Heavy Quark Diffusion coefficients in Magnetised Medium

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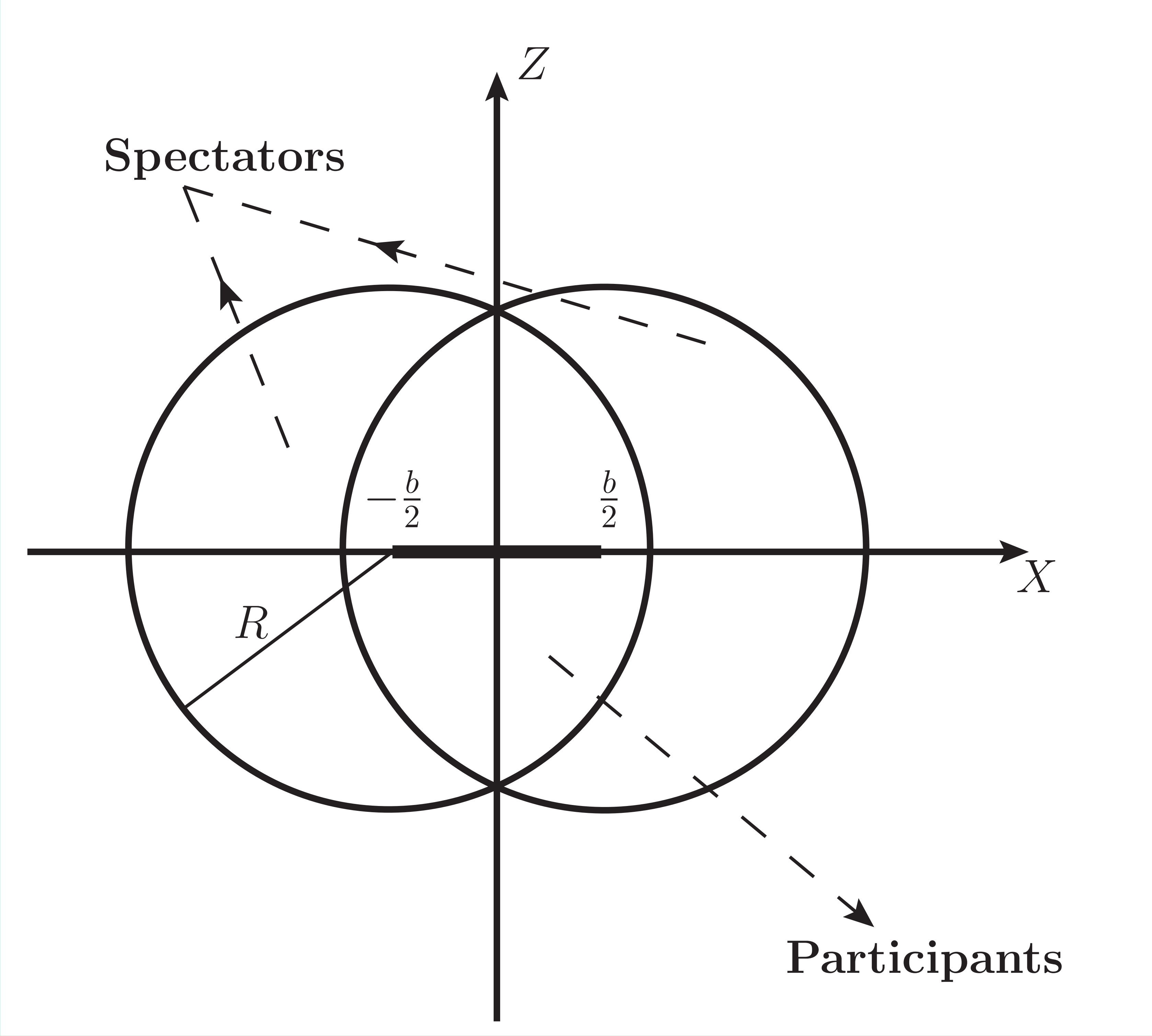
Strong-interaction matter under extreme conditions

Outline

- Introduction
- Approach
- Results
- Summary and Outlook

Magnetised medium



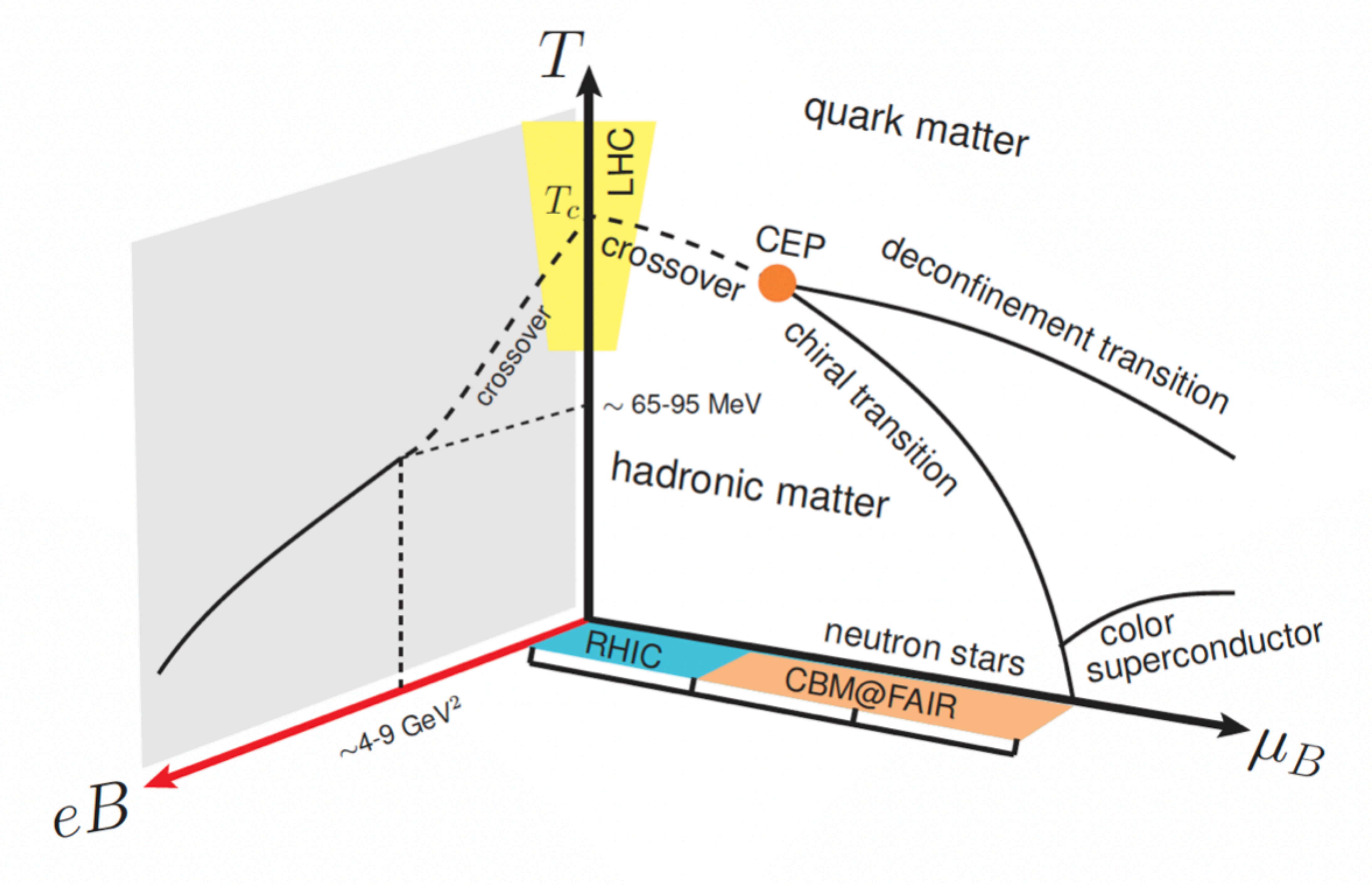
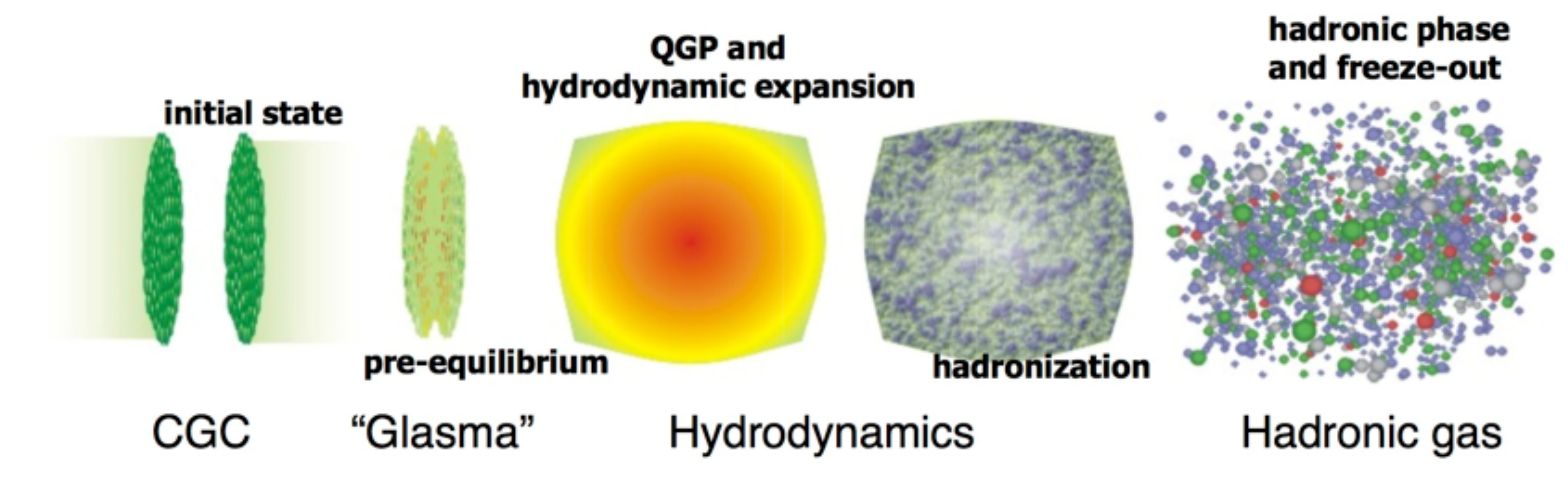


Fig 2 - Conjectured QCDpd with magnetic field

- **Strong magnetic fields are present in some stellar objects and non-central heavy ion collisions (e.g. $eB \sim \hat{O}(10)m_\pi^2$ at LHC).**

- **Introduces extra scale eB in the medium in addition to T, μ $\rightarrow$ triggers significant interest in theoretical understanding of the properties of a magnetised medium.**

Heavy quark as a QGP signature



- Large mass compared to T
- External to the bulk medium.

