QED Fermions in a noisy magnetic field background. The effective action approach

M. Loewe
Universidad San Sebastián, Santiago, Chile

In collaboarationwith Jorge David Castaño-Yepes (PUC), Enrique Muñoz (PUC) and Juan Cristóbal Rojas (UCN)

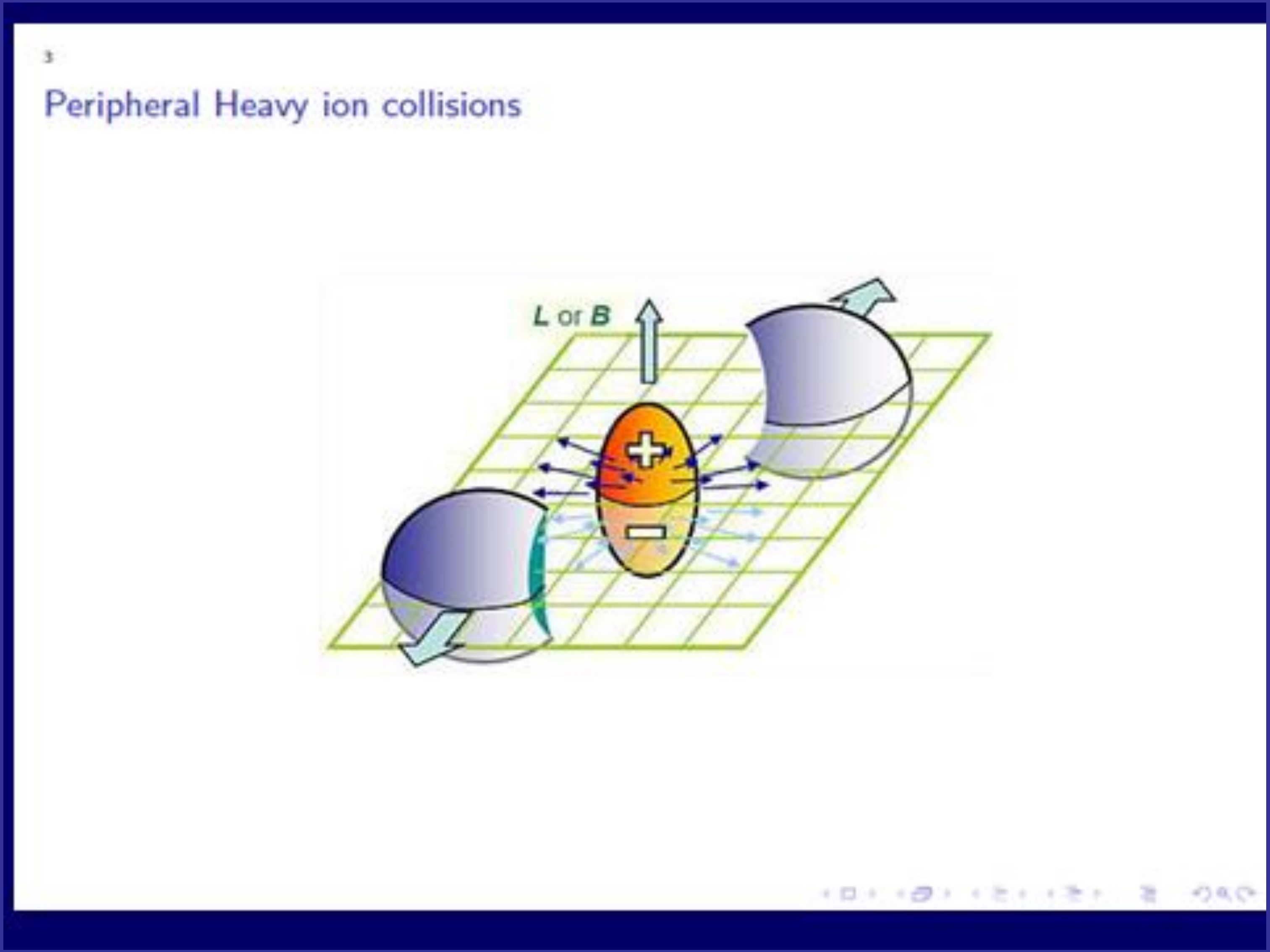
Workshop on Strongly Interacting Matter in Extreme Magnetic Fields.
Trento, Italy, 25-29 September 2023

This talk is based on the article:

“QED Fermions in a noisy magnetic field backgound: The effective action approach”, Jorge David Castaño-Yepes, M. Loewe, Enrique Muñoz, Juan Cristóbal Rojas;
arXiv: 2308.12249 [hep-th]

We aknowledge support from Fondecyt under grants: 1200483, 1190192, 1220035, 1190361

Magnetic Fields in perpheral heavy ion collisions. It is hard to believe that the field strength could be constant in the collision plane….



Effects of a constant and classical magnetic field background, in different physical scenarios, have been studied since the seminal work by J. Schwinger (Phys. Rev. 82, 664, 1951).

In all these studies, the background magnetic field is idealized as static and uniform: spatial anisotropies or fluctuations in its magnitude were normally disregarded.

Here we will consider a classical and static magnetic field background, as usual, possesing local random fluctuations by means of the replica method

The replica method will be used (M. Kardar, G. Parisi and Y. C. Zhang, P.R.L. 56, 889. 1986)

This work is an extension of a previous analysis, where spatial fluctuations of a classical magnetic field were considered from a perturbative perspective.
(Case presented in the previous talk by Enrique Muñoz)

Now, nevertheless, we abandon the perturbative treatment, going into a mean field description (in the spirit of the saddle point approach).

We also extend the previous discussion by allowing spatio-temporal fluctuation patterns. Physical consequences will be discussed.

