Meson mass calculation in a strong magnetic field and lattice constraints

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Presentation Outline

1. Brief Motivation

2. The two and three flavor Nambu-Jona-Lasinio (NJL) Model
   a) SU(2)-NJL Model
   b) SU(3)-NJL Model

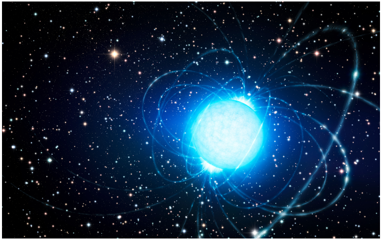
3. Magnetic Field Independent Regularization - MFIR

4. Meson Mass Calculation - SU(2)-NJL and SU(3)-NJL Models

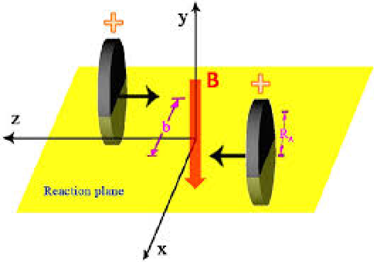
5. Conclusions and Future Perspectives

Motivation for the study of strong magnetic fields

- Magnetars: special class of neutron stars(surface field B $\sim 10^{15}$ Gauss)
- Non-central heavy-ion collisions (B $\sim 10^{20}$ Gauss)
  But problably the duration of the field is short ($\sim$ 1fm/c)



(a) Magnetar in the star cluster Westerlund 1



(b) collision Au+Au, b=10fm, COM-Energy = $\sqrt{s}$= 200 GeV

- To contribute to a better understanding of the Meson structure under strong magnetic fields
- Explore recent lattice QCD calculations of Meson Masses

credits: ESO, heavy-ion collision figure from: *"Electromagnetic fields and anomalous transports in heavy-ion collisions - A pedagogical review"*, Xu-Guang Huang - arxiv: 1509.04073

Lagrangian Density of the Two-flavor NJL model:

$$\mathcal{L} = \overline{\psi}\left(i\not{D} - \hat{m}\right)\psi + \mathcal{L}_{int,SU(2)} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

interaction terms: scalar-isoscalar + pseudoscalar-isovector , (4-point interaction)

$$\mathcal{L}_{int,SU(2)} = G\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\vec{\tau}\psi)^2\right]$$

$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ - eletromagnetic tensor field

$D^\mu = (i\partial^\mu - QA^\mu)$ - covariant derivative

$\vec{\tau}$ are isospin Pauli matrices

ψ is the quark fermion field,

$$\psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}, \ \hat{m} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}, \ Q = \begin{pmatrix} q_u = \frac{2}{3}e & 0 \\ 0 & q_d = -\frac{1}{3}e \end{pmatrix}.$$

We take m_u=m_d=m and use the Landau gauge: $A^\mu = (0, 0, Bx, 0) \rightarrow \vec{B} = B\hat{z}$

Lagrangian Density of the Three-flavor NJL model:

$$\mathcal{L} = \overline{\psi} \left(i\not{D} - \hat{m} \right) \psi + \mathcal{L}_{sym} + \mathcal{L}_{t' Hooft} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

interaction terms: $\mathcal{L}_{sym}$ + $\mathcal{L}_{t' Hooft}$, (4-point interaction + 6-point interaction)

$$\mathcal{L}_{sym} = G \sum_{a=0}^{8} \left[\left(\bar{\psi} \lambda_a \psi \right)^2 + \left(\bar{\psi} i \gamma_5 \lambda_a \psi \right)^2 \right] \quad (U(N_f)_L \times U(N_f)_R \text{ symmetric})$$

$$\mathcal{L}_{t' Hooft} = -K \left[\det{}_f \left(\bar{\psi} \left(1 + \gamma_5 \right) \psi \right) + \det{}_f \left(\bar{\psi} \left(1 - \gamma_5 \right) \psi \right) \right] \quad (\text{breaks } U_A(1) \text{ symmetry})$$

ψ is the quark fermion field , λ_a are the Gell-Mann matrices.

$$\psi = \begin{pmatrix} \psi_u \\ \psi_d \\ \psi_s \end{pmatrix} , \ \hat{m} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} , \ Q = \begin{pmatrix} q_u = \frac{2}{3} e & 0 & 0 \\ 0 & q_d = -\frac{1}{3} e & 0 \\ 0 & 0 & q_s = -\frac{1}{3} e \end{pmatrix} .$$

We take $m = m_u = m_d \neq m_s$

NJL Model as an Effective Model

NJL Lagrangian as a phenomenological effective model for the QCD:
$\rightarrow$ has to reflect the symmetries of the strong interaction!

Positive points:

- Invariant under global phase transformation $\rightarrow$ baryon number conservation
- chiral symmetric Lagrangian(in the limit m_u=m_d=0)
- spontaneous symmetry breaking mechanism (dynamical mass generation)
- The whole QCD phase diagram can be described using just one effective model

Negative points:

- Model is non-renormalizable (needs regularization, i. e., Λ-cutoff, part of the model)
- Interaction is not confining (no gluons or color charge)
  Meson mass is unstable against decay into a pair $q\bar{q}$ above a certain threshold.

