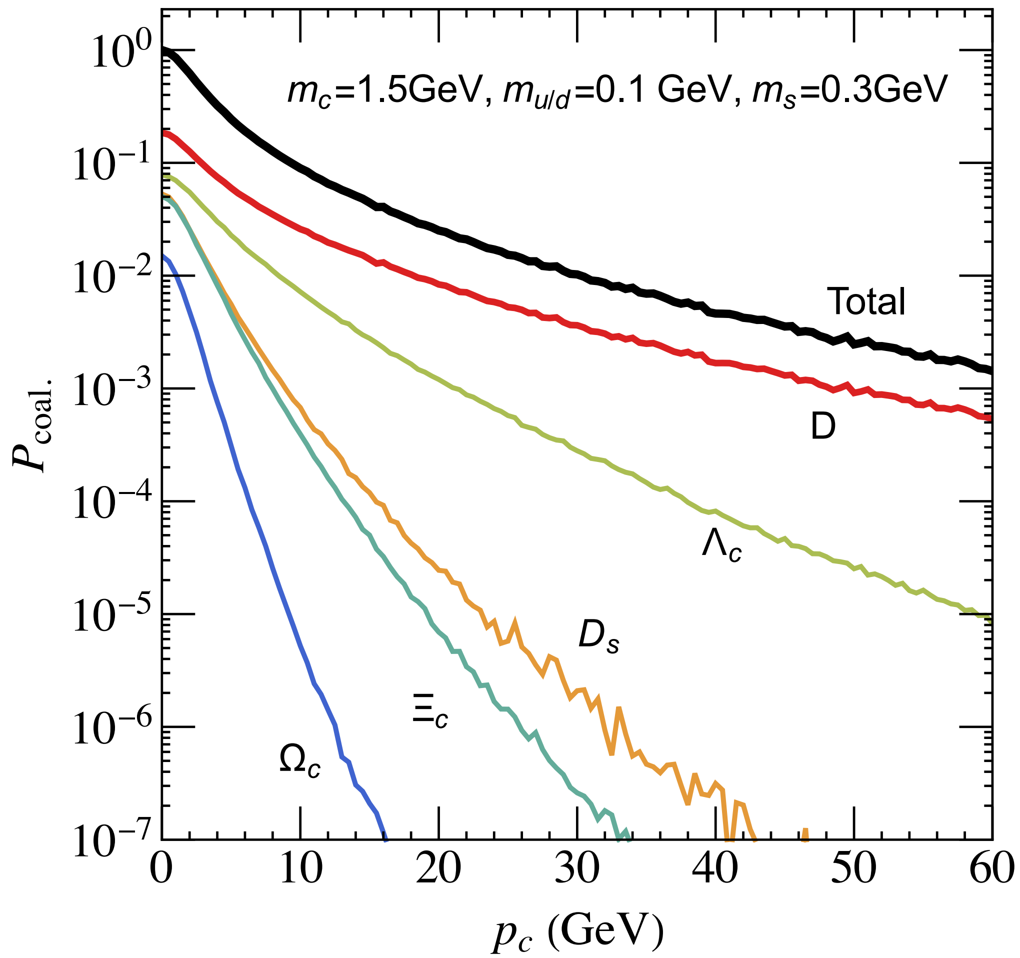
- *Hadronzation: Coalescence + Fragmentation.*

For ground states: $D^0, D^+, D_s^+, \Lambda_c, \Xi_c, \Omega_c, \ldots$

$$W(r,p) = 8e^{-\frac{r^2}{\sigma^2}-p^2\sigma^2}.$$

$$W(\rho,\lambda,p_\rho,p_\lambda) = 8^2 e^{-\frac{\rho^2}{\sigma_\rho^2}-p_\rho^2\sigma_\rho^2} e^{-\frac{\lambda^2}{\sigma_\lambda^2}-p_\lambda^2\sigma_\lambda^2}.$$

σ are given by their root mean radius!



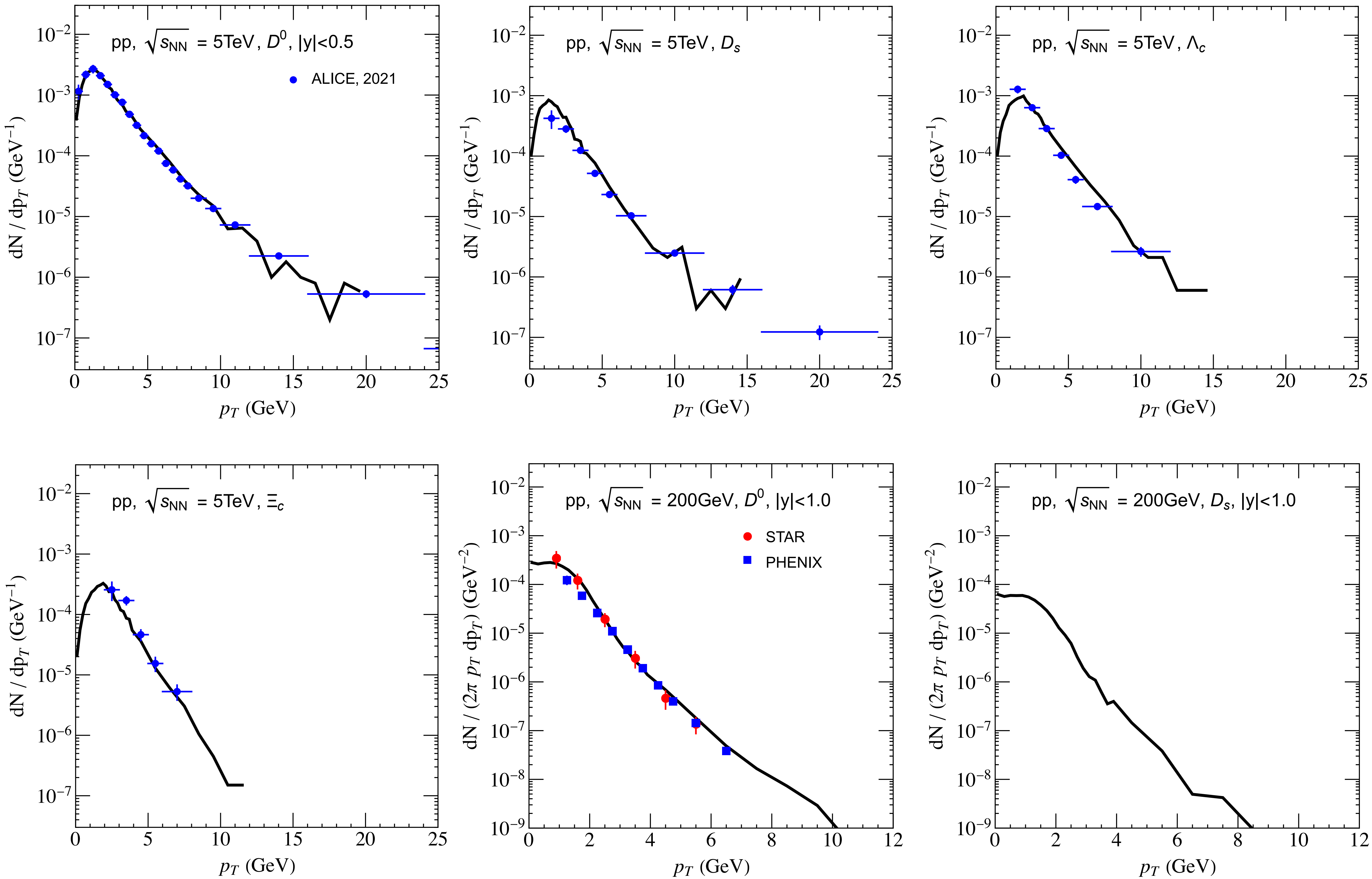
The contributions from all excited states (no matter they are P-or D-waves) are encoded in an overall momentum independent factor $\mathscr{F}$,which is given by the statistical model.

$$n_i = \frac{d_i}{2\pi^2} m_i^2 T_H K_2\left(\frac{m_i}{T_H}\right) ,$$

J. Zhao, J. Aichelin, K. Werner, P.B. Gossiaux, In preparation…

EPOS4+HQ

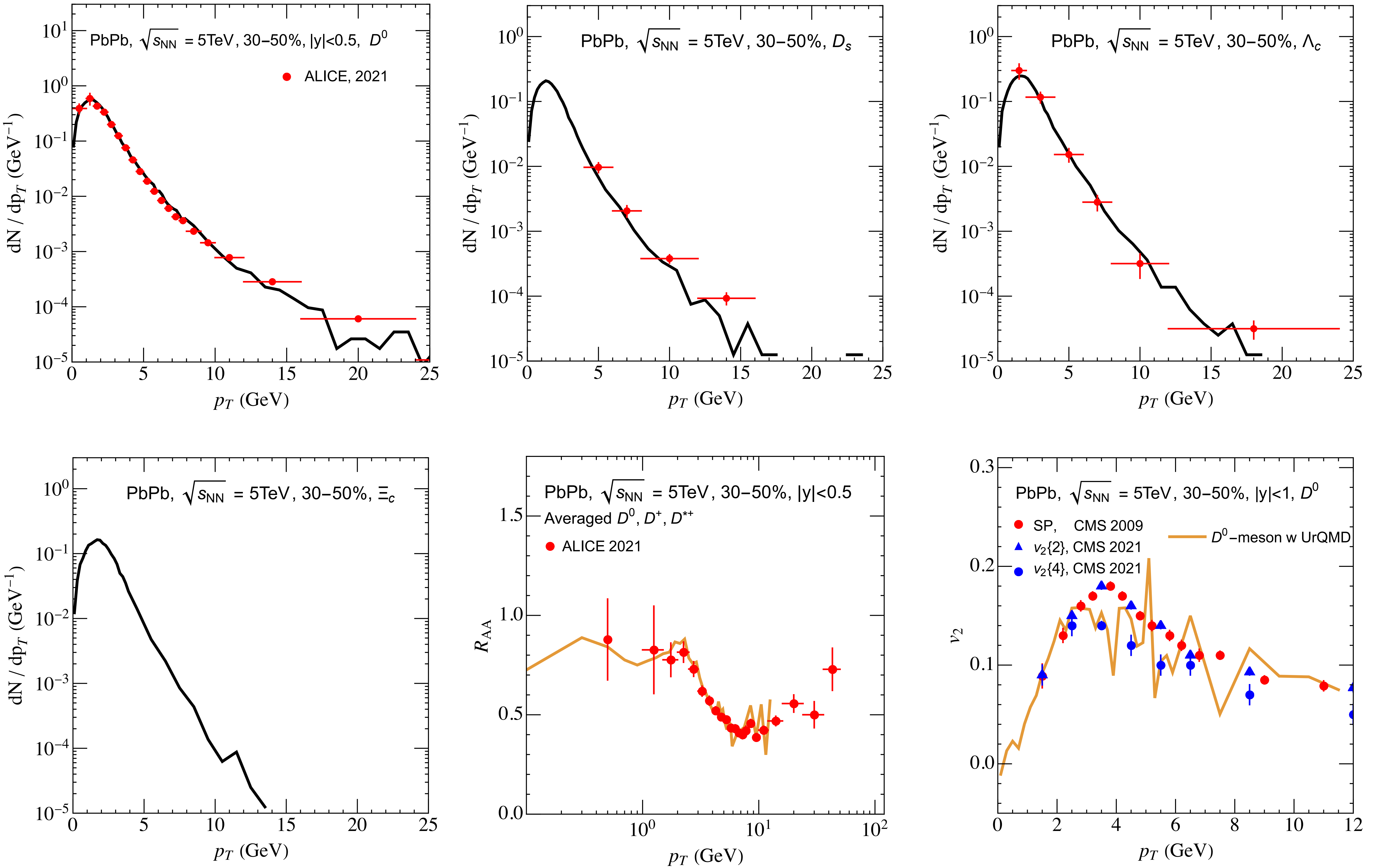
J. Zhao, J. Aichelin, K. Werner, P.B. Gossiaux, In preparation…



Charm hadrons in pp collisions at RHIC and LHC.

EPOS4+HQ

J. Zhao, J. Aichelin, K. Werner, P.B. Gossiaux, In preparation…



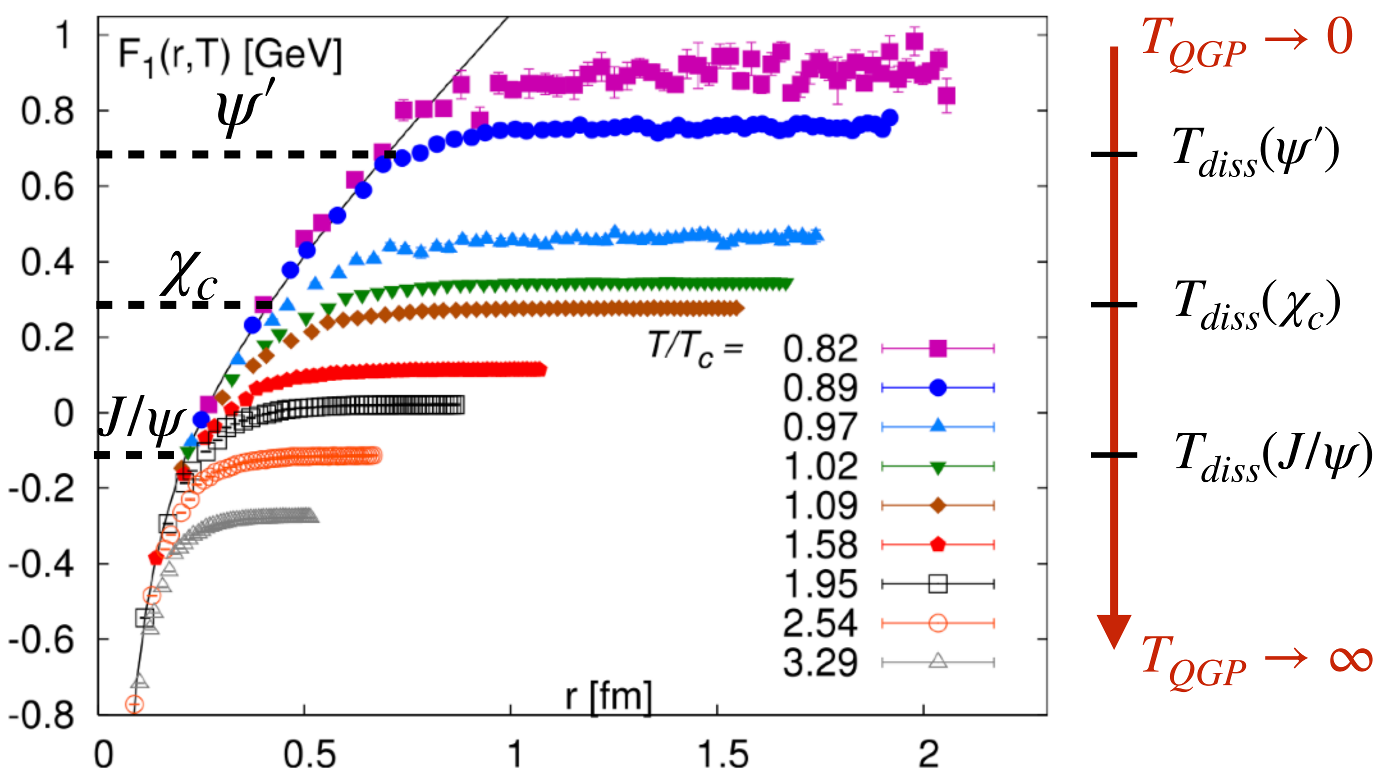
Charm hadrons in AA collisions at LHC.

