Mesons under strong magnetic field in the NJL model

N. N. Scoccola
Tandar Lab -CNEA– Buenos Aires

PLAN OF THE TALK

- Introduction
- Neutral and charged mesons of $J^{\pi} = 0^{\pm}$ and $1^{\pm}$ and their mixings under strong magnetic fields within the NJL model
- Numerical results
- Summary & Conclusions

Gomez Dumm, Noguera & NNS, Phys.Rev.D 108 (2023) 016012
Coppola, Gomez Dumm, Noguera & NNS, in preparation

Introduction

The effect of intense external magnetic fields on light mesons both at zero and finite T/μ has been studied in the framework of a variety of approaches to low-energy QCD. They include the NJL-like models, quark meson models, χPT, QCD sum rules, etc.

In addition, several results for the π and ρ meson masses have been obtained from LQCD calculations.

Most of the model calculations ignored the possible mixings induced by the presence of the magnetic fields. Recently, our group has started to investigate the role of those mixings with the NJL model. In two previous works (Carlomagno et al, Phys.Rev.D 106 (2022) 074002, Phys.Rev.D 108 (2023) 1, 016012) we have analyzed the possible π and ρ mixings.

However, as well-known, even at B=0 the axial vector mesons mix with the pseudocalar ones. We extend our previous analysis by incorporating the axial meson degrees of freedom.

Generalized NJL model at finite B

We start from the Lagrangian of the NJL model for 2 flavors in the presence of an external e.m. field

$$
\begin{aligned}
\mathcal{L} = \bar{\psi}(x)\left(i\not{D} - m_c\right)\psi(x) &+ g_S \sum_{b=0}^{3} \left\{\left[\bar{\psi}(x)\tau_b\psi(x)\right]^2 + \left[\bar{\psi}(x) i\gamma^5\tau_b\psi(x)\right]^2\right\} \\
&- g_V\left(\left[\bar{\psi}(x)\,\gamma^\mu\vec{\tau}\,\psi(x)\right]^2 + \left[\bar{\psi}(x)\,\gamma^\mu\gamma^5\vec{\tau}\,\psi(x)\right]^2\right) \\
&- g_{V_0}\left[\bar{\psi}(x)\,\gamma^\mu\,\psi(x)\right]^2 - g_{A_0}\left[\bar{\psi}(x)\,\gamma^\mu\,\psi(x)\right]^2 \\
&+ 2g_d \sum_{\varepsilon=\pm 1} \det[\bar{\psi}(x)(1+\varepsilon\,\gamma_5)\psi(x)]\,,
\end{aligned}
$$

S / P

V/A

t'Hooft

where

$$
D^\mu = \partial^\mu + i\hat{Q}\mathcal{A}_\mu \quad , \quad \hat{Q} = \mathrm{diag}(Q_u, Q_d) \quad , \quad Q_u = -2Q_d = 2e/3 \quad , \quad \tau_b = (1,\vec{\tau}) \quad , \quad \psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}
$$

We consider a constant and uniform magnetic field along the z-axis

$$\vec{B} = B\,\hat{x}_3$$

To write $\mathcal{A}_\mu$ common gauges are

$$\mathcal{A}^\mu(x) = B/2\,(0, -x^2, x^1, 0) \qquad SG$$
$$\mathcal{A}^\mu(x) = B\,(0, -x^2, 0, 0) \qquad LG1$$
$$\mathcal{A}^\mu(x) = B\,(0, 0, x^1, 0) \qquad LG2$$

We bosonize the fermionic theory, introducing $\sigma_b(x), \pi_b(x), \rho_b{}^\mu(x), a_b{}^\mu$ and integrating out the fermion fields. The bosonized Euclidean action reads

$$S_{\rm bos} = -i \ln\det i\mathcal{D} + \frac{1}{4g}\int d^4x \left[\sigma_0(x)\sigma_0(x) + \vec{\pi}(x)\cdot\vec{\pi}(x)\right]$$
$$+ \frac{1}{4g(1-2\alpha)}\int d^4x \left[\vec{\sigma}(x)\cdot\vec{\sigma}(x) + \pi_0(x)\pi_0(x)\right]$$
$$+ \frac{1}{4g_V}\int d^4x \left[\vec{\rho}^{\,\mu}(x)\cdot\vec{\rho}_\mu(x) + \vec{a}^{\,\mu}(x)\cdot\vec{a}_\mu(x)\right]$$
$$+ \frac{1}{4g_{V_0}}\int d^4x\, \rho_0{}^\mu(x)\rho_{0\mu}(x) + \frac{1}{4g_{A_0}}\int d^4x\, a_0{}^\mu(x)a_{0\mu}(x)$$

$$g = g_S + g_D$$
$$\alpha = \frac{g_D}{g_S + g_D}$$

where

$$i\mathcal{D}_{x,x'} = \delta^{(4)}(x-x')\left\{ i\not{D} - m_c - \sum_{b=0}^{3}\tau_b\left[\sigma_b(x) + i\gamma_5\pi_b(x) + \gamma^\mu\rho_{b\mu}(x) + \gamma^\mu a_{b\mu}(x)\right]\right\}$$

