Characteristics of QCD medium in the presence of a magnetic field

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Strongly interacting matter in extreme magnetic fields

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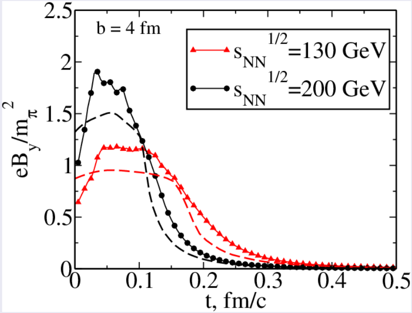
Production of a strong magnetic field in HICs

- A very strong magnetic field ($\approx m_\pi^2$ at RHIC and $\approx 10\, m_\pi^2$ at LHC) is generated in the direction perpendicular to the reaction plane, due to the relative motion of the ions themselves.
  ($m_\pi^2 = 1.96 \times 10^{-2}$ GeV^2 $\approx 10^{18}$ Gauss)

- A comparison with other terrestrial strengths: Earth $\approx 10^{-18}$ m_π^2, usual laboratory $\approx 10^{-13}$ m_π^2, max.

- A magnetar: $\approx 10^-5 - 10^-3$ m_π^2.

[V. Skokov et al, IJMP,2009]



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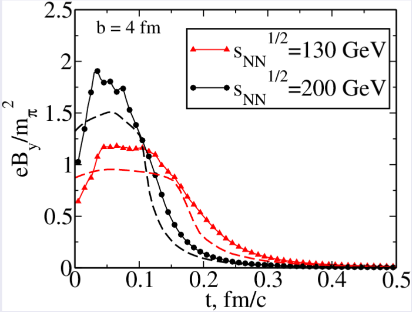
  ($m_\pi^2 = 1.96 \times 10^{-2}$ GeV$^2 \approx 10^{18}$ Gauss)

- The presence of an external field in the medium subsequently requires modification of the present theoretical tools.

[V. Skokov et al, IJMP,2009]

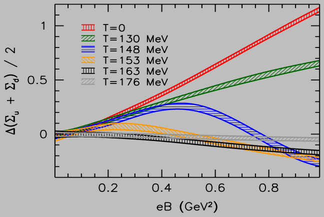


Outline:

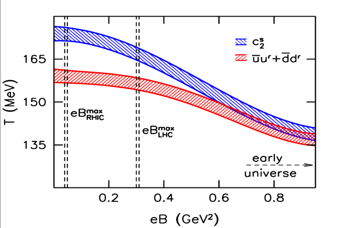
- QCD phase diagram in presence of a magnetic field and constraints on effective model
- Lepton pairs from a QCD medium in presence of a magnetic field using basic thermal field theoretical approach
- Conclusion

Phase diagram in presence of eB

- We observe magnetic catalysis (MC) and inverse magnetic catalysis (IMC) in presence of a magnetic field. [G. S. Bali et al., Phys. Rev. D 86, 071502]



- The PD in presence of eB looks as: [G. S. Bali et al., JHEP 02 (2012) 044]



- What happens if we increase the magnetic field further? [Endrődi 2015 and D'Elia et al., 2021]

Treatment with an effective model, namely the NJL

- The standard $2f$ Nambu–Jona-Lasinio (NJL) model: [Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345(1961); 124, 246(1961)]

$$\mathcal{L}_{\rm NJL} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \frac{G_S}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] \tag{1}$$

Treatment with an effective model, namely the NJL

- It can also be written as [M. Frank et al., Phys. Lett. B 562 (2003) 221-226]

$$\mathcal{L}_{\mathrm{NJL}} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2$$

$$\mathcal{L}_0 = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi$$

$$\mathcal{L}_1 = G_1\left\{(\bar{\psi}\psi)^2 + (\bar{\psi}\vec{\tau}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right\}$$

$$\mathcal{L}_2 = G_2\left\{(\bar{\psi}\psi)^2 - (\bar{\psi}\vec{\tau}\psi)^2 - (\bar{\psi}i\gamma_5\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right\}$$

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- The NJL Lagrangian [M. Frank et al., Phys. Lett. B 562 (2003) 221-226]

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- $\mathcal{L}_2$ explicitly breaks $U(1)_A$.
- Symmetry only allows the $\langle\bar\psi\psi\rangle$ condensate, which depends on $(G_1 + G_2)$.
- With μ_I or magnetic field as the $SU(2)$ symmetry is broken one can have $\langle\bar\psi\tau_3\psi\rangle$ which has a $(G_1 - G_2)$ dependence.
- G_1 and G_2 can be parameterized as $G_1 = (1-c)G_0/2$ and $G_2 = cG_0/2$
- $c = 1/2$ corresponds to the standard NJL model.