II. *Quarkonium*

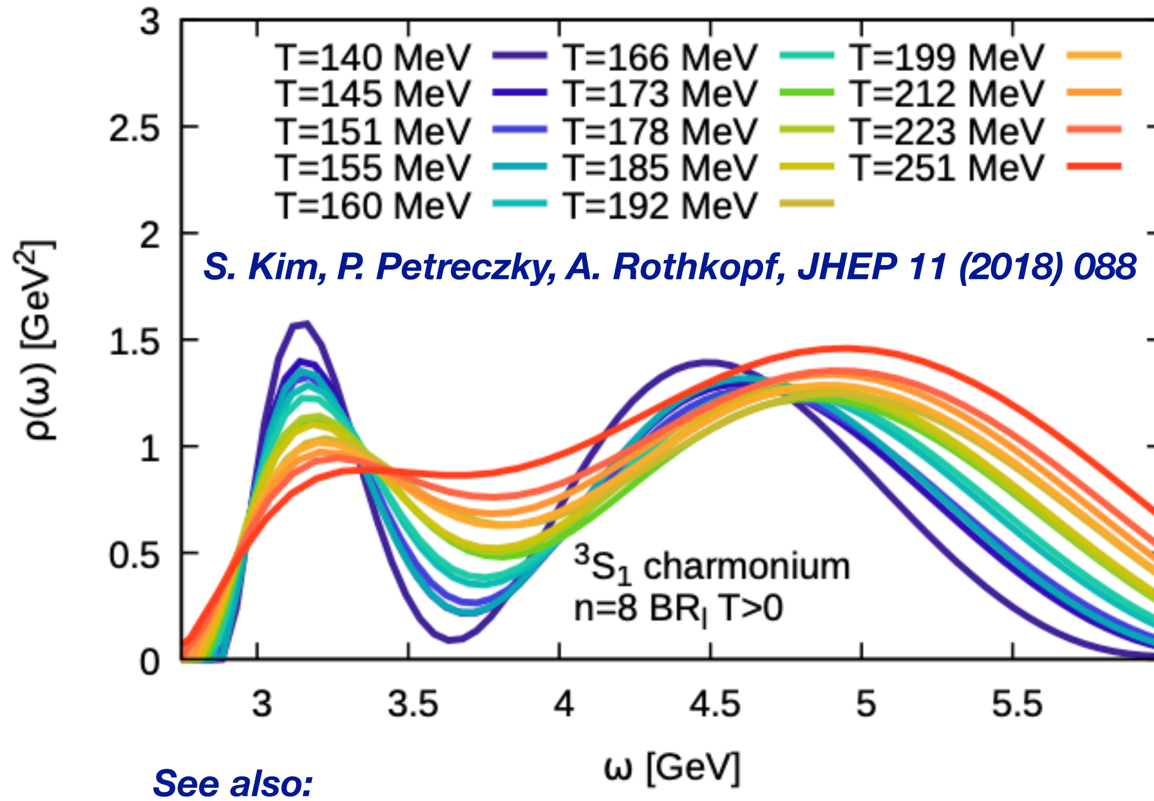
- *In-medium properties of quarkonium and complex potential*
- *Real-time evolution in hot QCD medium (backup slides)*

Quarkonium in the hot medium

Quarkonia suppression has been considered as a smoking gun of the QGP (Matsui, Satz at 1986, …)



P. Petreczky, J. Phys. G 37 (2010) 094009.



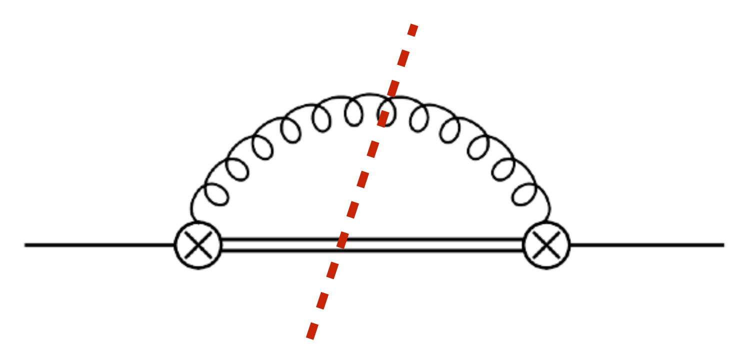
See also:
A.Ikeda, M. Asakawa, M. Kitazawa, PRD95, 014504 (2017)
M. Asakawa, T. Hatsuda and Y. Nakahara, PPNP. 46 (2001) 459. …

Besides the static color screening of heavy potential, quarkonia will obtaine the thermal width!

In the NRQCD/pNRQCD point of view:

Singlet-octet thermal break up —> gluon-dissociation

Landau damping—> inelastic scattering (quasifree limit)



N. Brambilla, M. Escobedo, J. Ghiglieri, M. Laine, O. Philipsen, P. Romatschke, M. Tassler, P. Petreczky, et al,
JHEP 03, 054 (2007). PRD 78, 014017 (2008). JHEP 09, 038 (2010). JHEP 1112 (2011) 116…

Heavy Quark Potential at finite temperature

- ***In the weak-coupling regime (High temperature—> HTL,…)*** $T,\, m_D \sim gT,\, g^2T,\, \Lambda_{QCD}$

Static Wilson loop in the imaginary-time: M. Laine, O. Philipsen, P. Romatschke, M. Tassler, JHEP 03 (2007) 054

$$V(r,T) \;=\; -\frac{g^2 C_F}{4\pi}\left[m_D + \frac{\exp(-m_D r)}{r}\right] - \frac{ig^2 T C_F}{4\pi}\phi(m_D r) \qquad \phi(x) = 2\int_0^\infty \frac{dz\, z}{(z^2+1)^2}\left[1 - \frac{\sin(zx)}{zx}\right]$$

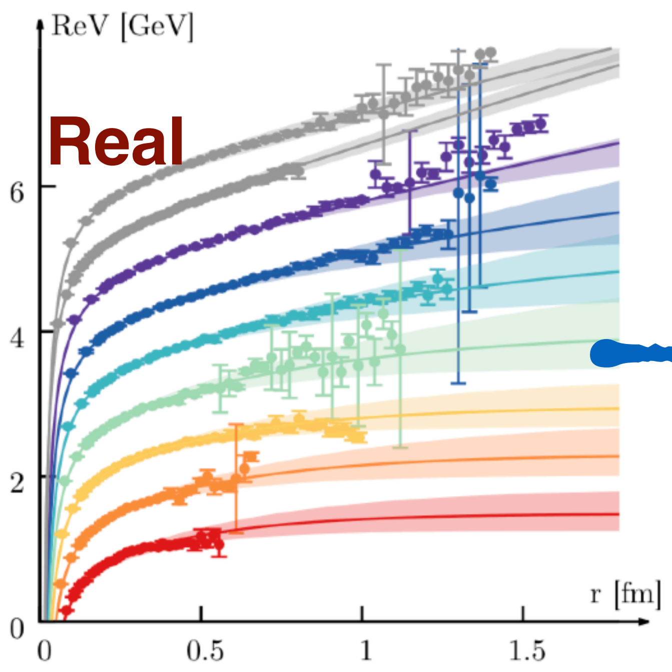
Real part shows strong Debye screening, identical to the singlet free energy.
Landau damping contribute to the imaginary part.

- ***In the strong-coupling regime (Lattice QCD, T-Matrix approach…)***

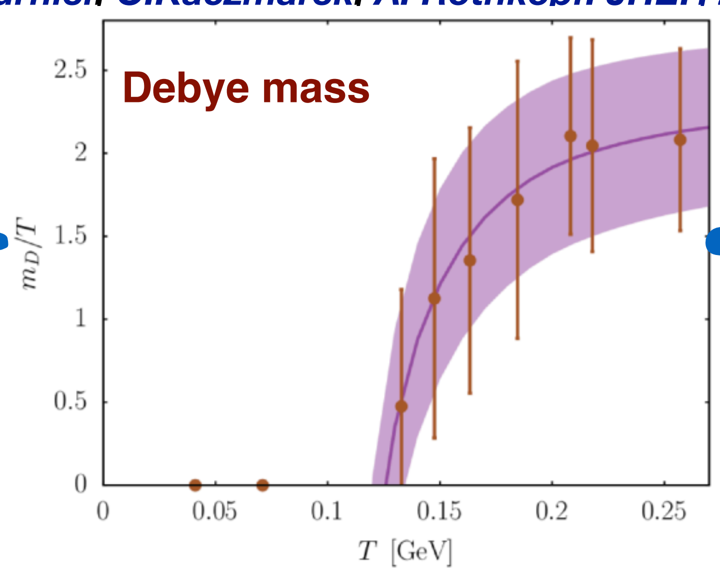
$$V(r) = \lim_{t\to\infty} \frac{i\partial_t W(t,r)}{W(t,r)}.$$

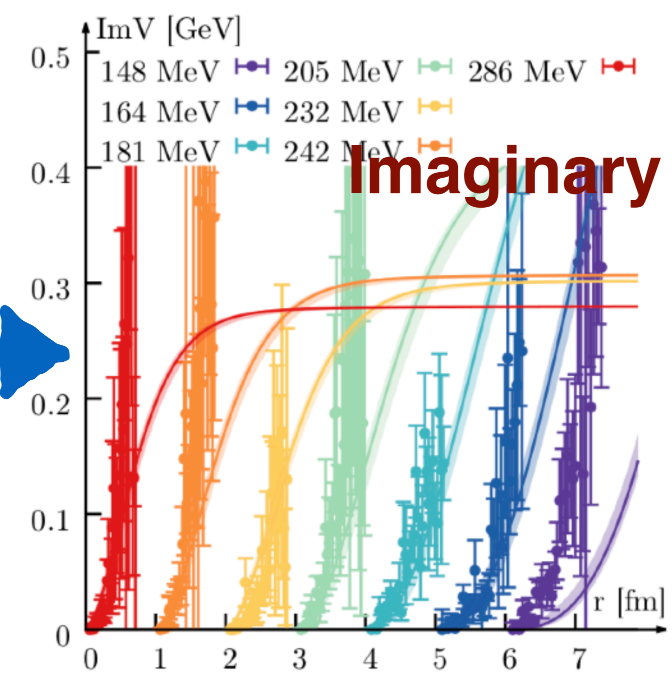
$$W(\tau) = \int d\omega e^{-\omega\tau}\rho(\omega) \leftrightarrow \int d\omega e^{-i\omega t}\rho(\omega) = W(t)$$

$$V(r) = \lim_{t\to\infty}\int d\omega\, \omega e^{-i\omega t}\rho(\omega,r) \Big/ \int d\omega\, e^{-i\omega t}\rho(\omega,r)$$



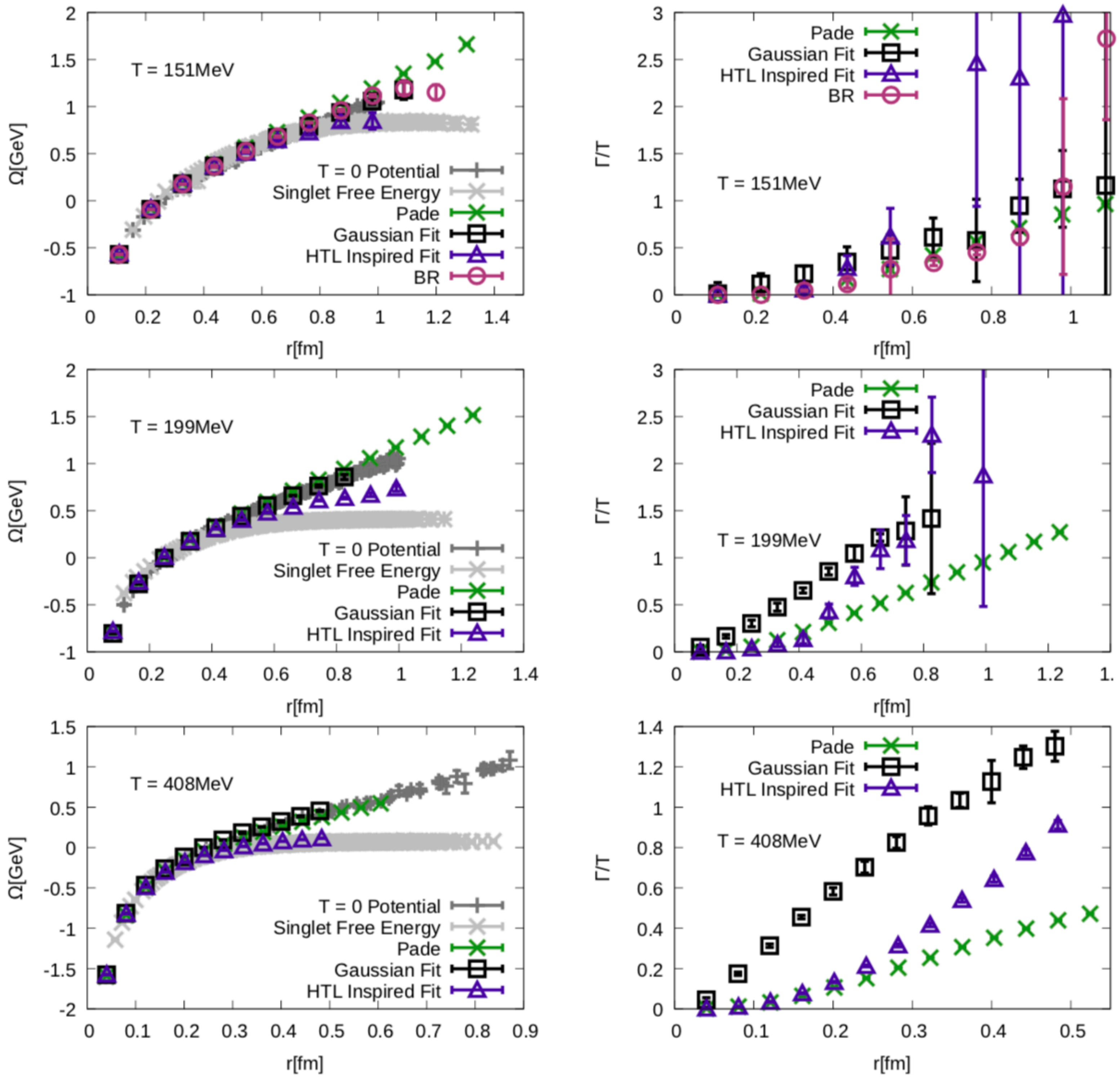
Lafferty, A. Rothkopf, Phys.Rev.D 101 (2020) 5.
Y.Burnier, O.Kaczmarek, A. Rothkopf, JHEP 2015.





Obvious screening for the real part potential, the imaginary part larger than HTL results (large uncertainty)

Heavy Quark Potential at finite temperature

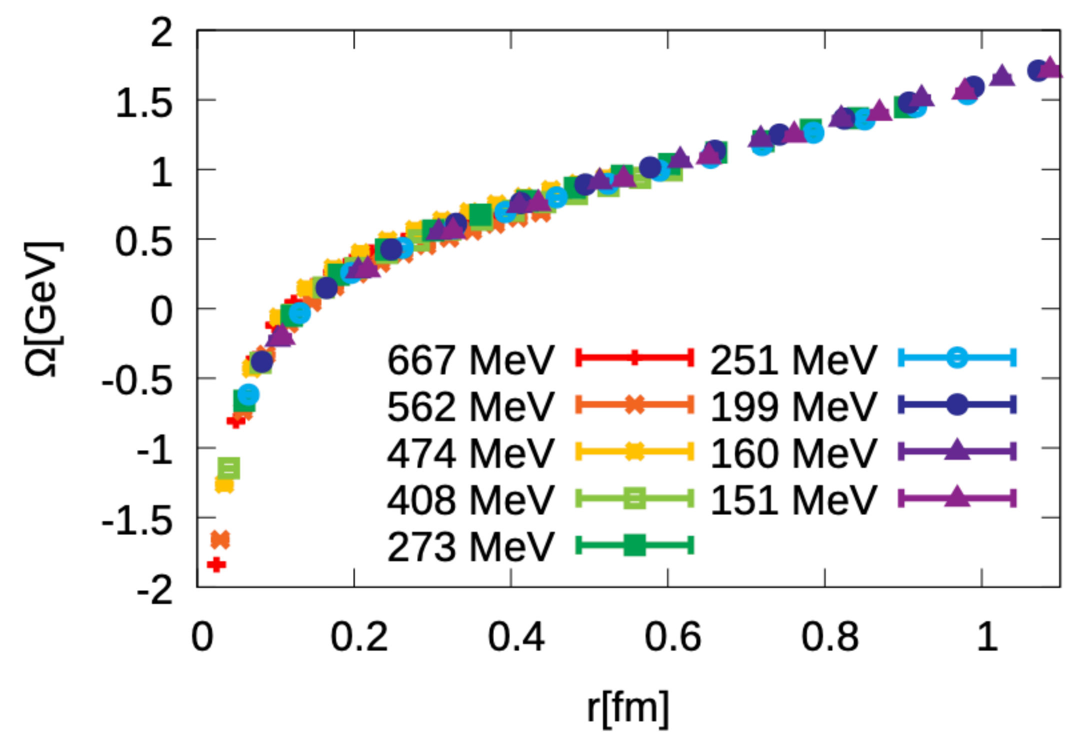


D. Bala, O. Kaczmarek, R. Larsen, S. Mukherjee, G. Parkar, P. Petreczky, A. Rothkopf, J. Weber. PRD105, 054513 (2022).

Extract the spectral functions from correlators with four different methods:

1. *Gaussian fit;*
2. *HTL inspired fit;*
3. *Pade fit;*
4. *Bayesian reconstruction (BR) method.*

Large difference caused by the extraction strategy !



