Short-range correlated van der Waals-type model at high density regime

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EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

Strongly interacting matter in extreme magnetic fields

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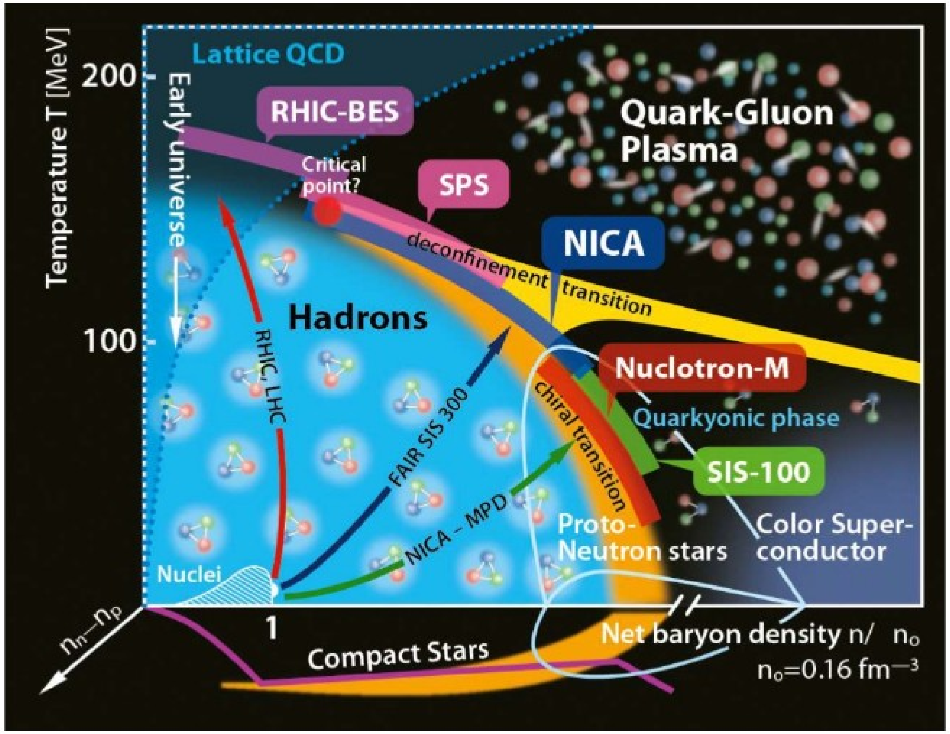
CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

CAPES

Outline

- Motivation
- Effective hadronic models
- van der Waals-type models for nuclear matter
- Short-range correlations (SRC)
- Clausis model with SRC
- Conclusions

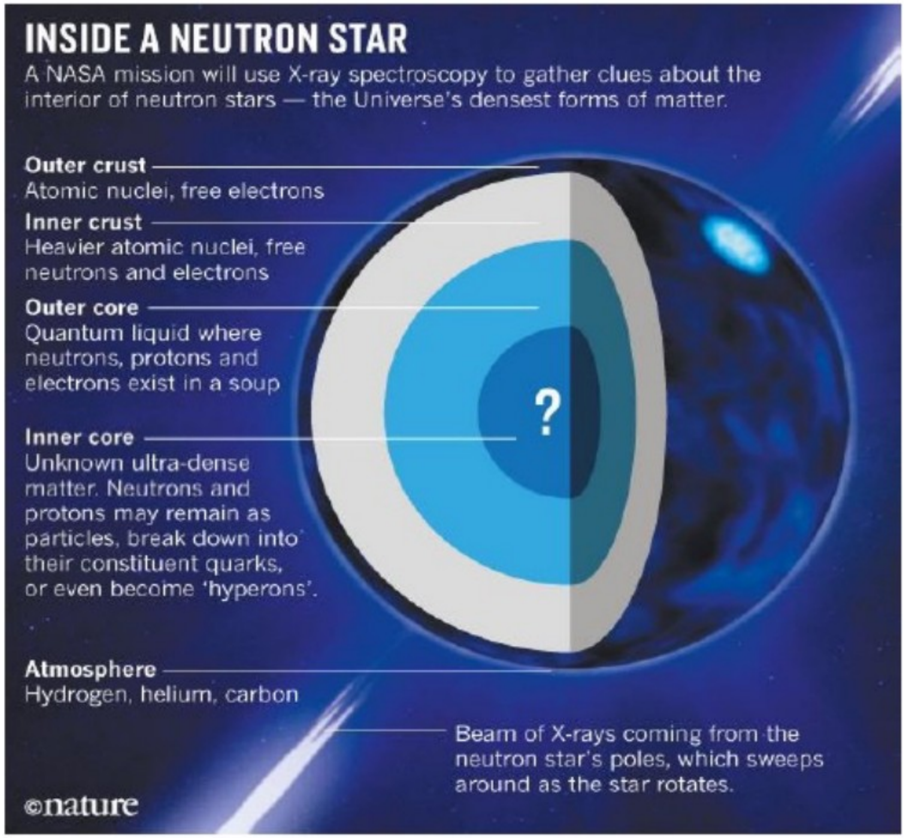
- The strongly interacting matter at extreme conditions is not fully understood (quark-gluon plasma, hyperons, color superconductivity, effect of magnetic fields…)



- No fundamental theories at low temperature and large densities. Effective models are useful in this context.

Motivation

- A natural high density regime: (neutron, quark, hybrid) stars



- One of the densest objects in the universe
- Around 6 to 8 times saturation density, $\rho_0 \approx 0.15 \text{ fm}^{-3} \approx 2.8 \times 10^{14} \text{ g/cm}^3$

- Different options for constructing hadronic models:

- Microscopic models (ab-initio) in which the nucleon-nucleon interaction is the starting point. Free parameters adjusted for reproducing deuteron data, for instance.

- Phenomenological models based on the mean-field approximation. Thermodynamics constructed from known data of infinite nuclear matter and/or finite nuclei.

- Examples:

  - Skyrme model:

$$\begin{aligned} V(\mathbf{r}_1,\mathbf{r}_2) &= t_0(1+x_0P_\sigma)\delta(\mathbf{r}) + \frac{1}{2}t_1(1+x_1P_\sigma)\left[\mathbf{k}'^2\delta(\mathbf{r})+\delta(\mathbf{r})\mathbf{k}^2\right] + t_2(1+x_2P_\sigma)\mathbf{k}'\cdot\delta(\mathbf{r})\mathbf{k} \\ &+ \frac{1}{6}t_3(1+x_3P_\sigma)\rho^\alpha(\mathbf{R})\delta(\mathbf{r}) + iW_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)[\mathbf{k}'\times\delta(\mathbf{r})\mathbf{k}] \end{aligned}$$

$$\mathbf{r} = \mathbf{r}_1-\mathbf{r}_2 \qquad P_\sigma = (1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)/2 \qquad \mathbf{R} = (\mathbf{r}_1+\mathbf{r}_2)/2 \qquad \mathbf{k} = (\vec{\nabla}_1-\vec{\nabla}_2)/2i$$

$$\Downarrow$$

$$\frac{\mathcal{E}}{\rho} = \frac{3\hbar^2}{10m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{2/3} + \frac{3}{8}t_0\rho + \frac{3}{80}\left[3t_1+(5+4x_2)t_2\right]\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{5/3} + \frac{1}{16}t_3\rho^{\alpha+1}$$

- Gogny model:

$$V(\mathbf{r}_1,\mathbf{r}_2) = \sum_{i=1,2} \left(W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau\right) e^{-r^2/\mu_i^2} + t_3\left(1 + x_3 P_\sigma\right)\rho^\alpha(\mathbf{R})\delta(\mathbf{r})$$
$$+ \; iW_0\left(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2\right)\left[\mathbf{k}' \times \delta(\mathbf{r})\mathbf{k}\right]$$