} $\rightarrow$ **Less Contamination**

- Generated at the early stage $\rightarrow$ **More Information**

In this talk: Heavy quark momentum diffusion coefficients

Heavy quark diffusion

- HQs experience drag forces as well as random kicks from the bulk medium.
- A widely adopted approach is to use the Langevin equations for describing HQ in-medium evolution.
- Essential theoretical inputs : HQ momentum diffusion coefficients → influence phenomenological modelings of predictions for experimental observables. (R_{AA} and v_2)

QGP Signatures

Static properties

(e.g. Heavy quark diffusion coefficients)

Dynamic properties

(e.g. Heavy quark spectra)

Approaches

Usually two approaches are taken in the literature to incorporate the theoretical modifications due to a magnetised medium

A. Through modification of the Debye mass :

- **Debye mass is related to the temporal part of the gluon self energy** Π_{00}
- $B = 0 \rightarrow$ **Most of the HQ observable can be expressed in such a way where the sole medium effect lies within the Debye mass** $m_D(T)$
- $B \neq 0 \rightarrow$ **Replace** $m_D(T)$ **by magnetised medium modified** $m'_D(T, eB)$

B. Through structural changes of the correlation functions :

- **Employing general structure of the gluon self energy** $\Pi_{\mu\nu}$ **for** $B \neq 0$
- **Evaluating the corresponding coefficients / form factors required**
- **Computing the HQ observable with the** eB **modified gluon CFs**

Static and Dynamic limits of Heavy Quark

$B = 0$

- **Static limit :** $M \gg T$

$$\langle \xi_i(t) \xi_j(t') \rangle = \kappa \ \delta_{ij} \delta(t - t')$$

Single diffusion coefficient κ

- **Dynamic limit :** $\gamma v \lesssim 1 \rightarrow p \lesssim M, \, M \gtrsim p \gg T$

$$\langle \xi_i(t) \xi_j(t') \rangle = \kappa_{ij}(\overrightarrow{p}) \ \delta(t - t')$$

where $$\kappa_{ij}(\overrightarrow{p}) = \kappa_L(p) \ \hat{p}_i \hat{p}_j + \kappa_T(p) \left(\delta_{ij} - \hat{p}_i \hat{p}_j \right)$$

Longitudinal (κ_L **) and Transverse (** κ_T **) diffusion coefficients.**

Static and Dynamic limits of Heavy Quark

$B \neq 0$

- **Static limit :** $M \gg (\sqrt{eB}, T)$

  Anisotropy given by $\vec{v}$ is now replaced by $\vec{B}$

  Longitudinal (κ_L) and Transverse (κ_T) diffusion coefficients.

- **Dynamic limit :** $M \gtrsim p > (\sqrt{eB}, T)$

  Two anisotropic directions - $\vec{v}$ and $\vec{B}$

  Case 1 : $\vec{v} \parallel \vec{B}$ **Diffusion coefficients** $\rightarrow \kappa_L,\ \kappa_T$

  Case 2 : $\vec{v} \perp \vec{B}$ **Diffusion coefficients** $\rightarrow \kappa_1,\ \kappa_2,\ \kappa_3$

Scattering / Interaction rate

$$\kappa_i(p) = \int d^3q \frac{d\ \Gamma(v)}{d^3q}\ q_i^2$$

- $2 \leftrightarrow 2$ **scattering processes in a finite temperature medium**

$$qH \to qH \text{ and } gH \to gH \ (q \to \text{quark}, g \to \text{gluon and } H \to \text{HQ}).$$

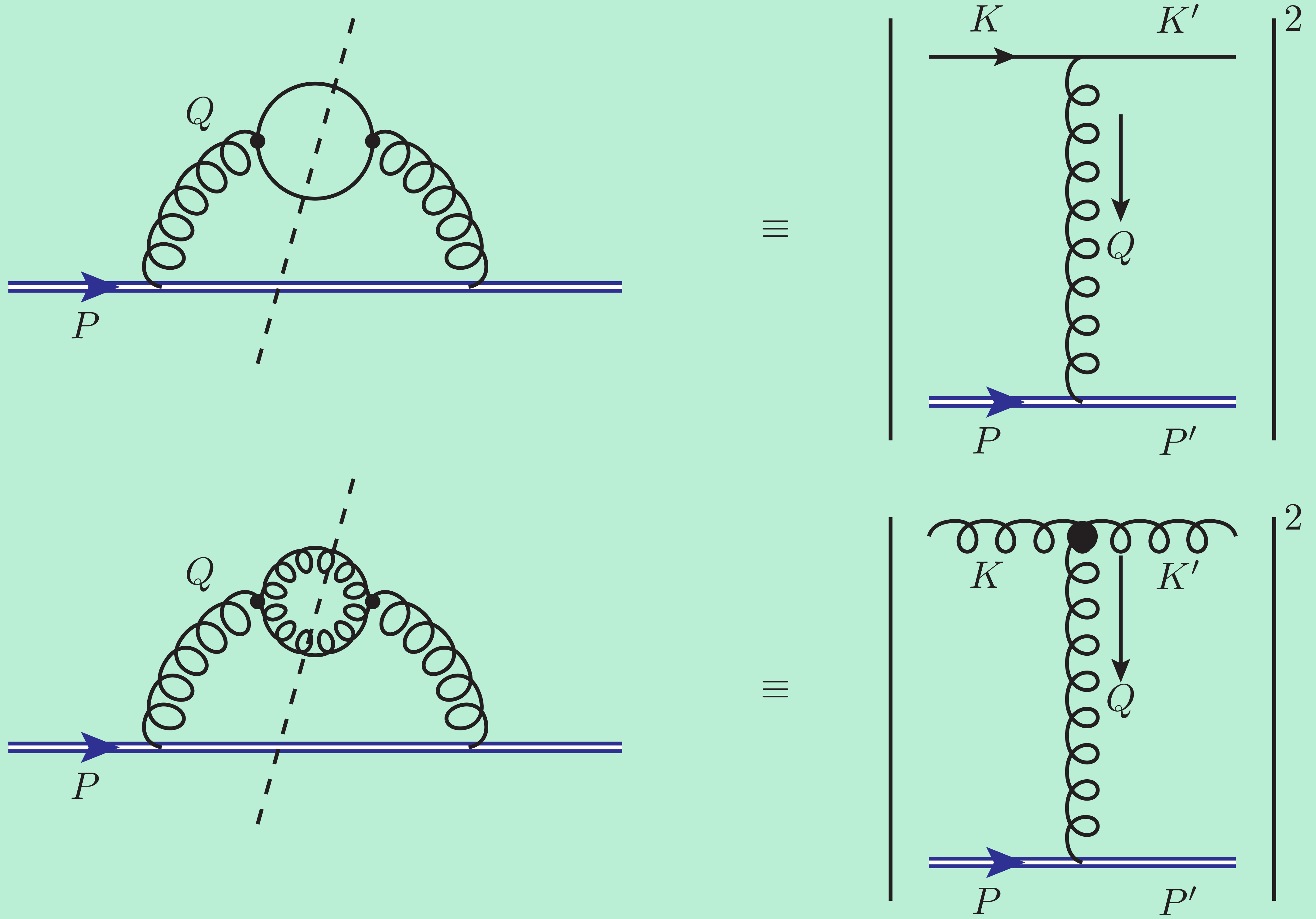
- **At leading order in strong coupling, these processes are dominated by t-channel gluon exchange. (Compton scattering is suppressed by a factor $Q^2/PK \equiv T/M$)**

Scattering / Interaction rate

An effective way of expressing Γ is in terms of the cut/imaginary part of the HQ self energy $\Sigma(P)$

$$\Gamma(P \equiv E, \mathbf{v}) = -\frac{1}{2E}\,\frac{1}{1+e^{-E/T}}\mathrm{Tr}\left[\left(\gamma_\mu P^\mu + M\right)\,\mathrm{Im}\,\Sigma(p_0 + i\epsilon, \mathbf{p})\right]$$

Weldon,
Phys Rev D 28, 1983.



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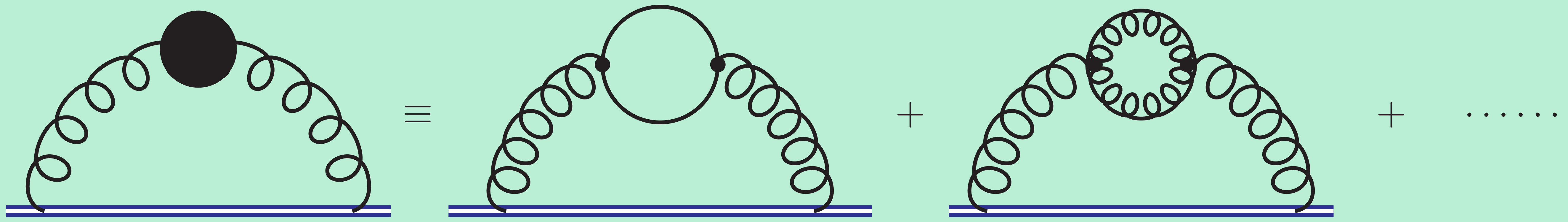

$$\Sigma(P) = ig^2 \int \frac{d^4Q}{(2\pi)^4} \mathscr{D}^{\mu\nu}(Q)\ \gamma_\mu\ S^s_m(P-Q)\ \gamma_\nu$$

Scattering / Interaction rate

$$\Gamma(P \equiv E, \mathbf{v}) = -\frac{1}{2E}\frac{1}{1+e^{-E/T}}\mathrm{Tr}\left[(\gamma_\mu P^\mu + M)\ \mathrm{Im}\ \Sigma(p_0 + i\epsilon, \mathbf{p})\right]$$

$$\Sigma(P) = ig^2 \int \frac{d^4Q}{(2\pi)^4} \mathscr{D}^{\mu\nu}(Q)\ \gamma_\mu\ S_m(P-Q)\ \gamma_\nu$$

$$S_m(K) = e^{-\frac{k_\perp^2}{|q_f B|}} \sum_{l=0}^{\infty} \frac{(-1)^l D_l(q_f B, K)}{K_\parallel^2 - M^2 - 2l q_f B},$$

- Schwinger, Phys Rev 82, 1951.
- Gusynin, Miransky, Shovkovy Nucl Phys B 462, 1996.

$$D_l(q_f B, K) = (\gamma_\mu K_\parallel^\mu + M)\left((1 - i\gamma^1\gamma^2)L_l\left(\frac{2k_\perp^2}{q_f B}\right) - (1 + i\gamma^1\gamma^2)L_{l-1}\left(\frac{2k_\perp^2}{q_f B}\right)\right) - 4(\gamma\cdot k)_\perp L_{l-1}^1\left(\frac{2k_\perp^2}{q_f B}\right)$$

