Neutron P-wave superfluids in neutron stars and magnetars

Shigehiro YASUI

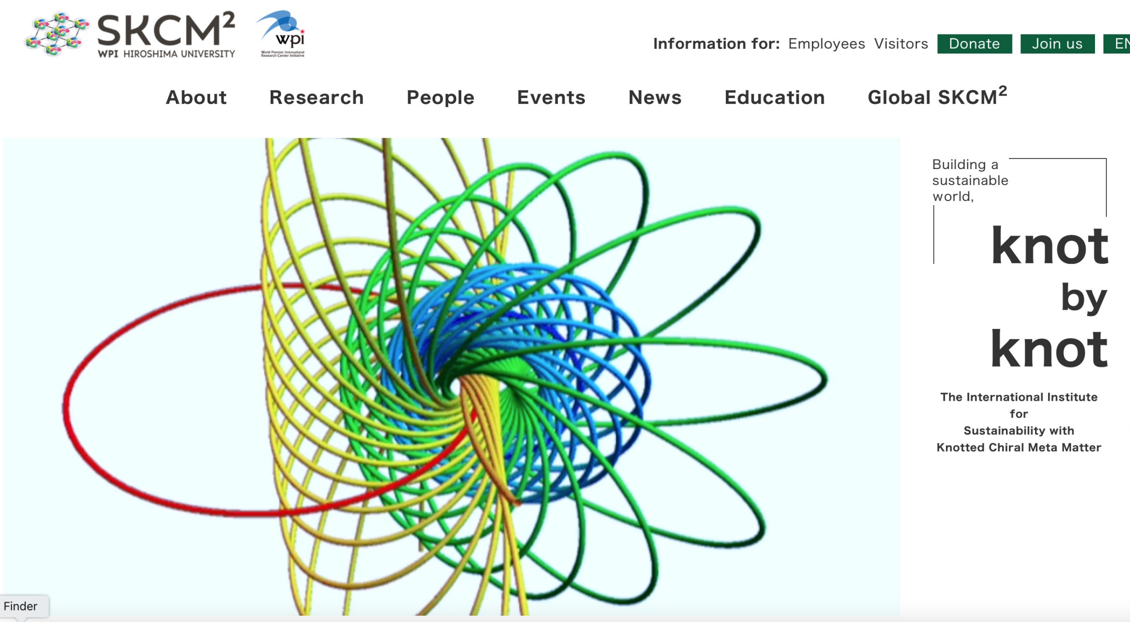
$\in$ Sasaki Lab $\subset$ WPI-SKCM^2 $\subset$ Hiroshima U.

Collaborators:
Muneto NITTA $\in$ Keio U. $\cap$ (WPI-SKCM^2 $\subset$ Hiroshima U.)
Daisuke INOTANI $\in$ Keio U.
Takeshi MIZUSHIMA $\in$ Osaka U.
Chandrasekhar CHATTERJEE $\in$ Keio U. → private company

Strongly interacting matter in extreme magnetic fields@ECT*, 25-29, Sep. 2023

International Institute for Sustainability with Knotted Chiral Meta Matter/**SKCM²**

World Premier International Research Center Initiative/WPI at Hiroshima University



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Building a sustainable world,

knot by knot

The International Institute for Sustainability with Knotted Chiral Meta Matter

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CU: コロラド大学(アメリカ)
UU: ユトレヒト大学(オランダ)
AS: 中央研究院(台湾)
UCam: ケンブリッジ大学(イギリス)
GT: ジョージア工科大学(アメリカ)
MIT: マサチューセッツ工科大学(アメリカ)
TIT: 東京工業大学
MPI: マックスプランク研究所（ドイツ）
UWr: ヴロツワフ大学(ポーランド)

✔ Cross-pollinates mathematical knot theory and chirality knowledge across disciplines and scales

✔ Creation of designable artificial knot-like particles that exhibit highly unusual and technologically useful properties

Hadron & nuclear physics group

PI: Chihiro SASAKI (HU, Uni. of Wroclaw)

PI: Kenta SHIGAKI (HU)

coPI: Chiho NONAKA (HU)

coPI: Muneto NITTA (HU, Keio Uni.)

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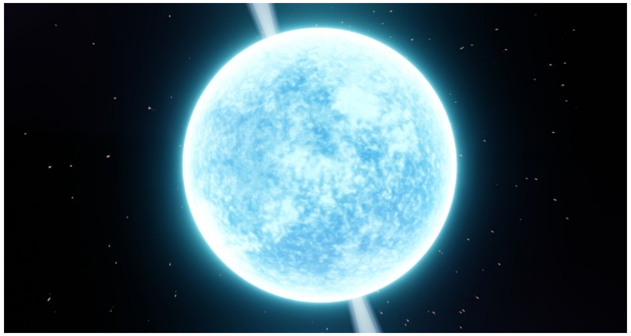
Neutron stars



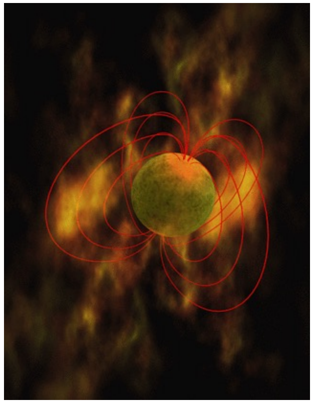
llustration: Shutterstock

- Radius: ~10km
- Mass: 1.4-2M_{SUN}
- Density: 10^{12} kg/cm^3
- Gravity: 10^{11} x Earth gravity
- Rotation period: 30-1/100 sec.
- Magnetic field: 10^{13}-10^{15}G (0.5G on Earth)
- Neutrino radiation
- Gravitation waves

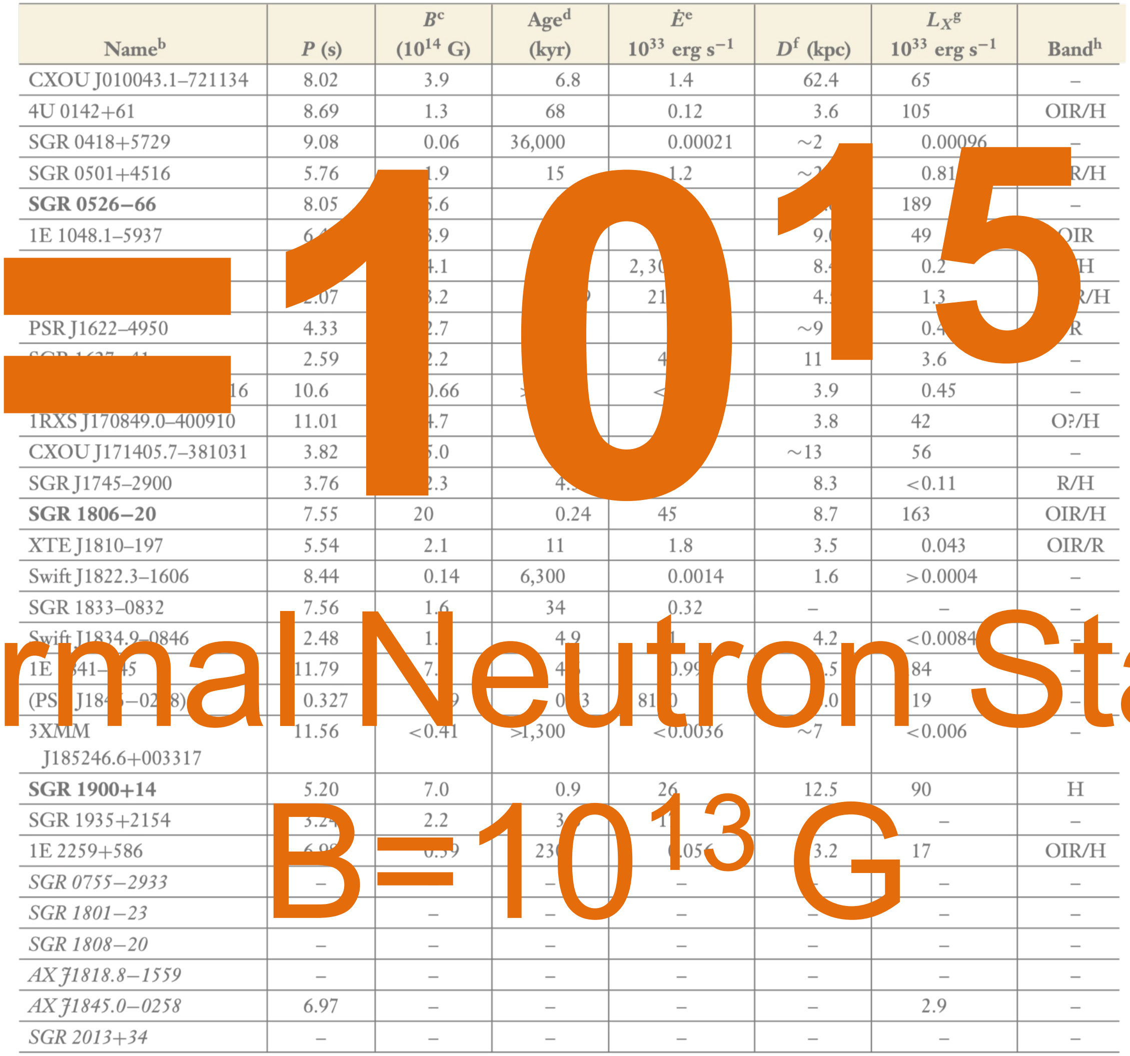
1. Introduction

Magnetar

Neutron star with strong magnetic fields



$B=10^{15}$ G



Name[b]	P (s)	B[c] (10^{14} G)	Age[d] (kyr)	$\dot{E}$[e] 10^{33} erg s^{-1}	D[f] (kpc)	L_X[g] 10^{33} erg s^{-1}	Band[h]
CXOU J010043.1–721134	8.02	3.9	6.8	1.4	62.4	65	–
4U 0142+61	8.69	1.3	68	0.12	3.6	105	OIR/H
SGR 0418+5729	9.08	0.06	36,000	0.00021	~2	0.00096	–
SGR 0501+4516	5.76	1.9	15	1.2	[illegible]	0.81	[illegible]
SGR 0526–66	8.05	[illegible]	[illegible]	[illegible]	[illegible]	189	–
1E 1048.1–5937	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	49	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.2	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.3	[illegible]
PSR J1622–4950	4.33	[illegible]	[illegible]	[illegible]	~9	[illegible]	[illegible]
[illegible]	2.59	[illegible]	[illegible]	[illegible]	11	3.6	–
[illegible]	10.6	[illegible]	[illegible]	[illegible]	3.9	0.45	–
1RXS J170849.0–400910	11.01	[illegible]	[illegible]	[illegible]	3.8	42	O?/H
CXOU J171405.7–381031	3.82	[illegible]	[illegible]	[illegible]	~13	56	–
SGR J1745–2900	3.76	[illegible]	[illegible]	[illegible]	8.3	<0.11	R/H
SGR 1806–20	7.55	20	0.24	45	8.7	163	OIR/H
XTE J1810–197	5.54	2.1	11	1.8	3.5	0.043	OIR/R
Swift J1822.3–1606	8.44	0.14	6,300	0.0014	1.6	>0.0004	–
SGR 1833–0832	7.56	1.6	34	0.32	–	–	–
Swift J1834.9–0846	2.48	[illegible]	4.9	[illegible]	4.2	<0.0084	[illegible]
[illegible]	11.79	[illegible]	[illegible]	[illegible]	[illegible]	84	[illegible]
[illegible]	0.327	[illegible]	[illegible]	[illegible]	[illegible]	19	[illegible]
3XMM J185246.6+003317	11.56	<0.41	>1,300	<0.0036	~7	<0.006	–
SGR 1900+14	5.20	7.0	0.9	26	12.5	90	H
SGR 1935+2154	[illegible]	2.2	[illegible]	[illegible]	[illegible]	–	–
1E 2259+586	[illegible]	[illegible]	[illegible]	[illegible]	3.2	17	OIR/H
SGR 0755–2933	–	[illegible]	–	–	[illegible]	–	–
SGR 1801–23	[illegible]	–	–	–	–	–	–
SGR 1808–20	–	–	–	–	–	–	–
AX J1818.8–1559	–	–	–	–	–	–	–
AX J1845.0–0258	6.97	–	–	–	–	2.9	–
SGR 2013+34	–	–	–	–	–	–	–

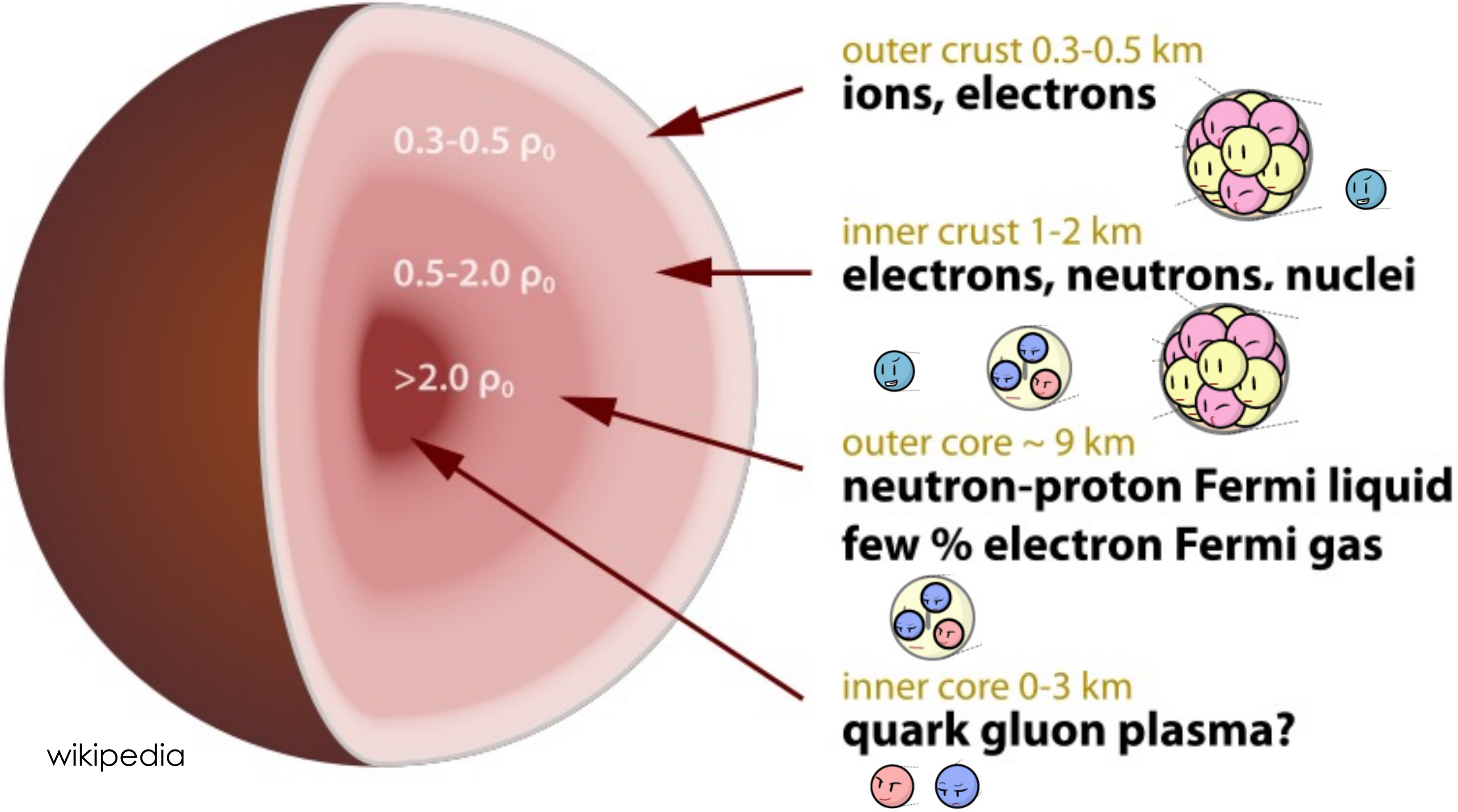
Normal Neutron Stars:

$B=10^{13}$ G

V. M. Kaspi and A. M. Beloborodov, Annual Review of Astronomy and Astrophysics 55, 261 (2017).

1. Introduction

Inside of neutron stars？



wikipedia

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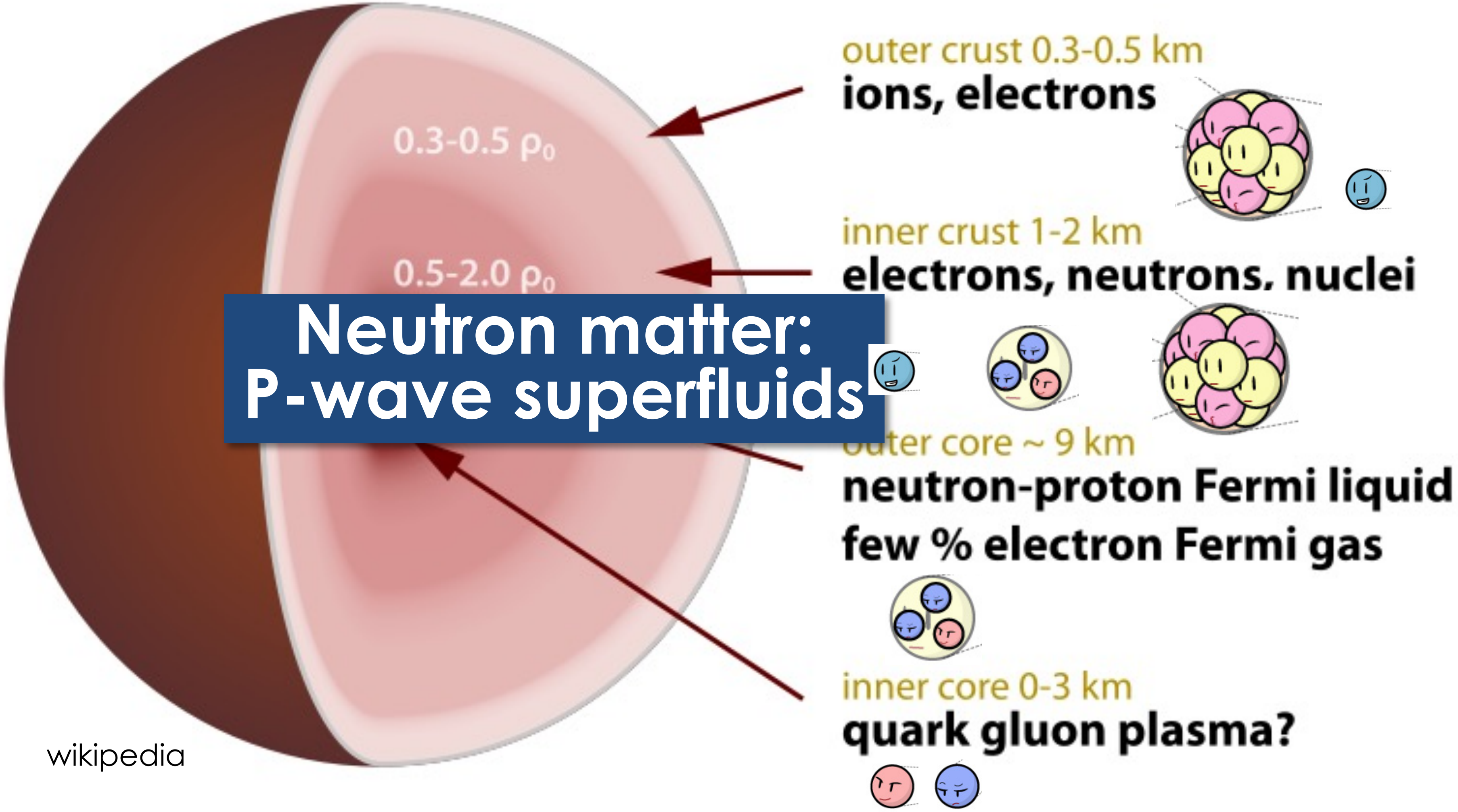
Inside of neutron stars？

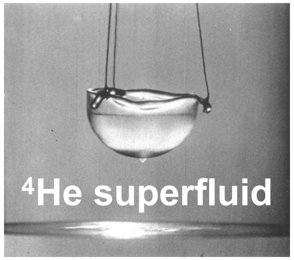
HiggsTan.com



Neutron matter:
P-wave superfluids

wikipedia

1. Introduction



Neutron (fermion)→Superfluid

Note：Electrically charged fermion→superconductor (electric current)

1937：^{4}He atom (boson) superfluids (experiment)

1957：Bardeen-Cooper-Schrieffer (BCS) theory

1972：^{3}He atom (fermion) superfluids (experiment)

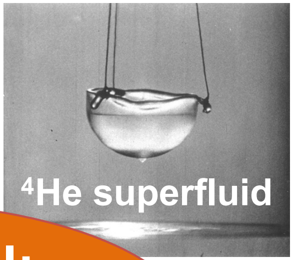
spin triplet-P-wave Cooper pair (spin-fluctuation interaction？)

1995：Ru atom (boson) superfluids (experiment)

2003：^{40}K atom Bose-Einstein condensate (experiment)

...

1. Introduction



^{4}He superfluid

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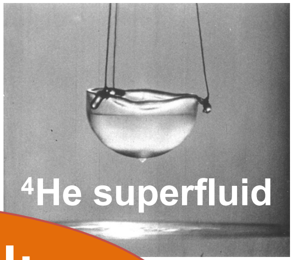
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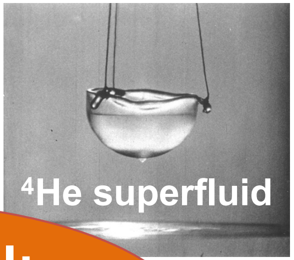
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1968 Tabakin: LS force can realize neutron $^3P_2$ superfluids!

F. Tabakin, Phys. Rev. **174**, 1208 (1968).
M. Hoffberg, A. E. Glassgold, R. W. Richardson, M. Ruderman, Phys. Rev. Lett. **24**, 775 (1970).
R. Tamagaki, Prog. Theor. Phys. **44**, 905 (1970).
T. Takatsuka, R. Tamagaki, Prog. Theor. Phys. **46**, 114 (1971).
T. Takatsuka, Prog. Theor. Phys. **47**, 1062 (1972).
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R. W. Richardson, Phys. Rev. D **5**, 1883 (1972).
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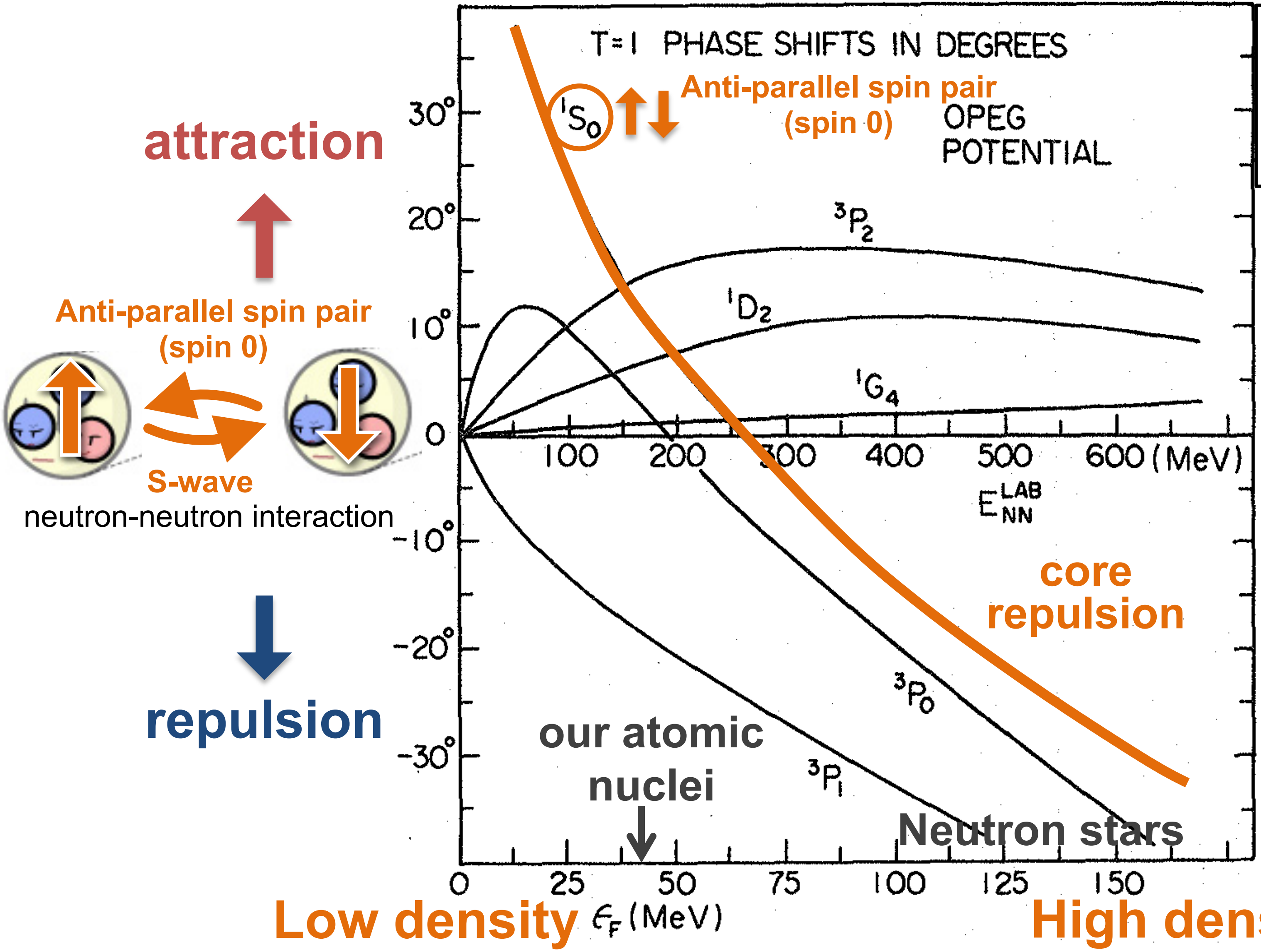
neutron-neutron interaction

Attraction and repulsion in various scattering channels (exp.)

Takatsuka, Tamagaki, PTP Suppl. 112, 27 (1993)

$^{2S+1}L_J$

S: spin
L: angular momentum
J: spin+angular momentum



Anti-parallel spin pair (spin 0)

attraction

Anti-parallel spin pair (spin 0)

S-wave

neutron-neutron interaction

repulsion

core repulsion

our atomic nuclei

Neutron stars

Low density

High density

2. Neutron $^3P_2$ superfluids (nuclear physics)

neutron-neutron interaction

Attraction and repulsion in various scattering channels (exp.)