Lattice QCD with dynamical fermions indicate no screening in static quark-antiquark potential !

Also supported by:

S. Shi, K. Zhou, J. Zhao, S. Mukherjee, and P. Zhuang. PRD 105 (2022) 1, 1.

X. Du, S. Liu, R. Rapp. Phys.Lett.B 796 (2019) 20-25.

Probe Heavy Quark Potential in experiments

Without regeneration

$$i\hbar\frac{\partial}{\partial t}\psi(r,t) = [-\frac{\hbar^2}{2m_\mu}\frac{\partial^2}{\partial r^2} + V(r,T) + \frac{L(L+1)\hbar^2}{2m_\mu r^2}]\psi(r,t)$$

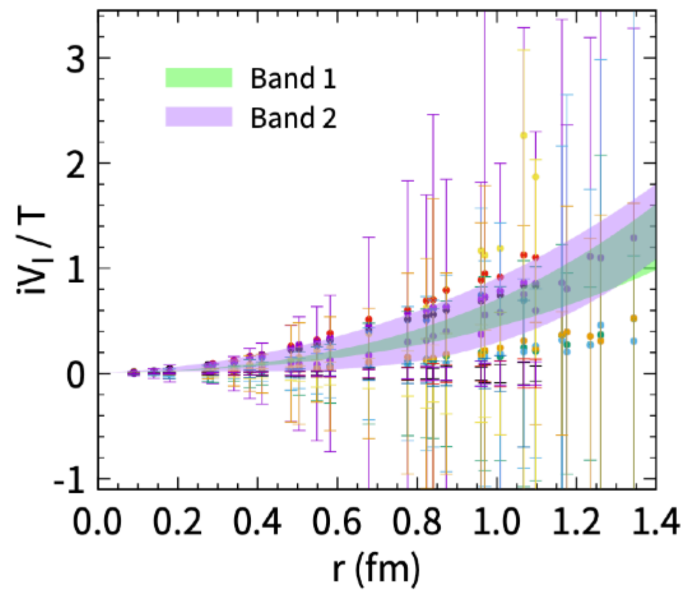
L. Wen and B. Chen. Phys.Lett.B 839 (2023) 137774.

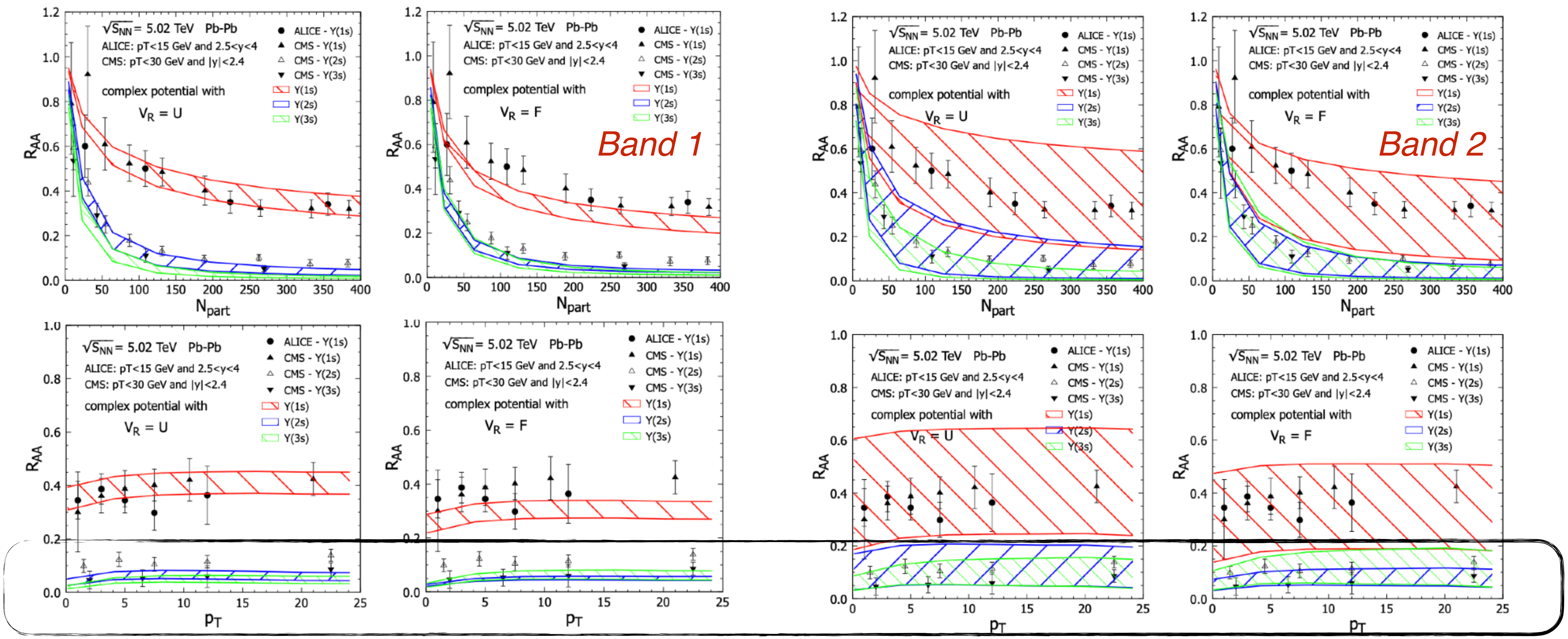
Free energy:

$$F(r,T) = -\frac{\alpha}{r}e^{-m_D r} + \frac{\sigma}{m_D}(1-e^{-m_D r})$$

Internal energy:

$$U(r,T) = -\frac{\alpha}{r}(1+m_D r)e^{-m_D r} + \frac{2\sigma}{m_D}[1-e^{-m_D r}] - \sigma r e^{-m_D r}$$

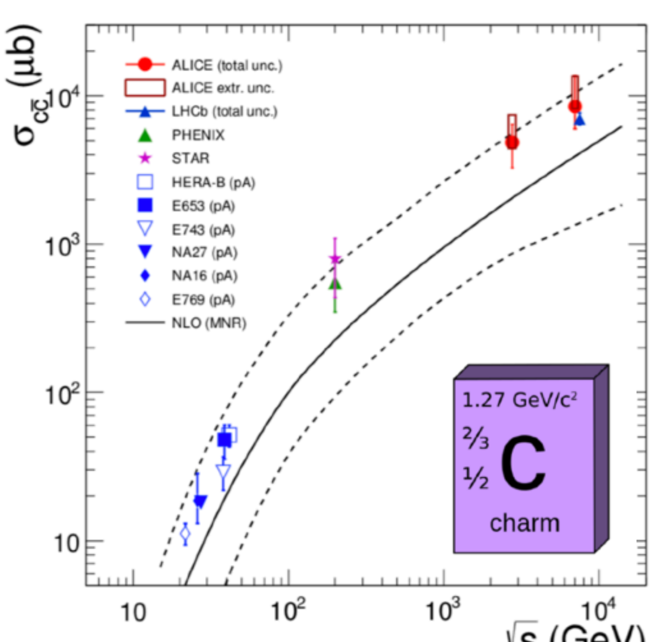




The yield ratio of excited states may supply a chance to distinglush this two different potentials!

III. *Heavy flavor rare/exotic states*

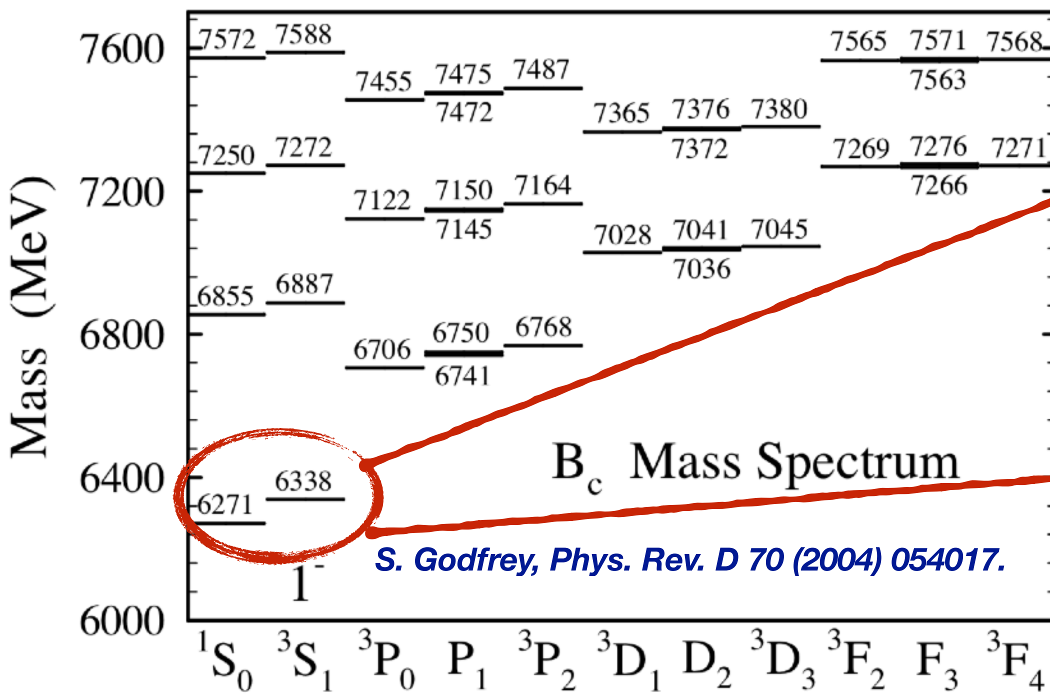
- *Searching for rare charmed hadrons in the most "charming" system.*
- *Probe the inner structure of tetraquark states*



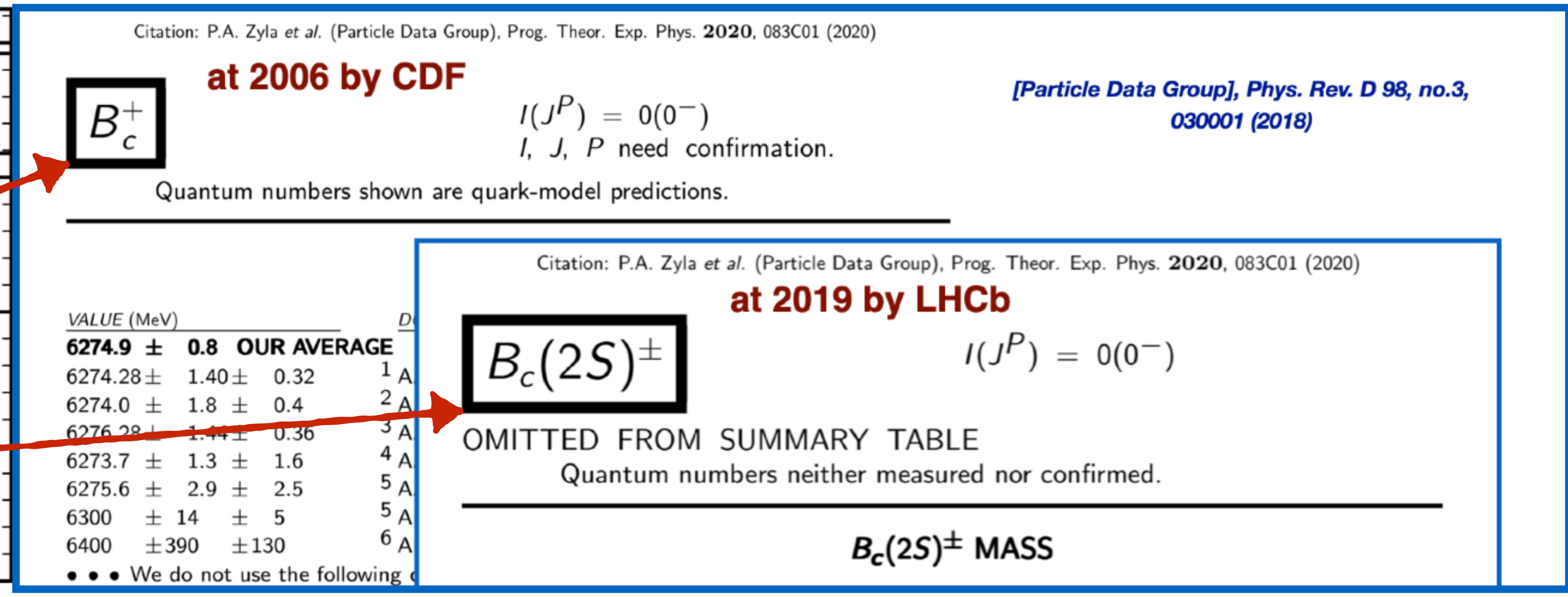
$N_{c\bar{c}} \sim T_A T_B \, \sigma_{c\bar{c}} \sim o(100)$ *charm quarks in the QGP at LHC!*

$$\Xi_{cc},\ \Omega_{ccc},\ B_c,\ X(3872),\ X(6900), \ldots$$

B_c in heavy-ion collisions



S. Godfrey, Phys. Rev. D 70 (2004) 054017.



Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

at 2006 by CDF

B_c^+

$I(J^P) = 0(0^-)$

I, J, P need confirmation.

Quantum numbers shown are quark-model predictions.