We proceed by expanding the bosonized action in powers of the fluctuations $\delta\sigma_b(x)$, $\delta\pi_b(x)$, $\delta\rho_b^\mu(x)$, $\delta a_b^\mu(x)$ around the corresponding mean field (MF) values. We assume that only $\tau_a\bar{\sigma}_a = diag(\bar{\sigma}_u\,, \bar{\sigma}_d)$ is non-vanishing. Thus we write

$$\mathcal{D}_{x,x'} = \mathrm{diag}\left(\mathcal{D}_{x,x'}^{MF,u}, \mathcal{D}_{x,x'}^{MF,d}\right) + \delta\mathcal{D}_{x,x'}\ .$$

where $\quad i\mathcal{D}_{x,x'}^{MF,f} = \delta^{(4)}(x-x')\left(i\,\not{D} - M_f\right)\qquad$ with $\quad M_f = m_c + \bar{\sigma}_f$

The effective action is $\quad S_{\mathrm{bos}} = S_{\mathrm{bos}}^{MF} + S_{\mathrm{bos}}^{quad} + \ldots$

At MF level we have

$$\frac{S_{\mathrm{bos}}^{MF}}{V^{(4)}} = \frac{(1-\alpha)(\bar{\sigma}_u{}^2 + \bar{\sigma}_d{}^2) - 2\alpha\,\bar{\sigma}_u\bar{\sigma}_d}{8g(1-2\alpha)} - \frac{N_c}{V^{(4)}} \sum_{f=u,d} \int d^4x\, d^4x'\, tr_D \ln\left(\mathcal{S}_{x,x'}^{MF,f}\right)^{-1},$$

MF quark propagator in presence of mag. field

For MF quark propagator we use

$$S^{MF,f}_{x,x'} = \exp\left[i\Phi_f(x,x') \right] \tilde{S}^{MF,f}(x-x')$$

Schwinger phase (SP)

$$\Phi_f(x,x') = Q_f \int_x^{x'} d\xi_\mu \, \mathcal{A}^\mu(\xi)$$

Gauge dependent

Translational and gauge invariant part

$$\tilde{S}^{MF,f}(x-x') = \int_p e^{ip(x-x')} \tilde{S}^f_p$$

In proper time

$$\tilde{S}^f_p = \int_0^\infty d\tau \, e^{-\tau\varphi} \left[\left(M_f - p_\parallel \cdot \gamma_\parallel\right)\left(1 + i s_f \gamma^1\gamma^2 \tanh(\tau B_f)\right) - \frac{p_\perp \cdot \gamma_\perp}{\cosh^2(\tau B_f)} \right]$$

$$\varphi = M_f{}^2 + p_\parallel^2 + \frac{\tanh(\tau B_f)}{\tau B_f} p_\perp^2 \;;\; B_f = |Q_f B| \;;\; s_f = sign\left[Q_f B\right]$$

To regularize the MF-action we sum and subtract the B=0 contribution. The B-dependent piece turns out to be finite. The B=0 one is regularized introducing cutoff Λ (Magnetic Field Independent Regularization – MFIR)

The MF effective masses M_f are obtained from the coupled set of gap equations

$$\partial S^{MF,reg}_{bos} \big/ \partial M_u = \partial S^{MF,reg}_{bos} \big/ \partial M_d = 0$$

Gauge independent
(SP cancel in calculation of condensates)

MESON MASSES IN NJL AT FINITE MAGNETIC FIELD

At quadratic level the neutral meson contribution is

$$S_{\text{bos}}^{\text{quad,neutral}} = \frac{1}{2}\int d^4x\, d^4x' \sum_{M,M'} \delta M(x)\ \mathcal{G}_{MM'}(x,x')\ \delta M'(x')$$

$M = \sigma_b, \pi_b, \rho_b^\mu, a_b^\mu$

with $b = 0,3$

$(\sigma_0, \pi_0, \rho_0, a_0, \sigma_3, \pi_3, \rho_3, a_3)$ correspond to $(\sigma, "\eta", \omega, f_1, a_0^0, \pi^0, \rho^0, a_1^0)$ for B=0

The inverse meson propagator $\mathcal{G}_{MM'}(x,x')$ is

$$\mathcal{G}_{MM'}(x,x') = \frac{1}{2}\frac{\delta_{MM'}}{g_M}\ \delta^{(4)}(x-x') + \mathcal{J}_{MM'}(x,x'),$$

$$\frac{\delta_{MM'}}{g_M} = \begin{cases} 1/g & ; M = M' = \sigma_0, \sigma_3 \\ 1/[g(1-2\alpha)] & ; M = M' = \sigma_3, \pi_0 \\ -\eta^{\mu\nu}/g_V & ; M, M' = \rho_3^\mu, a_3^\nu \\ -\eta^{\mu\nu}/g_{V_0} & ; M, M' = \rho_0^\mu, \rho_0^\nu \\ -\eta^{\mu\nu}/g_{A_0} & ; M, M' = a_0^\mu, a_0^\nu \end{cases}$$

$$\mathcal{J}_{MM'}(x,x') = c_{MM'}^{uu}(x',x) + \varepsilon_M\, \varepsilon_{M'}\ c_{MM'}^{dd}(x',x)$$

where $\varepsilon_M = +1(-1)$ for b=0 (3) and

$$c_{MM'}^{ff'}(x,x') = N_c\ \text{tr}_D\left[\mathcal{S}_{x,x'}^{MF,f}\,\Gamma^{M'}\,\mathcal{S}_{x',x}^{MF,f'}\,\Gamma^{M}\right]$$

$$\eta^{\mu\nu} = diag(1,-1,-1,-1)$$

$$\Gamma^M = \begin{cases} 1 & \text{for } M = \sigma_0, \sigma_3 \\ i\gamma^5 & \text{for } M = \pi_0, \pi_3 \\ \gamma^\mu & \text{for } M = \rho_0^\mu, \rho_3^\mu \\ \gamma^\mu\gamma^5 & \text{for } M = a_0^\mu, a_3^\mu \end{cases}$$

