Novel Transport Effects

Dirac Materials in Parallel Electric and Magnetic Fields

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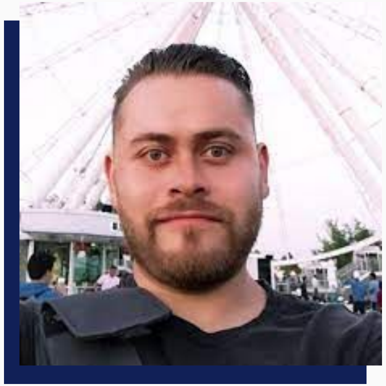
Collaborators



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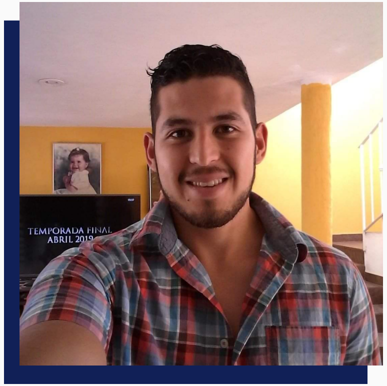

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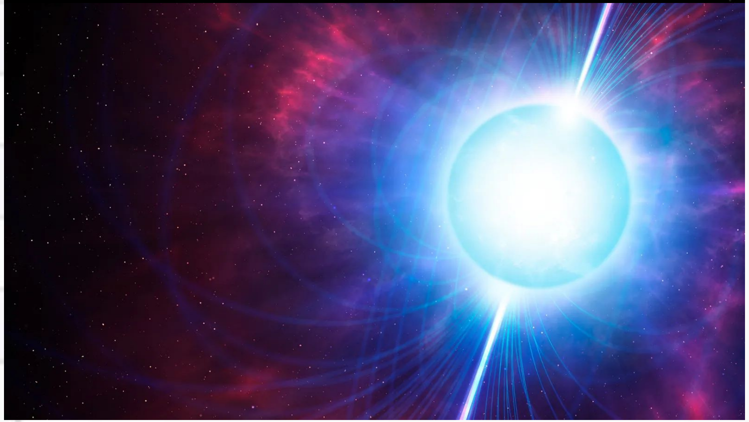
Motivation

Magnetic fields everywhere



Magnetic fields are found at all scales

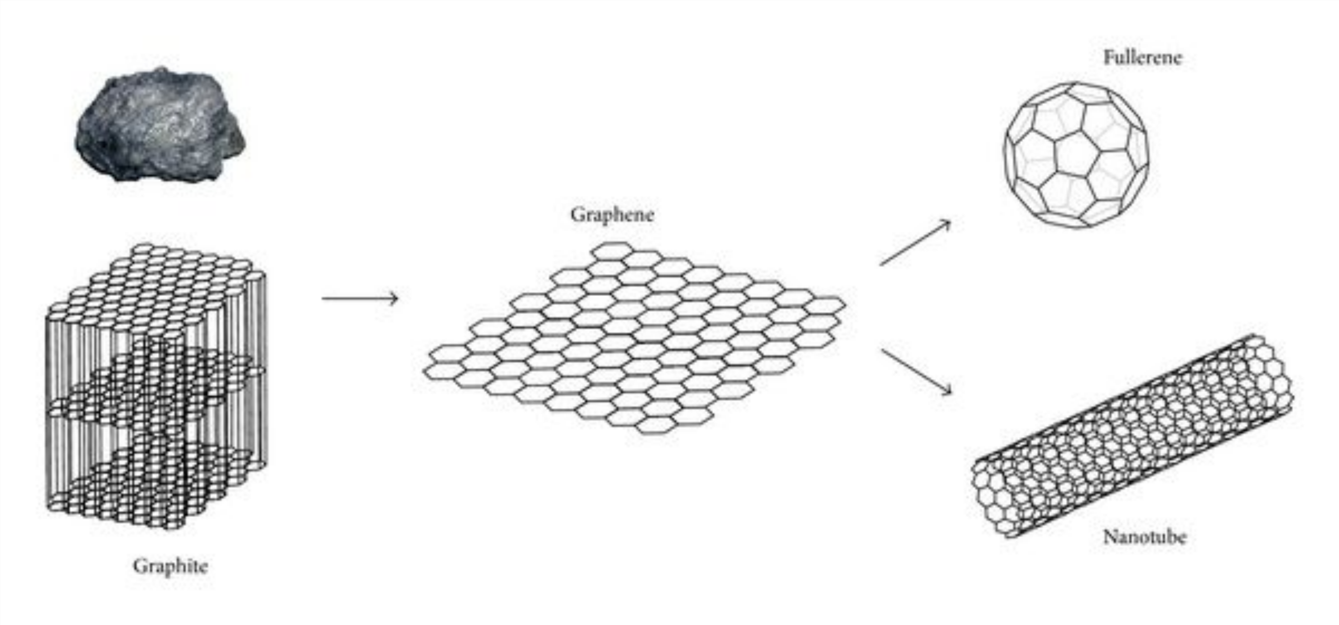
- Cosmic scale
- Astophysical scale
- Human scale
- Atomic and subatomic scale