↓

$$\frac{\mathcal{E}}{\rho} = \frac{3\hbar^2}{20m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{2/3}\left[(1+\delta)^{5/3} + (1-\delta)^{5/3}\right] + \frac{1}{8}t_3\rho^{\alpha+1}\left[3 - (2x_3+1)\delta^2\right] + \frac{1}{2}\sum_{i=1,2}\mu_i^3\pi^{3/2}\rho\left[\mathcal{A}_i + \mathcal{B}_i\delta^2\right]$$
$$- \sum_{\mathrm{i}=1,2}\frac{1}{2k_F^3\mu_i^3}\left\{\mathcal{C}_i\left[\mathsf{e}(k_{Fn}\mu_i) + \mathsf{e}(k_{Fp}\mu_i)\right] - \mathcal{D}_i\bar{\mathsf{e}}(k_{Fn}\mu_i, k_{Fp}\mu_i)\right\}$$

$$\mathsf{e}(\eta) = \frac{\sqrt{\pi}}{2}\eta^3\mathrm{erf}(\eta) + \left(\frac{\eta^2}{2} - 1\right)e^{-\eta^2} - \frac{3\eta^2}{2} + 1\,, \qquad \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$$

- Relativistic mean-field models:

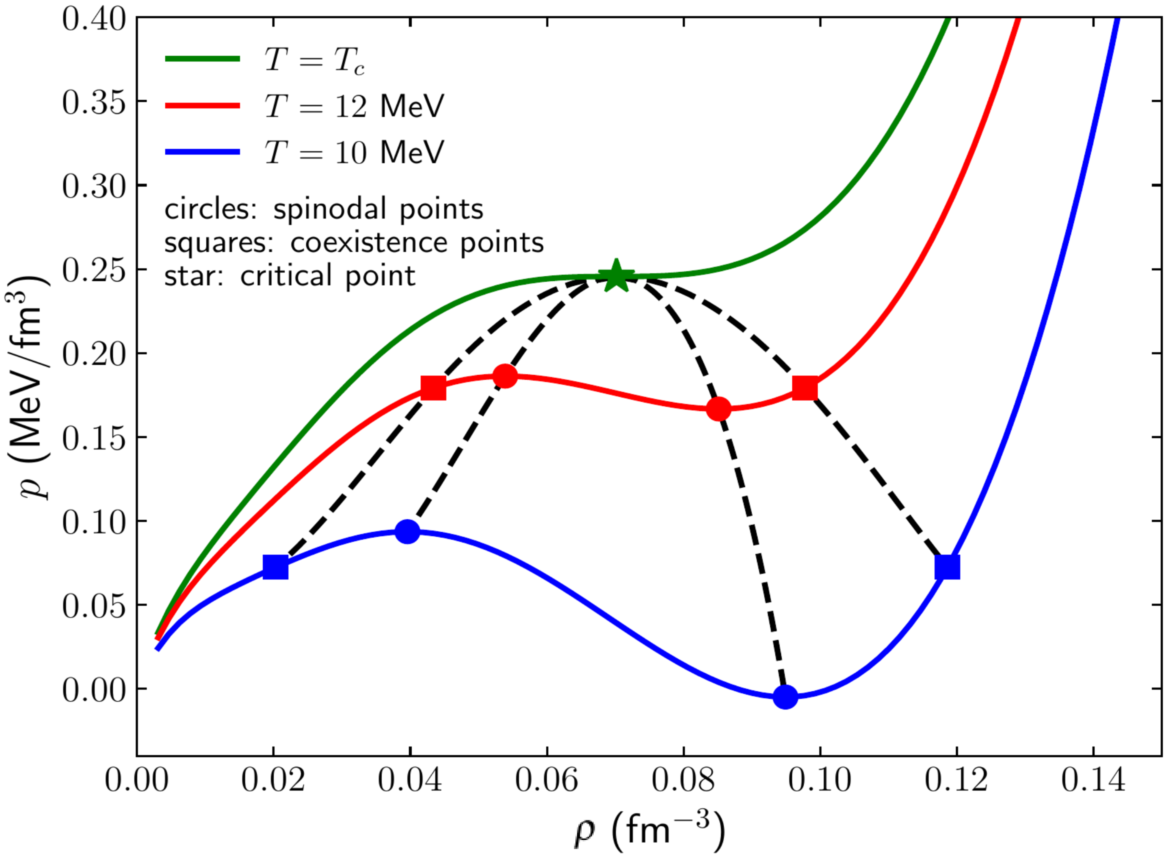
$$\mathcal{L} = \mathcal{L}_{nm} + \mathcal{L}_{\sigma} + \mathcal{L}_{\omega} + \mathcal{L}_{\rho} + \mathcal{L}_{\sigma\omega\rho},$$

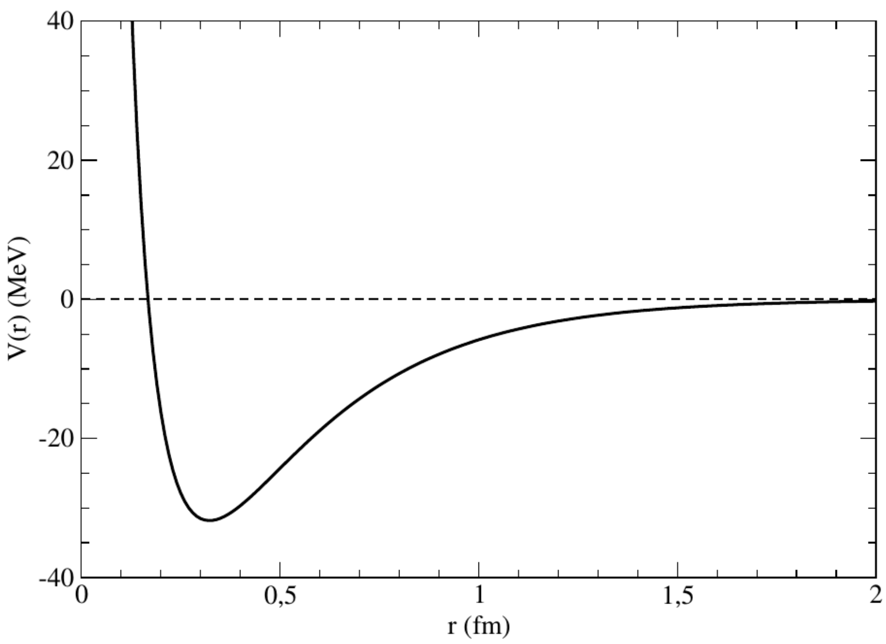
$$\begin{aligned}
\mathcal{L}_{nm} &= \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - M)\psi - g_{\sigma}\sigma\overline{\psi}\psi - g_{\omega}\overline{\psi}\gamma^{\mu}\omega_{\mu}\psi - \frac{g_{\rho}}{2}\overline{\psi}\gamma^{\mu}\vec{\rho}_{\mu}\vec{\tau}\psi, \\
\mathcal{L}_{\sigma} &= \frac{1}{2}(\partial^{\mu}\sigma\partial_{\mu}\sigma - m_{\sigma}^{2}\sigma^{2}) - \frac{A}{3}\sigma^{3} - \frac{B}{4}\sigma^{4}, \\
\mathcal{L}_{\omega} &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_{\omega}^{2}\omega_{\mu}\omega^{\mu} + \frac{c}{4}(g_{\omega}^{2}\omega_{\mu}\omega^{\mu})^{2}, \\
\mathcal{L}_{\rho} &= -\frac{1}{4}\vec{B}^{\mu\nu}\vec{B}_{\mu\nu} + \frac{1}{2}m_{\rho}^{2}\vec{\rho}_{\mu}\vec{\rho}^{\mu} \\
\mathcal{L}_{\sigma\omega\rho} &= -g_{\sigma}g_{\omega}^{2}\sigma\omega_{\mu}\omega^{\mu}\left(\alpha_{1} - \frac{1}{2}\alpha_{1}'g_{\sigma}\sigma\right) - g_{\sigma}g_{\rho}^{2}\sigma\vec{\rho}_{\mu}\vec{\rho}^{\mu}\left(\alpha_{2} - \frac{1}{2}\alpha_{2}'g_{\sigma}\sigma\right) + \frac{1}{2}\alpha_{3}'g_{\omega}^{2}g_{\rho}^{2}\omega_{\mu}\omega^{\mu}\vec{\rho}_{\mu}\vec{\rho}^{\mu}
\end{aligned}$$