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$^{2S+1}L_J$

S: spin
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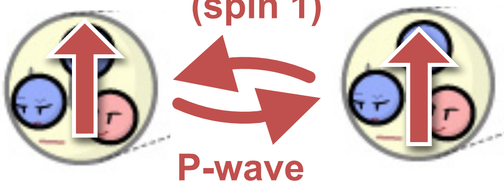
attraction

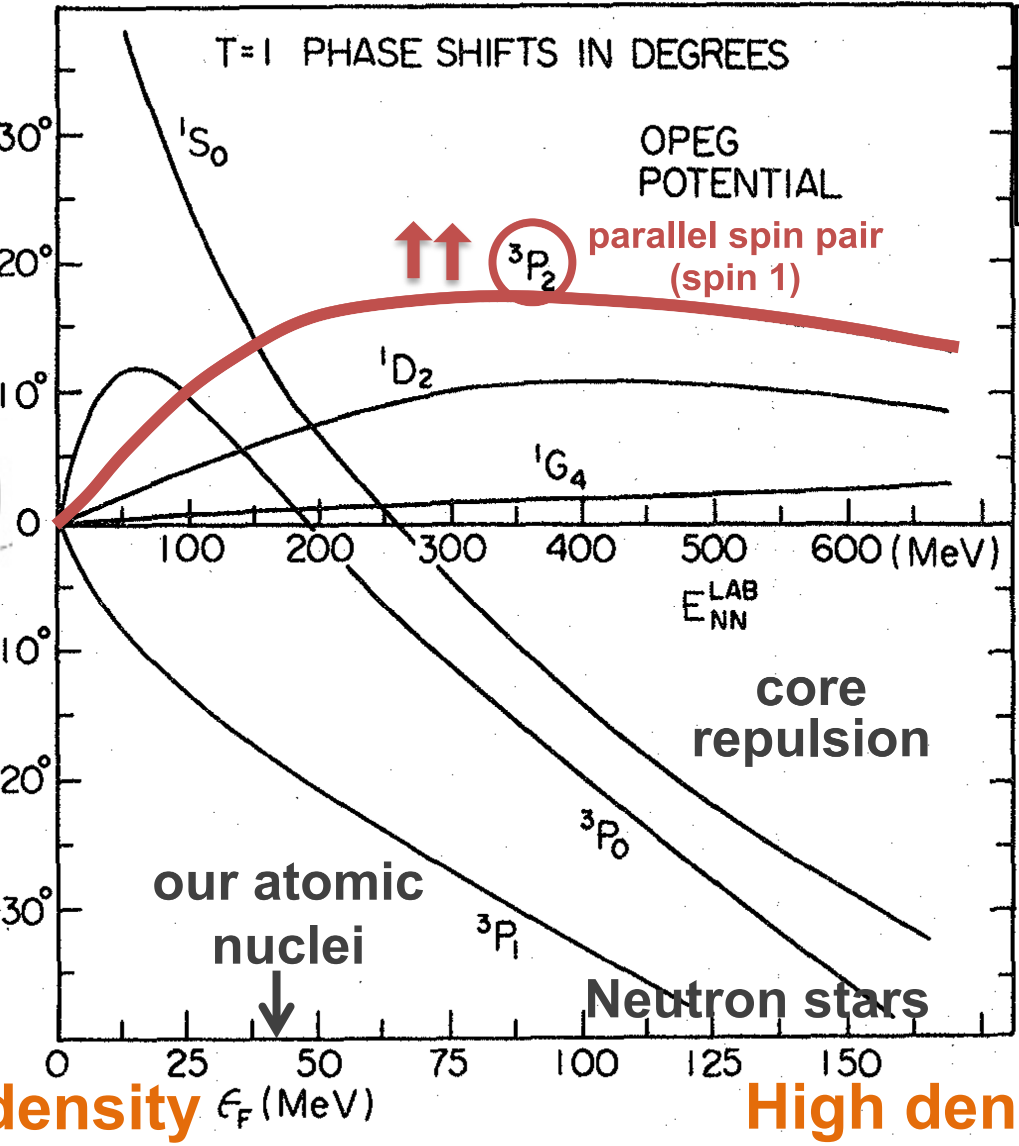
parallel spin pair (spin 1)



P-wave

neutron-neutron interaction

repulsion



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Neutron stars

Low density

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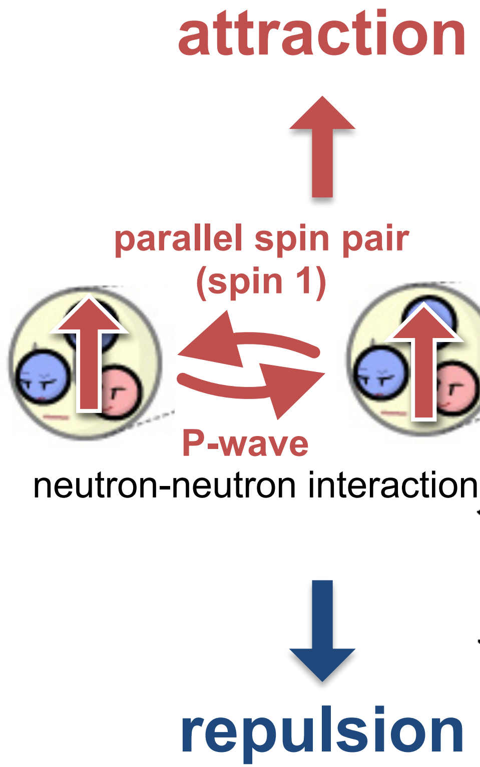
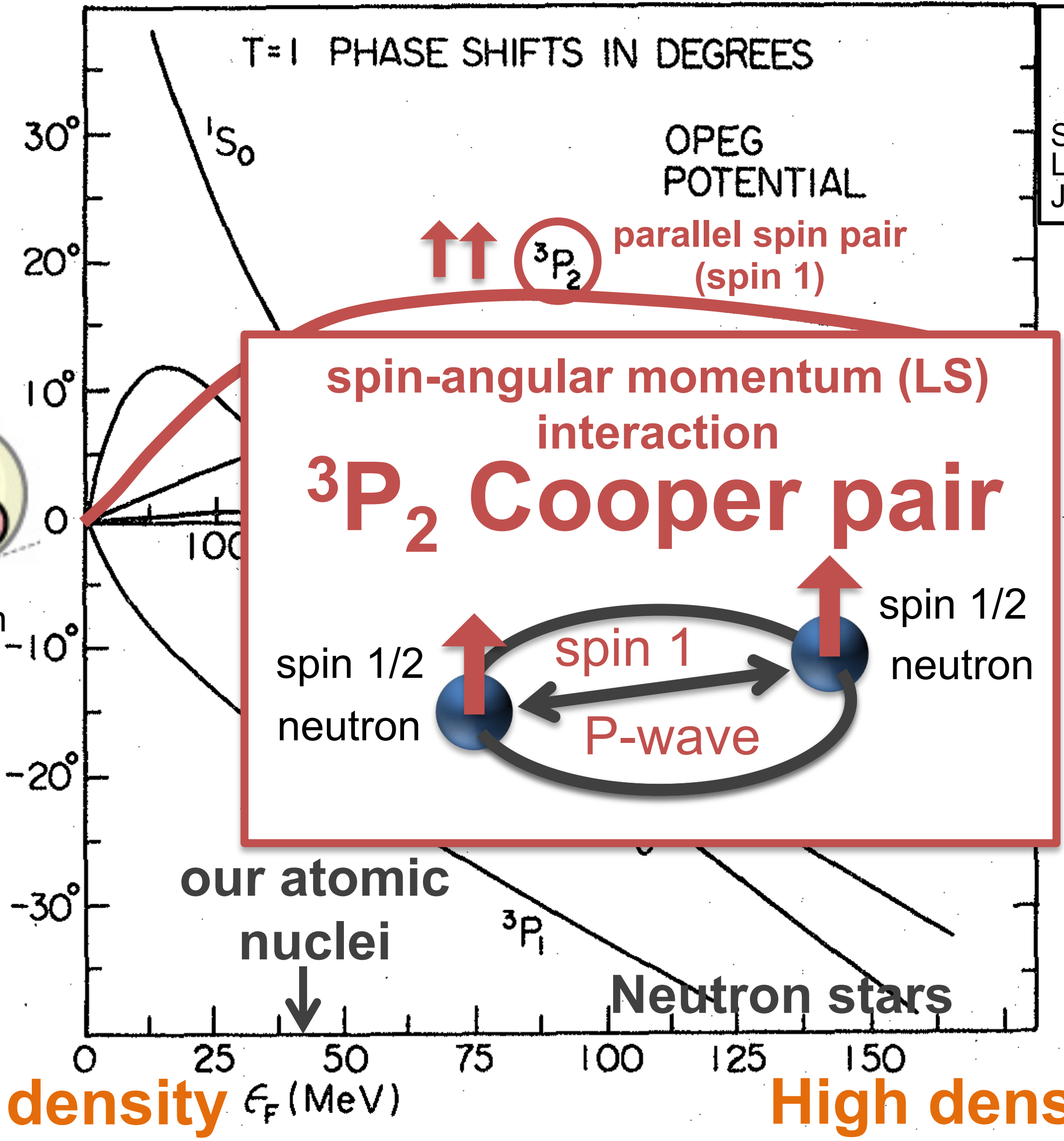




$^{2S+1}L_J$

S: spin
L: angular momentum
J: spin+angular momentum

Notice: J=0,1,2 are mixed in case of cold atomic gas like ^3He due to the absence of pure LS force.

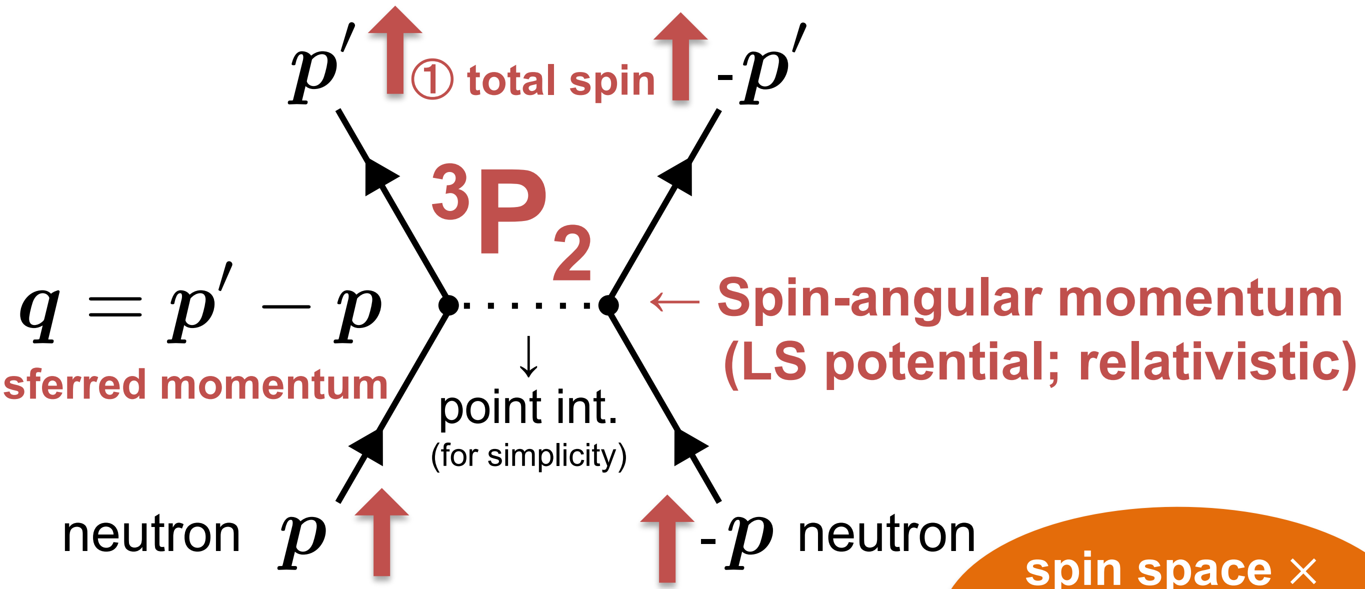
Notice: Microscopically, the $^3P_2$-$^3F_2$ mixing by tensor force induces a strong attraction (Takatsuka (1972)).

Low density

High density

2. Neutron $^3P_2$ superfluids (nuclear physics)

$^3P_2$: Most attractive channels between two neutrons at high density



Tensor-type condensate

symmetric & traceless
(J=2)

$$\boxed{\frac{1}{2}\left(s^a q^b + s^b q^a\right) - \frac{\delta^{ab}}{3}\boldsymbol{s}\cdot\boldsymbol{q}}$$

spin space × momentum space
$(a, b = x, y, z)$

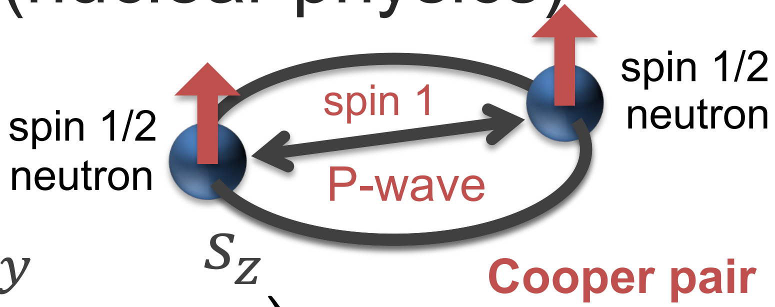
Neutron $^3P_2$ superfluids →

- **Tolerance to strong magnetic fields?**
- **Rapid neutrino cooling?**
- **Topological stars?**

2. Neutron $^3P_2$ superfluids (nuclear physics)

Tabakin (1968), Hoffenberg, Glassgold, Richardson, Ruderman (1970), Tamagaki (1970), Takatsuka, Tamagaki (1971), Takatsuka (1972), ...

$$\boxed{{}^3P_2}$$



Order parameter
(neutron pair condensate)

$$A(t,\boldsymbol{x}) = A_0 \begin{array}{cc} \begin{matrix} s_x & \quad s_y & \quad s_z \end{matrix} & \\ \begin{pmatrix} r & 0 & 0 \\ 0 & -(1+r) & 0 \\ 0 & 0 & 1 \end{pmatrix} & \begin{matrix} q_x \\ q_y \\ q_z \end{matrix} \end{array}$$

symmetric & traceless
(2J+1→# d.o.f. 5)

amplitude

spin (s_x, s_y, s_z)
× momentum
(q_x, q_y, q_z)

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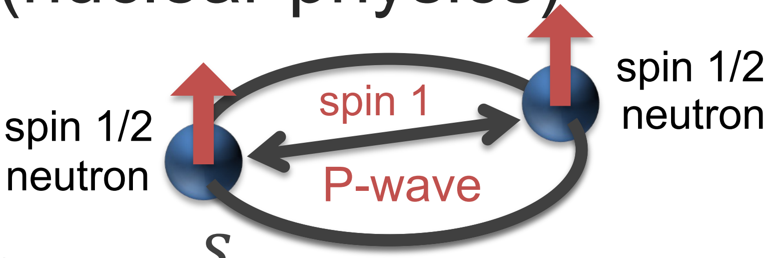
$$\boxed{^3P_2}$$



Cooper pair

Order parameter
(neutron pair condensate)

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symmetric & traceless
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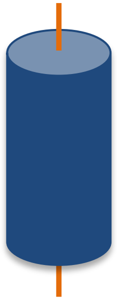
amplitude

spin (s_x, s_y, s_z)
× momentum
(q_x, q_y, q_z)

internal parameter $-1 \leq r \leq -1/2$

O(2)

cylinder
円柱

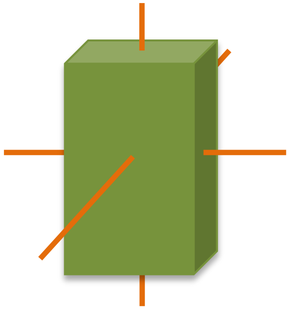


UN: uniaxial nematic
$(r = -1/2)$

$$\begin{pmatrix} -1/2 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

D_2

rectangular
直方体

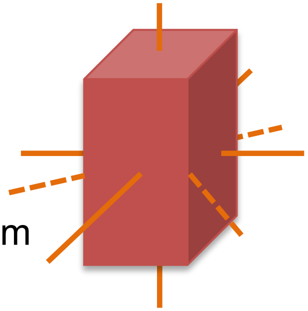


D_2-BN: D_2 biaxial nematic
(-1<r<-1/2)

$$\begin{pmatrix} r & 0 & 0 \\ 0 & -1-r & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

D_4

square prism
正四角柱



D_4-BN: D_4 biaxial nematic
$(r = -1)$

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2. Neutron $^3P_2$ superfluids (nuclear physics)

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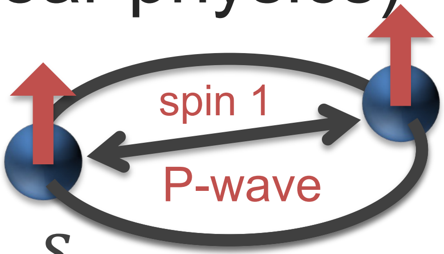
$\boxed{^3P_2}$



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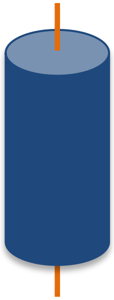
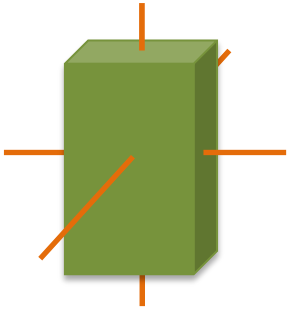


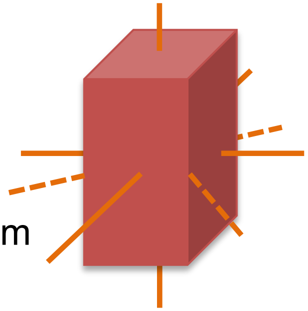
D_2

rectangular
直方体



D_4

square prism
正四角柱



UN: uniaxial nematic
$(r = -1/2)$

D_2-BN: D_2 biaxial nematic
$(-1 < r < -1/2)$

D_4-BN: D_4 biaxial nematic
$(r = -1)$

$\mathrm{U}(1) \times \mathrm{SO}(3)_{\mathrm{L+S}} \to \mathrm{O}(2)$

$\mathrm{U}(1) \times \mathrm{SO}(3)_{\mathrm{L+S}} \to \mathrm{D}_2$

$\mathrm{U}(1) \times \mathrm{SO}(3)_{\mathrm{L+S}} \to \mathrm{D}_4$

total phase

LS potential symmetry (rotation in S and L)

Spontaneous symmetry breaking

2. Neutron $^3P_2$ superfluids (nuclear physics)

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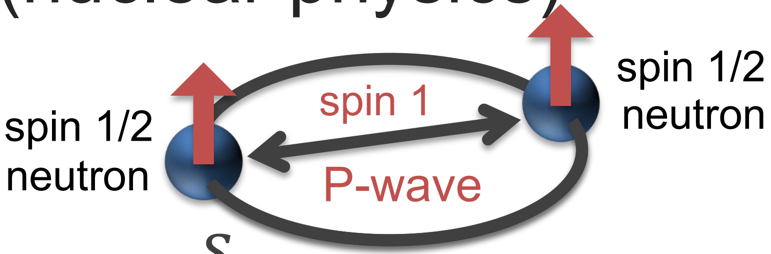
$$\boxed{{}^3P_2}$$



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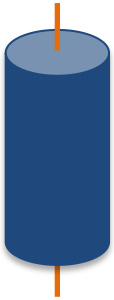
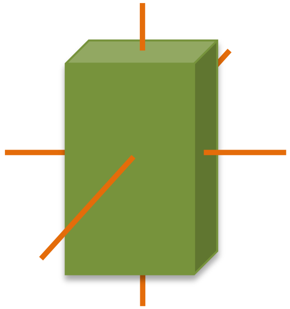


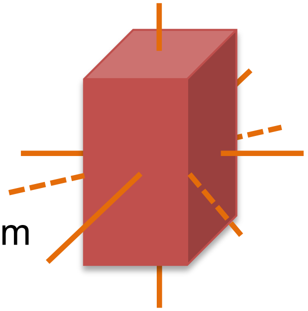
cylinder
円柱

D_2



rectangular
直方体

D_4



square prism
正四角柱

UN: uniaxial nematic
$(r = -1/2)$

D_2-BN: D_2 biaxial nematic
$(-1 < r < -1/2)$

D_4-BN: D_4 biaxial nematic
$(r = -1)$

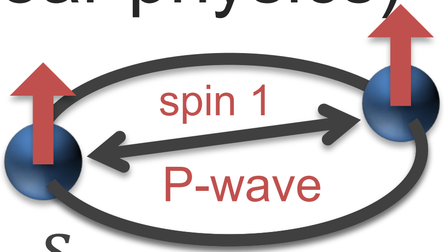
Phase	H	G/H	π_0	π_1	π_2	π_3	π_4
UN	O(2)	$U(1) \times [SO(3)/O(2)]$	0	$\mathbb{Z} \oplus \mathbb{Z}_2$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}_2$
D_2 BN	D_2	$U(1) \times [SO(3)/D_2]$	0	$\mathbb{Z} \oplus \mathbb{Q}$	0	$\mathbb{Z}$	$\mathbb{Z}_2$
D_4 BN	D_4	$[U(1) \times SO(3)]/D_4$	0	$\mathbb{Z} \times_h D_4^*$	0	$\mathbb{Z}$	$\mathbb{Z}_2$

←**homotopy group**

2. Neutron $^3P_2$ superfluids (nuclear physics)

Tabakin (1968), Hoffenberg, Glassgold, Richardson, Ruderman (1970), Tamagaki (1970), Takatsuka, Tamagaki (1971), Takatsuka (1972), ...

$$\boxed{^3P_2}$$



Cooper pair

Order parameter
(neutron pair condensate)

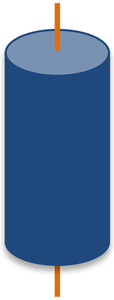
$$A(t,\boldsymbol{x}) = A_0 \begin{array}{cc} \begin{array}{ccc} s_x & s_y & s_z \end{array} & \\ \begin{pmatrix} r & 0 & 0 \\ 0 & -(1+r) & 0 \\ 0 & 0 & 1 \end{pmatrix} & \begin{array}{c} q_x \\ q_y \\ q_z \end{array} \end{array}$$

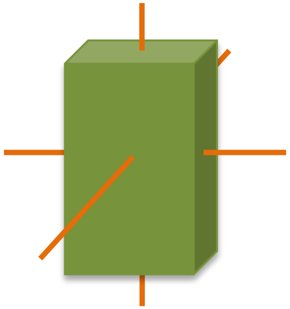
symmetric & traceless (2J+1→# d.o.f. 5)

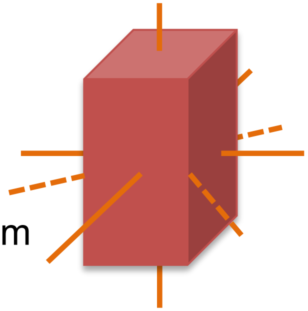
amplitude

spin (s_x, s_y, s_z) × momentum (q_x, q_y, q_z)

internal parameter $-1 \leq r \leq -1/2$

O(2)

cylinder
円柱

D_2

rectangular
直方体

D_4

square prism
正四角柱

UN: uniaxial nematic $(r=-1/2)$

D_2-BN: D_2 biaxial nematic $(-1<r<-1/2)$

D_4-BN: D_4 biaxial nematic $(r=-1)$

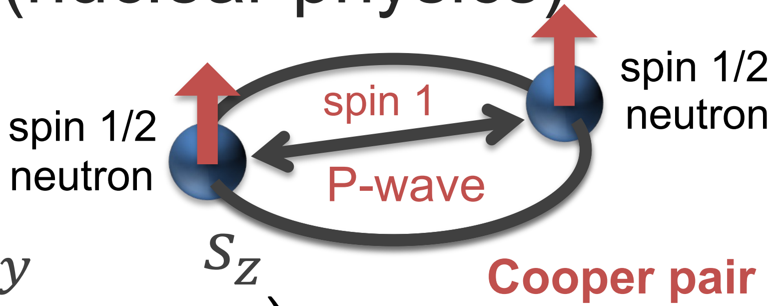
Topological matter

Quantized vortex: K. Masuda M. Nitta, PRC93, 035804 (2016)
Gapless Majorana fermions: T. Mizushima, K. Masuda, M. Nitta, PRB95, 140503 (2017)
Soliton on vortex: C. Chatterjee, M. Haberichter, M. Nitta, PRC96, 055807 (2017)
Half-quantized non-Abelian vortex: K. Masuda, M. Nitta, PTEP2020, 013D01 (2020)
Y. Masaki, T. Mizushima, M. Nitta, Phys. Rev. B105, L220503 (2022)
and more ...

2. Neutron $^3P_2$ superfluids (nuclear physics)

Tabakin (1968), Hoffenberg, Glassgold, Richardson, Ruderman (1970), Tamagaki (1970), Takatsuka, Tamagaki (1971), Takatsuka (1972), ...

$\boxed{^3P_2}$



Order parameter
(neutron pair condensate)

$$A(t,\boldsymbol{x}) = A_0 \begin{array}{cc} \begin{array}{ccc} s_x & s_y & s_z \end{array} & \\ \left(\begin{array}{ccc} r & 0 & 0 \\ 0 & -(1+r) & 0 \\ 0 & 0 & 1 \end{array}\right) & \begin{array}{c} q_x \\ q_y \\ q_z \end{array} \end{array}$$

symmetric & traceless
(2J+1→# d.o.f. 5)

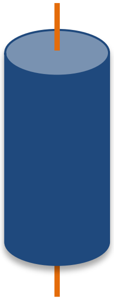
amplitude

spin (s_x, s_y, s_z)
× momentum
(q_x, q_y, q_z)

internal parameter $-1 \leq r \leq -1/2$

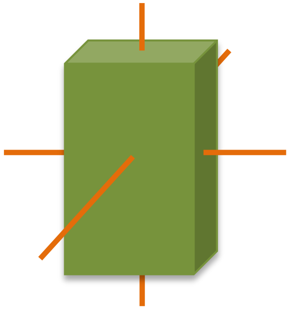
O(2)

cylinder
円柱



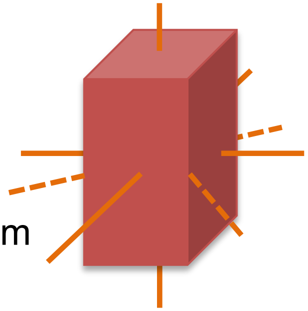
D_2

rectangular
直方体



D_4

square prism
正四角柱



UN: uniaxial nematic
$(r = -1/2)$

D_2-BN: D_2 biaxial nematic
$(-1 < r < -1/2)$

D_4-BN: D_4 biaxial nematic
$(r = -1)$

Neutron Star = Topological Star (!?)

2. Neutron $^3P_2$ superfluids (nuclear physics)

Hamiltonian

Fermion theory

$$\mathcal{H} = \int d\boldsymbol{r}\psi_a^\dagger(\boldsymbol{r})\xi_{ab}(-i\boldsymbol{\nabla})\psi_b(\boldsymbol{r})$$

$$+\frac{1}{2}\int d\boldsymbol{r}_1\int d\boldsymbol{r}_2\boxed{V_{a,b}^{c,d}(\boldsymbol{r}_{12})}\psi_a^\dagger(\boldsymbol{r}_1)\psi_b^\dagger(\boldsymbol{r}_2)\psi_c(\boldsymbol{r}_2)\psi_d(\boldsymbol{r}_1)$$

L･S potential

attractive $^3P_2$ channel (LS potential)

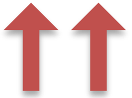
$$\xi(\boldsymbol{k}) = \xi_0(\boldsymbol{k}) - \frac{1}{2}\gamma_{\rm n}\boldsymbol{\sigma}\cdot\boldsymbol{B}$$

spin-magnetic field coupling

Bogoliubov-de Gennes (BdG) theory

Fermion

F. Tabakin, Single Phys. Rev. 174, 1208 (1968)
M. Hoffberg, A. E. Glassgold, R. W. Richardson, M. Ruderman, Phys. R[illegible] (1970)
R. Tamagaki, Progress of Theoretical Physics 44, 905 (1970)
T. Takatsuka, R. Tamagaki, Progress of Theoretical Physics 46, 114 (1971)
T. Takatsuka, R. Tamagaki, Prog. Theor. Phys. Suppl. 112, 27 (1993)
M. Baldo, J. Cugnon, A. Lejeune, U. Lombardo, Nucl. Phys. A536, 349 (1992)
O. Elgaroy, L. Engvik, M. Hjorth-Jensen, E. Osnes, Nucl. Phys. A607, 425 (1996)
V. A. Khodel, V. V. Khodel, J. W. Clark, Phys. Rev. Lett. 81, 3828 (1998)
M. Baldo, O. Elgaroey, L. Engvik, M. Hjorth- Jensen, H. J. Schulze, Phys. Rev. C58, 1921 (1998)
V. V. Khodel, V. A. Khodel, J. W. Clark, Nucl. Phys. A679, 827 (2001)
M. V. Zverev, J. W. Clark, V. A. Khodel, Nucl. Phys. A720, 20 (2003)
S. Maurizio, J. W. Holt, P. Finelli, Phys. Rev. C90, 044003 (2014)
S. K. Bogner, R. J. Furnstahl, A. Schwenk, Prog. Part. Nucl. Phys. 65, 94 (2010)
S. Srinivas and S. Ramanan, Phys. Rev. C94, 064303 (2016)
T. Mizushima, K. Masuda, M. Nitta, Phys. Rev. C93, 035804 (2016)
T. Mizushima, K. Masuda, M. Nitta, Phys. Rev. B95, 140503 (R) ()2017
T. Mizushima, S. Yasui, M. Nitta, Phys. Rev. Research2, 013194 (2020)
T. Mizushima, S. Yasui, D. Inotani, M. Nitta, Phy. Rev. C104, 045803 (2021)
...