For lowest Landau level (LLL), $l = 0$:

$$S_m^{LLL}(K) = e^{-\frac{k_\perp^2}{|q_f B|}} \frac{(\gamma_\mu K_\parallel^\mu + M)}{K_\parallel^2 - M^2}(1 - i\gamma^1\gamma^2)$$

Scattering / Interaction rate

$$\Gamma(P \equiv E, \mathbf{v}) = -\frac{1}{2E}\frac{1}{1+e^{-E/T}}\mathrm{Tr}\left[(\gamma_\mu P^\mu + M)\ \mathrm{Im}\ \Sigma(p_0 + i\epsilon, \mathbf{p})\right]$$

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- **Hattori, Ikatora, Ann Phys 330, 2013.**
- **Chao, Yu, Huang Phys Rev D 90, 2014.**
- **Mueller, Bonnet, Fischer Phys Rev D 89, 2014.**
- **Ayala, Castaño-Yepes, Dominguez, Hernández-Ortiz, Hernández, Loewe, Manreza Paret, Zamora Rev Mex Fis 66, 2020.**
- **Ayala, Castaño-Yepes, Hernández, Martín, Zamora EPJA 57, 2021.**
- **Ayala, Castaño-Yepes, Loewe, Muñoz, Phys Rev D 104, 2021.**

Scattering / Interaction rate

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Karmakar, AB, Haque, Mustafa : 1804.11336

$$\mathscr{D}^{\mu\nu}(Q) = \frac{\xi Q^\mu Q^\nu}{Q^4} + \frac{(Q^2-d_3)\Delta_1^{\mu\nu}}{(Q^2-d_1)(Q^2-d_3)-d_4^2} + \frac{\Delta_2^{\mu\nu}}{Q^2-d_2} + \frac{(Q^2-d_1)\Delta_3^{\mu\nu}}{(Q^2-d_1)(Q^2-d_3)-d_4^2} + \frac{d_4\Delta_4^{\mu\nu}}{(Q^2-d_1)(Q^2-d_3)-d_4^2}$$

$$\Delta_1^{\mu\nu} = \frac{1}{\bar{u}^2}\bar{u}^\mu\bar{u}^\nu, \quad \Delta_2^{\mu\nu} = g_\perp - \frac{Q_\perp^\mu Q_\perp^\nu}{Q_\perp^2}, \quad \Delta_3^{\mu\nu} = \frac{\bar{n}^\mu\bar{n}^\nu}{\bar{n}^2}, \quad \Delta_4^{\mu\nu} = \frac{\bar{u}^\mu\bar{n}^\nu + \bar{u}^\nu\bar{n}^\mu}{\sqrt{\bar{u}^2}\sqrt{\bar{n}^2}},$$

Scattering / Interaction rate

$$\Gamma(P \equiv E, \mathbf{v}) = -\frac{1}{2E}\,\frac{1}{1+e^{-E/T}}\mathrm{Tr}\left[(\gamma_\mu P^\mu + M)\ \mathrm{Im}\ \Sigma(p_0 + i\epsilon, \mathbf{p})\right]$$

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$$\bar{u}^\mu = u^\mu - \frac{q_0 Q^\mu}{Q^2}, \quad \bar{n}^\mu = n^\mu - \frac{q_3 Q^\mu}{q^2} + \frac{q_0 q_3 u^\mu}{q^2}$$

Form factors

$$\mathscr{D}^{\mu\nu}(Q) = \frac{\xi Q^\mu Q^\nu}{Q^4} + \frac{(Q^2 - d_3)\Delta_1^{\mu\nu}}{(Q^2 - d_1)(Q^2 - d_3) - d_4^2} + \frac{\Delta_2^{\mu\nu}}{Q^2 - d_2} + \frac{(Q^2 - d_1)\Delta_3^{\mu\nu}}{(Q^2 - d_1)(Q^2 - d_3) - d_4^2} + \frac{d_4\Delta_4^{\mu\nu}}{(Q^2 - d_1)(Q^2 - d_3) - d_4^2}$$

$$d_1(Q) = \Delta_1^{\mu\nu}\Pi_{\mu\nu}(Q),\ d_2(Q) = \Delta_2^{\mu\nu}\Pi_{\mu\nu}(Q),\ d_3(Q) = \Delta_3^{\mu\nu}\Pi_{\mu\nu}(Q),\ d_4(Q) = \frac{1}{2}\Delta_4^{\mu\nu}\Pi_{\mu\nu}(Q)$$

$$\Pi^g_{\mu\nu}(Q) = -\frac{N_c g^2 T^2}{3}\int \frac{d\Omega}{4\pi}\left(\frac{q_0 \hat{K}_\mu \hat{K}_\nu}{\hat{K}\cdot Q} - g_{\mu 0} g_{\nu 0}\right)$$

$$\Pi_{\mu\nu} = \Pi^m_{\mu\nu} + \Pi^g_{\mu\nu}$$

$$\Pi^m_{\mu\nu}(Q) = \sum_f \frac{ig^2}{2}\int \frac{d^4K}{(2\pi)^4}\mathbf{Tr}\left\{\gamma_\mu S_m(K)\gamma_\nu S_m(R)\right\}$$

$$= \sum_f \frac{ig^2}{2}\int \frac{d^4K}{(2\pi)^4}\, e^{-\frac{k_\perp^2 + r_\perp^2}{q_f B}} \sum_{l=0}^{\infty}\sum_{l'=0}^{\infty}(-1)^{l+l'}\left(\frac{1}{K_\parallel^2 - m_f^2 - 2lq_fB}\right)\left(\frac{1}{R_\parallel^2 - m_f^2 - 2l'q_fB}\right)$$

$$\mathbf{Tr}\left\{\gamma_\mu\ D_l(q_fB, K)\ \gamma_\nu\ D_{l'}(q_fB, R)\right\}$$

$B \neq 0$, static limit result

$$\kappa_L^{(s)} = \sum_{l=0}^{\infty} \frac{(-1)^l\, 2\pi g^2 TM}{\sqrt{M^2 + 2l|q_f B|}} \int \frac{d^3q}{(2\pi)^3} q_3^2 e^{-q_\perp^2/|q_f B|} \left[\frac{(m_D^g)^2 (L_l(\xi_q^\perp) - L_{l-1}(\xi_q^\perp))}{2q(q^2 + (m_D')^2)^2} \right]; \quad \xi_q^\perp = \frac{2q_\perp^2}{q_f B}$$

$$\kappa_T^{(s)} = \sum_{l=0}^{\infty} \frac{(-1)^l \pi g^2 TM}{\sqrt{M^2 + 2l|q_f B|}} \int \frac{d^3q}{(2\pi)^3} q_\perp^2 e^{-q_\perp^2/|q_f B|} \left[\frac{\left(\frac{1}{q}(m_D^g)^2 + \delta(q_3) \sum_f \delta m_{D,f}^2\right)(L_l(\xi_q^\perp) - L_{l-1}(\xi_q^\perp))}{2(q^2 + (m_D')^2)^2} \right]$$

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- **What happens when one approximates by just modifying the Debye mass ?**

$B \neq 0$, static limit result

$$\kappa_L^{(s)} = \sum_{l=0}^{\infty} \frac{(-1)^l\, 2\pi g^2 TM}{\sqrt{M^2 + 2l\,|\,q_f B\,|}} \int \frac{d^3q}{(2\pi)^3} q_3^2 e^{-q_\perp^2/|q_f B|} \left[\frac{(m_D^g)^2 (L_l(\xi_q^\perp) - L_{l-1}(\xi_q^\perp))}{2q(q^2 + (m_D')^2)^2} \right]; \quad \xi_q^\perp = \frac{2q_\perp^2}{q_f B}$$

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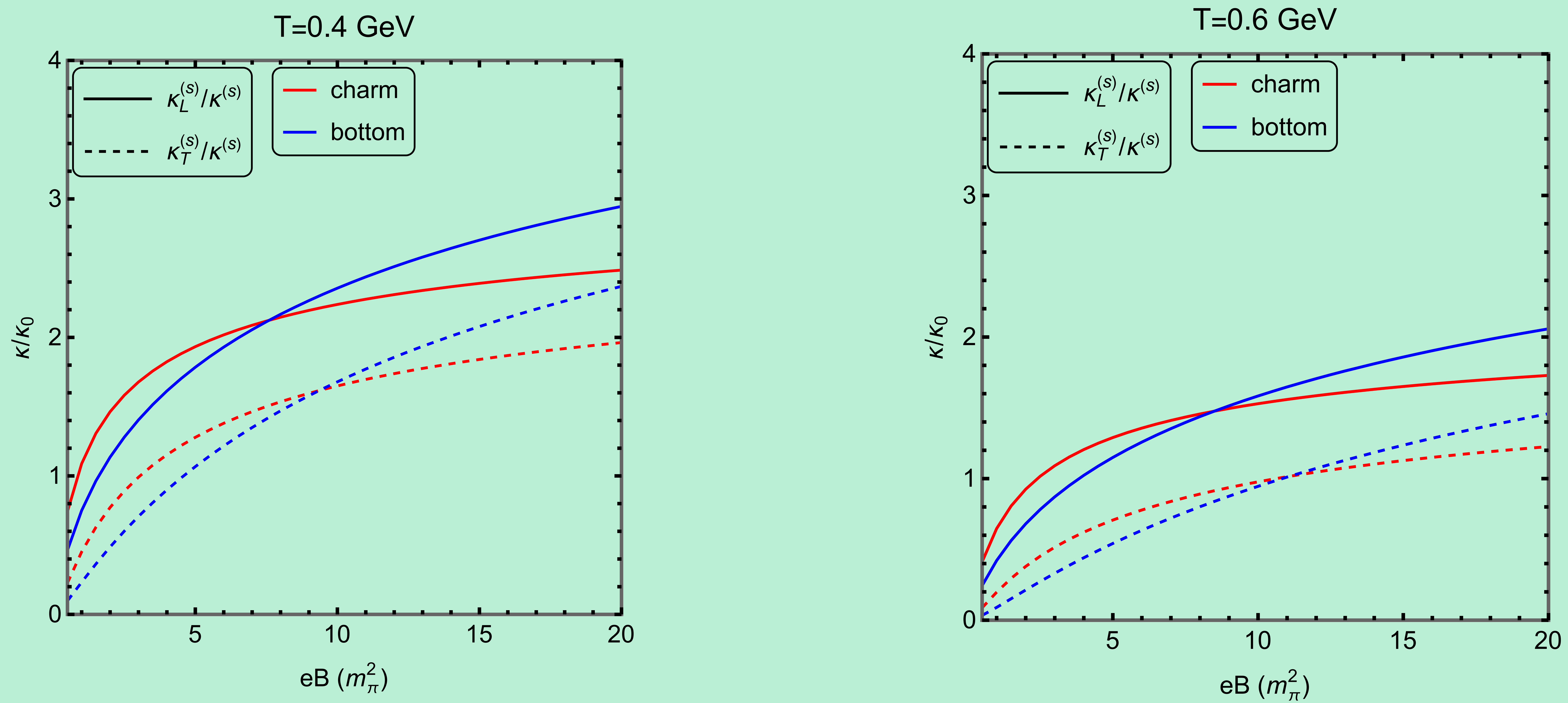
- **What happens when one approximates by just modifying the Debye mass ?**

$$\kappa_L^{(s)'} = 2\pi g^2 T \int \frac{d^3q}{(2\pi)^3} \left[\frac{q_3^2 (m_D')^2}{2q(q^2 + (m_D')^2)^2} \right],$$

