

Nonequilibrium dynamics of the quark condensate under a strong magnetic field

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Strongly Interacting Matter in Extreme Magnetic Fields

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Motivation

1. Time dependence is a common feature of strong magnetic fields affecting QCD properties in the early universe, magnetars, and heavy-ion collisions
2. The quark condensate is a prominent QCD property affected by a strong magnetic field
3. Time scales of the condensate dynamics are important for hadron production in heavy-ion collisions
4. What is the effect of a magnetic field on the quark condensate dynamics?
5. Here: formalism to tackle time dependence based on a nonequilibrium QFT calculation and first estimates

Work with Carlsson Miller, Symmetry 13, 551 (2021) + Arthur Frazon (in progress)

Phase change - time dependence

Typical situation:

- A system is forced to change from a thermodynamic equilibrium phase to another, out-of-equilibrium phase
- Evolution to new equilibrium through spatial fluctuations that take the system (initially homogeneous) through a sequence of highly (not in equilibrium) inhomogeneous states

Theory: coarse-graining

Rational:

- It is difficult/impossible to describe the system with microscopic d.o.f.
- Focus on a small number of semi-macroscopic variables; **order parameters φ**
- Dynamics of φ is slower than that of the microscopic degrees of freedom; **described by Ginzburg-Landau-Langevin type of equations**



From A. Zee book

Dynamical equations

Landau: system's state characterized by a macroscopic free energy $F[\varphi]$

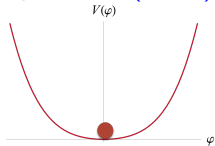
Example: $F[\varphi] = \int d^3x [\kappa(\nabla\varphi)^2 + V(\varphi)], \quad V(\varphi) = \frac{1}{2} r \varphi^2 + \frac{1}{4} u \varphi^4$

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Equilibrium ($r > 0$):



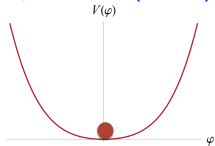
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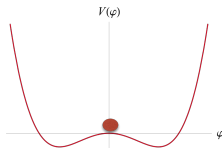
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Equilibrium ($r > 0$):



$$\frac{\delta F}{\delta \varphi} = 0$$

Out-of-equilibrium ($r = -|r| < 0$):



$$\varphi(x) \rightarrow \varphi(x, t) : \quad \eta \frac{\partial \varphi}{\partial t} = -\frac{\delta F}{\delta \varphi}$$

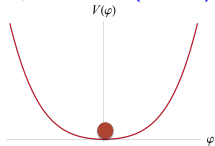
Near equilibrium: $\tilde{\varphi}(k, t) \approx e^{(|r|-k^2)t/\eta}$

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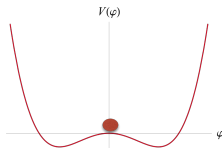
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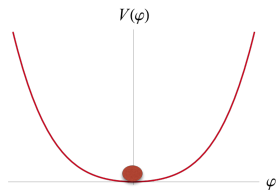


$$\varphi(x) \rightarrow \varphi(x, t) : \quad \eta \frac{\partial \varphi}{\partial t} = -\frac{\delta F}{\delta \varphi}$$

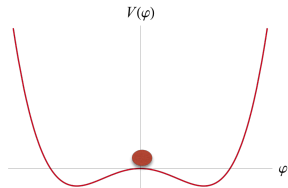
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Purely diffusive dynamics - no (thermal) fluctuations