⬇

$$\begin{aligned}
\mathcal{E} &= \frac{1}{2}m_{\sigma}^{2}\sigma^{2} + \frac{A}{3}\sigma^{3} + \frac{B}{4}\sigma^{4} - \frac{1}{2}m_{\omega}^{2}\omega_{0}^{2} - \frac{c}{4}(g_{\omega}^{2}\omega_{0}^{2})^{2} - \frac{1}{2}m_{\rho}^{2}\bar{\rho}_{0}^{2} + g_{\omega}\omega_{0}\rho + \frac{g_{\rho}}{2}\bar{\rho}_{0}\rho_{3} \\
&+ g_{\sigma}g_{\omega}^{2}\sigma\omega_{0}^{2}\left(\alpha_{1} - \frac{1}{2}\alpha_{1}'g_{\sigma}\sigma\right) + g_{\sigma}g_{\rho}^{2}\sigma\bar{\rho}_{0}^{2}\left(\alpha_{2} - \frac{1}{2}\alpha_{2}'g_{\sigma}\sigma\right) - \frac{1}{2}\alpha_{3}'g_{\omega}^{2}g_{\rho}^{2}\omega_{0}^{2}\bar{\rho}_{0}^{2} \\
&+ \mathcal{E}_{cin}^{p} + \mathcal{E}_{cin}^{n},
\end{aligned}$$

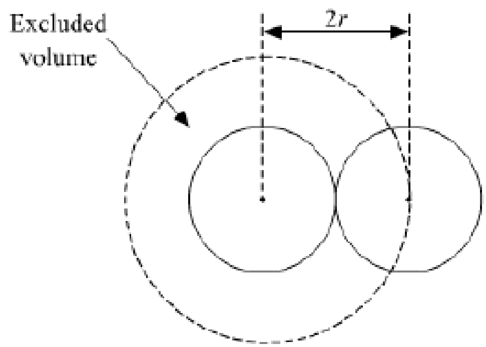
- Typical van der Waals behavior





- Classical van der Waals model

$$P(T,V,N) = \frac{NT}{V - bN} - a\frac{N^2}{V^2}$$



$$V \to V - bN, \qquad b = 4\frac{4\pi r^3}{3} \qquad P \to P - a\frac{N^2}{V^2}$$

- repulsion (excluded volume)

- attraction

- Is it possible to convert this model for describing nuclear matter?

- Yes, it is:

$$\mathcal{E}(\rho) = (1 - b\rho)\mathcal{E}^*_{\rm id}(\rho^*) - a\rho^2, \quad P(\rho) = P^*_{\rm id}(\rho^*) - a\rho^2,$$

$$\mathcal{E}^*_{\rm id}(\rho^*) = \frac{\gamma}{2\pi^2}\int_0^{k_F^*} dk\, k^2(k^2 + M^2)^{1/2}, \qquad P^*_{\rm id}(\rho^*) = \frac{\gamma}{6\pi^2}\int_0^{k_F^*} \frac{dk\, k^4}{(k^2 + M^2)^{1/2}}.$$

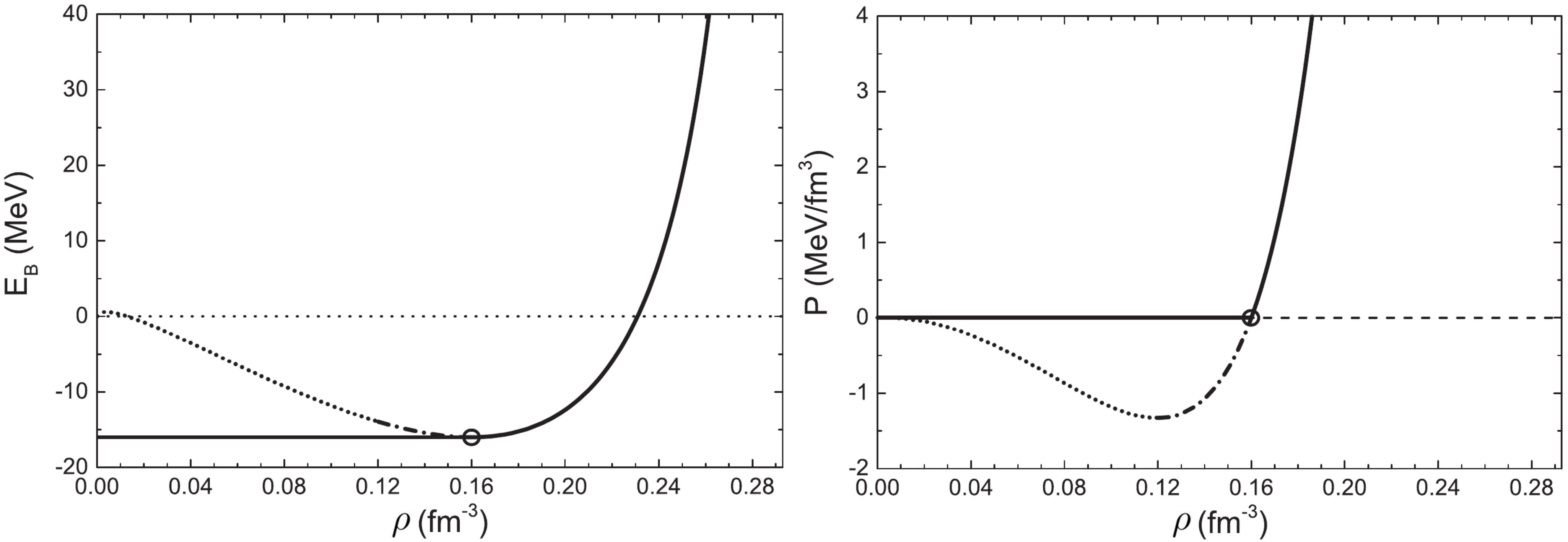
where $k_F^* = (6\pi^2\rho^*/\gamma)^{\frac{1}{3}}$, $\rho^* = \rho/(1 - b\rho)$. M is the nucleon rest mass.

- 2 free parameters to find: a, b.

- Found by imposing the correct values for binding energy and saturation density

V. Vovochenko, D. V. Anchishkin, M. I. Gorenstein; PRC91, 064314 (2015).

- Result:



$$E_B \equiv \frac{\mathcal{E}}{\rho} - M$$

V. Vovochenko, D. V. Anchishkin, M. I. Gorenstein; PRC91, 064314 (2015).

- Generalizing the expressions: $b \to b(\rho) \quad a \to a(\rho)$.

$$\mathcal{E}(\rho) = [1 - b(\rho)\rho]\mathcal{E}^*_{\rm id}(\rho^*) - a(\rho)\rho^2, \quad P(\rho) = P^*_{\rm id} - a(\rho)\rho^2 + \rho\Sigma(\rho),$$

- rearrangement term: $\Sigma(\rho) = b'\rho P^*_{\rm id} - a'\rho^2$.