The Model: QED in the presence of a classical and static magnetic field possesing random spatial fluctuations

$$A^\mu(x) \to A^\mu(x) + A^\mu_{\rm BG}(x) + \delta A^\mu_{\rm BG}(x).$$

We consider a white noise spatio-temporal fluctuation with respect to the mean value, i.e.

$$\begin{aligned} \langle \delta A^j_{\rm BG}(x) \delta A^k_{\rm BG}(x') \rangle_\Delta &= \Delta_B \delta_{j,k} \delta^{(4)}(x - x'), \\ \langle \delta A^\mu_{\rm BG}(x) \rangle_\Delta &= 0. \end{aligned}$$

As it is well known, these statistical properties are represented by a Gaussian functional distribution (which is natural because of the Central Limit theorem)

$$dP\left[\delta A^{\mu}_{\mathrm{BG}}\right] = \mathcal{N} e^{-\int d^4x \frac{\left[\delta A^{\mu}_{\mathrm{BG}}(x)\right]^2}{2\Delta_B}} \mathcal{D}\left[\delta A^{\mu}_{\mathrm{BG}}(x)\right]$$

In this way, we have the following decomposition

$$\mathcal{L} = \mathcal{L}_{\mathrm{FBG}} + \mathcal{L}_{\mathrm{NBG}},$$

$$\mathcal{L}_{\mathrm{FBG}} = \bar{\psi}\left(\mathrm{i}\partial\!\!\!/ - eA\!\!\!/_{\mathrm{BG}} - eA\!\!\!/ - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

$$\mathcal{L}_{\mathrm{NBG}} = \bar{\psi}\left(-e\delta A\!\!\!/_{\mathrm{BG}}\right)\psi.$$

The idea is that any quantity of interest will be averaged over the magnetic fluctuations

$$\langle \hat{O} \rangle_{\Delta} = \int dP \, [\delta A^{\mu}_{\mathrm{BG}}] \, \hat{O} \, [\delta A^{\mu}_{\mathrm{BG}}]$$

In particular, in QFT we are interested in lnZ (generator of connected diagrams)

The generating functional in absence of sources, forgetting about Photons, is given by

$$Z[A] = \int \mathcal{D}[\bar{\psi}, \psi] e^{\mathrm{i} \int d^4x [\mathcal{L}_{\mathrm{FBG}} + \mathcal{L}_{\mathrm{NBG}}]}$$

The physics of the system will be determined by a statistical average over the magnetic fluctuations of ln Z

$$\overline{\ln Z}$$

This is achieved in terms of the Replica Trick introduced by Parisi and coworkers

$$\overline{\ln Z[A]} = \lim_{n \to 0} \frac{\overline{Z^n[A]} - 1}{n}$$

The statistical average is given in terms of the Gaussian functional measure.

Z^n is obtained by the incorporation of replica components for the fermion fields. $\psi(x) \to \psi^a(x)$ $1 \leq a \leq n.$

$$\langle Z^n[A]\rangle_\Delta = \int \prod_{a=1}^{n} \mathcal{D}[\bar{\psi}^a, \psi^a] \int \mathcal{D}[\delta A_{\mathrm{BG}}^{\mu}]\, e^{-\int d^4x \frac{\left[\delta A_{\mathrm{BG}}^{\mu}(x)\right]^2}{2\Delta_B}} \times e^{\mathrm{i}\int d^4x \sum_{a=1}^{n}\left(\mathcal{L}_{\mathrm{FBG}}[\bar{\psi}^a,\psi^a]+\mathcal{L}_{DBG}[\bar{\psi}^a,\psi^a]\right)}$$

$$= \int \prod_{a=1}^{n} \mathcal{D}[\bar{\psi}^a, \psi^a]\, e^{\mathrm{i}\bar{S}[\bar{\psi}^a,\psi^a;A]}$$

After perfoming the integral over the magnetic fluctuations

$$\mathrm{i}\,\bar{S}\left[\bar{\psi}^a, \psi^a; A\right] = \mathrm{i}\int d^4x \left(\sum_a \bar{\psi}^a\left(\mathrm{i}\partial\!\!\!/ - eA\!\!\!/_{\mathrm{BG}} - eA\!\!\!/ - m_f\right)\psi^a - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$
$$- \frac{e^2\Delta_B}{2}\int d^4x \int d^4y \sum_{a,b}\sum_{j=1}^{3}\bar{\psi}^a(x)\gamma^j\psi^a(x)\bar{\psi}^b(y)\gamma_j\psi^b(y).$$

It reminds us the NJL-action (or the Fermi theory)

We have ended up with an effective interaction between vector currents, associated to different replica, with a coupling constant proportional to the fluctuation amplitude Δ_B .

The "free" part of the action corresponds to fermions in the presence of the average background classical field $A^{\mu}_{BG}(x)$.

We choose the background to represent a constant magnetic field in the Z direction

$$\mathbf{B} = \hat{e}_3 B,$$

And use the symmetric gauge

$$A^{\mu}_{\mathrm{BG}}(x) = \frac{1}{2}(0, -Bx^2, Bx^1, 0).$$

We know the Schwinger proper time representation of the "free"-Fermion Propagator dressed by the background field. No photons will be considered in the analysis. Now, some technicalities…

To deal with this effective action we will introduce the Hubbard-Stratonovich transformation via a set of complex auxiliary scalar fields $Q_i(x)$ using a gaussian integral identity