02

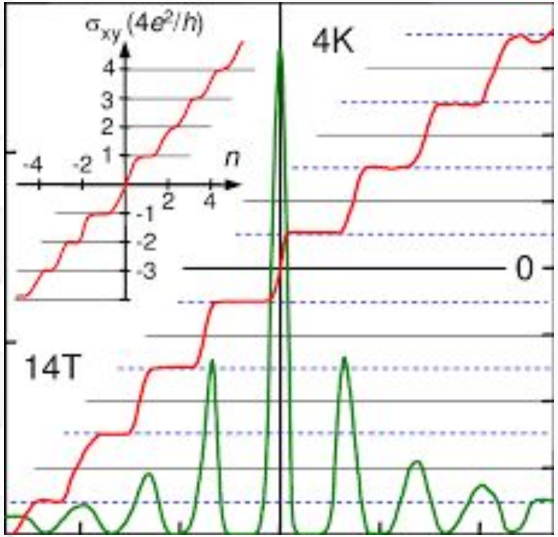
Bridges HEP-SS

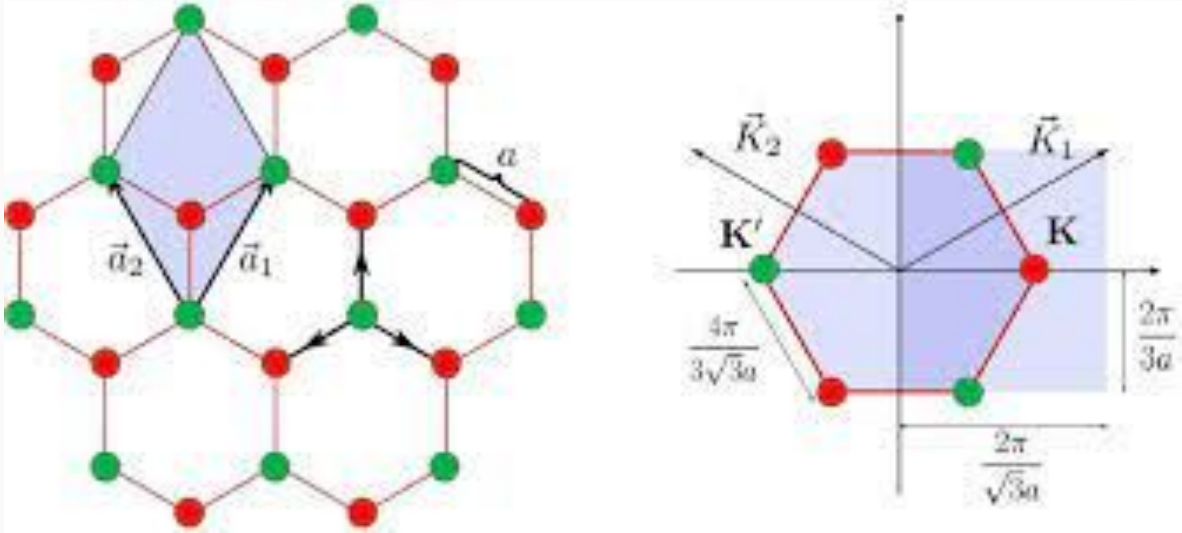
Graphene and Dirac–Weyl materials

Graphene

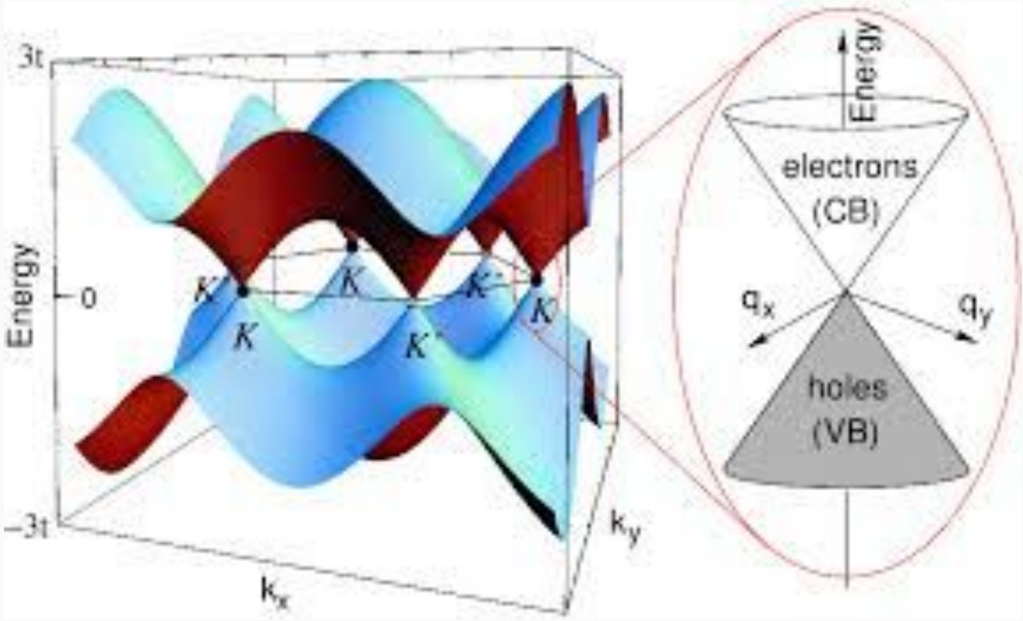


Graphene