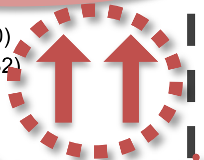


Cooper pair (fermion pair)

Ginzburg-Landau (GL) theory

Boson

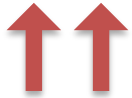
R. W. Richardson, Phys. Rev. D5, 1883 (1972)
J. A. Sauls and J. Serene, Phys. Rev. D17, 1524 (1978)
P. Muzikar, J. A. Sauls, . W. Serene, Phys. Rev. D21, 1494 (1980)
J. A. Sauls, D. L. Stein, J. W. Serene, , Phys. Rev. D25, 967 (1982)
V. Z. Vulovic, J. A. Sauls, Phys. Rev. D29, 2705 (1984)
K. Masuda, M. Nitta, Phys. Rev. C93, 035804 (2016)
K. Masuda and M. Nitta, PTEP2020, 013D01 (2020)
S. Yasui, C. Chatterjee, and M. Nitta, Phys. Rev. C101, 025204 (2020)
S. Yasui, M. Nitta, Phys. Rev. C101, 015207 (2020)
S. Yasui, C. Chatterjee, M. Nitta, Phys. Rev. C99, 035213 (2019)
S. Yasui, C. Chatterjee, M. Kobayashi, M. Nitta, Phys. Rev. C100, 025204 (2019)
T. Mizushima, S. Yasui, M. Nitta, Phys. Rev. Research2, 013194 (2020)
S. Yasui, D. Inotani, M. Nitta, Phys. Rev. C101, 055806 (2020)
T. Mizushima, S. Yasui, D. Inotani, M. Nitta, Phy. Rev. C104, 045803 (2021)
...



Cooper pair (boson-like)

2. Neutron $^3P_2$ superfluids (nuclear physics)

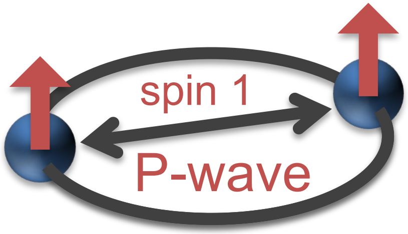
Bogoliubov-de Gennes (BdG) theory



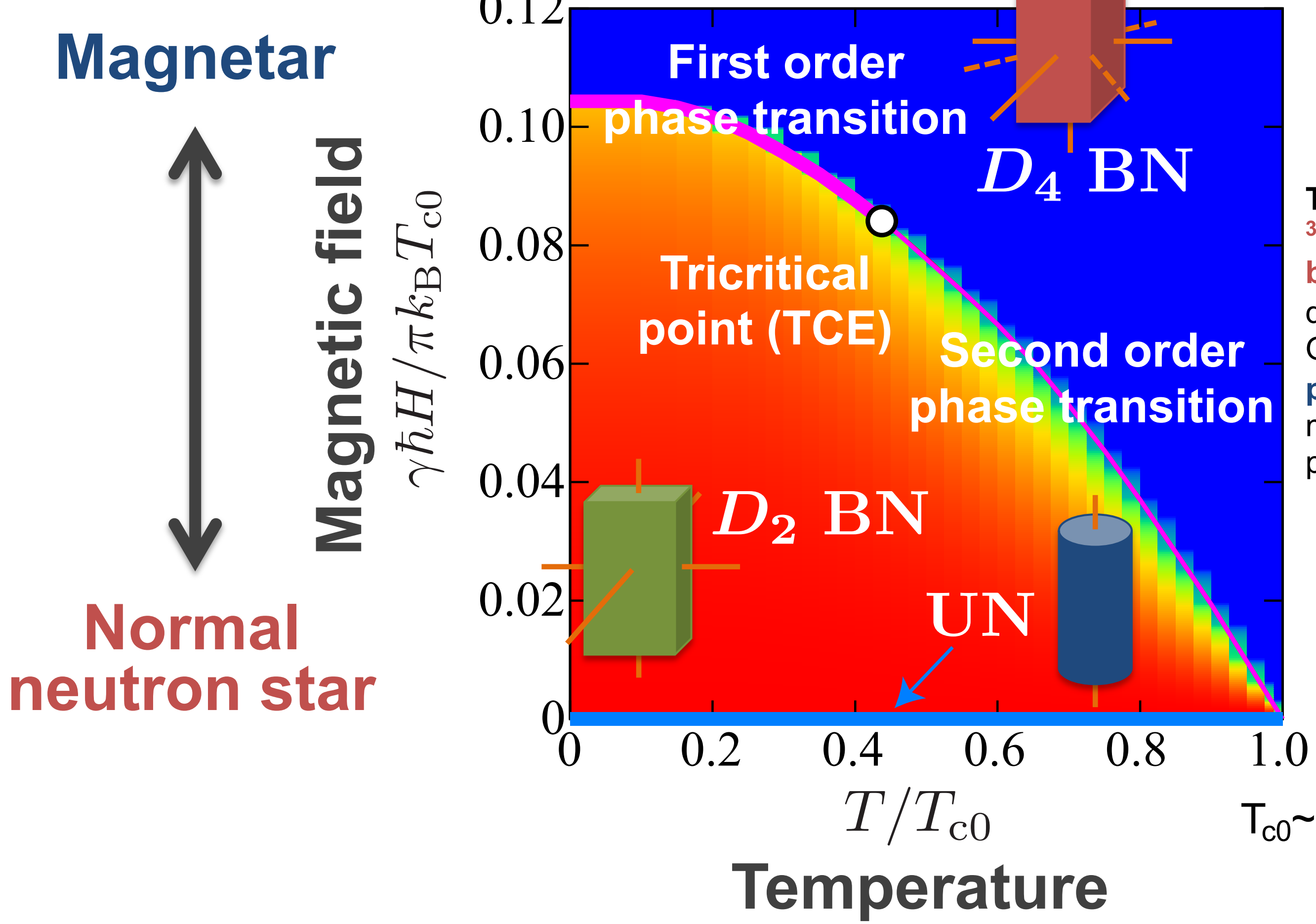
Cooper pair particle-hole basis

(fermion pair)

$$\mathscr{H}(\boldsymbol{k}) = \begin{pmatrix} \varepsilon(\boldsymbol{k}) & \overset{\text{gap}}{i\boldsymbol{\sigma}\cdot\boldsymbol{d}(\boldsymbol{k})\sigma_2} \\ \underset{\text{gap}}{i\sigma_2\boldsymbol{\sigma}\cdot\boldsymbol{d}^*(-\boldsymbol{k})} & -\varepsilon^{\mathrm{T}}(-\boldsymbol{k}) \end{pmatrix}$$



Cooper pair



T. Mizushima, K. Masuda, M. Nitta, Phys. Rev. B95, 140503(R) (2017)

Tolerance to magnetic fields:
$^3P_2$ Cooper pairings are **not broken** by strong magnetic fields due to the parallel spin pairs. On the other hand, **$^1S_0$ Cooper pairings are broken** by strong magnetic fields due to the anti-parallel spin pairs.

T_{c0}~0.2 MeV~10^{10} Kelvin

2. Neutron $^3P_2$ superfluids (nuclear physics)

Ginzburg-Landau (GL) theory

(A: tensor-type order parameter, B: magnetic field)

Tabakin (1968), Hoffenberg, Glassgold, Richardson, Ruderman (1970), Tamagaki (1970), Takatsuka, Tamagaki (1971), Takatsuka (1972), ...

boson-like field $\boxed{A_{ab} \sim \psi S^a \nabla^b \psi}$ a,b=1,2,3

$$f = A^2 + A^4 + A^6 + A^8 + B^2A^2 + B^4A^2 + B^2A^4 + \ldots$$

$$f[A] = K^{(0)}\left(\nabla_{xi}A^{ba*}\nabla_{xi}A^{ab} + \nabla_{xi}A^{ia*}\nabla_{xj}A^{aj} + \nabla_{xi}A^{ja*}\nabla_{xj}A^{ai}\right)$$

A^2 → kinetic term

A^2 → L.O.

$$+\alpha^{(0)}(\mathrm{tr}A^*A) + \beta^{(0)}\left((\mathrm{tr}\,A^*A)^2 - (\mathrm{tr}\,A^{*2}A^2)\right)$$

A^4 → SO(5) symmetry (pseudo Nambu−Goldstone boson)

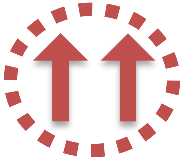
$$+\gamma^{(0)}\Big(-3(\mathrm{tr}\,A^*A)(\mathrm{tr}\,A^2)(\mathrm{tr}\,A^{*2}) + 4(\mathrm{tr}\,A^*A)^3 + 6(\mathrm{tr}\,A^*A)(\mathrm{tr}\,A^{*2}A^2) + 12(\mathrm{tr}\,A^*A)(\mathrm{tr}\,A^*AA^*A)$$
$$-6(\mathrm{tr}\,A^{*2})(\mathrm{tr}\,A^*A^3) - 6(\mathrm{tr}\,A^2)(\mathrm{tr}\,A^{*3}A) - 12(\mathrm{tr}\,A^{*3}A^3) + 12(\mathrm{tr}\,A^{*2}A^2A^*A) + 8(\mathrm{tr}\,A^*AA^*AA^*A)\Big)$$

A^6 → SO(5) symmetry breaking

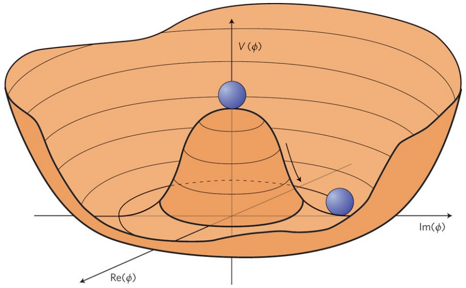
$$+\delta^{(0)}\Big((\mathrm{tr}\,A^{*2})^2(\mathrm{tr}\,A^2)^2 + 2(\mathrm{tr}\,A^{*2})^2(\mathrm{tr}\,A^4) - 8(\mathrm{tr}\,A^{*2})(\mathrm{tr}\,A^*AA^*A)(\mathrm{tr}\,A^2) - 8(\mathrm{tr}\,A^{*2})(\mathrm{tr}\,A^*A)^2(\mathrm{tr}\,A^2)$$
$$-32(\mathrm{tr}\,A^{*2})(\mathrm{tr}\,A^*A)(\mathrm{tr}\,A^*A^3) - 32(\mathrm{tr}\,A^{*2})(\mathrm{tr}\,A^*AA^*A^3) - 16(\mathrm{tr}\,A^{*2})(\mathrm{tr}\,A^*A^2A^*A^2)$$
$$+2(\mathrm{tr}\,A^{*4})(\mathrm{tr}\,A^2)^2 + 4(\mathrm{tr}\,A^{*4})(\mathrm{tr}\,A^4) - 32(\mathrm{tr}\,A^{*3}A)(\mathrm{tr}\,A^*A)(\mathrm{tr}\,A^2)$$
$$-64(\mathrm{tr}\,A^{*3}A)(\mathrm{tr}\,A^*A^3) - 32(\mathrm{tr}\,A^{*3}AA^*A)(\mathrm{tr}\,A^2) - 64(\mathrm{tr}\,A^{*3}A^2A^*A^2) - 64(\mathrm{tr}\,A^{*3}A^3)(\mathrm{tr}\,A^*A)$$
$$-64(\mathrm{tr}\,A^{*2}AA^{*2}A^3) - 64(\mathrm{tr}\,A^{*2}AA^*A^2)(\mathrm{tr}\,A^*A) + 16(\mathrm{tr}\,A^{*2}A^2)^2 + 32(\mathrm{tr}\,A^{*2}A^2)(\mathrm{tr}\,A^*A)^2$$
$$+32(\mathrm{tr}\,A^{*2}A^2)(\mathrm{tr}\,A^*AA^*A) + 64(\mathrm{tr}\,A^{*2}A^2A^{*2}A^2) - 16(\mathrm{tr}\,A^{*2}AA^{*2}A)(\mathrm{tr}\,A^2) + 8(\mathrm{tr}\,A^*A)^4$$
$$+48(\mathrm{tr}\,A^*A)^2(\mathrm{tr}\,A^*AA^*A) + 192(\mathrm{tr}\,A^*A)(\mathrm{tr}\,A^*AA^{*2}A^2) + 64(\mathrm{tr}\,A^*A)(\mathrm{tr}\,A^*AA^*AA^*A)$$
$$-128(\mathrm{tr}\,A^*AA^{*3}A^3) + 64(\mathrm{tr}\,A^*AA^{*2}AA^*A^2) + 24(\mathrm{tr}\,A^*AA^*A)^2 + 128(\mathrm{tr}\,A^*AA^*AA^{*2}A^2)$$
$$+48(\mathrm{tr}\,A^*AA^*AA^*AA^*A)\Big)$$

A^8

- Tricritical point
- Global stability



Cooper pair (boson-like)



B^2A^2 → L.O.

$$+\beta^{(2)}\boldsymbol{B}^tA^*A\boldsymbol{B} + \beta^{(4)}|\boldsymbol{B}|^2\boldsymbol{B}^tA^*A\boldsymbol{B}$$

B^4A^2 → Magnetic field higher order

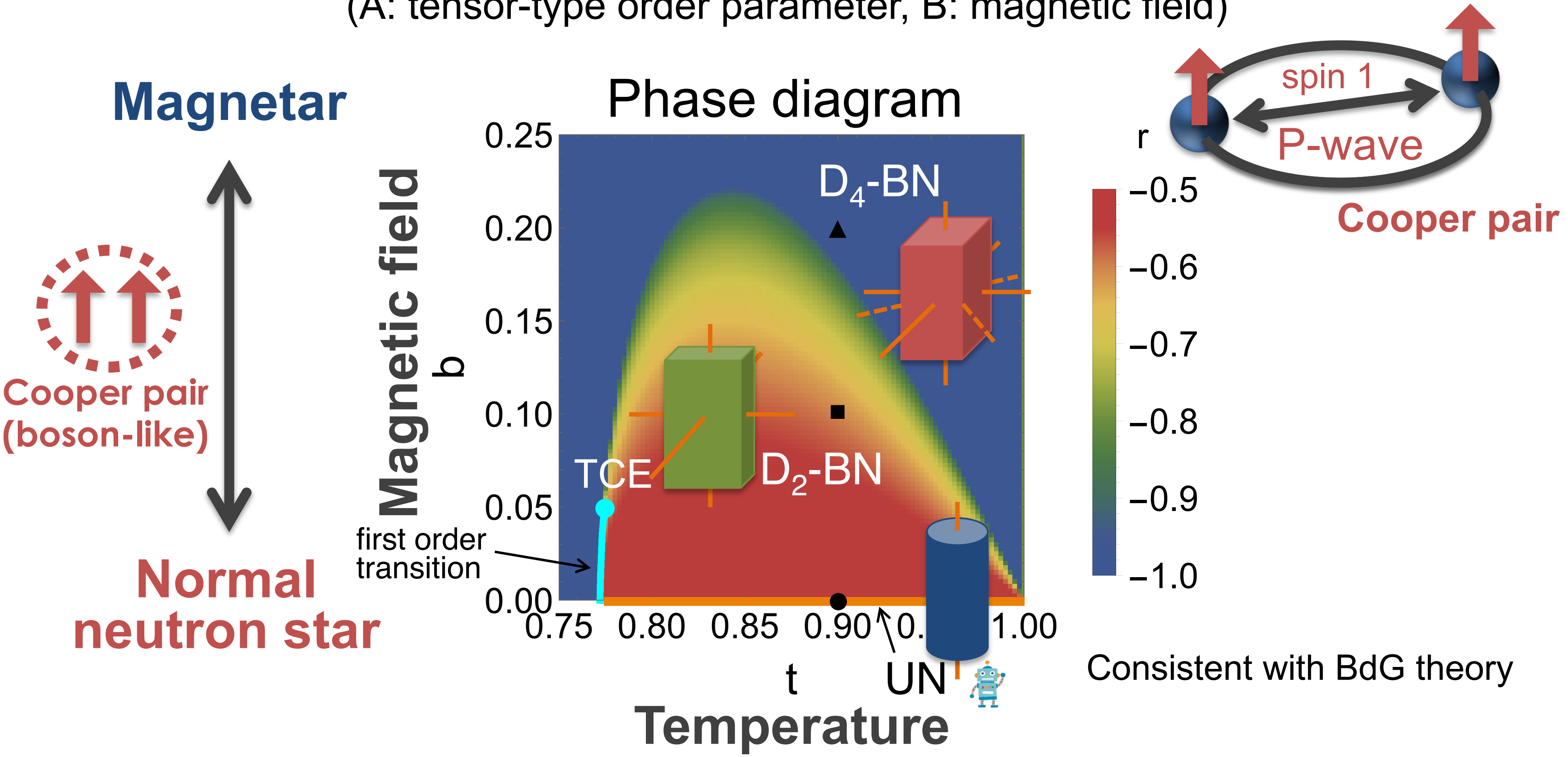
$$+\gamma^{(2)}\Big(-2\,|\boldsymbol{B}|^2(\mathrm{tr}\,A^2)(\mathrm{tr}\,A^{*2}) - 4\,|\boldsymbol{B}|^2(\mathrm{tr}\,A^*A)^2 + 4\,|\boldsymbol{B}|^2(\mathrm{tr}\,A^*AA^*A) + 8\,|\boldsymbol{B}|^2(\mathrm{tr}\,A^{*2}A^2)$$
$$+\boldsymbol{B}^tA^2\boldsymbol{B}(\mathrm{tr}\,A^{*2}) - 8\,\boldsymbol{B}^tA^*A\boldsymbol{B}(\mathrm{tr}\,A^*A) + \boldsymbol{B}^tA^{*2}\boldsymbol{B}(\mathrm{tr}\,A^2) + 2\,\boldsymbol{B}^tAA^{*2}A\boldsymbol{B}$$
$$+2\,\boldsymbol{B}^tA^*A^2A^*\boldsymbol{B} - 8\,\boldsymbol{B}^tA^*AA^*A\boldsymbol{B} - 8\,\boldsymbol{B}^tA^{*2}A^2\boldsymbol{B}\Big)$$

B^2A^4 → Magnetic field higher order

2. Neutron $^3P_2$ superfluids (nuclear physics)

Ginzburg-Landau (GL) theory

(A: tensor-type order parameter, B: magnetic field)



Consistent with BdG theory

magnetic field	zero	weak	strong
bulk phase	UN	D_2-BN	D_4-BN

S. Yasui, C. Chatterjee, M. Kobayashi, and M. Nitta, Phys. Rev. C100, 025204 (2019)
T. Mizushima, S. Yasui and M. Nitta, Phys. Rev. Research 2, 013194 (2020)

Contents

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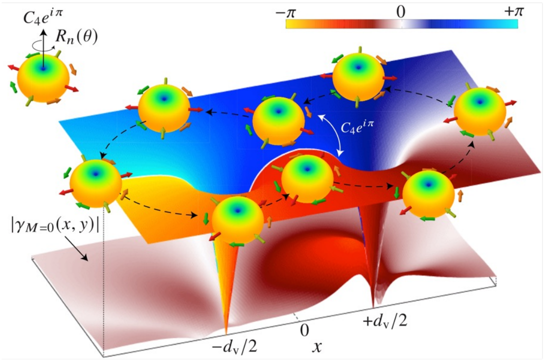
3. Topo-defects in neutron stars

Various phases in neutron stars and magnetars

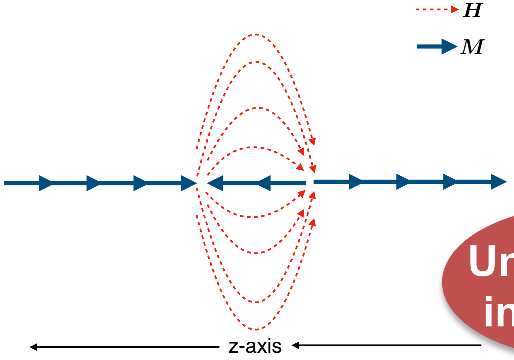
Half-integer vortex

Y. Masaki, T. Mizushima, M. Nitta,
PRB105, L220503 (2022)



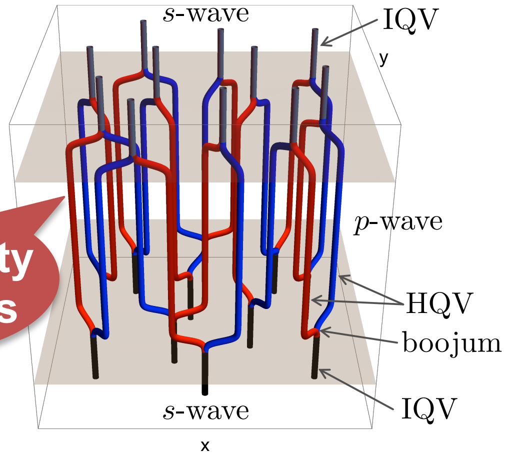
Soliton excitation

C. Chatterjee, M. Haberichter, M. Nitta,
PRC96, 055807 (2017)



Vortex networks

G. Marmoni, S. Yasui, M. Nitta,
arXiv:2010/09032 [astro-ph.HE]

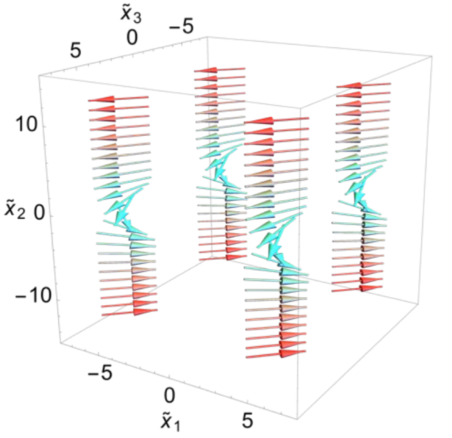


Universality in glitches

Domain walls

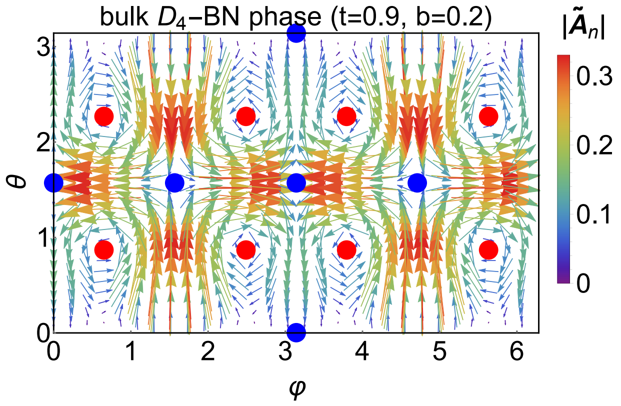
S. Yasui, M. Nitta, PRC101, 015207 (2020)

W_2^{13} in bulk D_4–BN phase (t=0.9,b=0.2)



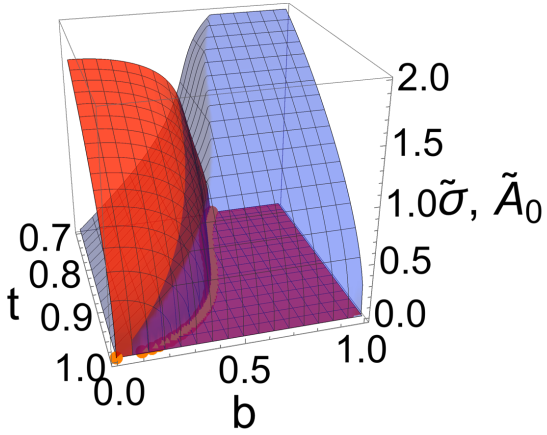
Topo-defects on surface

S. Yasui, C. Chatterjee, M. Nitta, PRC101, 025204 (2020)



$^1S_0$-$^3P_2$ mixing phase

S. Yasui, D. Inotani, M. Nitta,
PRC101, 055806 (2020)



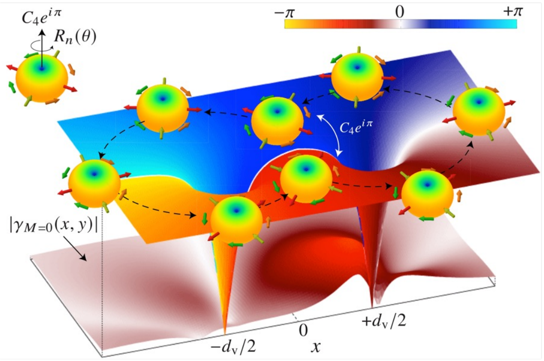
... etc.

3. Topo-defects in neutron stars

Various phases in neutron stars and magnetars

Half-integer vortex

Y. Masaki, T. Mizushima, M. Nitta,
PRB105, L220503 (2022)



Soliton excitation

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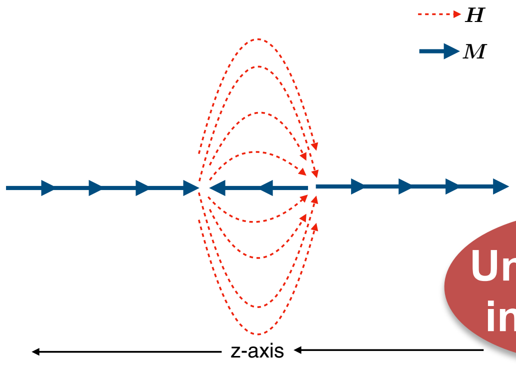
PRC96, 055807 (2017)



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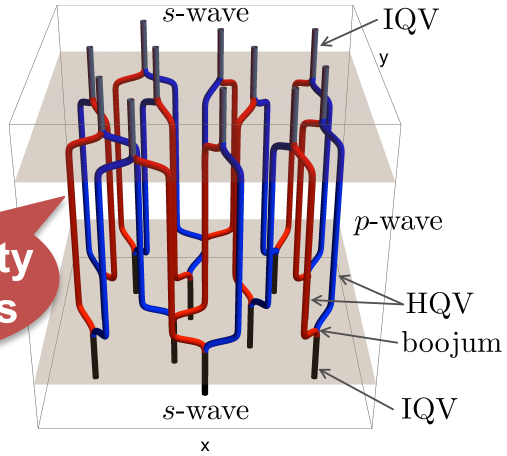

Universality in glitches

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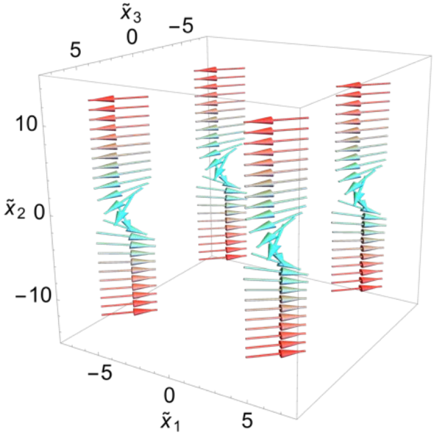
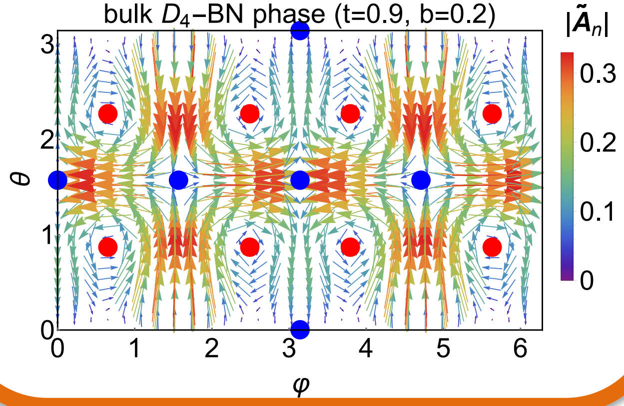


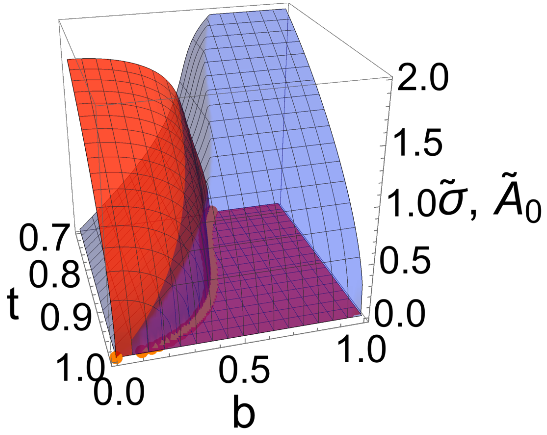
Topo-defects on surface

S. Yasui, C. Chatterjee, M. Nitta, PRC101, 025204 (2020)



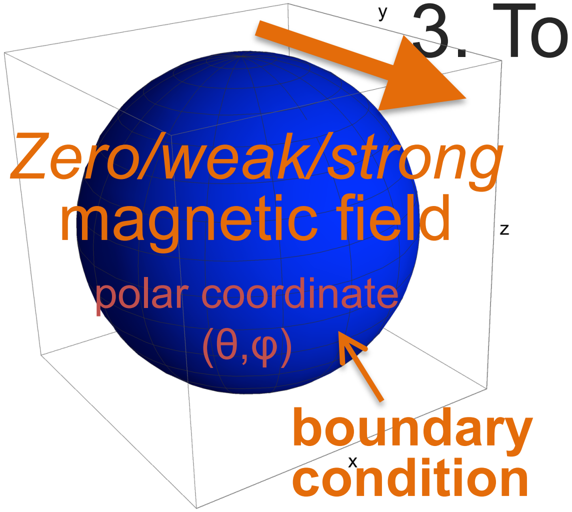
$^1S_0$-$^3P_2$ mixing phase

S. Yasui, D. Inotani, M. Nitta,
PRC101, 055806 (2020)



... etc.