- Carnahan-Starling (CS): $b(\rho) = \frac{1}{\rho} - \frac{1}{\rho}\exp\left[\frac{-(4 - \frac{3b\rho}{4})\frac{b\rho}{4}}{\left(1 - \frac{b\rho}{4}\right)^2}\right]$

- other real gases:

$$a(\rho) = a \text{ (vdW)},$$

$$a(\rho) = \frac{a}{b\rho}\ln(1 + b\rho) \text{ Redlich–Kwong–Soave (RKS)},$$

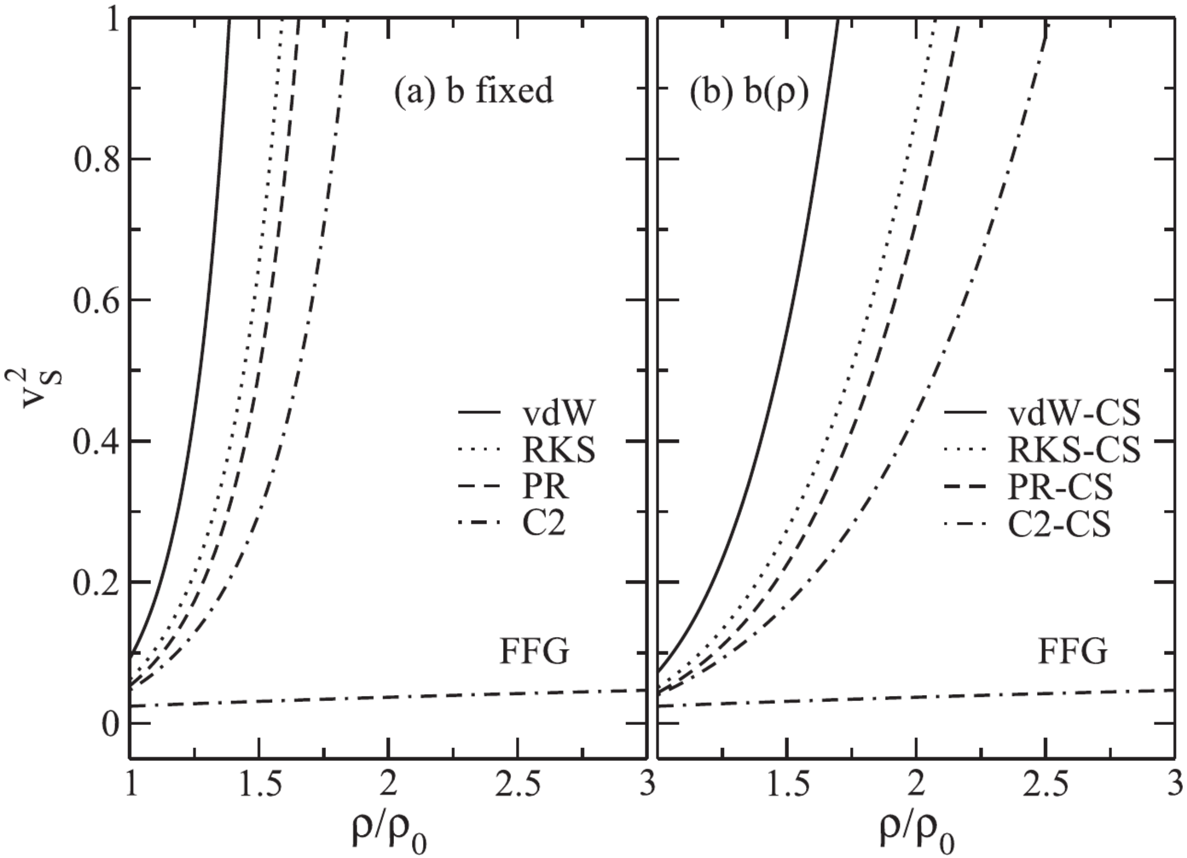
$$a(\rho) = \frac{a}{2\sqrt{2}b\rho}\ln\left[\frac{1 + b\rho(1 + \sqrt{2})}{1 + b\rho(1 - \sqrt{2})}\right] \text{Peng–Robinson (PR)},$$

$$a(\rho) = \frac{a}{1 + b\rho} \text{ Clausius-2 (C2)}$$

- Some problems:

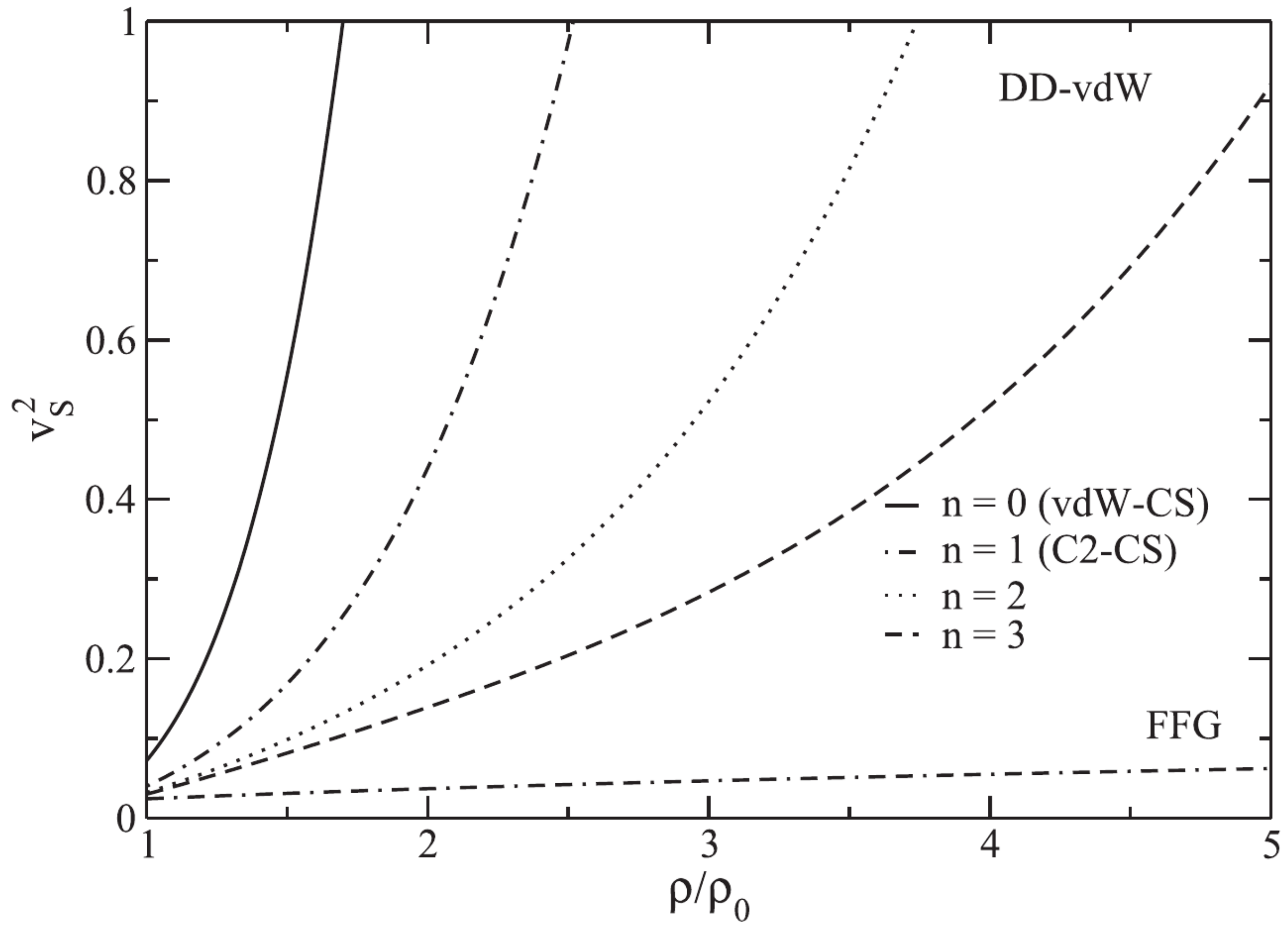
V. Vovochenko, PRC96, 015206 (2017).