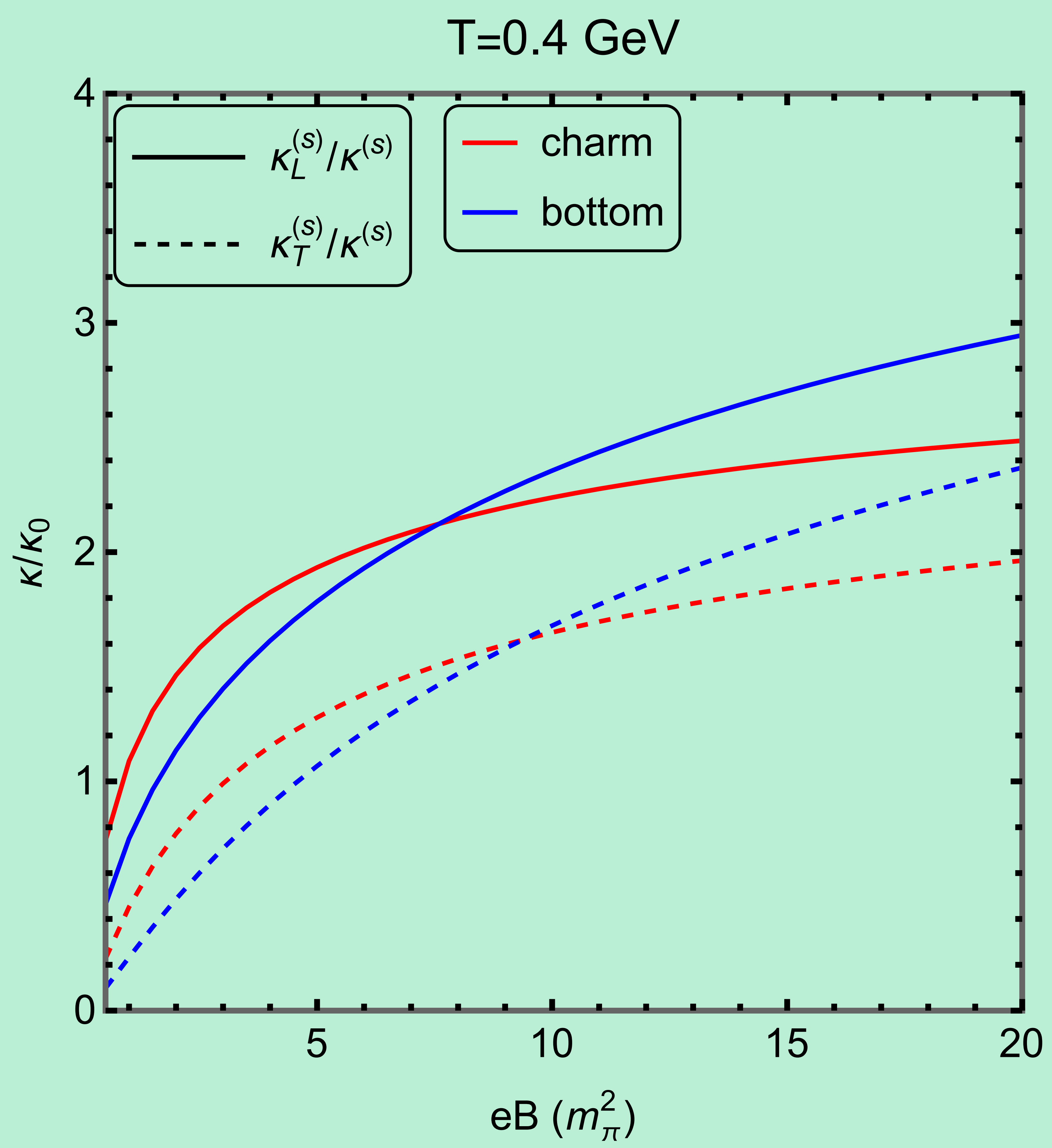
$$\kappa_T^{(s)'} = \pi g^2 T \int \frac{d^3q}{(2\pi)^3} \left[\frac{q_\perp^2 (m_D')^2}{2q(q^2 + (m_D')^2)^2} \right].$$

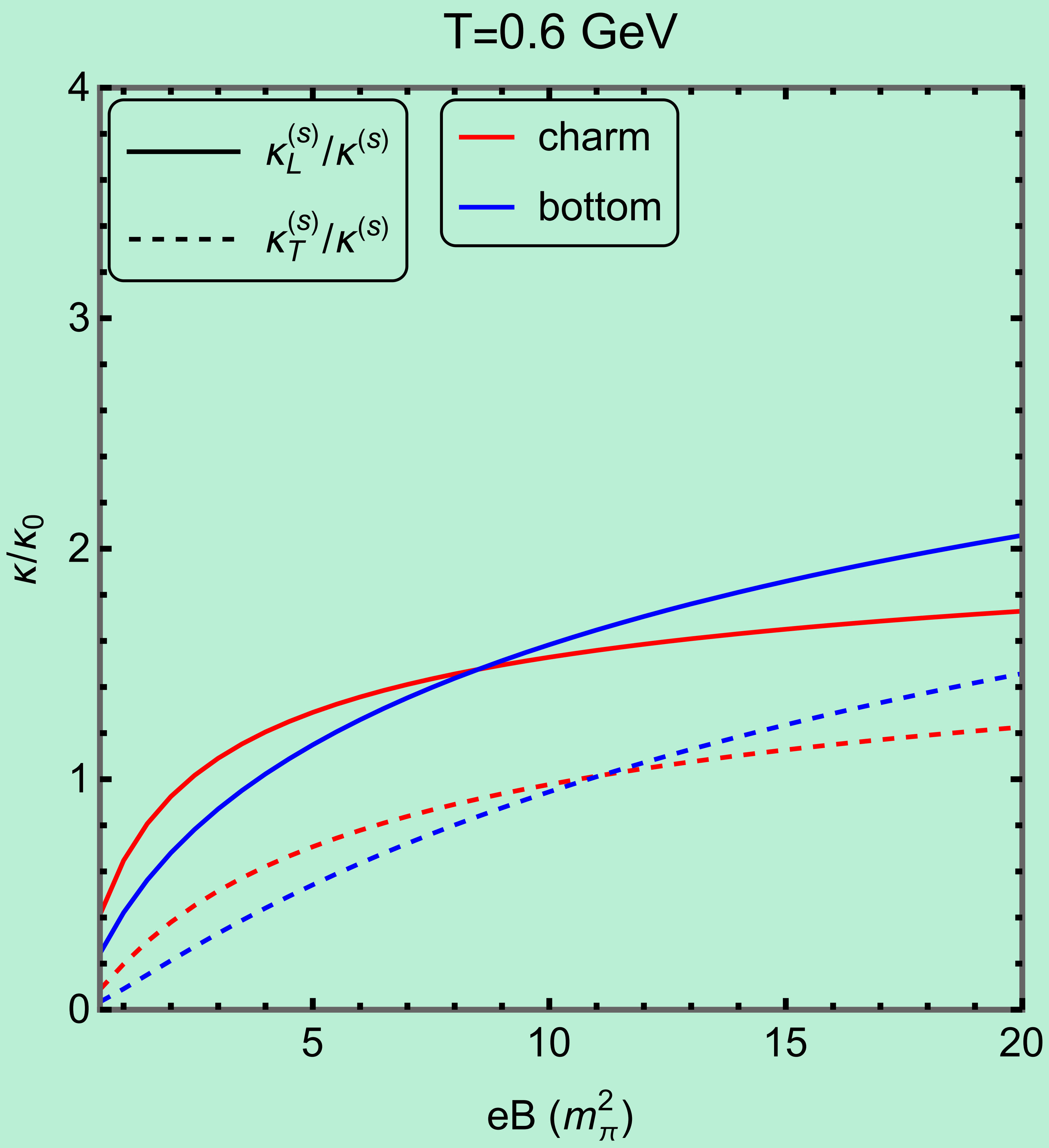
$B \neq 0$, static limit result



The magnetised medium modified exact results (κ) has been scaled with respect to the $eB = 0$ result (κ_0), variation of which with respect to eB has been shown for longitudinal (solid lines) and transverse (dashed lines) HQ momentum diffusion coefficients within the static limit of both charm (red curves) and bottom (blue curves) quarks.