3. Topo-defects in neutron stars

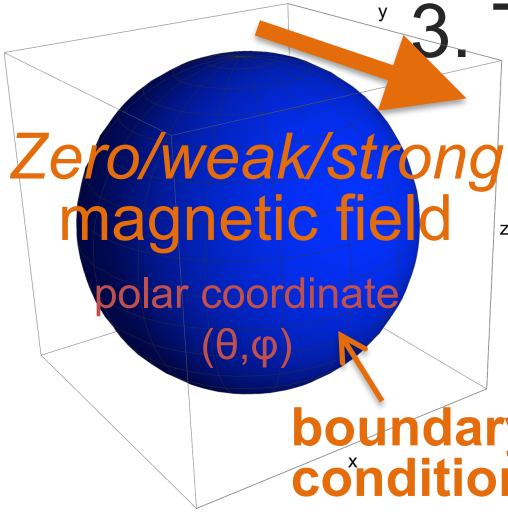


Zero/weak/strong
magnetic field

polar coordinate
(θ,φ)

boundary condition

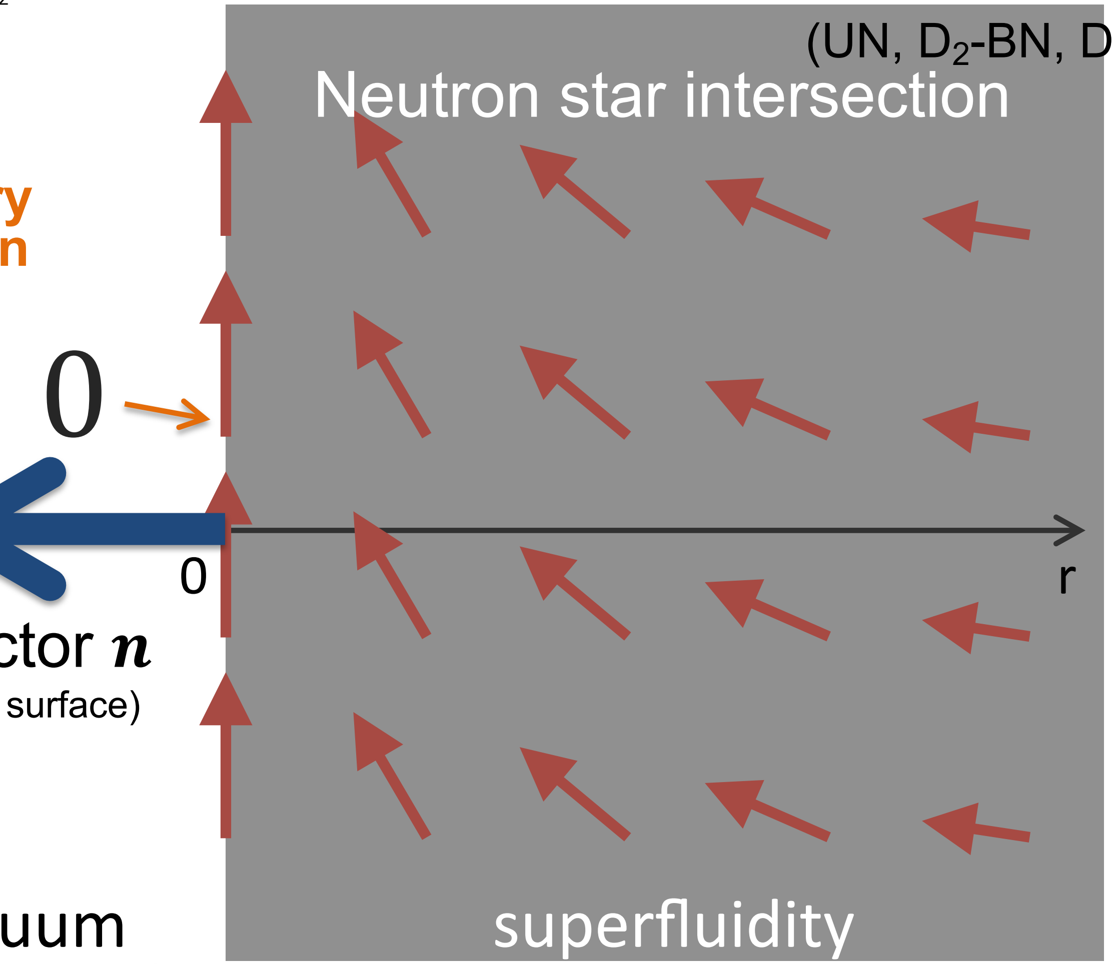
3. Topo-defects in neutron stars



Zero/weak/strong
magnetic field

polar coordinate
(θ,φ)

boundary
condition



Surface

Center
(UN, D_2-BN, D_4-BN phases)

Neutron star intersection

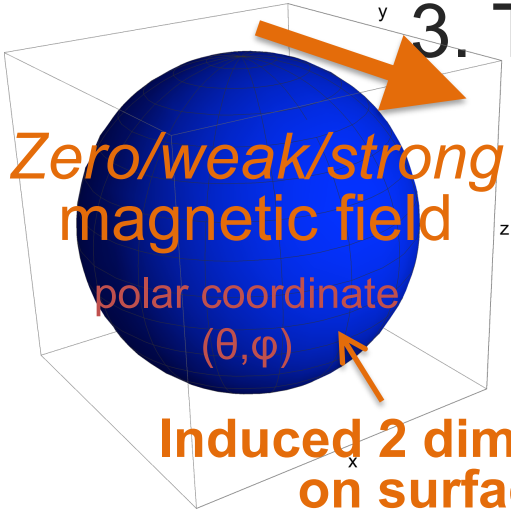
$$\boldsymbol{n}^t A \boldsymbol{n} = 0$$

normal vector $\boldsymbol{n}$
(perpendicular to surface)

vacuum
(or other phase)

superfluidity

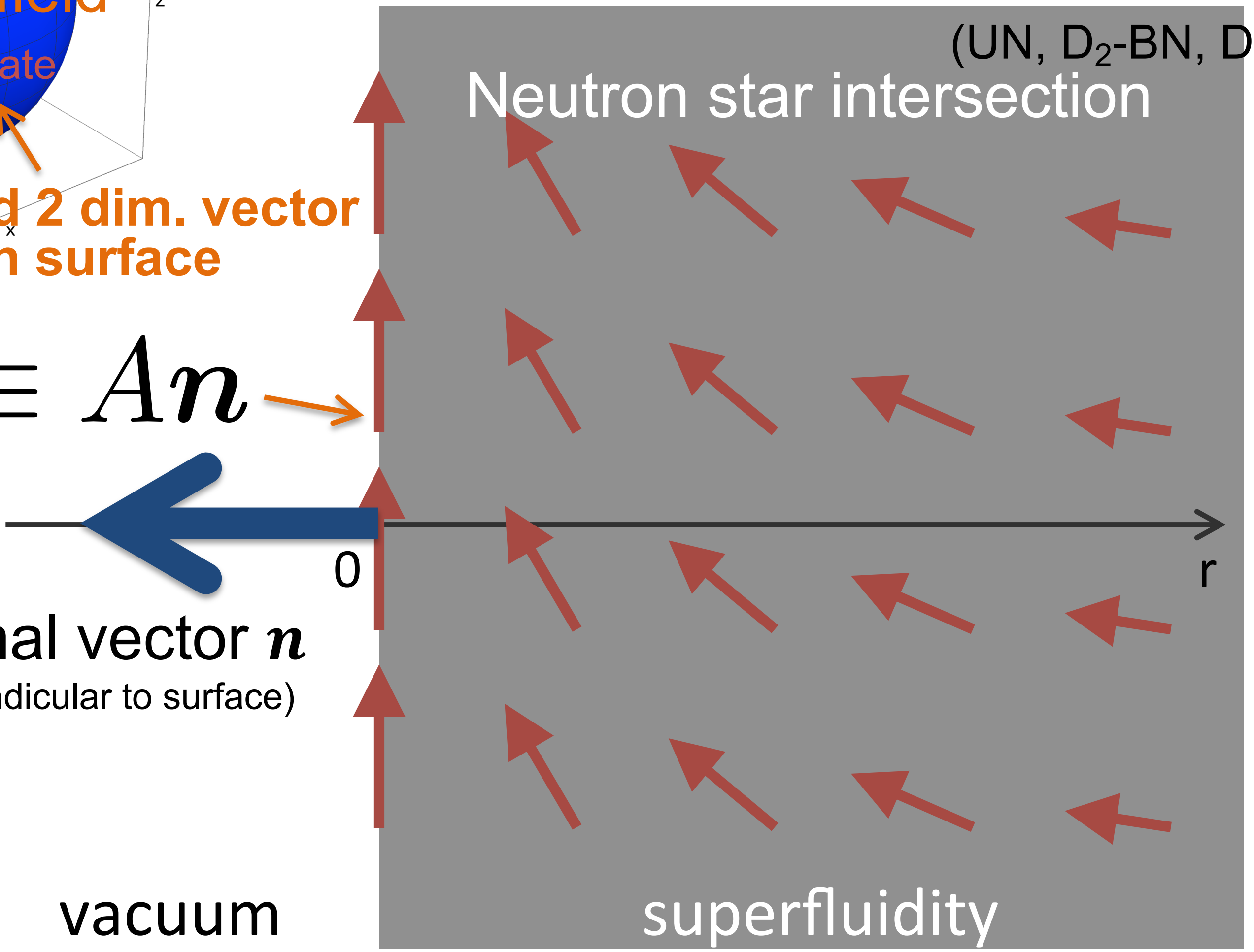
3. Topo-defects in neutron stars



Zero/weak/strong magnetic field

polar coordinate (θ,φ)

Induced 2 dim. vector on surface



Surface

Center

(UN, D_2-BN, D_4-BN phases)

Neutron star intersection

$$\boldsymbol{A}_n \equiv A\boldsymbol{n}$$

normal vector $\boldsymbol{n}$

(perpendicular to surface)

vacuum

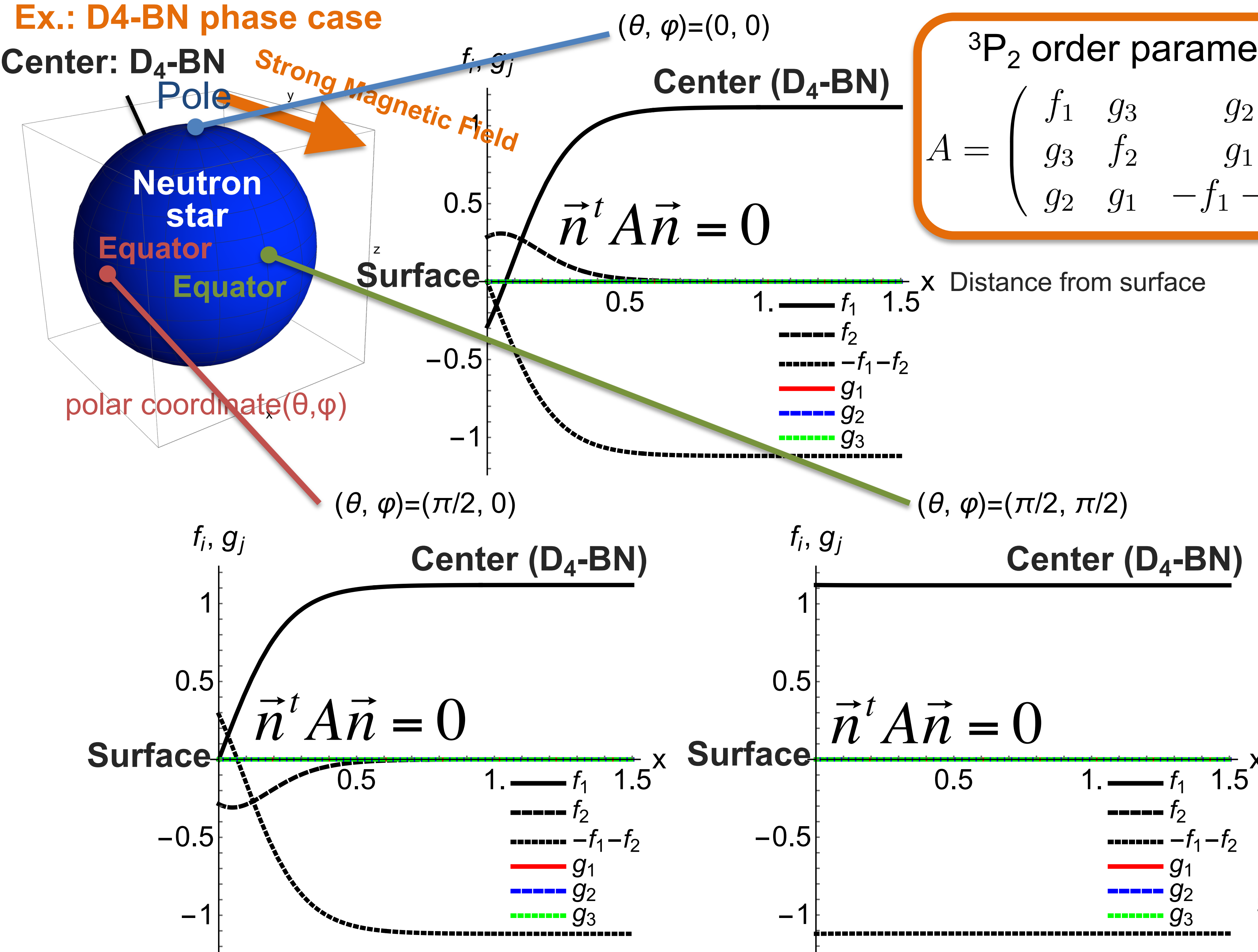
(or other phase)

superfluidity

3. Topo-defects in neutron stars

Ex.: D4-BN phase case

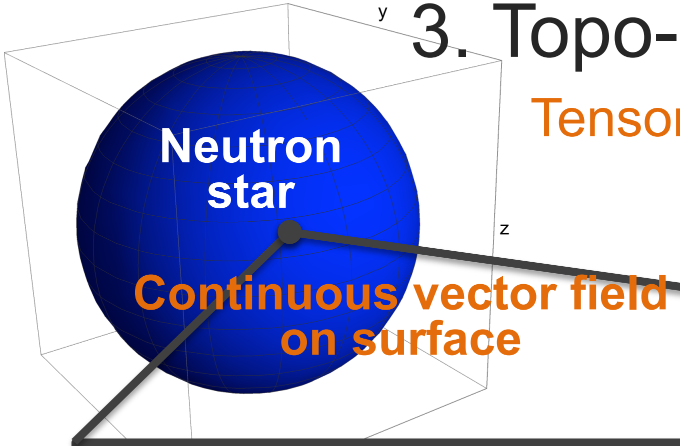
Center: D_4-BN



$^3P_2$ order parameter

$$A = \begin{pmatrix} f_1 & g_3 & g_2 \\ g_3 & f_2 & g_1 \\ g_2 & g_1 & -f_1 - f_2 \end{pmatrix}$$

3. Topo-defects in neutron stars



Tensor field projected on surface

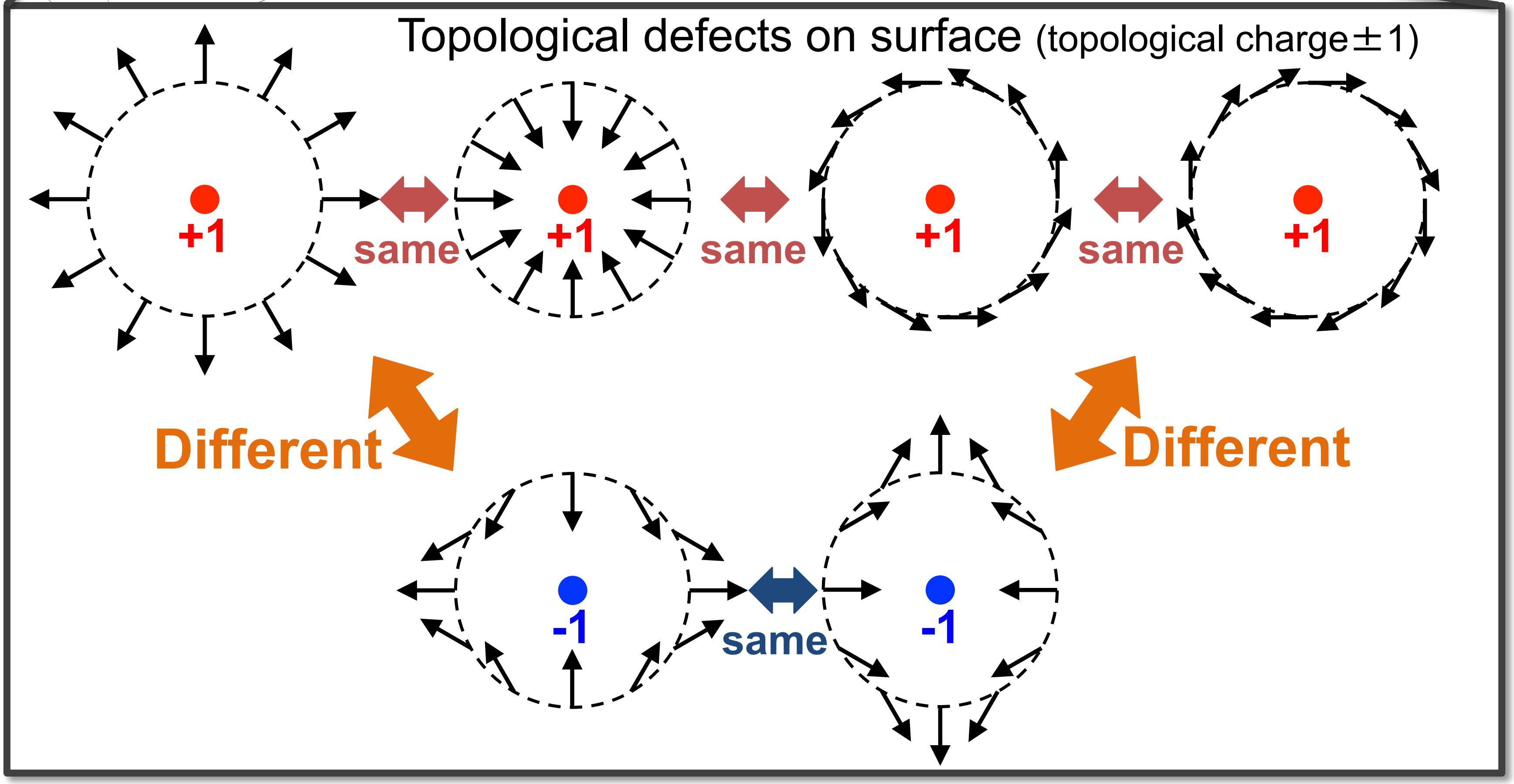
$$\boldsymbol{A}_n \equiv A\boldsymbol{n}$$

vector tensor

Defect=Vortex

$(S^1\times S^1)/Z_2$

$\boldsymbol{n}^t A\boldsymbol{n} = \boldsymbol{0}$ is invariant under rotation around $\boldsymbol{n}$ axis and U(1) phase in A field



3. Topo-defects in neutron stars

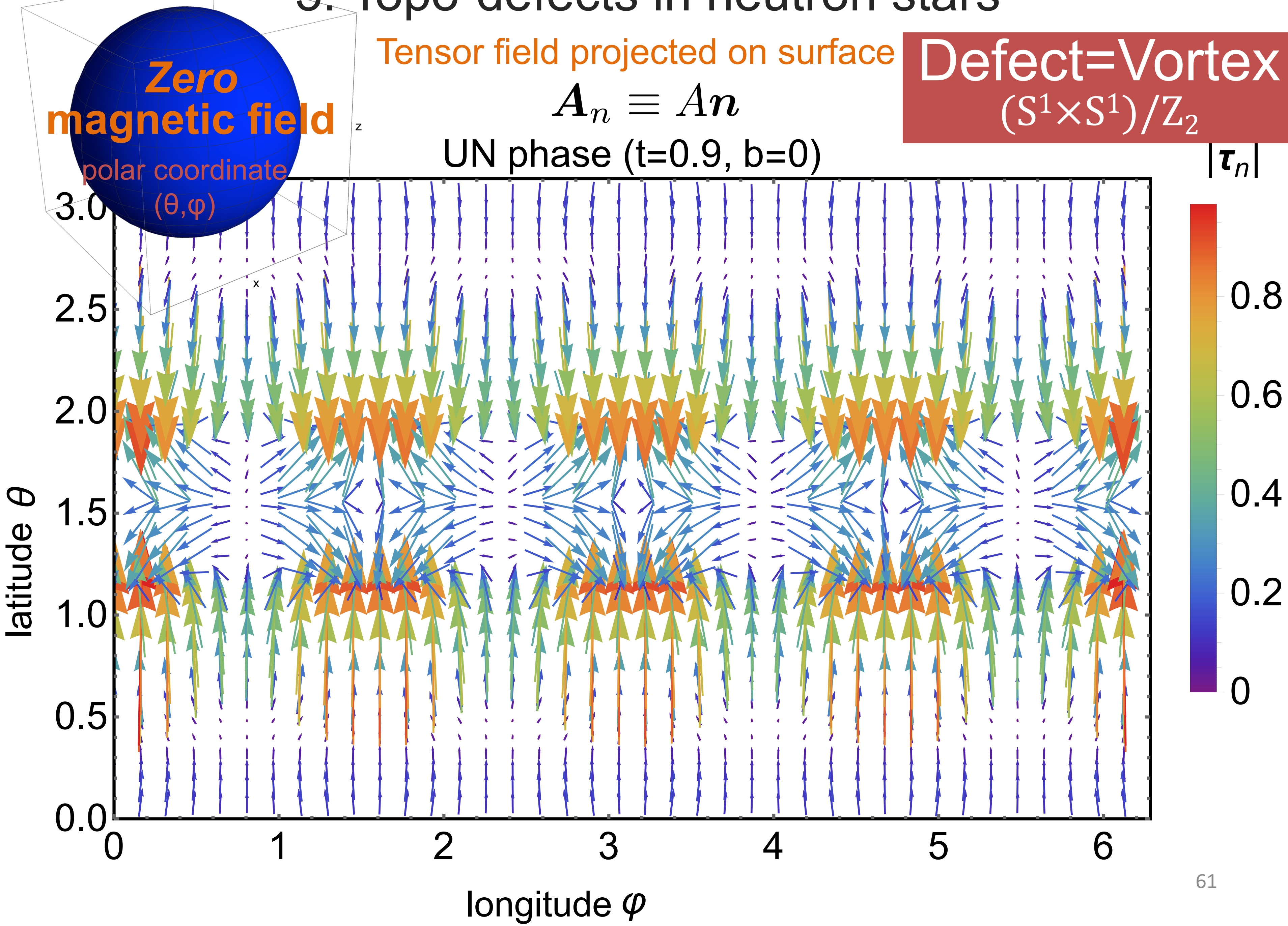
Center: UN phase

Tensor field projected on surface

$$\boldsymbol{A}_n \equiv A\boldsymbol{n}$$

Defect=Vortex

$(S^1 \times S^1)/Z_2$



Zero magnetic field

polar coordinate (θ,φ)

UN phase (t=0.9, b=0)