Model	K_0 (MeV)
VDW	763
RKS	518
PR	443
Clausius	390
VDW-CS	601
RKS-CS	421
PR-CS	359
Clausius-CS	333



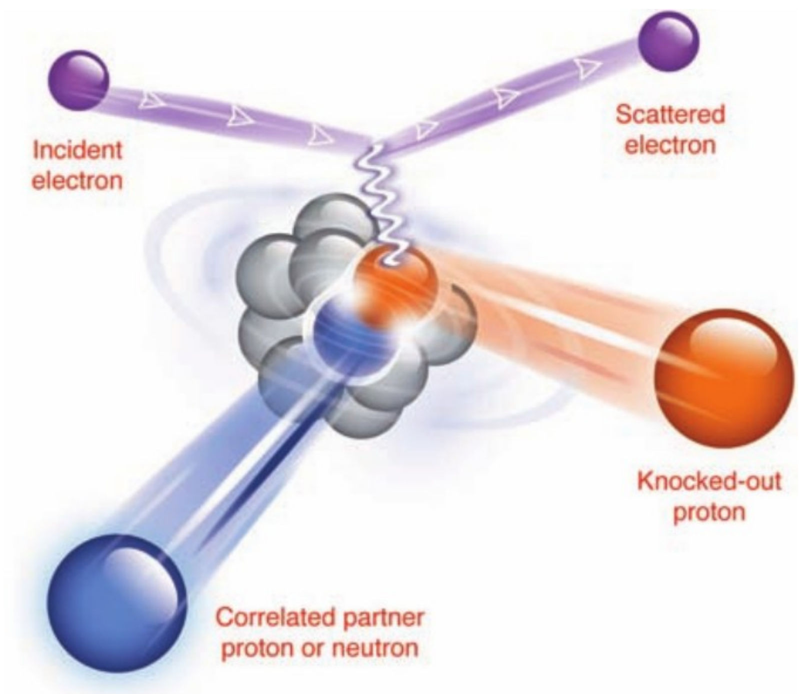
O. Lourenço, M. Dutra, C. H. Lenzi, M. Bhuyan, S. K. Biswal e B. M Santos, Astrophys. J. 882, 67 (2019).

- Proposal: $a(\rho) = \dfrac{a}{(1+b\rho)^n}.$



O. Lourenço, M. Dutra, C. H. Lenzi, M. Bhuyan, S. K. Biswal e B. M Santos, Astrophys. J. 882, 67 (2019).

- Inclusion of short-range correlations (SRC) in the hadronic side:



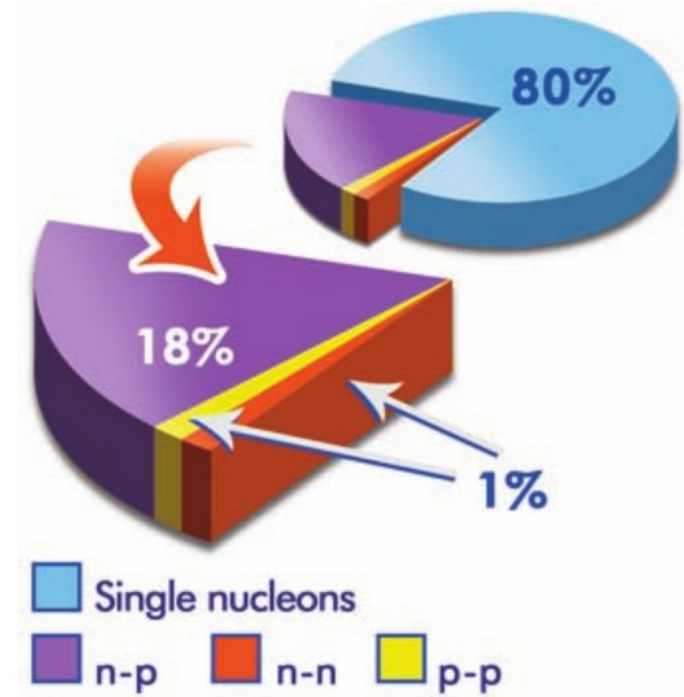
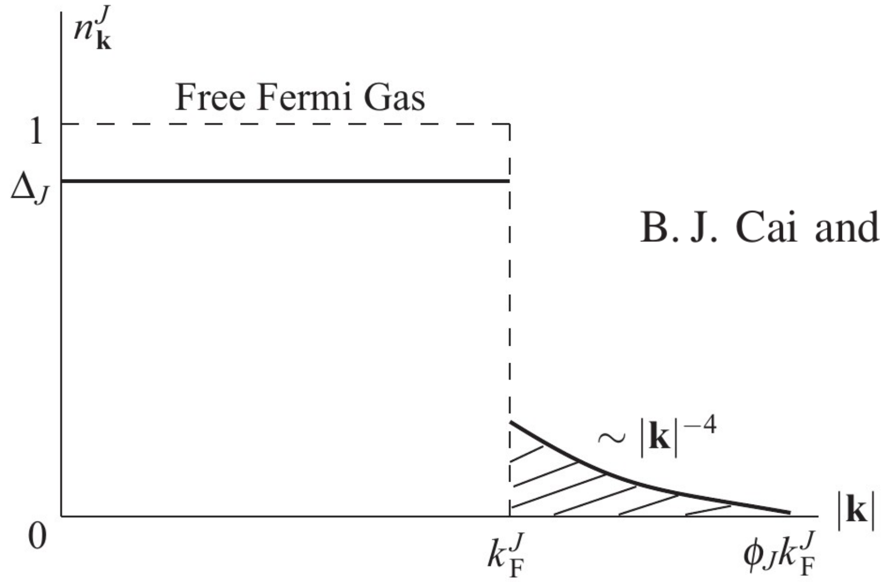


Fig. 3. The average fraction of nucleons in the various initial-state configurations of ^{12}C.

R. Subedi *et al.*, Science **320**, 1476 (2008)

- Consequence in single-nucleon momentum distribution:



B. J. Cai and B. A. Li, Phys. Rev. C **93**, 014619 (2016)

- modification in momentum integrals of relativistic mean-field (RMF) models:

$$\mathcal{E}_{\text{kin}}^{p,n} = \frac{\gamma}{2\pi^2}\int_0^{k_{F\,p,n}} k^2\left(k^2 + M_{p,n}^{*2}\right)^{1/2} dk \longrightarrow \mathcal{E}_{\text{kin}}^{n,p} = \frac{\gamma\Delta_{n,p}}{2\pi^2}\int_0^{k_{F\,n,p}} k^2 dk (k^2 + M^{*2})^{1/2} + \frac{\gamma C_{n,p}}{2\pi^2}\int_{k_{F\,n,p}}^{\phi_{n,p}k_{F\,n,p}} \frac{k_{F\,n,p}^4}{k^2} dk (k^2 + M^{*2})^{1/2}$$

$$P_{\text{kin}}^{p,n} = \frac{\gamma}{6\pi^2}\int_0^{k_{F\,p,n}} \frac{k^4 dk}{\left(k^2 + M_{p,n}^{*2}\right)^{1/2}} \longrightarrow P_{\text{kin}}^{n,p} = \frac{\gamma\Delta_{n,p}}{6\pi^2}\int_0^{k_{F\,n,p}} \frac{k^4 dk}{(k^2 + M^{*2})^{1/2}} + \frac{\gamma C_{n,p}}{6\pi^2}\int_{k_{F\,n,p}}^{\phi_{n,p}k_{F\,n,p}} \frac{k_{F\,n,p}^4 dk}{(k^2 + M^{*2})^{1/2}}$$

- Energy density and pressure:

$$\epsilon(\rho, y_p) = [1 - \rho\mathcal{B}(\rho)]\left[\epsilon^{\star p}_{kin(SRC)} + \epsilon^{\star n}_{kin(SRC)}\right] - \rho^2\mathcal{A}(\rho) + d(2y_p - 1)^2\rho^2,$$

$$p(\rho, y_p) = p^{\star p}_{kin(SRC)} + p^{\star n}_{kin(SRC)} - \rho^2\mathcal{A}(\rho) + \rho\Sigma_{SRC}(\rho, y_p) + d(2y_p - 1)^2\rho^2,$$

$$\Sigma_{SRC}(\rho, y_p) = \rho\mathcal{B}'\left[p^{\star p}_{kin(SRC)} + p^{\star n}_{kin(SRC)}\right] - \rho^2\mathcal{A}',$$

$$\mathcal{B}(\rho) = \frac{1}{\rho} - \frac{1}{\rho}\exp\left[-\frac{b\rho}{4}\frac{\left(4 - \frac{3b\rho}{4}\right)}{\left(1 - \frac{b\rho}{4}\right)^2}\right],$$