The charged meson contribution is (for $Q=+e$, similar for $Q=-e$)

$$S_{\rm bos}^{\rm quad,+} = \frac{1}{2}\int d^4x\, d^4x' \sum_{M,M'} \delta M(x)^\dagger\ \mathcal{G}_{MM'}(x,x')\ \delta M'(x')$$

$$M = \pi^+, \rho^{+\mu}, a^{+\mu}$$

The inverse meson propagator $\mathcal{G}_{MM'}(x,x')$ is

$$\mathcal{G}_{MM'}(x,x') = \frac{1}{2}\frac{\delta_{MM'}}{g_M}\ \delta^{(4)}(x-x') + \mathcal{J}_{MM'}(x,x')\,,$$

$$\frac{\delta_{MM'}}{g_M} = \begin{cases} 1/\mathrm{g} & \text{for } M = M' = \pi \\ 1/[g(1-2\alpha)] & \text{for } M = M' = \sigma \\ -\eta^{\mu\nu}/\mathrm{g}_V & \text{for } M, M' = \rho^\mu, a^\mu \end{cases}$$

$$\mathcal{J}_{MM'}(x,x') = 2\, c_{MM'}^{ud}(x',x)$$

where

$$c_{MM'}^{ff'}(x,x') = N_c\ \mathrm{tr}_D\left[\mathcal{S}_{x,x'}^{MF,f}\,\Gamma^{M'}\mathcal{S}_{x',x}^{MF,f'}\,\Gamma^{M}\right]$$

Same polarization function as before

For neutral f=f'=u or d

For positive charged f=u, f'=d

We concentrate on the calculation a generic polarization function

$$c^{ff'}_{MM'}(x,x') = N_c \, \mathrm{tr}_D\left[\mathcal{S}^{MF,f}_{x,x'}\Gamma^{M'}\mathcal{S}^{MF,f'}_{x',x}\Gamma^{M}\right]$$

where

$$S^{MF,f}_{x,x'} = \exp\left[i\Phi_f(x,x')\right]\int_p e^{ip(x-x')}\tilde{S}^f_p$$

Replacing we get

$$c^{ff'}_{MM'}(x,x') = e^{i\Phi_M(x,x')}\int_v e^{iv(x-x')}\,\hat{c}^{ff'}_{MM'}(v)$$

where

$$\hat{c}^{ff'}_{MM'}(v) = N_c \int_p \mathrm{tr}_D\left[\tilde{S}^f_{p+v/2}\Gamma^{M'}\tilde{S}^{f'}_{p-v/2}\Gamma^{M}\right]$$

$$\Phi_M(x,x') = \Phi_f(x,x') + \Phi_{f'}(x',x)$$

Meson SP (finite for charged, zero for neutral)

Gauge invariant part (only gauge and translational part of propagator appear)

In what follows we consider the particular case M=M'=π . Similar for other cases but Lorentz structure more complicated. We introduce functions $\mathcal{F}_Q(x,\bar{q})$ and define the "transform" of $c^{ff'}_{\pi\pi}(x,x')$

$$c^{ff'}_{\pi\pi}(\bar{q},\bar{q}') = \int_{xx'}\left[\mathcal{F}_Q(x,\bar{q})\right]^* c^{ff'}_{\pi\pi}(x,x')\ \mathcal{F}_Q(x',\bar{q}')$$

Which are the functions $\mathcal{F}_Q(x,\bar{q})$? They appear in the expansion of the fields in the presence of B

Neutral $$\phi_{\pi^0}(x) = \int \frac{d^3q}{(2\pi)^3 2E_{\pi^0}} \left[a(\vec{q}) e^{-iq\cdot x} + h.c.\right],$$ $\bar{q} = (q^0, q^1, q^2, q^3)$

Charged $$\phi_{\pi^+}(x) = \sum_{\ell=0}^{\infty} \sum_{\chi} \int \frac{dq^3}{(2\pi)^3 2E_{\pi^\sigma}} \left[a(\breve{q})\, \mathbb{F}(x,\bar{q}) + h.c.\right],$$ $\bar{q} = (q^0, \ell, \chi, q^3)$

Gauge dependent Ritus-like function

Landau level

Gauge dependent quantum n.

Replacing $c_{\pi\pi}^{ff'}(x,x')$ in terms of $\hat{c}_{\pi\pi}^{ff'}(v) = \hat{c}_{\pi\pi}^{ff'}(v_\perp^2, v_{||}^2)$ (due rot. $x^1 - x^2$ and boost x^0-x^3) we get

$$c_{\pi\pi}^{ff'}(\bar{q},\bar{q}') = \int_v \hat{c}_{\pi\pi}^{ff'}(v_\perp^2, v_{||}^2)\, h_Q(\bar{q},\bar{q}', v)$$

where

$$h_Q(\bar{q},\bar{q}', v) = \int_{xx'} \left[\mathcal{F}_Q(x,\bar{q})\right]^* \mathcal{F}_Q(x',\bar{q}')\, e^{iv(x-x')}\, e^{i\Phi_M(x,x')}$$

Gauge invariant !