[Particle Data Group], Phys. Rev. D 98, no.3, 030001 (2018)

VALUE (MeV)	
6274.9 ± 0.8 OUR AVERAGE	
6274.28 ± 1.40 ± 0.32	1 A
6274.0 ± 1.8 ± 0.4	2 A
6276.28 ± 1.44 ± 0.36	3 A
6273.7 ± 1.3 ± 1.6	4 A
6275.6 ± 2.9 ± 2.5	5 A
6300 ± 14 ± 5	5 A
6400 ± 390 ± 130	6 A

• • • We do not use the following

Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

at 2019 by LHCb

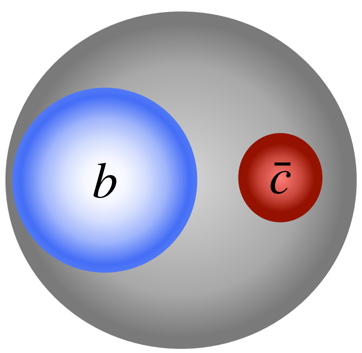
$B_c(2S)^\pm$

$I(J^P) = 0(0^-)$

OMITTED FROM SUMMARY TABLE

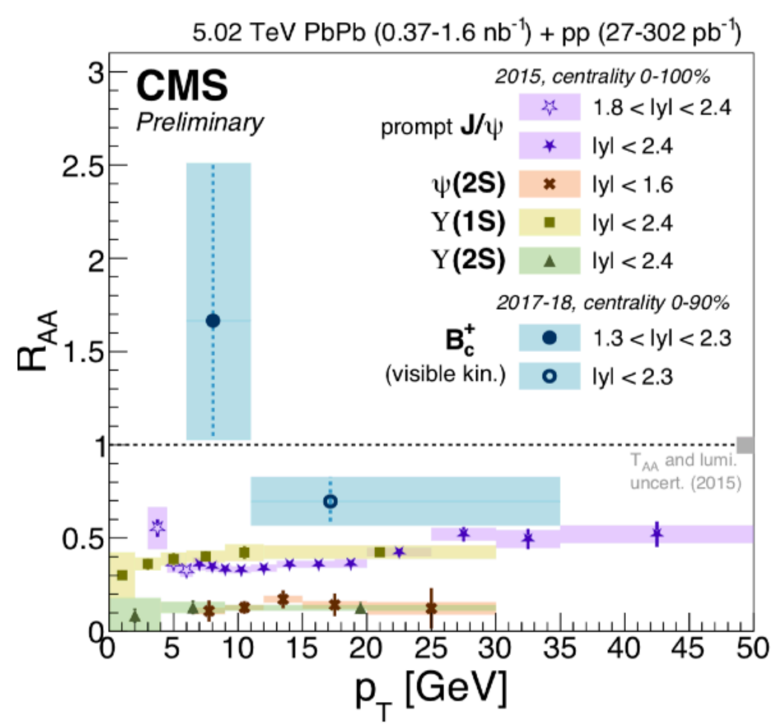
Quantum numbers neither measured nor confirmed.

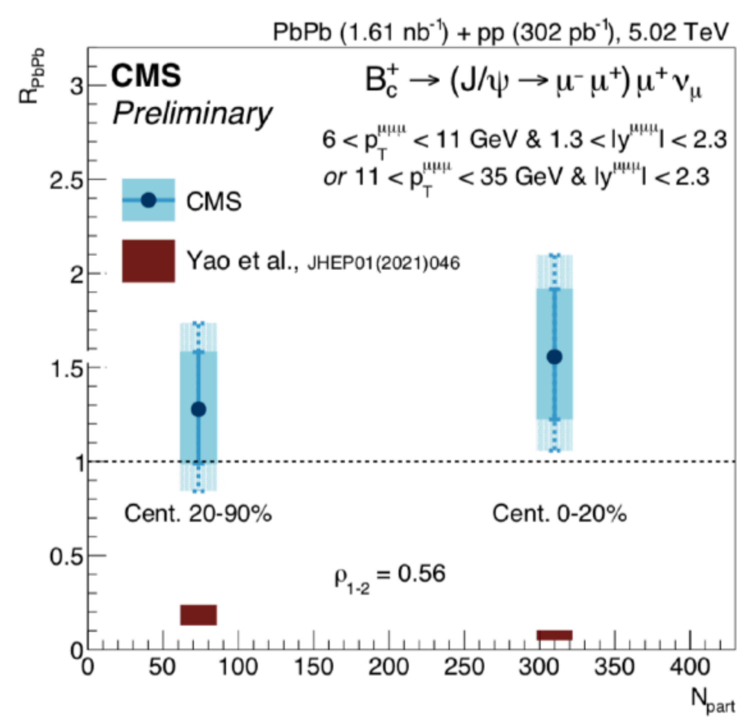
$B_c(2S)^\pm$ MASS



It's hard to produce a pair of $c\bar{c}$ and a pair of $b\bar{b}$ in one event of e^+e^- and pp collisions!

First observation of B_C mesons in heavy-ion collisions !



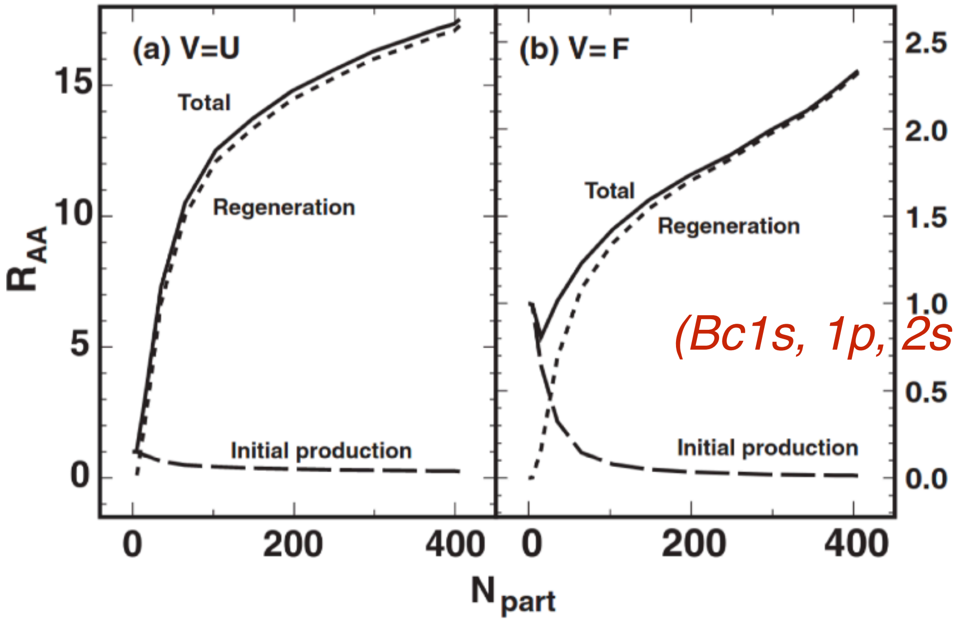


CMS Collaboration, PRL.128 (2022) 25, 252301.

$R_{AA} > 1$ indicate the production of B_c is largely enhanced in heavy-ion collisions!

B_c in heavy-ion collisions

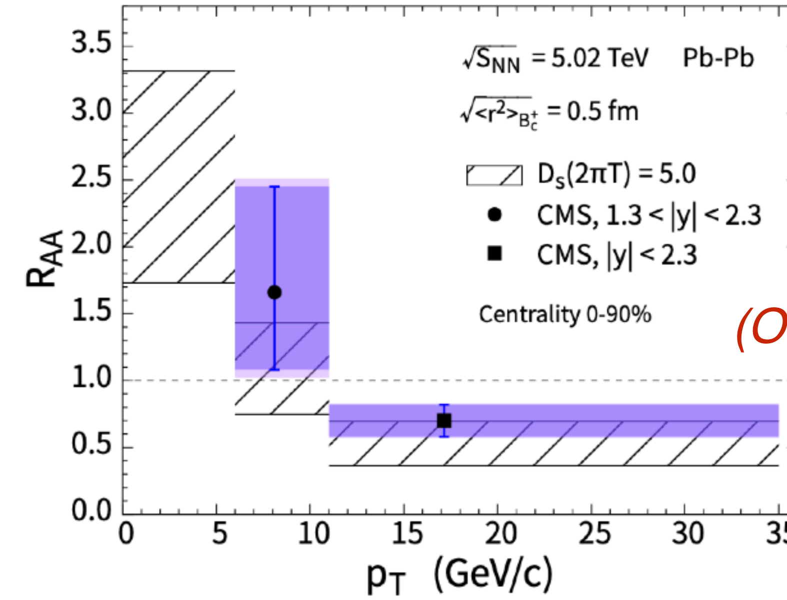
Boltzmann equation + thermal b and c dis.



(Bc1s, 1p, 2s states)

Y. Liu, C.Greiner, A. Kostyuk, PRC 87 (2013), 014910

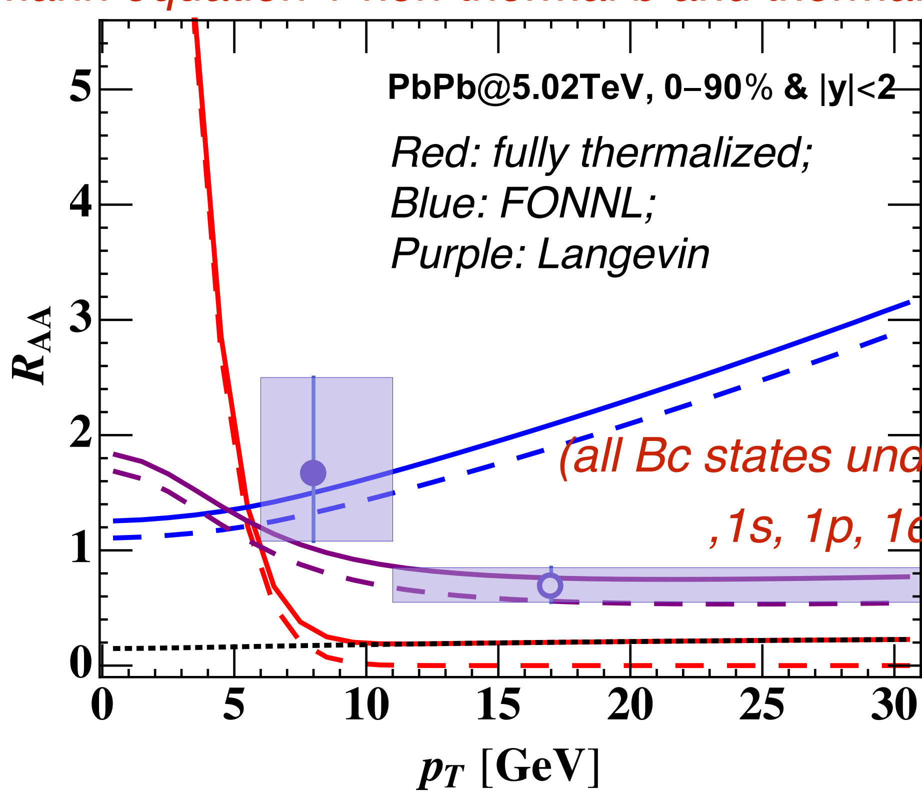
Coalescence + non-thermal b and c quark dis.



(Only Bc1s, 2s states)

B. Chen, L. Wen, and Y. Liu, PLB 834 (2022) 137448

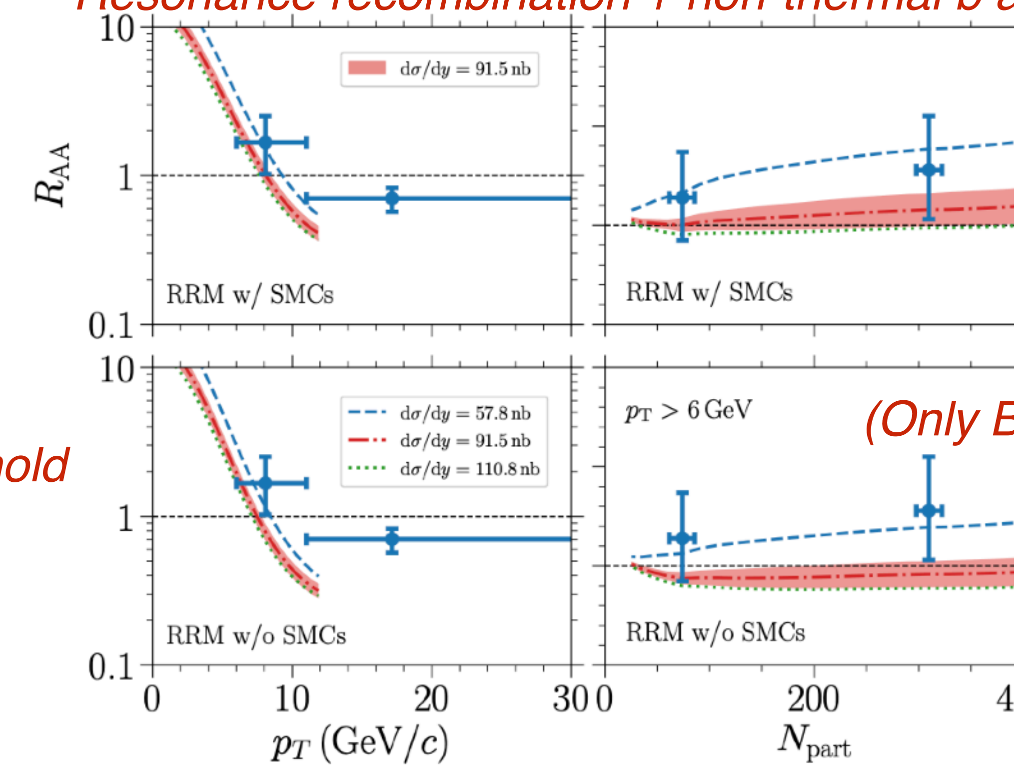
Boltzmann equation + non-thermal b and thermal c dis.



(all Bc states under threshold ,1s, 1p, 1d, 2s)

J. Zhao and P. Zhuang, arXiv: 2209.13275

Resonance recombination + non-thermal b and c quark dis.

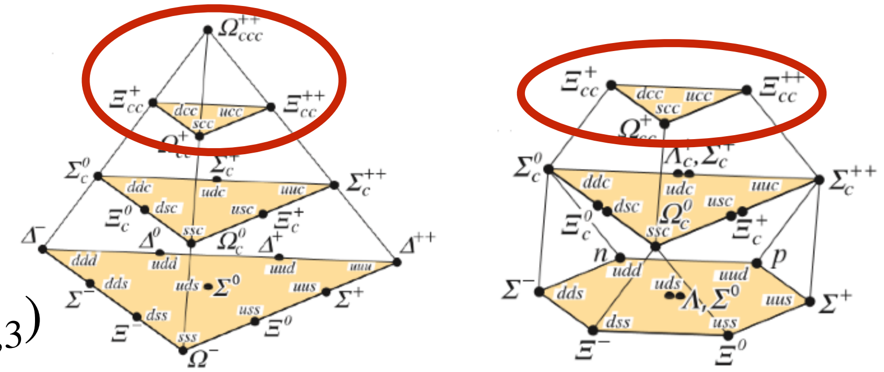


(Only Bc1s, 1p states)

B. Wu, Zh. Tang, M. He, and R.Rapp, arXiv: 2302.11511

Significant regeneration contributions with non-thermal bottom and charm quark!