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- G_1 and G_2 can be parameterized as $G_1 = (1-c)G_0/2$ and $G_2 = cG_0/2$
- Existing estimation of c in presence of μ_I and eB. [Frank et al, 2003; Boomsma et al, 2010]

Treatment with an effective model, namely the NJL

- The standard $3f$ Nambu–Jona-Lasinio (NJL) model:[Klevansky, 1992; Hatsuda & Kunihiro, 1994; Buballa, 2005]

$$\mathcal{L}_{\mathrm{NJL}} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2$$

$$\mathcal{L}_0 = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi$$

$$\mathcal{L}_1 = \frac{G_S}{2}\sum_{a=0}^{8}[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma_5\lambda_a\psi)^2]$$

$$\mathcal{L}_2 = -G_D\{\det[\bar{\psi}(1+\gamma_5)\psi] + \det[\bar{\psi}(1-\gamma_5)\psi]\}.$$

NJL model in presence of a magnetic field

- The Lagrangian in presence of eB [D. P. Menezes et al., Phys. Rev. C 79, 035807 (2009)]

$$\mathcal{L}_{\text{NJL}} = \bar{\psi}(i\gamma_\mu D^\mu - m_0)\psi + \mathcal{L}_1 + \mathcal{L}_2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu},$$

  with $D^\mu = \partial^\mu - iqA^\mu$ ($q_u = 2/3e$ and $q_d = -1/3e$) and $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$.

- Due to eB there are two important modifications.

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- I) The dispersion relation gets modified as,

$$E_f(B) = [M_f^2 + p_z^2 + 2k|q_f|B]^{1/2}$$

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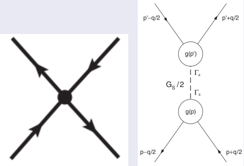
with $D^\mu = \partial^\mu - iqA^\mu$ ($q_u = 2/3e$ and $q_d = -1/3e$) and $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$.

- II) The integral over the three momenta gets modified as,

$$\int \frac{d^3p}{(2\pi)^3} \rightarrow \frac{|q_f|B}{2\pi} \sum_{k=0}^{\infty} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi}$$

Non-local NJL model

Local vs non-local interaction



- $j_a(x) = \int d^4z\, g(z)\, \bar{\psi}(x+z/2)\Gamma_a\psi(x-z/2)$
- Non-local: Intuitively can be thought of as an alternate prescription for regularisation.

Non-local NJL model

The constituent mass is found from the gap equation: $M(p) = m_q + \mathcal{C}(p)\bar{\sigma}$ with the mean field $\bar{\sigma} = \langle \sigma \rangle$

By the principle of least action we can get the mean field: $$\bar{\sigma} = 8N_c G \int \frac{d^4p}{(2\pi)^4} \mathcal{C}(p) \frac{M(p)}{p^2 + M^2(p)}$$

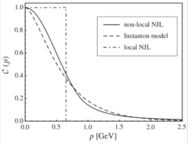
How to get the condensate: $$\langle \bar{\psi}\psi \rangle = -4N_f N_c \int \frac{d^4p}{(2\pi)^4} \left[\frac{M(p)}{p^2 + M^2(p)} - \frac{m_q}{p^2 + m_q^2} \right]$$

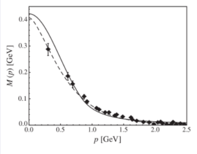
Differences with the local version? $M = -G\langle \bar{\psi}\psi \rangle = \bar{\sigma}$

- But it does more than that.