For neutral mesons

$$h_{Q=0}(q,q',v) = (2\pi)^4\delta^{(4)}(q-q')\,(2\pi)^4\delta^{(4)}(q-v)$$

And

$$c_{\pi\pi}^{ff'}(q,q') = (2\pi)^4\delta^{(4)}(q-q')\,C_{\pi\pi}^{ff'}(q_\perp,q_{||})$$

with

$$C_{\pi\pi}^{ff'}(q_\perp,q_{||}) = \hat{c}_{\pi\pi}^{ff'}(q_\perp^2,q_{||}^2)\,|$$

Diagonal in “Fourier” space

For charged mesons

$$h_{Q=e}(\bar{q},\bar{q}',v) = (2\pi)^4\,\delta_{\chi\chi'}\,\delta^{(2)}(q_{||}-q'_{||})(2\pi)^2\,\delta^{(2)}(q_{||}-v_{||})f_{\ell\ell'}(v_\perp)$$

with

$$f_{\ell\ell'}(v_\perp) = \frac{4\pi(-i)^{\ell+\ell'}}{B_e}\sqrt{\frac{\ell\,!}{\ell\,'!}}\left(\frac{2}{B_e}\right)^{\frac{\ell'-\ell}{2}} L_\ell^{\ell'-\ell}\left(\frac{2v_\perp^2}{B_e}\right)\; exp\left[-v_\perp^2\,/\,B_e\right]\; exp\left[is(\ell-\ell')\phi_\perp\right]$$

$$B_e = |eB|,\; s = \text{sign}(eB)$$

$$v_\perp = |\,v_\perp\,|\,(\cos\phi_\perp,\sin\phi_\perp)$$

for all three gauges (SG, LG1, LG2) !

Then we get

$$c_{\pi\pi}^{ff'}(\bar{q},\bar{q}') = (2\pi)^4\;\delta_{\chi\chi'}\,\delta_{\ell\ell'}\,\delta^{(2)}(q_{||}-q'_{||})\;C_{\pi\pi}^{ff'}(\ell,q_{||})$$

Diagonal in “Ritus” space

$$C_{\pi\pi}^{ff'}(\ell,q_{||}) = \int_0^\infty dv_\perp^2\;\;\hat{c}_{\pi\pi}^{ff'}(v_\perp^2,q_{||}^2)\;\;\rho_\ell(v_\perp^2)$$

$$\rho_\ell(v_\perp^2) = \frac{(-1)^\ell}{B_e}e^{-v_\perp^2/B_e}L_\ell\left(\frac{2v_\perp^2}{B_e}\right)$$

Even for $\ell = 0$ (Lowest LL) we cannot set $v_\perp$=0. Zero−point motion

The same can be shown for all other cases. Thus,

Neutral mesons: $\mathcal{G}_{MM'}(x,x')$ becomes diagonal (i.e. $\mathrm{G}_{MM'}(q_\perp,q_{||})$) when transformed to "Fourier" space

Charged meson: $\mathcal{G}_{MM'}(x,x')$ becomes diagonal (i.e. $\mathrm{G}_{MM'}(\ell,\ q_{||})$) when transformed to "Ritus" space

When vector or axial vectors are involved the Minkowski structure gets complicated. For example,

Neutral ρ

$$G_{\rho^\mu\rho^\nu}(q_\perp,q_{||}) = d^{(0)}_{\rho\rho,1}\eta^{\mu\nu}_{||} + d^{(0)}_{\rho\rho,2}\eta^{\mu\nu}_{\perp} + d^{(0)}_{\rho\rho,3}q^\mu_{||}q^\nu_{||} + d^{(0)}_{\rho\rho,4}q^\mu_{\perp}q^\nu_{\perp} + d^{(0)}_{\rho\rho,5}\left(q^\mu_\perp q^\nu_{||} + q^\mu_{||}q^\nu_\perp\right)$$

$d^{(0)}_{\rho\rho,k}(q^2_\perp,q^2_{||})$ are PT integrals which depend on B and quark masses

Charged ρ

$$G_{\rho^\mu\rho^\nu}(\ell,q_{||}) = d^{(+)}_{\rho\rho,1}\eta^{\mu\nu}_{||} + d^{(+)}_{\rho\rho,2}\eta^{\mu\nu}_{\perp} + d^{(+)}_{\rho\rho,3}\Pi^\mu_{||}\Pi^{\nu*}_{||} + d^{(+)}_{\rho\rho,4}\Pi^\mu_\perp\Pi^{\nu,*}_\perp + d^{(+)}_{\rho\rho,5}\left(\Pi^\mu_{||}\Pi^{\nu,*}_\perp + \Pi^\mu_\perp\Pi^{\nu*}_{||}\right) - d^{(+)}_{\rho\rho,6}\, isF^{\mu\nu} + d^{(+)}_{\rho\rho,7}\; is\left(F^{\mu\alpha}\Pi_{\alpha\perp}\Pi^{\nu*}_{||} + \Pi^\mu_{||}\Pi^*_{\alpha\perp}F^{\alpha\nu}\right)$$

$d^{(+)}_{\rho\rho,k}(\ell,q^2_{||})$ are PT integrals which depend on B and quark masses

$$\Pi^\mu = \left(q^0, i\sqrt{\frac{B_e}{2}}\left(\sqrt{\ell+1}-\sqrt{\ell}\right), -s\sqrt{\frac{B_e}{2}}\left(\sqrt{\ell+1}+\sqrt{\ell}\right), q^3\right)$$

