



The Photon/Gluon Polarization Tensor in a Fluctuating Magnetic Field

Background

Jorge David Castaño Yepes

Pontificia Universidad Católica de Chile

In collaboration with:

Enrique Muñoz and Marcelo Loewe



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PRELIMINARY

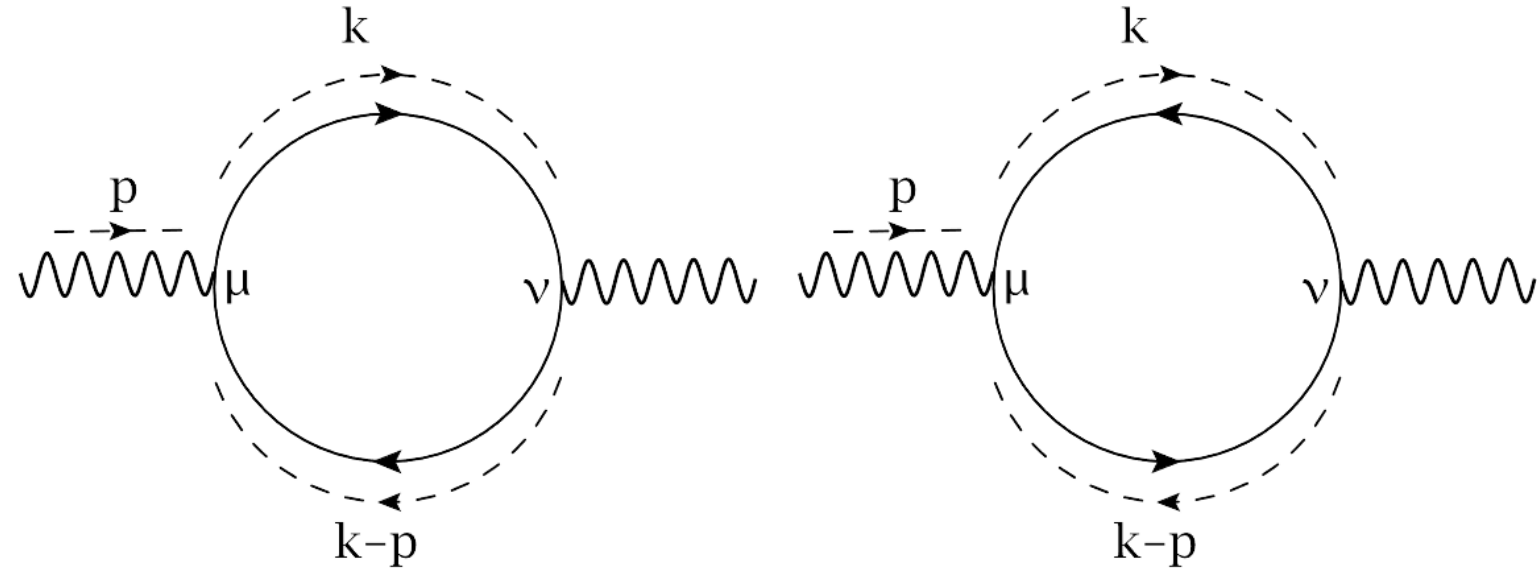
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Enrique Muñoz and Marcelo Loewe

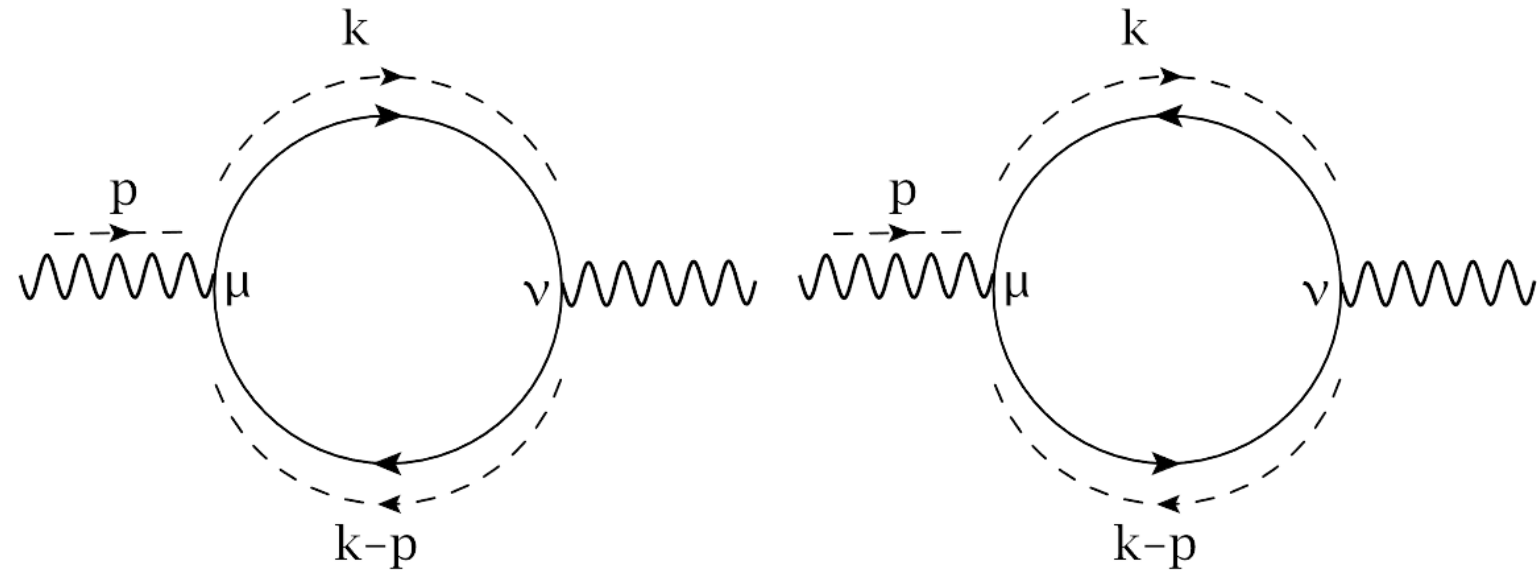
Outline

- Why this?
- The Replica formalism (again)
- The Photon/Gluon Polarization Tensor in a Fluctuating Magnetic Field Background
- Conclusions

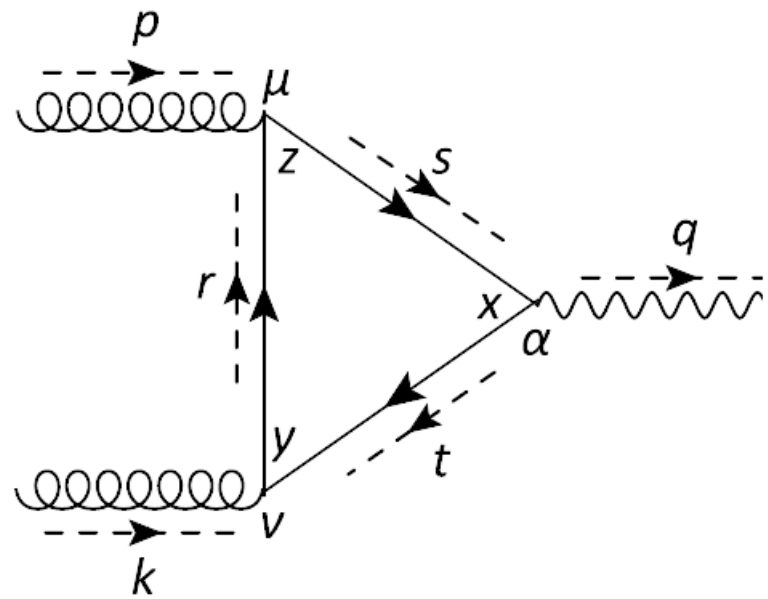
Why this?