NJL in the Mean Field Approximation

MFA $\rightarrow$ linearization of the $\mathcal{L}_{NJL}$ interaction terms disregarding quadratic fluctuations:

$$\hat{O} \equiv \langle \hat{O} \rangle + \underbrace{(\hat{O} - \langle \hat{O} \rangle)}_{\delta \hat{O}} = \langle \hat{O} \rangle + \delta \hat{O}$$

$$\hat{O}_1 \hat{O}_2 \equiv (\langle \hat{O}_1 \rangle + \delta \hat{O}_1)(\langle \hat{O}_2 \rangle + \delta \hat{O}_2) = \langle \hat{O}_1 \rangle \langle \hat{O}_2 \rangle + \langle \hat{O}_1 \rangle \delta \hat{O}_2 + \delta \hat{O}_2 \langle \hat{O}_2 \rangle + \cancel{\delta \hat{O}_1 \delta \hat{O}_2}$$

$$\rightarrow \hat{O}_1 \hat{O}_2 \approx \hat{O}_1 \langle \hat{O}_2 \rangle \; + \; \langle \hat{O}_1 \rangle \hat{O}_2 \; - \; \langle \hat{O}_1 \rangle \langle \hat{O}_2 \rangle \, ,$$

$$\rightarrow \hat{O}_1 \hat{O}_2 \hat{O}_3 \approx \hat{O}_1 \langle \hat{O}_2 \rangle \langle \hat{O}_3 \rangle \; + \; \langle \hat{O}_1 \rangle \hat{O}_2 \langle \hat{O}_3 \rangle \; + \; \langle \hat{O}_1 \rangle \langle \hat{O}_2 \rangle \hat{O}_3 - 2 \langle \hat{O}_1 \rangle \langle \hat{O}_2 \rangle \langle \hat{O}_3 \rangle \, .$$

Two-Flavor NJL Lagrangian Density in the MFA

$$(\bar{\psi}\psi)^2 \cong 2 \left\langle \bar{\psi}\psi \right\rangle \bar{\psi}\psi - \left\langle \bar{\psi}\psi \right\rangle^2 \quad , \quad (\bar{\psi} i \gamma_5 \vec{\tau} \psi)^2 \cong 0 \quad , \quad \left(\langle \bar{\psi} i \gamma_5 \vec{\tau} \psi \rangle = 0 \text{ (symmetry)} \right) \qquad (1)$$

$$\mathcal{L}_{int,SU(2)} = G \left[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \vec{\tau} \psi)^2 \right] \rightarrow \mathcal{L}^{MFA}_{int,SU(2)} = G \left[2 \, \langle \bar{\psi}\psi \rangle \, \bar{\psi}\psi - \langle \bar{\psi}\psi \rangle^2 \right]$$

$$\mathcal{L}^{MFA}_{NJL} = \bar{\psi} \left[i \not{\partial} - \underbrace{\left(m - 2G \left\langle \bar{\psi}\psi \right\rangle \right)}_{M} \right] \psi - G \left\langle \bar{\psi}\psi \right\rangle^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} = \bar{\psi} \, (i \not{\partial} - M) \, \psi - \frac{(M-m)^2}{4G} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$M \equiv m - 2G \left\langle \bar{\psi}\psi \right\rangle$, CONSTITUENT QUARK MASS (GAP EQUATION)

Three-Flavor NJL Lagrangian Density in the MFA

$$\mathcal{L}_{sym} = G\sum_{a=0}^{8}\left[\left(\bar{\psi}\lambda_a\psi\right)^2 + \left(\bar{\psi}i\gamma_5\lambda_a\psi\right)^2\right]$$

$$\to \mathcal{L}^{MFA}_{sym} = 4G\left[\phi_u\,\psi_u^\dagger\psi_u + \phi_d\,\psi_d^\dagger\psi_d + \phi_s\,\psi_s^\dagger\psi_s - \frac{1}{2}(\phi_u^2+\phi_d^2+\phi_s^2)\right] ,$$

$$\mathcal{L}_{t'Hooft} = -K\left[\det{}_f\left(\bar{\psi}\left(1+\gamma_5\right)\psi\right) + \det{}_f\left(\bar{\psi}\left(1-\gamma_5\right)\psi\right)\right]$$

$$\to \mathcal{L}^{MFA}_{t'Hooft} = -2K(\phi_d\phi_s\,\bar{\psi}_u\psi_u \;+\; \phi_u\phi_s\,\bar{\psi}_d\psi_d \;+\; \phi_u\phi_d\,\bar{\psi}_s\psi_s - 2\phi_u\phi_d\phi_s) ,$$

$$\mathcal{L}^{MFA} = \bar{\psi}\left(\gamma_\mu\left(i\partial^\mu - QA^\mu\right) - \hat{M}\right)\psi - 2G(\phi_u^2+\phi_d^2+\phi_s^2) + 4K\phi_u\phi_d\phi_s ,$$

$\hat{M} = diag(M_u, M_d, M_s)$, $\phi_f \equiv \langle\bar{\psi}_f\psi_f\rangle$, $\mathrm{f} \in (\mathrm{u},\mathrm{d},\mathrm{s})$, quark condensate

$M_i = m_i - 4G\phi_i + 2K\phi_j\phi_k$, (i,j,k) any permutation of $(\mathrm{i},\mathrm{j},\mathrm{k})$

$$M_u = m_u - 4G\,\phi_u + 2K\phi_d\phi_s$$
$$M_d = m_d - 4G\,\phi_d + 2K\phi_u\phi_s$$
$$M_s = m_s - 4G\,\phi_s + 2K\phi_u\phi_d$$

Quark Condensate Calculation

Propagator in Minkowski Space

$$i\mathcal{S}_f^M(x,x') \equiv \langle 0|T[\psi_f(x)\bar{\psi}_f(x')]|0\rangle \quad , \quad \text{(quark propagator)} \quad ,$$

$$\phi_f = \left\langle \bar{\psi}_f \psi_f \right\rangle = -N_c \lim_{x_0' \to x_0^+} \lim_{\vec{x}' \to \vec{x}} \mathrm{tr}_D \left[i\mathcal{S}_f^M(x,x') \right] = -N_c \, \mathrm{tr}_D \left[i\mathcal{S}_f^M(x,x) \right] \quad ,$$