Plays the role of "momentum"

The $G_{MM'}$ involving vector/axial vector mesons have to be contracted with polarization vectors. A convenient choice for these vectors is

Neutral

$$\epsilon_0^\mu(\vec{q},1) = \frac{1}{\sqrt{2}\,\tilde{m}_\perp \tilde{m}_{2\perp}}\left[q_+\left(E,0,0,q^3\right) + \tilde{m}_\perp^2\left(0,1,i,0\right)\right]$$

$$\epsilon_0^\mu(\vec{q},2) = \frac{1}{\tilde{m}_\perp}\left(q^3,0,0,E\right)$$

$$\epsilon_0^\mu(\vec{q},3) = \frac{1}{\sqrt{2}\,m\tilde{m}_{2\perp}}\left[q_-\left(E,0,0,q^3\right) + \frac{q_+^* q_-}{2}\left(0,1,i,0\right) + \tilde{m}_{2\perp}^2\left(0,1,-i,0\right)\right]$$

$$\epsilon_0^\mu(\vec{q},L) = \frac{1}{m}\left(E,q^1,q^2,q^3\right)$$

$$q_\pm = q^1 \pm iq^2$$

$$\tilde{m}_\perp = \sqrt{m^2 + q_\perp^2}$$

$$\tilde{m}_{2\perp} = \sqrt{m^2 + q_\perp^2 / 2}$$

$$E = \sqrt{m^2 + q_\perp^2 + q_3^2}$$

At rest $\epsilon_0^\mu(\vec{0},1) = \left(0,1,i,0\right)/\sqrt{2}$ $\quad \epsilon_0^\mu(\vec{0},2) = \left(0,0,0,1\right)$ $\quad \epsilon_0^\mu(\vec{0},3) = \left(0,1,-i,0\right)/\sqrt{2}$ $\quad \epsilon_0^\mu(\vec{0},L) = \left(1,0,0,0\right)$

Charged

$$\epsilon_+^\mu(\ell,q^3,1) = \frac{1}{\sqrt{2}\,m_\perp m_{2\perp}}\left[\Pi_+\left(E,0,0,q^3\right) + m_\perp^2\left(0,1,is,0\right)\right] \qquad \ell \geq -1$$

$$\epsilon_+^\mu(\ell,q^3,2) = \frac{1}{m_\perp}\left(q^3,0,0,E\right) \qquad \ell \geq 0$$

$$\epsilon_+^\mu(\ell,q^3,3) = \frac{1}{\sqrt{2}\,m m_{2\perp}}\left[\Pi_-\left(E,0,0,p^3\right) + \frac{\Pi_+^*\Pi_-}{2}\left(0,1,is,0\right) + m_{2\perp}^2\left(0,1,-is,0\right)\right] \qquad \ell \geq 1$$

$$\epsilon_+^\mu(\ell,q^3,L) = \Pi^\mu(\ell,q_{||})/m \qquad \ell \geq 0$$

$$\Pi_+ = -i\sqrt{2(\ell+1)B_e}\ ;\ \Pi_- = i\sqrt{2\ell B_e}\ ;\ m_\perp = \sqrt{m^2 + (2\ell+1)B_e}\ ;\ m_{2\perp} = \sqrt{m^2 + \ell B_e}\ ;\ E = \sqrt{m^2 + (2\ell+1)B_e + \left(q^3\right)^2}$$

The inverse propagator turns out to be of the form $G_{NN'}$ where

Neutral mesons	N,N' = σ_b , π_b , $\rho_{b,L}$, $\rho_{b,1}$, $\rho_{b,2}$, $\rho_{b,3}$, $a_{b,L}$, $a_{b,1}$, $a_{b,2}$ $a_{b,3}$ with b=0,3	20 states
Charged mesons	N,N' =σ^+ , π^+ , ρ_L^+ , ρ_1^+ , ρ_2^+ , ρ_3^+ , a_L^+ , a_1^+ , a_2^+ a_3^+	10 states

The matrix elements $G_{NN'}$ have to regularized. We use MFIR (sum regularized "B=0" contribution and subtract unregularized "B=0" on).

Masses m are found by looking for solutions of

Neutral mesons

$$\det G(q_\perp, q_\parallel)\Big|_{q^0=m,\vec{q}=\vec{0}} = 0$$

Charged mesons

$$\det G(\ell, q_\parallel)\Big|_{q^0=E,q^3=0} = 0 \qquad E = \sqrt{m^2 + (2\ell+1)B_e}$$

The matrix G separates in blocks. We note that system should be invariant under parity P and a rotation R in π around the 3-axis (direction of $\vec{B}$). We call the combined operation $\Sigma_3 = R_3(\pi)\,P$. Applying this transformation to the different states we see

$$\Sigma_3\,|\,M(\bar{q})> = \phi^M_{P_3}\,|\,M(\Sigma_3\,\bar{q})> \quad \text{where} \quad \phi^M_{\Sigma_3} = \begin{cases} 1 & M = \sigma, \rho_L, \rho_1, \rho_3, a_2 \\ -1 & M = \pi, \rho_2, a_L, a_1, a_3 \end{cases}$$

Σ acting on $\bar{q}$ changes q^3 by $-q^3$

States with different $\phi^M_{\Sigma_3}$ cannot mix !