Phase change, fluctuations



$$T > T_c$$



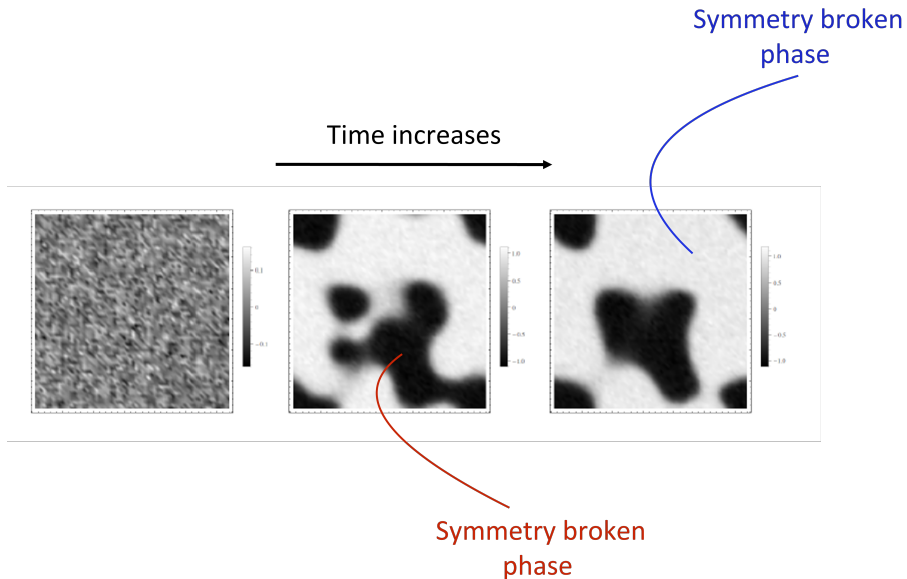
$$T < T_c$$

$$\eta \frac{\partial \varphi}{\partial t} = -\frac{\delta F}{\delta \varphi} + \xi(x, t) \quad \leftarrow \text{Ginzburg-Landau-Langevin (GLL) equation}$$

$$\langle \xi(x, t) \xi(x', t') \rangle = \sqrt{2\eta T} \delta(x - x') \delta(t - t')$$

Fluctuation-dissipation theorem

Phase change



Dynamical universality classes

Dynamical phase changes/transitions
can be classified in universality classes according
to the nature and couplings of the order parameters*

- Model A: nonconserved order parameter
- Model B: conserved order parameter
- Model C: nonconserved and conserved order parameters
-

P. C. Hohenberg & M.I Halperin, Rev. Mod. Phys. 49, 435 (1977).

Derivation of GLLeq - Linear σ model*

$$\mathcal{L} = \bar{q}[i\not{\partial} - g(\sigma + i\gamma_5\boldsymbol{\tau} \cdot \boldsymbol{\pi})]q + \frac{1}{2} [\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\boldsymbol{\pi} \cdot \partial^\mu\boldsymbol{\pi}] - U(\sigma, \boldsymbol{\pi})$$

$$U(\sigma, \boldsymbol{\pi}) = \frac{\lambda}{4}(\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2 - h_q\sigma - U_0$$

Parameters: $v^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda^2}$, $m_\sigma^2 = 2\lambda^2 f_\pi^2 + m_\pi^2$, $h_q = f_\pi m_\pi^2$, $m_q = g\langle\sigma\rangle$, $U_0: U(0,0) = 0$

Crossover at $T \simeq 150$ MeV: $g = 3.3$

Quark condensate: $m\langle\bar{q}q\rangle_{\text{QCD}} = -h_q\langle\sigma\rangle$

Coupling magnetic field: $\partial_\mu \rightarrow D_\mu = \partial_\mu + iqA_\mu$ (for the charged fields)

The scenario, approximations

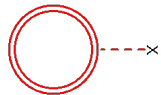
- QGP scales for temperature (T) and magnetic field (B)
- Perturbation around local equilibrium (medium at local T and B)
- No expansion & energy transfer field and medium
- Source of dissipation η : $\sigma \rightarrow \bar{q}q$ (no pions)
- Look at qualitative changes due to strong B
- Analytical understanding

Closed time path formalism

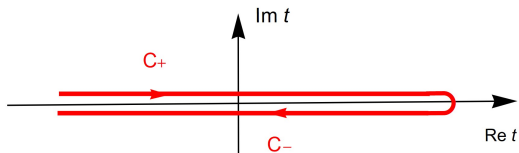
Semiclassical effective action:

$$\Gamma[\sigma, S] = \Gamma_{\text{cl}}[\sigma] + i \text{Tr} \ln S - i \text{Tr} (i \not{D} - m_0) S + \Gamma_2[\sigma, S]$$

$$\Gamma_2[\sigma, S] = g \int_{\mathcal{C}} d^4x \text{tr} [S^{++}(x, x)\sigma^+(x) + S^{--}(x, x)\sigma^-(x)]$$



- σ and S defined on the Schwinger-Keldysh contour (CTP contour), σ^\pm , S^\pm
- σ^\pm , S^\pm are not independent, they are equal at some large time (CTP b.c.)



