QED Fermions in a noisy magnetic field background

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Enrique Muñoz

Pontificia Universidad Católica de Chile

Co-authors: J. Castaño, M. Loewe, J.-C. Rojas, R. Zamora

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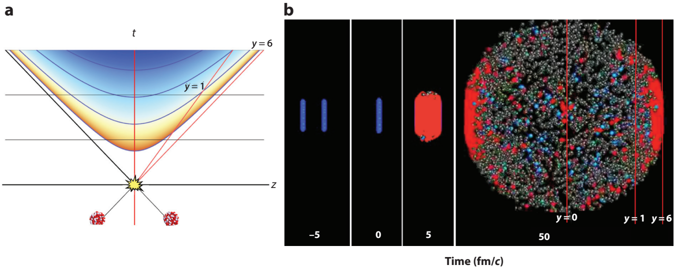
Heavy Ion Collissions (HIC)

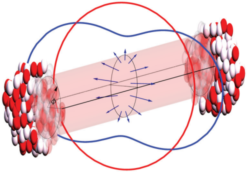


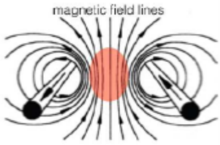
a) Space-time picture of a HIC, color indicates T of the plasma formed

b) Snapshots of a central 2.76 TeV Pb+Pb collison. Blue and grey are hadrons, red is the quark-gluon plasma
http://web.mit.edu/mithig/movies/LHCanmation.mov

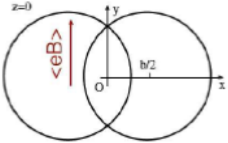
Busza, Rajagopal, van der Schee, Annu. Rev. Nucl. Part. Sci. 2018. 68:339-76

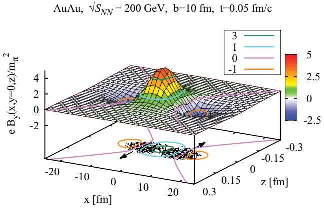
Magnetic fields in HIC

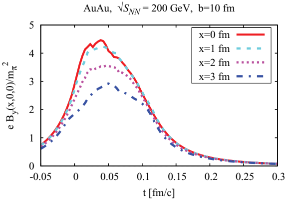




Very intense magnetic fields at initial times







Voronyuk, Toneev, Cassing, Bratkovskaya, Konchakovski, Voloshin, Phys. Rev. C 83, 054911 (2011)

Motivation

- The effect of a constant “classical” magnetic field background has been studied since the seminal work of Schwinger (Phys. Rev. 82, 664 (1951))
- In most theoretical studies, the background magnetic field is idealized as static and uniform
- In non-central HIC scenarios, strong magnetic fields emerge in comparatively small regions of space, with spatial anisotropies and fluctuations
- We here propose a statistical model to study the effects of such fluctuations

Inhomogeneus magnetic fields: Statistical model

QED gauge fields $A^\mu(x)$, involving three physically different contributions

$$\boxed{A^\mu(x) \to A^\mu(x) + A^\mu_{\mathrm{BG}}(x) + \delta A^\mu_{\mathrm{BG}}(\mathbf{x})}$$

Here, ”BG” stands for classical background in contrast with photons $A^\mu(x)$. We shall assume the following statistical properties for the BG fluctuation

$$\begin{aligned} \overline{\delta A^j_{\mathrm{BG}}(\mathbf{x})\delta A^k_{\mathrm{BG}}(\mathbf{x}')} &= \Delta_B \delta_{j,k}\delta^{(3)}(\mathbf{x}-\mathbf{x}') \\ \overline{\delta A^\mu_{\mathrm{BG}}(\mathbf{x})} &= 0 \end{aligned}$$

Phenomenological scenario for the magnetic fluctuations

- Very strong magnetic fields $\mathbf{B} = \nabla \times \mathbf{A}_{BG}$ are generated locally within a small spatial region $L \sim \sqrt{\sigma}$
- On average $\langle \mathbf{B} \rangle = \hat{e}_3\, B$, but smaller transverse components δB_x and δB_y exist such that field fluctuations are estimated on the order of $(\delta B)^2 \sim (\delta B_x)^2 + (\delta B_y)^2$.
- Therefore, by dimensional analysis

$$\boxed{\Delta_B \sim (\delta B)^2\; L^5 \sim (\delta B)^2\; \sigma^{5/2}}$$

Further estimations

- The fraction f of the geometrical cross-section σ_{geom}, defined by a circle with a radius of $r_1 + r_2 = 2R$ in a maximum peripheral collision, and the cross-section σ_b for a peripheral collision with impact parameter b

$$f = \frac{\sigma_b}{\sigma_{\text{geom}}} = \left(\frac{N_{\text{part}}}{2N}\right)^{2/3}$$

- The nuclear radius $r_A = r_0 N^{1/3}$, where N is the number of nucleons per ion and $r_0 \sim 1.25$ fm.