Charged mesons

$$G = G_{(-)} \otimes G_{(+)} \qquad \begin{cases} G_{(-)} \to \pi, \rho_2, a_L, a_1, a_3 \\ G_{(+)} \to \sigma, \rho_L, \rho_1, \rho_3, a_2 \end{cases}$$

For $\ell = -1$ only red
For $\ell = 0$ red + blue

We set $\vec{q} = 0$ to determine m that S_z is a good quantum number. Further separation

Neutral mesons

$$G = \prod_{S_z=0,\pm1} G_{(-,S_z)} \otimes G_{(+,S_z)} \qquad b=0,3 \qquad \begin{cases} G_{(-,0)} \to \pi_b, \rho_{b,2}, a_{b,L} \\ G_{(-,1)} \to a_{b,1} \\ G_{(-,-1)} \to a_{b,3} \\ G_{(+,0)} \to \sigma_b, \rho_{b,L}, a_{b,2} \\ G_{(+,1)} \to \rho_{b,1} \\ G_{(+,-1)} \to \rho_{b,3} \end{cases}$$

$G_{(-,0)}$: Channel of lowest ``π''

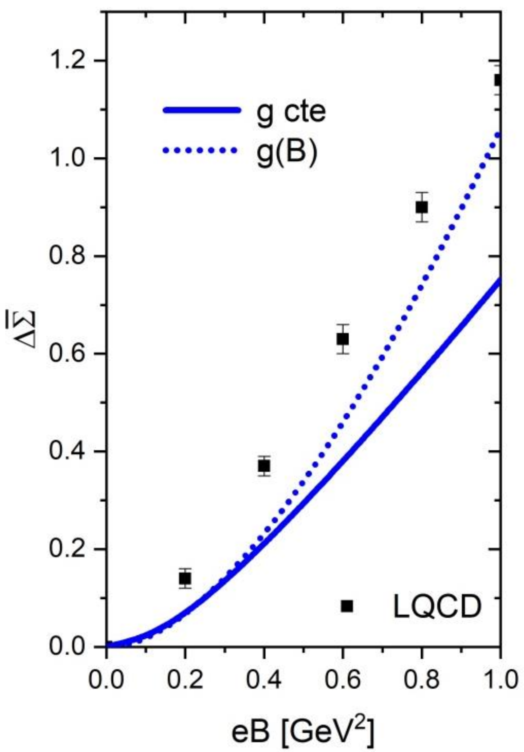
$G_{(-,1)}, G_{(-,-1)}$: $a_{||}$

$G_{(+,1)}, G_{(+,-1)}$: $\rho_{||}$

Results

We use PT regularization for B=0 with the values of g, α, $g_V = g_{V0} = g_{A0}$, Λ and m_c fixed to reproduce the *B*=0 values of f_π=92.4 MeV, m_π=139 MeV, $m_\rho = m_\omega =$ 775 MeV, $m_{"\eta"}$=548 MeV together with *M*=400 MeV. We get m_{a_1}=1.05 GeV with Γ_{a_1}=0.5 GeV

Magnetic catalysis



$$\Delta\bar{\Sigma}_B^f = (\Delta\Sigma_B^u + \Delta\Sigma_B^d)/2$$

$$\Sigma_B^f = \frac{2m_c}{S^4}\left[\langle \bar{q}_f q_f \rangle_B - \langle \bar{q}_f q_f \rangle_0\right] + 1$$

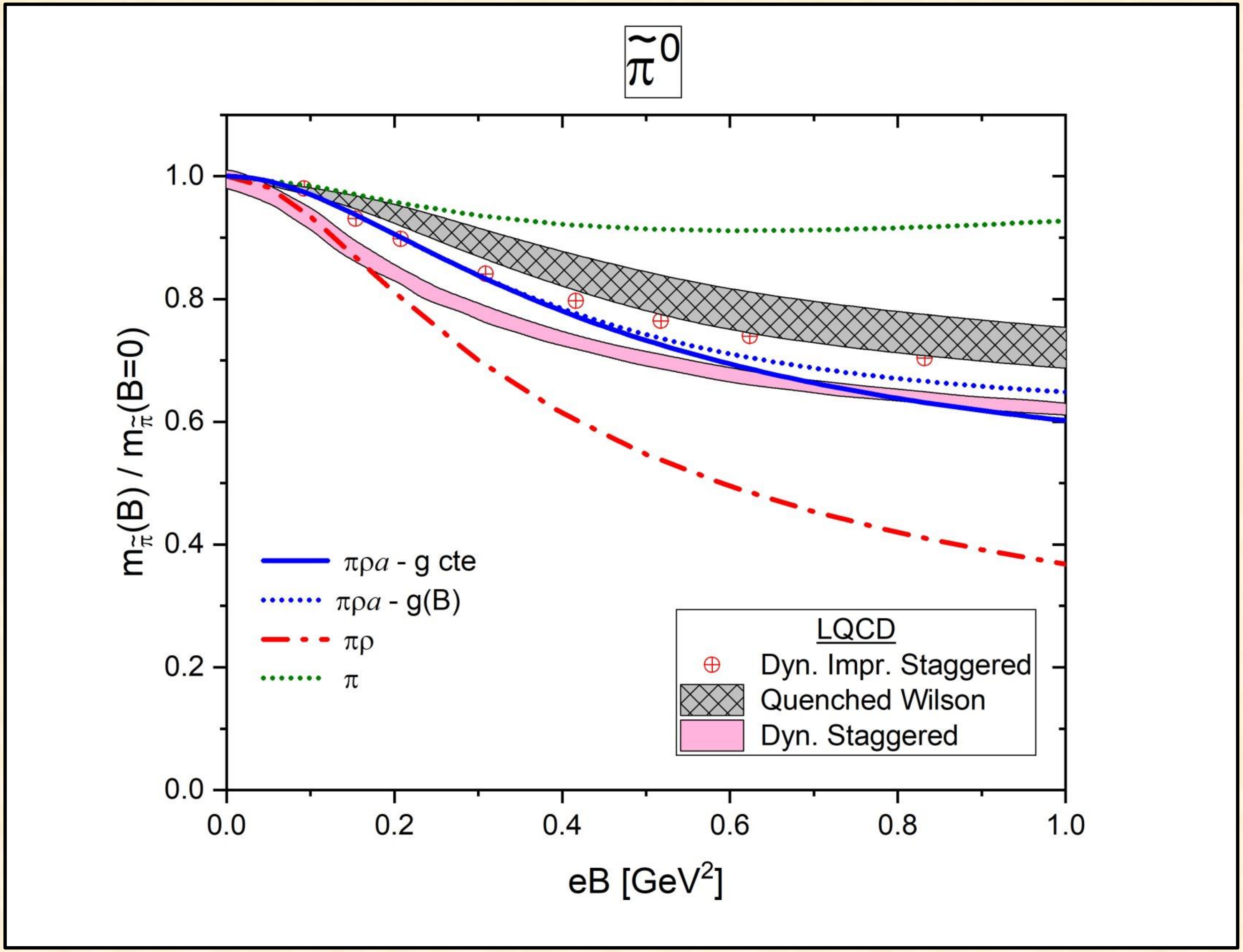
$$S = (135 \times 86)^{1/2}\,\mathrm{MeV}$$