$\sigma_{xy}\ (4e^2/h)$
4K
14T
n
0

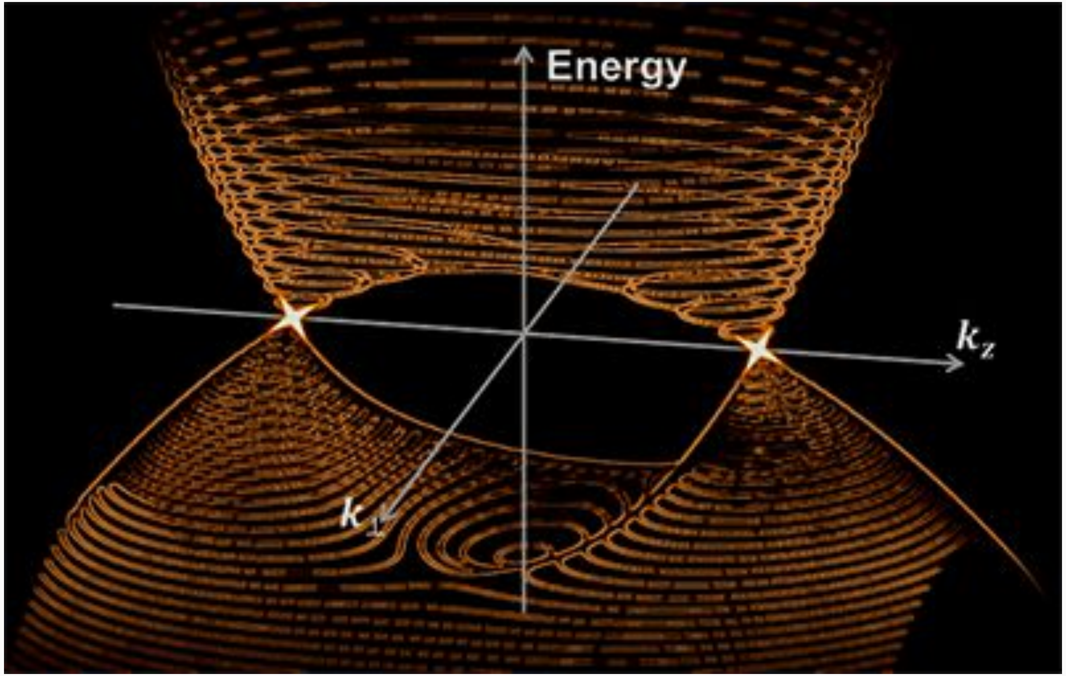

$\vec{a}_2$
$\vec{a}_1$
a
$\vec{K}_2$
$\vec{K}_1$
$\mathbf{K}'$
$\mathbf{K}$
$\frac{4\pi}{3\sqrt{3}a}$
$\frac{2\pi}{3a}$
$\frac{2\pi}{\sqrt{3}a}$