- From the previous expressions

$$\Delta_B \sim \pi^{5/2} \, (\delta B)^2 \, r_0^5 N^{5/3} \left(\frac{N_{\text{part}}}{2N} \right)^{5/3}$$

- In peripheral heavy-ion collisions, the magnetic fluctuations in the transverse plane $|e \, \delta B| \sim m_\pi^2/4$
- For an Au+Au collision with $N = 197$, and if $N_{\text{part}}/N = 1/2$,

$$\Delta \equiv e^2 \Delta_B \sim 2.6\text{MeV}^{-1}$$

- For less central collisions with $N_{\text{part}}/N = 1/8$

$$\Delta \sim 0.26 \text{ MeV}^{-1}$$

Functional distribution of the magnetic fluctuations

The statistical properties for the magnetic fluctuations are reproduced by a Gaussian functional distribution

$$dP\left[\delta A^{\mu}_{\mathrm{BG}}\right] = \mathcal{N} e^{-\int d^3x \frac{\left[\delta A^{\mu}_{\mathrm{BG}}(\mathbf{x})\right]^2}{2\Delta_B}} \mathcal{D}\left[\delta A^{\mu}_{\mathrm{BG}}(\mathbf{x})\right]$$

The ensemble-average of over such fluctuations is defined by

$$\overline{\mathcal{O}(x; A_{BG})} = \int dP[\delta A^{\mu}_{BG}]\mathcal{O}(x; A_{BG} + \delta A_{BG})$$

Connected 2k-point correlations

As usual, the physical properties are characterized by connected 2k-point correlation functions

$$\begin{aligned} G(x_1,\ldots,x_{2k};A_{BG}) &= \langle T\psi(x_1)\ldots\bar{\psi}(x_{2k})\rangle_c \\ &= \left(-i\frac{\delta}{\delta\bar{J}(x_1)}\right)\ldots\left(i\frac{\delta}{\delta J(x_{2k})}\right)\ln Z[\bar{J},J;A_{BG}]\Big|_{J=\bar{J}=0} \end{aligned}$$

We are interested in the ensemble-average of such correlation functions over the magnetic background fluctuations with respect to its mean value $A^{\mu}_{BG}+\delta A^{\mu}_{BG}$

Connected 2k-point correlations

The ensemble-average of such functions over the magnetic background fluctuations with respect to its mean value $A_{BG}^{\mu} + \delta A_{BG}^{\mu}$

$$\boxed{\overline{G(x_1,\ldots,x_{2k};A_{BG})} = \left(-i\frac{\delta}{\delta\bar{J}(x_1)}\right)\ldots\left(i\frac{\delta}{\delta J(x_{2k})}\right)\overline{\ln Z[\bar{J},J;A_{BG}]}\Big|_{J=\bar{J}=0}}$$

clearly depends on the corresponding average of the logarithm of the generating functional

$$\overline{\ln Z[\bar{J},J;A_{BG}]} \neq \ln \overline{Z[\bar{J},J;A_{BG}]}$$

The Replica Method

The basic idea in the Replica Method is to apply the identity [Mèzard and Parisi, (1991); Kardar, Parisi and Zhang, (1986)]

$$\overline{\ln Z[A_{BG}]} = \lim_{n\to 0} \frac{\overline{Z^n[A_{BG}]} - 1}{n}$$

Initially developed in the context of spin-glasses, and latter applied in quantum field theory for disordered condensed matter systems.
In this context, n-replicas of the original system are defined

$$\psi(x) \to \psi^a(x) \qquad 1 \leq a \leq n$$

The Lagrangian

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The Lagrangian for this model is a superposition of two terms

$$\boxed{\mathcal{L} = \mathcal{L}_{\mathrm{FBG}} + \mathcal{L}_{\mathrm{NBG}}}$$

