Pion coupling to Constituent quarks and the Yukawa potential at weak magnetic field

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Workshop on Strongly Interacting Matter
in Extreme Magnetic Fields
ECT* Trento, 28th September 2023

In part: in collaboration Cristian Villavicencio and Marcelo Loewe
(What started just after the workshop in ICTP-SAIFR-IFT - São Paulo 2022)

* FL Braghin, M Loewe, C Villavicencio,
The Yukawa potential under weak magnetic field,
arXiv:2308.12122

* FL Braghin, WF de Sousa,
Form factors for pions couplings to constituent quarks under weak magnetic field,
J. Phys. G: Nucl. Part. Phys. 47 (2020) 045110

Group at UFG (2022-2023):
FLB, W.F. de Sousa, I.deM. Froldi, D.de A. Camargos

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CNPq

Presentation Outline

1. Motivations/context
2. Magnetic field corrections
   - Pion propagator under B
   - Meson- constituent quarks under B field
   - Quark-antiquark interaction- dynamical calculation
   - B dependent masses at $(eB) \sim 0.1 M_q^2$
   - Fourier transforms: The magnetic field corrections
3. Numerical results
4. Summary

Motivations, context

Strong magnetic fields in strongly interacting matter:
* (Peripheric) relativistic heavy ion collisions: reduced strength
* Magnetars, dense stars

How to understand hadrons/quark+gluons in a magnetic field ?
* given that they are not even fully understood in the vacuum
Manifestations of quark/gluon degrees of freedom
in the Hadron/Nuclear observable world ?

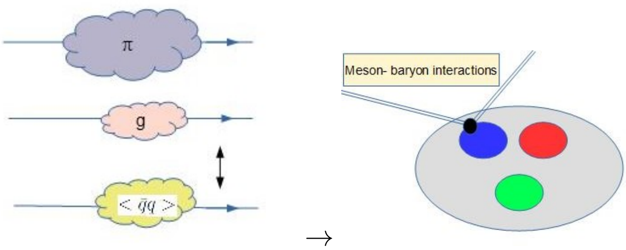
To account for everything at once: only lattice-QCD
* Analytical calculations still important to identify/ keep track of microscopic specific effects $\rightarrow$ effective models and calculations

Particle (hadron) properties and dynamics may be modified:
* Masses (as poles of two point GF)
* Interactions - it helps to decompose a complicate dynamics

The constituent quark model

Since GellMann/Zweig: Morpurgo, Dalitz, De Rujula etal, Lavelle, and many other

(Low energy) QCD effective models: **global hadron properties**



* Dynamical Chiral Symmetry Breaking: $< \bar{q}q >$ masses/couplings

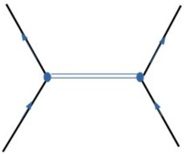
* How to understand nucleon and nuclear interactions from QCD ...

The Yukawa potential:

1934-35 H. Yukawa improved the Heisenberg proposal
Predicting a *heavy quantum / mesotron* for nucleon-interactions

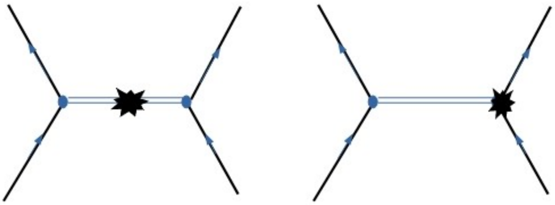
From the Klein-Gordon equation / one-pion exchange / etc

$$V(R) = -\frac{g^2}{4\pi}\frac{e^{-m_\pi R}}{R} \quad \leftarrow \quad V(R) = \mathcal{F}(\tilde{V}(\vec{Q}))$$



Magnetic field corrections

$\left(\frac{eB}{m_\pi^2}\right)^2$ $\left(\frac{eB}{M^{*2}}\right)^2$ (pion PS coupling)