John Hubbard

Ruslan Stratonovich

$$e^{-\frac{e^2\Delta_B}{2}\int d^4x\int d^4y\sum_{a,b}^{n}\sum_{j=1}^{3}\bar{\psi}^a(x)\gamma^j\psi^a(x)\bar{\psi}^b(y)\gamma_j\psi^b(y)}$$

$$= \mathcal{N}\left[\prod_{j=1}^{3}\int \mathcal{D}Q_j(x)\mathcal{D}Q_j^*(x)\right]$$

$$e^{-\frac{2}{\Delta_B}\int d^4x|Q_j(x)|^2+ie\int d^4xQ_j(x)\sum_{a=1}^{n}\bar{\psi}^a(x)\gamma^j\psi^a(x)-ie\int d^4xQ_j^*(x)\sum_{a=1}^{n}\bar{\psi}^a(x)\gamma^j\psi^a(x)}$$

Inserting this transformation into the replicated n^{th} power of the generating functional that gives us

$$\langle Z^n[A]\rangle_\Delta$$

We obtain

$$
\begin{aligned}
\langle Z^n[A_{\mathrm{BG}}]\rangle_\Delta &= \mathcal{N}\left[\prod_{j=1}^{3}\int \mathcal{D}Q_j(x)\mathcal{D}Q_j^*(x)\right] e^{-\frac{2}{\Delta_B}\int d^4x\,|Q_j(x)|^2} \\
&\quad\times\left[\prod_{a=1}^{n}\int \mathcal{D}[\bar{\psi}^a,\psi^a]\right] e^{\mathrm{i}\int d^4x\left\{\sum_{a=1}^{n}\bar{\psi}^a(x)\left(\mathrm{i}\partial\!\!\!/-eA\!\!\!/_{\mathrm{BG}}-m_f-e\gamma^j(Q_j-Q_j^*)\right)\psi^a(x)\right\}} \\
&= \mathcal{N}\left[\prod_{j=1}^{3}\int \mathcal{D}Q_j(x)\mathcal{D}Q_j^*(x)\right] e^{-\frac{2}{\Delta_B}\int d^4x\,|Q_j(x)|^2}\left[\det\left(\mathrm{i}\partial\!\!\!/-eA\!\!\!/_{\mathrm{BG}}-m_f-e\gamma^j\left(Q_j-Q_j^*\right)\right)\right]^n \\
&= \mathcal{N}\left[\prod_{j=1}^{3}\int \mathcal{D}Q_j(x)\mathcal{D}Q_j^*(x)\right] e^{-\frac{2}{\Delta_B}\int d^4x\,|Q_j(x)|^2+n\,\mathrm{Tr}\ln\left[\mathrm{i}\partial\!\!\!/-eA\!\!\!/_{\mathrm{BG}}-m_f-e\gamma^j\left(Q_j-Q_j^*\right)\right]}
\end{aligned}
$$

We will return to this expression later

The effective action can be written as

$$\mathrm{i}\, S_{\text{eff}}[A_{\text{BG}}] - \mathrm{i}\, S_0[A_{\text{BG}}] = \langle \ln Z[A_{\text{BG}}] \rangle_\Delta - \ln Z_0[A_{\text{BG}}]$$

$$= \mathcal{N} \left[\prod_{j=1}^{3} \int \mathcal{D}Q_j(x) \mathcal{D}Q_j^*(x) \right] e^{-\frac{2}{\Delta_B} \int d^4x\, |Q_j(x)|^2}$$

$$\lim_{n \to 0} \frac{1}{n} \left[e^{n \operatorname{Tr} \ln \left[\mathrm{i}\not\partial - e \not A_{\text{BG}} - m_f - e\gamma^j (Q_j - Q_j^*) \right]} - 1 \right]$$

$$= \langle \operatorname{Tr} \ln \left(\mathrm{i}\not\partial - e\not A_{\text{BG}} - m_f - e\gamma^j (Q_j - Q_j^*) \right) \rangle_\Delta - \operatorname{Tr} \ln \left(\mathrm{i}\not\partial - e\not A_{\text{BG}} - m_f \right)$$

$\langle (\cdot) \rangle_\Delta$ Denotes the average over the Gaussian functional measure of the complex fields Q_j

In terms of the inverse fermion propagator in the presence of the bakground field

$$S_{\mathrm{F}}^{-1}(x-y) = \left(\mathrm{i}\partial\!\!\!/ - eA\!\!\!/_{\mathrm{BG}} - m_f\right)_x \delta^{(4)}(x-y)$$

the effective action can be written as

$$\begin{aligned} \mathrm{i}\, S_{\mathrm{eff}}[A_{\mathrm{BG}}] &= \langle \mathrm{Tr}\ln\left(\mathrm{i}\partial\!\!\!/ - eA\!\!\!/_{\mathrm{BG}} - m_f - e\gamma^j(Q_j - Q_j^*)\right)\rangle_\Delta \\ &= \langle \mathrm{Tr}\ln\left(S_{\mathrm{F}}^{-1} - e\gamma^j(Q_j - Q_j^*)\right)\rangle_\Delta \end{aligned}$$

Since

$$\mathrm{i}\, S_0[A_{\mathrm{BG}}] = \mathrm{Tr}\ln S_{\mathrm{F}}^{-1}$$

$$\begin{aligned} \mathrm{i}\, S_{\mathrm{eff}}[A_{\mathrm{BG}}] &- \mathrm{i}\, S_0[A_{\mathrm{BG}}] \\ &= \langle \mathrm{Tr}\ln\left(1 - eS_{\mathrm{F}}\gamma^j(Q_j - Q_j^*)\right)\rangle_\Delta \end{aligned}$$

Effects of magnetic disorder are encoded in the last term.