Fermions immersed in the average BG

$$\boxed{\mathcal{L}_{\mathrm{FBG}} = \bar{\psi}\left(\mathrm{i}\partial\!\!\!/ - e A\!\!\!/_{\mathrm{BG}} - e A\!\!\!/ - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

Fermions interacting with the classical background noise (NBG), represented by the spatial fluctuations $\delta A^{\mu}_{BG}(x)$

$$\boxed{\mathcal{L}_{\mathrm{NBG}} = \bar{\psi}\left(-e\delta A\!\!\!/_{\mathrm{BG}}\right)\psi}$$

The ensemble-averaged functional

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We perform the statistical average over classical BG fluctuations under the Gaussian functional measure $dP[\delta A^{\mu}_{BG}]$,

$$
\begin{aligned}
\overline{Z^n[A_{BG}]} &= \int \prod_{a=1}^{n} \mathcal{D}[\bar{\psi}^a, \psi^a] \int \mathcal{D}\left[\delta A^{\mu}_{\mathrm{BG}}\right] e^{-\int d^3x \frac{\left[\delta A^{\mu}_{\mathrm{BG}}(\mathbf{x})\right]^2}{2\Delta_B}} \\
&\quad \times e^{\mathrm{i}\int d^4x \sum_{a=1}^{n}\left(\mathcal{L}_{\mathrm{FBG}}[\bar{\psi}^a,\psi^a]+\mathcal{L}_{\mathrm{NBG}}[\bar{\psi}^a,\psi^a]\right)} \\
&= \int \prod_{a=1}^{n} \mathcal{D}[\bar{\psi}^a, \psi^a] e^{\mathrm{i}\bar{S}[\bar{\psi}^a,\psi^a;A_{BG}]}
\end{aligned}
$$

The ensemble-averaged action

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The statistical average leads to an effective fermion-fermion interaction proportional to the magnitude of the BG magnetic fluctuations self-correlation Δ_B

$$\bar{S}\left[\bar{\psi}^a, \psi^a; A_{BG}\right] = \int d^4x \left(\sum_a \bar{\psi}^a \left(\mathrm{i}\partial\!\!\!/ - eA\!\!\!/_{\mathrm{BG}} - eA\!\!\!/ - m \right) \psi^a - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)$$

$$+\mathrm{i}\frac{e^2\Delta_B}{2} \underbrace{\int d^4x \int d^4y \sum_{a,b} \sum_{j=1}^{3} \bar{\psi}^a(x)\gamma^j\psi^a(x)\bar{\psi}^b(y)\gamma_j\psi^b(y)\delta^3(\mathbf{x}-\mathbf{y})}_{Effective\ Fermion-Fermion\ interaction}$$

In what follows, we shall neglect photons $A^\mu = 0$ and will focus on the fermions in the classical BG magnetic field $\mathbf{B} = \hat{e}^3 B$

$$\boxed{A_{BG}^{\mu} = \frac{B}{2}(0, -x^2, x^1, 0)}$$

The Schwinger propagator

- The propagator for the average BG magnetic field $\mathbf{B} = \nabla \times \mathbf{A}_{\rm BG}$

$$S_F(x,x') = e^{i\Phi_{A_{BG}}(x,x')} \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-x')} S_F(k)$$

$$[S_{\rm F}(k)]_{a,b} = -{\rm i}\delta_{a,b} \int_0^\infty \frac{d\tau}{\cos(eB\tau)} e^{{\rm i}\tau\left(k_\parallel^2 - \mathbf{k}_\perp^2 \frac{\tan(eB\tau)}{eB\tau} - m^2 + {\rm i}\epsilon\right)}$$
$$\times \left\{ \left[\cos(eB\tau) + \gamma^1\gamma^2 \sin(eB\tau)\right](m + \not{k}_\parallel) + \frac{\not{k}_\perp}{\cos(eB\tau)} \right\}$$

- The metric tensor is splitted into two subspaces $g^{\mu\nu} = g_\parallel^{\mu\nu} + g_\perp^{\mu\nu}$, such that

$$\begin{aligned} \not{k} &= \not{k}_\perp + \not{k}_\parallel \\ k^2 &= k_\parallel^2 - \mathbf{k}_\perp^2 \end{aligned}$$