Multi-heavy baryons in heavy-ion collisions

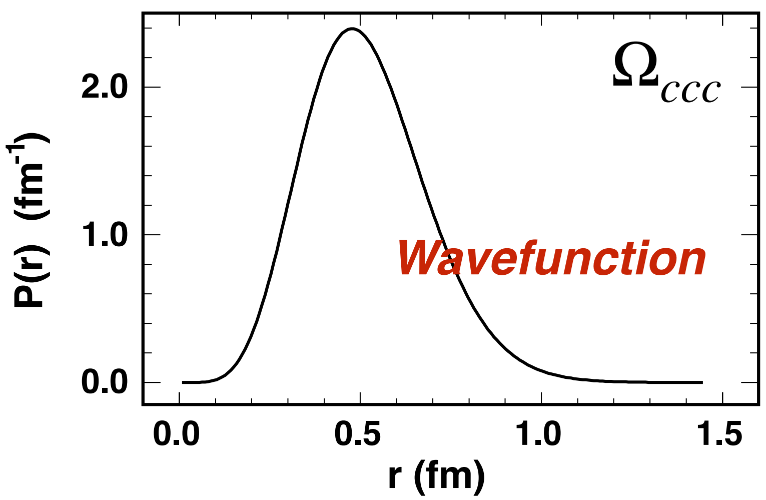


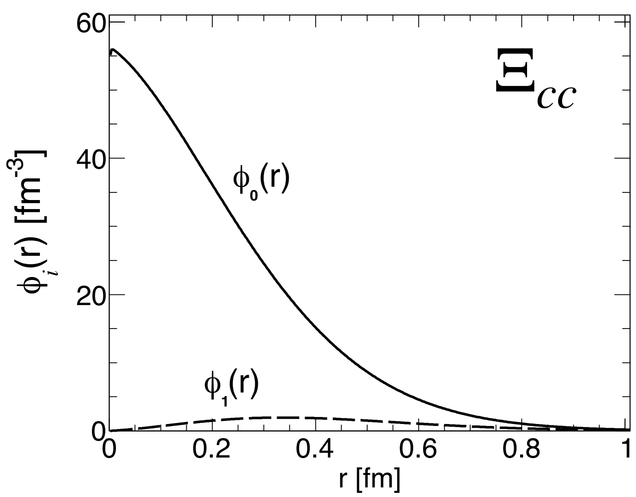
Coalescence mechanism:

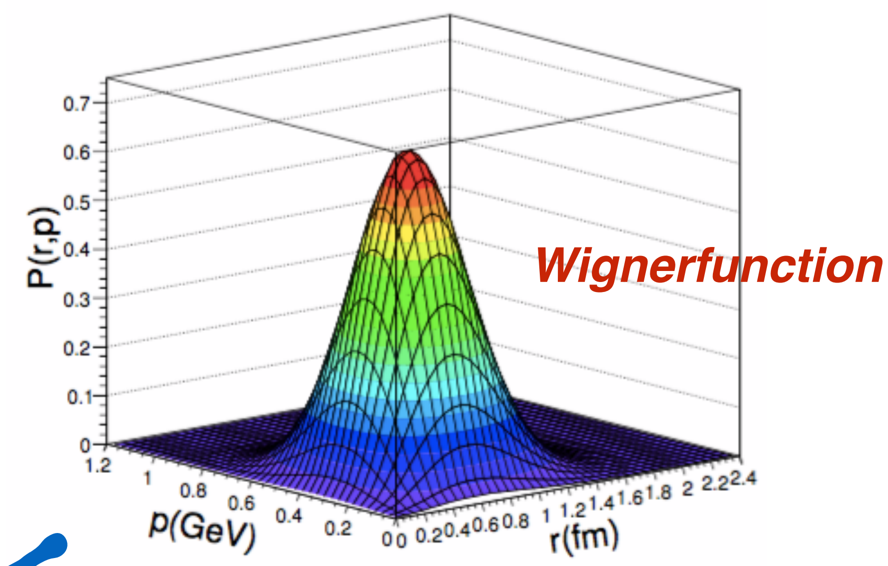
$$\frac{dN}{d^2\mathbf{P}_T dy} = C\int \prod_{i=1}^{3} \frac{d^3p_i}{(2\pi)^3 E_i} p_i \cdot d\sigma_i \; f(r_1, p_i) W(\mathbf{x}, \mathbf{p}) \delta^{(2)}(P_T - p_{T,1} - p_{T,2} - p_{T,3})$$

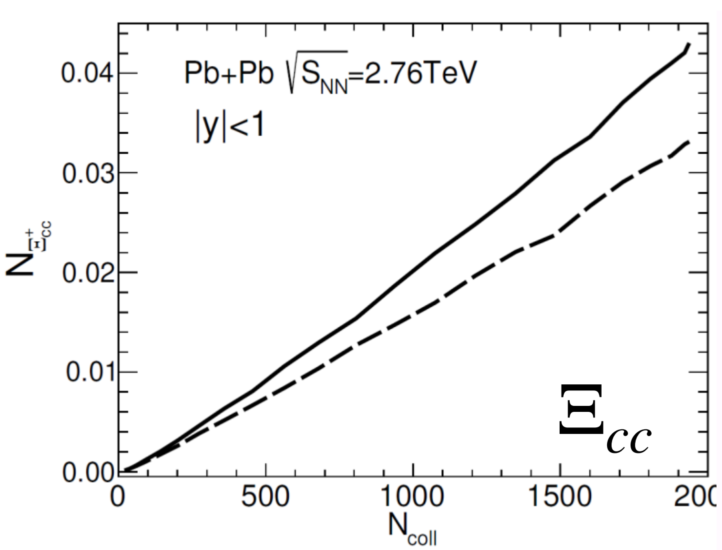
The Wigner function is determined by the wavefunction (solve 3-body Schrödinger equation). Instead of taking a Gaussian distribution with the width as a free parameter.

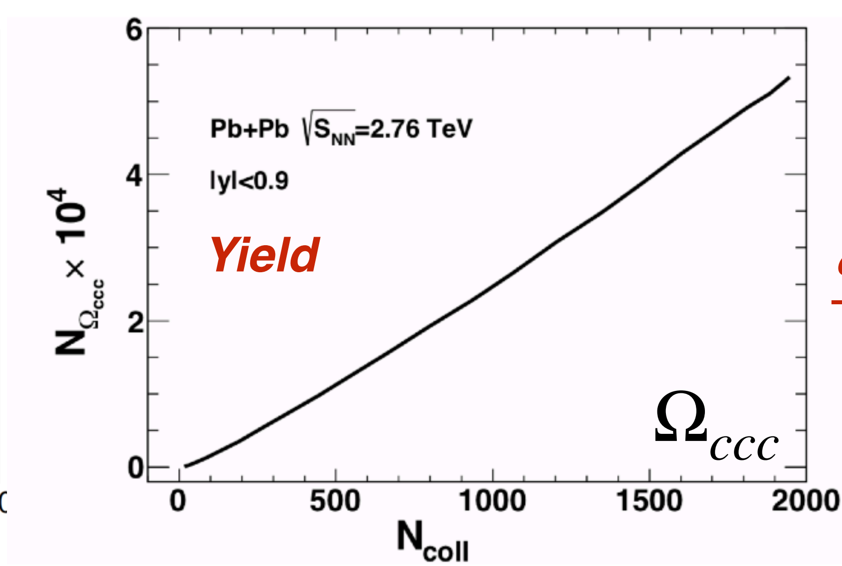
$$W(r,p) = \int d^4y e^{-ipy} \psi(r + \frac{y}{2}) \psi(r - \frac{y}{2})$$











$$\sigma_{AA}^{eff} \equiv \frac{N_{AA}}{N_{coll}\Delta\eta}\sigma_{pp}^{inel}$$

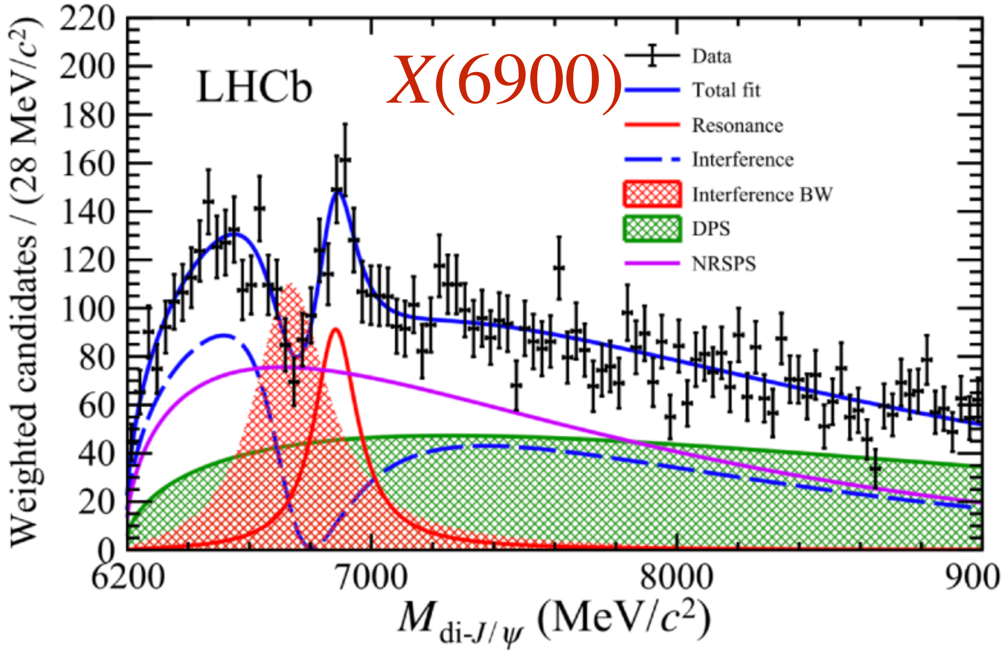
$$\frac{\sigma_{AA}^{eff}(\Omega_{ccc})}{\sigma_{pp}(\Omega_{ccc})} : \frac{\sigma_{AA}^{eff}(\Xi_{cc})}{\sigma_{pp}(\Xi_{cc})} : \frac{\sigma_{AA}^{eff}(J/\psi)}{\sigma_{pp}(J/\psi)} \approx 10^2 : 10^1 : 10^0 .$$

J. Zhao, H. He, Y. Liu, P. Zhuang. PLB 746(2015); PLB 771 (2017) 349-353; Few Body Syst. 58 (2017) 2, 100.

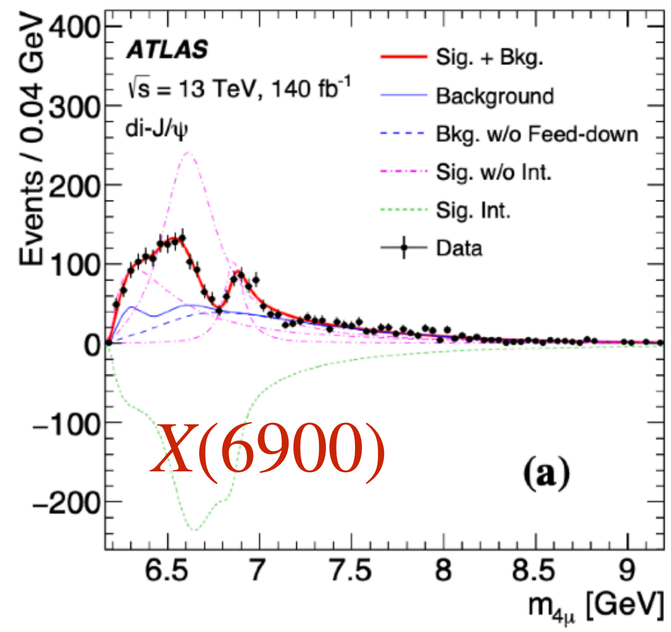
Also supported by the statistical model: ***A. Andronic et al, JHEP 07 (2021) 035.***

Fully-heavy Tetraquark

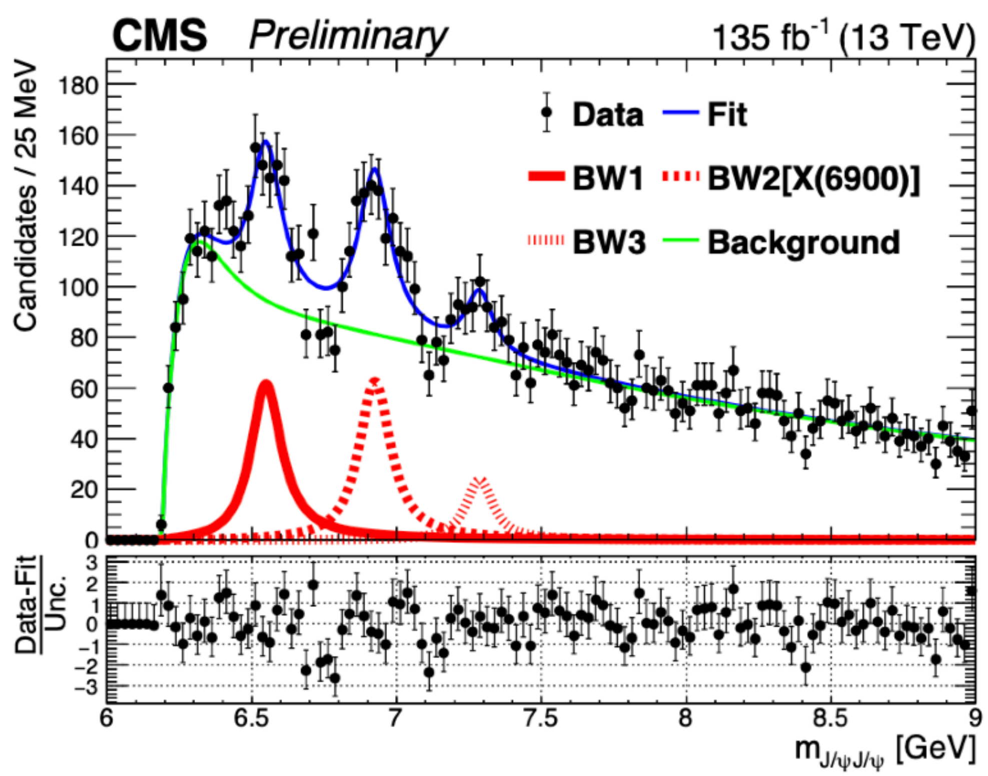
LHCb, Science Bulletin, 2020, 65(23)1983-1993



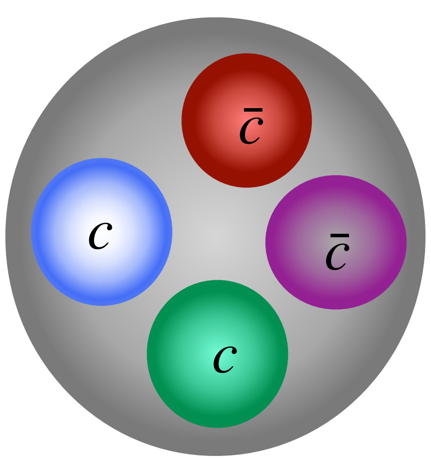
ATLAS, arXiv: 2304.08962



CMS PAS BPH-21-003

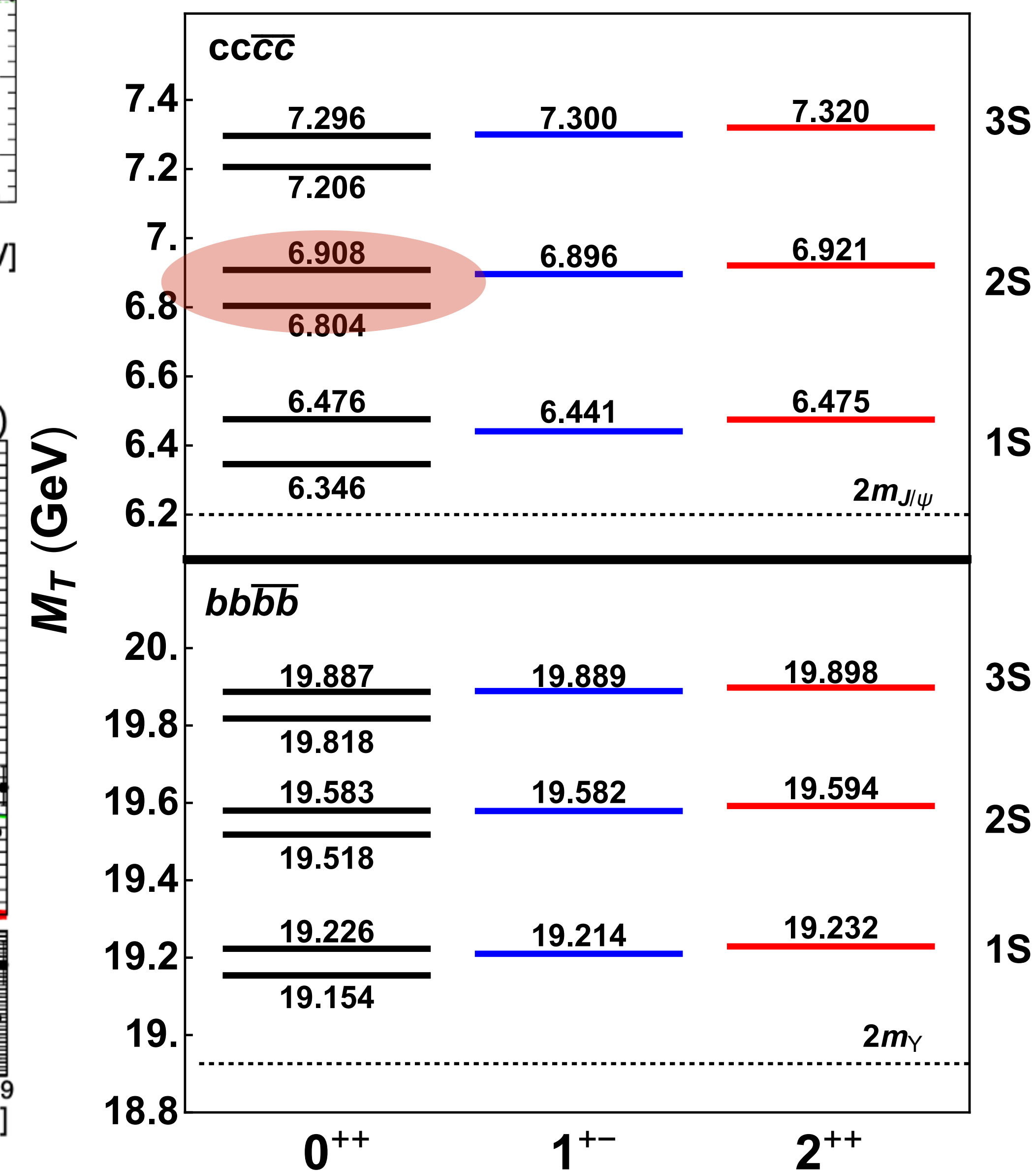


$X(6600), X(6900), X(7300)$



Solve four-body Schrödinger equation with Cornell + spin-spin interaction.