* Besides that: corrections to Pion and Constituent quark masses:

$$\Delta M \sim \frac{(eB)}{M}$$

Spin 0 field propagator under B
Weak magnetic field limit $(eB) << m_\pi^2$

$$
\begin{aligned}
iD^B(Q) &\simeq \frac{i}{Q^2 - m^2}\left[1 - (eB)^2\left(\frac{1}{(Q^2-m^2)^2} + \frac{2Q_\perp^2}{(Q^2-m^2)^3}\right)\right] \\
&\equiv D_0(Q) + (eB)^2 D_1^B(Q, Q_\perp) \qquad (1)
\end{aligned}
$$

A. Ayala, A. Sanchez, G. Piccinelli, S. Sahu, Phys. Rev. D71, (2005).

It generates isotropic and anisotropic corrections

Pion PS coupling to constituent quarks: pion form factor

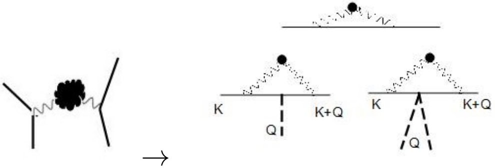
*For the usual Yukawa potential:
Pion couples to the pseudoscalar current
(although the axial coupling also leads to Yukawa potential)

* HOwever constituent quarks are better defined with vector current (gluons)

* A method for calculating the pion form fator is reminded next

* Magnetic field contribution for the form factor:

How to obtain constituent quarks dynamically: quark determinant



$$Z[\eta,\bar\eta] = N \int \mathcal{D}[\bar\psi,\psi]$$

$$\exp\ i\int d^4x \quad \left[\bar\psi\left(i\not{D} - m\right)\psi - \frac{g^2}{2}\int_y j^\beta_\mu(x)\tilde{R}^{\mu\nu}_{\beta\alpha}(x-y)j^\alpha_\nu(y) + \bar\psi\eta + \bar\eta\psi\right],$$

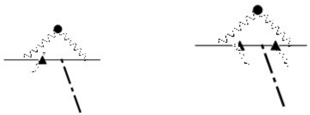
color quark current $j^\mu_\alpha = \bar\psi\lambda_\alpha\gamma^\mu\psi$,
$i, j, k = 0, ...(N_f^2 - 1)$ for U($N_f = 2$), $\alpha, \beta ... = 1, ...(N_c^2 - 1)$

Fierz transformation $\rightarrow$ all flavor-Dirac (and color) channels
Auxiliary fields: suitable for quark-antiquark states

FLB: Phys.Rev. D (2018,2019), Journ.Phys. G (2020)

Meson- constituent quarks under B field

Expansion of quark determinant (some ambiguities-symmetries)



$$
\begin{aligned}
S_{0,c}(k) &= S_0(k) + S_1(k)(eB_0) \\
&= \frac{\not{k} + M^*}{k^2 - {M^*}^2} + i\gamma_1\gamma_2 \frac{(\gamma_0 k^0 - \gamma_3 k^3 + M^*)}{(k^2 - {M^*}^2)^2}(eB_0). \qquad (2)
\end{aligned}
$$

T.-K. Chyi, et al, Phys. Rev. D 62, (2000).

Coupling constants ($K = Q = 0$) or ($Q^2 = M_\pi^2$) ..
Correct order of magnitude (fixing one coupling constant)
* Pseudoscalar, scalar, vector and axial pion couplings vacuum/B

F.L.B., W.F.deS., Journ. Phys. G (2020)

At the end: one has the following Lagrangian term SU(2)

$$\mathcal{L}_{\pi-Q(B)} = c_i \, F_{ps}^{B}(Q,K) \, P_i(Q)(\bar{\psi} i \gamma_5 \lambda_i \psi)^{\dagger}, \qquad i = 1,2,3 \tag{3}$$

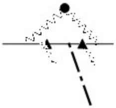
$c_1 = c_2 = -4/9$ and $c_3 = 5/9$.

$$F_{ps}^{B}(Q,K) = \left(\frac{eB_0}{M^2}\right)^2 \left[F_{ps}^{B,iso}(Q,K) + F_{ps}^{B,ani}(Q,K)\right] \tag{4}$$

Being that, form factor = integral in internal momentum

$$F_{ps}^{B}(Q,K) \to F_{ps}^{B}(Q^2, K^2, K \cdot Q)$$

For the Yukawa potential
(static) Constituent quarks: on shell $K^2 = M_q^2$
Off Shell pions : $Q^2 = -\vec{Q}^2$