Integrate out the quarks

$$\frac{\delta\Gamma[\sigma, S]}{\delta S^{ab}(x, y)} = 0$$



$$(i\not{D} - g\sigma_0(x)) S^{ab}(x, y) - \int_C d^4z \frac{\delta\Gamma_2[\sigma, S]}{\delta S^{ac}(x, z)} S^{cb}(z, y) = i\delta^{ab}\delta^{(4)}(x - y)$$

Very difficult to solve (even numerically): expand around local equilibrium

$$\sigma^a(x) = \sigma_0^a(x) + \delta\sigma^a(x)$$

$$S^{ab}(x, y) = S_{\text{thm}}^{ab}(x, y) + \delta S^{ab}(x, y) + \delta^2 S^{ab}(x, y) + \dots$$

where $\frac{\delta\Gamma_{\text{cl}}}{\delta\sigma_0^a(x)} = -g\text{Tr}S^{aa}(x, x)$ and $[i\not{D} - m_0 - g\sigma_0(x)] S_{\text{thm}}^{ab}(x, y) = -i\delta^{ab}\delta^{(4)}(x - y)$

GLL equation

- Dissipation ($\sigma \rightarrow \bar{q}q$): imaginary part of $\Gamma[\sigma, S]$
- Variation of $\Gamma[\sigma, S]$ w.r.t. σ to obtain e.o.m. not possible
- Solution: use Feynman-Vernon trick, identify the imaginary part with a noise source coupling linearly to the field
- Obtain real action, variation w.r.t. σ leads to GLL equation

$$\partial_\mu \partial^\mu \sigma(x) + \frac{\delta U[\sigma]}{\delta \sigma(x)} + g\rho_s(\sigma_0) - D_\sigma(x) = \xi_\sigma(x)$$

GLL equation

$$\partial_\mu \partial^\mu \sigma(x) + \frac{\delta U[\sigma]}{\delta \sigma(x)} + g\rho_s(\sigma_0) - D_\sigma(x) = \xi_\sigma(x)$$

Scalar Density: $\rho_s(\sigma_0) = \text{tr } S_{thm}^{++}(x, x)$

Dissipation kernel: $D_\sigma(x) = ig^2 \int d^4y \theta(x^0 - y^0) M(x, y) \delta\sigma(y) \leftarrow \text{Memory}$

$$M(x, y) = \text{tr} \left[S_{thm}^{+-}(x, y) S_{thm}^{-+}(y, x) - S_{thm}^{-+}(x, y) S_{thm}^{+-}(y, x) \right]$$

Colored noise: $\langle \xi_\sigma(x) \rangle_\xi = 0$ and $\langle \xi_\sigma(x) \xi_\sigma(y) \rangle_\xi = N(x, y)$

$$N(x, y) = -\frac{1}{2} g^2 \text{tr} \left[S_{thm}^{+-}(x, y) S_{thm}^{-+}(y, x) + S_{thm}^{-+}(x, y) S_{thm}^{+-}(y, x) \right]$$

$\langle \dots \rangle_\xi$: functional average with prob. distr. $P[\xi] = \exp \left[-\frac{1}{2} \int d^4x d^4y \xi(x) N^{-1}(x, y) \xi(y) \right]$

Quark propagator $S_{\text{thm}}^{ab}(x, y)$

Structure of $\rho_s(x)$, $M(x, y)$ and $N(x, y)$

— Schwinger phase drops out, use Fourier transform

In the lowest Landau level (LLL) approximation:

$$S_{\text{thm}}^{++}(p) = e^{-\mathbf{p}_{\perp}^2/|q_f B|} A(p) \left[\frac{i}{p_{\parallel}^2 - m_q^2 + i\epsilon} - 2\pi n_F(p_0) \delta(p_{\parallel}^2 - m_q^2) \right]$$

$$S_{\text{thm}}^{+-}(p) = e^{-\mathbf{p}_{\perp}^2/|q_f B|} A(p) 2\pi \delta(p_{\parallel}^2 - m_q^2) [\theta(-p_0) - n_F(p_0)],$$

$$S_{\text{thm}}^{-+}(p) = e^{-\mathbf{p}_{\perp}^2/|q_f B|} A(p) 2\pi \delta(p_{\parallel}^2 - m_q^2) [\theta(p_0) - n_F(p_0)]$$

$$S_{\text{thm}}^{--}(p) = e^{-\mathbf{p}_{\perp}^2/|q_f B|} A(p) \left[\frac{-i}{p_{\parallel}^2 - m_q^2 - i\epsilon} - 2\pi n_F(p_0) \delta(p_{\parallel}^2 - m_q^2) \right]$$

where $A(p) = (\not{p}_{\parallel} + m_q) [1 + i\gamma^1 \gamma^2 \text{sign}(qB)]$ and $n_F(p_0) = \frac{1}{e^{|p_0|/T} + 1}$