Before we proceed, it is relevant to have an idea about Δ_B.
In heavy ion collisions there is a natural length scale

$$L \sim \sqrt{\sigma},$$

By dimensional analysis, $[\Delta_B]$ MeV^{-2} On top of the big magnetic field in the z direction, we have also magnetic fluctuations δB_x and δB_y. We can estimate the magnetic fluctuation, within a small region, being of the order

$$(\delta B)^2 \sim (\delta B_x)^2 + (\delta B_y)^2.$$

Many collisions occur at different points in space with a time-spam of the order

$$\delta\tau \sim L'/c$$

In this way we estimate

$$\Delta_B \sim (\delta B)^2\ L^5 L' \sim (\delta B)^2\ \sigma^{5/2} L'.$$

The effective cross-section corresponds to a fraction f of the area of perfectly central collisions between two nuclei of radius r_A

$$\sigma = f \pi r_A^2.$$

f can be interpreted as the fracion of: 1) the geometrical cross-section (σ_{geom}) considered as the area of the circle with radius $r_1 + r_2 = 2R$ in maximum peripheral collision and 2) the cross-section for a peripheral collision with impact parameter b (σ_b)

$$f = \frac{\sigma_b}{\sigma_{\text{geom}}} = \left(\frac{N_{\text{part}}}{2N} \right)^{2/3},$$

The nuclear radius obeys

$r_A = r_0 N^{1/3}$, with $r_0 \sim 10^{-3}$ MeV^{-1}

N_{part} is the number of nucleons participating in the collision. The magnetic field fluctuations are approximately $|e\,\delta B| \sim m_\pi^2/4$. In Au-Au collision with N=197, if $N_{part}/N = ½$ (with $L' \sim r_A$) we obtain

$$e^2\Delta_B \sim 1.6 \times 10^{-6}\,\mathrm{MeV}^{-2}$$

For less central conditions, $N_{part}/N = 1/8$, we get $e^2\Delta_B \sim 1.6 \times 10^{-7}\ \mathrm{MeV}^{-2}$.

It is convenient to use the dimensionless parameter $\tilde{\Delta} = e^2\Delta_B m_f^2$

which varies between $\Delta_{\rm proton} \sim 0.16 - 1.6$, for the proton mass and

$\Delta_{\rm quark} \sim 0.018 - 0.18$. for a quark mass.

Saddle Point Approximation (Mean Field) for the Effective Action

Let us write explicitly our previous expression

$$\mathrm{i}\,S_{\mathrm{eff}}[A_{\mathrm{BG}}] - \mathrm{i}\,S_0[A_{\mathrm{BG}}] = \mathcal{N}\left[\prod_{j=1}^{3}\int \mathcal{D}Q_j(x)\mathcal{D}Q_j^*(x)\right] e^{-\frac{2}{\Delta_B}\int d^4x\,|Q_j(x)|^2 + \ln\left[\mathrm{Tr}\ln\left(\mathbf{1} - eS_{\mathrm{F}}\gamma^j(Q_j - Q_j^*)\right)\right]}$$

The saddle point of the exponent is determined from

$$\frac{\delta}{\delta Q_j(x)}\left\{-\frac{2}{\Delta_B}\int d^4y Q_l(y)Q_l^*(y) + \ln\left[\mathrm{Tr}\ln\left[\mathbf{1} - eS_{\mathrm{F}}\gamma^l(Q_l - Q_l^*)\right]\right]\right\} = 0,$$

$$\frac{\delta}{\delta Q_j^*(x)}\left\{-\frac{2}{\Delta_B}\int d^4y Q_l(y)Q_l^*(y) + \ln\left[\mathrm{Tr}\ln\left[\mathbf{1} - eS_{\mathrm{F}}\gamma^l(Q_l - Q_l^*)\right]\right]\right\} = 0.$$

This gives us

$$Q_j^* = -e\frac{\Delta_B}{2}\frac{\mathrm{Tr}\left[S_F\gamma^j\left(\mathbf{1} - eS_F\gamma^l(Q_l - Q_l^*)\right)^{-1}\right]}{\mathrm{Tr}\ln\left[\mathbf{1} - eS_F\gamma^l(Q_l - Q_l^*)\right]}$$

You can verify

$$\begin{aligned} Q_j + Q_j^* &= 0 \\ q_j \equiv Q_j - Q_j^* &= 2\,\mathrm{i}\,\mathrm{Im}Q_j \neq 0, \end{aligned}$$

$$q_j = e\Delta_B\frac{\mathrm{Tr}\left[S_F\gamma^j\left(\mathbf{1} - eS_F\gamma^l q_l\right)^{-1}\right]}{\mathrm{Tr}\ln\left[\mathbf{1} - eS_F\gamma^l q_l\right]}.$$

We will use q_j

Expanding the numerator in a geometric series and the numerator in a Taylor series, keeping terms up to third order in the q_j coefficients in the numerator while retaining up to second order in the numerator we get

$$q_j = e\Delta_B \frac{e\mathcal{M}^{jl} q_l + e^3 \mathcal{M}^{jlmn} q_l q_m q_n}{\frac{e^2}{2}\mathcal{M}^{mn} q_m q_n},$$

Where we have introduced

$$\begin{aligned} \mathcal{M}^{jl} &= \mathrm{Tr}\left[S_{\mathrm{F}}\gamma^j S_{\mathrm{F}}\gamma^l\right] \\ &= \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left[S_{\mathrm{F}}(k)\gamma^j S_{\mathrm{F}}(-k)\gamma^l\right], \end{aligned}$$

$$\mathcal{M}^{jlmn} = \mathrm{Tr}\left[S_{\mathrm{F}}\gamma^j S_{\mathrm{F}}\gamma^l S_{\mathrm{F}}\gamma^m S_{\mathrm{F}}\gamma^n\right]$$

$$= \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left[S_{\mathrm{F}}(k)\gamma^j S_{\mathrm{F}}(k)\gamma^l S_{\mathrm{F}}(k)\gamma^m S_{\mathrm{F}}(k)\gamma^n\right]$$

Our equation for q_j can be casted as a quasi-linear system of equations.