Graphene



$$H_g = v_F \vec{\sigma} \cdot \vec{p}$$

Dirac-Weyl semimetals



$$H_g = v_F \vec{\sigma} \cdot (\vec{p} - e\vec{A}) + e\phi$$

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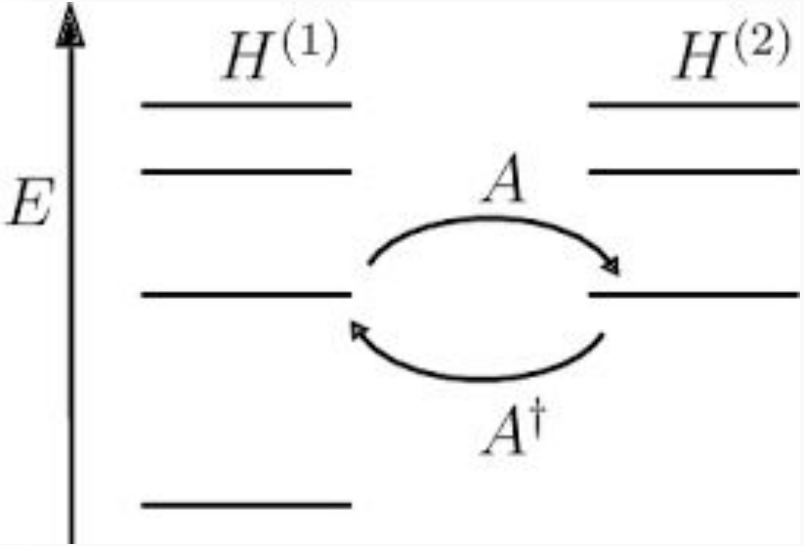
SUSY-QM

Factorization of the Dirac Hamiltonian

SUSY-QM



$$H^{(1)} = A^\dagger A,$$

$$H^{(2)} = AA^\dagger$$

$$A = \frac{d}{dx} + w(x),$$

$$A^\dagger = -\frac{d}{dx} + w(x)$$

Dirac Equation in External Fields

$$\gamma^\mu \pi_\mu \Psi_D = 0$$

$$(\gamma \cdot \pi)^2 = \pi^2 + \frac{1}{2}\sigma^{\mu\nu} F_{\mu\nu}$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

Dirac Equation in External Fields

PHYSICS REPORTS

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Physics Reports 251 (1995) 267–385

Supersymmetry and quantum mechanics

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(i) $B = \text{constant}$ (ii) $B = -a\,\text{sech}^2 y$ (iii) $B = -a\sec^2 y(-\pi/2 \le y \le \pi/2)$ (iv) $B = -c_2 \exp(-y)$.

Dirac Equation in External Fields

- *Am.J.Phys.* 78 (2010) 700-707
- Phys.Rev.D 82 (2010) 016004
- 2018 Mater. Res. Express 5 065607
- Phys.Scripta 97 (2022) 9, 095203,

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Axion-QED

General considerations

Dirac Equation in External Fields

$$\mathbf{B} = B(x)\hat{\mathbf{e}}_z, \qquad \mathbf{E} = E(z)\hat{\mathbf{e}}_z$$

$$\mathbf{A} = A(x)\hat{\mathbf{e}}_y, \qquad \phi = \phi(z)$$

$$B(x) = \frac{dA(x)}{dx}, \qquad E(z) = -\frac{d\phi(z)}{dz}$$

Dirac Equation in External Fields

$$\Psi_D(t,x,y,z) = e^{i(\varepsilon t + ky)}\Psi(x,z).$$

$$\begin{pmatrix} 0 & 0 & -i\partial_z + \phi(z) + \varepsilon & -i\partial_x - ik - iA(x) \\ 0 & 0 & -i\partial_x + ik + iA(x) & i\partial_z + \phi(z) + \varepsilon \\ i\partial_z + \phi(z) + \varepsilon & i\partial_x + ik + iA(x) & 0 & 0 \\ i\partial_x - ik - iA(x) & -i\partial_z + \phi(z) & 0 & 0 \end{pmatrix} \Psi(x,z) = 0.$$

Dirac Equation in External Fields

$$\left[\partial_z^2 + (\phi(z)+\varepsilon)^2 \pm i\frac{d\phi(z)}{dz} + \partial_x^2 - (k + A(x))^2 - \frac{dA(x)}{dx}\right]\psi_\downarrow^\pm(x,z) = 0$$

$$\left[\partial_z^2 + (\phi(z)+\varepsilon)^2 \mp i\frac{d\phi(z)}{dz} + \partial_x^2 - (k + A(x))^2 + \frac{dA(x)}{dx}\right]\psi_\uparrow^\pm(x,z) = 0,$$

$$\psi_L(x,z) = \begin{pmatrix} \psi_\downarrow^-(x,z) \\ \psi_\uparrow^-(x,z) \\ 0 \\ 0 \end{pmatrix}, \quad \psi_R(x,z) = \begin{pmatrix} 0 \\ 0 \\ \psi_\downarrow^+(x,z) \\ \psi_\uparrow^+(x,z) \end{pmatrix}, \quad \Psi(x,z) = \psi_L(x,z) + \psi_R(x,z).$$

