Strong magnetic fields and the inner crust of neutron stars

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Strongly interacting matter in extreme magnetic fields

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FCT Fundação para a Ciência e a Tecnologia
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Neutron stars

check eg N. K. Glendenning, Compact Stars: Nuclear Physics, Particle Physics, and General Relativity (Springer, 2000)

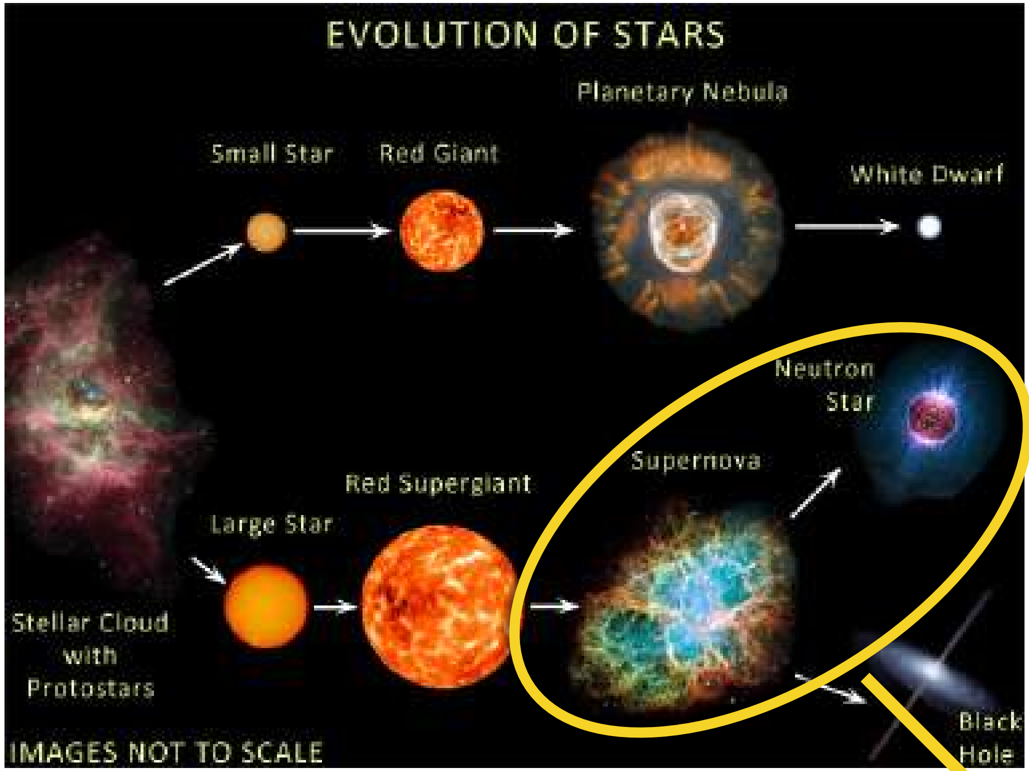
R~10 km; M~1.5 M$\odot$

- Constituted by **catalized cold stellar matter**
- Central densities can reach several ρ_0
- Strongly asymmetric matter, very neutron-rich
- Onion-like structured objects with:

- Surface: ^{56}Fe, P=0
- Outer crust: Neutron rich nuclei embedded in electron sea
- Inner crust: Above neutron drip density, nucleons form geometrical structures (non-spherical: **pasta phases**) embedded in neutron and electron background gas.
- Core: Uniform matter, in the centre exotic matter may exist.

Where do these heavy clusters form?

in http://essayweb.net/astronomy/blackhole.shtml



in https://www.ligo.org/detections/GW170817.php
Credit: Soares-Santos et al. and DES Collab

NS mergers

scenarios where these clusters are important:
supernovae, NS mergers, (crust of) neutron stars

Why are these clusters important?