Momentum space GLL equation

$$\frac{\partial^2 \sigma(t, \mathbf{p})}{\partial t^2} + \mathbf{p}^2 \sigma(t, \mathbf{p}) + \eta(\mathbf{p}) \frac{\partial \sigma(t, \mathbf{p})}{\partial t} + F_\sigma(t, \mathbf{p}) = \xi_\sigma(t, \mathbf{p})$$

$$\eta(\mathbf{p}) = g^2 \frac{1}{2E_\sigma(\mathbf{p})} M(\mathbf{p}) \quad \leftarrow M(\mathbf{p}) = M(E_\sigma, \mathbf{p}), \quad E_\sigma = \sqrt{\mathbf{p}^2 + m_\sigma^2}$$

$$F_\sigma(t, \mathbf{p}) = \int d^3x e^{-i\mathbf{p}\cdot\mathbf{x}} \left[\frac{\delta U[\sigma]}{\delta \sigma(t, \mathbf{x})} + g \rho_s(\sigma_0) \right]$$

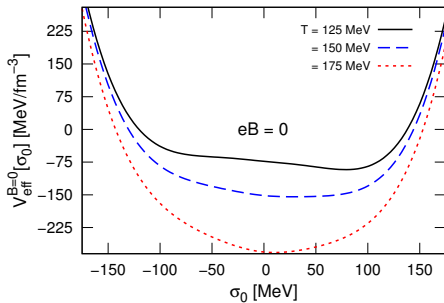
$$\langle \xi_\sigma(t, \mathbf{p}) \xi_\sigma(t', \mathbf{p}') \rangle_\xi = (2\pi)^3 \delta(\mathbf{p} + \mathbf{p}') N(t - t', \mathbf{p})$$

$\eta = 0$: “classical” equation of motion

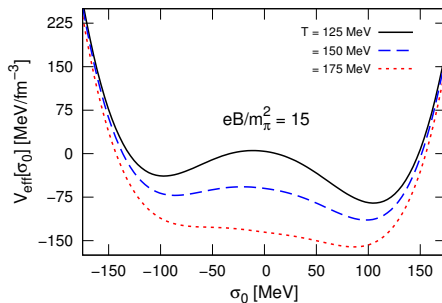
$\eta \neq 0$: slows the dynamics

Equilibrium*

$B = 0$



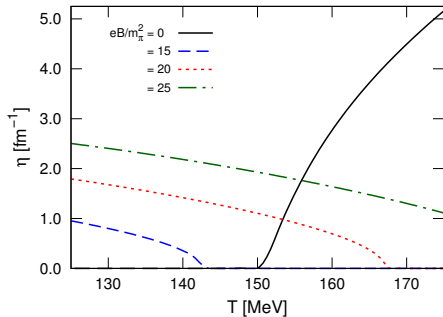
$B \neq 0$



E.S. Fraga & A.J. Mizher, Phys. Rev. D 78, 025016 (2008).

Zero-mode η

Recall, source of dissipation is $\sigma \rightarrow \bar{q}q$



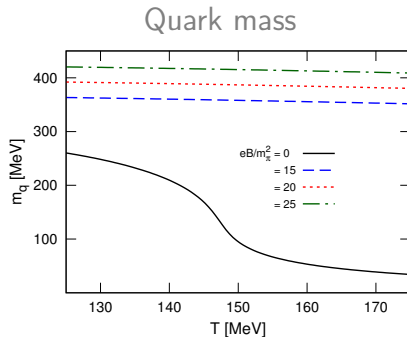
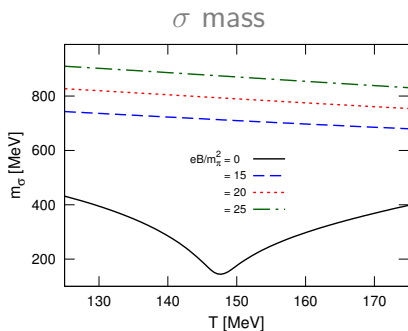
$$\underline{B = 0} \quad \eta_0 = g^2 \frac{2N_c}{\pi} [1 - 2n_F(m_\sigma/2)] \frac{1}{m_\sigma^2} (m_\sigma^2 - 4m_q^2)^{3/2}$$

$$\underline{B \neq 0} \quad \eta_B = g^2 \frac{N_c}{4\pi} [1 - 2n_F(m_\sigma/2)] (eB) \frac{1}{m_\sigma^2} \sqrt{m_\sigma^2 - 4m_q^2} \quad (LLL)$$