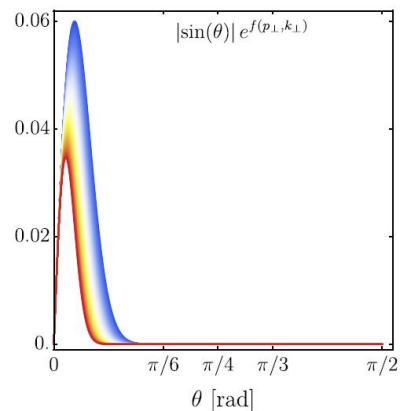
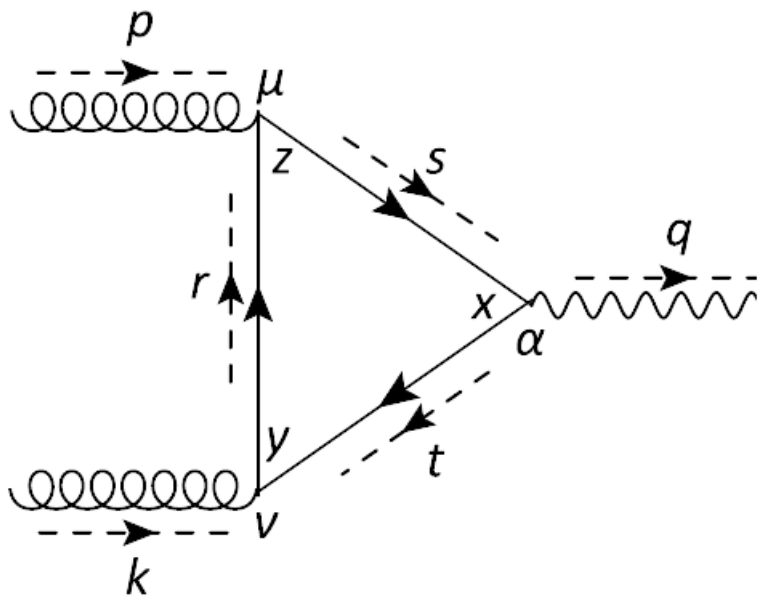


Why this?

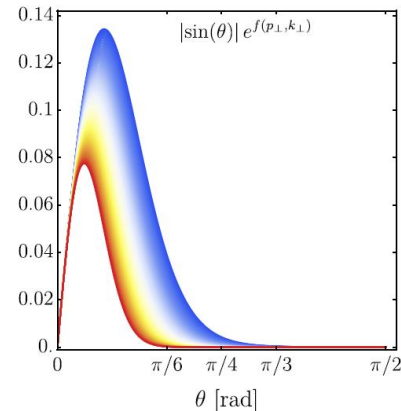


Because this

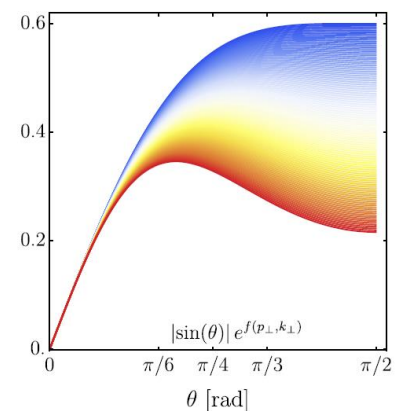




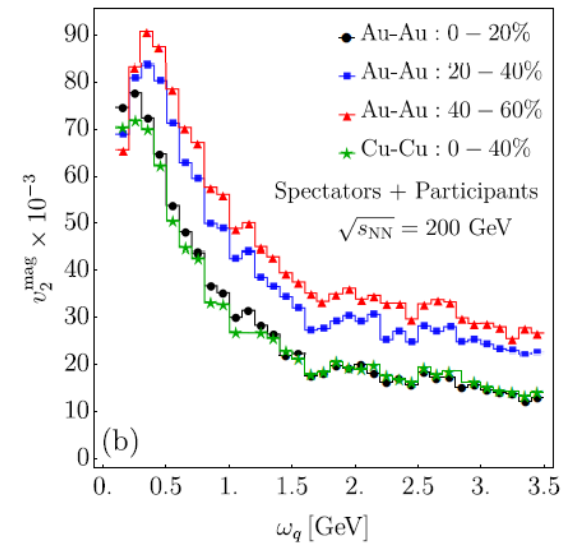
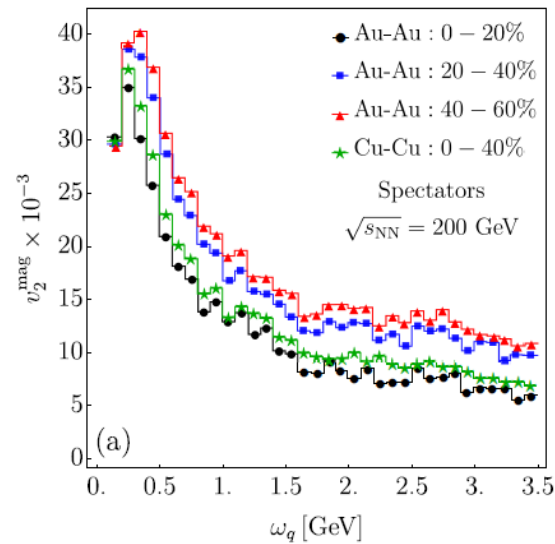
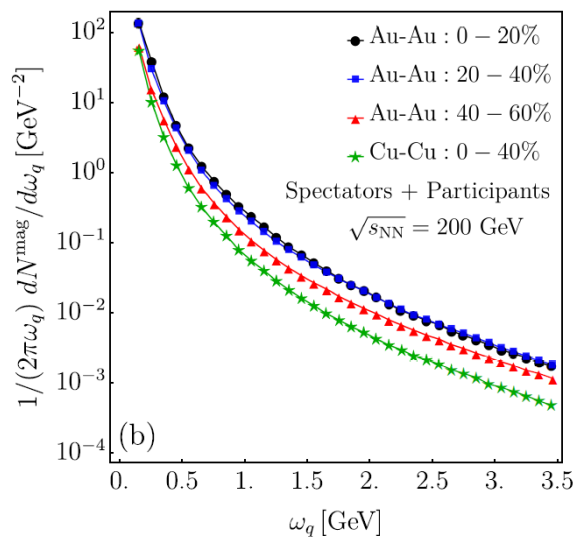
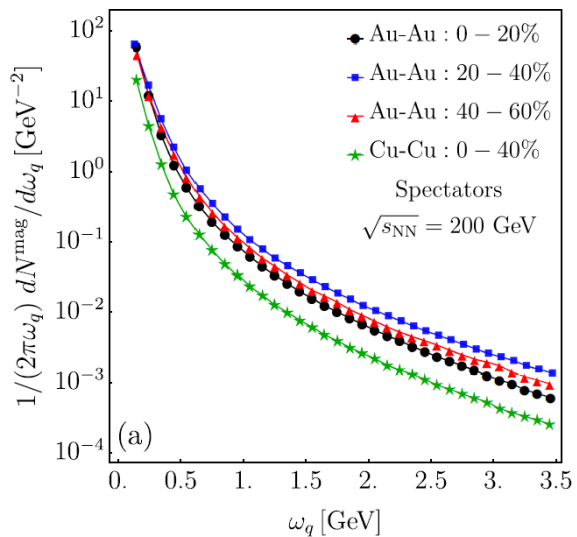
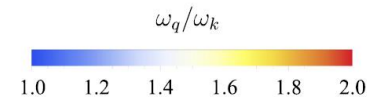
(a) $\omega_k = 1 \text{ GeV}$, $|q_f B| = m_\pi^2$



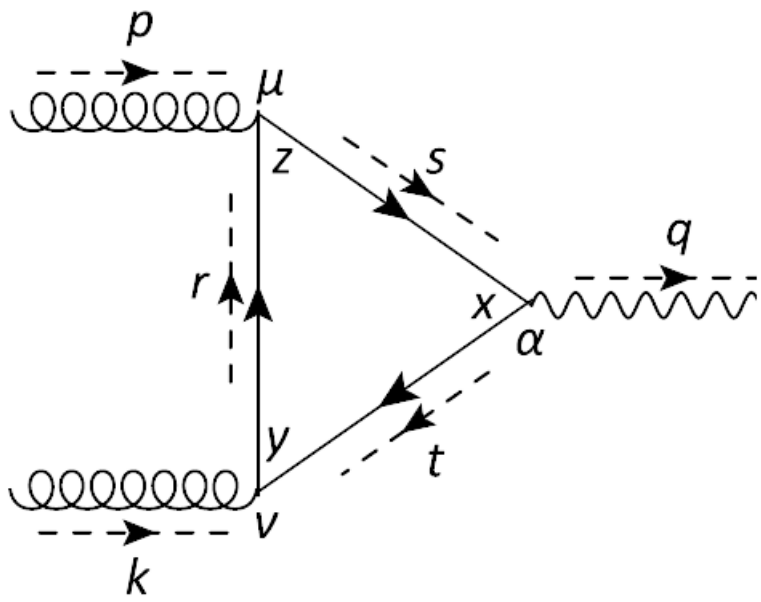
(b) $\omega_k = 1 \text{ GeV}$, $|q_f B| = 5 m_\pi^2$