Dirac Equation in External Fields

$$H_A^{\pm} = -\partial_x^2 + (k + A(x))^2 \pm \frac{dA(x)}{dx}, \quad H_\phi^{\pm} = -\partial_z^2 + [i(\phi(z) + \varepsilon)]^2 \pm i\frac{d\phi(z)}{dz},$$

$$\mathrm{w}_A = k + A(x), \quad \mathrm{w}_\phi = i(\varepsilon + \phi(z))$$

$$[H_A^- + H_\phi^{\pm}]\psi_\uparrow^{\pm} = 0, \quad \left[H_A^+ + H_\phi^{\mp}\right]\psi_\downarrow^{\pm} = 0$$

Dirac Equation in External Fields

$$\psi_{\uparrow}^{\pm} = \chi_{\uparrow}^{-}(x)\zeta_{\uparrow}^{\pm}(z), \quad \psi_{\downarrow}^{\pm} = \chi_{\downarrow}^{+}(x)\zeta_{\downarrow}^{\mp}(z),$$

$$H_A^{\pm}\chi_{\uparrow\downarrow}^{\pm} = \varepsilon_A \chi_{\uparrow\downarrow}^{\pm}, \quad H_\phi^{\pm}\zeta_{\uparrow\downarrow}^{\pm} = \varepsilon_\phi \zeta_{\uparrow\downarrow}^{\pm}.$$