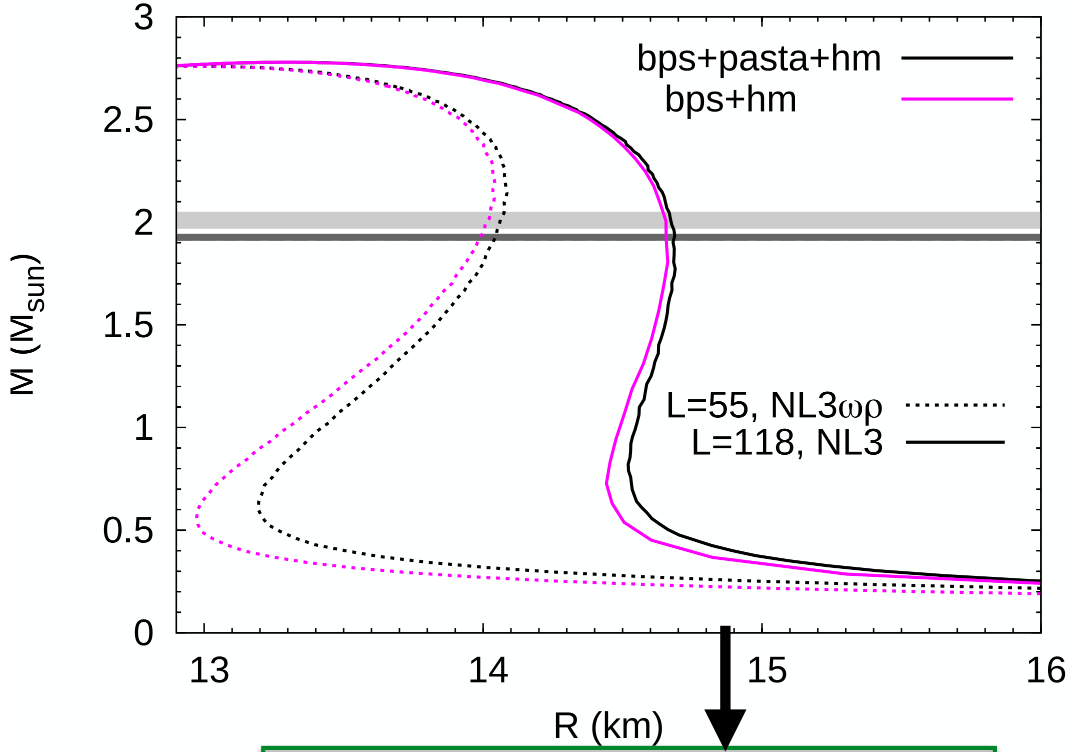
- They influence supernova properties: the clusters can modify the neutrino transport, affecting the cooling of the proto-neutron star and/or binary and accreting systems.
- They may be essential to describe the glitch mechanism. (sudden change in star's rotation)

- Magnetars (neutron stars with very strong magnetic fields, $\approx 10^{15} G$ at the surface) may have an inner crust even more complex, as we will see.

- They do have an effect in R (which then will be reflected in other properties such as the tidal deformability):