$$\left(\Delta_B \mathcal{M} + \tilde{\mathcal{M}}[\mathbf{q}]\right) \mathbf{q} = 0,$$

$$\left[\tilde{\mathcal{M}}[\mathbf{q}]\right]^{jl} \equiv \left(-\frac{1}{2}\delta^{jl}\mathcal{M}^{mn} + e^2 \Delta_B \mathcal{M}^{jlmn}\right) q_m q_n$$

Nontrivial solutions might exist, provided that the matrix coefficient is singular. Secular equation.

$$\det\left(\Delta_B \mathcal{M} + \tilde{\mathcal{M}}[\mathbf{q}]\right) = 0.$$

Then we need to work with Schwinger´s propagator, which participates in the definition of our matrices.

$$[S_{\mathrm{F}}(k)]_{a,b} = -\mathrm{i}\delta_{a,b} \int_0^\infty \frac{d\tau}{\cos(eB\tau)} e^{\mathrm{i}\tau\left(k_\parallel^2 - \mathbf{k}_\perp^2 \frac{\tan(eB\tau)}{eB\tau} - m^2 + \mathrm{i}\epsilon\right)}$$
$$\times \left\{ \left[\cos(eB\tau) + \mathrm{i}\gamma^1\gamma^2 \sin(eB\tau)\right] (m + \not{k}_\parallel) + \frac{\not{k}_\perp}{\cos(eB\tau)} \right\}$$

Which is diagonal in the replica indeces

It is natural to decompose $g^{\mu\nu} = g_\parallel^{\mu\nu} + g_\perp^{\mu\nu}$ such that

$$g_\parallel^{\mu\nu} = \mathrm{diag}(1, 0, 0, -1),$$
$$g_\perp^{\mu\nu} = \mathrm{diag}(0, -1, -1, 0),$$

$$\not{k} = \not{k}_\perp + \not{k}_\parallel$$

$$k^2 = k_\parallel^2 - \mathbf{k}_\perp^2$$

$$k_\parallel^2 = k_0^2 - k_3^2,$$

Important remarks: If we consider our previous expression for $\langle Z^n[A_{\mathrm{BG}}]\rangle_\Delta$

(Page 15)

before integrating out the fermion fields, we obtain via the saddle point. the mean expectation value

$$Q_j = \mathrm{i}e\Delta_B\langle\langle\bar{\psi}\gamma_j\psi\rangle\rangle_\Delta = -Q_j^*,$$

showing that the physical meaning of our order parameters q_j corresponds to components of a vector current for the fermions, j=1,2,3

So, if we find non-trivial solutions for q_j, this means that fermionic vector currents will appear in our system

A second important remarks concerns the behavior of the fermionic propagator in the presence of the Δ_B term.

The differential equation for the propagator in the presence of magnetic noise is given by

$$\left(\mathrm{i}\partial\!\!\!/ - e A\!\!\!/_{\mathrm{BG}} - m_f - e q\!\!\!/\right)_x S_{\mathrm{F},\Delta}(x-y) = \mathrm{i}\delta^{(4)}(x-y).$$

It is easy to verify that

$$S_{\mathrm{F},\Delta}(x-y) = e^{-\mathrm{i}eq\cdot(x-y)} S_{\mathrm{F}}(x-y)$$

satisfies the previous equation. Since q is imaginary, we conclude that at the level of the disorder-averaged propagator, the phase factor introduces an exponential damping effect

Results and Conclusions

It is convenient to introduce dimensionless quantities

$$\Delta = e^2 \Delta_B m_f^2 ,$$
$$\mathcal{B} = \frac{eB}{m_f^2} ,$$

We will present results for the order parameter $\boldsymbol{q} = (q_1, q_2, q_3)$