$$\varepsilon_A = -\varepsilon_\phi.$$

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Example

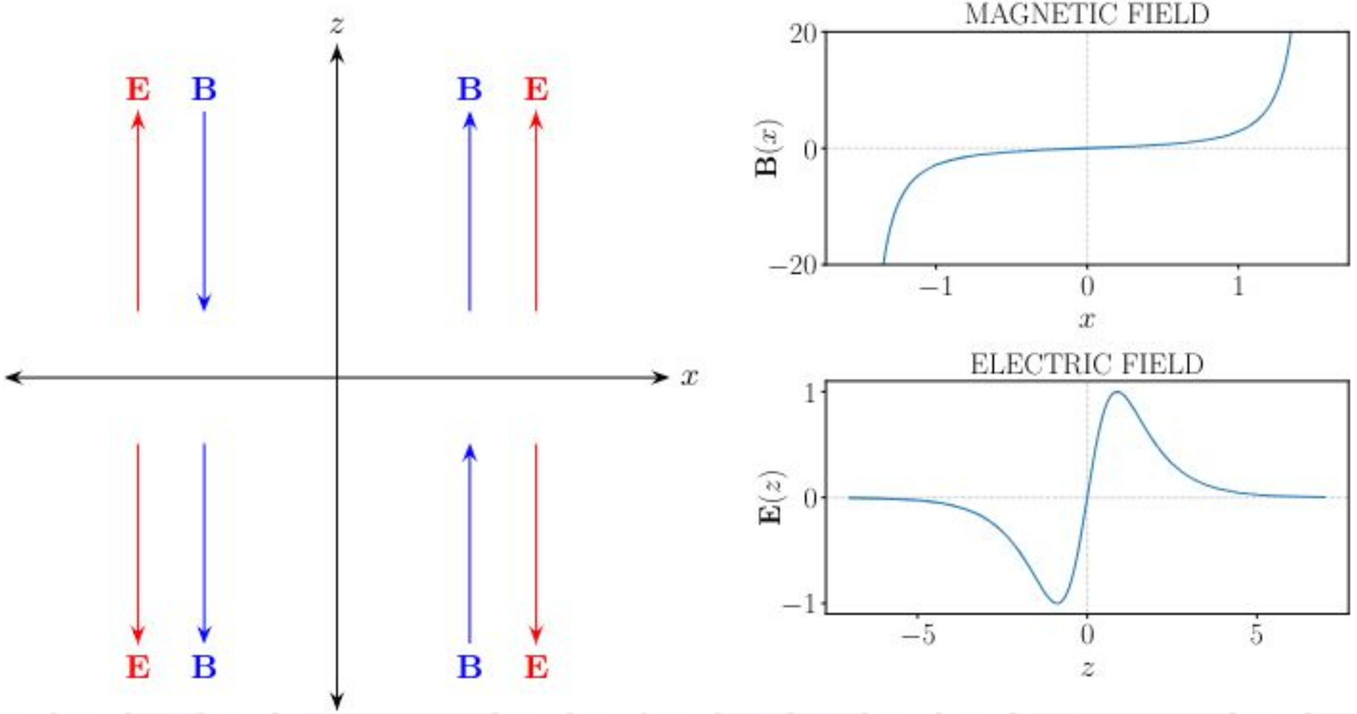
Analytical solution

Pöschl-Teller-like potentials

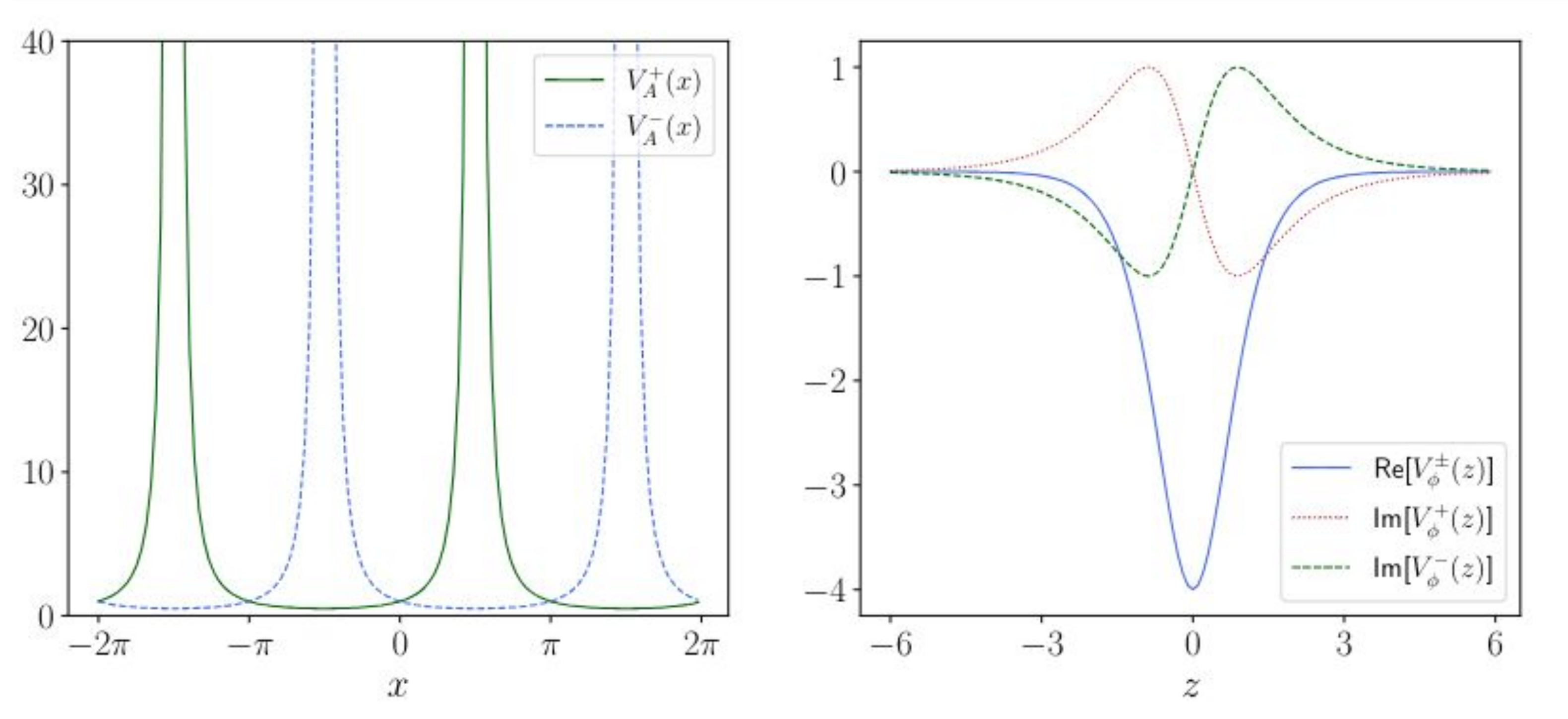
$$A(x) = \frac{B_0}{\nu}\sec(\nu x), \quad -\frac{\pi}{2} < \nu x < \frac{\pi}{2}; \; \phi(z) = \frac{E_0}{\mu}\mathrm{sech}(\mu z)$$

$$\begin{aligned} V_A^{\pm}(x) &= k^2 + 2kD_A\sec(\nu x) + D_A^2\sec^2(\nu x) \pm \nu D_A\sec(\nu x)\tan(\nu x), \\ V_\phi^{\pm}(z) &= -\varepsilon^2 - 2\varepsilon D_\phi \mathrm{sech}(\mu z) - D_\phi^2 \mathrm{sech}^2(\mu z) \mp i\mu D_\phi \mathrm{sech}(\mu z)\tanh(\mu z) \end{aligned}$$