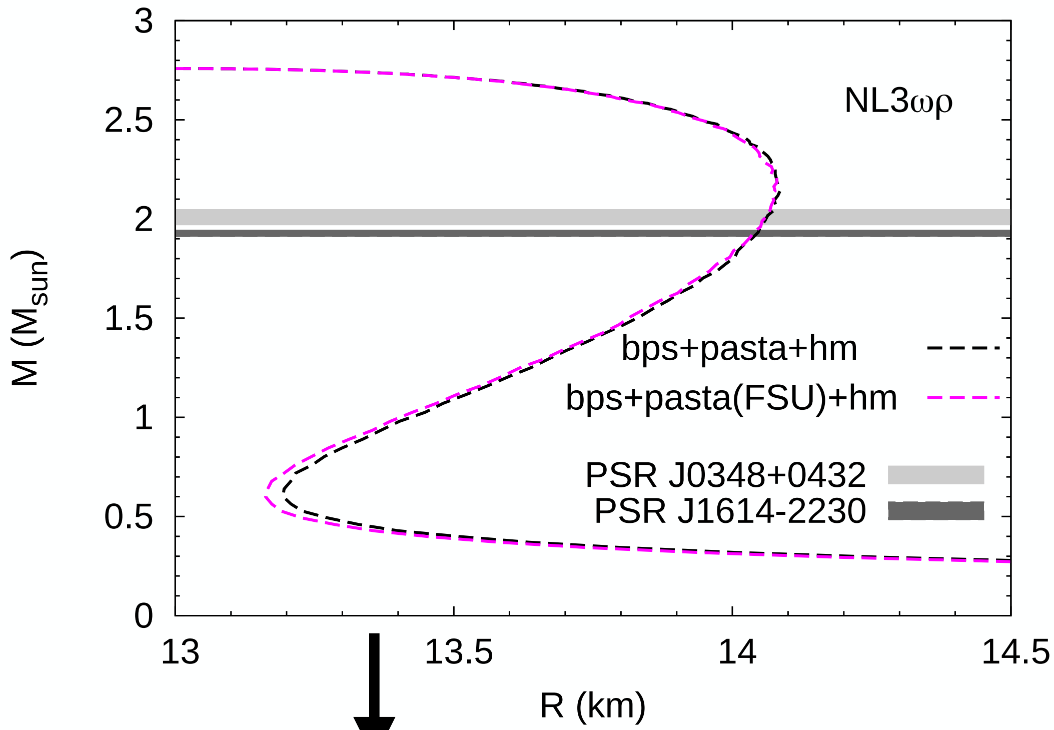
PRC 94, 015808 2016

a) effect of pasta:



No effect on Mmax, but effect on the radius!

b) effect of different inner crust EoS with L close to core EoS:



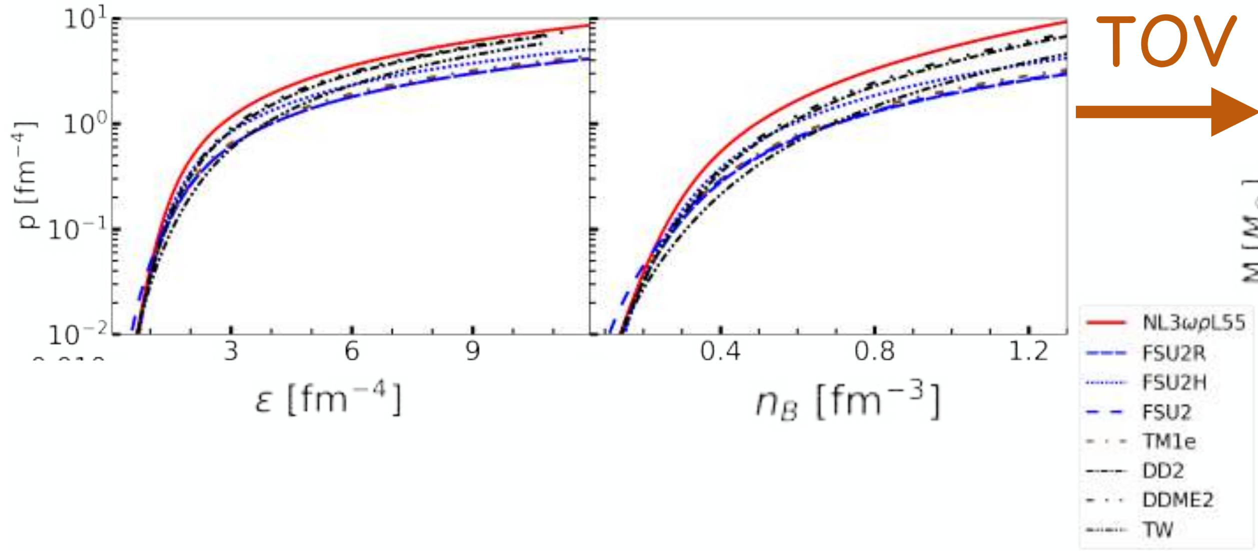
The error on the determination of the radius is negligible for all masses.

For **1.4M⊙ stars**, the RMF models that passed the **experimental and observational constrains** predict **R=13.6 ± 0.3 km,** with a crust thickness of **ΔR=1.36 ± 0.06km**.