Propagator in Euclidean Coordinates: $x_4 = ix_0 \quad ; \quad p_4 = -ip_0$

$$\phi_f = -N_c \, \mathrm{tr}_D \left[\mathcal{S}_f(x,x)\right] \; .$$

(Dressed) Fermion Propagator in Schwinger PT Representation

$$\mathcal{S}_f(x,x') = e^{i\Phi_f(x,x')} \int \frac{d^4p}{(2\pi)^4} \, \tilde{S}_f(p) \, e^{ip\cdot(x-x')} \quad , \quad \underbrace{\Phi_f(x,x') = \frac{q_f B}{2}(x_1 + x_1')(x_2 - x_2')}_{\text{Schwinger phase}} ,$$

$$\tilde{S}_f(p) = \int_0^\infty d\tau \exp\left[-\tau\left(M_f^2 + p_\parallel^2 + p_\perp^2 \frac{\tanh(B_f\tau)}{\tau B_f} - i\epsilon\right)\right]$$

$$\times \left[\left(M_f - p_\parallel \cdot \gamma_\parallel\right)\left(1 + i\, s_f \gamma_1\gamma_2 \tanh(B_f\tau)\right) - \frac{p_\perp \cdot \gamma_\perp}{\cosh^2(B_f\tau)} \right] , \; s_f = \mathrm{sgn}(q_f), B_f = |Bq_f|$$

$$\gamma_\parallel = (\gamma_3, \gamma_4) \, , \; \gamma_\perp = (\gamma_1, \gamma_2) \quad , \quad p_\parallel = (p_3, p_4) \quad , \quad p_\perp = (p_1, p_2) \, ,$$

$$p_\parallel \cdot \gamma_\parallel = p_3\gamma_3 + p_4\gamma_4 \quad , \quad p_\perp \cdot \gamma_\perp = p_1\gamma_1 + p_2\gamma_2 \, , \quad d^4p \equiv d^4p_E = dp_1 dp_2 dp_3 dp_4$$

(Dressed) Fermion Propagator in Landau Level Representation

$$S_f(x,x') = e^{i\Phi_f(x,x')} \int \frac{d^4p}{(2\pi)^4}\, \tilde{S}_f(p)\, e^{ip\cdot(x-x')} .$$

- $\Phi_f(x,x') \equiv Q_f \int_x^{x'} dy_\mu A^\mu(y)$ is the Schwinger phase, $f = u, d, s$
- The integral is along a straight line connecting x and x'

$$\tilde{S}_f(p) = \tilde{S}_f(p_{\|}, p_\perp) = e^{-\frac{p_\perp^2}{\beta_f}} \sum_{k=0}^{\infty} \frac{(-1)^k\, D_{f,k}(p_{\|}, p_\perp)}{p_{\|}^2 + M_f^2 + 2\beta_f\, k}$$

$$D_{f,k}(p_{\|}, p_\perp) = \left\{ \left[(p\gamma)_{\|} + M_f\right] \left[\Pi_- L_k\left(2\,\frac{p_\perp^2}{\beta_q}\right) + \Pi_+ L_{k-1}\left(2\,\frac{p_\perp^2}{\beta_q}\right)\right] + 4(p\cdot\gamma)_\perp \left[L_k\left(2\,\frac{p_\perp^2}{\beta_q}\right) - L_{k-1}\left(2\,\frac{p_\perp^2}{\beta_q}\right)\right]\right\}$$

$$(p\cdot\gamma)_{\|} = p_4\,\gamma_4 + p_3\,\gamma_3 \quad , \quad ,(p\cdot\gamma)_\perp = p_1\,\gamma_1 + p_2\,\gamma_2 \quad , \quad p_{\|}^2 = p_4^2 + p_3^2 \quad , \quad p_\perp^2 = (p_1^2 + p_2^2) \quad ,$$

$$\Pi_\pm = \frac{1}{2}(\mathbb{I} \pm i s_f\, \gamma^1\gamma^2) \quad , \quad \beta_f = |Q_f| B \quad , \quad s_f = sign(Q_f B) \quad , \quad d^2p_{\|} = dp_4 dp_3$$

(ref. V. P. Gusynin, V. A. Miransky and I. A. Shovkovy, Nucl. Phys. B 462 (1996) 249)

Condensate $B \neq 0$

$$\phi_f = \langle \bar{\psi}_f \psi_f \rangle = -N_c \, \mathrm{tr}_D \, [\mathcal{S}_f(x,x)] = -N_c \underbrace{e^{i\Phi_f(x,x)}}_{=1} \int \frac{d^4p}{(2\pi)^4} \, \mathrm{tr}_D \left[\tilde{S}_f(p) \right] .$$

$$\mathrm{tr}_D \left[\tilde{S}_f(p) \right] = 4M_f \int_0^\infty d\tau \exp \left[-\tau \left(M_f^2 + p_\parallel^2 + p_\perp^2 \frac{\tanh(B_f\tau)}{\tau B_f} - i\epsilon \right) \right] ,$$

$$\phi_f = -4M_f N_c \int_0^\infty d\tau \, e^{-\tau M_f^2} \int \frac{d^4p}{(2\pi)^4} \exp \left[-\tau \left(p_\parallel^2 + p_\perp^2 \frac{\tanh(B_f\tau)}{\tau B_f} - i\epsilon \right) \right]$$

$$\phi_f = -M_f N_c \frac{B_f}{2\pi^{3/2}} \int_0^\infty d\tau \, e^{-\tau M^2} \frac{\coth(B_f\tau)}{\tau^{1/2}} \int_{-\infty}^{\infty} \frac{dp_4}{(2\pi)} \exp \left[-\tau p_4^2 \right]$$

$$\phi_f^B = -M_f N_c \frac{B_f}{(2\pi)^2} \int_0^\infty d\tau \, e^{-\tau M_f^2} \frac{\coth(B_f\tau)}{\tau} \quad (T = 0 \, , \ PT) \, .$$

$$\phi_f^B = -M_f N_c \frac{B_f}{(2\pi)^2} \sum_{n=0}^{\infty} (2 - \delta_{n0}) \int_{-\infty}^{\infty} dp_3 \frac{1}{\sqrt{p_3^2 + M_f^2 + 2B_f n}} \quad (T = 0 \, , \ Landau) \, .$$