Pöschl-Teller-like potentials



Pöschl-Teller-like potentials



Pöschl-Teller-like potentials

$$\zeta_n^{\pm}(u) = (1-u)^{\frac{1}{4}\left(1-\sqrt{1+4S_\phi(S_\phi\pm1)}\right)}(1+u)^{\frac{1}{4}\left(1-\sqrt{1+4S_\phi(S_\phi\mp1)}\right)}$$

$$\times P_n^{\left(-\frac{1}{2}\sqrt{1+4S_\phi(S_\phi\pm1)},-\frac{1}{2}\sqrt{1+4S_\phi(S_\phi\mp1)}\right)}(u), \quad S_\phi = D_\phi/\mu.$$

$$\varepsilon_\phi = -\mu^2\left[n(n-Q_\phi+1)+\frac{(1-Q_\phi)^2}{4}\right].$$

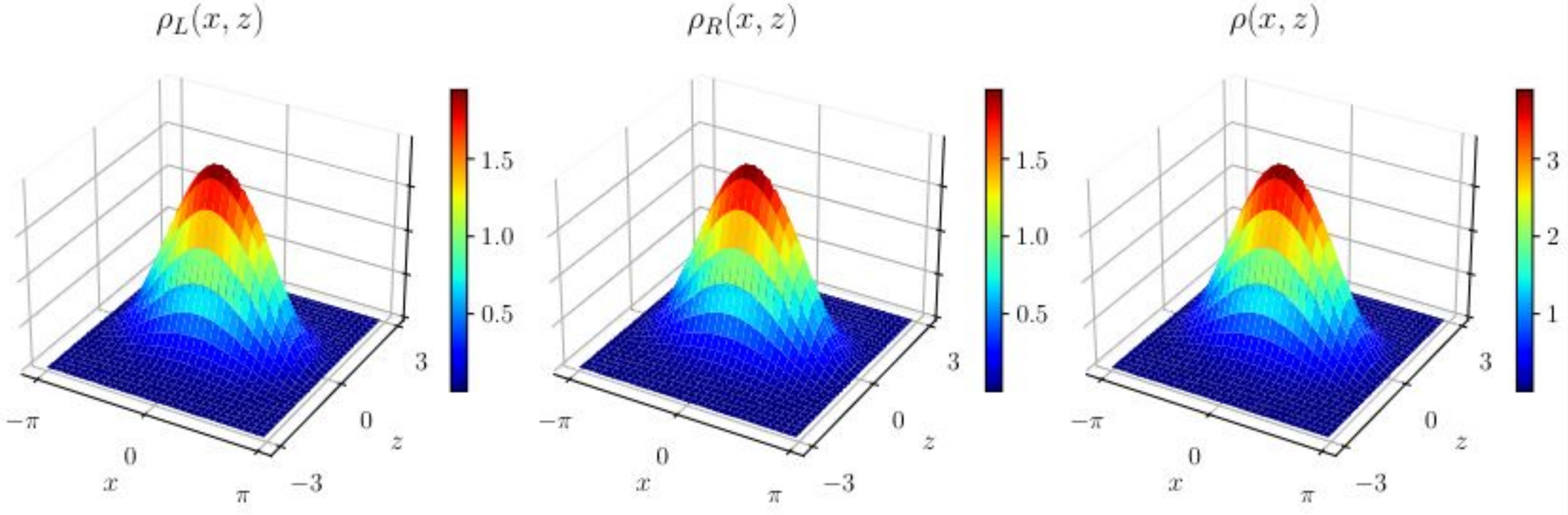
Pöschl-Teller-like potentials

$$\chi_n^{\pm}(u) = \quad (1-u)^{\frac{1}{4}\left(1+\sqrt{1+4S_A(S_A\pm1)}\right)}(1+u)^{\frac{1}{4}\left(1+\sqrt{1+4S_A(S_A\mp1)}\right)} \times P_n^{\left(\frac{1}{2}\sqrt{1+4S_A(S_A\pm1)},\frac{1}{2}\sqrt{1+4S_A(S_A\mp1)}\right)}(u),$$