$$\left(\sum_{i=1}^{4}\frac{\widehat{\mathbf{q}}_i^2}{2m}+\sum_{i<j}V_{ij}(|\mathbf{r}_{ij}|)\right)\Psi=E\Psi,$$

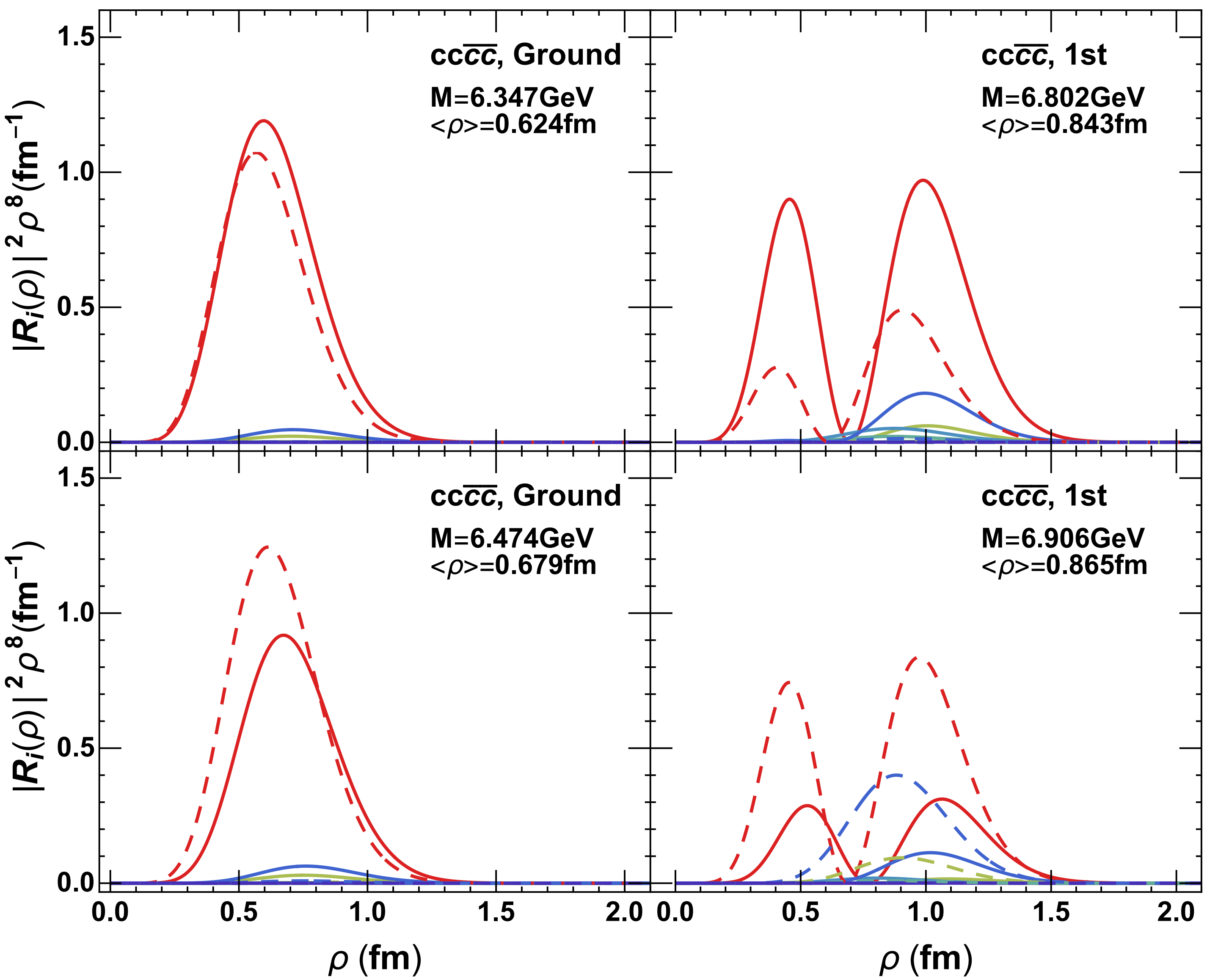


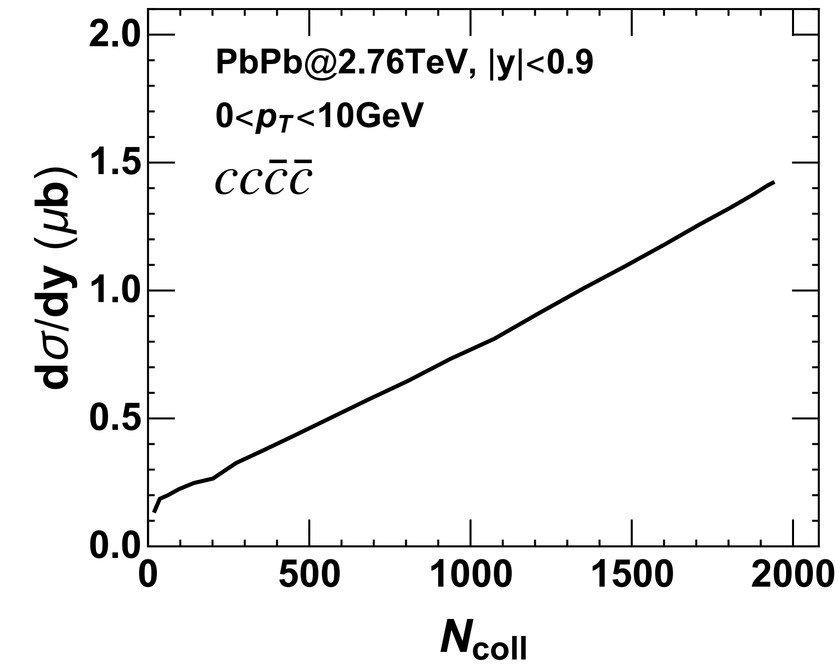
J. Zhao, S.Shi, and P. Zhuang, PRD 102 (2020) 11, 114001

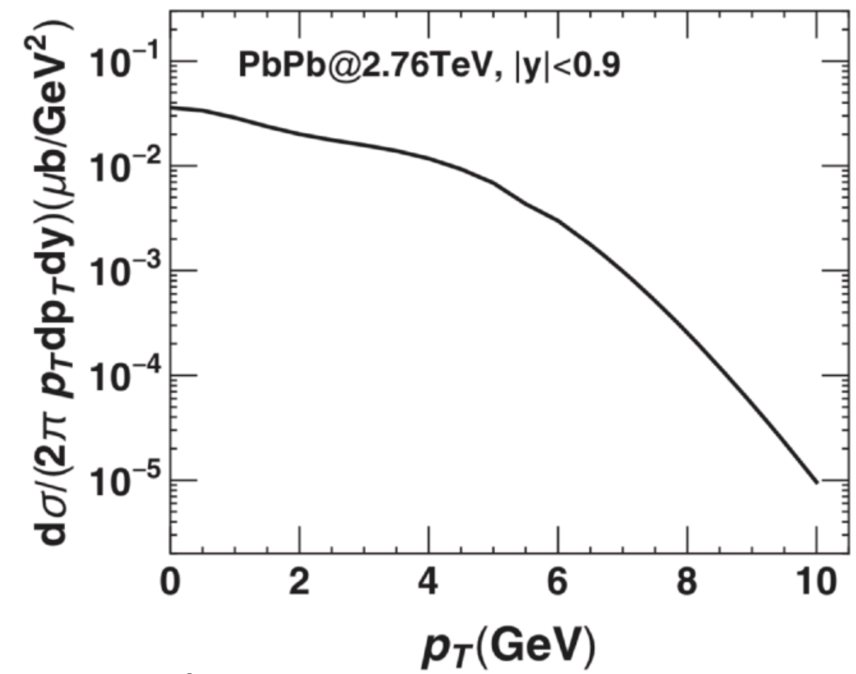
- *The mass spectra can be explained by the potential model !*

Fully-heavy Tetraquark

Fully-heavy tetraquark production in heavy-ion collisions : Coalescence model







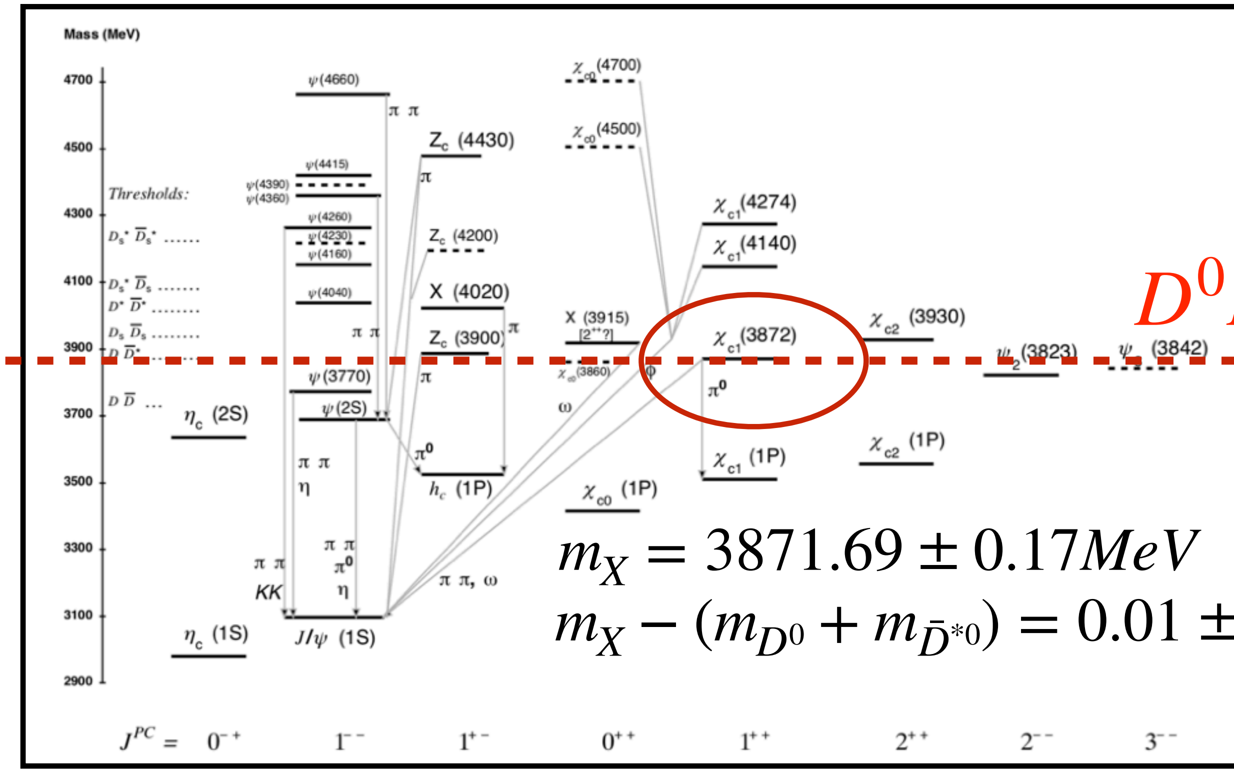
Coalescence probability: Wigner function <— Wavefunction

- $\left.\frac{d\sigma}{N_{coll}dy}\right|_{AA} \approx 770pb$ in AA at 5.02TeV is much larger than $\left.\frac{d\sigma}{dy}\right|_{pp} = 78pb$
- *The four-lepton decay ($X(cc\bar{c}\bar{c}) \to l_1^+ l_2^- l_3^+ l_4^-$), well separated from the bulk back ground !*

J. Zhao, S.Shi, and P. Zhuang, Phys.Rev.D 102 (2020) 11, 114001

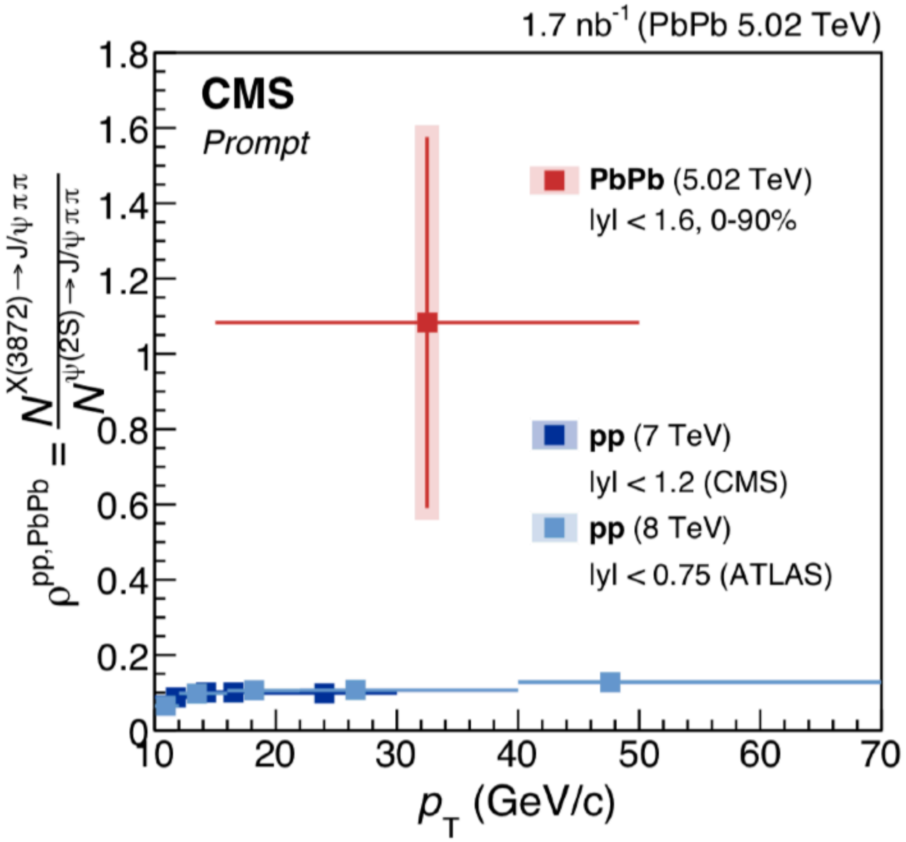
X(3872)

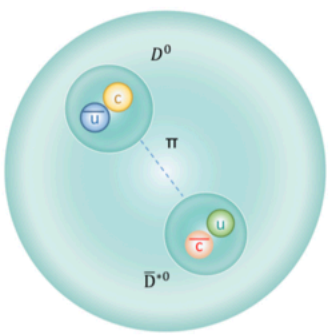
First observed by Belle collaboration (2003)