An alternative representation

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- The propagator can be expressed (exactly) in terms of a single "master" integral and its derivatives

$$[S_{\rm F}(k)]_{a,b} = -\mathrm{i}\delta_{a,b}\left[\left(m + \not{k}_{\parallel}\right)\mathcal{A}_1 + \gamma^1\gamma^2\left(m + \not{k}_{\parallel}\right)\mathcal{A}_2 + \mathcal{A}_3\not{k}_{\perp}\right]$$

$$\begin{aligned}
\mathcal{A}_1(k,B) &= \int_0^\infty d\tau\, e^{\mathrm{i}\tau\left(k_\parallel^2 - m^2 + \mathrm{i}\epsilon\right) - \mathrm{i}\frac{\mathbf{k}_\perp^2}{eB}\tan(eB\tau)} \\
\mathcal{A}_2(k,B) &= \int_0^\infty d\tau\, \tan(eB\tau)\, e^{\mathrm{i}\tau\left(k_\parallel^2 - t_B(\tau)\mathbf{k}_\perp^2 - m^2 + \mathrm{i}\epsilon\right)} = \mathrm{i}eB\frac{\partial\mathcal{A}_1}{\partial(\mathbf{k}_\perp^2)} \\
\mathcal{A}_3(k,B) &= \int_0^\infty \frac{d\tau}{\cos^2(eB\tau)}\, e^{\mathrm{i}\tau\left(k_\parallel^2 - t_B(\tau)\mathbf{k}_\perp^2 - m^2 + \mathrm{i}\epsilon\right)} \\
&= \mathcal{A}_1 + (\mathrm{i}eB)^2\frac{\partial^2\mathcal{A}_1}{\partial(\mathbf{k}_\perp^2)^2}
\end{aligned}$$

The inverse propagator

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- Clearly as $B \to 0$, we have

$$\begin{aligned} \lim_{B\to 0} \mathcal{A}_1(k,B) &= \lim_{B\to 0} \mathcal{A}_3(k,B) = \frac{\mathrm{i}}{k^2 - m^2 + \mathrm{i}\epsilon} \\ \lim_{B\to 0} \mathcal{A}_2(k,B) &= 0 \end{aligned}$$

- The inverse propagator

$$\boxed{\hat{S}_F^{-1}(k) = \frac{\mathrm{i}}{\mathcal{D}(k)}\left[\left(m - \not{k}_{\parallel}\right)\mathcal{A}_1 - \gamma^1\gamma^2\left(m - \not{k}_{\parallel}\right)\mathcal{A}_2 - \mathcal{A}_3\not{k}_{\perp}\right]}$$

$$\mathcal{D}(k) = \mathcal{A}_3^2\mathbf{k}_{\perp}^2 - \left(\mathcal{A}_1^2 - \mathcal{A}_2^2\right)\left(k_{\parallel}^2 - m^2\right)$$

Feynman Diagrams for perturbation theory

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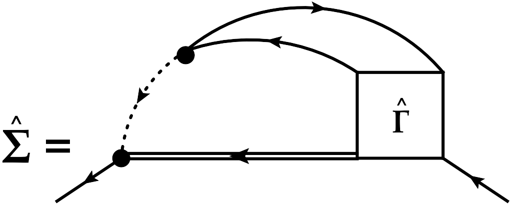
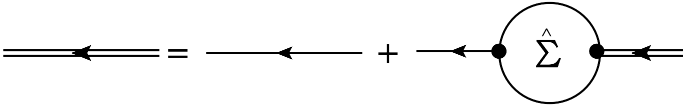
- Selfenergy Skeleton Diagram



- Dyson equation for the dressed propagator



$$\hat{S}_{\Delta}^{-1}(k) = \hat{S}_{F}^{-1}(k) - \hat{\Sigma}_{\Delta}(k)$$

The selfenergy at first-order in Δ_B

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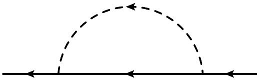
- The self-energy diagram at first-order in $\Delta = e^2\Delta_B$