Condensate - Finite Temperature

$$\phi_f = -M_f N_c \frac{B_f}{2\pi^{3/2}} \int_0^\infty d\tau\, e^{-\tau M_f^2} \frac{\coth(B_f\tau)}{\tau^{1/2}} \int_{-\infty}^{\infty} \frac{dp_4}{(2\pi)} \exp\left[-\tau p_4^2\right] .$$

$$\text{Matsubara formalism : } p_4 \to \omega_n = (2n+1)\pi T \quad ; \quad \int_{-\infty}^{\infty} \frac{dp_4}{(2\pi)} \to T \sum_{n=-\infty}^{n=\infty}$$

$$\phi_f = -\, M_f N_c \frac{B_f}{2\pi^{3/2}} \int_0^\infty d\tau\, e^{-\tau M_f^2} \frac{\coth(B_f\tau)}{\tau^{1/2}}\, T \sum_{n=-\infty}^{n=\infty} \exp\left[-\tau\omega_n^2\right] .$$

$$\sum_{n=-\infty}^{n=\infty} e^{-\tau\omega_n^2} = \frac{1}{\tau^{1/2}\, 2\pi^{1/2} T} \left(1 + 2\sum_{k=1}^{\infty} e^{-\frac{k^2}{4\tau T^2}} (-1)^k\right) .$$

$$\phi_f = \phi_f^B + \phi_f^{B,T} ,$$

$$\phi_f^B = -M_f N_c \frac{B_f}{4\pi^2} \int_0^\infty \frac{d\tau}{\tau} e^{-\tau M_f^2} \coth(B_f\tau) \quad (\text{Infinity : has to be regularized}) ,$$

$$\phi_f^{B,T} = -M_f\, N_c\, 2 \frac{B_f}{4\pi^2} \sum_{k=1}^{\infty} (-1)^k \int_0^\infty \frac{d\tau}{\tau} e^{-\tau M_f^2} \coth(B_f\tau) e^{-\frac{k^2}{4\tau T^2}} \quad (\text{finite}) .$$

Magnetic Field Independent Regularization - MFIR

Condensate - MFIR

$$\phi_f^B \equiv (\phi_f^B - \lim_{B\to 0} \phi_f^B) + \lim_{B\to 0} \phi_f^B = \underbrace{\phi_f^{mag}}_{finite} + \underbrace{\phi_f^0}_{infinity} ,$$

$$\phi_f^0 \equiv \lim_{B\to 0} \phi_f^B = -M_f N_c \lim_{B\to 0} \frac{B_f}{4\pi^2} \int_0^\infty \frac{d\tau}{\tau} e^{-\tau M_f^2} \coth(B_f \tau) = -M_f N_c \frac{1}{4\pi^2} \int_0^\infty \frac{d\tau}{\tau^2} e^{-M_f^2 \tau}$$

$$\phi_f^{mag} \equiv -M_f N_c \frac{1}{4\pi^2} \int_0^\infty \frac{d\tau}{\tau^2} e^{-M_f^2 \tau} (B_f \tau \coth(B_f \tau) - 1) , \quad \coth x \sim \frac{1}{x} + \frac{x}{3} + O(x^3)$$

ϕ_f^0 Regularization

$$\phi_f^{0\ reg} = -\frac{M_f N_c}{4\pi^2} \int_{1/\Lambda^2}^\infty \frac{d\tau}{\tau^2} e^{-M_f^2 \tau} \quad \text{(proper-time regularization)} ,$$

$$\phi_f^0 = -\frac{M_f N_c}{4\pi^2} \int_0^\infty \frac{d\tau}{\tau^2} e^{-M_f^2 \tau} = -\frac{M_f N_c}{\pi^2} \int_0^\infty dp\, p^2 \frac{1}{\pi^{1/2}} \int_0^\infty \frac{d\tau}{\tau^{1/2}} e^{-(M_f^2 + p^2)\tau} = -\frac{M_f N_c}{\pi^2} \int_0^\infty dp \frac{p^2}{E_p} ,$$

$$\phi_f^{0\ reg} = -\frac{M_f N_c}{\pi^2} \int_0^\Lambda dp \frac{p^2}{E_p} , \quad E_p = \sqrt{M_f^2 + p^2} \quad , \text{(3D-cutoff regularization)} .$$

P. Bakshi, R. A. Cover and G. Kalman, Phys. Rev. D14, 2532 (1976).

A Proper Regularization is Crucial

$$\Sigma_f(B,T) = \frac{2m}{m_\pi^2 f_\pi^2}\left[\phi_f^{BT} - \phi_f^{00}\right] + 1\ ,\ \Delta\Sigma_f(B,T) = \Sigma_f(B,T) - \Sigma_f(B=0,T) = \frac{2m}{m_\pi^2 f_\pi^2}\left[\phi_f^{BT} - \phi_f^{0T}\right]$$

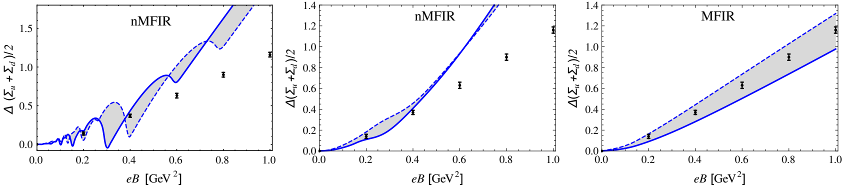


Figure: Results for the Fermi-Dirac cut-off and Lorenztian form factor compared with lattice results. nMFIR ≡ non-MFIR . Parameterization with -$\phi_0^{1/3}$= -$((\phi_u^0+\phi_d^0)/2))^{-1/3}$= 220-260 MeV (Lattice results: Bali et al, PRD 86, 071502(R) (2012)).

$$\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{dp_3}{2\pi} \rightarrow \sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{dp_3}{2\pi}\, U_\Lambda(p_3^2 + 2n\,B_f)\ ,$$

$$U_\Lambda^{FD}(x) = \frac{1}{2}\left[1 - \tanh\left(\frac{\frac{x}{\Lambda}-1}{\alpha}\right)\right]\ (\mathrm{Fermi-Dirac})\ ,\ U_\Lambda^{(LorN)}(x) = \left[1 + \left(\frac{x}{\Lambda^2}\right)^N\right]^{-1}\ (\mathrm{Lorenztian})$$

Sidney S. Avancini, Ricardo L. S. Farias, Norberto N. Scoccola, William R. Tavares, “NJL-type models in the presence of intense magnetic fields: the role of the regularization prescription”, PRD99 (2019), 116002.