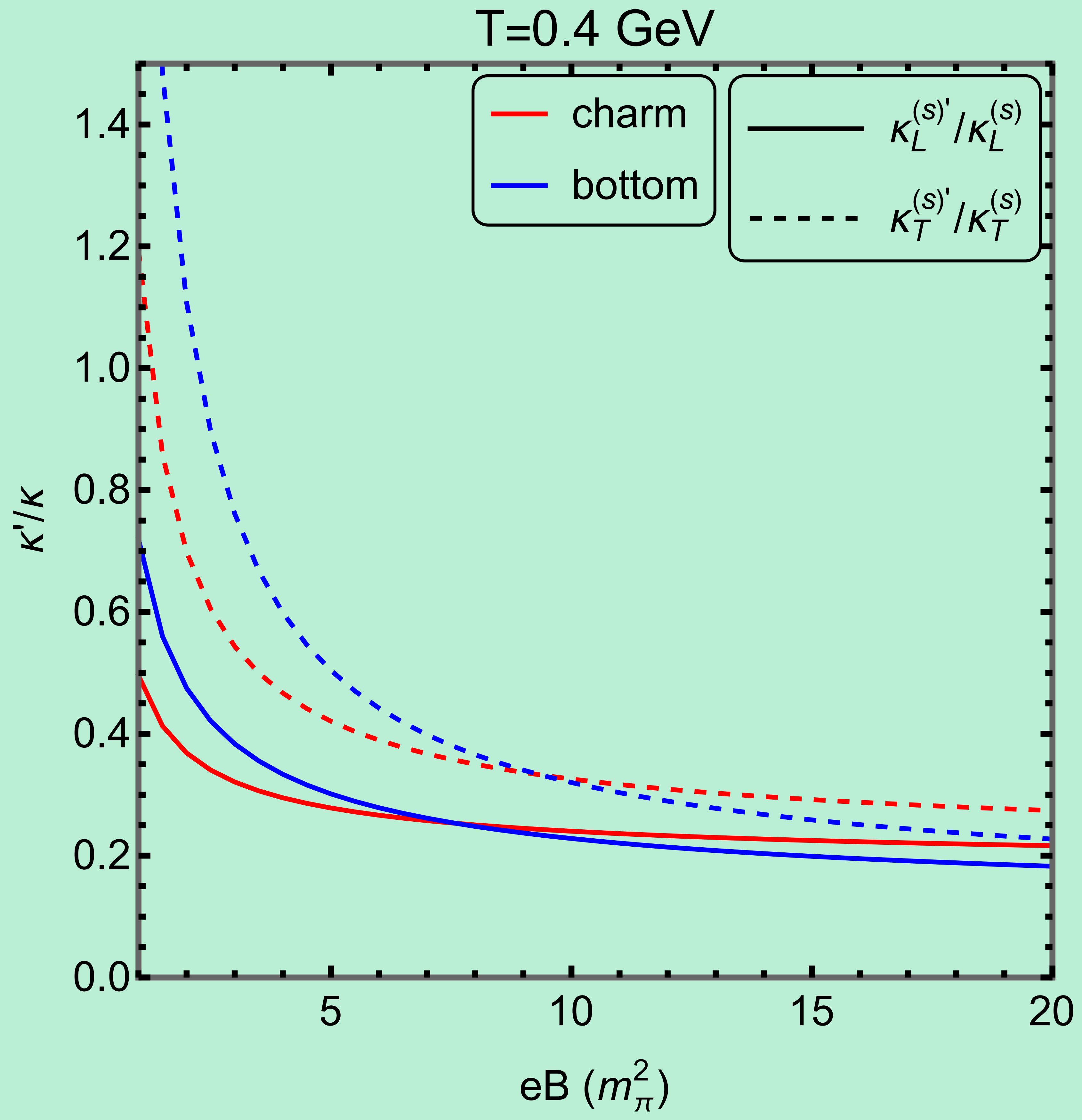
$B \neq 0$, static limit result

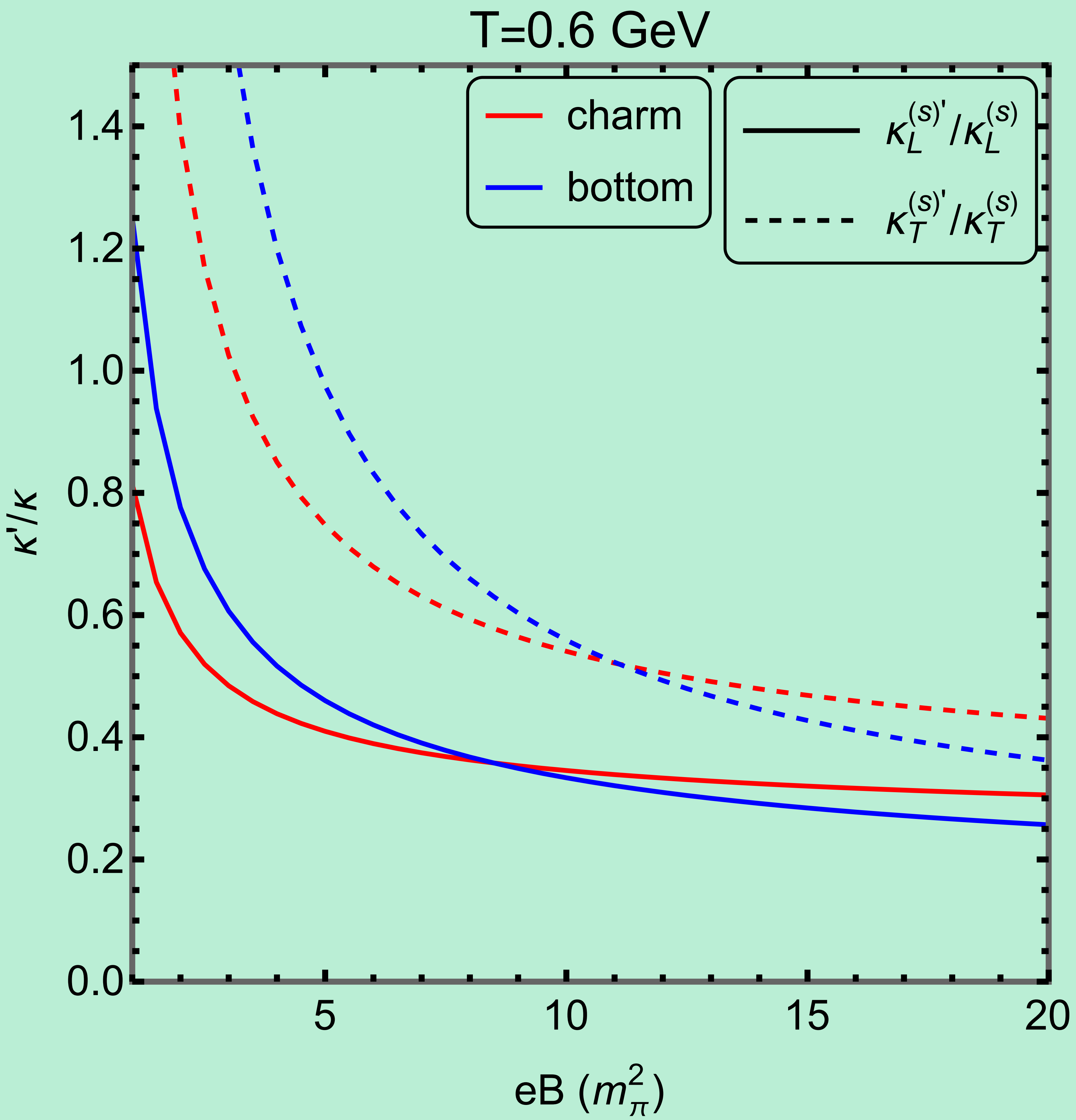




- Rate of increase for $\kappa_L/\kappa_T \rightarrow$ Low $eB >$ High eB. (More evident for charm quarks)
- $\kappa_L > \kappa_T \rightarrow$ dominant gluonic contribution in the t-channel scatterings