(c) $\omega_k = 0.1 \text{ GeV}$, $|q_f B| = m_\pi^2$



$$p^\mu = \left(\frac{\omega_p}{\omega_q} \right) q^\mu, \quad k^\mu = \left(\frac{\omega_k}{\omega_q} \right) q^\mu, \quad \rightarrow \sigma_{gg} \propto \frac{1}{\mathbf{p}_1 - \mathbf{p}_2}$$

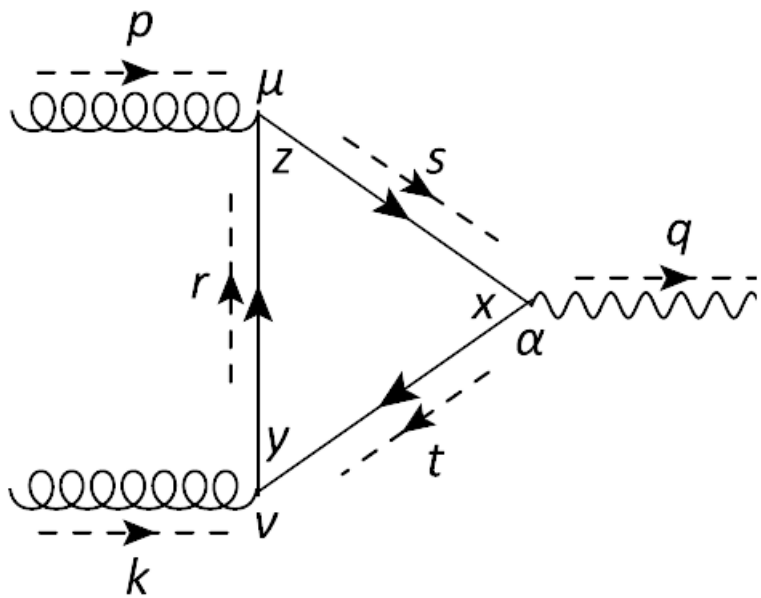


$$k^\mu = \left(\frac{\omega_k}{\omega_q} \right) q^\mu,$$

$$p^\mu = \left(\frac{\omega_p}{\omega_q} \right) q^\mu,$$

Can photons/gluons develop a magnetic mass?

$$\text{angle}(p^\mu, k^\mu) \neq 0$$



$$k^\mu = \left(\frac{\omega_k}{\omega_q} \right) q^\mu,$$

Can photons/gluons develop a magnetic mass?

$$p^\mu = \left(\frac{\omega_p}{\omega_q} \right) q^\mu,$$

$$\text{angle}(p^\mu, k^\mu) \neq 0$$

Effects of Intense Magnetic Fields, High Temperature and Density on QCD-Related Phenomena

Jorge David Castaño-Yepes

The effects of high-temperature, dense systems, and strong magnetic fields on Quantum Chromodynamics related phenomena are studied in different perspectives: in the high-temperature and densities, the QCD-phase diagram from the Linear Sigma Model point of view is computed. In particular, the CEP location and transition lines for chiral restoration were found. In the intense magnetic fields and high-gluon density, the gluon fusion and gluon splitting channels are proposed as new photon production sources to better explain the experimental data in the so-called "photon puzzle". Finally, the thermal and magnetic screening properties for gluons encoded into the polarization tensor were addressed in terms of the Debye mass.

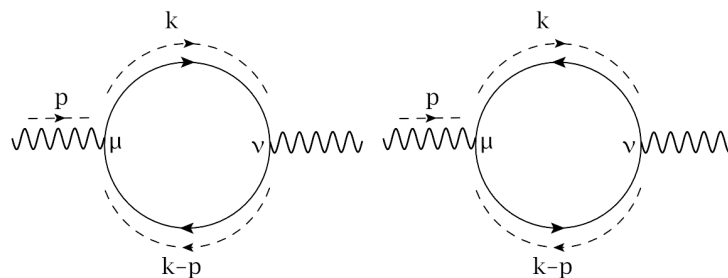
Comments: Ph.D. Thesis. 183 pages. Title and some typos corrected

Subjects: **High Energy Physics - Phenomenology (hep-ph)**

Cite as: [arXiv:2103.12898](https://arxiv.org/abs/2103.12898) [hep-ph]

(or [arXiv:2103.12898v2](https://arxiv.org/abs/2103.12898v2) [hep-ph] for this version)

<https://doi.org/10.48550/arXiv.2103.12898> 



In a presence of B

- It played a central role in our research
- We didn't find such a mass
- Are there other mechanisms to generate it?

Gluon polarization tensor in a magnetized medium: Analytic approach starting from the sum over Landau levels

Alejandro Ayala^{1,2}, Jorge David Castaño-Yepes^{1,*}, M. Loewe^{3,2,4} and Enrique Muñoz^{3,5}

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<https://doi.org/10.1140/epja/s10050-021-00429-4>

**THE EUROPEAN
PHYSICAL JOURNAL A**



Regular Article - Theoretical Physics

Gluon polarization tensor and dispersion relation in a weakly magnetized medium

Alejandro Ayala^{1,2}, Jorge David Castaño-Yepes¹, L. A. Hernández^{1,2,3,4,a}, Jordi Salinas San Martín¹, R. Zamora^{5,6}

Revista Mexicana de Física **66** (4) 446–461

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Thermal corrections to the gluon magnetic Debye mass

Alejandro Ayala^{a,b}, Jorge David Castaño-Yepes^a, C. A. Dominguez^b, S. Hernández-Ortiz^a, L. A. Hernández^{a,b}, M. Loewe^{b,c,d}, D. Manreza Paret^e and R. Zamora^{f,g}

The Replica formalism

QED fermions in a noisy magnetic field background

Jorge David Castaño-Yepes^{1,*}, Marcelo Loewe^{1,2,3,4,†}, Enrique Muñoz^{1,5,‡},
 Juan Cristóbal Rojas^{6,§} and Renato Zamora^{7,8,||}

$$A^\mu(x) \rightarrow A^\mu(x) + A_{\text{BG}}^\mu(x) + \delta A_{\text{BG}}^\mu(\mathbf{x}).$$

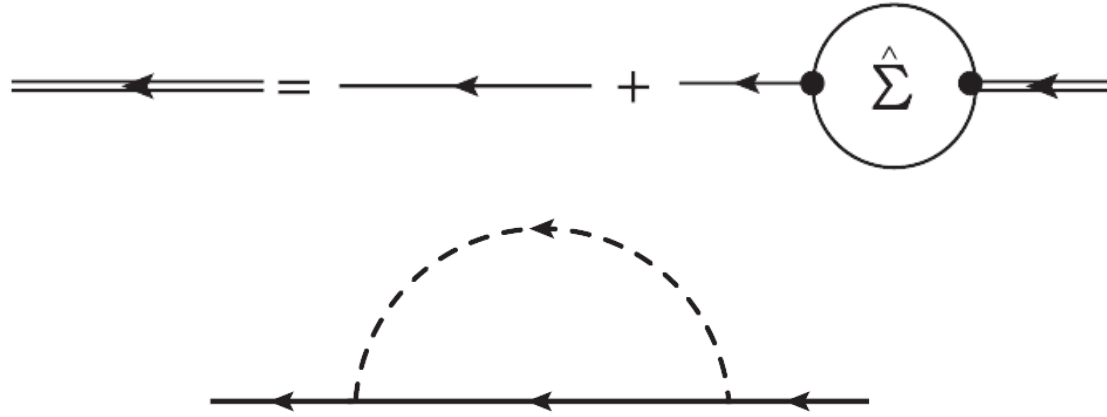
$$\overline{\ln Z[A]} = \lim_{n \rightarrow 0} \frac{\overline{Z^n[A]} - 1}{n}.$$

$$\begin{aligned} \langle \delta A_{\text{BG}}^j(\mathbf{x}) \delta A_{\text{BG}}^k(\mathbf{x}') \rangle &= \Delta_B \delta_{j,k} \delta^3(\mathbf{x} - \mathbf{x}'), & \bar{S}[\bar{\psi}^a, \psi^a; A] &= \int d^4x \left(\sum_a \bar{\psi}^a (i\not{\partial} - e\not{A}_{\text{BG}} - e\not{A} - m) \psi^a - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) \\ \langle \delta A_{\text{BG}}^\mu(\mathbf{x}) \rangle &= 0. & &+ i \frac{e^2 \Delta_B}{2} \int d^4x \int d^4y \sum_{a,b} \sum_{j=1}^3 \bar{\psi}^a(x) \gamma^j \psi^a(x) \bar{\psi}^b(y) \gamma_j \psi^b(y) \delta^3(\mathbf{x} - \mathbf{y}). \end{aligned}$$