Lattice Results - Inverse Magnetic Catalysis

$$\Sigma_f(B,T) = \frac{2m}{m_\pi^2 f_\pi^2}\left[\phi_f^{BT} - \phi_f^{00}\right] + 1 \quad , \quad \Delta\Sigma_f(B,T) = \Sigma_f(B,T) - \Sigma_f(B=0,T) = \frac{2m}{m_\pi^2 f_\pi^2}\left[\phi_f^{BT} - \phi_f^{0T}\right]$$

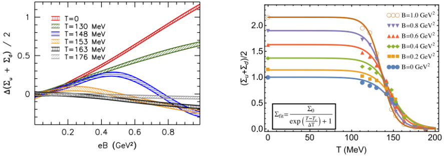


Figure: Lattice Results

$$G(B,T) = c(B)\left[1 - \frac{1}{1 + e^{\beta(B)[T_a(B) - T]}}\right] + s(B).$$

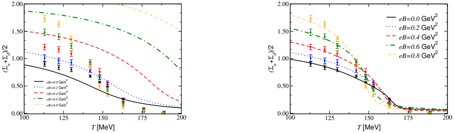


Figure: SU(2)-NJL results, G = constant (left), $G(B,T)$ (right)

G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz, and A. Schafer, PR D 86, 071502(R) (2012)
R. L. S. Farias, V. S. Timóteo, S. S. Avancini, M. B. Pinto and G. Krein, EPJA 53, 101 (2017) (SU(2))

Meson Mass Calculation (Bosonizing the Action)

Euclidean Action

$$S_E = \int d^4x\, \mathcal{L}_E^{NJL} = \int d^4x \left[\bar{\psi}\left(-i\,\not{D} + \hat{m}\right)\psi - G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi} i\gamma_5 \vec{\tau}\psi\right)^2\right]\right]$$

Partition Function

$$\mathcal{Z} = \int \mathcal{D}\bar{\psi}\, \mathcal{D}\psi\; e^{-S_E} = \int \mathcal{D}\bar{\psi}\, \mathcal{D}\psi\; e^{-\int d^4x\left[\bar{\psi}\left(-i\,\not{D}+\hat{m}\right)\psi - G\left[\left(\bar{\psi}\psi\right)^2+\left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right]\right]}$$

Defining the auxilliary scalar $s(x)$ and pseudo-scalar $\vec{P}(x)$ fields:

$$\int \mathcal{D}s(x)\; \delta[s(x) - f(x)]\; F[s(x)] \equiv F[f(x)] \quad , \quad \text{(Dirac } \delta \text{ functional)}$$

$$\delta[s(x) - \bar{\psi}(x)\psi(x)] = \int \mathcal{D}\,\sigma(x)\; e^{\int dx^4 \sigma(x)[s(x) - \bar{\psi}(x)\psi(x)]}$$

$$\delta[\vec{P}(x) - \bar{\psi}(x) i\gamma_5\vec{\tau}\psi(x)] = \int \mathcal{D}\,\vec{\pi}(x)\; e^{\int dx^4 \vec{\pi}(x)[\vec{P}(x) - \bar{\psi}(x)i\gamma_5\vec{\tau}\psi(x)]}$$

$$\mathcal{Z} \equiv \int \mathcal{D}\sigma \int \mathcal{D}\vec{\pi} \int \mathcal{D}\bar{\psi}\,\mathcal{D}\psi\; e^{-\int d^4x\left[\bar{\psi}\left(-i\,\not{D}+\hat{m}+\sigma(x)+i\gamma_5\vec{\pi}(x)\vec{\tau}\right)\psi\right]}$$

$$\times \int \mathcal{D}s \int \mathcal{D}\vec{P}\; e^{\int d^4x\,\left(\sigma(x)s(x)+\vec{\pi}(x)\vec{P}(x)+G\left(s(x)^2+\vec{P}(x)^2\right)\right)}$$

$$= \int \mathcal{D}\sigma \int \mathcal{D}\vec{\pi} \int \mathcal{D}\bar{\psi}\,\mathcal{D}\psi\; e^{-\int d^4x\left[\bar{\psi}\left(-i\,\not{D}+\hat{m}+\sigma(x)+i\gamma_5\vec{\pi}(x)\vec{\tau}\right)\psi\right]} \times \int \mathcal{D}s \int \mathcal{D}\vec{P} e^{\,f[\sigma,s,\vec{\pi},\vec{P}]}$$