EoS and Constraints

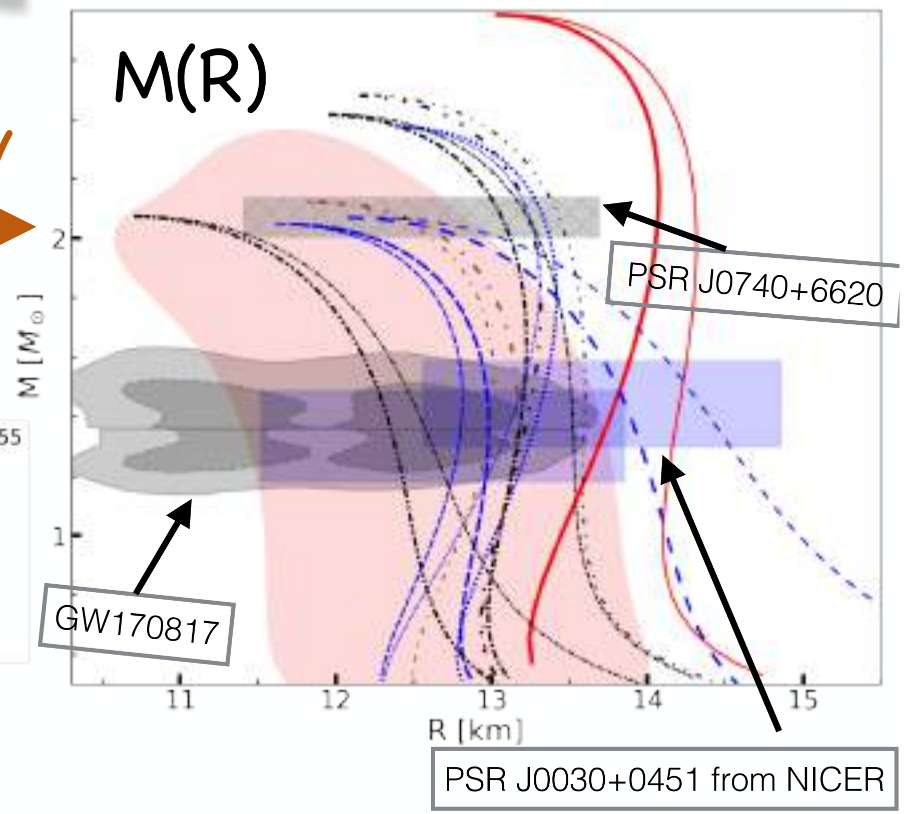
Malik and Pais, EPJA 58, 154 (2022)

EoS



TOV

M(R)



Many EoS models in literature, like e.g. phenomenological models, whose parameters are fitted to nuclei properties, such as **RMF**, or **Skyrme.**

check CompOSE:
https://compose.obspm.fr/

Solution: Need Constraints (Experiments, Observations, Microscopic calculations)

Magnetars

- Online catalogue: http://www.physics.mcgill.ca/~pulsar/magnetar/main.html.

Artist's impression of the magnetar in star cluster Westerlund 1. (Image credit: ESO/L. Calçada)

- Neutrons stars with very strong magnetic fields (B)
- At the surface, B up to $10^{14} \sim 10^{15}$ G
- Long spin periods, 2 $\sim$ 20 s.
- About 30 objects observed.

In this talk:

- Inner crust EoS under strong B within a RMF framework
- Compute crust via CP, CLD, and dynamical spinodal approaches
- Results (B-field cause extension of inner crust)

Theoretical Framework

Non-linear Walecka Model

mesons: mediation of nuclear force

$$\mathcal{L} = \sum_{i=p,n} \mathcal{L}_i + \mathcal{L}_e + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{\omega\rho} + \mathcal{L}_A$$

nucleons — electrons — mesons — non-linear mixing coupling — electromagnetic

$$\mathcal{L}_i = \bar{\psi}_i \left[\gamma_\mu i D^\mu - M^*\right] \psi_i$$

$$\mathcal{L}_e = \bar{\psi}_e \left[\gamma_\mu \left(i\partial^\mu + eA^\mu\right) - m_e\right] \psi_e$$

with $M^* = M - g_s\phi,$

$$\mathcal{L}_\sigma = \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2 - \frac{1}{3}\kappa\phi^3 - \frac{1}{12}\lambda\phi^4\right)$$

$$\mathcal{L}_\omega = -\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_v^2 V_\mu V^\mu + \frac{1}{4!}\xi g_v^4 (V_\mu V^\mu)^2$$

$$\mathcal{L}_\rho = -\frac{1}{4}\mathbf{B}_{\mu\nu}\cdot\mathbf{B}^{\mu\nu} + \frac{1}{2}m_\rho^2 \boldsymbol{b}_\mu \cdot \boldsymbol{b}^\mu$$

non-linear mixing coupling term:
responsible for density dependence of Esym

$$\mathcal{L}_{\omega\rho} = g_{\omega\rho} g_\rho^2 g_v^2 V_\mu V^\mu \mathbf{b}_\nu \cdot \mathbf{b}^\nu$$