The Replica formalism

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SELF-ENERGY AT ORDER Δ

$$iS_{\Delta}^{(\pm)}(p) = C(p) \left(m + \gamma_0 p^0 + \frac{\gamma_3 p^3}{z(p)} \right) \mathcal{P}^{(\pm)}(p)$$

$$C(p) = i \frac{z(p) e^{-\mathbf{p}_{\perp}^2 / |qB|}}{p_{\parallel}^2 - z^2(p) m^2},$$





$$z(p) = 1 + \frac{3}{4} \frac{\Delta |qB| e^{-\mathbf{p}_{\perp}^2 / |qB|}}{\pi \sqrt{p_0^2 - m^2}}$$

$$z_3(p) = \frac{1}{z(p)} \left(1 + \frac{\Delta |qB| e^{-\mathbf{p}_{\perp}^2 / |qB|}}{4\pi \sqrt{p_0^2 - m^2}} \right)$$

$$\mathcal{P}^{(\pm)}(p) = [1 + \text{sign}(|qB|) i z_3(p) \gamma^1 \gamma^2]$$

The Replica formalism

QED fermions in a noisy magnetic field background

Jorge David Castaño-Yepes ^{1,*} Marcelo Loewe ^{1,2,3,4,†} Enrique Muñoz ^{1,5,‡}
 Juan Cristóbal Rojas,^{6,§} and Renato Zamora ^{7,8,||}

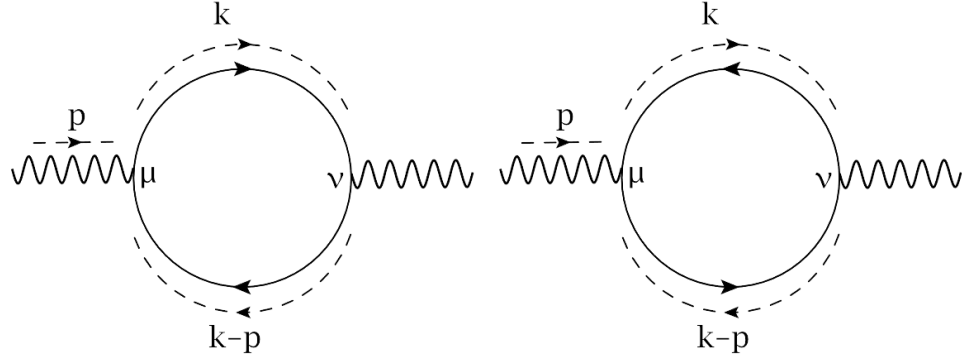
PROPAGATOR AT ORDER Δ

$$\begin{aligned}
 iS_{\Delta}^{(\pm)}(p) &= iS_0^{(\pm)}(p) \mp i\Delta \left(\frac{|qB|}{2\pi} \right) \frac{e^{-2\mathbf{p}_{\perp}^2/|qB|}}{(p_{\parallel}^2 - m^2)\sqrt{p_0^2 - m^2}} (\not{p}_{\parallel} + m) i\gamma^1 \gamma^2 - i\Delta \left(\frac{3|qB|}{2\pi} \right) \frac{e^{-2\mathbf{p}_{\perp}^2/|qB|}}{(p_{\parallel}^2 - m^2)\sqrt{p_0^2 - m^2}} p_3 \gamma^3 \mathcal{O}^{(\pm)} \\
 &+ i\Delta \left(\frac{3|qB|}{2\pi} \right) \frac{(p_{\parallel}^2 + m^2)e^{-2\mathbf{p}_{\perp}^2/|qB|}}{(p_{\parallel}^2 - m^2)^2 \sqrt{p_0^2 - m^2}} (\not{p}_{\parallel} + m) \mathcal{O}^{(\pm)} + \mathcal{O}(\Delta^2)
 \end{aligned}$$

$$iS_0^{(\pm)}(p) = 2i \frac{e^{-\mathbf{p}_{\perp}^2/|qB|}}{p_{\parallel}^2 - m^2} (\not{p}_{\parallel} + m) \mathcal{O}^{(\pm)}$$

$$\mathcal{O}^{(\pm)} = \frac{1}{2} (1 \pm i\gamma^1 \gamma^2).$$

The Photon/Gluon Polarization Tensor in a Fluctuating Magnetic Field Background



$$\begin{aligned}
 i\Pi_{\Delta}^{\mu\nu} &= -\frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left\{ i q \gamma^{\nu} i S_{\Delta}^{(-)}(k) i q \gamma^{\mu} i S_{\Delta}^{(-)}(k-p) \right\} \\
 &\quad - \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left\{ i q \gamma^{\nu} i S_{\Delta}^{(+)}(-k+p) i q \gamma^{\mu} i S_{\Delta}^{(+)}(-k) \right\}
 \end{aligned}$$

$$iS_{\Delta}^{(\pm)}(p) = iS_0^{(\pm)}(p) + i\Delta \left(\frac{|qB|}{2\pi} \right) \left[\Sigma_1(p)(\not{p}_{\parallel} + m)\mathcal{O}^{(\pm)} - \Sigma_2(p)\gamma^3\mathcal{O}^{(\pm)} \pm \Sigma_3(p)i\gamma^1(\not{p}_{\parallel} + m)\gamma^2 \right] + \mathcal{O}(\Delta^2)$$

$$i\Pi_{\Delta}^{\mu\nu} = i\Pi_0^{\mu\nu} + i \frac{q^2 |qB| \Delta}{4\pi} \sum_{i=1}^{12} \int \frac{d^4 k}{(2\pi)^4} t_i^{\mu\nu}(k)$$

The Photon/Gluon Polarization Tensor in a Fluctuating Magnetic Field Background

$$i\Pi_{\Delta}^{\mu\nu} = i\Pi_0^{\mu\nu} + i\frac{q^2|qB|\Delta}{4\pi} \sum_{i=1}^{12} \int \frac{d^4k}{(2\pi)^4} t_i^{\mu\nu}(k)$$

$$t_1^{\mu\nu} = \Sigma_1(k-p)\text{Tr} \left\{ \gamma^\nu iS_0^{(-)}(k) \gamma^\mu (\not{k}_{\parallel} - \not{p}_{\parallel} + m) \mathcal{O}^{(-)} \right\},$$

$$t_7^{\mu\nu} = \Sigma_1(-k)\text{Tr} \left\{ \gamma^\nu iS_0^{(+)}(-k+p) \gamma^\mu (-\not{k}_{\parallel} + m) \mathcal{O}^{(+)} \right\},$$

$$t_2^{\mu\nu} = -\Sigma_2(k-p)\text{Tr} \left\{ \gamma^\nu iS_0^{(-)}(k) \gamma^\mu \gamma^3 \mathcal{O}^{(-)} \right\},$$

$$t_8^{\mu\nu} = -\Sigma_2(-k)\text{Tr} \left\{ \gamma^\nu iS_0^{(+)}(-k+p) \gamma^\mu \gamma^3 \mathcal{O}^{(+)} \right\},$$

$$t_3^{\mu\nu} = -i\Sigma_3(k-p)\text{Tr} \left\{ \gamma^\nu iS_0^{(-)}(k) \gamma^\mu (\not{k}_{\parallel} - \not{p}_{\parallel} + m) \gamma^1 \gamma^2 \right\},$$

$$t_9^{\mu\nu} = i\Sigma_3(-k)\text{Tr} \left\{ \gamma^\nu iS_0^{(+)}(-k+p) \gamma^\mu (-\not{k}_{\parallel} + m) \gamma^1 \gamma^2 \right\},$$

$$t_4^{\mu\nu} = \Sigma_1(k)\text{Tr} \left\{ \gamma^\nu (\not{k}_{\parallel} + m) \mathcal{O}^{(-)} \gamma^\mu iS_0^{(-)}(k-p) \right\},$$

$$t_{10}^{\mu\nu} = \Sigma_1(-k+p)\text{Tr} \left\{ \gamma^\nu (-\not{k}_{\parallel} + \not{p}_{\parallel} + m) \mathcal{O}^{(+)} \gamma^\mu iS_0^{(+)}(-k) \right\},$$

$$t_5^{\mu\nu} = -\Sigma_2(k)\text{Tr} \left\{ \gamma^\nu \gamma^3 \mathcal{O}^{(-)} \gamma^\mu iS_0^{(-)}(k-p) \right\},$$

$$t_{11}^{\mu\nu} = -\Sigma_2(-k+p)\text{Tr} \left\{ \gamma^\nu \gamma^3 \mathcal{O}^{(+)} \gamma^\mu iS_0^{(+)}(-k) \right\},$$

$$t_6^{\mu\nu} = -i\Sigma_3(k)\text{Tr} \left\{ \gamma^\nu (\not{k}_{\parallel} + m) \gamma^1 \gamma^2 \gamma^\mu iS_0^{(-)}(k-p) \right\},$$

$$t_{12}^{\mu\nu} = i\Sigma_3(-k+p)\text{Tr} \left\{ \gamma^\nu \gamma^1 \gamma^2 \gamma^\mu (-\not{k}_{\parallel} + \not{p}_{\parallel} + m) iS_0^{(+)}(-k) \right\}.$$