Non-local NJL model

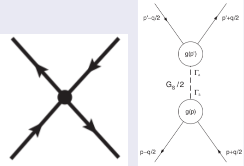
Hell et al., 2009





In presence of eB

Local vs non-local interaction



- IMC effect in local NJL model, eB dependent coupling constant, $G(eB, T)$. Farias et al., 2014, 2016, Ferreira et al., 2014
- IMC effect in non-local version. Pagura et al., 2016
- We have considered $g(p)$ to be Gaussian in nature

$$g(p) = \exp[-p^2/\Lambda^2]$$

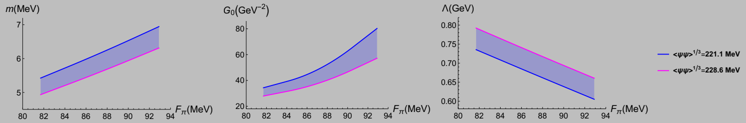
Parameter dependence

- There are three parameters of the model that needs to be fixed: m, G_0 and Λ (sets the scale of the theory).
- Condensate and F_π taken from LQCD calculation with $m_\pi = 135\text{MeV}$.[H. Fukaya et al.(JLQCD), Phys. Rev. D 77, 074503 (2008)]

$$\langle\bar{\psi}\psi\rangle^{1/3}|_{\mu=2\,\text{GeV}} = 240(4)\,\text{MeV and}$$
$$F_\pi = 87.3(5.6)\,\text{MeV}.$$

- We have used perturbative RG running to obtain the condensate at 1 GeV following the. [L. Giusti et al., Nucl. Phys. B, 538:249-277, 1999]

$$\langle\bar{\psi}\psi\rangle^{1/3}|_{\mu=1\,\text{GeV}} = 224.8(3.7)\,\text{MeV and}$$



Parameter dependence

- Central and the four corner parameter sets:

	$\langle\bar{\psi}_f\psi_f\rangle^{1/3}$(MeV)	m_π(MeV)	F_π(MeV)	$F_{\pi,0}$(MeV)	m(MeV)	G_0(GeV^{-2})	Λ(MeV)
Parameter Set CC	224.8	135	87.3	84.25	5.87	43.34	697.22
Parameter Set HH	228.6	135	92.9	90.63	6.31	57.15	660.46
Parameter Set HL	228.6	135	81.7	77.04	4.94	27.90	792.22
Parameter Set LH	221.1	135	92.9	91.00	6.94	80.26	605.05
Parameter Set LL	221.1	135	81.7	77.61	5.42	34.32	735.38

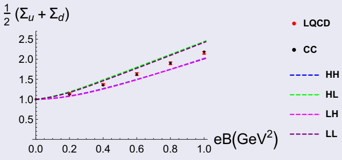
- We will see that only some of the parameter sets will eventually survive and can give desired results.

  a) Can reproduce the phase diagram in $T - eB$ plane

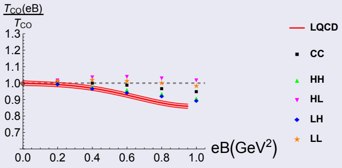
  b) And can generate phenomenologically reasonable mass for η^*, which is an isoscalar pseudoscalar having mass roughly of the order of 400 MeV.

Condensate average & phase diagram in NJL model

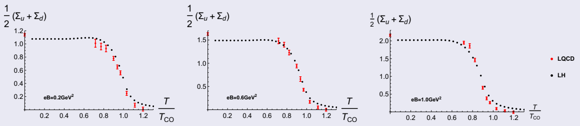
Condensate average at zero T



Phase diagram in $T - eB$ plane



Condensate average at nonzero T



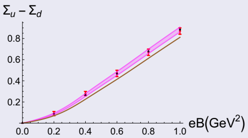
CAI et al., PRD 2021

Constraint from η^*

- To choose between HH and LH we evoke η^* phenomenology.
- η^* mass is very sensitive to c. At $c = 0$, π^0 and η^* become degenerate.
- Considering η^* to be admixture of η and η' of three flavor, we can impose that $M_{\eta^*} > 400$ MeV.
- The above constrain set the lower bound to be $c \geq 0.12$.
- We eliminate HH as the fitted c is less than the lower bound.