- Analytical expression

$$\hat{\Sigma}_\Delta(q) = (\mathrm{i}\Delta)\int\frac{d^3p}{(2\pi)^3}\gamma^j\hat{S}_F(p+q;p_0=0)\gamma_j$$

$$= \frac{\mathrm{i}(\mathrm{i}\Delta)}{(2\pi)^3}\left[3\left(\gamma^0 q_0-m\right)\widetilde{\mathcal{A}}_1(q_0)-\gamma^1\gamma^2(\mathrm{i}\pi eB)\left(m-q_0\gamma^0\right)\widetilde{\mathcal{A}}_2(q_0)\right]$$

$$\widetilde{\mathcal{A}}_1(q_0) \equiv \int d^3p\mathcal{A}_1(q_0,p_3;\mathbf{p}_\perp)$$

$$\widetilde{\mathcal{A}}_2(q_0) \equiv \int_{-\infty}^{+\infty} dp_3\mathcal{A}_1(q_0,p_3;\mathbf{p}_\perp=0)$$

The (inverse) dressed propagator

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- After Dyson's equation

$$\hat{S}_{\Delta}^{-1}(k) = \hat{S}_{\mathrm{F}}^{-1}(k) - \hat{\Sigma}_{\Delta}$$

- From the "free" propagator in the average BG field

$$\hat{S}_F^{-1}(q)=\frac{\mathrm{i}}{\mathcal{D}(q)}\left[\left(m-\not{q}_{\|}\right)\mathcal{A}_1(q)-\gamma^1\gamma^2\left(m-\not{q}_{\|}\right)\mathcal{A}_2(q)-\mathrm{i}\mathcal{A}_3(q)\not{q}_{\perp}\right]$$

- The (inverse) dressed propagator is given by

$$\boxed{\hat{S}_{\Delta}^{-1}(q)=\frac{\mathrm{i}z}{\mathcal{D}(q)}\left[\left(m-\tilde{\not{q}}_{\|}\right)\mathcal{A}_1(q)-z_3\gamma^1\gamma^2\left(m-\tilde{\not{q}}_{\|}\right)\mathcal{A}_2(q)-\mathrm{i}\mathcal{A}_3(q)\tilde{\not{q}}_{\perp}\right]}$$

- Here, we defined the momenta $\tilde{q}^{\mu} = (q^0, z^{-1}\mathbf{q})$, with an effective refractive index $v'/c = z^{-1}$ due to the magnetic fluctuations.

Renormalization factors

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- Wavefunction renormalization factor and refractive index

$$z = 1 + \frac{3\mathrm{i}\Delta}{(2\pi)^3}\frac{\widetilde{\mathcal{A}}_1(q_0)}{\mathcal{A}_1(q)}\mathcal{D}(q) \qquad \frac{v'}{c} = z^{-1}$$

- Charge renormalization factor

$$z_3 = \frac{1 - \frac{\mathrm{i}\pi(\mathrm{i}\Delta)(eB)}{(2\pi)^3}\frac{\widetilde{\mathcal{A}}_2(q_0)}{\mathcal{A}_2(q)}\mathcal{D}(q)}{1 + \frac{3\mathrm{i}\Delta}{(2\pi)^3}\frac{\widetilde{\mathcal{A}}_1(q_0)}{\mathcal{A}_1(q)}\mathcal{D}(q)}$$

- Dressed propagator

$$\boxed{S_\Delta(q) = -\mathrm{i}z^{-1}\frac{\mathcal{D}(q)}{\tilde{\mathcal{D}}(q)}\left[\left(m + \tilde{\not{q}}_{\parallel}\right)\mathcal{A}_1(q) + z_3\gamma^1\gamma^2\left(m + \tilde{\not{q}}_{\parallel}\right)\mathcal{A}_2(q) + \mathcal{A}_3(q)\tilde{\not{q}}_{\perp}\right]}$$

Different magnetic field regimes

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- Very weak field limit $eB/m^2 \ll 1$

$$\mathcal{A}_1(k,B) - \mathcal{A}_1(k,0) = \frac{-2\mathrm{i}\,(eB)^2\,\mathbf{k}_\perp^2}{[k^2 - m^2 + \mathrm{i}\epsilon]^4} + O((eB)^4)$$

- Intermediate field intensity: Landau levels ($x = \mathbf{k}_\perp^2/eB$)

$$\mathcal{A}_1(k) = \mathrm{i}\frac{e^{-x}}{\mathcal{D}_\parallel}\left[1 + \sum_{n=1}^{\infty}\frac{(-1)^n\left[L_n^0(2x) - L_{n-1}^0(2x)\right]}{1 - 2n\frac{eB}{\mathcal{D}_\parallel}}\right]$$