→ Clausius-CS model with SRC: CCS-SRC

$$\mathcal{A}(\rho) = \frac{a}{1 + c\rho}.$$

- Free constants: a, b, c, d. Nuclear bulk parameters: binding energy, saturation density, incompressibility and symmetry energy

- Sound velocity:

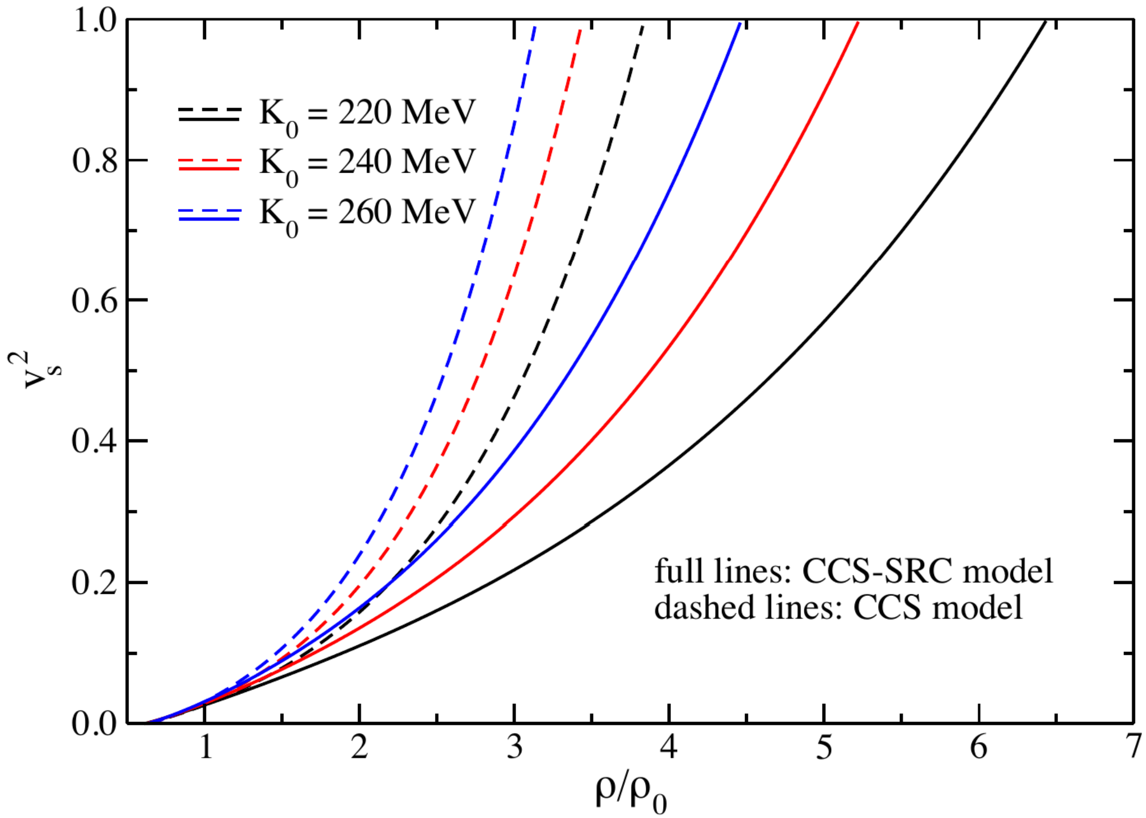


Figure 5. Squared sound velocity as a function of ρ/ρ_0 for different parametrizations of the CCS model with (full lines) and without (dashed lines) SRC included. Curves for symmetric nuclear matter with $\rho_0 = 0.15$ fm^{-3} and $B_0 = -16$ MeV.

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- Density dependence of the pressure:

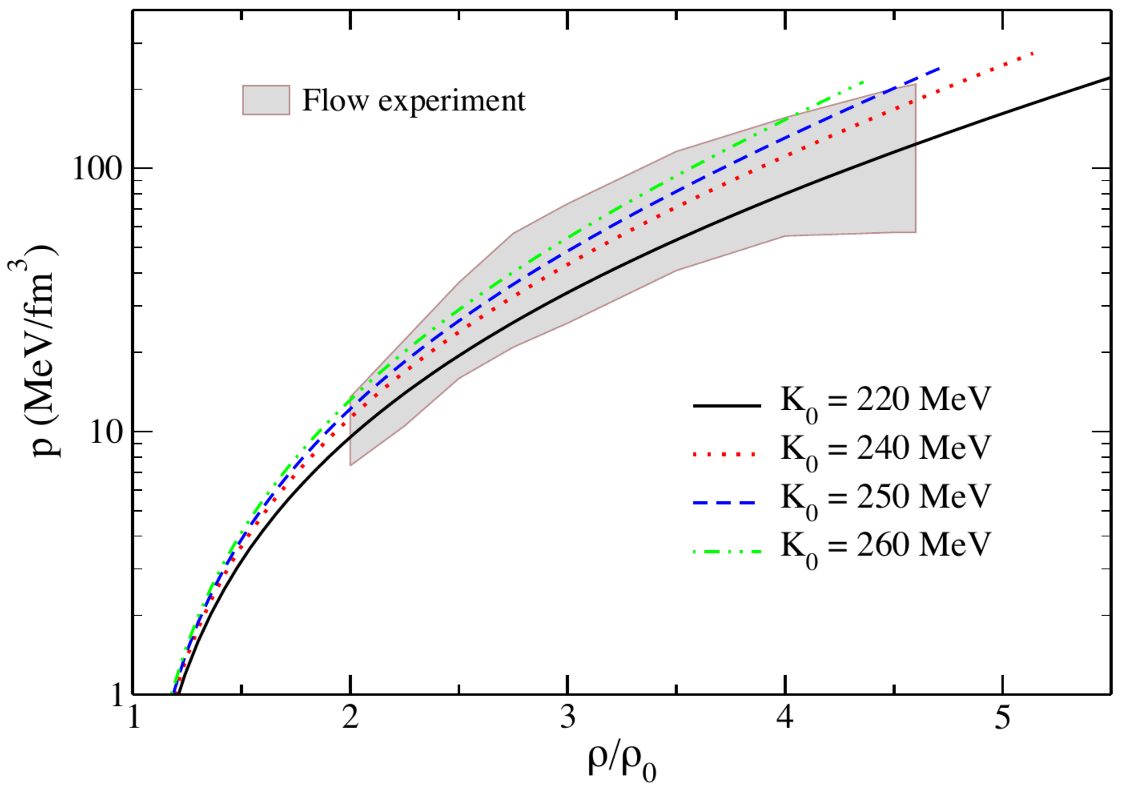

Figure 4. Pressure versus ρ/ρ_0 for different parametrizations of the CCS-SRC model. Curves for symmetric nuclear matter with $\rho_0 = 0.15$ fm^{-3} and $B_0 = -16$ MeV. Band: Flow constraint extracted from (Danielewicz et al. 2002a).