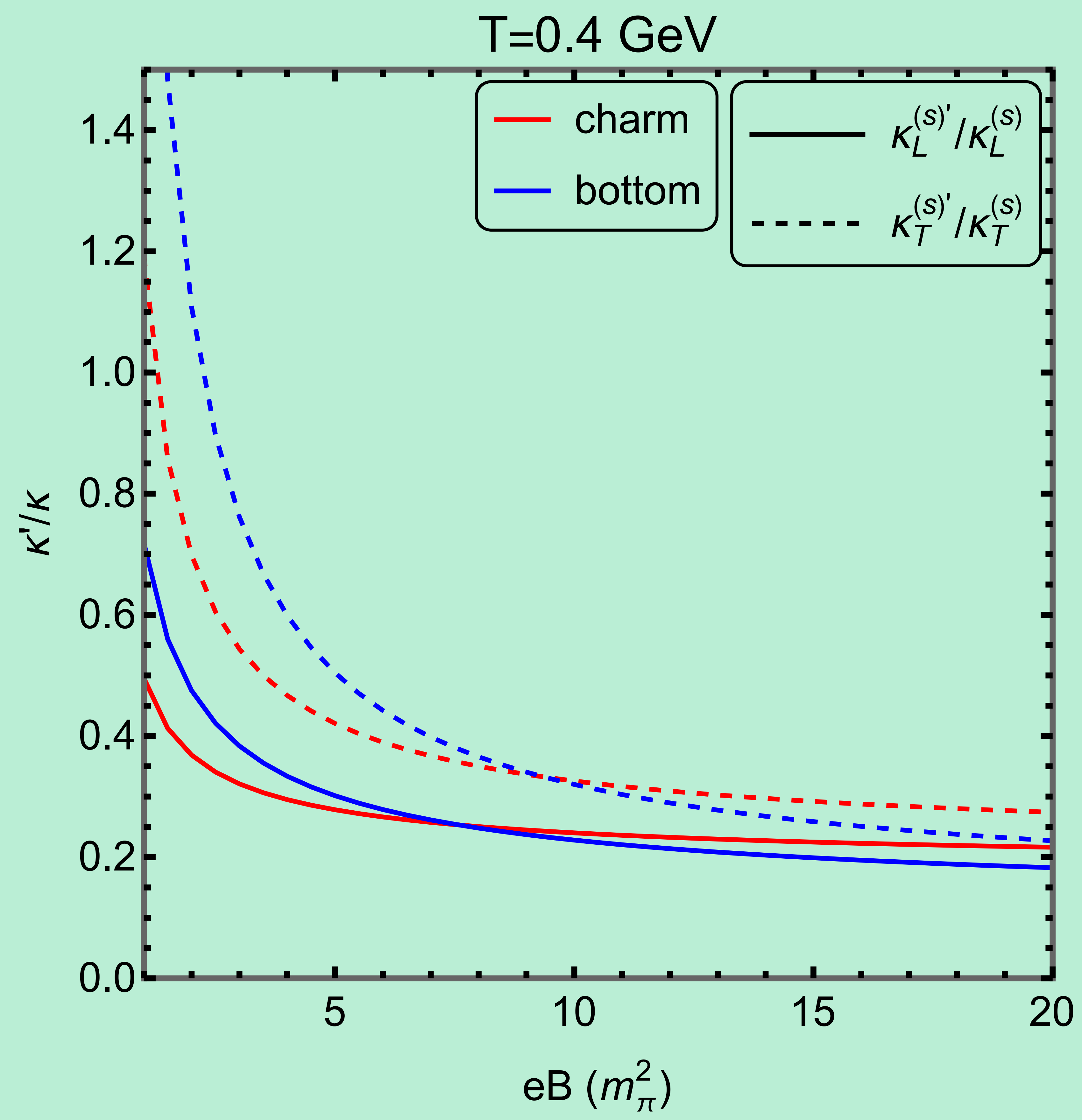
Comparison between two approaches

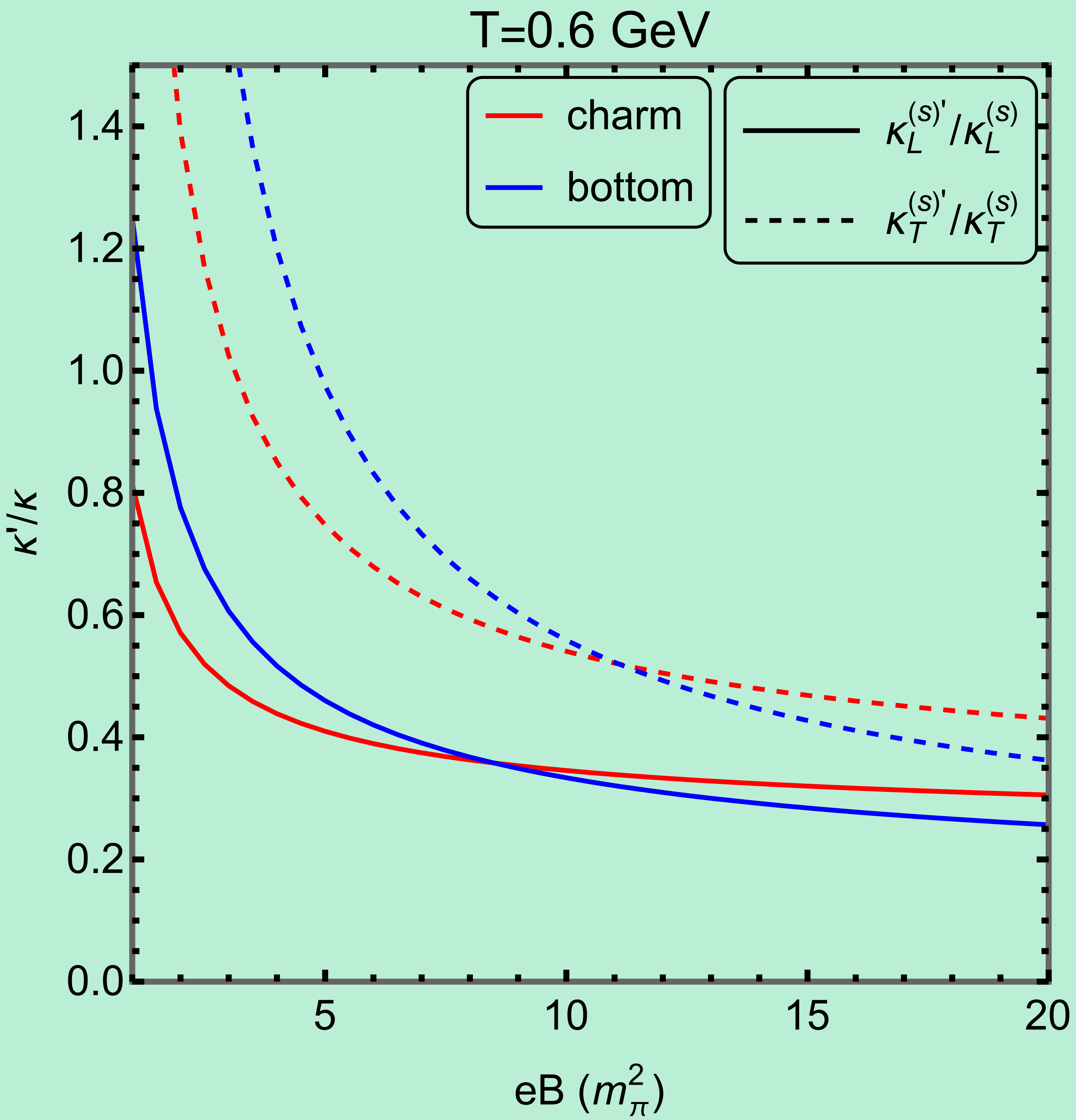




Variation of the ratio between the Debye mass approximated results (κ') and the exact results (κ) with respect to eB has been shown for longitudinal (solid lines) and transverse (dashed lines) HQ momentum diffusion coefficients within the static limit of both charm (red curves) and bottom (blue curves) quarks.

Comparison between two approaches





- Debye mass approximated results underestimate the exact results for larger values of eB and overestimate them for smaller values of eB. (More prominent in the case of bottom quarks)

Estimations beyond the static limit of HQ

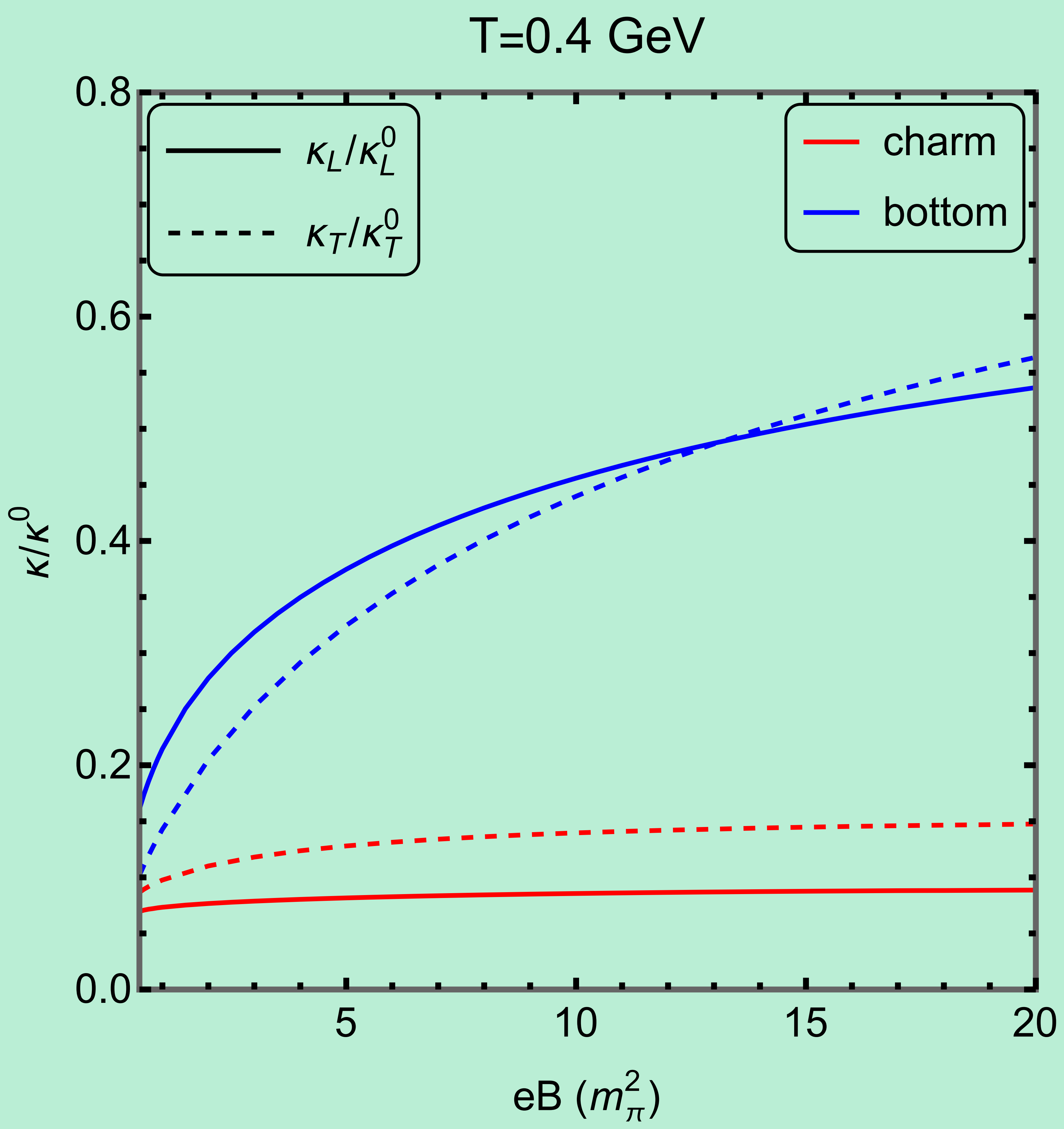
$$M > p \gg T$$

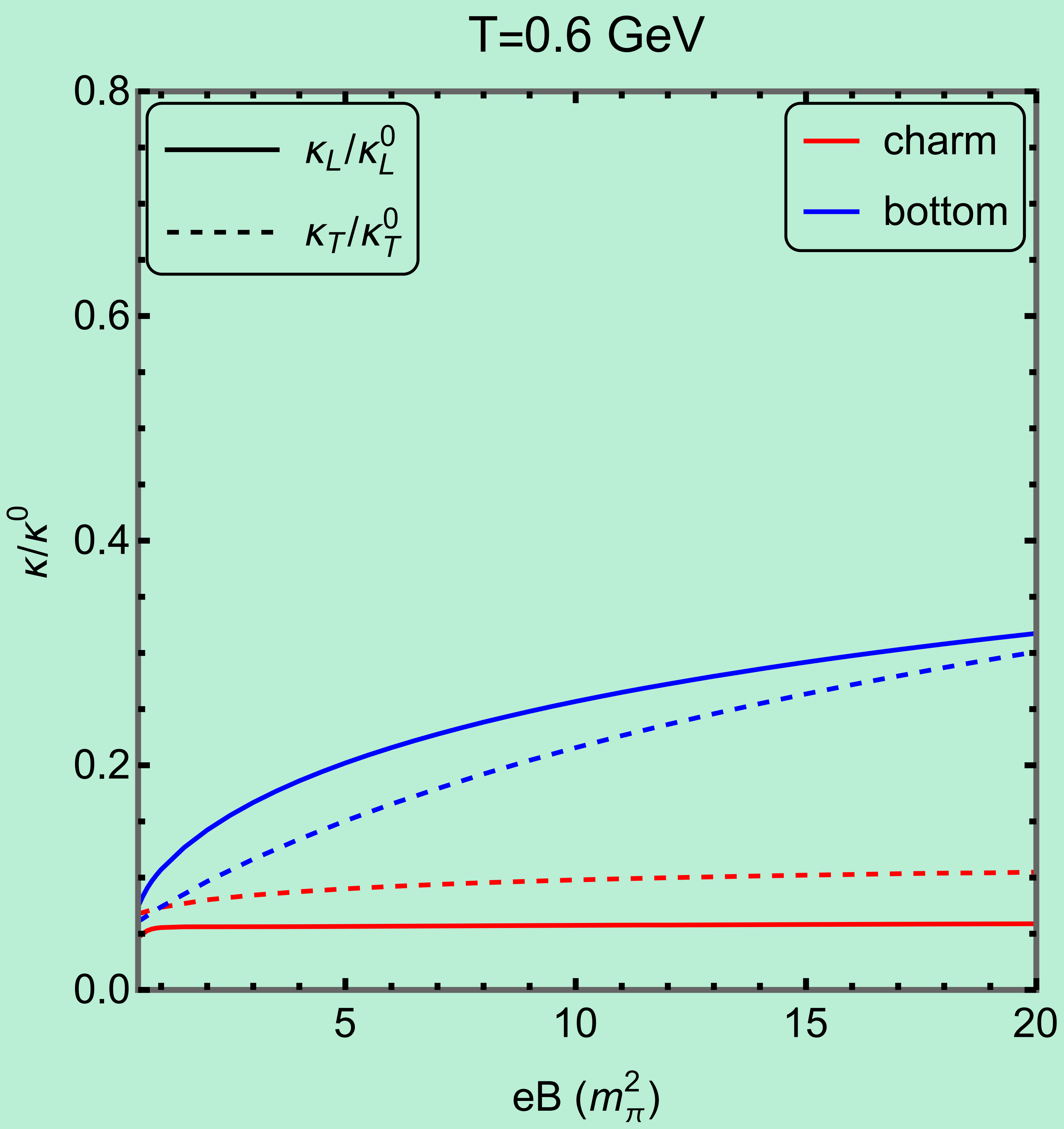
Case 1 : $\overrightarrow{v} \parallel \overrightarrow{B} \quad \rightarrow p_3 \neq 0$