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

	B/A (MeV)	ρ_0 (fm^{-3})	M^*/M	K (MeV)	$\mathcal{E}_{\rm sym}$ (MeV)	L (MeV)
NL3	16.24	0.148	0.60	270	37.34	118
NL3$\omega\rho$	16.24	0.148	0.60	270	31.66	55

Non-linear Walecka Model (cont)

We always consider $A^\mu = (0, 0, Bx, 0)$ and we define $B^* = B/B_e^c$, with
$B_e^c = 4.414 \times 10^{13}\,\mathrm{G}$

- The vector densities are given by

$$\rho_p = \frac{q_p B}{2\pi^2} \sum_{\nu=0}^{\nu_{\max}^p} g_s k_{F,\nu}^p\,,$$

$$\rho_n = \frac{k_F^{n\,3}}{3\pi^2}\,,$$

$$\rho_e = \frac{|e| B}{2\pi^2} \sum_{\nu=0}^{\nu_{\max}^e} g_s k_{F,\nu}^e\,,$$

with

$$\nu_{\max}^e = \frac{E_F^{e2} - m_e^2}{2|q_e|B}\,,$$

$$\nu_{\max}^p = \frac{E_F^{p2} - M^{*2}}{2 q_p B}\,.$$

the maximum number of Landau levels, for which the square of the Fermi momentum of the particle is still positive.

- And the bulk energy density by $\mathcal{E} = \mathcal{E}_f + \mathcal{E}_p + \mathcal{E}_n\,,$ with

$$\mathcal{E}_f = \frac{m_\omega^2}{2} V_0^2 + \frac{\xi g_v^4}{8} V_0^4 + \frac{m_\rho^2}{2} b_0^2 + \frac{m_\sigma^2}{2} \phi_0^2 + \frac{\kappa}{6} \phi_0^3 + \frac{\lambda}{24} \phi_0^4 + 3\lambda_{\omega\rho} g_\rho^2 g_\omega^2 V_0^2 b_0^2\,,$$

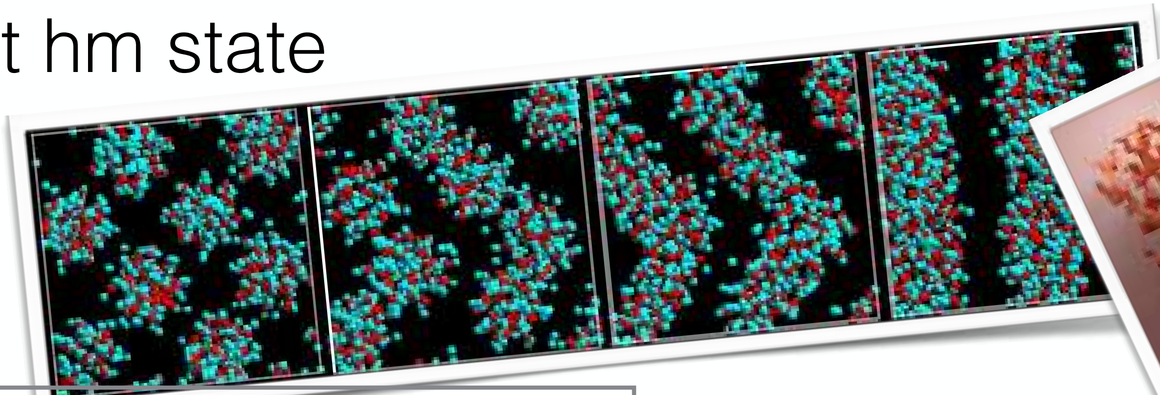
$$\mathcal{E}_n = \frac{1}{4\pi^2} \left[k_F^n E_F^{n3} - \frac{1}{2} M^* \left(M^* k_F^n E_F^n + M^{*3} \ln \left| \frac{k_F^n + E_F^n}{M^*} \right| \right) \right],$$