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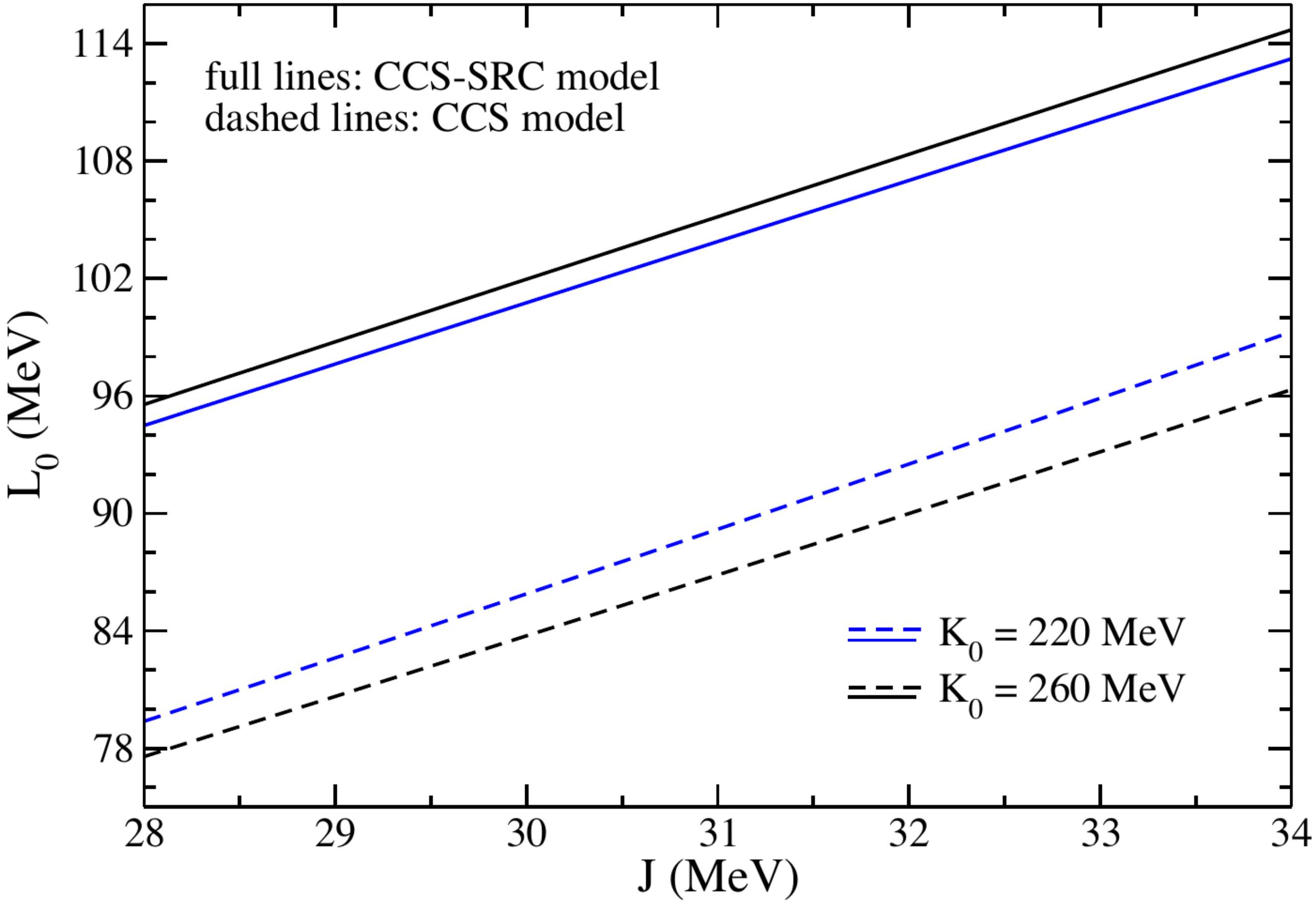
- Symmetry energy and slope:



$$\mathcal{S}(\rho) = \frac{1}{8}\frac{\partial^2(\mathcal{E}/\rho)}{\partial y^2}\bigg|_{y=\frac{1}{2}}$$

$J = \mathcal{S}(\rho_0)$ (symmetry energy at $\rho = \rho_0$),

$L_0 = 3\rho_0 \left(\frac{\partial \mathcal{S}}{\partial \rho}\right)_{\rho=\rho_0}$ (slope of $\mathcal{S}$),

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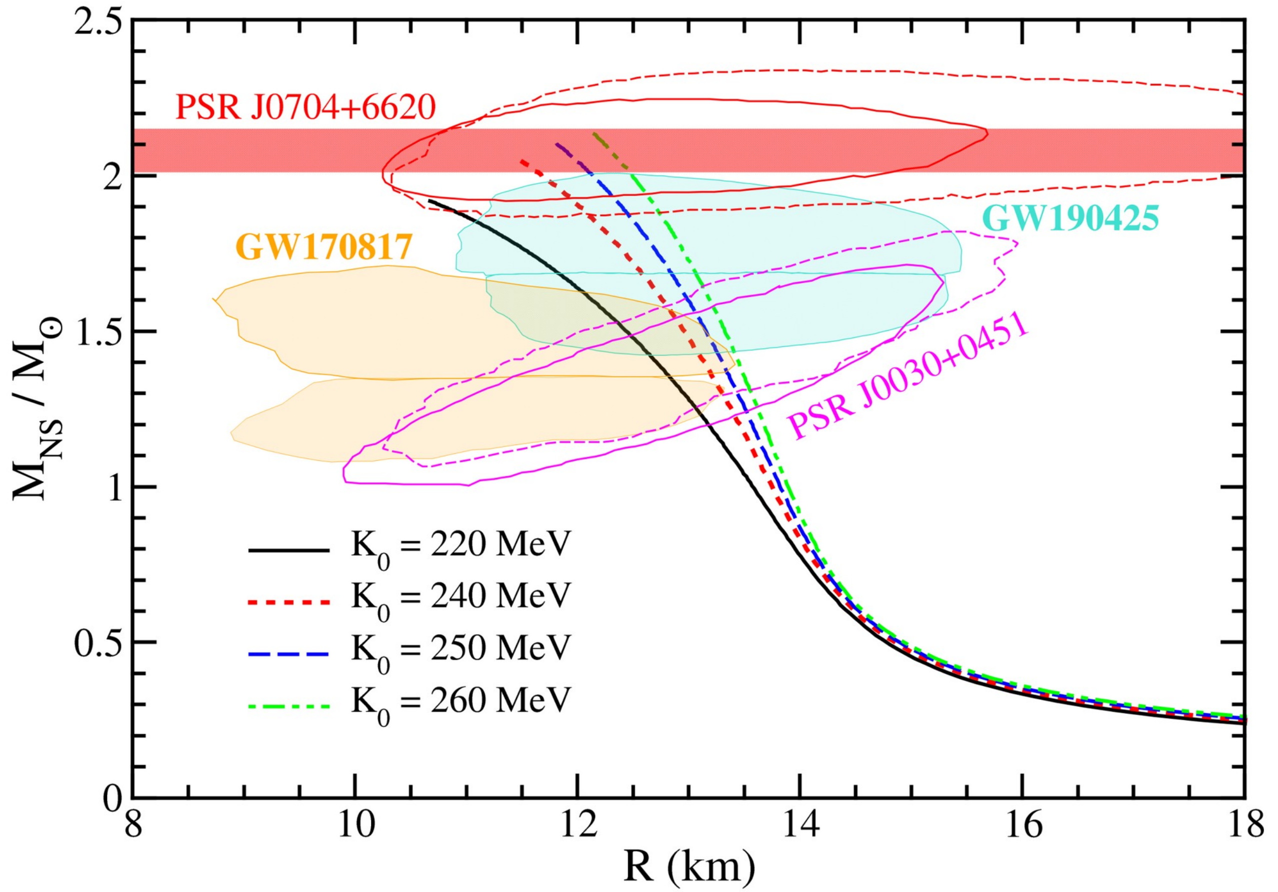
- Mass-radius profiles of neutron stars. TOV equations:

$$\frac{dP(r)}{dr} = -\frac{[\mathcal{E}(r)+P(r)]\left[m(r)+4\pi r^3 P(r)\right]}{r^2\left[1-\frac{2m(r)}{r}\right]} \qquad \frac{dm(r)}{dr} = 4\pi r^2 \mathcal{E}(r), \qquad \begin{array}{ll} P(0)=P_c & m(0)=0 \\ P(R)=0 & m(R)\equiv M_{\rm NS} \end{array}$$