3. Topo-defects in neutron stars

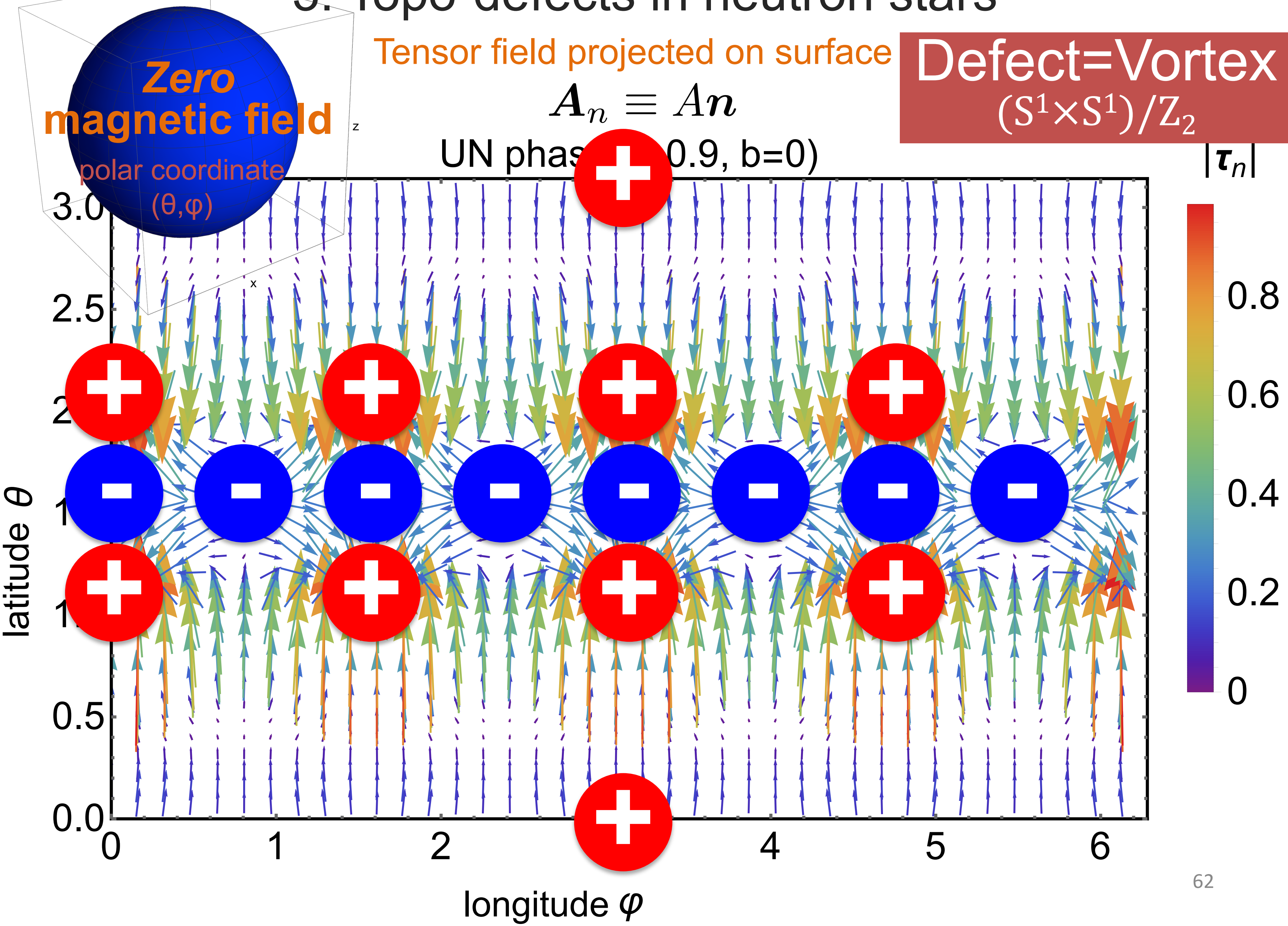
Center: UN phase

Tensor field projected on surface

$$\boldsymbol{A}_n \equiv A\boldsymbol{n}$$

Defect=Vortex

$(S^1 \times S^1)/Z_2$



3. Topo-defects in neutron stars

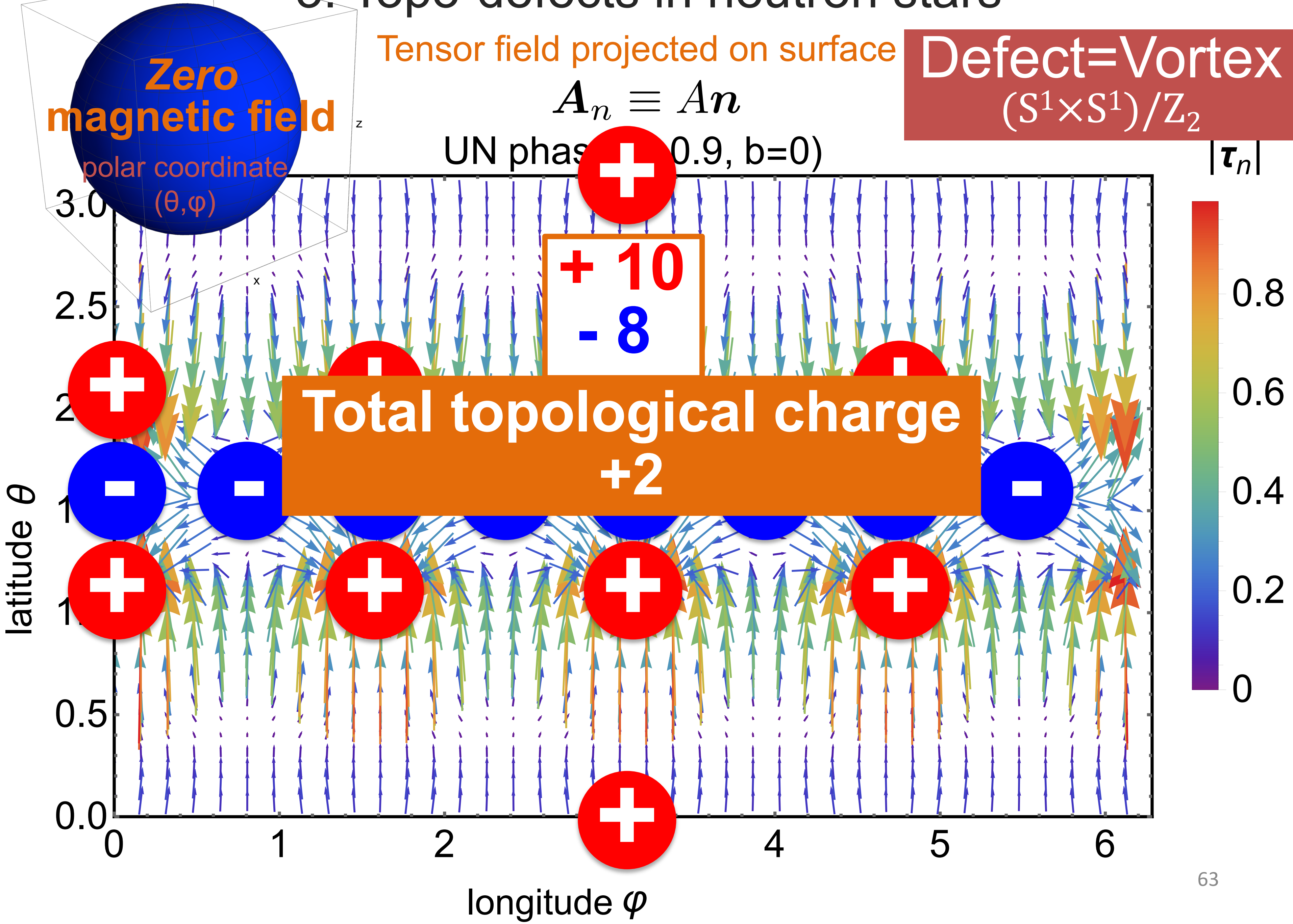
Center: UN phase

Tensor field projected on surface

$$\boldsymbol{A}_n \equiv A\boldsymbol{n}$$

Defect=Vortex

$(S^1 \times S^1)/Z_2$



Zero magnetic field

polar coordinate (θ,φ)

+ 10
- 8

Total topological charge +2

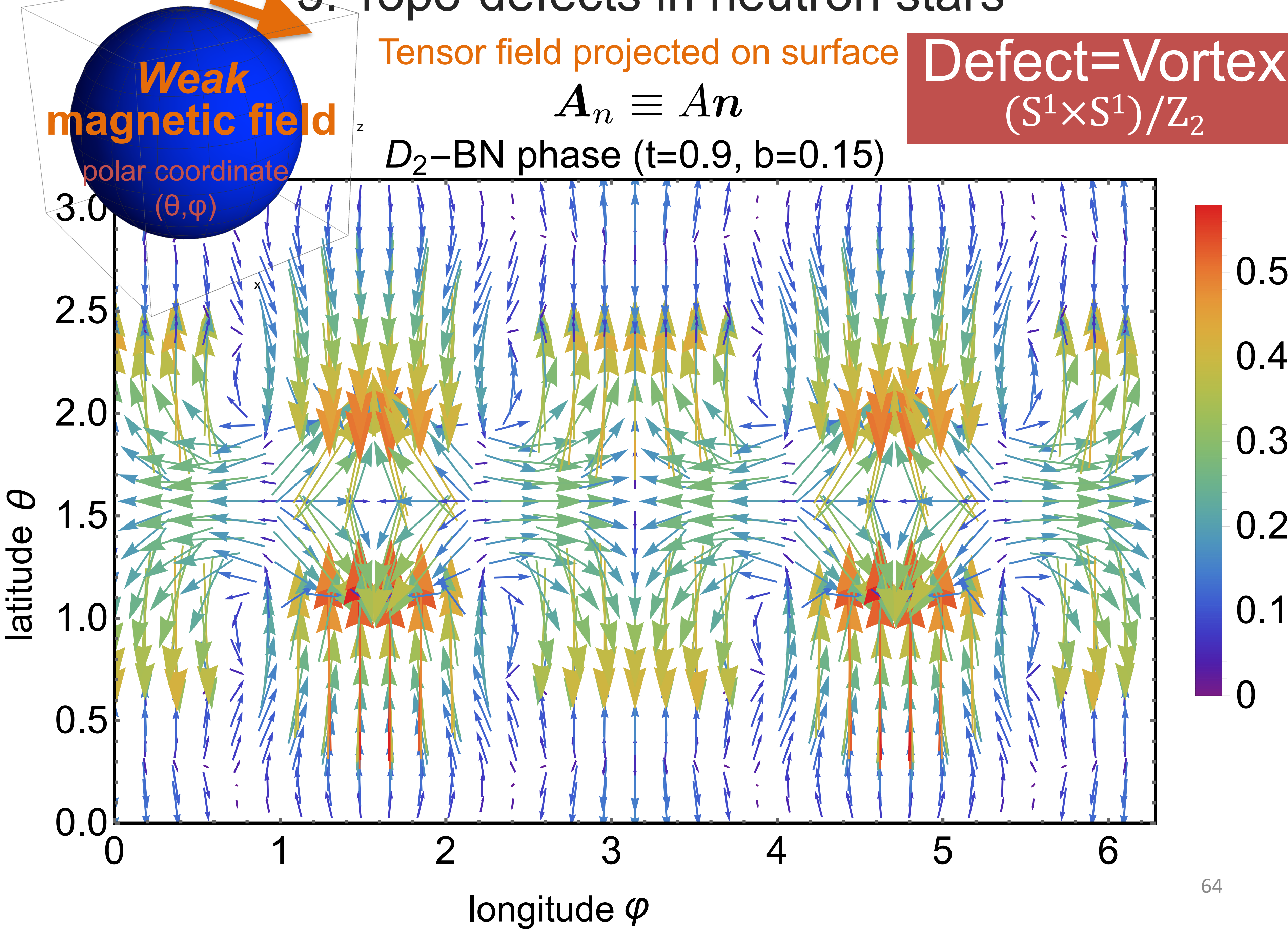
3. Topo-defects in neutron stars

Tensor field projected on surface

$$\boldsymbol{A}_n \equiv A\boldsymbol{n}$$

Defect=Vortex

$(S^1 \times S^1)/Z_2$



3. Topo-defects in neutron stars

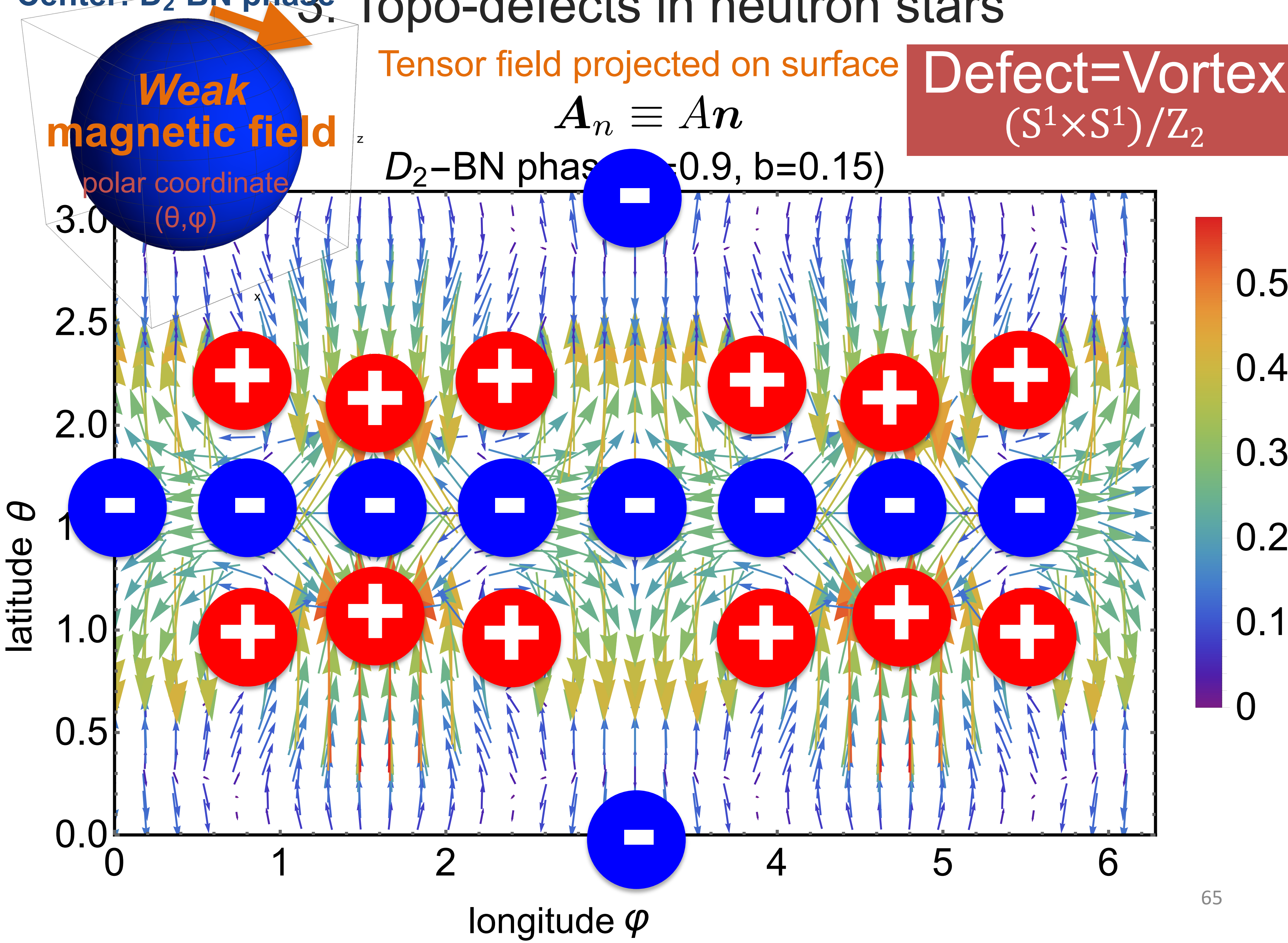
Center: D_2-BN phase

Tensor field projected on surface

$$\boldsymbol{A}_n \equiv A\boldsymbol{n}$$

Defect=Vortex

$(S^1 \times S^1)/Z_2$



3. Topo-defects in neutron stars

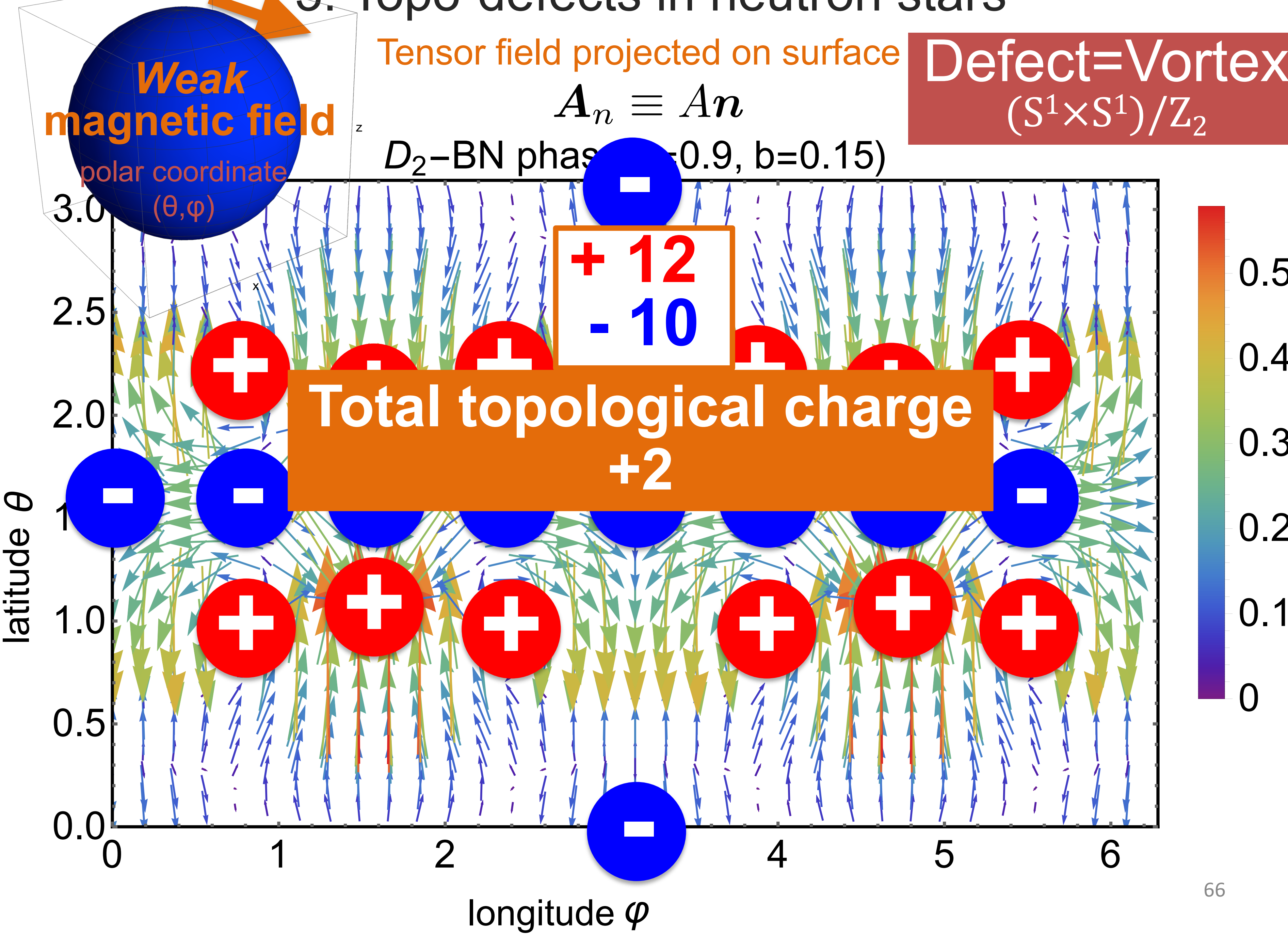
Center: D_2-BN phase

Defect=Vortex

$(S^1 \times S^1)/Z_2$

Tensor field projected on surface

$$\boldsymbol{A}_n \equiv A\boldsymbol{n}$$



+ 12

- 10

Total topological charge +2

3. Topo-defects in neutron stars

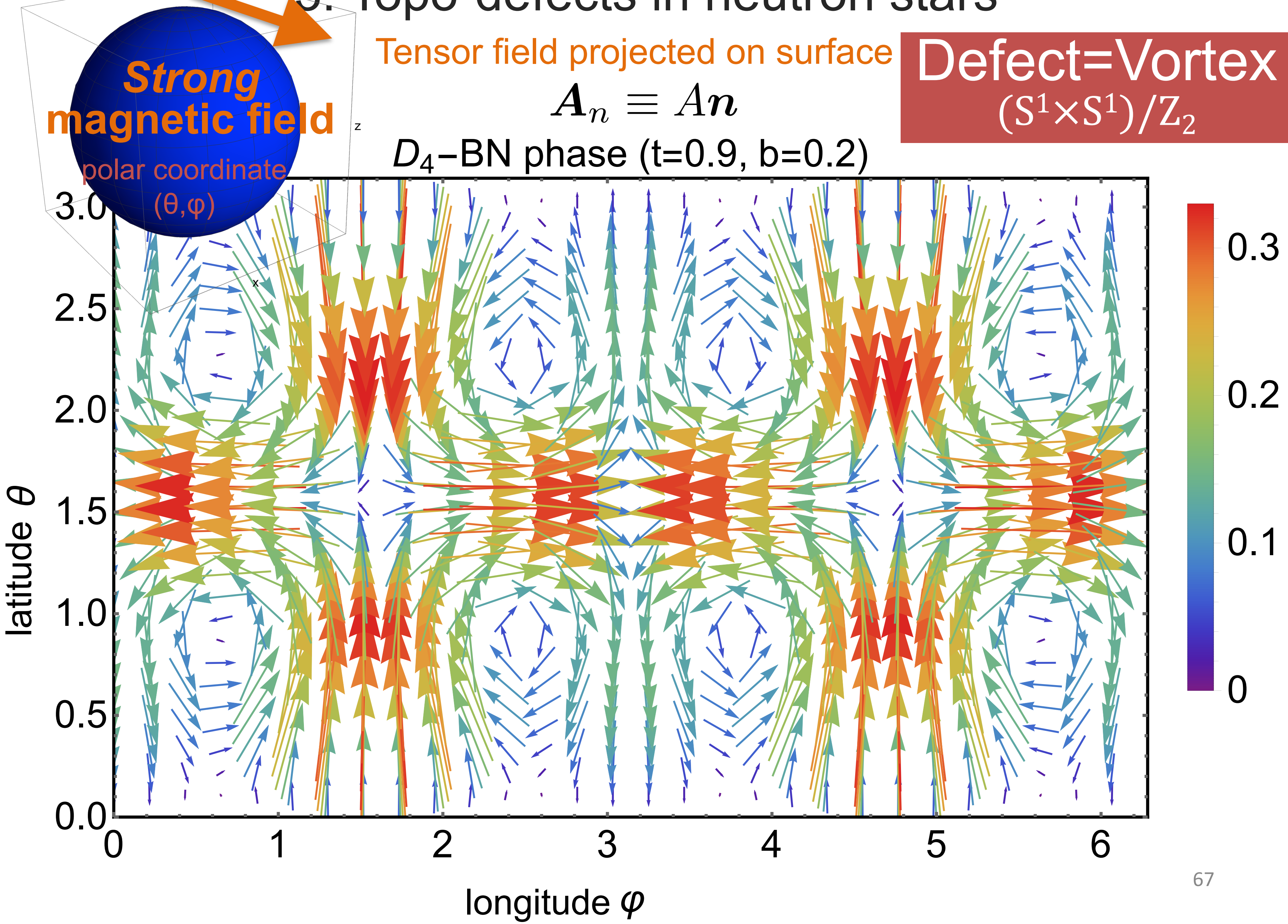
Center: D_4-BN phase

Tensor field projected on surface

$$\boldsymbol{A}_n \equiv A\boldsymbol{n}$$

Defect=Vortex

$(S^1 \times S^1)/Z_2$



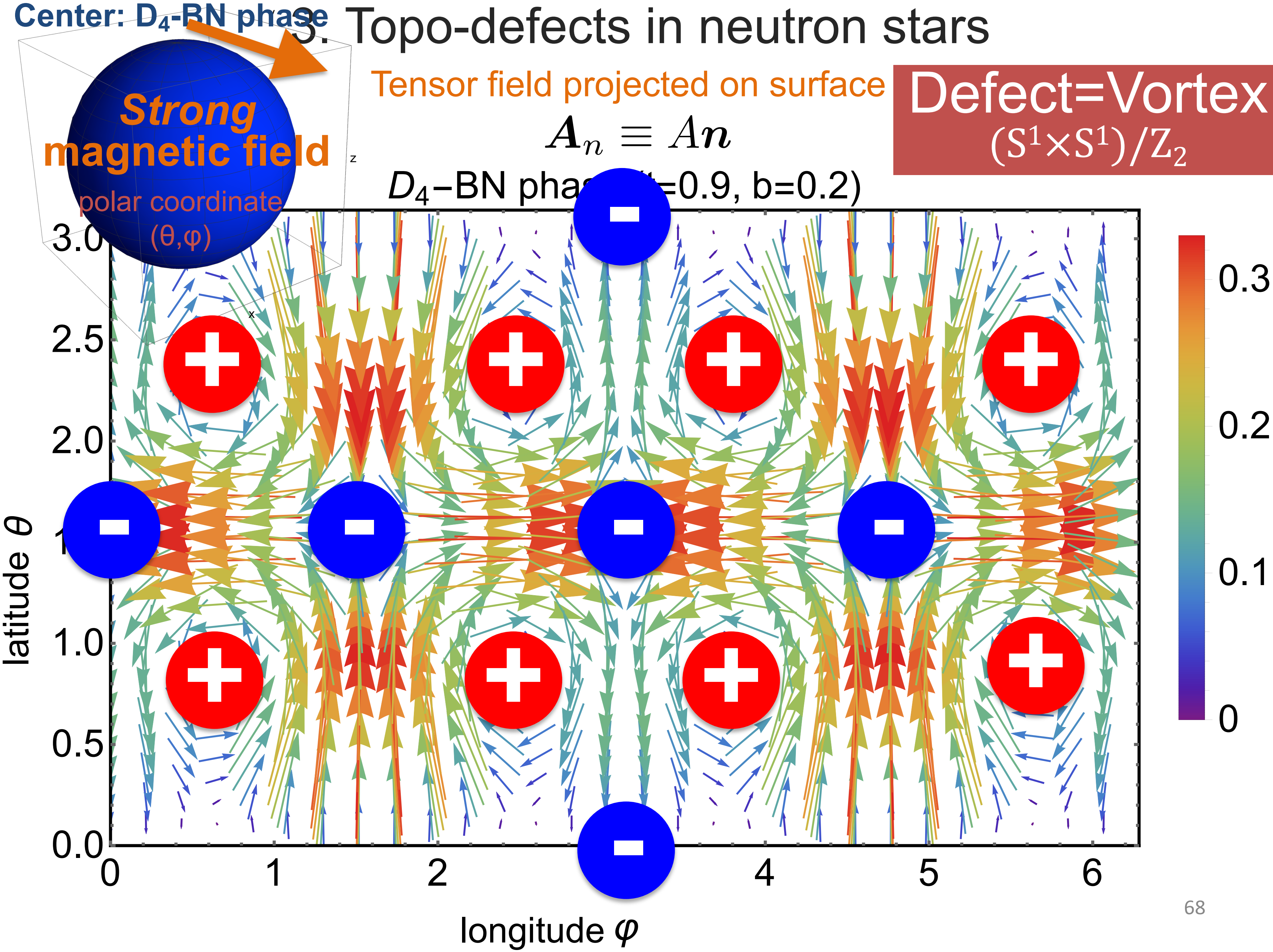
3. Topo-defects in neutron stars

Defect=Vortex

$(S^1\times S^1)/Z_2$

Tensor field projected on surface

$$\boldsymbol{A}_n \equiv A\boldsymbol{n}$$



3. Topo-defects in neutron stars

Center: D_4-BN phase

Strong magnetic field

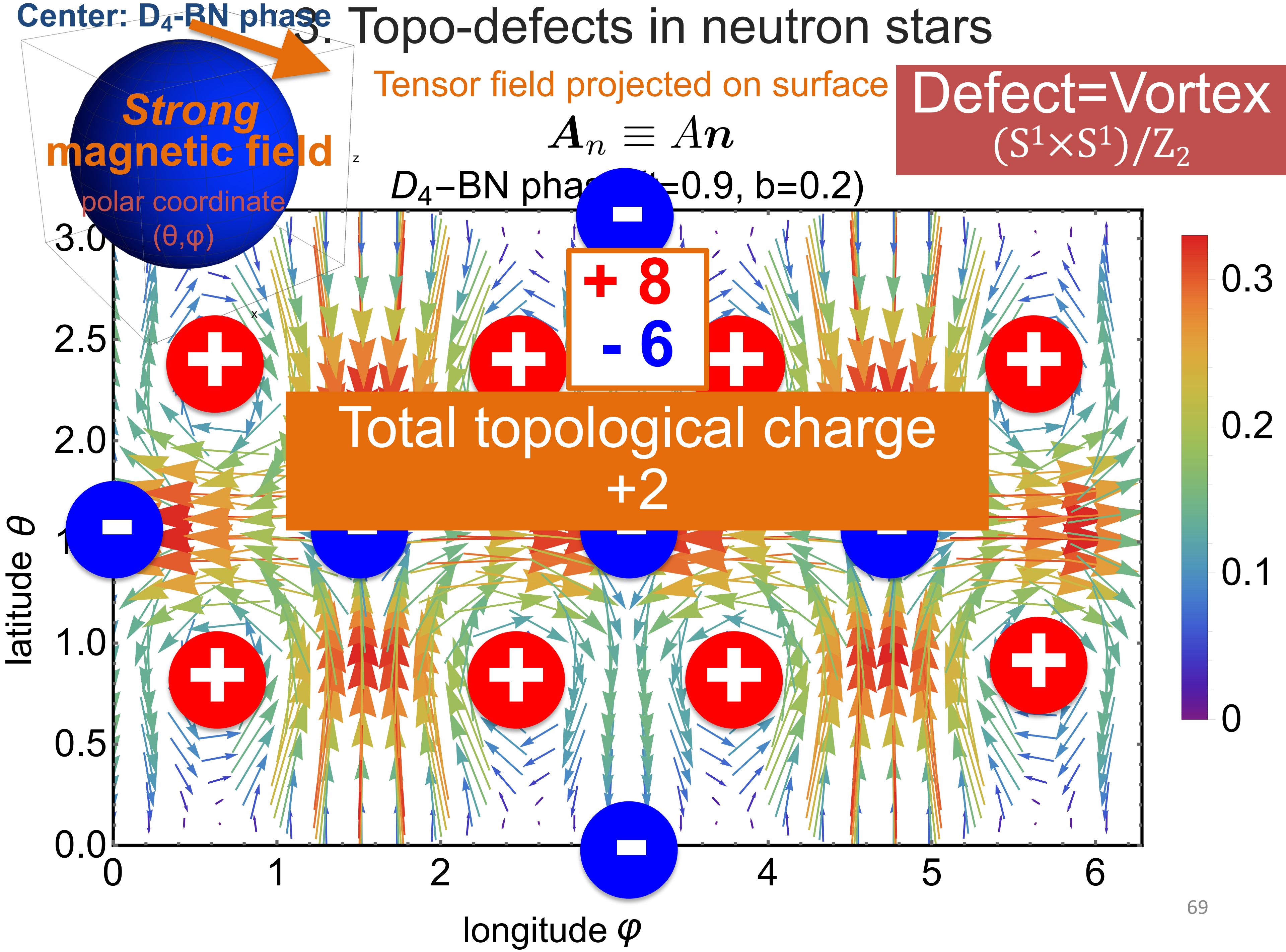
polar coordinate (θ,φ)