Fitting of c and condensate difference

Condensate diff at zero T



Fitted c values

	c	χ^2 per DoF
Parameter Set CC	0.276 ± 0.068	0.211
Parameter Set HH	0.044 ± 0.079	0.149
Parameter Set HL	0.374 ± 0.051	0.290
Parameter Set LH	0.149 ± 0.103	0.634
Parameter Set LL	0.465 ± 0.062	0.551

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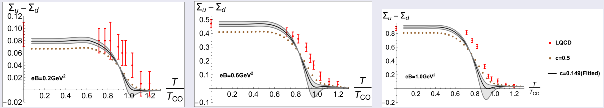

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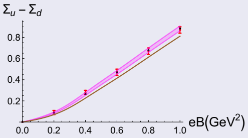

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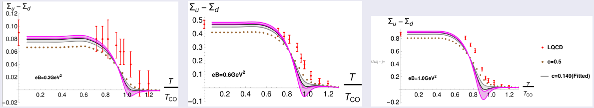
With restrictions $M_{\eta^*} > 400$ MeV



CAI et al., PRD 2021

EM probes, particularly leptons:

- Photons and leptons (virtual photons) can probe the interior of a QCD medium.
- They are produced from multiple stages.
- They can be used to extract information from the hot and dense matter.
- We are particularly interested in the thermal leptons.

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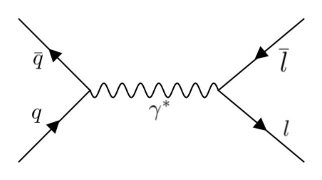
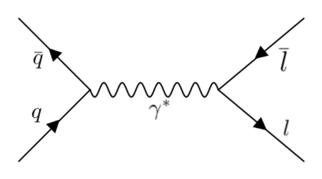
Lepton pair

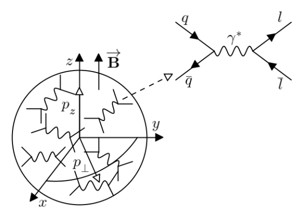


Lepton pair from the magnetised medium:

Lepton pair



A magnetised medium

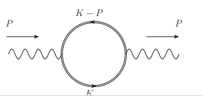


- A straightforward question is to ask whether the dilepton production will be affected by the magnetic field.

Methodology:

- This dilepton rate (DR) is given by [H. A. Weldon PRD 42, 2384].

$$\frac{dN}{d^4X d^4P} \equiv \frac{dR}{d^4P} = \frac{\alpha_{\mathsf{EM}}}{12\pi^3}\frac{1}{P^2}\frac{1}{e^{p_0/T}-1}\sum_{f=u,d}\frac{1}{\pi}\,\mathsf{Im}\,\Pi^{\mu}_{\mu,f}(P). \quad (1)$$



- We can use the fermionic propagator, depending on the scenarios, to write down the one loop electromagnetic (EM) polarization tensor

$$\Pi^{\mu\nu}_f(P) = -iN_c q_f^2\int\frac{d^4K}{(2\pi)^4}\mathsf{Tr}\left[\gamma^\mu S^B_f(K)\gamma^\nu S^B_f(K-P)\right]. \quad (2)$$

Fermionic propagator in presence of eB:

- Schwinger propagator in momentum space as

$$S_f^{(B)}(K) = \exp\left(-\frac{k_\perp^2}{|q_f B|}\right) \sum_{\ell=0}^{\infty} (-1)^\ell \frac{D_\ell(K, q_f B)}{k_\shortparallel^2 - 2\ell|q_f B| - m_f^2 + i\epsilon}\,, \tag{3}$$

where

$$D_\ell(K, q_f B) = (\not{k}_\shortparallel + m_f)\left\{L_\ell\left(\frac{2k_\perp^2}{|q_f B|}\right)\left[\mathbb{1} - i\gamma^1\gamma^2 \mathsf{sgn}(q_f B)\right] - L_{\ell-1}\left(\frac{2k_\perp^2}{|q_f B|}\right)\left[\mathbb{1} + i\gamma^1\gamma^2 \mathsf{sgn}(q_f B)\right]\right\} + 4\not{k}_\perp L_{\ell-1}^1\left(\frac{2k_\perp^2}{|q_f B|}\right). \tag{4}$$

- m_f and q_f are the mass and charge of the fermion of flavor f, respectively; ℓ denotes the Landau level index.