Case 2 : $\overrightarrow{v} \perp \overrightarrow{B} \quad \rightarrow p_\perp = \sqrt{p_1^2 + p_2^2} \neq 0$

$(p_1 \neq 0, \quad p_2 = 0)$

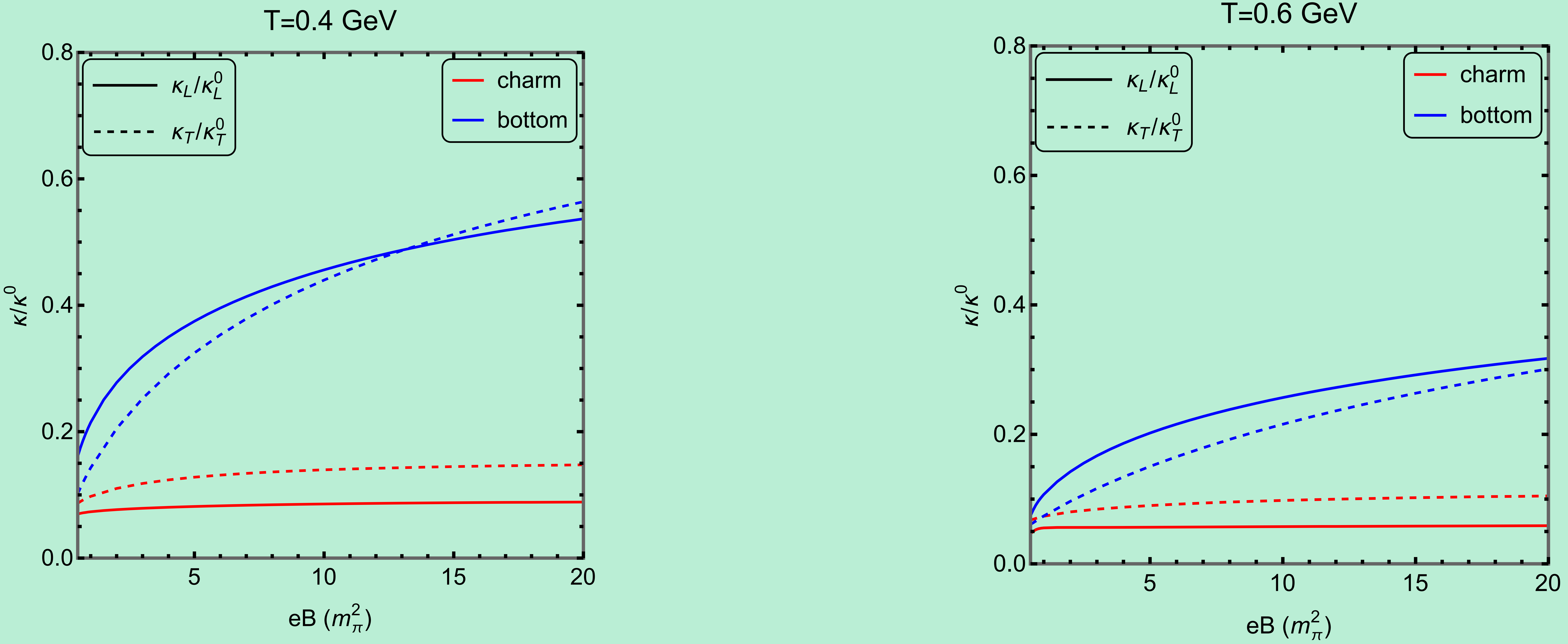
$B \neq 0, \vec{v} \parallel \vec{B}$ case





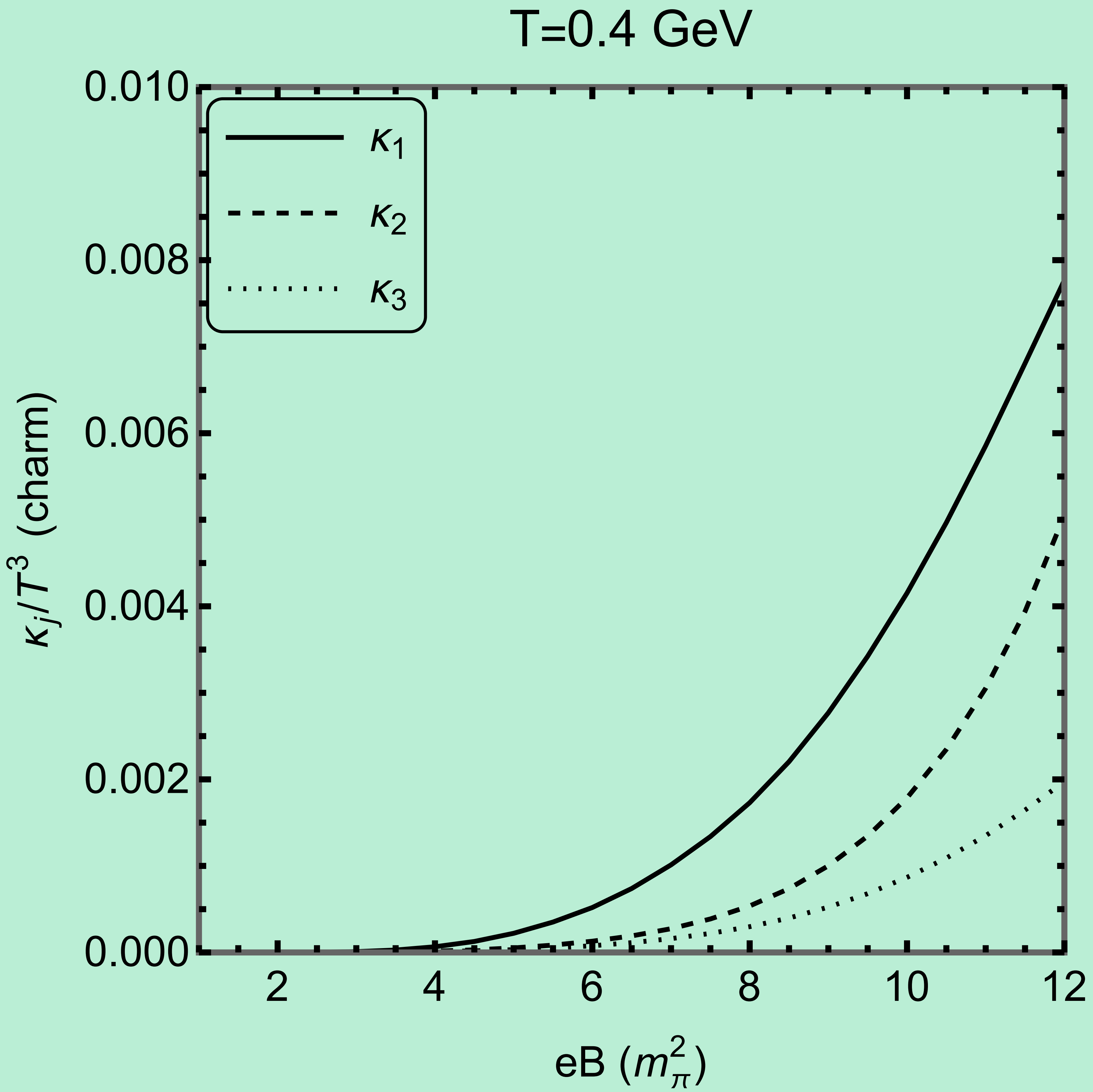
Variation of the longitudinal and transverse momentum diffusion coefficients for charm and bottom quarks with external magnetic field for two different values of temperatures. The magnetized momentum diffusion coefficients are scaled with respect to their eB = 0 counterparts.

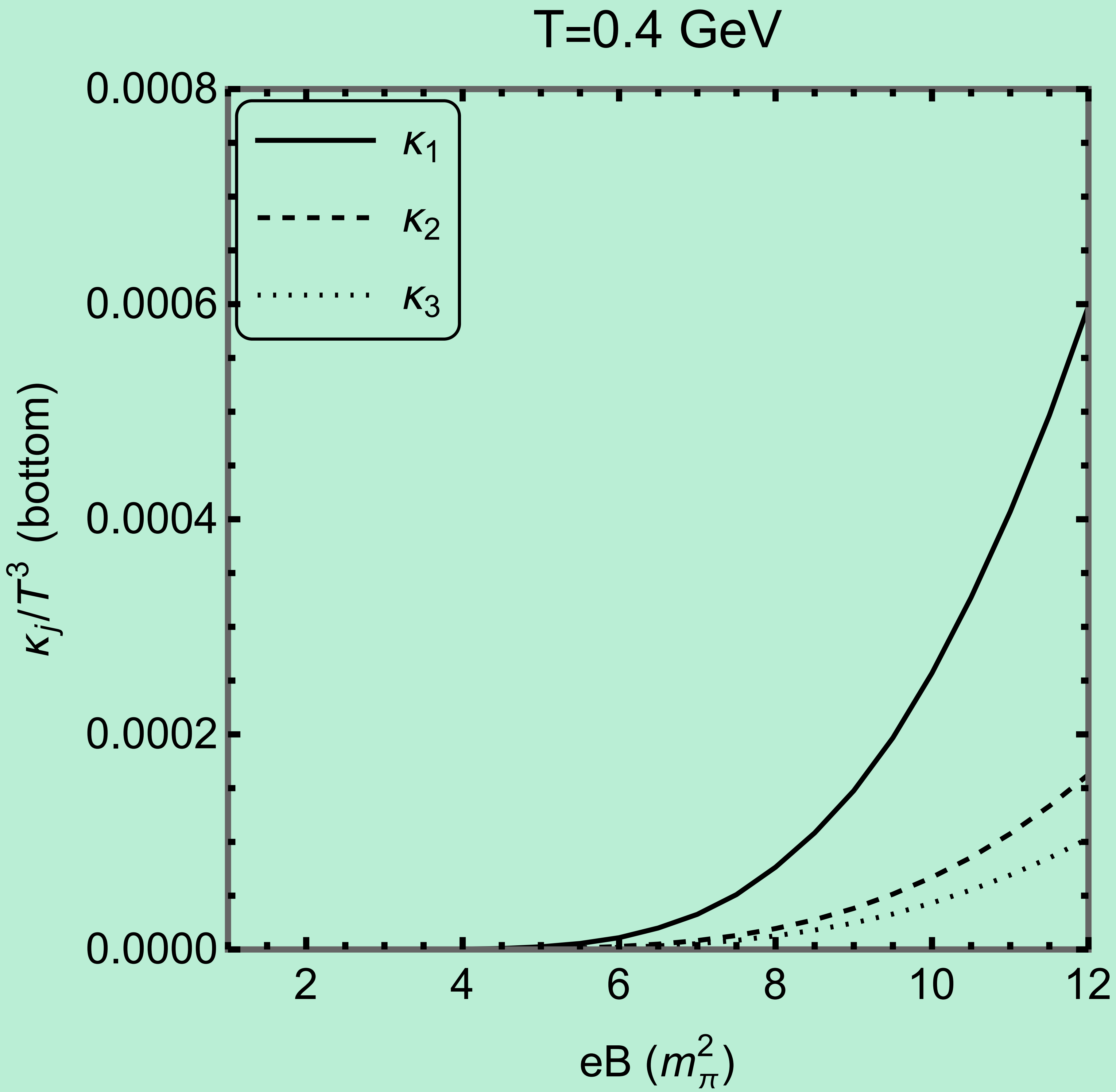
$B \neq 0, \vec{v} \parallel \vec{B}$ case