The Photon/Gluon Polarization Tensor in a Fluctuating Magnetic Field Background

After some “trazology”

$$T_1^{\mu\nu} = \left(\mathcal{I}_1^{(+)} + \mathcal{I}_1^{(-)} \right) P_{\parallel}^{\mu\nu} + \left(\mathcal{J}_1^{(+)} + \mathcal{J}_1^{(-)} \right) g_{\parallel}^{\mu\nu} + \left(\mathcal{K}_1^{(+)} + \mathcal{K}_1^{(-)} \right) \frac{p_{\parallel}^{\mu} p_{\parallel}^{\nu}}{m^2}$$

$$P_{\parallel} = g_{\parallel}^{\mu\nu} - \frac{p_{\parallel}^{\mu} p_{\parallel}^{\nu}}{p_{\parallel}^2}$$

$$T_2^{\mu\nu} = \left(\mathcal{I}_2^{(+)} + \mathcal{I}_2^{(-)} \right) \left(g_{\parallel}^{\mu\nu} + 2\delta_3^{\mu} \delta_3^{\nu} \right) + \left(\mathcal{J}_2^{(+)} + \mathcal{J}_2^{(-)} \right) \left(g_{\parallel}^{\mu\nu} - \frac{p_{\parallel}^{\mu} \delta_3^{\nu} + p_{\parallel}^{\nu} \delta_3^{\mu}}{p_3} \right)$$

$$T_3^{\mu\nu} = \left(\mathcal{I}_3^{(+)} + \mathcal{I}_3^{(-)} \right) \left(P_{\parallel}^{\mu\nu} - g_{\perp}^{\mu\nu} \right) + \left(\mathcal{J}_3^{(+)} + \mathcal{J}_3^{(-)} \right) \left(g_{\parallel}^{\mu\nu} - g_{\perp}^{\mu\nu} \right) + \left(\mathcal{K}_3^{(+)} + \mathcal{K}_3^{(-)} \right) \frac{p_{\parallel}^{\mu} p_{\parallel}^{\nu}}{m^2}$$

The Photon/Gluon Polarization Tensor in a Fluctuating Magnetic Field Background

After some “trazology”

$$\mathcal{I}_1^{(\pm)} \equiv -\frac{8i\pi|qB|}{(2\pi)^4} \exp\left(-\frac{2\mathbf{p}_\perp^2}{3|qB|}\right) \omega^2 \sin^2 \theta \int_0^\infty \frac{\lambda^2 d\lambda}{(1+\lambda)^5} \int \frac{d^2 \ell_E}{(2\pi)^2} \frac{\left(\ell \pm \frac{\lambda}{1+\lambda} p\right)_E^2 - m^2}{\sqrt{\left(\ell_4 \mp \frac{i\lambda}{1+\lambda} \omega\right)^2 + m^2} \left[\ell_E^2 + \frac{\lambda}{(1+\lambda)^2} \omega^2 \sin^2 \theta + m^2\right]^3}$$

$$\mathcal{J}_1^{(\pm)} \equiv \frac{8i\pi|qB|m^2}{(2\pi)^4} \exp\left(-\frac{2\mathbf{p}_\perp^2}{3|qB|}\right) \int_0^\infty \frac{\lambda d\lambda}{(1+\lambda)^3} \int \frac{d^2 \ell_E}{(2\pi)^2} \frac{\left(\ell \pm \frac{\lambda}{1+\lambda} p\right)_E^2 - m^2}{\sqrt{\left(\ell_4 \mp \frac{i\lambda}{1+\lambda} \omega\right)^2 + m^2} \left[\ell_E^2 + \frac{\lambda}{(1+\lambda)^2} \omega^2 \sin^2 \theta + m^2\right]^3}$$

$$\mathcal{K}_1^{(\pm)} \equiv -\frac{8i\pi|qB|}{(2\pi)^4} \exp\left(-\frac{2\mathbf{p}_\perp^2}{3|qB|}\right) \int_0^\infty \frac{\lambda^2 d\lambda}{(1+\lambda)^5} \int \frac{d^2 \ell_E}{(2\pi)^2} \frac{\left(\ell \pm \frac{\lambda}{1+\lambda} p\right)_E^2 - m^2}{\sqrt{\left(\ell_4 \mp \frac{i\lambda}{1+\lambda} \omega\right)^2 + m^2} \left[\ell_E^2 + \frac{\lambda}{(1+\lambda)^2} \omega^2 \sin^2 \theta + m^2\right]^3},$$

The Photon/Gluon Polarization Tensor in a Fluctuating Magnetic Field Background

After some “trazology”

$$T_1^{\mu\nu} = \left(\mathcal{I}_1^{(+)} + \mathcal{I}_1^{(-)} \right) P_{\parallel}^{\mu\nu} + \left(\mathcal{J}_1^{(+)} + \mathcal{J}_1^{(-)} \right) g_{\parallel}^{\mu\nu} + \left(\mathcal{K}_1^{(+)} + \mathcal{K}_1^{(-)} \right) \frac{p_{\parallel}^{\mu} p_{\parallel}^{\nu}}{m^2}$$

$$P_{\parallel} = g_{\parallel}^{\mu\nu} - \frac{p_{\parallel}^{\mu} p_{\parallel}^{\nu}}{p_{\parallel}^2}$$

$$T_2^{\mu\nu} = \left(\mathcal{I}_2^{(+)} + \mathcal{I}_2^{(-)} \right) \left(g_{\parallel}^{\mu\nu} + 2\delta_3^{\mu} \delta_3^{\nu} \right) + \left(\mathcal{J}_2^{(+)} + \mathcal{J}_2^{(-)} \right) \left(g_{\parallel}^{\mu\nu} - \frac{p_{\parallel}^{\mu} \delta_3^{\nu} + p_{\parallel}^{\nu} \delta_3^{\mu}}{p_3} \right)$$

$$T_3^{\mu\nu} = \left(\mathcal{I}_3^{(+)} + \mathcal{I}_3^{(-)} \right) \left(P_{\parallel}^{\mu\nu} - g_{\perp}^{\mu\nu} \right) + \left(\mathcal{J}_3^{(+)} + \mathcal{J}_3^{(-)} \right) \left(g_{\parallel}^{\mu\nu} - g_{\perp}^{\mu\nu} \right) + \left(\mathcal{K}_3^{(+)} + \mathcal{K}_3^{(-)} \right) \frac{p_{\parallel}^{\mu} p_{\parallel}^{\nu}}{m^2}$$

Consequences:

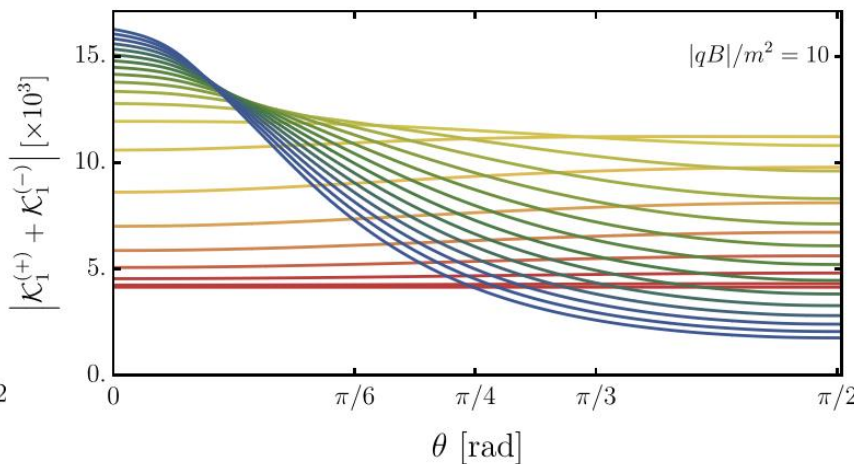
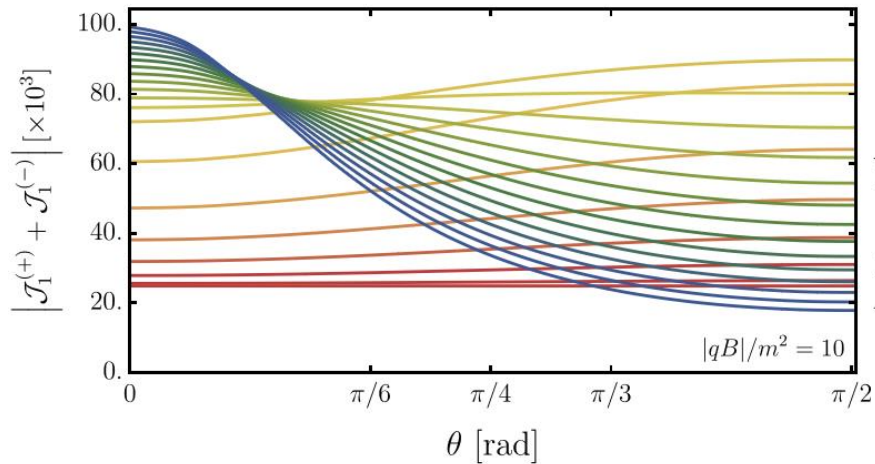
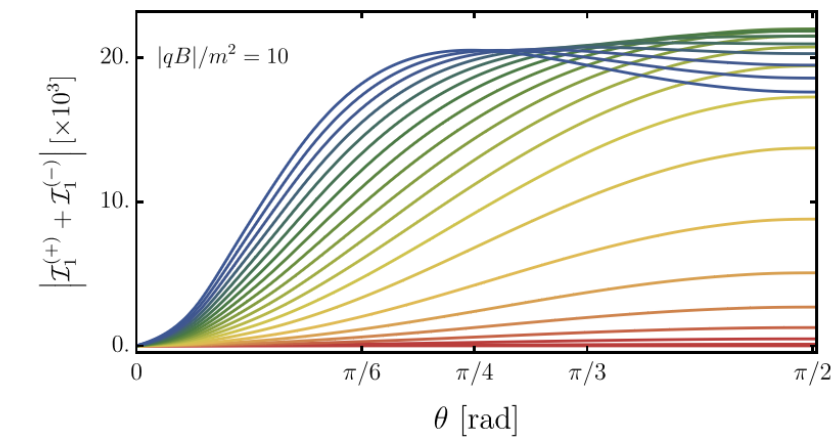
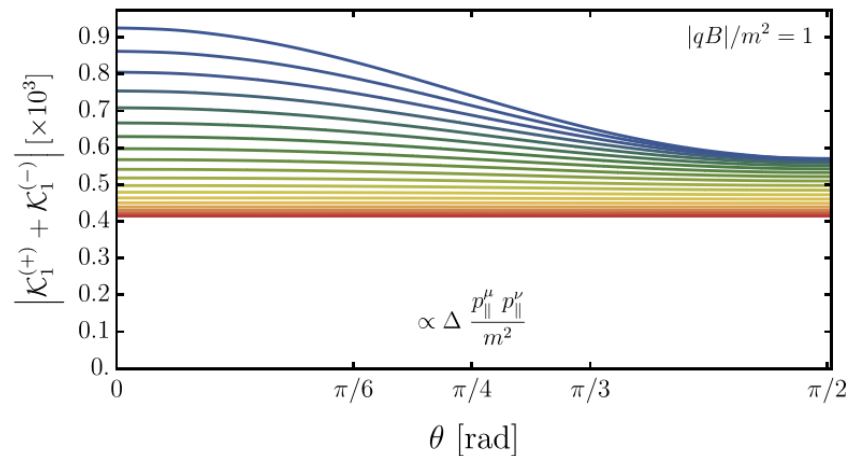
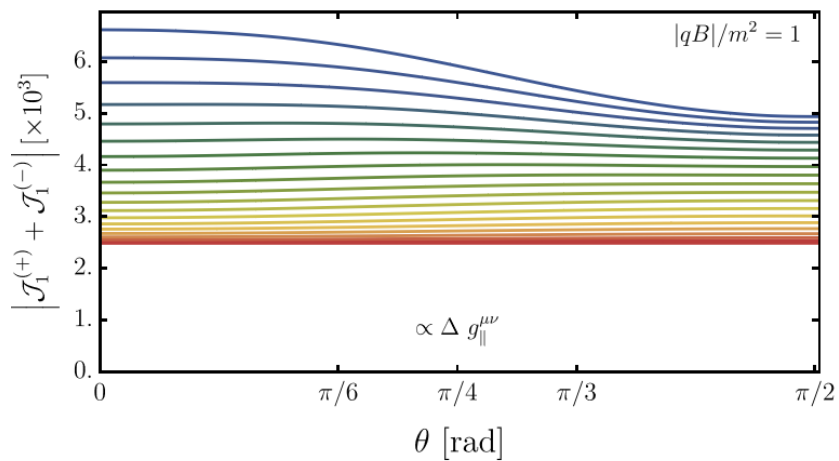
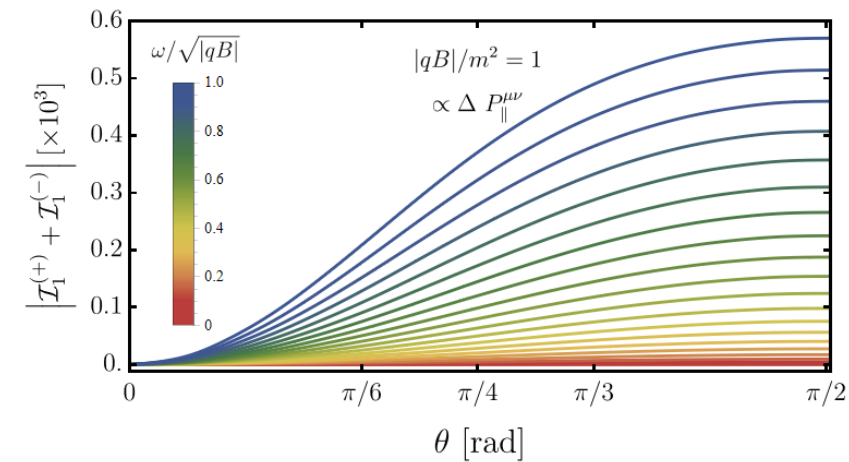
- If the integrals are non-zero, Ward identity is not preserved:

$$i\Pi_{\Delta}^{\mu\nu} = i\Pi_0^{\mu\nu} + i \frac{q^2 |qB| \Delta}{4\pi} \sum_{i=1}^{12} \int \frac{d^4 k}{(2\pi)^4} t_i^{\mu\nu}(k)$$

$$\Pi_0^{\mu\nu} \propto g_{\parallel}^{\mu\nu} - \frac{p_{\parallel}^{\mu} p_{\parallel}^{\nu}}{p_{\parallel}^2} \rightarrow p_{\mu} \Pi_0^{\mu\nu} = 0$$