Meson Mass Calculation

SPA - Stationary Phase Approximation (or steepest descent method)

$$\int \mathcal{D}s \int \mathcal{D}\vec{P}\; e^{\,f[\sigma,s,\vec{\pi},\vec{P}]} \sim \int \mathcal{D}s \int \mathcal{D}\vec{P}\; e^{\,f[\sigma,s,\vec{\pi},\vec{P}]\big|_{s=\bar{s},\vec{P}=\bar{\vec{P}}}}$$

$$\frac{\delta}{\delta s(x)} f[\sigma, s, \vec{\pi}, \vec{P}]\Big|_{s=\bar{s}} = 0 \to \bar{s}(x) = -\frac{\sigma(x)}{2G} \;,\; \frac{\delta}{\delta \vec{P}(x)} f[\sigma, s, \vec{\pi}, \vec{P}]\Big|_{\vec{P}=\bar{\vec{P}}} = 0 \to \bar{\vec{P}}(x) = -\frac{\vec{\pi}(x)}{2G}$$

$$\mathcal{Z} \equiv \int \mathcal{D}\sigma \int \mathcal{D}\vec{\pi}\; e^{-\left[-\ln\det\tilde{D} + \frac{1}{4G}\int d^4x\,(\sigma(x)\sigma(x) + \vec{\pi}(x)\cdot\vec{\pi}(x))\right]} \equiv \int \mathcal{D}\sigma \int \mathcal{D}\vec{\pi} e^{-S_{bos}}$$

$$\tilde{D} \equiv \delta(x - x')\left[\bar{\psi}\left(-i\,\not{D} + \hat{m} + \sigma(x) + i\gamma_5\vec{\pi}(x)\vec{\tau}\right)\psi\right]$$

$$S_{bos} = -\ln\det\tilde{D} + \frac{1}{4G}\int d^4x\,(\sigma(x)\sigma(x) + \vec{\pi}(x)\cdot\vec{\pi}(x)) \quad \text{(Bosonized Euclidean Action)}$$

Expanding $\sigma(x)$ and $\vec{\pi}(x)$ around their Mean Field values:

$$\sigma(x) = \bar{\sigma} + \delta\sigma(x) \;,\; \bar{\sigma} = \langle\sigma(x)\rangle \;;\; \vec{\pi}(x) = \delta\vec{\pi}(x) \;,\; \underbrace{\bar{\vec{\pi}} = \langle\bar{\vec{\pi}}(x)\rangle}_{=0(symmetry)} \;,\; \pi_0 = \pi_3 \,,\; \pi^{\pm} = \frac{(\pi_1 \mp i\pi_2)}{\sqrt{2}}$$

$$\tilde{D} = \underbrace{\delta(x - x')\begin{pmatrix} -i(\not{\partial} - iq_u\not{A}) + M & 0 \\ 0 & -i(\not{\partial} - iq_d\not{A}) + M \end{pmatrix}}_{\tilde{D}_{MFA}} + \underbrace{\delta(x - x')\begin{pmatrix} \delta\sigma + i\gamma_5\delta\pi_0 & \sqrt{2}\,i\gamma_5\delta\pi^+ \\ \sqrt{2}\,i\gamma_5\delta\pi^- & \delta\sigma - i\gamma_5\delta\pi_0 \end{pmatrix}}_{\delta\tilde{D}}$$

SU(2)-NJL: Pole Meson Mass Calculation

Expanding the bosonized action around the MF values:

$$S_{bos} = S_{bos}^{MFA} + S_{bos}^{quad}$$

$$S_{bos}^{MFA} = -\frac{N_c}{V^{(4)}} \sum_{f=u,d} \int d^4x\, d^4x'\, tr_D \ln(\mathcal{S}_f)^{-1}(x,x') + \frac{\bar{\sigma}^2}{4G}$$

$$S_{bos}^{quad} = \frac{1}{2} \sum_{M=\sigma,\pi^0,\pi^\pm} \int d^4x \int d^4x' \delta M(x)^* \left[\frac{1}{2G}\delta^{(4)}(x-x') - J_M(x,x')\right] \delta M(x')$$

$$J_{\pi^0}(x,x') = \sum_{f=u,d} N_c\, tr_D \left[\mathcal{S}_f(x,x')\gamma_5 \mathcal{S}_f(x',x)\gamma_5\right]$$

$$J_{\pi^\pm}(x,x') = 2N_c\, tr_D \left[\mathcal{S}_u(x,x')\, \gamma_5\, \mathcal{S}_d(x',x)\, \gamma_5\right]$$

$$S_{\pi^0}^{quad} = \frac{1}{2} \int d^4x \int d^4x'\, \delta\pi_0(x)^* \left[\frac{1}{2G}\delta^{(4)}(x-x') - J_{\pi^0}(x,x')\right] \delta\pi_0(x')$$

$$= \frac{1}{2} \int d^4q\, \delta\pi_0(-q) \left[\frac{1}{2G} - J_{\pi^0}(q_\perp^2, q_\parallel^2)\right] \delta\pi_0(q)$$

$$J_{\pi^0}(q_\perp^2, q_\parallel^2) = \sum_{f=u,d} N_c \int \frac{d^4p}{(2\pi)^4}\, tr_D \left[\tilde{S}_f(p+\frac{q}{2})\, \gamma_5 \tilde{S}_f(p-\frac{q}{2})\gamma_5\right] = \sum_{f=u,d} c_{ff}(q)$$

(ref. M. Coppola, D. G. Dumm and N. Scoccola, PLB 782 (2018), 155.)

SU(3)-NJL Model: Meson Pole Mass Calculation

$$S_E = \int d^4x \left[\bar{\psi}\left(-i\not{D} + \hat{m}\right)\psi - G\sum_{a=0}^{8}\left[\left(\bar{\psi}\lambda_a\psi\right)^2 + \left(\bar{\psi}i\gamma_5\lambda_a\psi\right)^2\right] + K\left(d_+ + d_-\right)\right] ,$$

$$\psi = \left(\psi_u, \psi_d, \psi_s\right)^T \ , \ d_\pm = \det\left[\bar{\psi}\left(1 \pm \gamma_5\right)\psi\right] \ \text{and} \ \hat{m} = \text{diag}\left(m_u, m_d, m_s\right)$$

Expanding the bosonized action around the MF values:

$$S_{bos} = S_{bos}^{MFA} + S_{bos}^{quad}$$

$$S_{bos}^{MFA} = -\frac{N_c}{V^{(4)}} \sum_{f=u,d,s} \int d^4x\, d^4x' \ tr_D \ \ln(\mathcal{S}_f)^{-1}(x, x') + 2G \sum_{f=u,d,s} \phi_f^2 - 4K\,\phi_u\,\phi_d\,\phi_s\,,$$

$$S_{bos}^{quad} = \frac{1}{2}\int d^4x' d^4x \sum_{P,P'} \delta P^*(x)\,\mathcal{G}_{P,P'}(x,x')\,\delta P'(x')\,, \quad P, P' = \pi_3, \pi^\pm, K^0, \bar{K}^0, K^\pm, \eta_0, \eta_8.$$