$$\mathcal{E}_p = \frac{q_p B}{4\pi^2} \sum_{\nu=0}^{\nu_{\max}} g_s \left[k_{F,\nu}^p E_F^p + \left(M^{*2} + 2\nu q_p B \right) \cdot \ln \left| \frac{k_{F,\nu}^p + E_F^p}{\sqrt{M^{*2} + 2\nu q_p B}} \right| \right].$$

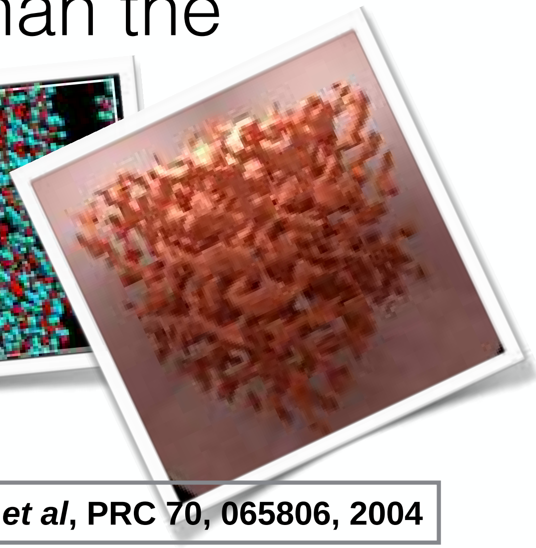
The pasta phases

- Competition between Coulomb and nuclear forces leads to frustrated system
- Geometrical structures, the **pasta phases**, evolve with density until they melt ⟶ **crust-core transition**
- Criterium: pasta free energy must be lower than the correspondent hm state

QMD calculations:

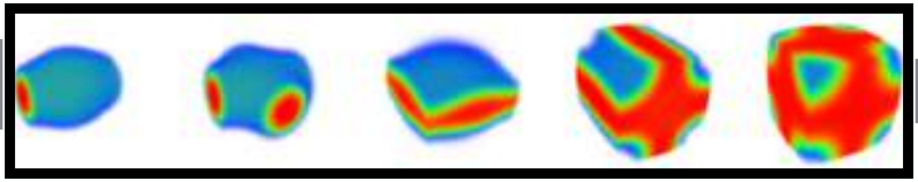


G. Watanabe *et al*, PRL 103, 121101, 2009



C. J. Horowitz *et al*, PRC 70, 065806, 2004

3D-SHF calculation:



Pais and Stone, PRL 109, 151101, 2012

Pasta phases - calculation (I)

check e.g. PRC 91, 055801 2015

• Coexistence Phase (CP) approximation:

- Separated regions of higher (pasta phases) and lower density (background nucleon gas).
- Gibbs equilibrium conditions:

$$\mu_p^I = \mu_p^{II}$$
$$\mu_n^I = \mu_n^{II}$$
$$P^I = P^{II}$$

- Finite size effects are taken into account by a surface and a Coulomb terms in the energy density, after the coexisting phases are achieved.
- The total energy density and proton fraction of the system are given by

$$\mathcal{E} = f\mathcal{E}^I + (1-f)\mathcal{E}^{II} + \mathcal{E}_{Coul} + \mathcal{E}_{surf} + \mathcal{E}_e$$
$$\rho_p = \rho_e = f\rho_p^I + (1-f)\rho_p^{II}$$

- By minimizing the surface the Coulomb energies w.r.t size of cluster:

$$\varepsilon_{\rm surf} = 2\varepsilon_{\rm Coul}$$

Pasta phases - calculation (III)

- Compressible Liquid Drop (CLD) approximation:

The total free energy density is minimized, including the surface and Coulomb terms.