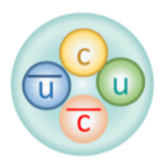
$m_X = 3871.69 \pm 0.17 MeV$

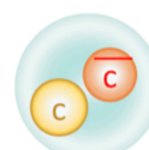
$m_X - (m_{D^0} + m_{\bar{D}^{*0}}) = 0.01 \pm 0.27 MeV$

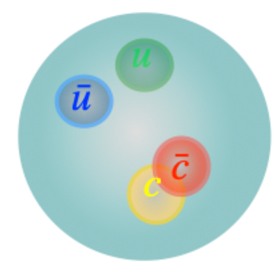


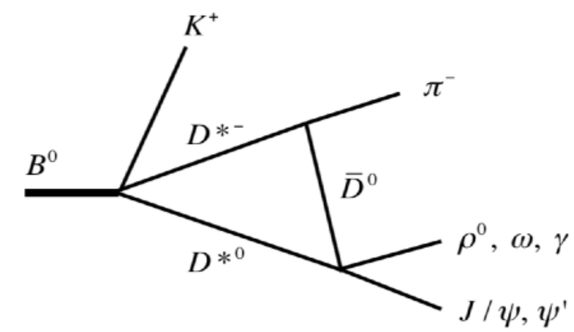


Y. Lee's slides









Hadronic molecule	Tetraquark	Hybrid	Charmonium	Hadro-quarkonium	Triangle singularities	…
$(Q\bar{q})_1 + (\bar{Q}q)_1$	$(Q\bar{Q}q\bar{q})_1$	$(Q\bar{Q})_8 + g$	$(Q\bar{Q})_1$	$(Q\bar{Q})_1 + (q\bar{q})_1$ $(Q\bar{Q})_8 + (q\bar{q})_8$		

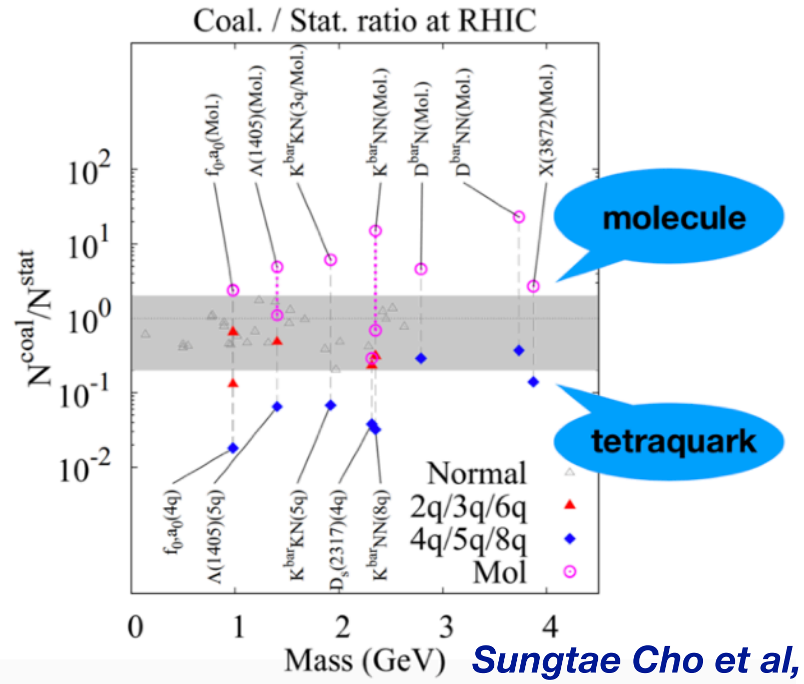
First evidence of X(3872) production in heavy-ion collisions, only one point at pT ~ 30GeV and with large uncertainty!

CMS Collaboration, PRL. 128 (2022) 3, 032001

Can HIC help us to understand its inner structure ?

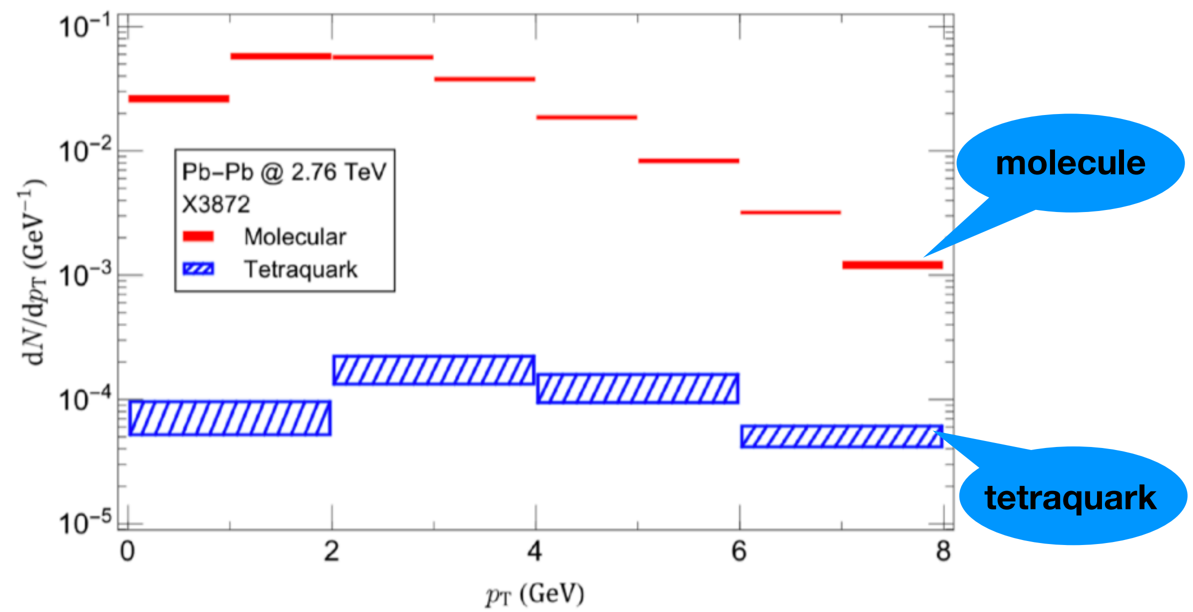
X(3872)

Coalescence



Sungtae Cho et al,
Phys. Rev. Lett. 106 (2011) 212001.

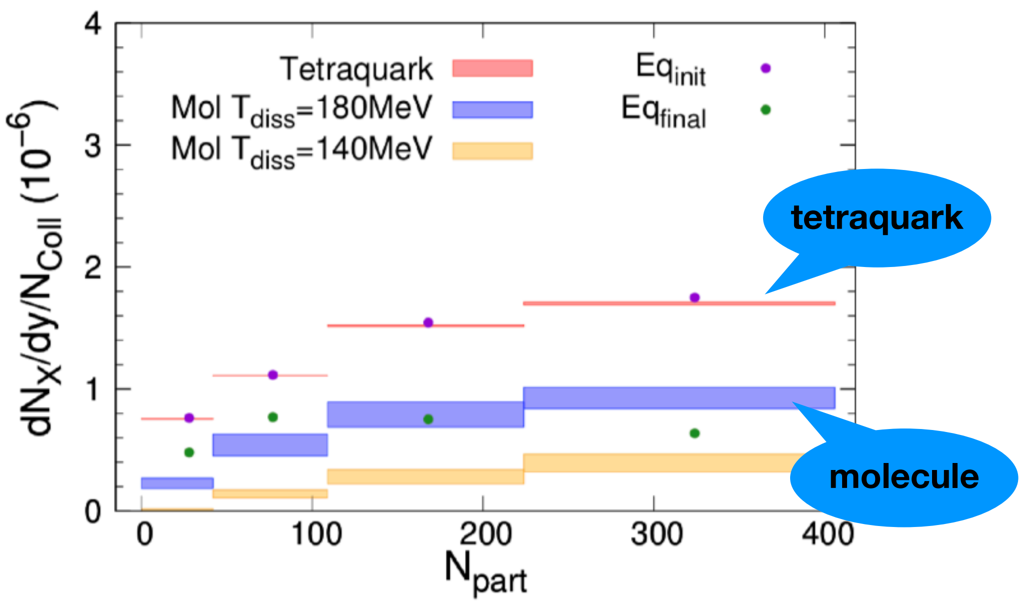
AMPT+invariant mass&radius



H. Zhang, J. Liao, E. Wang, Q. Wang, H. Xing,
Phys.Rev.Lett. 126 (2021) 1, 012301.

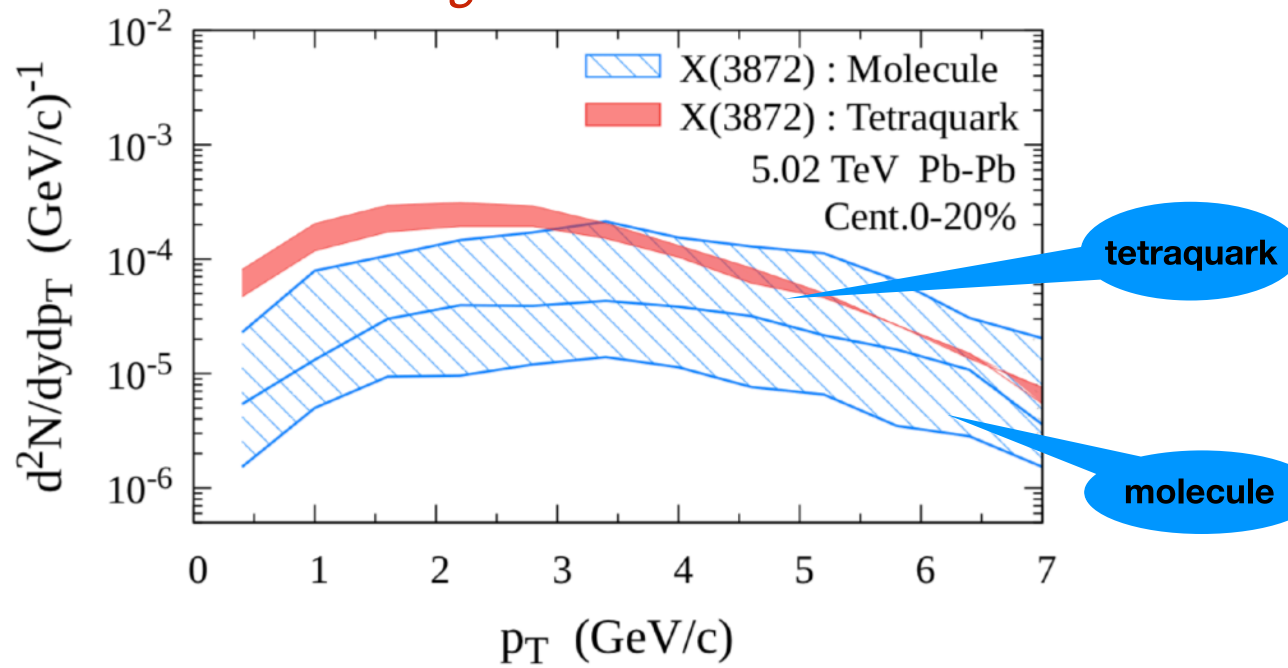
NMolecule
>
NTetraquark

Transport(Rate Eq.)



B. Wu, X. Du, M. Sibila, R. Rapp,
Eur. Phys. J. A 57 (2021) 4, 122.

Langevin+Coalescence



B. Chen, L. Jiang, X. Liu, Y. Liu, J. Zhao.
Phys.Rev.C 105 (2022) 5, 054901

NTetraquark
>
NMolecule

- *Production (at low pT) in heavy-ion collisions: Reveal the inner structure of X(3872)*
- *Dissociation of loosely bound molecular states in hadronic phase is important !*
- *Hadronic correlation (eg. DDbar) may reflect the interaction and possible molecular structure.*

Summary

I. *Open Heavy Flavor*

- *Heavy quark can help us to understand the hadronization mechanism in the QGP. Model comparsion is very important to go forward.*

- *EPOS4+HQ can give a good description of all heavy flavor hadrons production in both pp and AA, from RHIC to LHC.*

II. *Quarkonium*

- *The in-medium properties mostly can be absorbed in the finite-temperature potential, which with both real and imaginary part. Recent lattice QCD results and theoretical study show a very weak color screening but a large imaginary part.*

III. *Heavy flavor rare/exotic states*

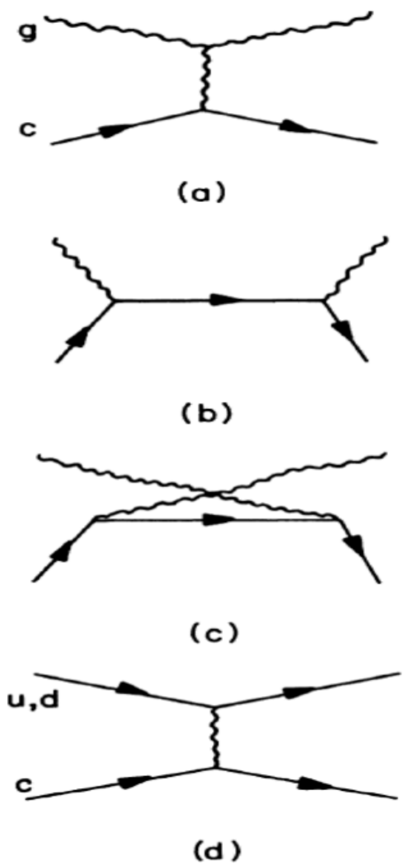
- *The production of* B_c, Ξ_{cc}, Ω_{ccc}, $X(6900)$ *are largely enhanced in heavy ion collisions!*
- *It is possible to probe the inner structure of tetraquark states, such as X(3872), in heavy ion collisions.*

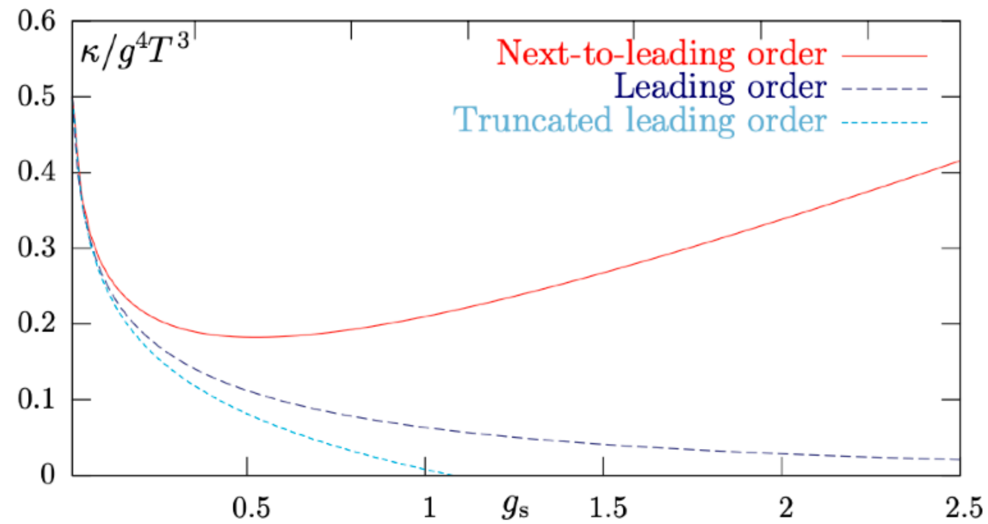
Looking forward to more experimental results!

Thanks for your attention!