Work done so far:

- So far dilepton rate from a magnetised medium has been calculated in different articles using different techniques. [A. Bandyopadhyay et al, Snigdha Ghosh et al, N. Sadooghi et al, X. Wang et al].

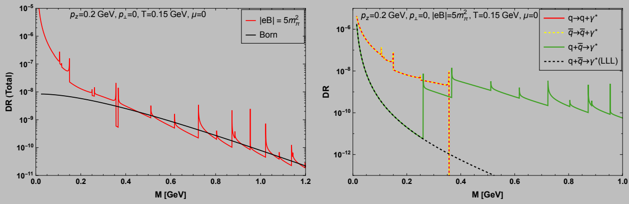
- Many of the calculations used different approximations, particularly either strong or weak magnetic field approximations.

- For arbitrary magnetic field, either parallel (p_z) or perpendicular ($p_\perp$) component taken to be zero.

Calculational novelty in the present effort:

- We have relaxed all approximations related to the field and external momentum.
- It is easy to grasp because of the simplicity in ITF.
- There is similar work done which talks about the ellipticity of the lepton pairs as well. X. Wang and I. Shovkovy, PRD, 2022

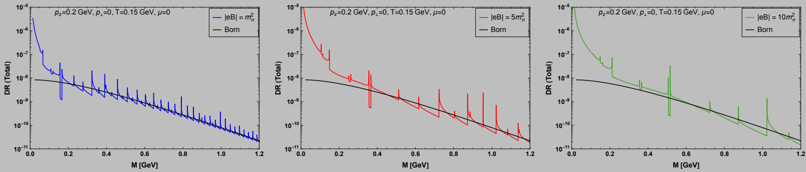
Effect of magnetic field on DR:



- In the left panel we have the plot of DR as a function of invariant mass for $eB = 5\,m_\pi^2$. In the right panel the contribution coming from different processes are shown separately.

A. Das, A. Bandyopadhyay, CAI, PRD 2022

Effect of magnetic field on DR:



A. Das, A. Bandyopadhyay, CAI, PRD 2022

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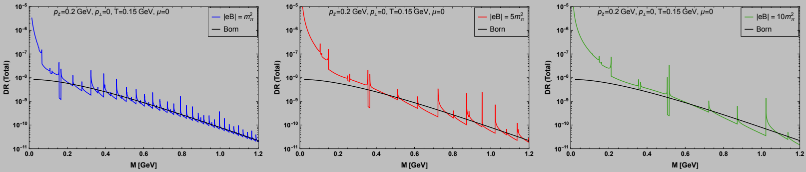
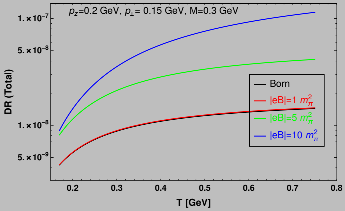




A. Das, A. Bandyopadhyay, CAI, R. Chatterjee (ongoing)

Rate to spectrum:

- The expression of dilepton rate,

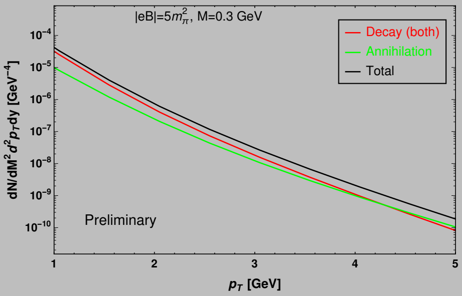
$$\frac{dN}{d^4X d^4P} \equiv \frac{dR}{d^4P} = \frac{\alpha_{\mathsf{EM}}}{12\pi^3}\frac{1}{P^2}\frac{1}{e^{p_0/T}-1}\sum_{f=u,d}\frac{1}{\pi}\,\mathsf{Im}\,\Pi^{\mu}_{\mu,f}(P). \quad (5)$$

- For convenience we make a variable transformation from $(p_0, p_x, p_y, p_z) \to (M, p_T, \phi, y)$