in two different cases

Case 1: $q_3^2 \equiv q_{\|}^2$, **with** $q_1 = q_2 = 0$

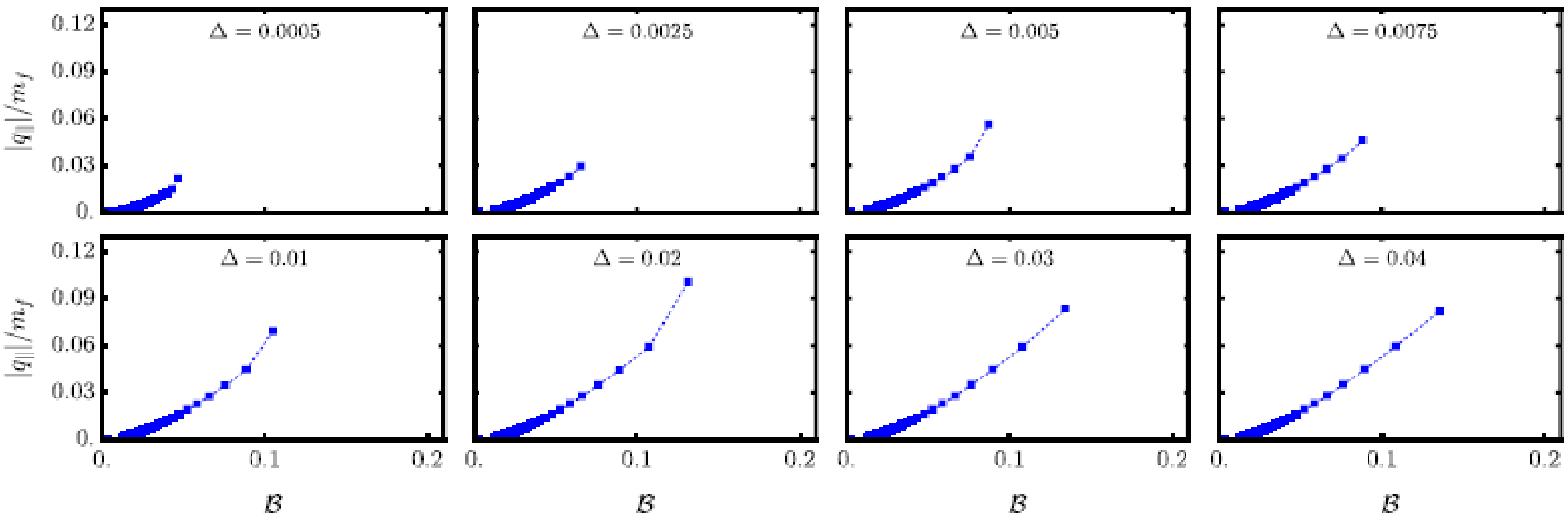
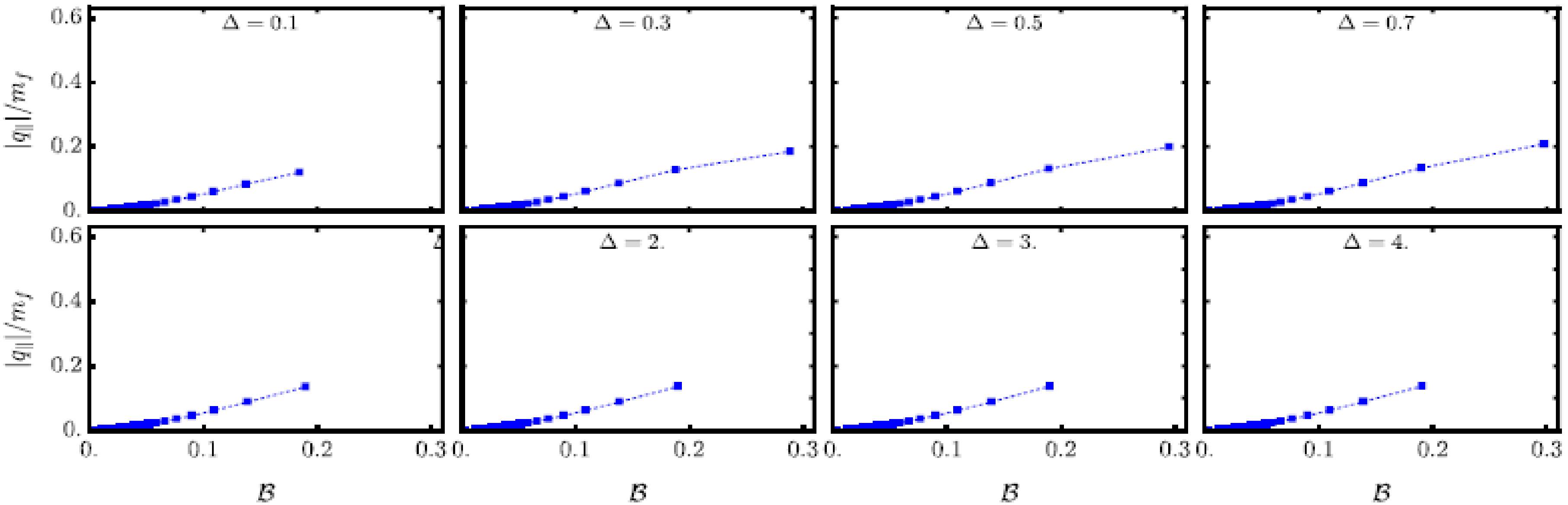


FIG. 1. Non-trivial solutions of Eq. (52) for the Case 1 as a function of $\mathcal{B}$ for $\Delta \in [0.1, 0.04]$. The dashed line represents the smooth envelope connecting the discrete non-trivial solutions.

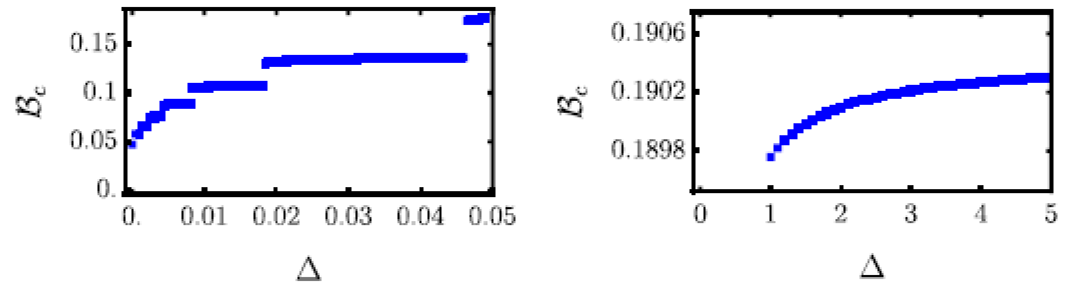
We find solutions for values of $\mathcal{B}$ only up to a certain value (critical field).

Beyond this value the only existing solution to the secular equation is the trivial one (absence of currents induced by the noise in the medium)

The same case for higher values of Δ



The behavior of the critica field



Similar results were found for **Case 2:** $q_1^2 = q_2^2 \equiv q_\perp^2$, **with** $q_3 = 0$