- Ultra-intense field $eB/m^2 \gg 1$ (LLL)

$$\mathcal{A}_1(k) = \mathrm{i}\frac{e^{-\mathbf{k}_\perp^2/eB}}{k_\parallel^2 - m^2}$$

Asymptotic results

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- Very weak field limit $eB/m^2 \ll 1$

$$\begin{aligned} z &= 1 + O(B^4) \\ z_3 &= 1 + O(B^4) \end{aligned}$$

- Ultra-intense field $eB/m^2 \gg 1$ (LLL)

$$\begin{aligned} z &= 1 + \frac{3}{4}\frac{\Delta(eB)e^{-\mathbf{q}_\perp^2/eB}}{\pi\sqrt{q_0^2 - m^2}} \\ z_3 &= \frac{1 + \frac{\Delta(eB)e^{-\mathbf{q}_\perp^2/(eB)}}{4\pi\sqrt{q_0^2-m^2}}}{1 + \frac{3}{4}\frac{\Delta(eB)e^{-\mathbf{q}_\perp^2/(eB)}}{\pi\sqrt{q_0^2-m^2}}} \\ \lim_{eB/m^2\rightarrow\infty} z_3 &= 1/3 \end{aligned}$$

Results

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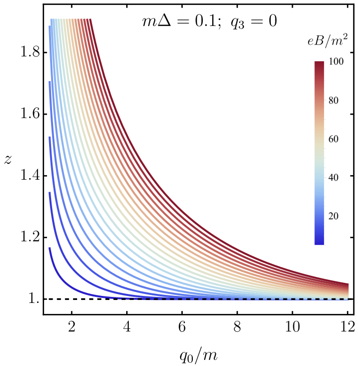

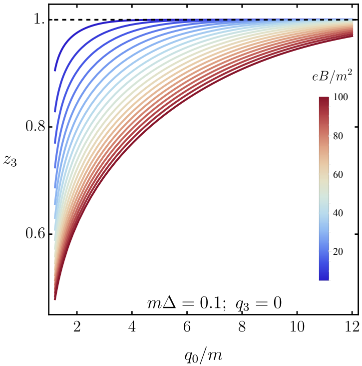

Clearly $z \rightarrow 1$ and $z_3 \rightarrow 1$ as $q_0/m \gg 1$: The quasi-particle renormalization due to magnetic fluctuations tends to be negligible at high energies, but it can be quite significant at low energy scales.

Results

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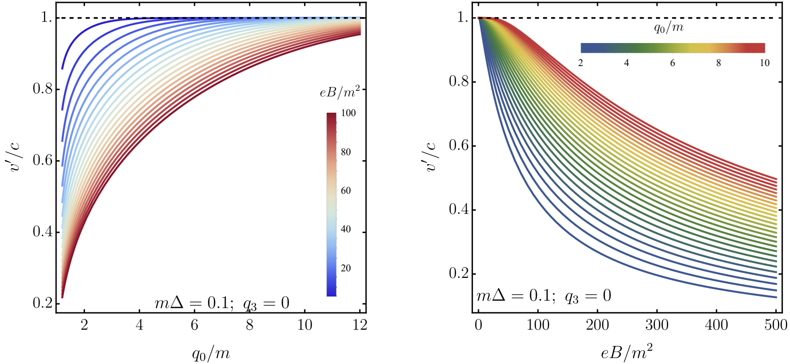
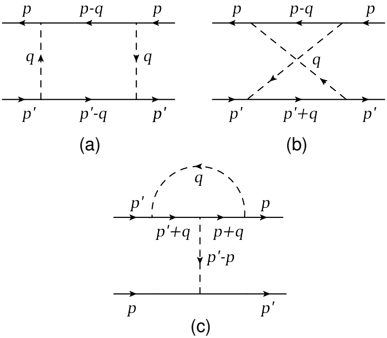


A similar dependence on the fluctuation renormalization is observed in the refraction index v'/c.