Tensor field projected on surface

$$\boldsymbol{A}_n \equiv A\boldsymbol{n}$$

Defect=Vortex

$(S^1 \times S^1)/Z_2$



+ 8
- 6

Total topological charge +2

3. Topo-defects in neutron stars

Poincaré-Hopf theorem (hairy ball theorem)

M: manifold (directed), v: vector field

$$\sum_{p \in M} \mathrm{index}_p\, v = \chi(M)$$

zero point of v
(index=±1)

Euler characteristic
χ=2 (sphere)

3. Topo-defects in neutron stars

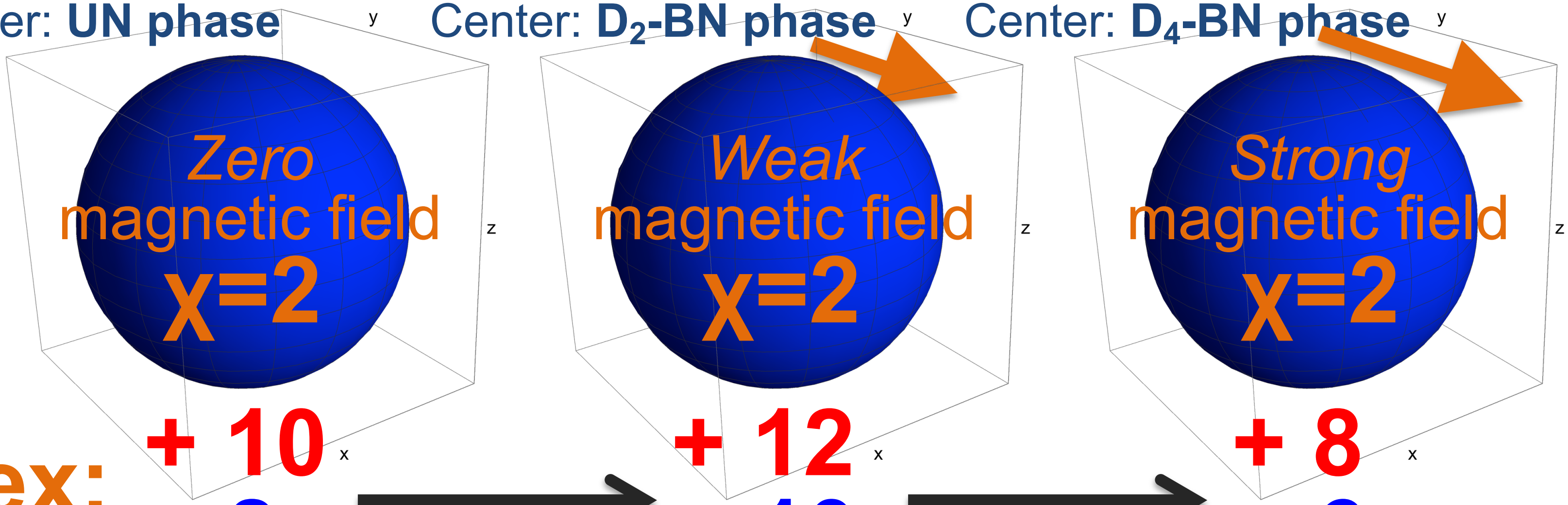
Poincaré-Hopf theorem (hairy ball theorem)

M: manifold (directed), v: vector field

$$\sum_{p\in M} \mathrm{index}_p\, v = \chi(M)$$

zero point of v
(index=±1)

Euler characteristic
χ=2 (sphere)



Center: **UN phase**

Zero magnetic field

χ=2

Center: **D_2-BN phase**

Weak magnetic field

χ=2

Center: **D_4-BN phase**

Strong magnetic field

χ=2

Index: + 10 - 8 → + 12 - 10 → + 8 - 6

3. Topo-defects in neutron stars

Poincaré-Hopf theorem (hairy ball theorem)

M: manifold (directed), v: vector field

$$\sum_{p\in M} \mathrm{index}_p\, v = \chi(M)$$

zero point of v (index=±1)

Euler characteristic $\chi=2$ (sphere)

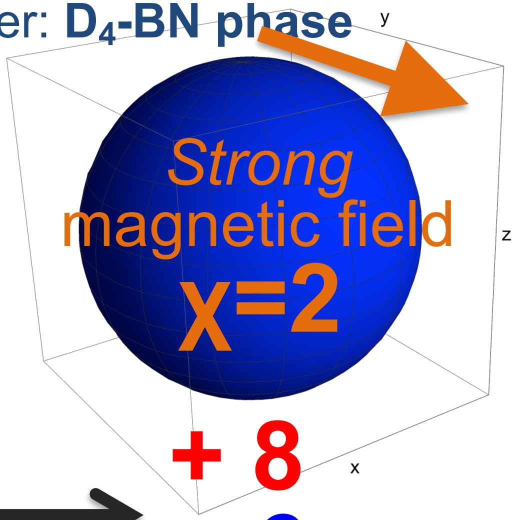
Center: **UN phase**

Zero magnetic field $\chi=2$

Center: **D_2-BN phase**

Weak magnetic field $\chi=2$

Center: **D_4-BN phase**



Strong magnetic field $\chi=2$

Index: + 10 / - 8 → **Creation of vortex pairs** → + 12 / - 10 → **Annihilation of vortex pairs** → + 8 / - 6

3. Topo-defects in neutron stars

What's there in neutron $^3P_2$ superfluids on neutron star?

Bulk + Boundary →

(Center) $(S^1 \times S^1)/Z_2$

3. Topo-defects in neutron stars

What's there in neutron $^3P_2$ superfluids on neutron star?

Poincaré-Hopf theorem

Bulk + Boundary → Vortex

(Center) $(S^1 \times S^1)/Z_2$ on surface

#(+) - #(-) = 2

3. Topo-defects in neutron stars

What's there in neutron $^3P_2$ superfluids on neutron star?

Poincaré-Hopf theorem

Bulk + Boundary → Vortex

(Center)

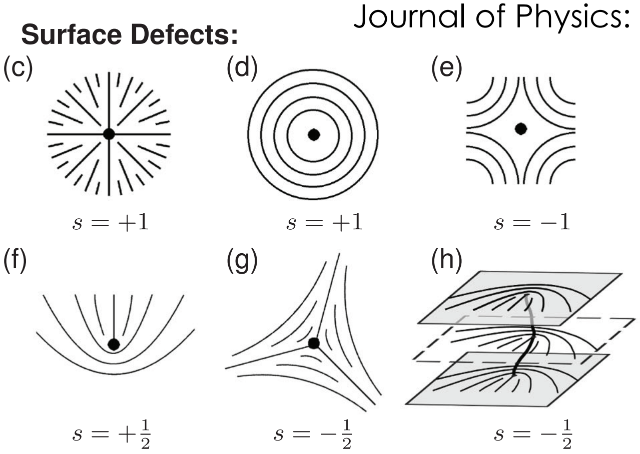
$(S^1 \times S^1)/Z_2$

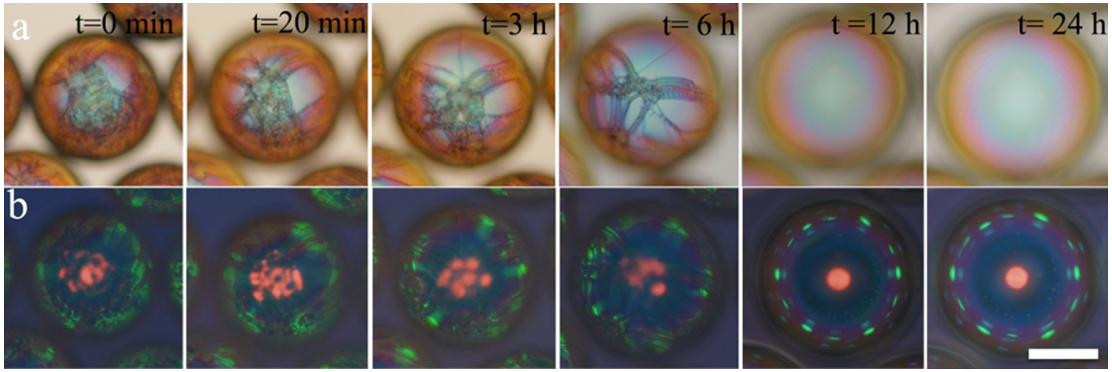
on surface

#(+) - #(-) = 2

Analogous to boojums (defects) in ^{3}He superfluids and liquid crystals

Cf. N. D. Mermin, Rev. Mod. Phys. 51, 591 (1979)
M. Urbanski, C. G. Reyes, J. Noh, A. Sharma, Y. Geng, V. S. R. Jampani, J. P. F. Lagerwall, Journal of Physics: Condensed Matter 29, 133003 (2017)





3. Topo-defects in neutron stars

What’s there in neutron $^3P_2$ superfluids on neutron star?

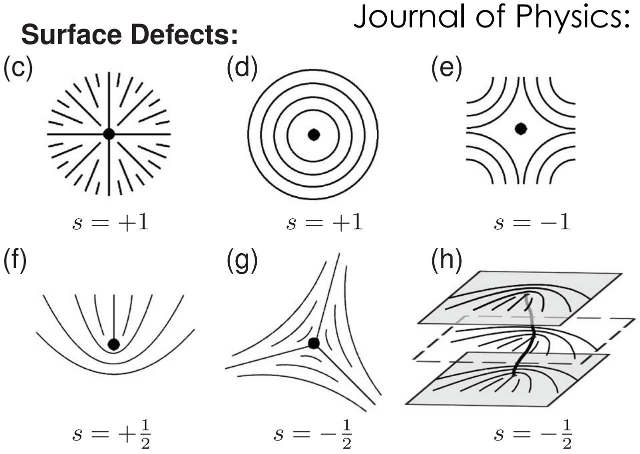
Poincaré-Hopf theorem

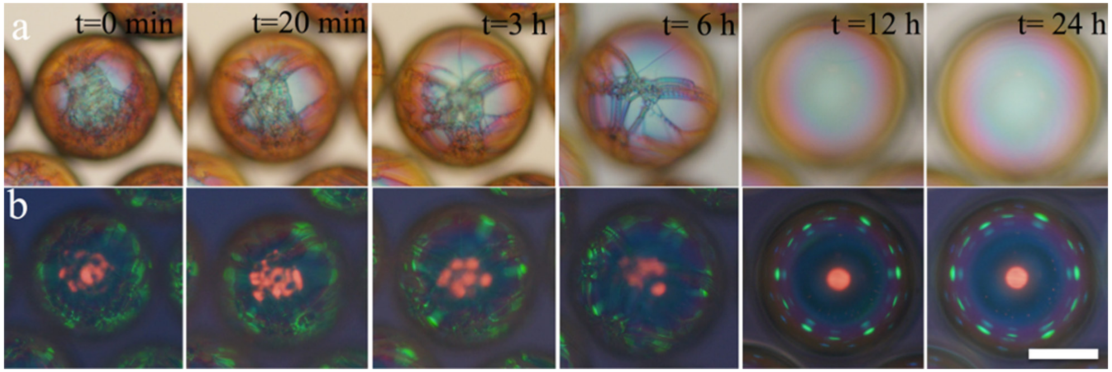
Bulk + Boundary → Vortex

(Center) $(S^1 \times S^1)/Z_2$ on surface $\#(+) - \#(-) = 2$

Analogous to boojums (defects) in ^{3}He superfluids and liquid crystals

Cf. N. D. Mermin, Rev. Mod. Phys. 51, 591 (1979)
M. Urbanski, C. G. Reyes, J. Noh, A. Sharma, Y. Geng, V. S. R. Jampani, J. P. F. Lagerwall, Journal of Physics: Condensed Matter 29, 133003 (2017)





Poincaré-Hopf theorem applied to nuclear and astrophysics!

4. Summary

① Neutron $^3P_2$ superfluids in neutron stars have various phases, such as UN, D_2-BN, and D_4-BN.

② Structures existing in neutron $^3P_2$ superfluids:

- Topological defects on surface
- Quasistable domain wall
- Quantum vortices (glitches)
- $^1S_0$-$^3P_2$ coexistence phase
- Mixed biaxial nematic (MBN) phase
- Ferromagnetic (FM) phase

③ Neutron star = topological star (!?)

④ We should explore *topology* in neutron stars!

Appendix

Open Challenge

Phase diagram？

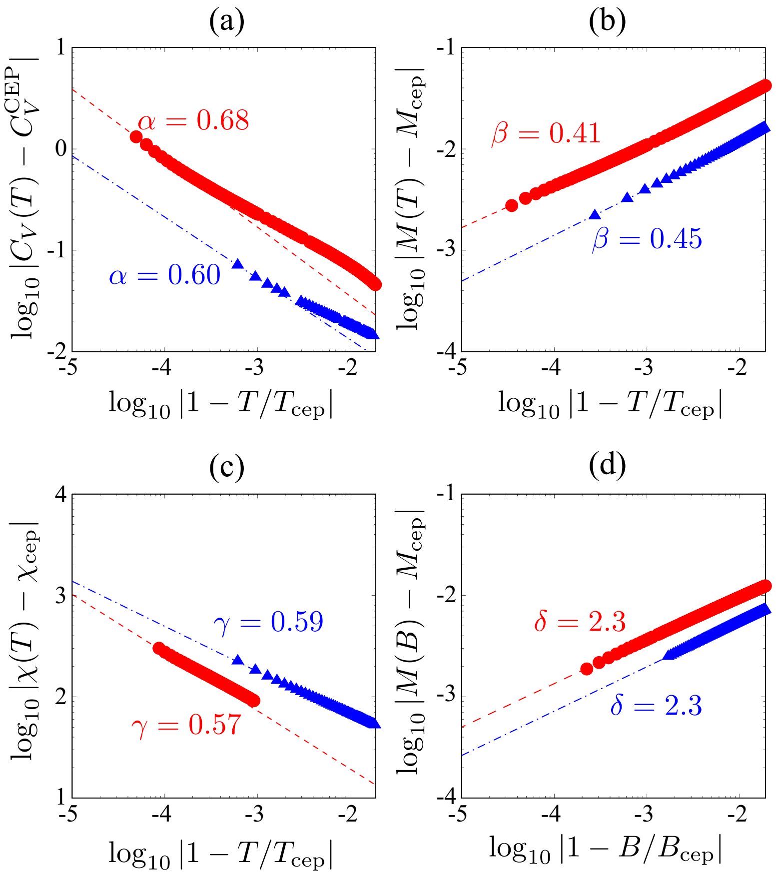
- Thermodynamic properties
- Transport coefficients (cooling process)
- Other New phases
- Hyperon matter
- Non-uniform phase (FFLO) D. Inotani, S. Yasui, T. Mizushima, M. Nitta, Phys. Rev. A103, 053308 (2021)

Topological objects？

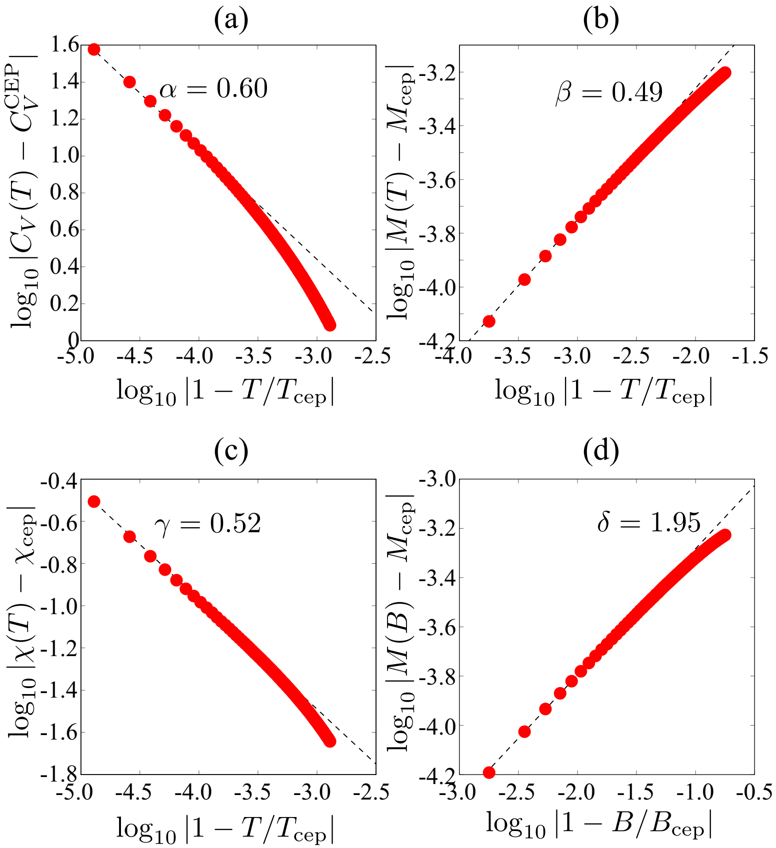
- Fractionally quantized vortices K. Masuda, M. Nitta, PRC93, 035804 (2016), PTEP202 (2020) 013
- Solitons in vortices C. Chatterjee, M. Haberichter, M. Nitta, PRC96, 055807 (2017)
- Gapless fermions T. Mizushima, K. Masuda, M. Nitta, PRB95, 140503 (2017)
- Boojum
  M. Cipriani, W. Vinci and M. Nitta, Phys. Rev. D 86, 121704 (2012)
  G. Alford, G. Baym, F. Fukushima, T. Hatsuda, M. Tachibana, Phys. Rev. D99, 036004 (2019)
  C. Chatterjee, M. Nitta, S. Yasui, Phys. Rev. D99, 034001 (2019)
  A. Cherman, S. Sen, L. G. Yaffe, Phys, Rev, D100, 034015 (2019)
  G. Maromorini, S. Yasui, M. Nitta, arXiv:2010.09032 [astro-ph.HE]
  …

臨界指数の評価

BdG



GL



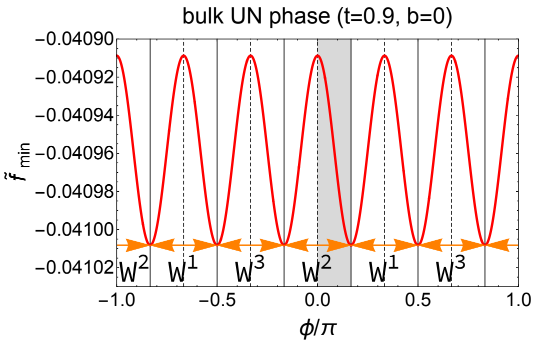
red, blue…Landau parameter $G_0^{(n)}$=-0.7, -0.4

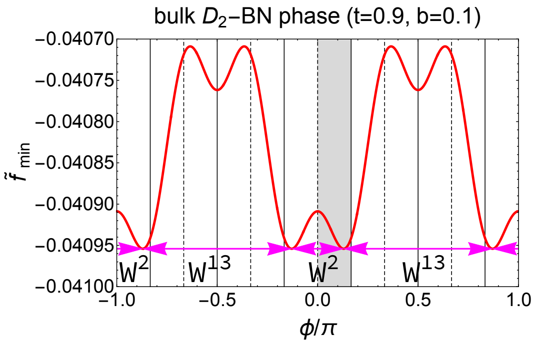
ドメインウォール

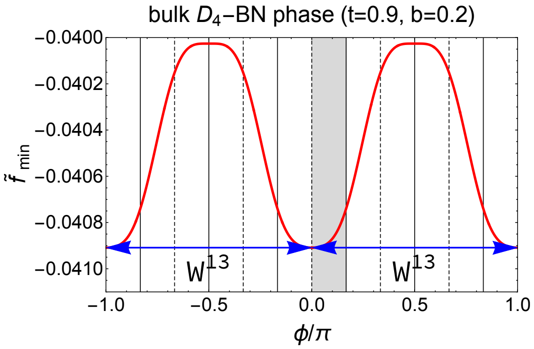
Surface energy density (summary picture in next page...)

bulk UN phase	$W^2(\mathrm{UN})$			$W^1(\mathrm{UN})$			$W^3(\mathrm{UN})$		
angle	$-1/6 \le (\phi \bmod \pi)/\pi < 1/6$			$1/6 \le (\phi \bmod \pi)/\pi < 1/2$			$1/2 \le (\phi \bmod \pi)/\pi < 5/6$		
direction	W^2_1	W^2_2	W^2_3	W^1_1	W^1_2	W^1_3	W^3_1	W^3_2	W^3_3
σ [keV/fm^2]	0.0154	0.0199	0.0154	0.0199	0.0154	0.0154	0.0154	0.0154	0.0199
bulk D$_2$-BN phase	$W^2(\mathrm{D_2BN})$			$W^{13}(\mathrm{D_2BN})$					
angle	$-0.129 \le (\phi \bmod \pi)/\pi < 0.129$			$0.129 \le (\phi \bmod \pi)/\pi < 0.870$					
direction	W^2_1	W^2_2	W^2_3	W^{13}_1		W^{13}_2		W^{13}_3	
σ [keV/fm^2]	0.0082	0.0107	0.0082	0.0722		0.0616		0.0722	
bulk D$_4$-BN phase	—			$W^{13}(\mathrm{D_4BN})$					
angle	—			$0 \le (\phi \bmod \pi)/\pi < 1$					
direction	—			W^{13}_1		W^{13}_2		W^{13}_3	
σ [keV/fm^2]	—			0.1533		0.1353		0.1533	

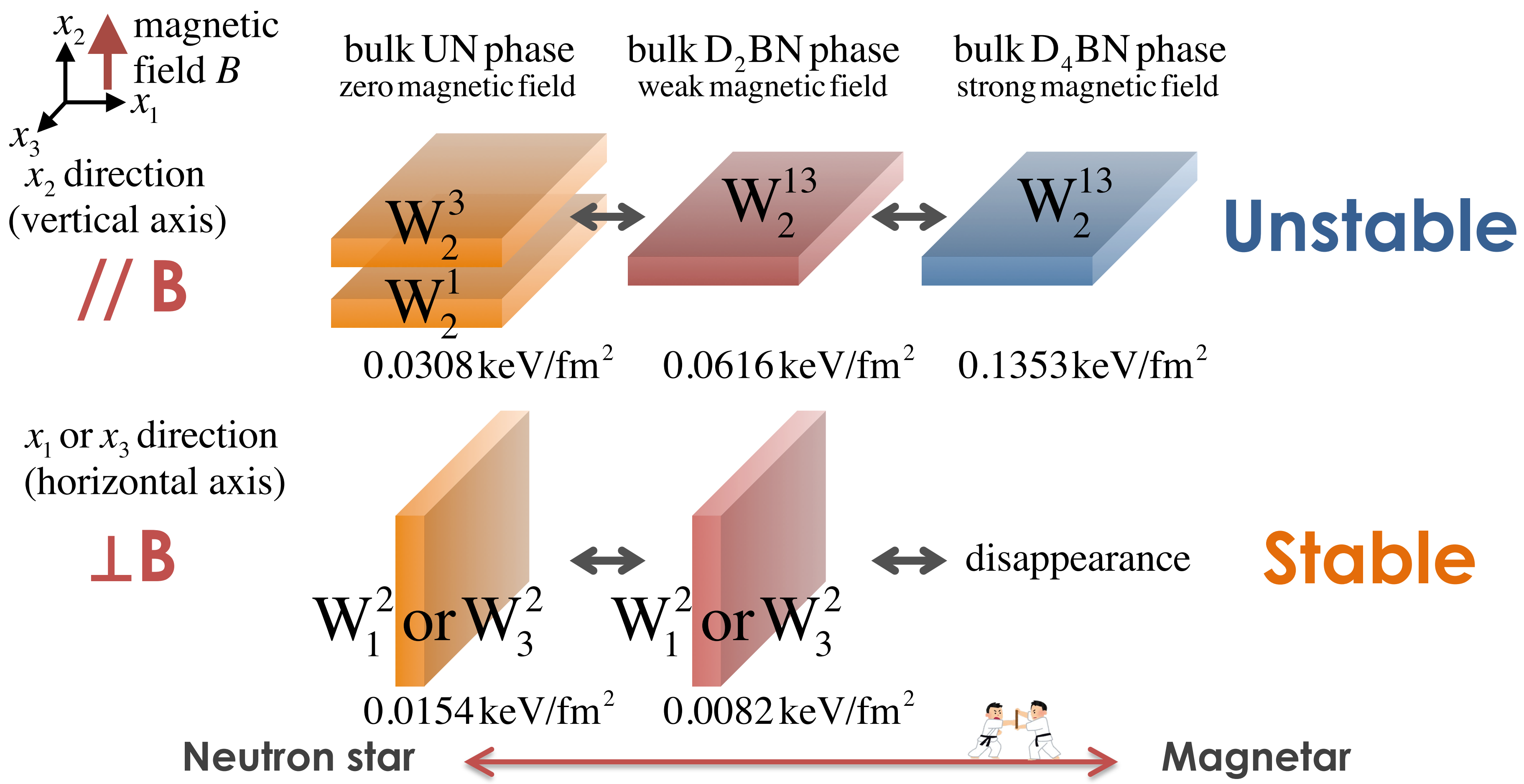
Neutron star ↔ **Magnetar**


bulk UN phase (t=0.9, b=0)


bulk D_2–BN phase (t=0.9, b=0.1)


bulk D_4–BN phase (t=0.9, b=0.2)

ドメインウォール



x_2 direction (vertical axis)

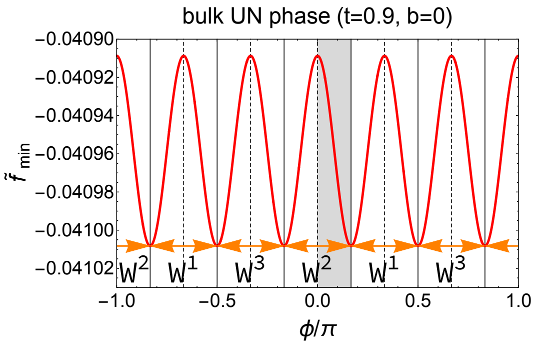
// B

x_1 or x_3 direction (horizontal axis)

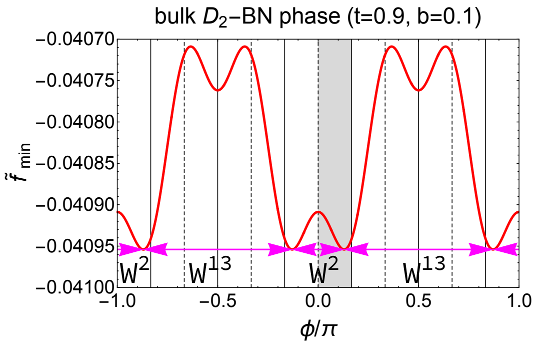
⊥B

	bulk UN phase zero magnetic field		bulk D_2BN phase weak magnetic field		bulk D_4BN phase strong magnetic field	
x_2 direction (vertical axis) // B	W_2^3, W_2^1 $0.0308\,\mathrm{keV/fm^2}$	↔	W_2^{13} $0.0616\,\mathrm{keV/fm^2}$	↔	W_2^{13} $0.1353\,\mathrm{keV/fm^2}$	**Unstable**
x_1 or x_3 direction (horizontal axis) ⊥B	W_1^2 or W_3^2 $0.0154\,\mathrm{keV/fm^2}$	↔	W_1^2 or W_3^2 $0.0082\,\mathrm{keV/fm^2}$	↔	disappearance	**Stable**