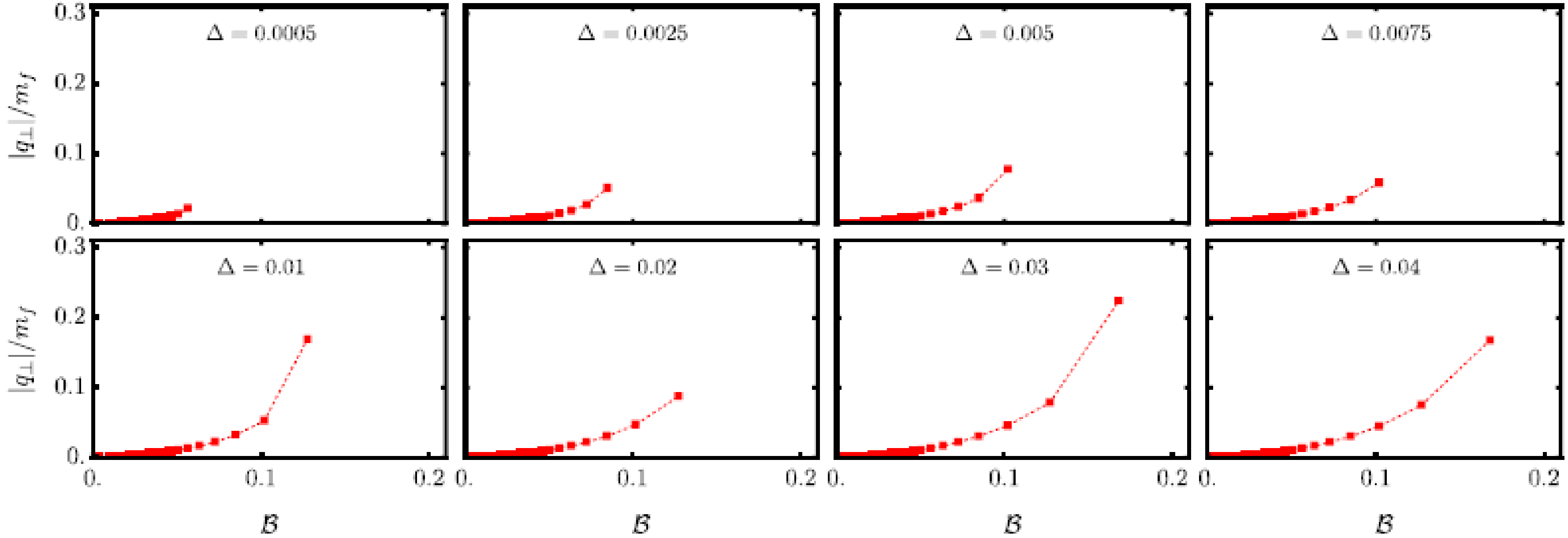
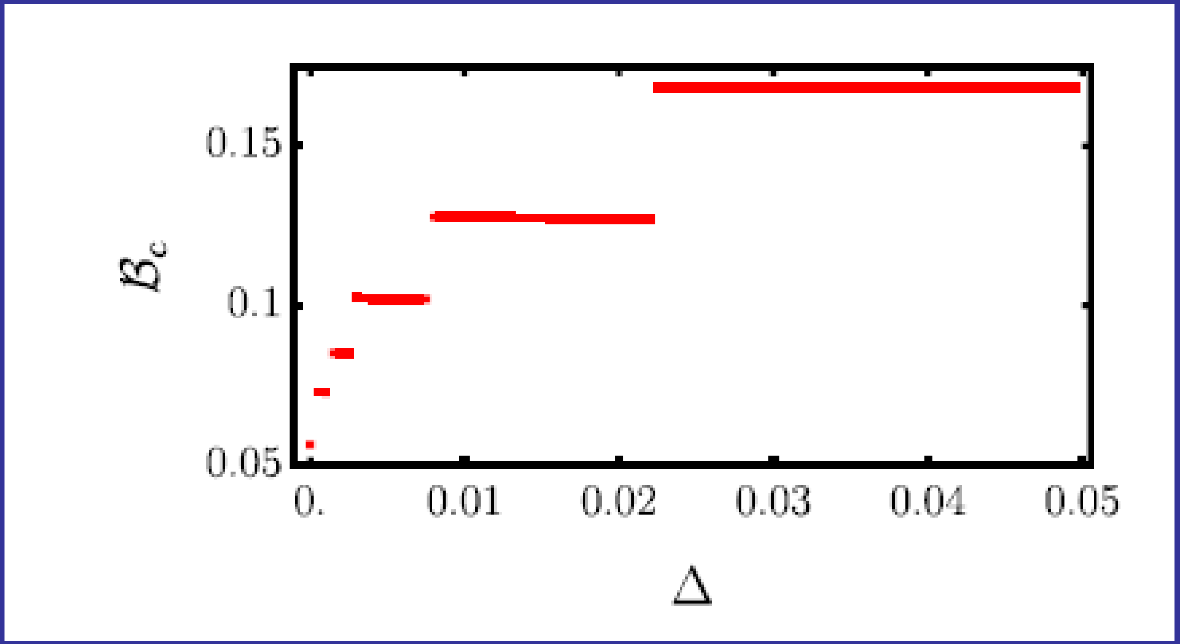


FIG. 5. Non-trivial solutions of Eq. (52) for the Case 2 as a function of $\mathcal{B}$ for $\Delta \in [0.1, 0.04]$. The dashed line is the smooth envelope connecting those discrete non-trivial solutions.