The equilibrium conditions become:

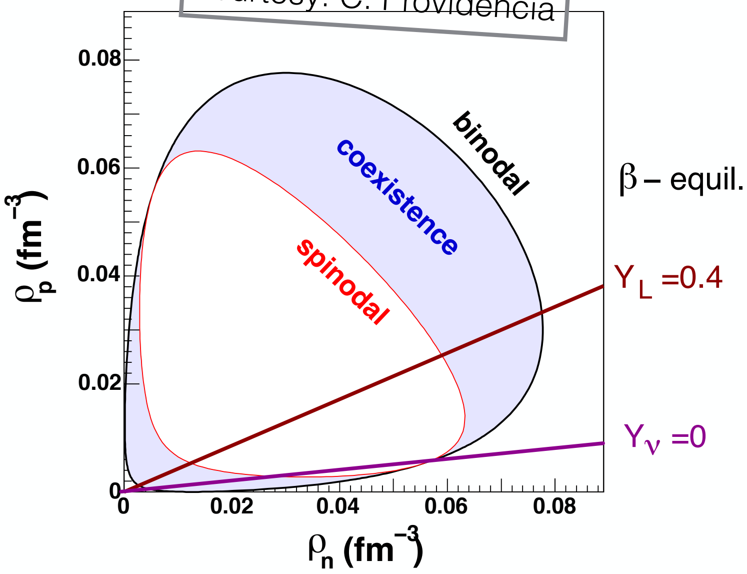
$$\mu_n^I = \mu_n^{II},$$

$$\mu_p^I = \mu_p^{II} - \frac{\epsilon_{surf}}{f(1-f)(\rho_p^I - \rho_p^{II})},$$

$$P^I = P^{II} - \frac{2\varepsilon_{Coul}}{(\rho_p^I - \rho_p^{II})}\left(\frac{\rho_p^I}{f} + \frac{\rho_p^{II}}{(1-f)}\right) + \varepsilon_{Coul}\left(\frac{3}{\alpha}\frac{\partial\alpha}{\partial f} + \frac{1}{\Phi}\frac{\partial\Phi}{\partial f}\right)$$

How to calculate transition density?