- The Born rate becomes,

$$\frac{dN}{d^4x M dM p_T dp_T dy d\phi} = \frac{5N_c\alpha^2_{\mathsf{EM}}}{108\pi^4}\frac{1}{\sqrt{M_T^2\sinh^2 y + p_T^2}}\frac{T}{e^{\frac{M_T\cosh y}{T}}-1}\left(1+\frac{2m_f^2}{M^2}\right)$$
$$\times \log\left(\frac{\left[\exp\left(-\frac{M_T\cosh y+\mu}{T}\right)+e^{-\omega_-/T}\right]\left[e^{-\mu/T}+e^{-\omega_+/T}\right]}{\left[\exp\left(-\frac{M_T\cosh y+\mu}{T}\right)+e^{-\omega_+/T}\right]\left[e^{-\mu/T}+e^{-\omega_-/T}\right]}\right), \quad (6)$$

Dilepton spectrum



- A smooth $2+1$ dimensional ideal relativistic hydrodynamical model calculation for a Au+Au collision at 200A GeV.
- Impact parameter ~ 6 fm and initial formation time of the plasma $\tau_0 = 0.17$ fm/c

A. Das, A. Bandyopadhyay, CAI, R. Chatterjee (ongoing)

Upshots I:

- Consistence lattice data has been used to constrain the strength of the axial $U(1)$ symmetry breaking strength (c).
- Low condensate value$\langle\bar{\psi}\psi\rangle$, and high F_{Π} produce a stronger IMC effect around the crossover temperature.
- Our estimated value for c, which is done for the first time to the best of our knowledge, is $\mathbf{0.149}$ within some error bars.
- This value is consistent with other approximated estimation of c, $0.1 - 0.2$.

Upshots II:

- We have calculated the dilepton rate for arbitrary strength of magnetic field with both non-zero values of p_z and $p_\perp$.
- We could break down the rate into the contributions coming from different processes and showed that it gets enhanced as compared to the Born rate in presence of eB, particularly at the lower end of the invariant mass.
- Outlook: It will be interesting to have an estimation of the dilepton spectrum as well as flow. That will facilitate to have a direct experimental comparision (ongoing work).

Collaborators:

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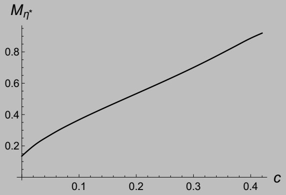
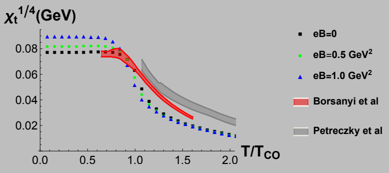
- Aritra Bandyopadhyay
- Rupa Chatterjee
- Aritra Das
- Rishi Sharma

Thank You

Variation of mass of η^*



Topological susceptibility:



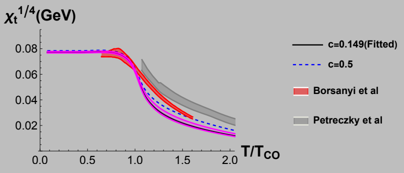


Figure: Top sus for LH parameter set

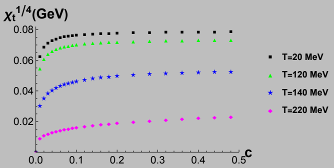
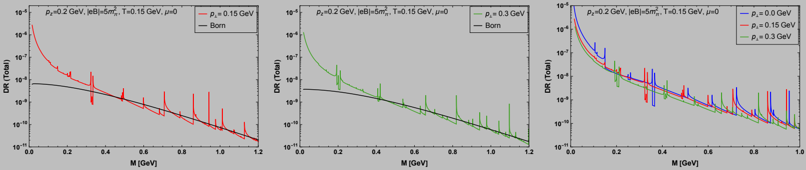


Figure: Top sus as a function of c.

[CAI et al., PRD 2021, S. Borsanyi et al., Nature 2016, Petreczky et al., PLB 2016]

Effect of the transverse momentum:



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A few words on the lifetime of the field

- The very high initial magnitude of this magnetic field then decreases very fast, being inversely proportional to the square of time(?).

  [A. Bzdak et al, PRL, 2013;

  K. Tuchin, PRC, 2013]

  [Z. Wang et al, PRC, 2022;

  STAR, PRC, 2022]

[V. Skokov et al, IJMP,2009]