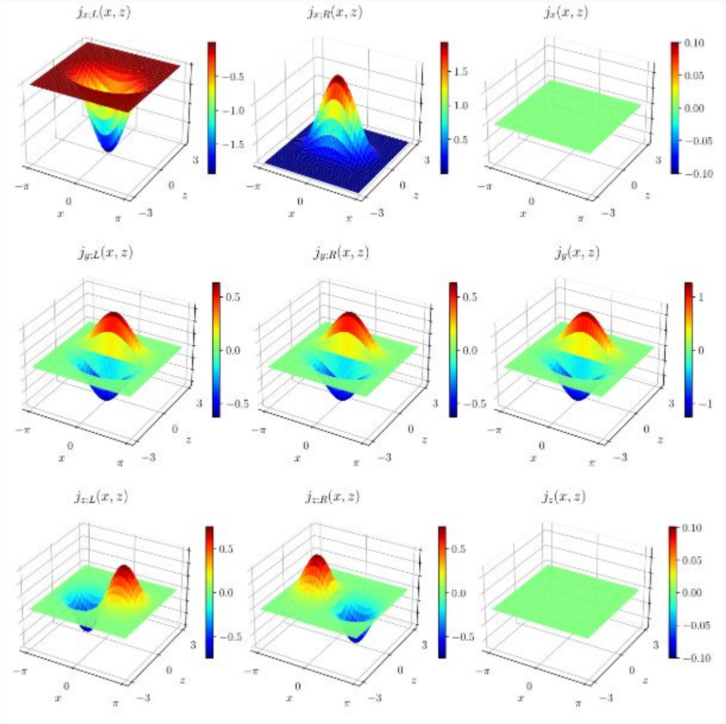
$$\varepsilon_A = \nu^2\left[n(n+Q_A+1)+\frac{(1+Q_A)^2}{4}\right], \quad Q_A = \frac{\sqrt{1+4S_A(S_A+1)}+\sqrt{1+4S_A(S_A-1)}}{2}.$$

$$k = n = 0$$

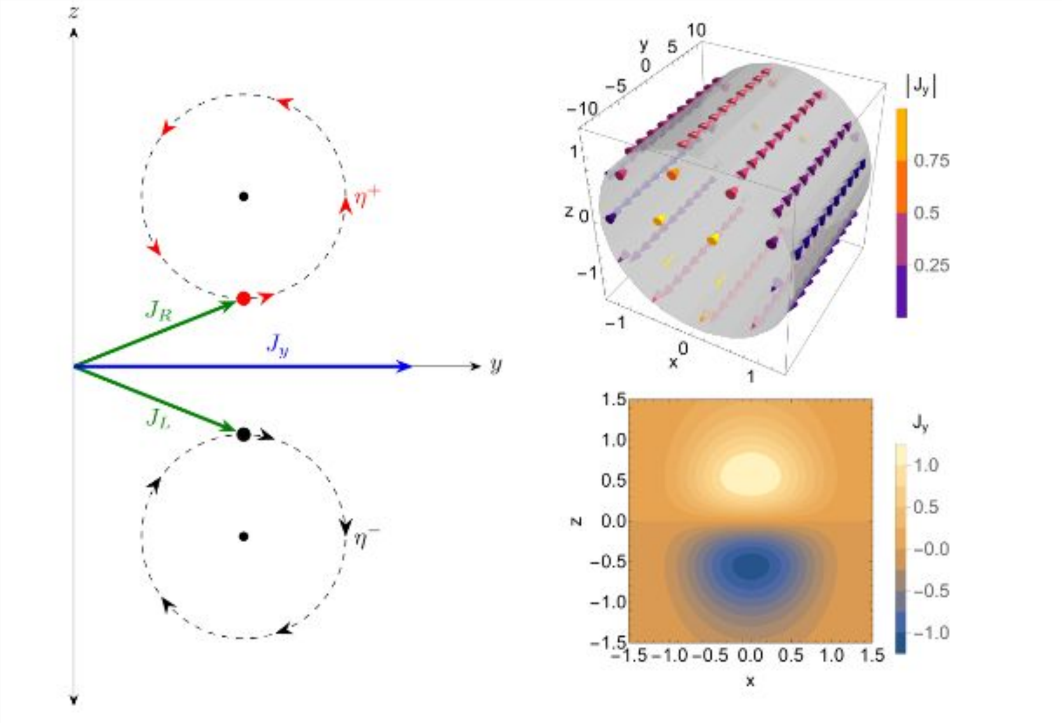
Pöschl-Teller-like potentials



Pöschl-Teller-like potentials



Pöschl-Teller-like potentials



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Concluding

Final Remarks

Conclusions

- Dirac Meterials in a non-uniform static parallel electric and magnetic field configuration
- Pöschl-Teller like potentials
- Zero mode analytically found
- Non-vanishing current density along y-axis
- PHE with 90° angle
- Driven by chiral symmetry
- Connection with JR model

Open questions

- Can the field configuration be achieved in experiments?
- Excited states?
- Other field configurations?

Thanks!

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