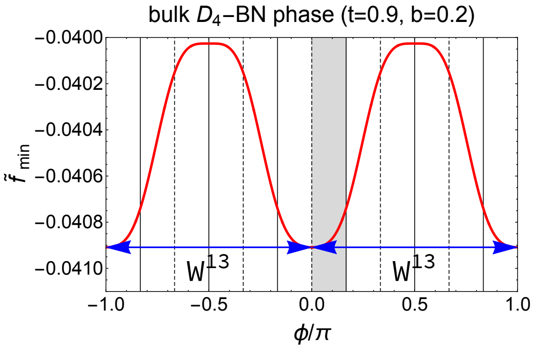
Neutron star ↔ **Magnetar**



bulk UN phase (t=0.9, b=0)

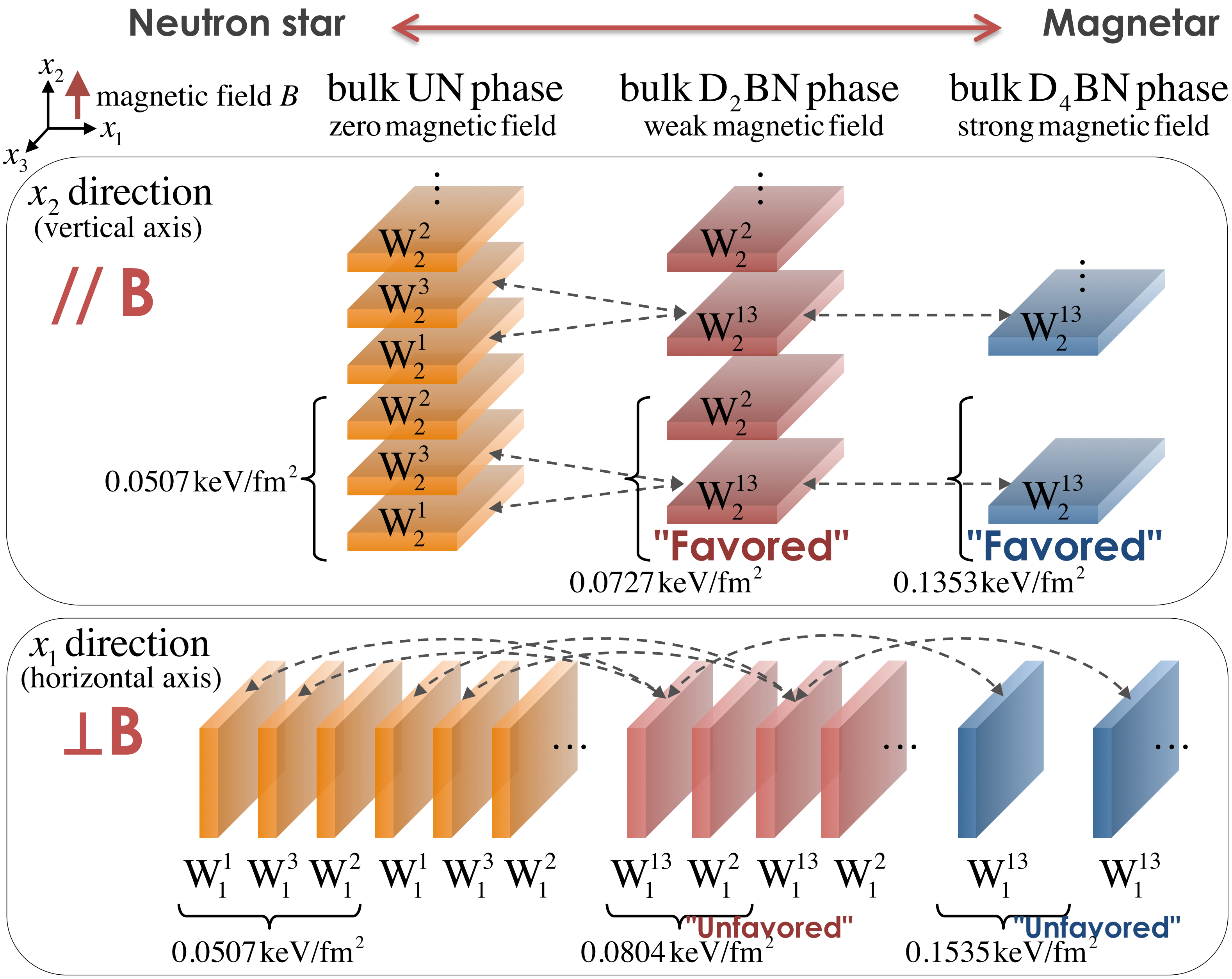


bulk D_2–BN phase (t=0.9, b=0.1)



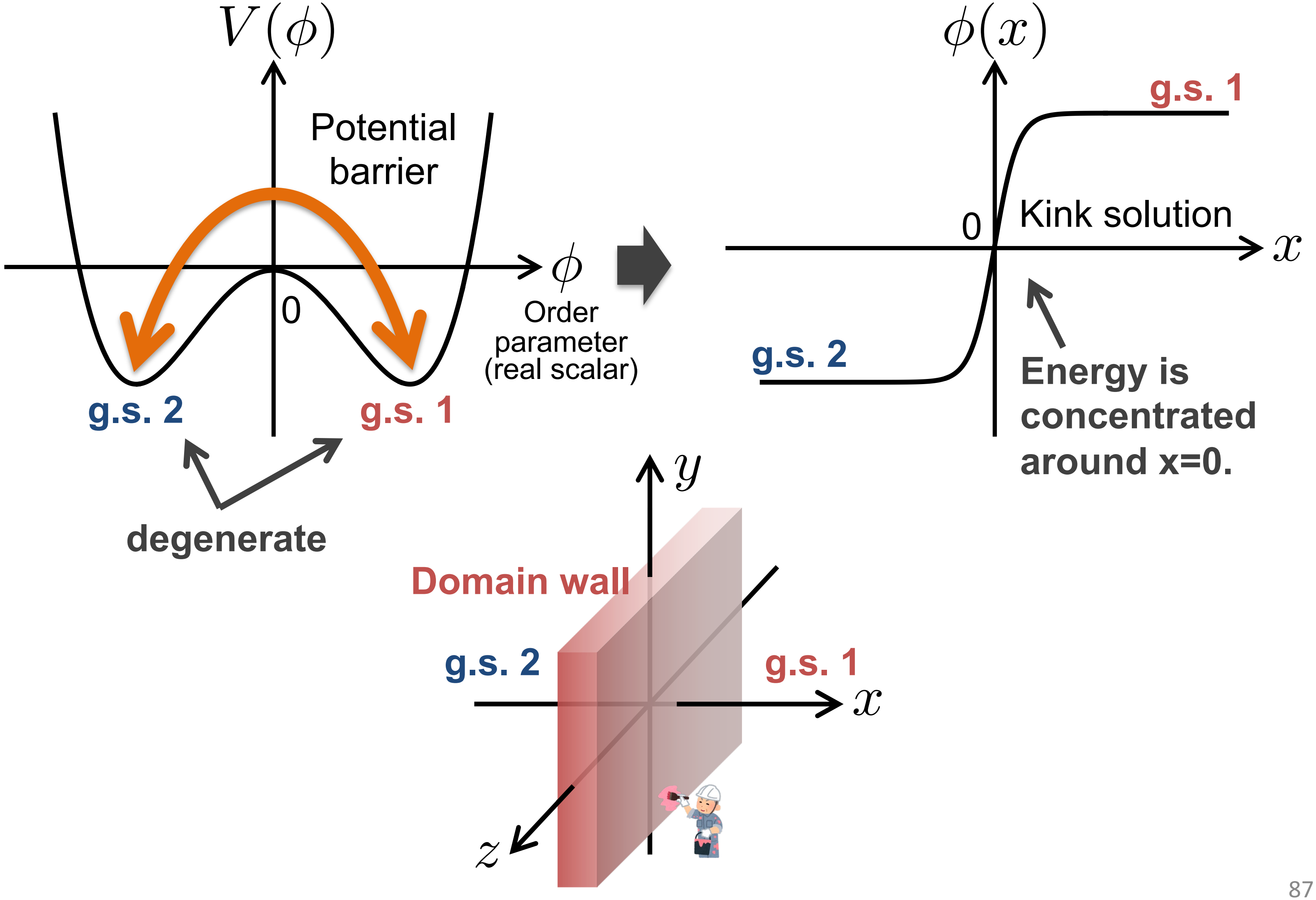
bulk D_4–BN phase (t=0.9, b=0.2)

ドメインウォール


Neutron star
Magnetar
x_2
x_1
x_3
magnetic field B
bulk UN phase
zero magnetic field
bulk D_2BN phase
weak magnetic field
bulk D_4BN phase
strong magnetic field
x_2 direction
(vertical axis)
// B
W_2^2
W_2^3
W_2^1
W_2^2
W_2^3
W_2^1
W_2^2
W_2^{13}
W_2^2
W_2^{13}
W_2^{13}
W_2^{13}
0.0507 keV/fm^2
"Favored"
0.0727 keV/fm^2
"Favored"
0.1353 keV/fm^2
x_1 direction
(horizontal axis)
⊥B
W_1^1 W_1^3 W_1^2 W_1^1 W_1^3 W_1^2
W_1^{13} W_1^2 W_1^{13} W_1^2
W_1^{13} W_1^{13}
0.0507 keV/fm^2
"Unfavored"
0.0804 keV/fm^2
"Unfavored"
0.1535 keV/fm^2

3.1 Domain wall

Domain wall: One-dimensional nonuniform solution connecting two different ground states (g.s.)



Energy is concentrated around x=0.

3.1 Domain wall

$^3P_2$ order parameter (spin × angular momentum)

Solution ansatz
(only diagonal)

$$A \propto \mathrm{diag}(f_1, f_2, f_3)$$

$$f_1 + f_2 + f_3 = 0$$

traceless condition

$$f_1 = \left(\frac{\cos\phi}{\sqrt{2}} - \frac{\sin\phi}{\sqrt{6}}\right) f_0, \quad f_2 = \sqrt{\frac{2}{3}}\left(\sin\phi\right) f_0, \quad f_3 = \left(-\frac{\cos\phi}{\sqrt{2}} - \frac{\sin\phi}{\sqrt{6}}\right) f_0$$

3.1 Domain wall

$^3P_2$ order parameter (spin × angular momentum)

Solution ansatz
(only diagonal)

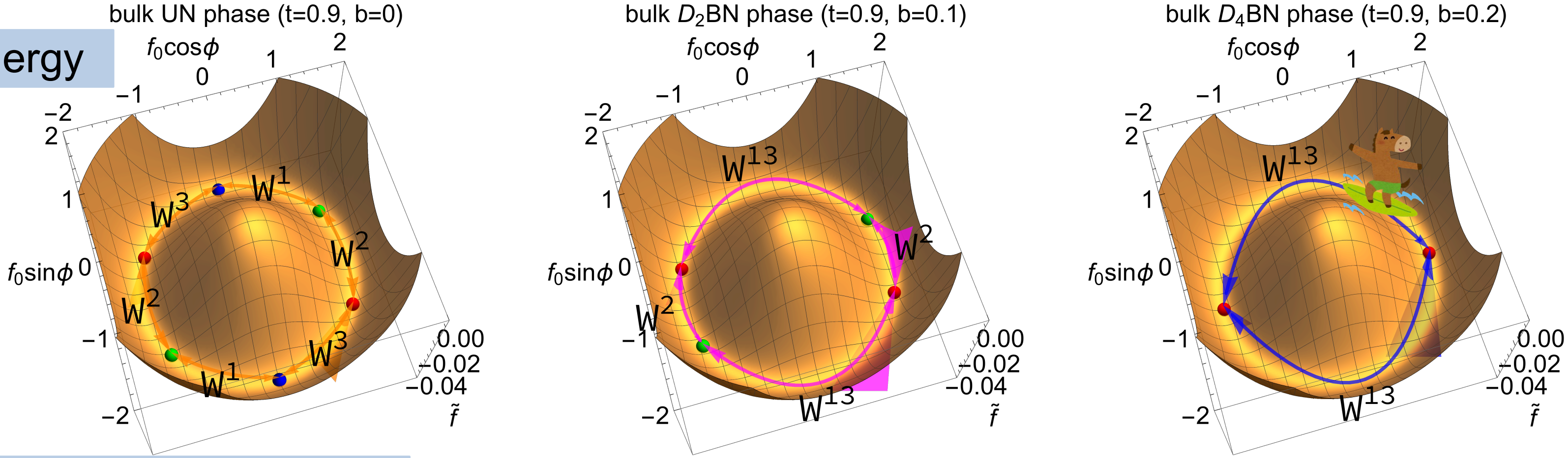
$$A \propto \mathrm{diag}(f_1, f_2, f_3)$$

$$f_1 + f_2 + f_3 = 0$$

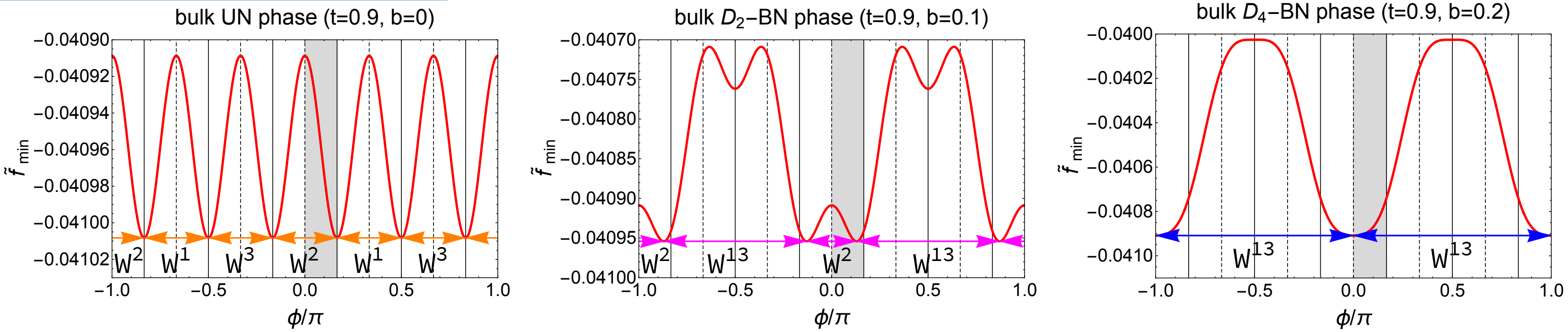
traceless condition

$$f_1 = \left(\frac{\cos\phi}{\sqrt{2}} - \frac{\sin\phi}{\sqrt{6}}\right) f_0, \quad f_2 = \sqrt{\frac{2}{3}}(\sin\phi) f_0, \quad f_3 = \left(-\frac{\cos\phi}{\sqrt{2}} - \frac{\sin\phi}{\sqrt{6}}\right) f_0$$

Free energy



Free energy (ϕ dependence)



Normal neutron star ↔ **Magnetar**

3.1 Domain wall

$^3P_2$ order parameter (spin × angular momentum)

Solution ansatz
(only diagonal)

$$A \propto \mathrm{diag}(f_1, f_2, f_3)$$

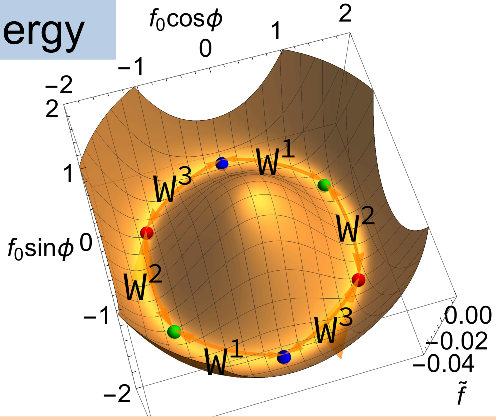
$$f_1 + f_2 + f_3 = 0$$

traceless condition

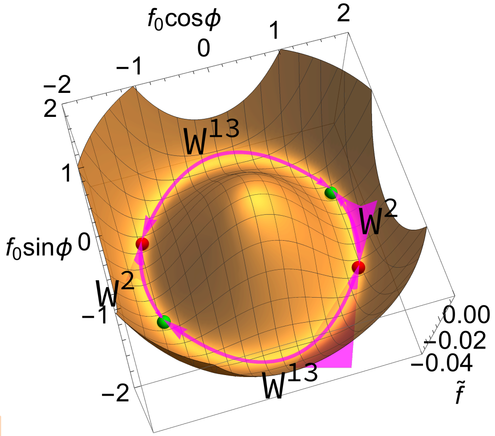
$$f_1 = \left(\frac{\cos\phi}{\sqrt{2}} - \frac{\sin\phi}{\sqrt{6}}\right) f_0, \quad f_2 = \sqrt{\frac{2}{3}}(\sin\phi) f_0, \quad f_3 = \left(-\frac{\cos\phi}{\sqrt{2}} - \frac{\sin\phi}{\sqrt{6}}\right) f_0$$

Free energy

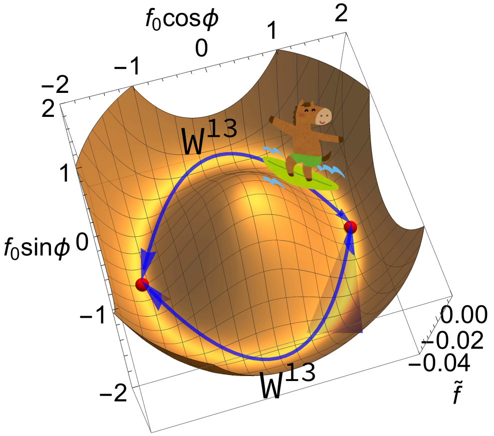
bulk UN phase (t=0.9, b=0)



bulk D_2BN phase (t=0.9, b=0.1)

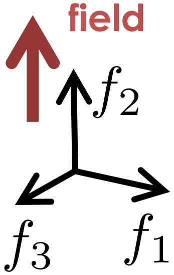


bulk D_4BN phase (t=0.9, b=0.2)

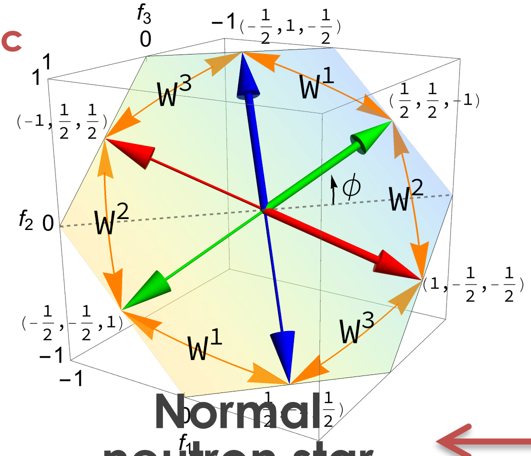


Order parameter: (f_1,f_2,f_3) vector

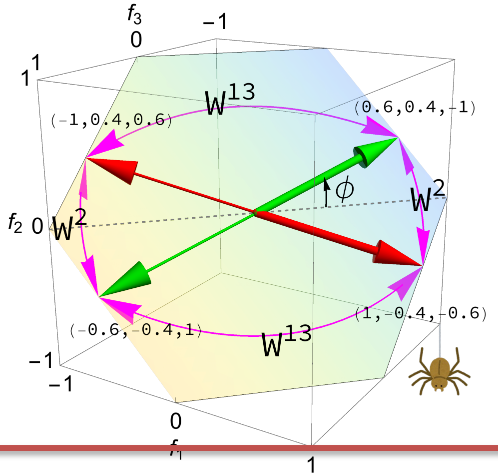
Magnetic field



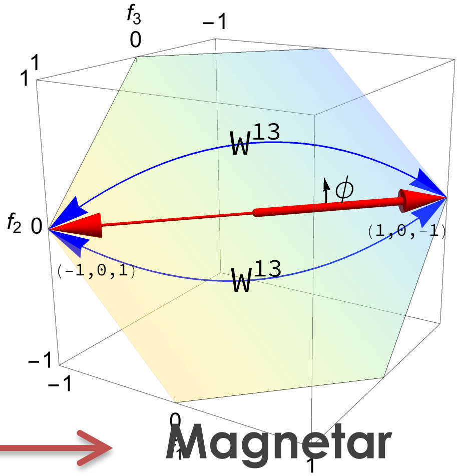
bulk UN phase (t=0.9,b=0)



bulk D_2−BN phase (t=0.9,b=0.1)



bulk D_4−BN phase (t=0.9,b=0.2)



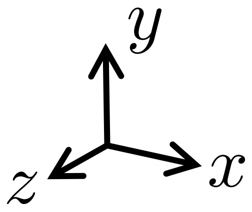
Normal neutron star ↔ **Magnetar**

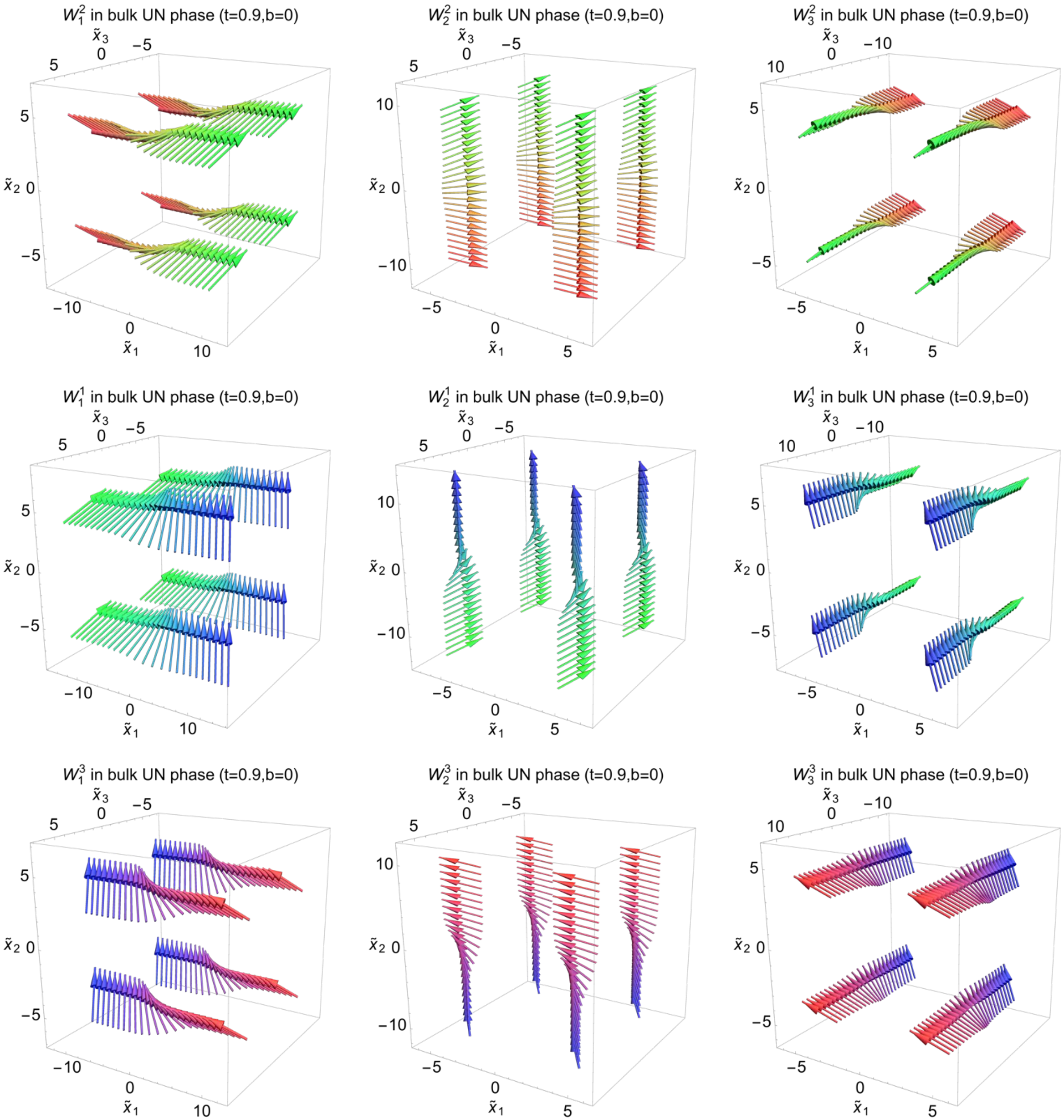
3.1 Domain wall

Domain wall formed by order parameter: (f_1,f_2,f_3) vector

UN phase

(normal neutron star)




W_1^2 in bulk UN phase (t=0.9,b=0)

W_2^2 in bulk UN phase (t=0.9,b=0)

W_3^2 in bulk UN phase (t=0.9,b=0)

W_1^1 in bulk UN phase (t=0.9,b=0)

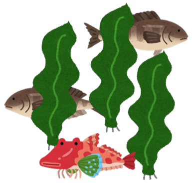
W_2^1 in bulk UN phase (t=0.9,b=0)

W_3^1 in bulk UN phase (t=0.9,b=0)

W_1^3 in bulk UN phase (t=0.9,b=0)

W_2^3 in bulk UN phase (t=0.9,b=0)

W_3^3 in bulk UN phase (t=0.9,b=0)

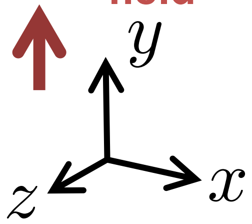


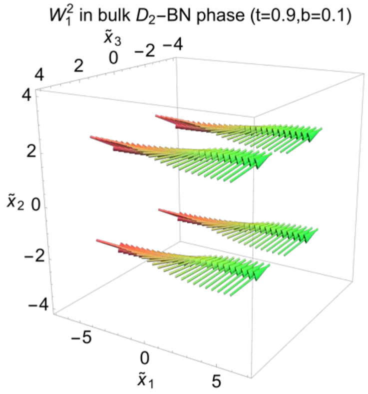
3.1 Domain wall

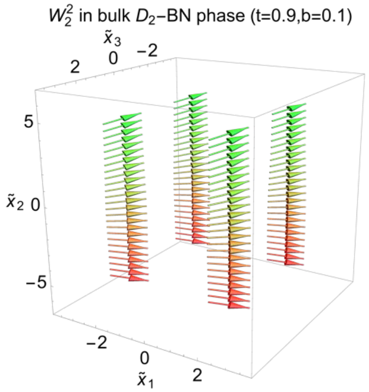
Domain wall formed by order parameter: (f_1,f_2,f_3) vector

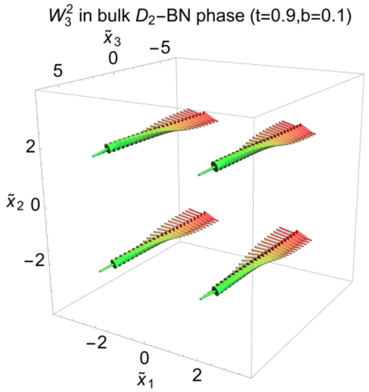
D_2-BN phase

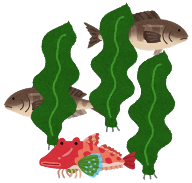
weak magnetic field

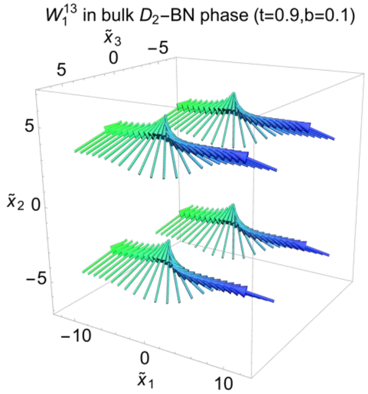


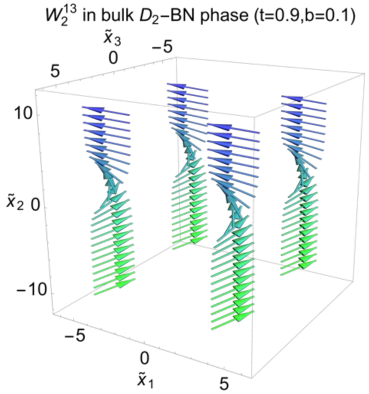

W_1^2 in bulk D_2–BN phase (t=0.9,b=0.1)