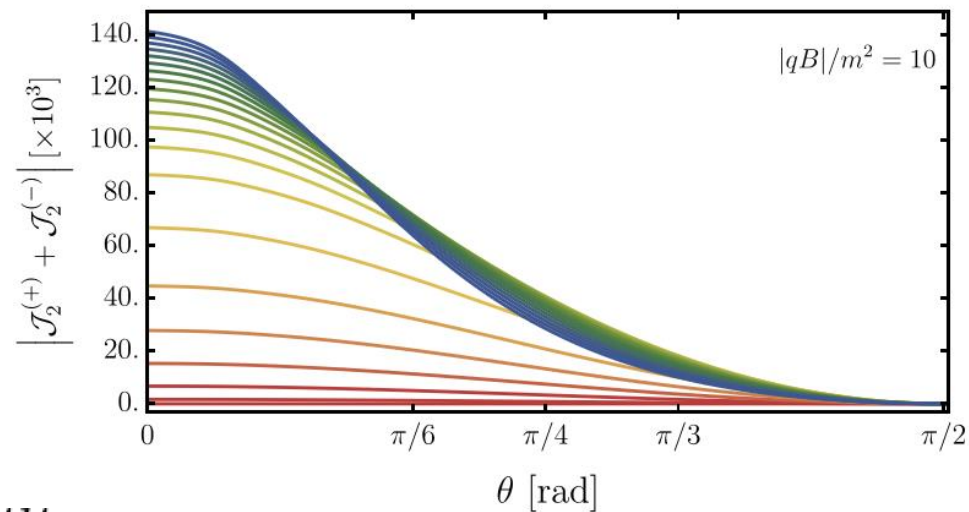
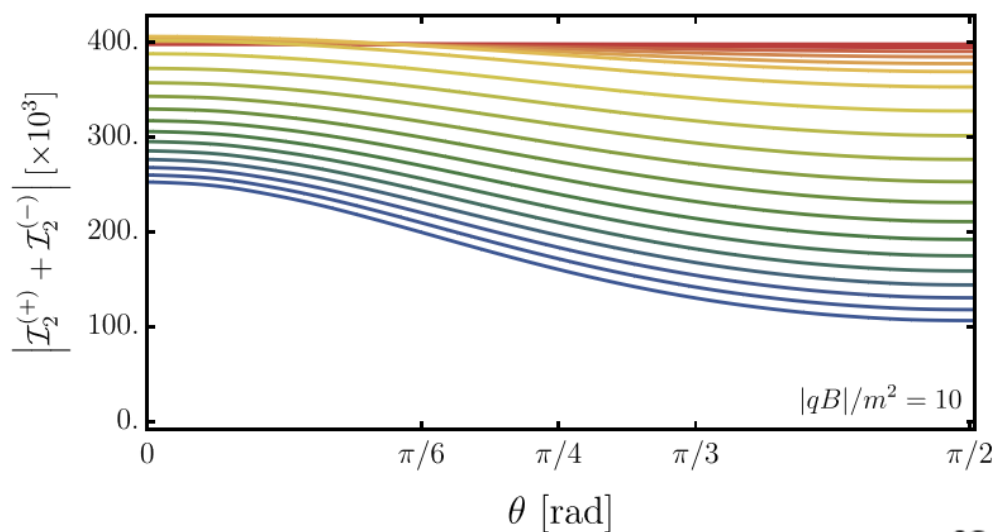
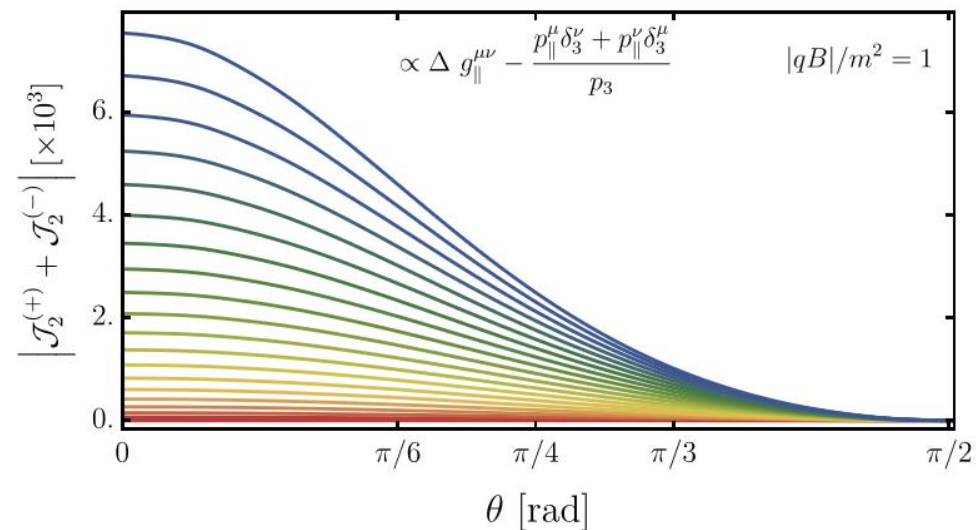
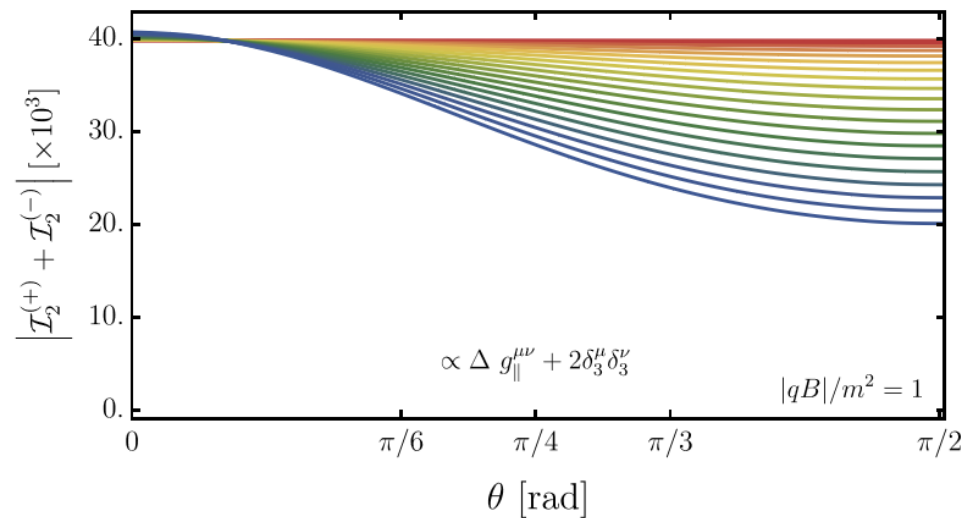
$$p_{\mu} T_i^{\mu\nu} \neq 0$$

$$T_1^{\mu\nu} = \left(\mathcal{I}_1^{(+)} + \mathcal{I}_1^{(-)} \right) P_{\parallel}^{\mu\nu} + \left(\mathcal{J}_1^{(+)} + \mathcal{J}_1^{(-)} \right) g_{\parallel}^{\mu\nu} + \left(\mathcal{K}_1^{(+)} + \mathcal{K}_1^{(-)} \right) \frac{p_{\parallel}^{\mu} p_{\parallel}^{\nu}}{m^2}$$