Sidney S. Avancini, Máximo Coppola, Norberto N. Scoccola, Joana C. Sodré, Phys. Rev. D 104, 094040 (2021)

π^0 Pole Mass Calculation - Polarization Function

$$c_{ff'}(q) \equiv c^B_{ff'}(q_\perp^2, q_\parallel^2) = N_c \int \frac{d^4p}{(2\pi)^4}\, tr_D \left[\tilde{S}_f(p+\frac{q}{2})\,\gamma_5\,\tilde{S}_{f'}(p-\frac{q}{2})\gamma_5\right] =$$

$$= 2N_c \int_0^\infty d\tau \int_0^\infty d\tau' \int \frac{d^4p}{(2\pi)^4}$$

$$\times \exp\left[-\tau\left(M^2 + p^2_{\parallel +} + p^2_{\perp +}\frac{\tanh(B_f\tau)}{\tau B_f}\right)\right] \times \exp\left[-\tau'\left(M^2 + p^2_{\parallel -} + p^2_{\perp -}\frac{\tanh(B_{f'}\tau')}{\tau' B_{f'}}\right)\right]$$

$$\times\, tr_D\Bigg(\left[(M - p_{\parallel +}\cdot\gamma_\parallel)\,\Pi^f_+(\tau) - \frac{p_{\perp +}\cdot\gamma_\perp}{\cosh^2(B_f\tau)}\right]\gamma_5$$

$$\times \left[(M - p_{\parallel -}\cdot\gamma_\parallel)\,\Pi^{f'}_+(\tau') - \frac{p_{\perp -}\cdot\gamma_\perp}{\cosh^2(B_{f'}\tau')}\right]\gamma_5\Bigg)$$

$$c_{ff'}(q) = \frac{N_c}{(2\pi)^2}\int_0^\infty dz \int_{-1}^1 dx\; e^{-\frac{z}{B_f}\left(M^2 + \frac{(1-x^2)}{4}\, q_\parallel^2\right)}$$

$$\times \left\{\left(M^2 - (1-x^2)\frac{q_\parallel^2}{4} + \frac{B_f}{z}\right) \times \coth z + \frac{1}{\sinh^2 z}\left[B_f - \frac{1}{2}\left(\coth z - \frac{\cosh zx}{\sinh z}\right) q_\perp^2\right]\right\}$$

$$c^B_{ff'}(q_\perp^2, q_\parallel^2) = \left(c^B_{ff'}(q_\perp^2, q_\parallel^2) - \lim_{B\to 0} c^B_{ff'}(q_\perp^2, q_\parallel^2)\right) + \lim_{B\to 0} c^B_{ff'}(q_\perp^2, q_\parallel^2)$$

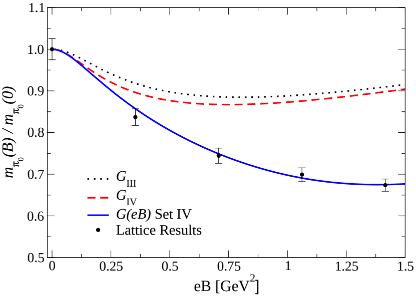
$$= c^{mag}_{ff'}(q_\perp^2, q_\parallel^2) + \lim_{B\to 0} c^B_{ff'}(q_\perp^2, q_\parallel^2) \quad \text{(MFIR)}.$$

Meson Pole Mass Calculation - SU(2)-NJL Results

The π^0 pole mass:

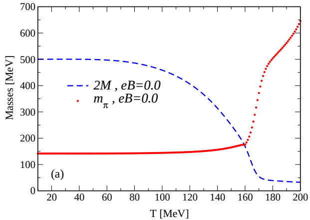
$$S_{\pi^0}^{quad} = \frac{1}{2}\int d^4q\, \delta\pi_0(-q)\left[\frac{1}{2G} - J_{\pi^0}(q_\perp^2, q_\parallel^2)\right]\delta\pi_0(q)\ ,$$

$$q_\perp^2 = 0\ ,\ \ q_\parallel^2 = -m_{\pi^0}^2\ ,\ \ \frac{1}{2G} - J_{\pi^0}(0, -m_{\pi^0}^2) = 0$$

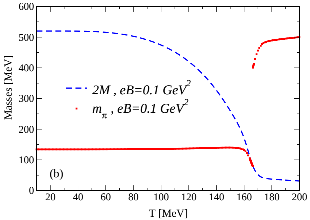


Sidney S. Avancini, Ricardo L.S. Farias, Marcus Benghi Pinto, William R. Tavares, Varese S. Timóteo, PLB 767, 247 (2017)

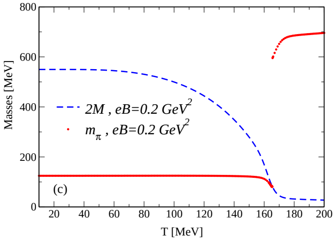
π^0 Meson Pole Mass Calculation - SU(2)-NJL Finite T Results



(a) π_0 pole mass, G(eB,T), eB=0



(b) π_0 pole mass, G(eB,T),eB=0.1GeV^2



(c) π_0 pole mass, G(eB,T), eB=0

Sidney S. Avancini, Ricardo L.S. Farias, William R. Tavares, PRD 99, 056009 (2019) neutral meson contribution to the quadratic action:

$$S^{quad}_{neut.mes} = \frac{1}{2}\int_q \sum_{A=K^0,\bar{K}^0} \Pi^*_A(-q)\,\mathcal{G}_A(q_\perp^2, q_\parallel^2)\,\Pi_A(q) + \frac{1}{2}\int_q \sum_{A,A'=\pi_0,\eta_0,\eta_8} \Pi^*_A(-q)\,\mathcal{G}_{A,A'}(q_\perp^2, q_\parallel^2)\,\Pi_{A'}(q)\,. \quad (2)$$