- Rate of increase for $\kappa_L/\kappa_T \to$ Low $eB >$ High eB. (More evident for charm quarks)
- Crossover reflects the behaviours of competing scales M, T and eB

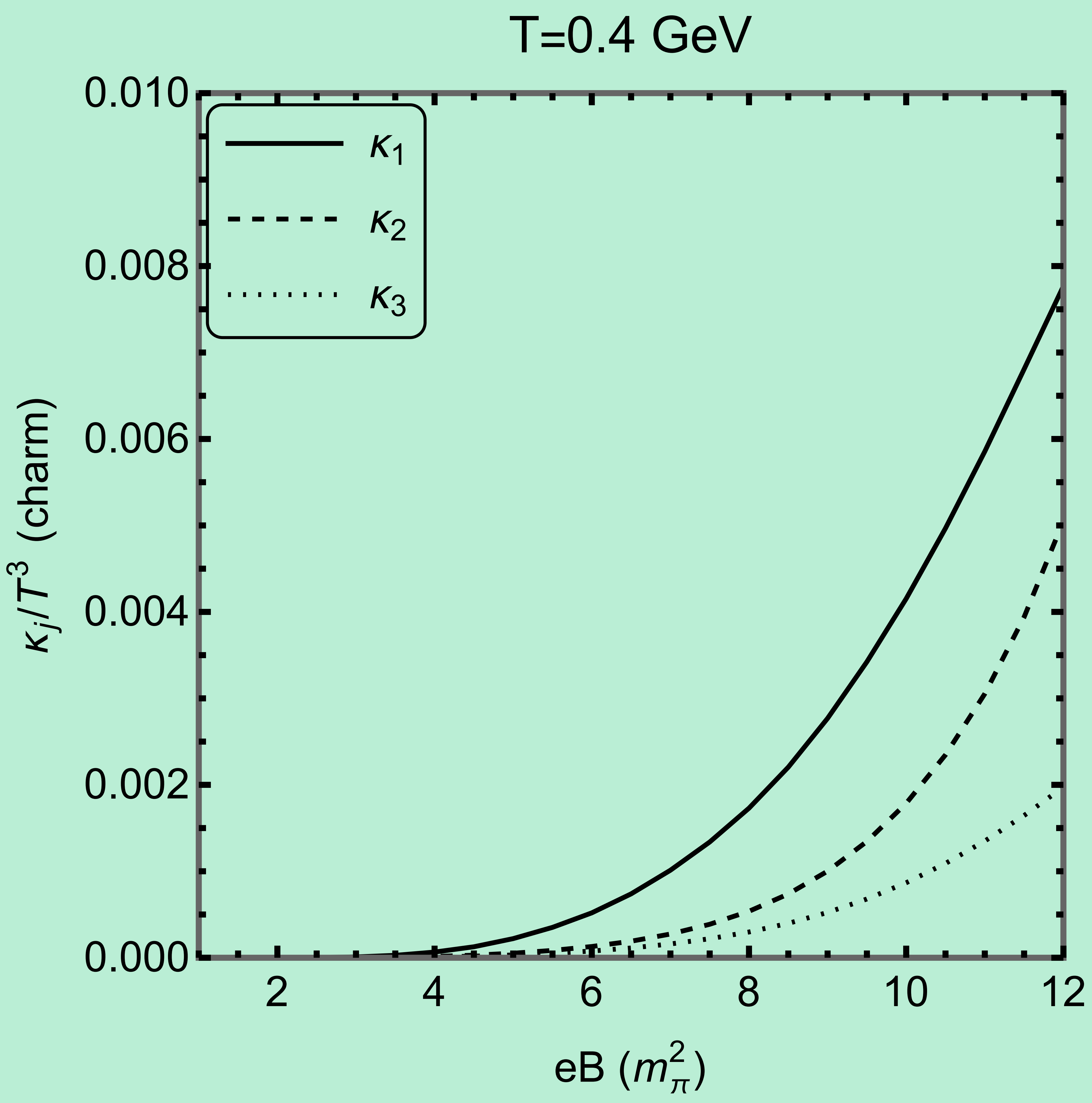
$B \neq 0, \vec{v} \perp \vec{B}$ case

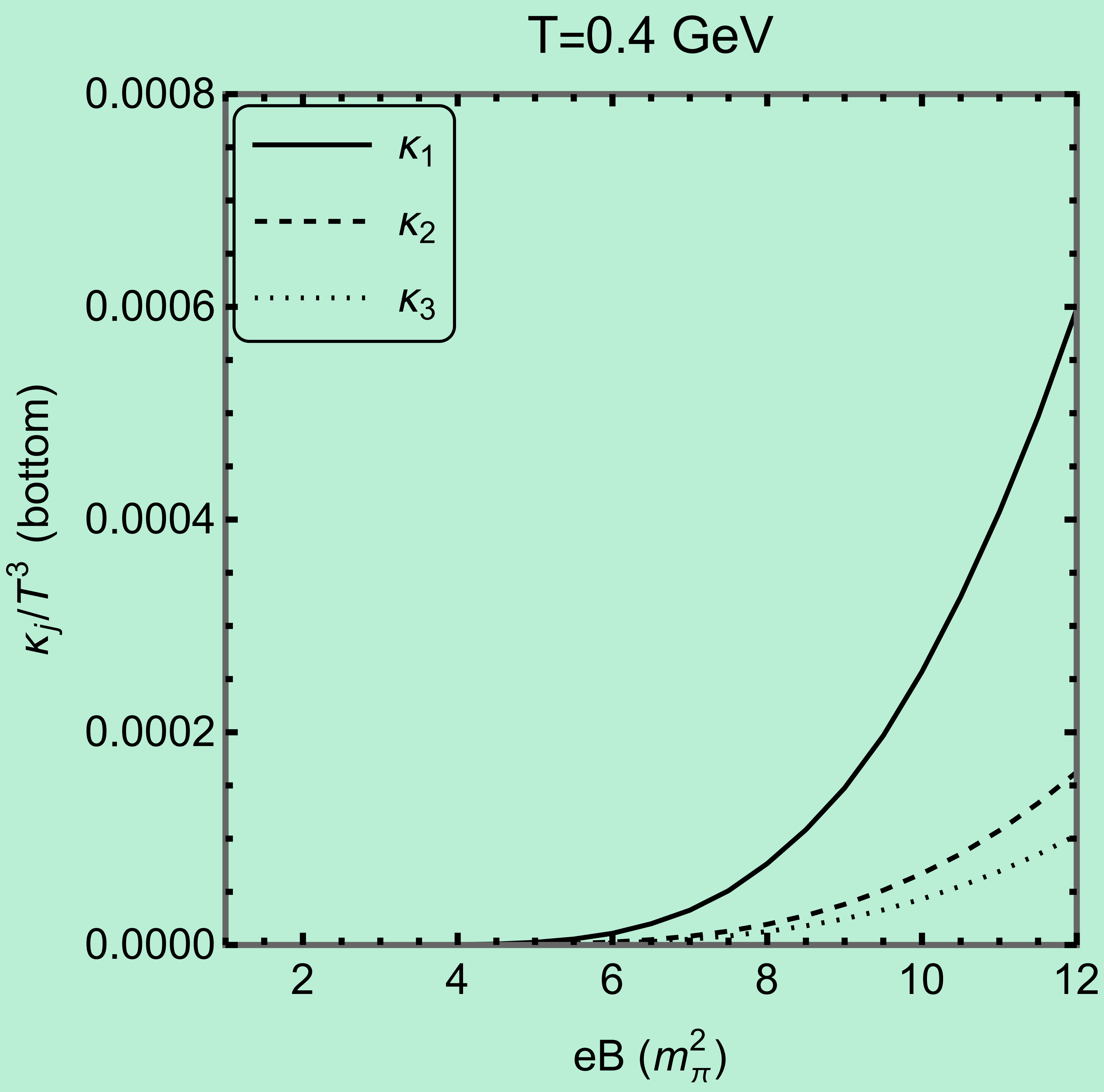




Variation of the transverse components κ1, κ2 and longitudinal component κ3 of the momentum diffusion coefficient for charm and bottom quarks with external magnetic field for a fixed value of temperature. The magnetized momentum diffusion coefficients are scaled with respect to their eB = 0 counterparts.

$B \neq 0$, $\vec{v} \perp \vec{B}$ case





- No saturating behaviours for κ's at higher values of eB, rate of change increases with increasing eB
- Transverse components dominate over the longitudinal component

Summary

- We attempt to study the HQ dynamics with arbitrary values of the external magnetic field.
- We evaluate the form factors for the general structure of the gluon correlation function including all Landau levels for the first time in literature.
- For most of the cases studied, eB dependence of κ is rapidly increasing for lower values of eB, whereas it becomes saturated for relatively higher values of eB.
- Even without the quark contributions, κ_L dominates over κ_T within the static limit of HQ.
- By comparing the results of an alternate approximated procedure with our exact results, we clearly emphasise the importance of employing the general structure of the gluon two-point correlation functions in a hot magnetised medium.

Limitations and future outlook

- **At the cost of getting analytic, gauge independent, simplified expressions, scale restrictions appearing because of the HTL approximation.** $(\alpha_s eB \ll T^2)$

- **Assuming massless quarks leads to vanishing quark contribution to** κ_L**.**

- **UV cut-off** q_{max} **dependence.**

- **Proper inclusion of the hard contribution is needed.**

- **Examining the HQ in-medium evolution and their consequences on elliptic flow.**

Thank you for your attention.