Just a comment on this diagram:
Schwinger phase can be gauged away and does not contribute
Ayala,Martines, Loewe, Yeomans, Zamora, PRD91 (2015)

(Effective) Gluon propagator

To calculate: need of a gluon propagator
To garantee as much as possible analytical calculation
An effective gluon propagator inspired in Cornwall 1981:

$$R(k) = \frac{K_g}{(k^2 - M_G^2)^2} \quad \text{for} \quad M_g \sim 0.5 \; GeV \tag{5}$$

$K_g = 8\pi\sigma$, the string tension
* This is Minkowski version of effective confining propagator

Here: K_g is fixed to reproduce the pion PS coupling to constituent quarks ($G_{ps} \simeq 13$) - as calculated by the same method used for pion form factor in Bfield

$$\sigma_{M_q=0.35GeV} \sim \frac{K_F}{8\pi} \sim 0.082 GeV^2 \qquad \sigma_{3M_q \sim 1.05GeV} \sim 0.7 GeV^2$$

* To fix G_{ps} is/as a renormalization condition
* Advantage: results are UV finite

B dependent masses shifts at $(eB) \sim 0.1 M_q^2$

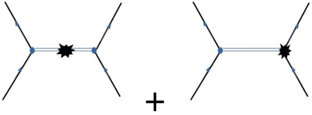
The behavior of the pion mass has been investigated:
Lattice QCD , effective models NJL-model:
many discussed in this workshop and more

$$
\begin{aligned}
m_{\pi^0}(B) &\sim (m_{\pi^0}^{(0)} - 0.020) GeV, \quad \rightarrow pinB \\
m_{\pi^\pm}(B) &\sim (m_{\pi^\pm}^{(0)} + 0.020) GeV, \quad \rightarrow picB
\end{aligned} \tag{6}
$$

The pion form factor depends on the quark constituent mass
$M_q \sim 0.35$ GeV

$$
M_q(B) \sim (M_q + 0.020) GeV, \quad \rightarrow MqB \tag{7}
$$

Fourier transforms: The magnetic field corrections



$$
\begin{aligned}
\tilde{V}(Q) &= G^B_{ps}(K,Q) D^B_\pi(Q,Q_\perp) G^B_{ps}(K,Q) \\
&= \left(g_{ps} + F^B_{ps}(K,Q) \frac{(eB)^2}{M^4} \right) \left(D_0(Q) + (eB)^2 D^B_1(Q,Q_\perp) \right) \\
&\times \left(g_{ps} + F^B_{ps}(K,Q) \frac{(eB)^2}{M^4} \right) + \ldots \\
&\simeq V_0(Q^2) + V^B_\pi(Q_z,Q_\perp) + V^B_{FF}(Q_z,Q_\perp) + \ldots \qquad (8)
\end{aligned}
$$

AND With the roles of Magnetic field
On the pion mass and constituent quark mass

$$V(\vec{R}) = \int \tilde{V}(\vec{Q}^2) e^{-i\vec{Q}\cdot\vec{R}} \frac{d^3Q}{(2\pi)^3}. \tag{9}$$

Results from each of the contributions:

Effects due to pion propagator

$$V_\pi^B(R, R_z) = -m\frac{g_{ps}^2(eB)^2}{32m^4}\left[\left(m|\vec{R}| + 1\right)e^{-m|\vec{R}|} + 2m^3\mathcal{J}_2(R, R_z)\right]$$

$$\mathcal{J}_2(R, R_z) \equiv \int dQ_\perp Q_\perp^3 e^{-R_z E_Q} J_0(R_\perp k_\perp) \times \left(\frac{R_z^3}{6E_Q^4} + \frac{R_z^2}{E_Q^5} + \frac{5R_z}{2E_Q^6} + \frac{5}{2E_Q^7}\right)$$

$$E_Q = \sqrt{\vec{Q}^2 + m^2}$$

$$\frac{I_1 I_2}{V_{Yuk}}, \qquad V_{Yuk} = -g_{ps}^2\frac{e^{-m_\pi R}}{4\pi R}$$