Inverse neutral kaon propagator is given by

$$\mathcal{G}_{K^0}(q_\perp^2, q_\parallel^2) = \mathcal{G}_{\bar{K}^0}(q_\perp^2, q_\parallel^2) = [2G - K\phi_u]^{-1} + c_{ds}(q_\perp^2, q_\parallel^2) \rightarrow \mathcal{G}_{K^0}(q_\perp^2 = 0, q_\parallel^2 = -m^2_{K^0}) = 0\,.$$

$$\mathcal{M} = \begin{pmatrix} \mathcal{G}_{\pi_3\pi_3} & \mathcal{G}_{\pi_3\eta_0} & \mathcal{G}_{\pi_3\eta_8} \\ \mathcal{G}_{\eta_0\pi_3} & \mathcal{G}_{\eta_0\eta_0} & \mathcal{G}_{\eta_0\eta_8} \\ \mathcal{G}_{\eta_8\pi_3} & \mathcal{G}_{\eta_0\eta_8} & \mathcal{G}_{\eta_8\eta_8} \end{pmatrix},$$

The physical meson π^0, η, η' pole-masses and widths $\rightarrow$ roots of

$$\det[\mathcal{M}(m_A, \Gamma_A)] = 0 \quad (3)$$

Sidney S. Avancini, Máximo Coppola, Norberto N. Scoccola, Joana C. Sodré, Phys. Rev. D 104, 094040 (2021)

SU(3) - Results

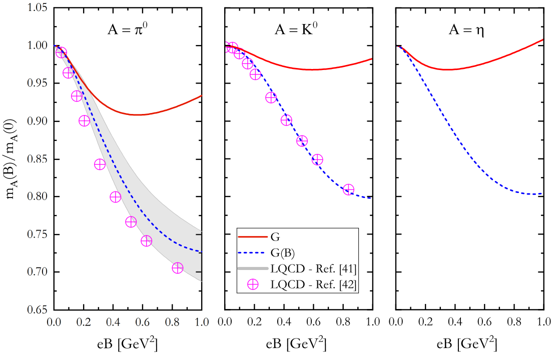


Figure: Normalized neutral meson masses as functions of *eB* for constant (red solid lines) and *B*-dependent (blue dashed lines) coupling *G*. LQCD results from Ref.[41] (grey band) and Ref.[42] (magenta circles) are added for comparison.

[41] G. Bali, B. Brandt, G. Endrődi, and B. Gläßle, Phys. Rev. D 97, 034505 (2018)
[42] H.-T. Ding, S.-T. Li, A. Tomiya, X.-D. Wang, and Y. Zhang, Phys. Rev. D 104, 014505 (2021).

SU(3) - Results

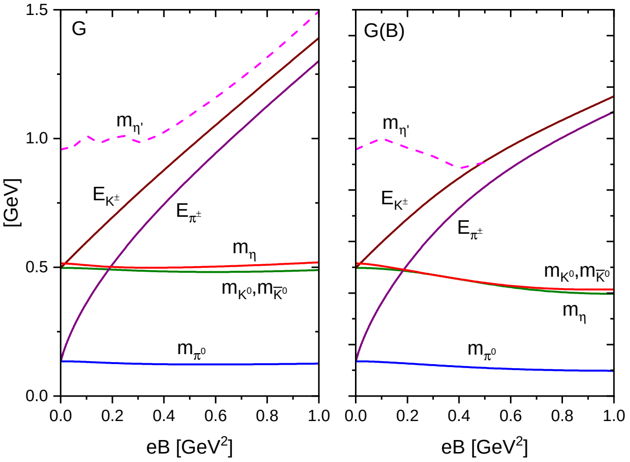
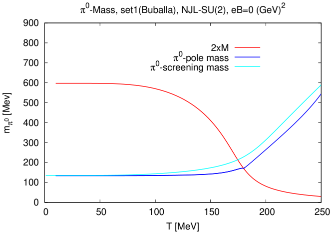


Figure: Masses as functions of *eB*.

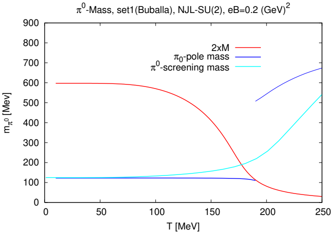
Meson Screening Mass (Work in Progress)

The π^0 screening mass:

$$\frac{1}{2G} - J_{\pi^0}(q_\perp^2, q_3^2) = 0 \ , \ \begin{cases} m_{sc,\parallel} \to q_3^2 = -m_{sc,\parallel}^2 \ , \ q_\perp^2 = 0, \\ m_{sc,\perp} \to q_\perp^2 = -m_{sc,\perp}^2 \ , \ q_3^2 = 0 \end{cases} .$$



(a) π_0 pole and parallel screening masses, eB=0



(b) π_0 pole and parallel screening masses, eB=0.2GeV^2

W. Florkowski and B. L. Friman, Acta Physica Polonica B, vol. 25(1994), p.49.

work in collaboation with: Max Coppola, Norberto Scoccola, Joana Sodré, William Tavares.

Conclusions

- The light pseudo-scalar meson masses in a strong magnetic field were obtained from the expansion of the bosonized Euclidean action (RPA calculation)
- For calculating the polarization functions, we use (for most of the calculations) the Schwinger proper-time formalism (performs the sum over the Landau levels analytically).
- We always use the MFIR regularization procedure, thus avoiding spurious solutions which are often found in the literature.
- Our calculation, when it is possible to compare, shows the same trend as the lattice
- It is a starting point for several generalizations

Collaborators (in Magnetized Matter):

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Norberto Scoccola, Máximo Coppola - (Argentina)

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