The critical field in this case is lower compared with the previous case

The crtical fields as function of Δ



As a final remark. In the firs case

$$\det\left(\mathbf{1}_3 + q_{\|}^2 \Delta_B^{-1} \mathcal{M}^{-1} \mathcal{C}_{\|}\right) = 0.$$

We can use

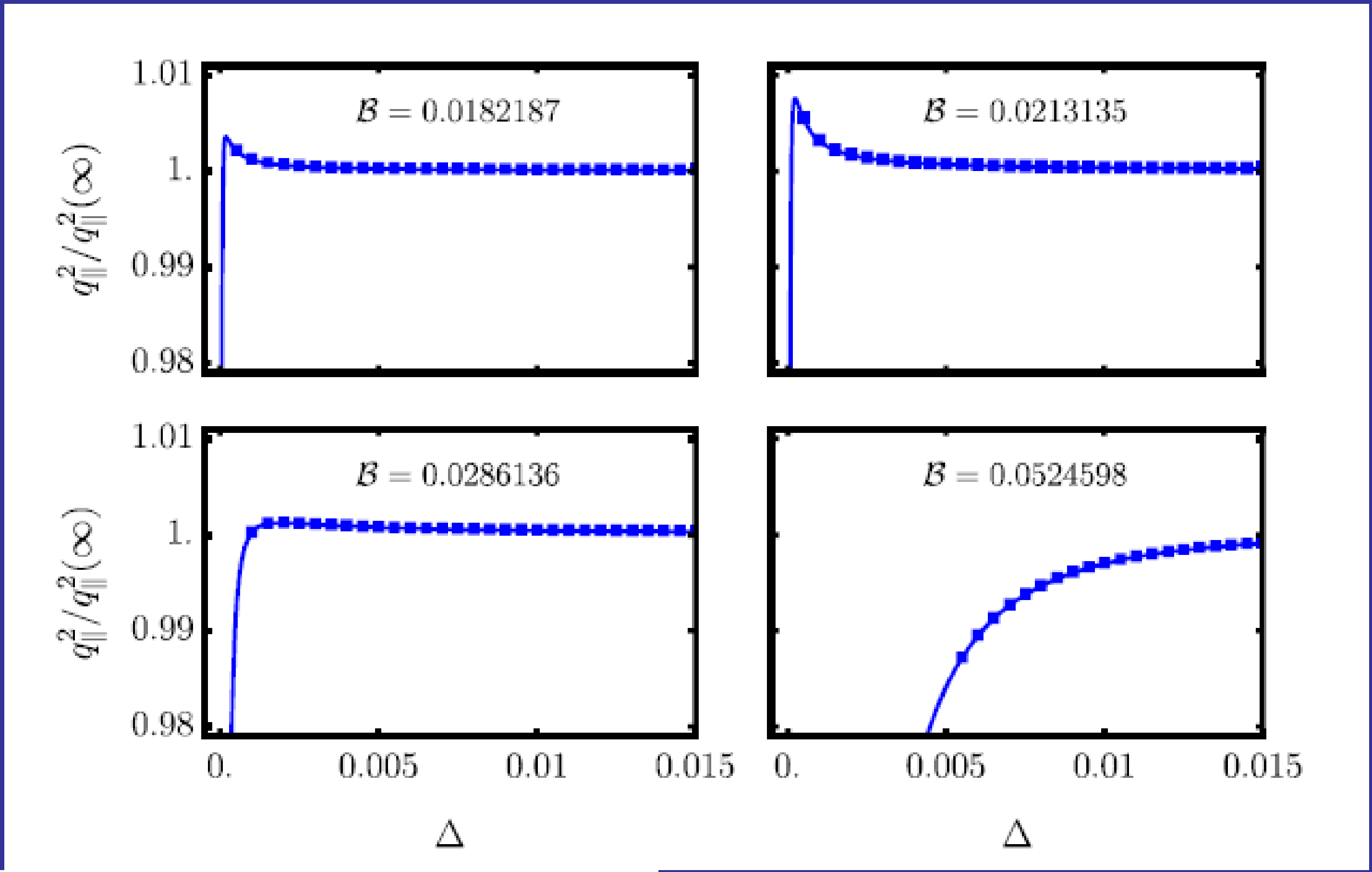
$$\det(\mathbf{1} + \epsilon X) = 1 + \epsilon \mathrm{tr} X + O(\epsilon^2)$$

$$\begin{aligned} q_{\|}^2 &= -\frac{\Delta_B}{\mathrm{tr}\left(\mathcal{M}^{-1}\mathcal{C}_{\|}\right)} \\ &= -\frac{\Delta_B}{-\frac{1}{2}\mathcal{M}^{33}\mathrm{tr}\left(\mathcal{M}^{-1}\right) + e^2\Delta_B \mathrm{tr}\left(\mathcal{M}^{-1}\tilde{\mathcal{M}}^{(33)}\right)} \\ &= \frac{\chi_{\|}\Delta_B}{1 + \mathcal{K}_{\|}\Delta_B}, \end{aligned}$$

with

$$\begin{aligned} \chi_{\|} &= \frac{2}{\mathcal{M}^{33}\,\mathrm{tr}\left(\mathcal{M}^{-1}\right)} \\ \mathcal{K}_{\|} &= \frac{-2e^2\mathrm{tr}\left(\mathcal{M}^{-1}\tilde{\mathcal{M}}^{(33)}\right)}{\mathcal{M}^{33}\,\mathrm{tr}\left(\mathcal{M}^{-1}\right)}, \end{aligned}$$

$$\left[\tilde{\mathcal{M}}^{(33)}\right]^{jl} \equiv \mathcal{M}^{jl33}$$



Discrete solutions for the order parameter (normalized by its asymptotic limit) as a function of Δ, for fixed $\mathcal{B}$ (filled squares).

1) A mean field analysis of the effective action in terms of auxiliary bosonic fields shows that the magnetic noise effects can be captured by an order parameter (q_j). This order parameter is the statistical ensemble average of the expectation value of the fermion vector current components. Non-trivial solutions for the order parameter signal break of the U(1) gauge symmetry.

2) Those solutions exist only for certain discrete values of the average background magnetic field. These values can be identified with the quantized Landau levels associated with the average magnetic field.

3) For a fixed value of Δ_B there is an upper critical value for the existence of a critical field beyond which there are no non-trivial solutions for the order parameter.

4) Schwinger's average propagator due to magnetic noise presents an exponential damping.   The order parameter represents, therefore, a finite screening length that leads to a weak location.

Thank you for your attention!!