- Diagrams contributing to the 4-point vertex



$$
\begin{aligned}
\hat{\Gamma}_{(a)} &= \int \frac{d^3q}{(2\pi)^3}\,\gamma^i S_F(p-q)\gamma^j \otimes \gamma_i S_F(p'-q)\gamma_j \\
\hat{\Gamma}_{(b)} &= \int \frac{d^3q}{(2\pi)^3}\,\gamma^i S_F(p-q)\gamma^j \otimes \gamma_i S_F(p'+q)\gamma_j \\
\hat{\Gamma}_{(c)} &= \int \frac{d^3q}{(2\pi)^3}\,\gamma^i S_F(p+q)\gamma^j \otimes \gamma_i S_F(p'-q)\gamma_j
\end{aligned}
$$

Vertex corrections

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$$\hat{\Gamma} = 2\hat{\Gamma}_{(a)} + 2\hat{\Gamma}_{(b)} + 4\hat{\Gamma}_{(c)} = \tilde{\Delta}(\bar{\psi}\gamma^i\psi)(\bar{\psi}\gamma^i\psi)$$

- Renormalized $\tilde{\Delta}$

$$\tilde{\Delta}=\Delta+2\Delta^2\Big(\mathcal{J}_2^{(-,-)}+\mathcal{J}_2^{(-,+)}+2\mathcal{J}_2^{(+,-)}+\big(1-\partial_x^2\big)(1-\partial_y^2)\mathcal{J}_3^{(-,-)}$$
$$+\big(1-\partial_x^2\big)(1-\partial_y^2)\mathcal{J}_3^{(-,+)}+2\big(1-\partial_x^2\big)(1-\partial_y^2)\mathcal{J}_3^{(+,-)}\Big)$$

- In terms of the integrals

$$\begin{aligned}
\mathcal{J}_1^{(\lambda,\sigma)}(p,p') &= \int \frac{d^3q}{(2\pi)^3}\mathcal{A}_1(p+\lambda q)\mathcal{A}_1(p'+\sigma q)\\
\mathcal{J}_2^{(\lambda,\sigma)}(p,p') &= \int \frac{d^3q}{(2\pi)^3}q_{\parallel}^2\mathcal{A}_1(p+\lambda q)\mathcal{A}_1(p'+\sigma q)\\
\mathcal{J}_3^{(\lambda,\sigma)}(p,p') &= \int \frac{d^3q}{(2\pi)^3}\mathbf{q}_{\perp}^2\mathcal{A}_1(p+\lambda q)\mathcal{A}_1(p'+\sigma q)
\end{aligned}$$

Vertex renormalization

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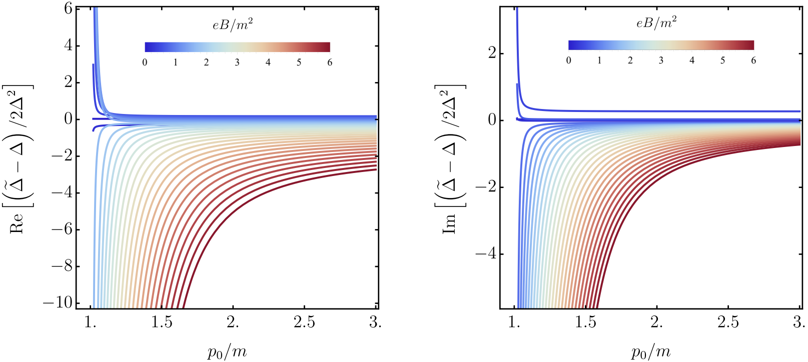

Conclusions and Prospects

- We studied the effects of white noise spatial fluctuations in an otherwise uniform background magnetic field, over the QED fermion propagator
- At first order in Δ, the propagator retains its free form, thus representing renormalized quasi-particles with the same mass $m' = m$, but propagating in a "dispersive medium" with an index of refraction $v'/c = z^{-1}$, and effective charge $e' = z_3 e$, where z and z_3 depend on the average field and its noise
- Low energy components in the propagator (long-wavelength) are more sensitive to the spatial distribution of the magnetic fluctuations, and hence experience a higher degree of decoherence, thus reducing $v'/c = z^{-1}$. In contrast, the high-energy Fourier modes are less sensitive to magnetic fluctuations.
- If $m\Delta \ll 1$ (i.e. for $m \ll 0.4\,\text{MeV}$), one may in principle neglect the magnetic fluctuation effects. However, if $m\Delta \sim 1$ (i.e. for $m \sim 0.4\,\text{MeV}$ or larger), those effects may become significant.
- Non-perturbative scenario: to be discussed in the next talk by M. Loewe