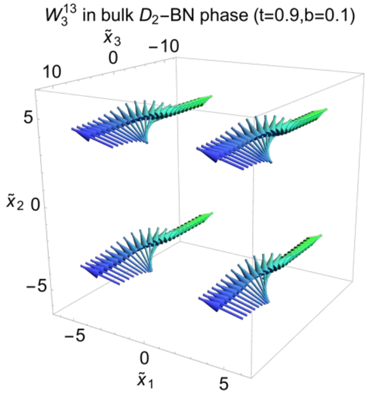

W_2^2 in bulk D_2–BN phase (t=0.9,b=0.1)


W_3^2 in bulk D_2–BN phase (t=0.9,b=0.1)




W_1^{13} in bulk D_2–BN phase (t=0.9,b=0.1)

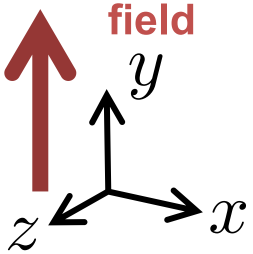

W_2^{13} in bulk D_2–BN phase (t=0.9,b=0.1)

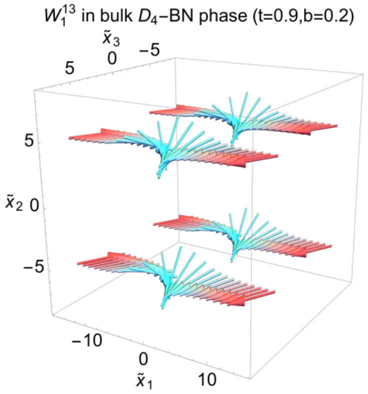

W_3^{13} in bulk D_2–BN phase (t=0.9,b=0.1)

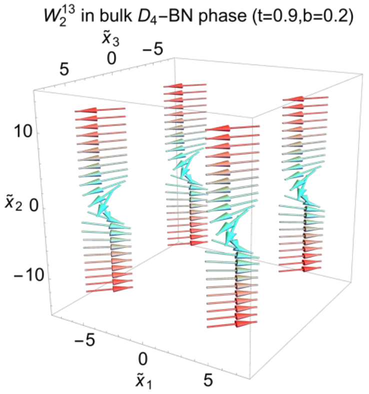
D_4-BN phase

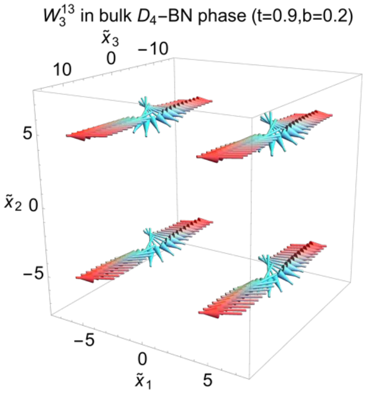
(Magnetar)

Strong magnetic field

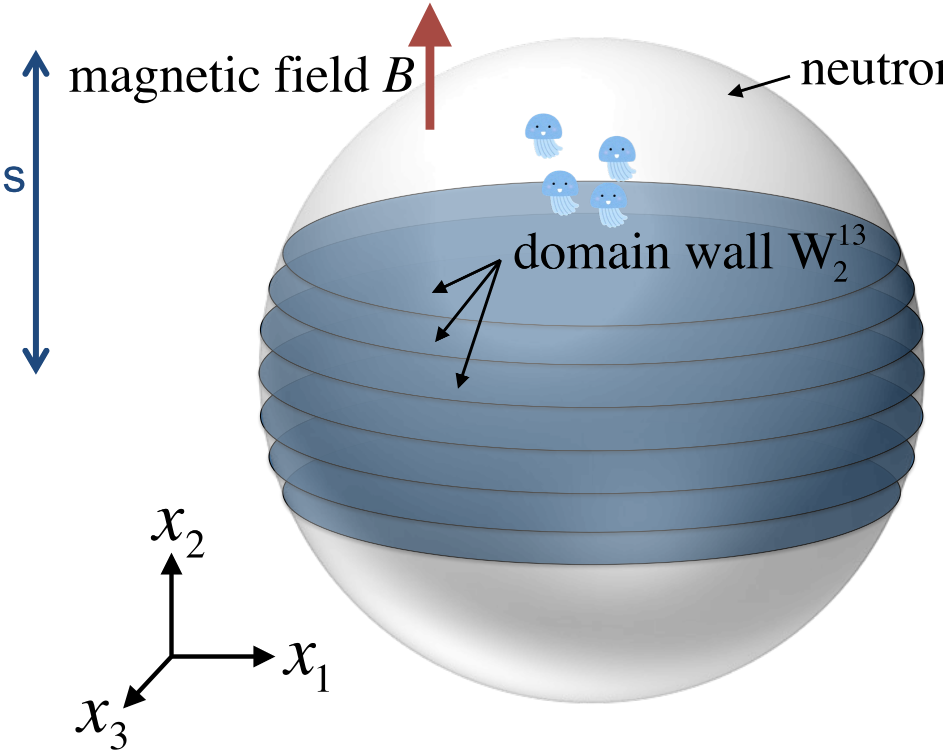



W_1^{13} in bulk D_4–BN phase (t=0.9,b=0.2)


W_2^{13} in bulk D_4–BN phase (t=0.9,b=0.2)

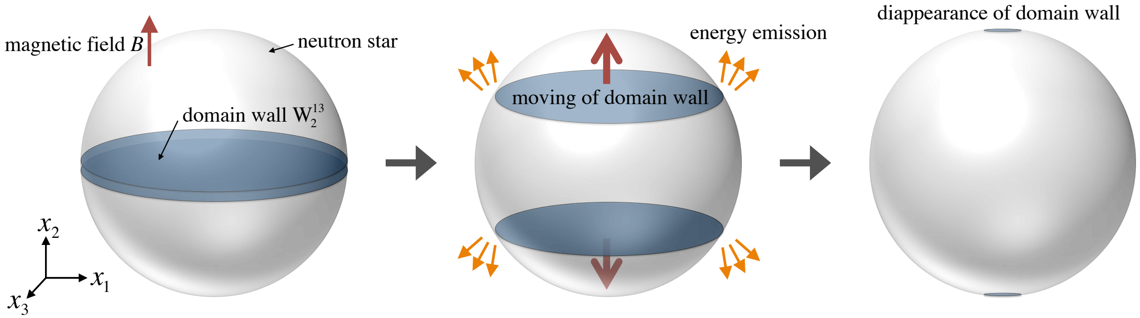

W_3^{13} in bulk D_4–BN phase (t=0.9,b=0.2)

3.1 Domain wall



Neutron star radius
～10km

Coherence length
～300 fm



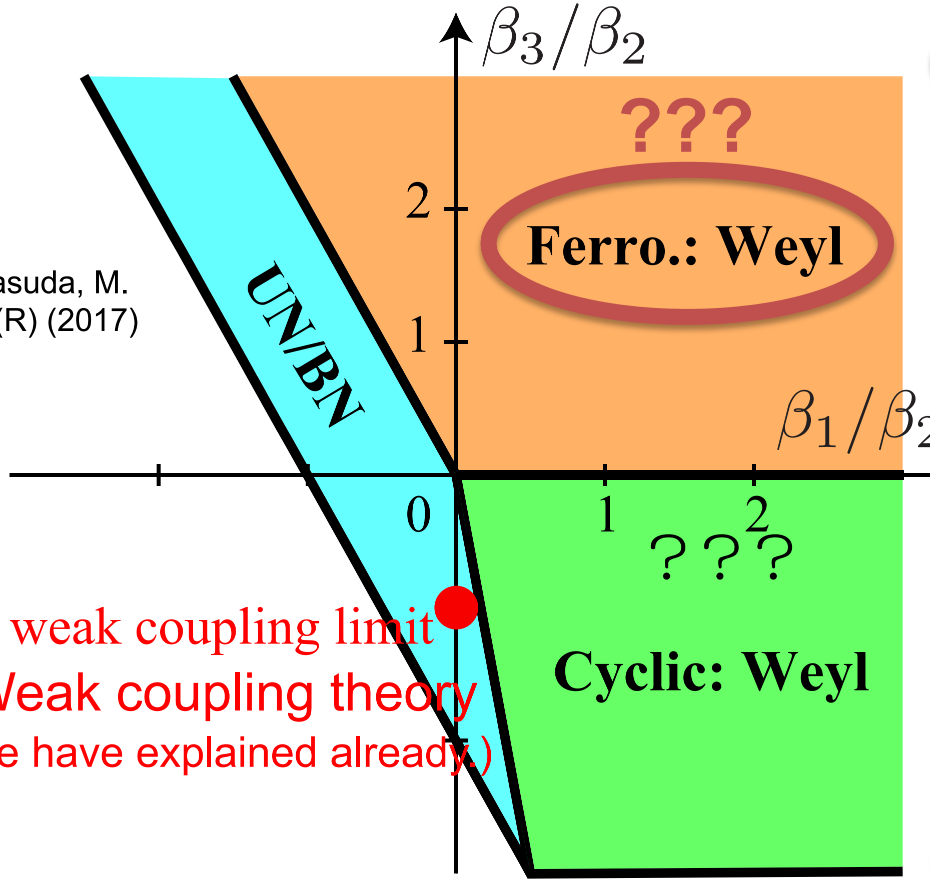
Released energy~ 10^{45} erg !

3.3 Ferromagnetic phase

Revisiting $^3P_2$ phase diagram by Ginzburg-Landau theory

Cf. V. Z. Vulovic, J. A. Sauls, PRD29, 2705 (1984)

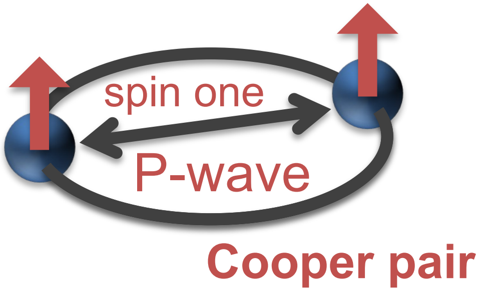
Taken from . Mizushima, K. Masuda, M. Nitta, Phys. Rev. B95, 140503(R) (2017)



weak coupling limit

Weak coupling theory

(we have explained already.)



β_1, β_2, β_3: Coefficients in GL theory

Important for magnetars!

So far we have discussed UN/BN phases.

Does *ferromagnetic* phase exist?

3.3 Ferromagnetic phase

What does cause spin polarization?

① Strong coupling

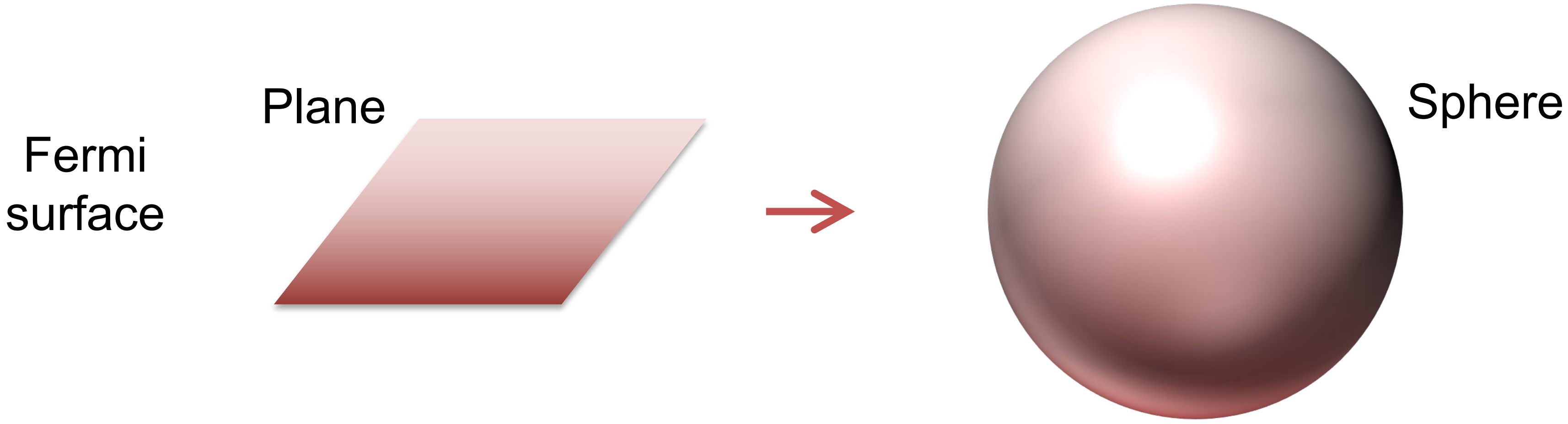
J. A. Sauls, J. W. Serene, PRD17, 1524 (1978),
V. Z. Vulovic, J. A. Sauls, PRD29, 2705 (1984),
D. N. Voskresensky, PRD101, 056011 (2020)

② Violation of particle-hole symmetry

T. Mizushima, D. Inotani, S. Yasui, M. Nitta , PRC104, 045803 (2021)

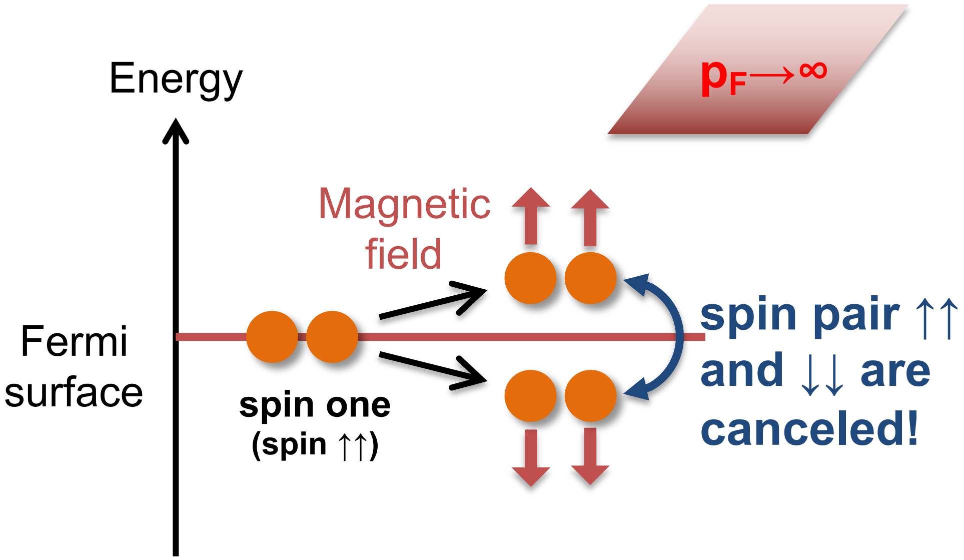
Fermi momentum (p_F) → ∞

Fermi momentum (p_F) : finite



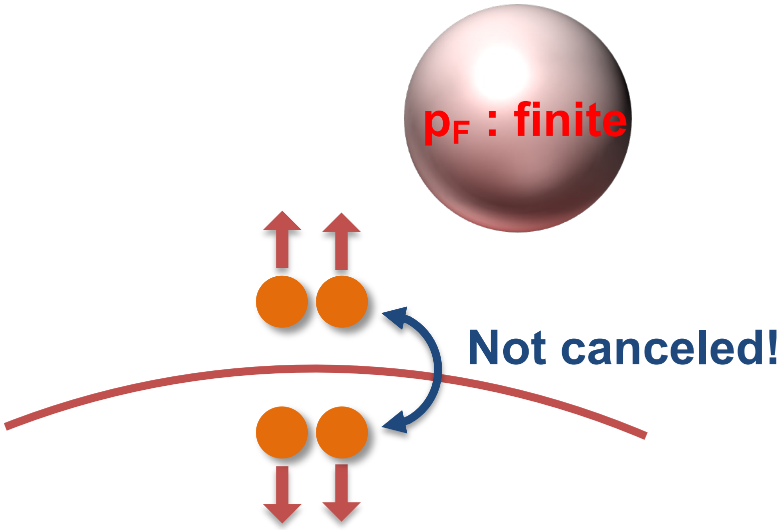
3.3 Ferromagnetic phase

What does cause spin polarization?



spin pair ↑↑ and ↓↓ are canceled!

↑↑ phase space = ↓↓ phase space



Not canceled!

↑↑ phase space > ↓↓ phase space

Fermi surface curvature can induce spin polarization.

3.3 Ferromagnetic phase

$^3P_2$ order parameter (spin × momentum)

Phase	O.P. [see Eq. (28)]	H	$R = G/H$	$\pi_1(R)$	$\#_{\mathrm{NG}}$	$\#_{\mathrm{qNG}}$ [66]
Uniaxial nematic	$r = -1/2$, $\kappa = 0$	$D_\infty \simeq \mathrm{O}(2)$	$\mathrm{U}(1) \times \mathbb{R}P^2$	$\mathbb{Z} \oplus \mathbb{Z}_2$ [43, 67]	3	2
Biaxial nematic	$r \in (-1, -1/2)$, $\kappa = 0$	D_2	$\mathrm{U}(1) \times \mathrm{SO}(3)/D_2$	$\mathbb{Z} \oplus \mathbb{Q}$ [43, 67]	4	1
	$r = -1$, $\kappa = 0$	D_4	$[\mathrm{U}(1) \times \mathrm{SO}(3)]/D_4$	$\mathbb{Z} \times_h D_4^*$ [43, 44, 65]	4	1
Cyclic	$r = e^{i2\pi/3}$, $\kappa = 0$	T	$[\mathrm{U}(1) \times \mathrm{SO}(3)]/T$	$\mathbb{Z} \times_h T^*$ [65, 68–70]	3	—
Magnetized biaxial nematic	$r \in (-1, -1/2)$, $\kappa \in (0, 1)$	0	$\mathrm{SO}(3) \times \mathrm{U}(1)$	$\mathbb{Z}_2 \oplus \mathbb{Z}$	4	—
	$r = -1$, $\kappa \in (0, 1)$	C_4	$[\mathrm{U}(1) \times \mathrm{SO}(3)]/\mathbb{Z}_4$	$\mathbb{Z} \times_h C_4^*$	4	—
Ferromagnetic	$r = -1$, $\kappa = 1$	$\mathrm{U}(1)_{J_z+2\Phi}$	$\mathrm{SO}(3)_{J_z-2\Phi}/\mathbb{Z}_2$	$\mathbb{Z}_4$ [69, 71]	3	—
	Eq. (26)	$\mathrm{U}(1)_{J_z+\Phi}$	$\mathrm{SO}(3)_{J_z-\Phi}/\mathbb{Z}_2$	$\mathbb{Z}_4$ [69, 71]	3	—

Magnetized biaxial nematic (MBN) $\xrightarrow{\kappa \to 1}$ Ferromagnetic (FM)

$$\mathcal{A}_{\mu i} = \Delta \begin{pmatrix} 1 & \boxed{i\kappa} & 0 \\ \boxed{i\kappa} & r & 0 \\ 0 & 0 & -1-r \end{pmatrix}_{\mu i}$$

$\kappa \in (0, 1)$ New parameter (off-diagonal)

$$\mathcal{A}^{\mathrm{FM}}_{\mu i} = \Delta \begin{pmatrix} 1 & \pm i & 0 \\ \pm i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{\mu i}$$

$\kappa = 1$

$^3P_2$ Cooper pair spin polarization

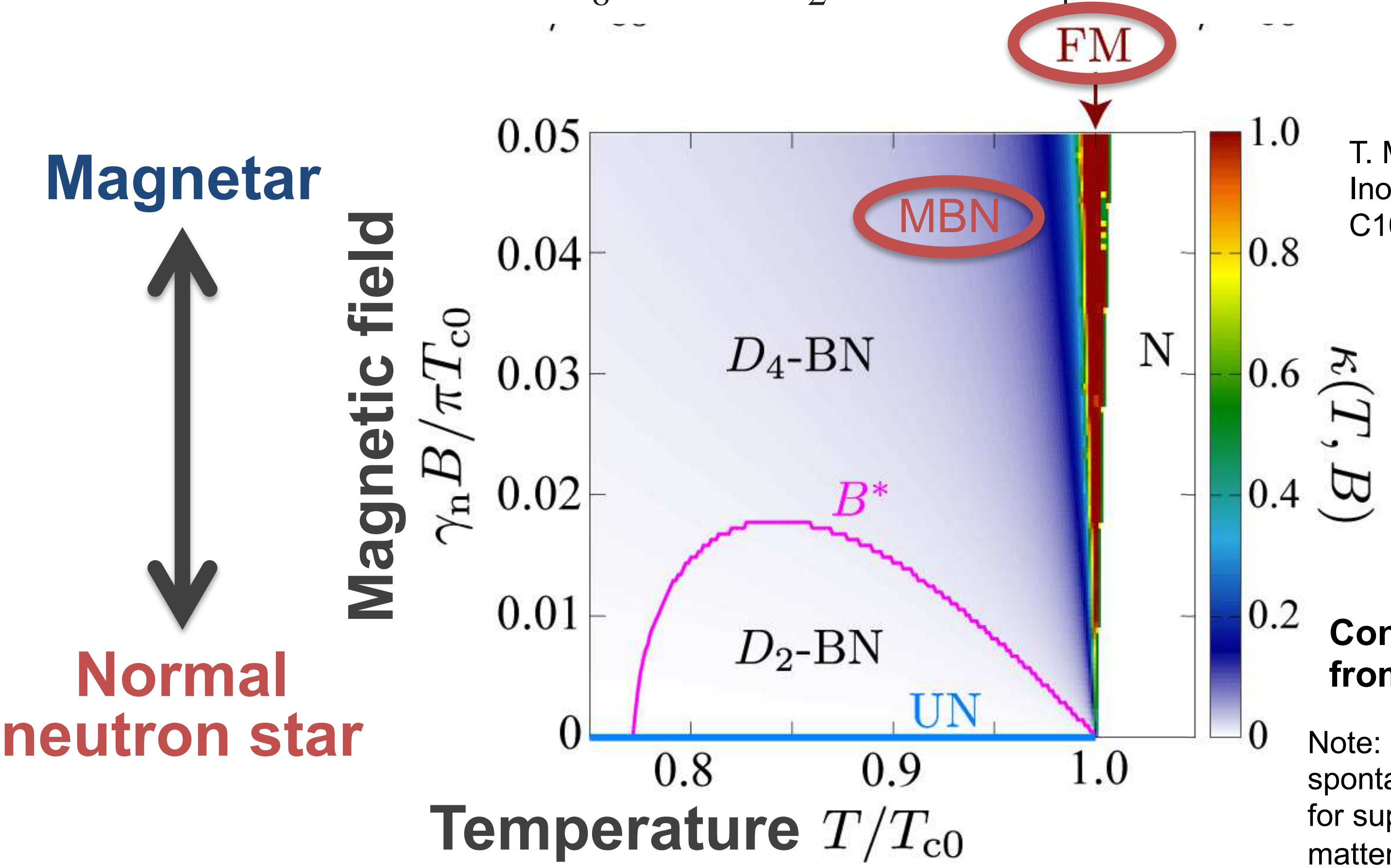
$$\langle S^z_{\mathrm{pair}} \rangle = 2\kappa(1-r)\Delta^2/3$$

κ≠0 → Spin polarization

3.3 Ferromagnetic phase

Ginzburg-Landau theory

$$f(\tau) = f_8^{(0)}(\tau) + f_2^{(\le 4)}(\tau) + f_4^{(\le 2)}(\tau) \qquad \tau_{\mu i} = \tau_0 \begin{pmatrix} r & \boxed{i\kappa} & 0 \\ \boxed{i\kappa} & 1 & 0 \\ 0 & 0 & -1-r \end{pmatrix}_{\mu i}$$



Magnetar

Normal neutron star

T. Mizushima, S. Yasui, D. Inotani, M. Nitta, Phys. Rev. C104, 045803 (2021)

Consistent with result from BdG equation

Note: "Ferromagnetism" is not spontaneous. This is a term used for superfluids in condensed matter physics.

Magnetized biaxial-nematic (MBN)/Ferromagnetic (FM)

Possible to observe in magnetars？

Glossary of basic terms

Spin (s) Intrinsic degrees of freedom in a particle. Spin 0 particle has only one component. Spin ½ particle has two components, $+\frac{1}{2}$ and $-\frac{1}{2}$, each of which is called up (↑) and down (↓) components. Spin 1 particle has three components, $+1$, 0, -1.

Fermion A particle with half-integer spin, e.g., $1/2$. One fermion can occupy one state only, and hence two fermions cannot occupy the same state. This is called the Pauli exclusion principle.

Boson A particle with integer spin, e.g., 0 or 1. (Infinitely) many boson can occupy one state. There is no exclusion rule for bosons.

Total spin (S) A sum of the spins of n particles: $\vec{S} = \vec{s}_1 + \vec{s}_2 + \cdots + \vec{s}_n$. In case of $n = 2$, for example, we consider a pair of fermions, each of which fermion has spin $s_1 = s_2 = 1/2$. In this case, we have either $S = 0$ or $S = 1$, each of which is called a *singlet* state and a *triplet* state.

Angular momentum (L) A discretized angular momentum in quantum mechanics: $L = 0, 1, 2, \ldots$ whose wave functions are given by harmonic spherical functions $Y_{LL_z}(\theta, \varphi)$ with $L_z = -L, -L + 1, \ldots, L - 1, L$ for polar coordinate (θ, φ) in three dimensional space. $L = 0, 1, 2, \ldots$ are called S-wave, P-wave, D-wave, …

Total angular momentum (J) A sum of the angular momentum (L) and the total spin (S): $\vec{J} = \vec{L} + \vec{S}$. For example, if $L = 1$ and $S = 1$, then $J = 0, 1, 2$. The state with J, L and S is denoted by ${}^{2S+1}L_J$ where L are conventionally denoted by $L = S, P, D, \ldots$ instead of $L = 0, 1, 2, \ldots$.

Proton (p) A composite fermion composed of three quarks (u,u,d). The spin of a proton is ½. A proton has an electric charge (+1), and a finite magnetic moment.

Neutron (n) A composite fermion composed of three quarks (u,d,d). The spin of a neutron is ½. A neutron has no electric charge, but has a finite magnetic moment.

Glossary of basic terms (continued)

Nucleon (N) Generic name indicating either proton and neutron, or both, often denoted by $N = \binom{p}{n}$.

Deuteron (d) A bound state formed by a proton and a neutron. The total spin is $S = 1$, and the angular momentum is $L = 0$ (S-wave). Their combination (channel) is denoted by ${}^1S_0$.

Nuclear force (potential) An interaction between two (or more) nucleons. The nuclear force can be attractive or repulsive, depending on the spin and angular momentum between two (and more) nucleons. It can make a bound state of nucleons, called an atomic nucleus.

LS force (potential) A nuclear force acting at higher energy scattering between two nucleons. It is proportional to the inner product of the angular momentum (L) and the total spin (S), $\propto \vec{L} \cdot \vec{S}$. Either attraction or repulsion is dependent on the channels: attractive for $J = 2$ and repulsive for $J = 0$ and 1. The high-energy scattering of neutrons gives an attraction in ${}^3P_2$ channel with $L = 1$ (P-wave) and $S = 1$ (triplet).

Bardeen-Cooper-Schrieffer (BCS) theory One of the most basic theory for superconductivity and superfluidity. A pair of fermion (the Cooper pair) near the Fermi surface makes a bosonic-state, and cause the quantum transition analogous to the Bose-Einstein condensate. As a result, a gap appears in the energy-momentum dispersion relations of the fermions. The original phase transformation, i.e., $\mathrm{U}(1)$ symmetry, is broken in the ground state due to the gap formation. This is one of the well-known examples of the spontaneous symmetry breaking.

Bogoliubov-de-Gennes (BdG) theory General theory for superconductivity and superfluidity, which includes the nonuniform wave-functions of fermions and be applicable to vortices for example.

Ginzburg-Landau (GL) theory A pair of fermions behave like a boson. The GL theory is obtained from the BdG theory by taking the fermionic degrees of freedom into account (integrating-out) in terms of the bosonic degrees of freedom.