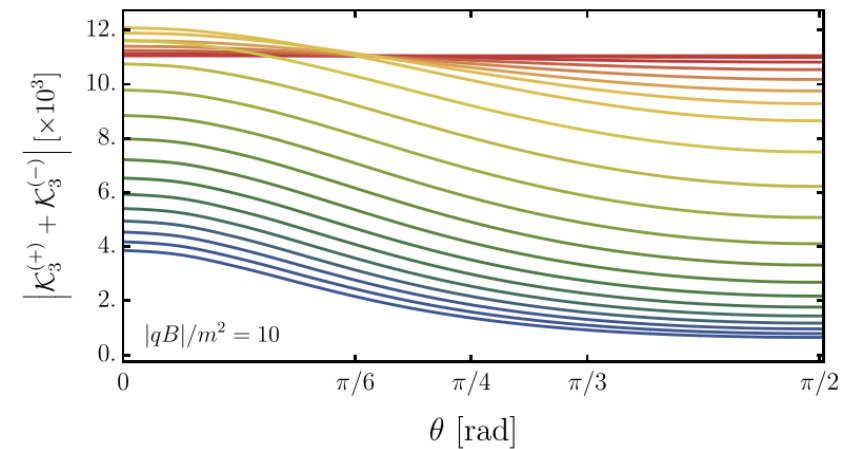
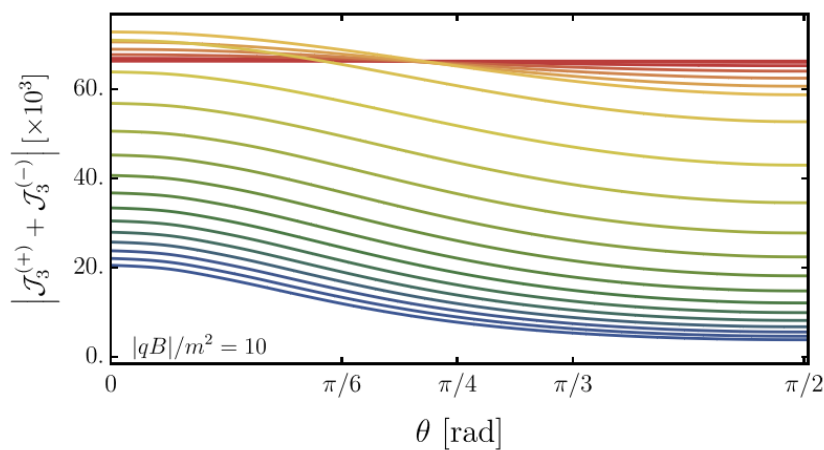
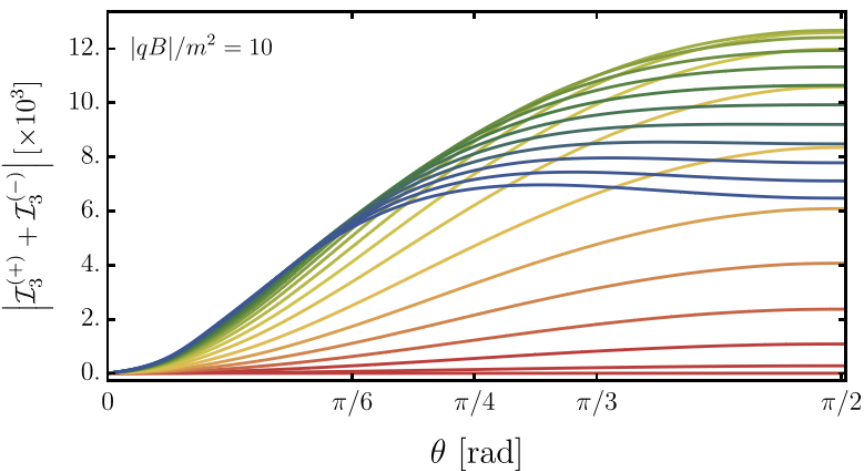
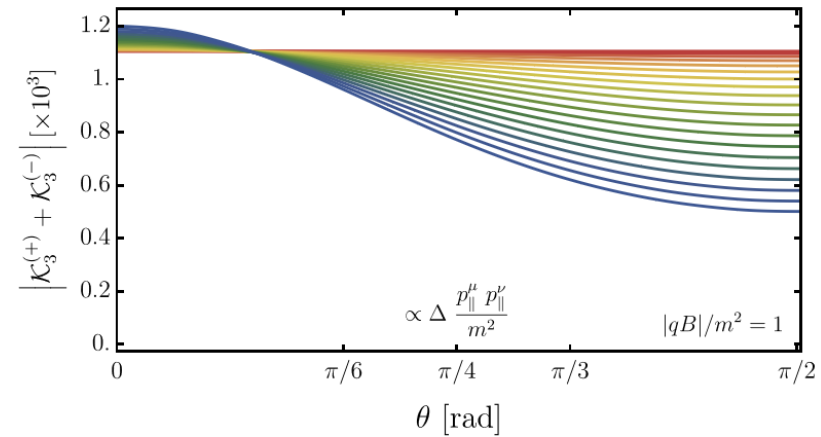
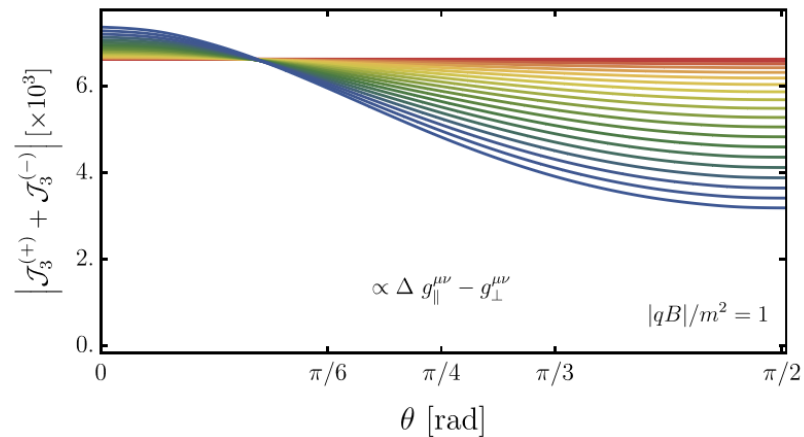
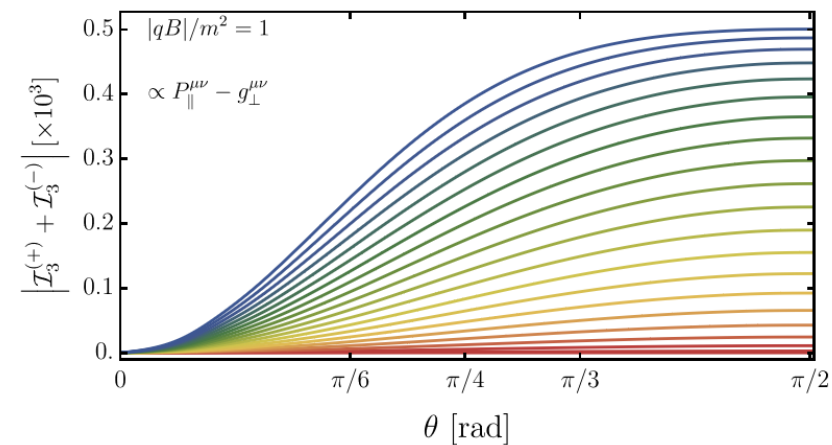
$$p_{\mu} T_1^{\mu\nu} \neq 0$$

$$T_2^{\mu\nu} = \left(\mathcal{I}_2^{(+)} + \mathcal{I}_2^{(-)} \right) \left(g_{\parallel}^{\mu\nu} + 2\delta_3^\mu \delta_3^\nu \right) + \left(\mathcal{J}_2^{(+)} + \mathcal{J}_2^{(-)} \right) \left(g_{\parallel}^{\mu\nu} - \frac{p_{\parallel}^\mu \delta_3^\nu + p_{\parallel}^\nu \delta_3^\mu}{p_3} \right)$$



$$p_\mu T_2^{\mu\nu} \neq 0$$

$$T_3^{\mu\nu} = \left(\mathcal{I}_3^{(+)} + \mathcal{I}_3^{(-)} \right) \left(P_{\parallel}^{\mu\nu} - g_{\perp}^{\mu\nu} \right) + \left(\mathcal{J}_3^{(+)} + \mathcal{J}_3^{(-)} \right) \left(g_{\parallel}^{\mu\nu} - g_{\perp}^{\mu\nu} \right) + \left(\mathcal{K}_3^{(+)} + \mathcal{K}_3^{(-)} \right) \frac{p_{\parallel}^{\mu} p_{\parallel}^{\nu}}{m^2}$$



$$p_{\mu} T_3^{\mu\nu} \neq 0$$

The Photon/Gluon Polarization Tensor in a Fluctuating Magnetic Field Background

After some “trazology”

$$T_1^{\mu\nu} = \left(\mathcal{I}_1^{(+)} + \mathcal{I}_1^{(-)} \right) P_{\parallel}^{\mu\nu} + \left(\mathcal{J}_1^{(+)} + \mathcal{J}_1^{(-)} \right) g_{\parallel}^{\mu\nu} + \left(\mathcal{K}_1^{(+)} + \mathcal{K}_1^{(-)} \right) \frac{p_{\parallel}^{\mu} p_{\parallel}^{\nu}}{m^2}$$

$$P_{\parallel} = g_{\parallel}^{\mu\nu} - \frac{p_{\parallel}^{\mu} p_{\parallel}^{\nu}}{p_{\parallel}^2}$$

$$T_2^{\mu\nu} = \left(\mathcal{I}_2^{(+)} + \mathcal{I}_2^{(-)} \right) \left(g_{\parallel}^{\mu\nu} + 2\delta_3^{\mu} \delta_3^{\nu} \right) + \left(\mathcal{J}_2^{(+)} + \mathcal{J}_2^{(-)} \right) \left(g_{\parallel}^{\mu\nu} - \frac{p_{\parallel}^{\mu} \delta_3^{\nu} + p_{\parallel}^{\nu} \delta_3^{\mu}}{p_3} \right)$$

$$T_3^{\mu\nu} = \left(\mathcal{I}_3^{(+)} + \mathcal{I}_3^{(-)} \right) \left(P_{\parallel}^{\mu\nu} - g_{\perp}^{\mu\nu} \right) + \left(\mathcal{J}_3^{(+)} + \mathcal{J}_3^{(-)} \right) \left(g_{\parallel}^{\mu\nu} - g_{\perp}^{\mu\nu} \right) + \left(\mathcal{K}_3^{(+)} + \mathcal{K}_3^{(-)} \right) \frac{p_{\parallel}^{\mu} p_{\parallel}^{\nu}}{m^2}$$

Consequences:

- If the integrals are non-zero, Ward identity is not preserved.
- If Ward identity is not preserved, can a photon acquire an effective mass due to magnetic quenched noise? [work in process].

Conclusions

- We applied the findings of the replica method to compute the one-loop gluon/photon polarization tensor.
- This leads to a non-transverse structure.
- The latter arises from the breaking of the $U(1)$ symmetry due to the effective potential induced by the magnetic noise.
- Conjecture: If the Ward identity is not satisfied, then there may be an effective magnetic-induced mass for photons and gluons.