Effects due to the pion form factor

$$\begin{aligned} V^B_{FF}(R) &= V_{iso}(R) + V_{ani}(R_z, R_\perp) \\ &= \left(\frac{eB_0}{M^2}\right)^2 \tilde{C}^B_{PS} C_i \int \frac{d^3Q}{(2\pi)^3} e^{-i\vec{Q}\cdot\vec{R}} \frac{I_4(\vec{Q}^2)}{\vec{Q}^2 + m^2} \\ &+ \left(\frac{eB_0}{M^2}\right)^2 \tilde{C}^B_{PS} C_i \int \frac{d^2Q_\perp dQ_z}{(2\pi)^3} e^{-i(Q_z R_z + Q_\perp \cdot R_\perp)} \frac{I_5(Q_\perp^2, Q^2)}{\vec{Q}^2 + m^2}, \end{aligned} \tag{10}$$

being $\vec{Q}^2 = Q_z^2 + Q_\perp^2$.

$$\frac{V_{45}}{V_{Yuk}} = \frac{V_{iso}(R) + V_{ani}(R, R_z)}{V_{Yuk}}, \qquad V_{Yuk} = -g_{ps}^2 \frac{e^{-m_\pi R}}{4\pi R}$$

Example of the resulting terms:

$$V_{iso} = C_{iso} \int \frac{Q^2\ dQ}{(2\pi)^2} \frac{(e^{-iQR} - e^{iQR})}{iQR} \frac{I_4(Q^2)}{\vec{Q}^2 + m_\pi^2} \equiv C_{iso}[\mathcal{F}_{4a}(R) + \mathcal{F}_{4b}(R)$$

by using: $|\vec{Q}| = \pm i\phi \equiv \pm i\sqrt{\frac{M^2(1-z)^2 + M_g^2 z}{y(1-y)}}$

y, z Feynman parameters

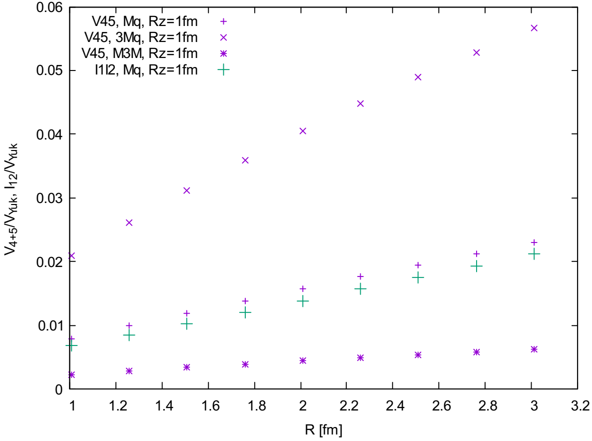
$$\mathcal{F}_{4a} = -\frac{1}{16\pi^3}\int_{y,z}[(1-y-z)yz]\left[\frac{2(e^{-\phi R} - e^{-m_\pi R})}{R(\phi^2 - m_\pi^2)^3} + \frac{Re^{-\phi R}}{4\phi^2(\phi^2 - m_\pi^2)} + \frac{e^{-\phi R}}{4\phi^3(\phi^2 - m_\pi^2)} + \frac{e^{-\phi R}}{\phi(\phi^2 - m_\pi^2)^2}\right] \quad (11)$$

$$\mathcal{F}_{4b} = \frac{-1}{32\pi^3}\int_y\int_z \frac{(1-y-z)yz}{[(y(1-y)]^4} F_{4b} \quad (12)$$

$$F_{4b} = \frac{R^2 e^{-\phi R}}{8\phi^3(m_\pi^2 - \phi^2)} + \frac{3Re^{-\phi R}}{8\phi^2(m_\pi^2 - \phi^2)} + \frac{3e^{-\phi R}}{8\phi^5(m_\pi^2 - \phi^2)} + \frac{3Re^{-\phi R}}{4\phi^2\left(m_\pi^2 - \phi^2\right)^2} - \frac{3e^{-\phi R}}{4\phi^3(m_\pi^2 - \phi^2)^2} + \frac{6e^{-\phi R}}{2\phi(m_\pi^2 - \phi^2)^3} + \frac{6\left(-e^{-\phi R} + e^{-m_\pi R}\right)}{(m_\pi^2 - \phi^2)^4} \quad (13)$$