$$< \bar{q}_f q_f >_{B,T} = \partial S_{bos}^{MF} / \partial m_c^f$$

$$< \bar{q}_f q_f >_{0,0} = (-227\ \mathrm{MeV})^3$$

$$\mathrm{g(B)/g} = \kappa_1 + (1-\kappa_1)\exp\left[-\kappa_2 (eB)^2\right]$$

Comparison of mass of lowest S_z=0 state with LQCD results



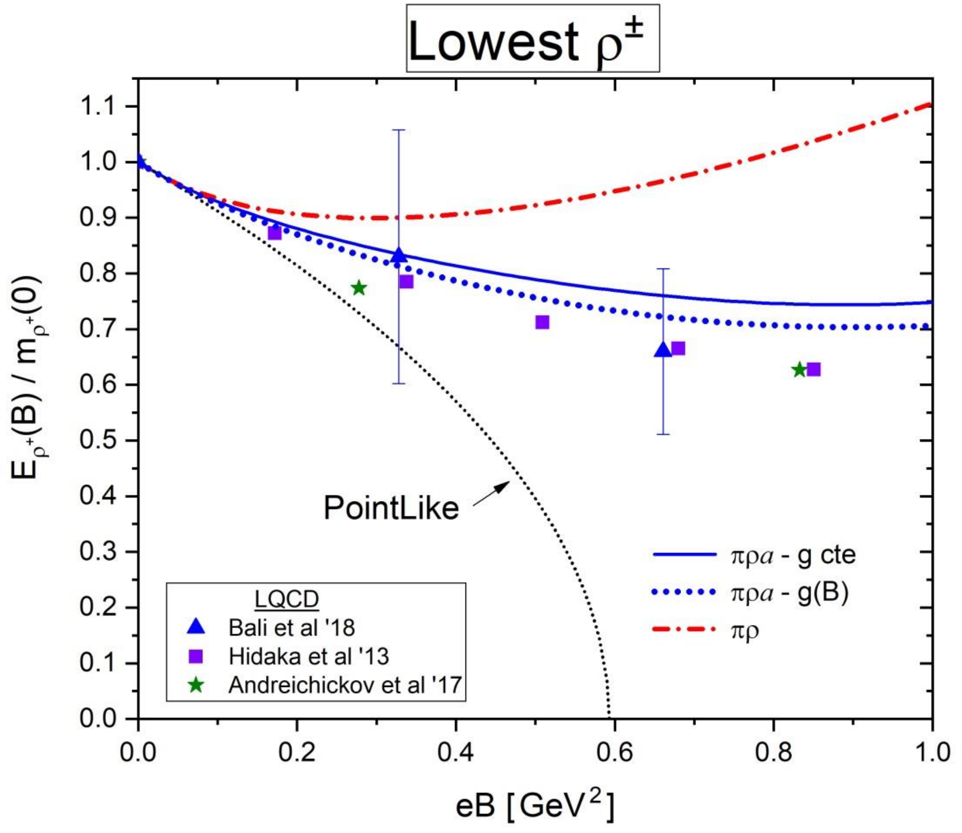
Spin-isospin composition of the “π"

eB[GeV^2]	C(π_0)	C(π_3)	C($\rho_{0,2}$)	C($\rho_{3,2}$)	C($a_{0,L}$)	C($a_{3,L}$)
0	0	0.9977	0	0	0	-0.0666
1	0.1414	0.9865	0.0574 i	0.01045 i	-0.0124	-0.0578