Collisional and radiative energy loss in the hot medium

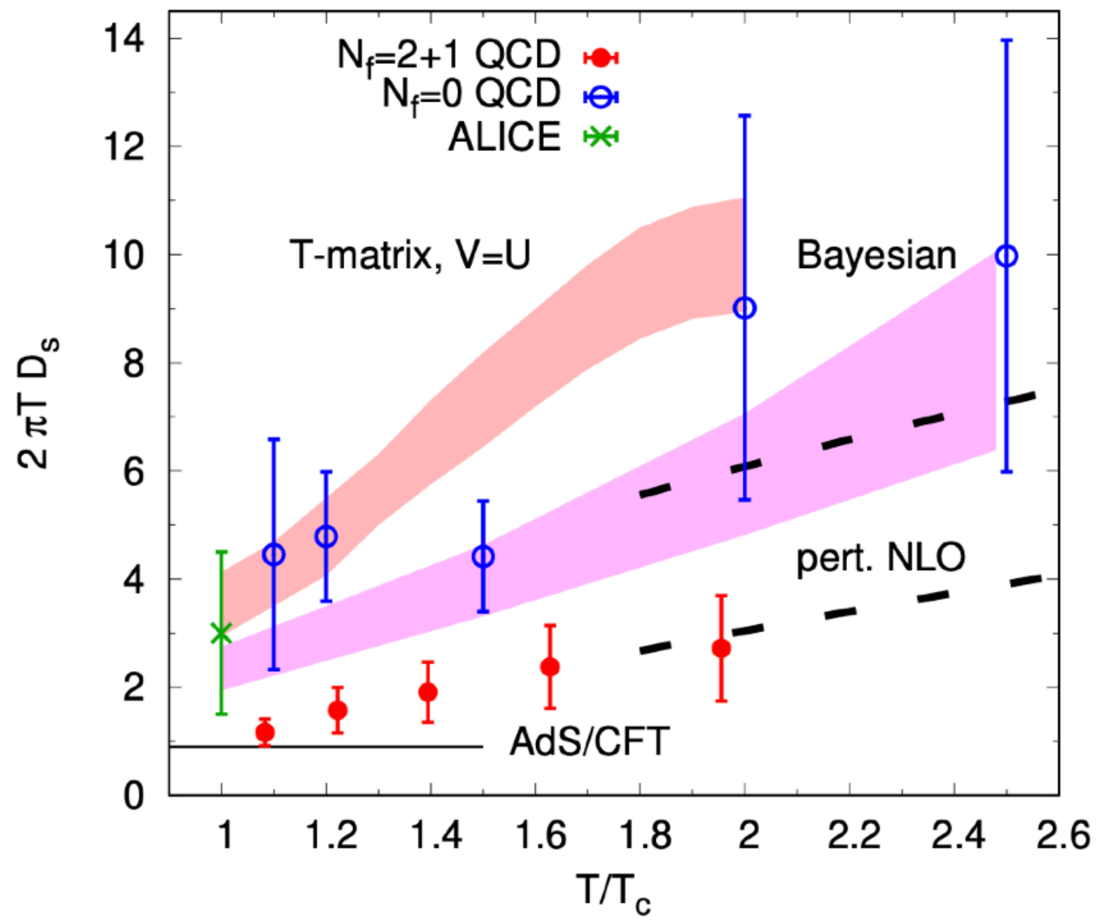
HQs suffer collisional and radiative energy loss in the QGP, can be simulated by Boltzmann/Langevin equations.





S. Caron-Huot, G.Moore, JHEP02(2008)081.

poor convergence of NLO!



non-perturbative method required (lattice QCD, T-matrix,..)

momentum diffusion coefficient:

$$\kappa = \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega}$$

$$D_s = 2T^2/\kappa$$

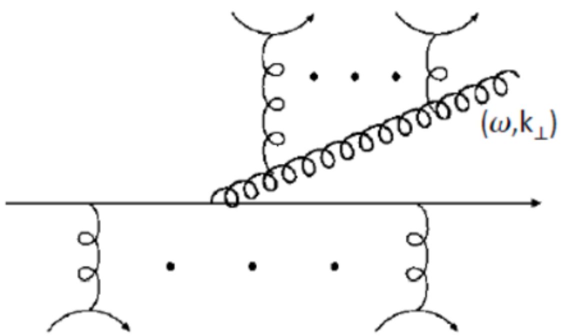
Olaf Kaczmarek et al, arXiv: 2302.08501

See also: O. Kaczmarek, HP2023, Wed. 09:40

Spatial diffusion coefficient is found to be significantly smaller than previous quenched lattice QCD and recent phenomenological estimates —> thermalizated easily!

Collisional and radiative energy loss in the hot medium

HQs suffer collisional and radiative energy loss in the QGP, can be simulated by Boltzmann/Langevin equations.



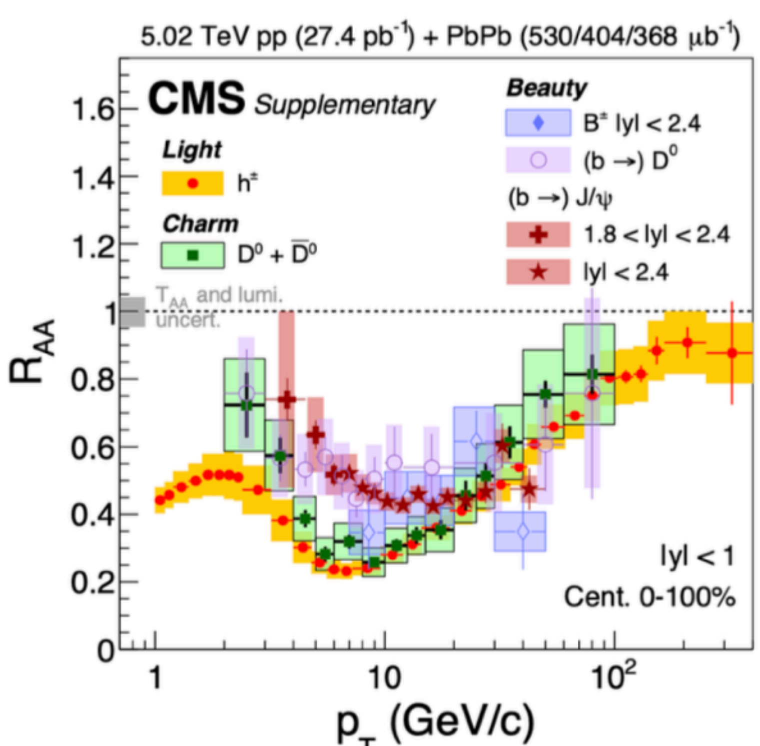
Many approaches have been developed to simulate the radiative energy loss, such as the GLV, Higher Twist, AMY, …

M. Gyulassy, P. Levai, and I. Vitev, Phys. Rev. Lett. 85, 5535 (2000)
B. Zhang, E. Wang, and X. Wang, Phys. Rev. Lett. 93, 072301 (2004) .
P. B. Arnold, G. D. Moore, and L. G. Yaffe, JHEP 06, 030 (2002).
P. Gossiaux, J. Aichelin, T. Gousset and V. Guiho,J.Phys.G 37 (2010) 094019.
.....

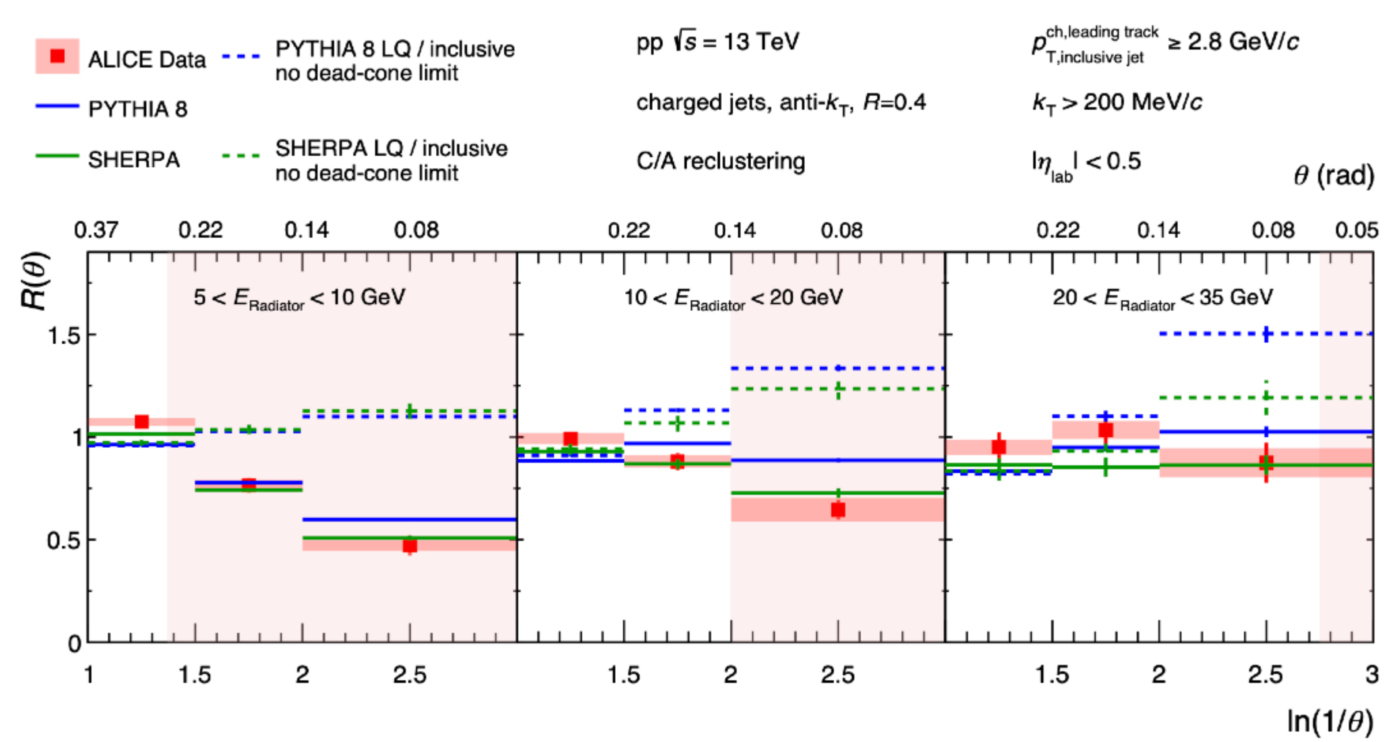
The radiative energy loss dominates at high pT while collisional energy loss play at low pT.

Energy loss hierarchy: $\Delta E_b < \Delta E_c < \Delta E_q < \Delta E_g$

- *Reflected to the* R_{AA}: $R_{AA}(bottom) > R_{AA}(charm)$
- *Dead cone:* $\theta_D = m_Q/E$

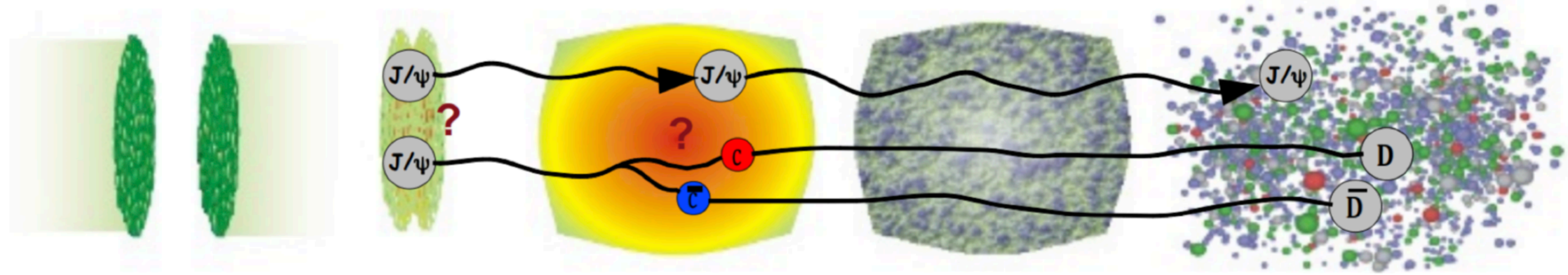


CMS, Phys. Rev. Lett. 123, 022001 (2019).



ALICE, Nature 605 (2022) 7910, 440-446.

Quarkonium real-time evolution in heavy-ion collisions



✦ ***Schrödinger approach***

For bottomonium, neglect regeneration, time-dependent Schrödinger equation.

With a complex potential given by Lattice (M.Strickland, A.Rothkopf…).

Include stochastic term, Schrödinger-Langevin equation(P.Gossiaux…).

✦ ***Transport approach***

(Boltzmann equation, THU model, P.Zhuang…; Rate equation, TAMU model, R.Rapp…)

Treat both dissociation and regeneration dynamically

Transition rates are given by cross-section, detail balance, heavy quark potential control the time and BE.

Developing a genuine first principles based framework of quarkonium real-time evolution !

Quantum effects/deal with resonance with cross-section?/…

✦ ***Open quantum system***

(N.Brambilla, M.Strickland, A.Rothkopf, Y.Akamatsu, M.Asakawa, P.Blaizot, P.Gossiaux, X.Yao, B.Müller.…)

J. Blaizot, M. Escobedo, JHEP. 2018, 34 (2018).

X. Yao and T. Mehen, et al. PRD 99 (2019) 096028;JHEP 21 (2020) 046.

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, PRD 96 (2017) 034021;PRD 100 (2019) 054025.

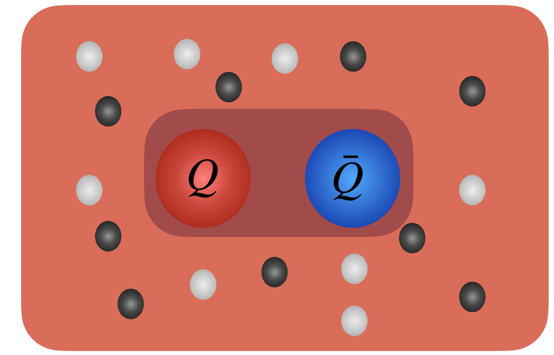
S. Delorme, T. Gousset, R, Katz, and P. Gossiaux,EPJ Web Conf. 259 (2022) 12001; EPJA 58 (2022) 10,198.

T. Miura,Y. Akamatsu, M. Asakawa, et al, PRD 87 (2013) 045016; PRD 91 (2015) 5, 056002.; PRD97 (2018), 014003.; PRD 101 (2020) 3,034011.

D. Villar, J. Zhao, J. Aichelin, and P. Gossiaux, arXiv: 2206.01308, PRC accepted.

Quarkonium real-time evolution in heavy-ion collisions

Open quantum system



$$\hat{H}_{tot} = \hat{H}_s \otimes I_e + I_s \otimes \hat{H}_e + \hat{H}_{int}\,,$$

Subsystem **Environment** **Interaction**

$$\frac{d\hat{\rho}_{tot}}{dt} = -\,i[\hat{H}_{tot}, \hat{\rho}_{tot}]$$ *von Neumann equation* $$\hat{\rho}_{tot} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

Trace over the environment degrees of freedom, reduced density matrix

$$i\hbar\dot{\hat{\rho}}_s(t) = \mathrm{Tr}_e[\hat{H}_{tot}, \hat{\rho}_{tot}] = [\hat{H}_s, \hat{\rho}_s] + \mathrm{Tr}_e[I_s \otimes \hat{H}_e + \hat{H}_{int}, \hat{\rho}_{tot}]$$ *Quantum master equation*

- *Separation of time-scales:*

  Environment relaxation time scale $\tau_e \sim 1/T$.

  Intrinsic time scale of subsystem $\tau_s \sim 1/\Delta E$.

  Subsystem relaxation time scale $\tau_r \sim 1/\Pi \sim m^2/T^3$.

- *Markovian approximation:* $\tau_e \ll \tau_r$ *, memory lose*

$$\dot{\hat{\rho}}_s(t) = -i[\hat{H}_s, \hat{\rho}_s] + \sum_{i=1}^{N} \gamma_i \left(L_i \hat{\rho}_s L_i^\dagger - \frac{1}{2} L_i^\dagger L_i \hat{\rho}_s - \frac{1}{2} \hat{\rho}_s L_i^\dagger L_i \right)$$ ***Lindblad equation***

Lindblad Equation

$\tau_s \ll \tau_r$ → *Boltzmann Equation*

Quantum optical regime

$\tau_e \ll \tau_s$ → Master Equation with transport coefficients; Non-linear Stochastic Schrödinger Equation

Quantum Brownian Motion

See review:

A. Rothkopf, Physics Reports 858 (2020) 1–117.