σ and quark masses

$\eta \neq 0$ when $m_\sigma > 2m_q$

B modifies minimum of V_{eff} ($\leftarrow m_q$) and its curvature ($\leftarrow m_\sigma$)



m_σ increases faster than m_q as the temperature decreases:

- η_B increases at low temperatures
- increase in η_B delays evolution of σ

Short-time dynamics

Linearized GLL equation:

$$\eta(\mathbf{p}_\perp) \frac{\partial \bar{\sigma}(t, \mathbf{p}_\perp)}{\partial t} - (\mu^2 - \mathbf{p}_\perp^2) \bar{\sigma}(t, \mathbf{p}_\perp) + g\rho_s(\sigma_0) - f_\pi m_\pi^2 = \bar{\xi}_\sigma(t, \mathbf{p}_\perp)$$

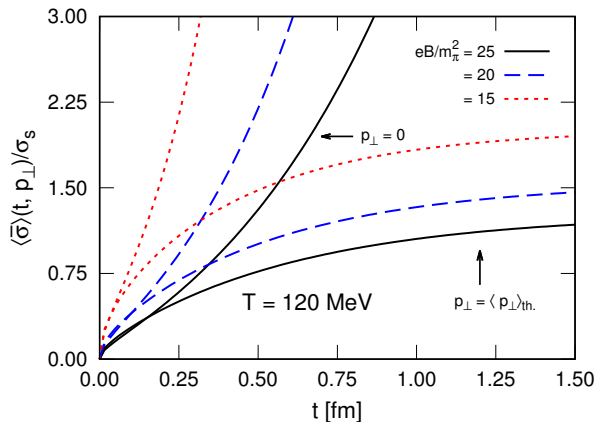
$$\mu^2 = \lambda \left(f_\pi^2 - \frac{m_\pi^2}{\lambda} \right), \quad \bar{\sigma} = \sigma/L^3, \quad \bar{\xi} = \xi/L^3$$

Quench from high to low T :

$$\langle \bar{\sigma}^2(t, \mathbf{p}_\perp^2) \rangle_\xi = \frac{[g\rho_s(\sigma_0) - f_\pi m_\pi^2]^2}{(\mu^2 - \mathbf{p}_\perp^2)^2} \left(e^{\lambda(\mathbf{p}_\perp) t/\tau_s} - 1 \right)^2 + \frac{E(\mathbf{p}_\perp) \coth(E(\mathbf{p}_\perp))}{L^3(\mu^2 - \mathbf{p}_\perp^2)} \left(e^{2\lambda(\mathbf{p}_\perp) t/\tau_s} - 1 \right)$$

$$\tau_s = \frac{\eta_B}{\mu^2} \quad \text{and} \quad \lambda(\mathbf{p}_\perp) = \frac{1 - \mathbf{p}_\perp^2/\mu^2}{\eta(\mathbf{p}_\perp)/\eta_B},$$

Short-time dynamics



$$\sigma_s = (g\rho_s - f_{\pi}m_{\pi}^2)/\mu^2$$

$$\langle \mathbf{p}_{\perp}^2 \rangle_{th.} = \frac{\int d^2 p_{\perp} p_{\perp}^2 n_B(\mathbf{p}_{\perp})}{\int d^2 p_{\perp} n_B(p_{\perp})}$$

$$n_B(p) = \frac{1}{e^{E_{\sigma}(p)/T} - 1}$$

Large B slows considerably the short-time dynamics

Present estimate: delays of $\simeq 1$ fm/c \leftarrow 1/10 of QGP lifetime

Conclusions & Perspectives

1. Presented a nonequilibrium QFT setup to tackle magnetic field effects on chiral dynamics, a first incursion into a complex many-body problem
2. Made first numerical estimates, short-time dynamics
3. Applications to HIC: beyond LLL approximation (weak fields), include pion dynamics, couple to hydrodynamics
4. Possibility to use the formalism in magnetars and early universe

Thank you

Funding