Spin-flavor composition of the “π"

eB[GeV^2]	C(π_u)	C(π_d)	C($\rho_{u,2}$)	C($\rho_{d,2}$)	C($a_{u,L}$)	C($a_{d,L}$)
0	0.7055	-0.7055	0	0	-0.0471	0.0471
1	0.7975	-0.5976	0.0480 i	0.0332 i	-0.0496	0.03211

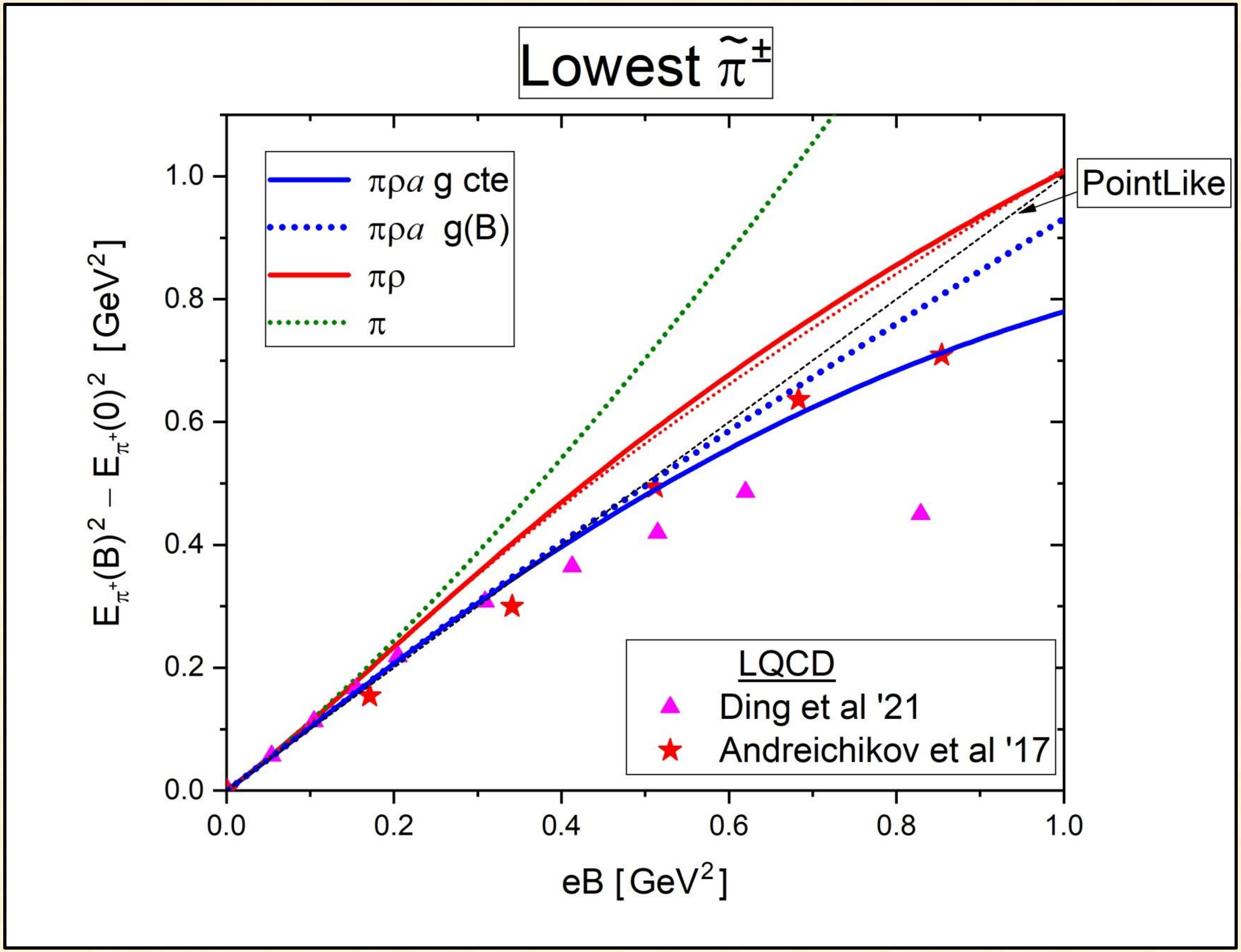
CHARGED MESON SECTOR



PointLike

$$E_\rho = \sqrt{\left(m_\rho^{B=0}\right)^2 - eB}$$

Our result differs from previous SU2 NJL calculations (Liu et al '15; Cao '19) which find $E_{\rho^+} = 0$ at a certain $eB \approx 0.4\ GeV^2$. They set ρ meson at rest ($\vec{q}$ =0) and neglect Schwinger Phase.



Inclusion of axials brings results closer to LQCD

Summary and Conclusions

- We consider the mixings induced by B on the masses of some light mesons in an extended NJL model with scalar, pseudoscalar, vector and axial vector interactions.
- We obtain expressions for **all** polarization functions for unequal fermion masses in both neutral and charged case. Gauge independent.
- Symmetries separate inverse propagator matrix in blocks.
- The effect of the mixing on mass of that state (the \`pion´) is non-negligible. Inclusion of vector and axial vector meson improve agreement with LQCD. Effect of B-dependent couplings on neutral meson masses rather small
- The mass of lowest ρ^+ decreases at low B but stabilizes at a non-zero value. Different from other NJL calculations which do not treat properly SP
- Effect of $\pi\rho a$ mixing on mass of lowest charged \`pion´ non-negligible. Gets NJL results closer to those of LQCD.