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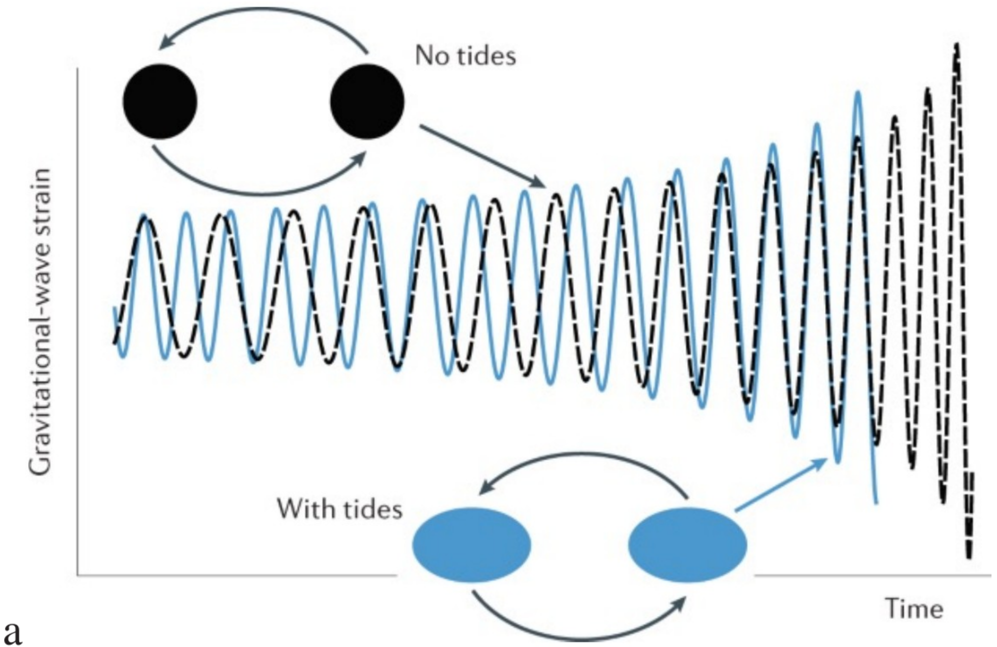
- Tidal deformability: quantifies how easily the star is deformed when subject to an external tidal field.

$$\Lambda = 2k_2/(3C^5), \text{ with } C = M/R$$

$$k_2 = \frac{8C^5}{5}(1-2C)^2[2+2C(y_R-1)-y_R] \times \{2C[6-3y_R+3C(5y_R-8)] + 4C^3[13-11y_R+C(3y_R-2)+2C^2(1+y_R)] + 3(1-2C)^2[2-y_R+2C(y_R-1)]\ln(1-2C)\}^{-1},$$

$r(dy/dr) + y^2 + yF(r) + r^2Q(r) = 0$, solved as part of a coupled system also containing the TOV equations.



Yunes et al. , 2022

$$F(r) = \frac{1 - 4\pi r^2[\epsilon(r) - p(r)]}{g(r)},$$

$$Q(r) = \frac{4\pi}{g(r)}\left[5\epsilon(r) + 9p(r) + \frac{\epsilon(r)+p(r)}{v_s^2(r)} - \frac{6}{4\pi r^2}\right] - 4\left[\frac{m(r)+4\pi r^3 p(r)}{r^2 g(r)}\right]^2,$$

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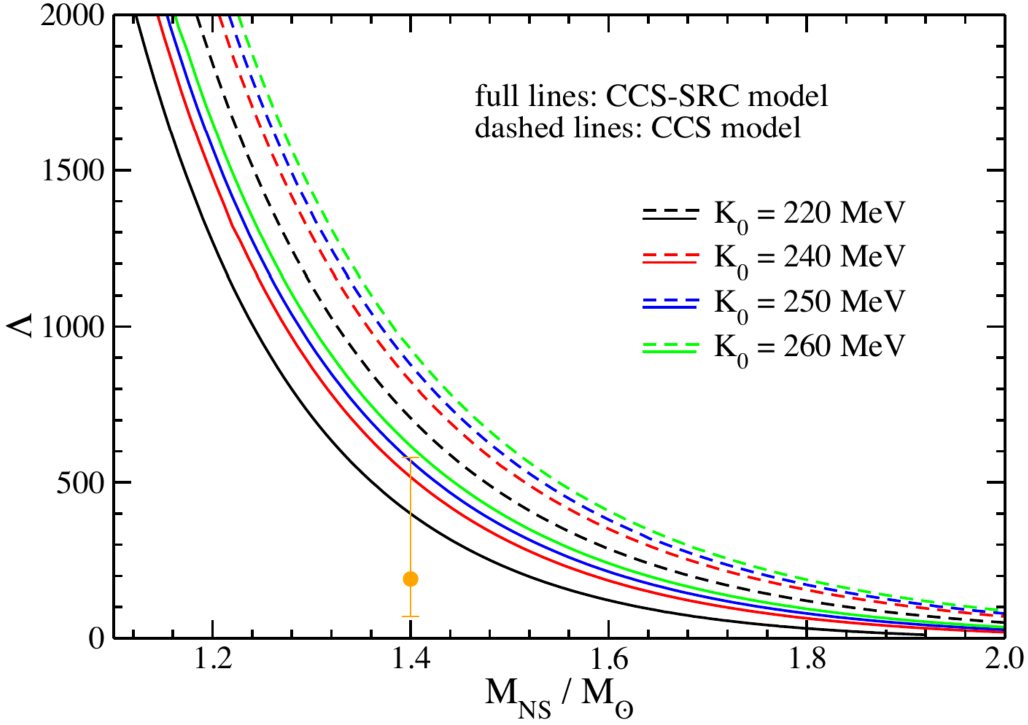
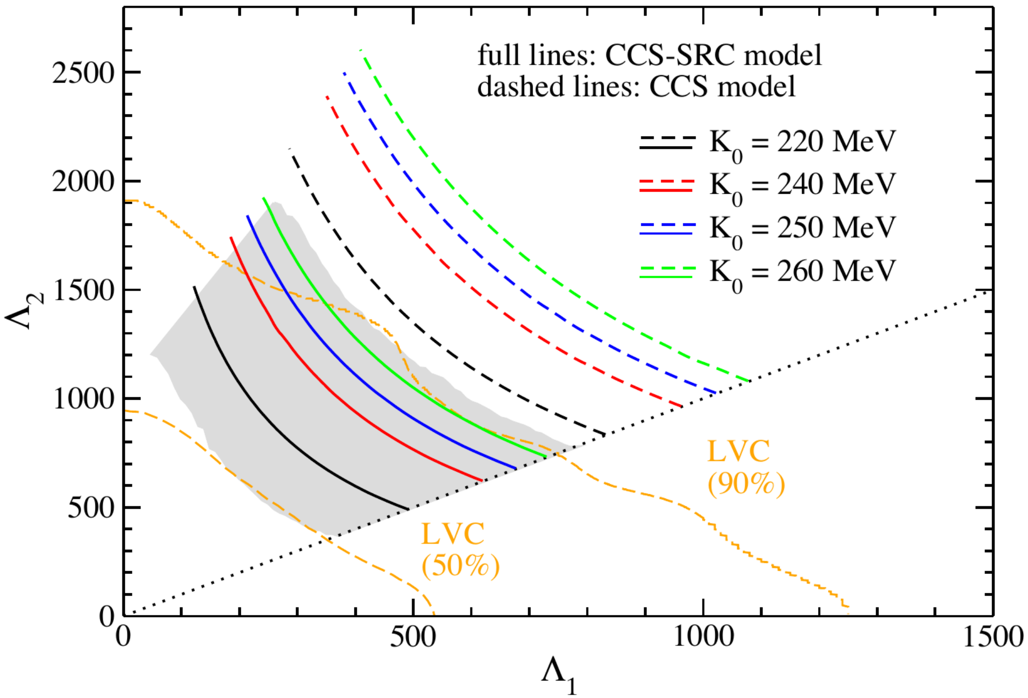
- Results:





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Conclusions

- short-range correlations shift the break of causality to a higher density region.
- short-range correlations increase the symmetry energy slope.
- it is possible to generate parametrizations in agreement with observational data from NICER and LIGO/Virgo collaboration.
- tidal deformabilities related to the GW170817 event also reproduced.

Thank you very much !