$$V_{ani}(R_z, R_\perp) = C_{iso} \int \frac{d^2 Q_\perp dQ_z}{(2\pi)^3} e^{-i(Q_z \cdot R_z + Q_\perp \cdot R_\perp)} \frac{I_5(\vec{Q}^2, \vec{Q}_\perp^2)}{\vec{Q}^2 + M_\pi^2} \quad (14)$$

Numerical Results: separate contributions



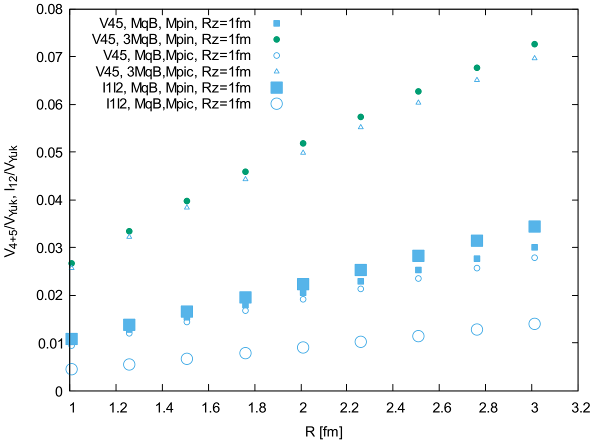
$V45 \rightarrow$ pion form factor (iso + ani)

$I_1 I_2 \rightarrow$ pion propagator (iso + ani)

$3Mq \rightarrow \quad M_q \sim 3 \times 0.35$GeV

$M3M \rightarrow$ to fix g_{ps} and after letting $3M_q$ for trivial form factors

Numerical Results: separate contributions



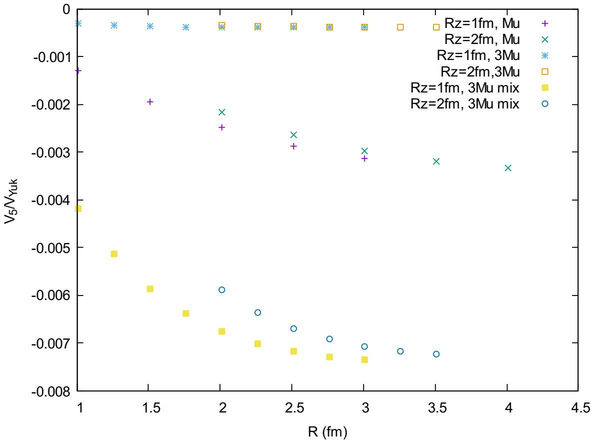
* Constituent Quark mass with magnetic field correction

* Non degenerate pion masses B=0 ($\Delta m_\pi = 4$MeV), FF$_{\pi^\pm,\pi^0}$

$V45 \to$ pion form factor (iso + ani)

$I_1 I_2 \to$ pion propagator (iso + ani)

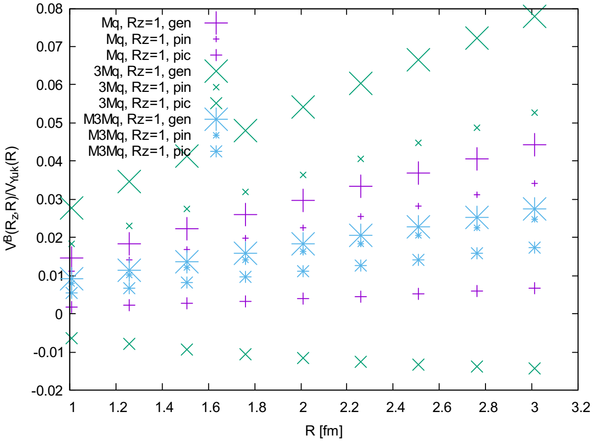
Numerical Results: anisotropic V_5 FF



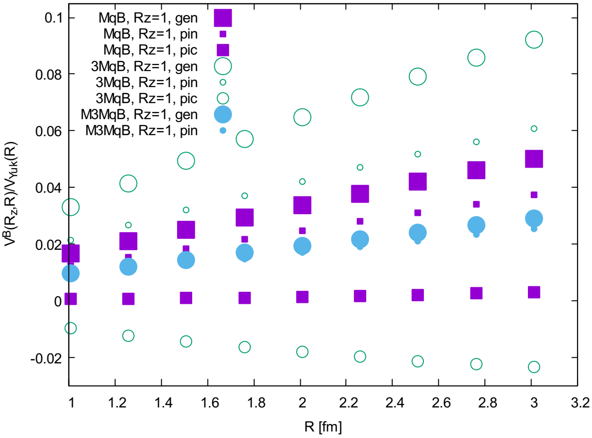
* Constituent Quark mass B
* Non degenerate pion masses at B

$FF_{\pi^\pm,\pi^0}$ (pin+ pic)

Numerical Results: all contributions



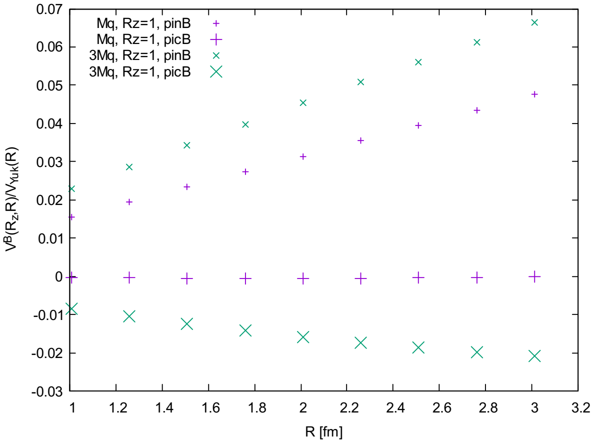
* Constituent Quark mass B=0
* Non degenerate pion masses B=0 ($\Delta m_\pi = 4$MeV) FF$_{\pi^\pm,\pi^0}$



* Constituent Quark mass B
* Non degenerate pion masses B=0 ($\Delta m_\pi = 4$MeV)

$FF_{\pi^\pm,\pi^0}$ (pin+ pic) and *FF* (gen)

Numerical Results: all contributions

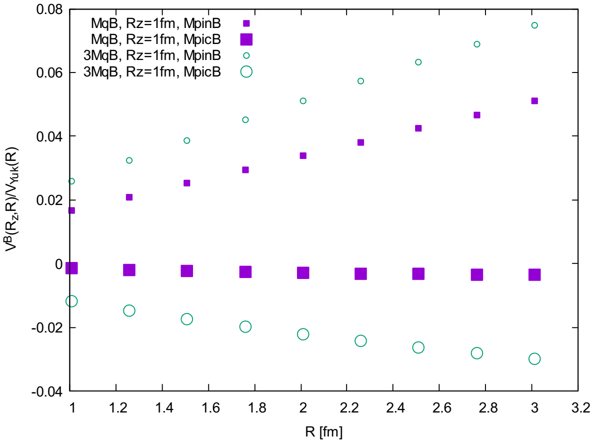


* Constituent Quark mass B=0

* Non degenerate pion masses at B

$FF_{\pi^\pm,\pi^0}$ (pin+ pic)

Numerical Results: all contributions



* Constituent Quark mass B
* Non degenerate pion masses at B

$FF_{\pi^\pm,\pi^0}$ (pin+ pic)

Summary

- Pion propagator / Form factor / B field on mass
  $\rightarrow$ corrections of the same order of magnitude
- Neutral and charged pions exchange $\rightarrow$ different behaviors (B)
- Sizeable effects of quark and gluon effective masses on $V(R)$
- Missing: B-effect on gluon propagator

On going/planned:

- Effect of other couplings/particles couplings to constituent quarks
- (Similar effects at B=0) finite baryonic density, finite temperature, etc
- What does it happen to the deuteron?

Thank you for your attention!