courtesy: C. Providência

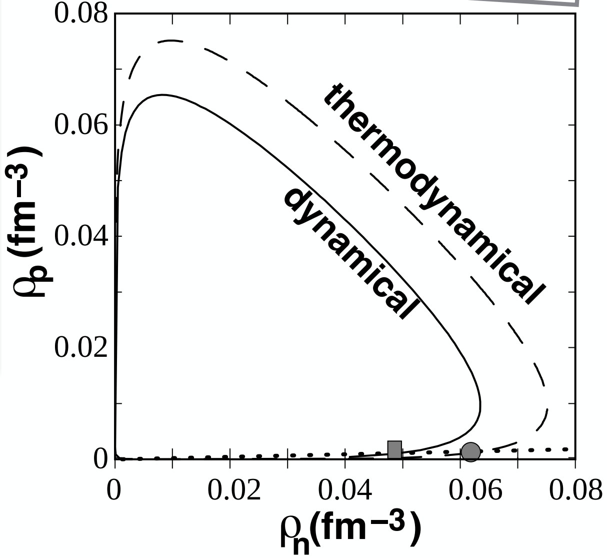


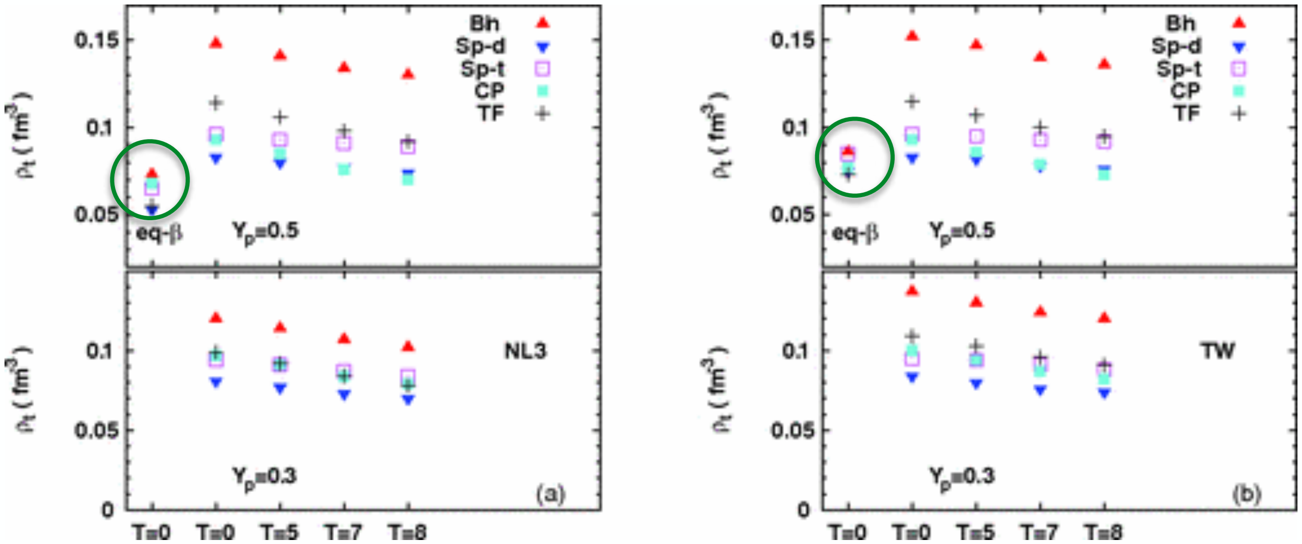
1) Get the instability region:

- Dynamical spinodal or
- Thermodynamical spinodal

2) Intersect EoS with that boundary to get ρ_t

courtesy: C. Providência



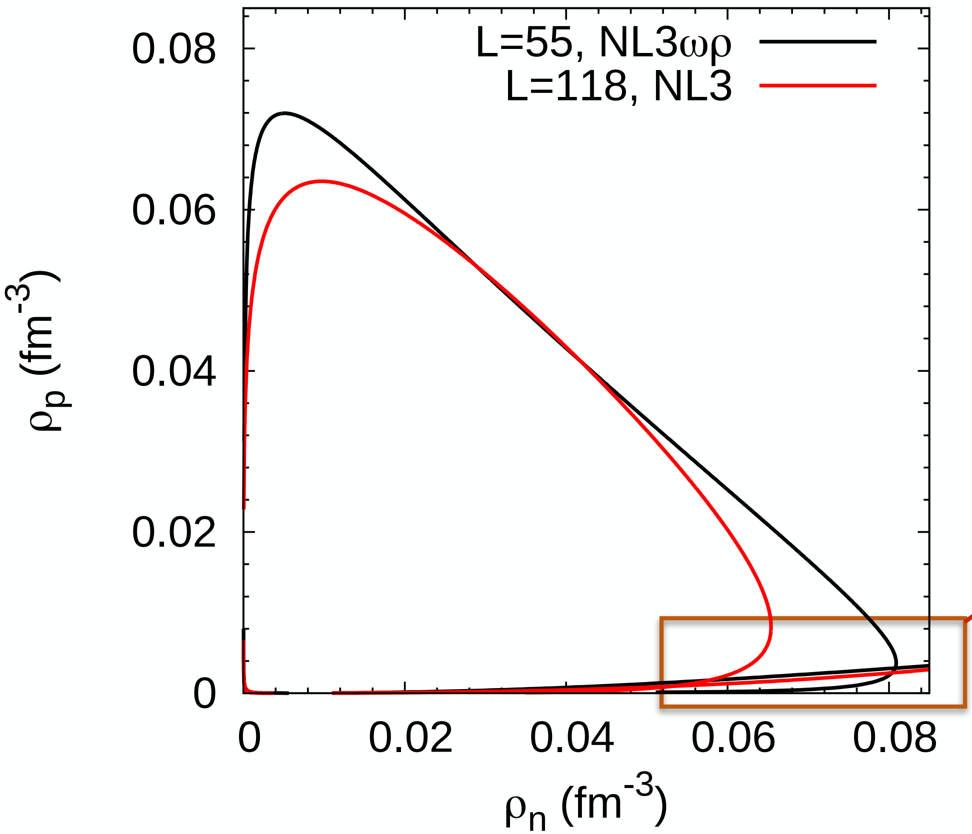


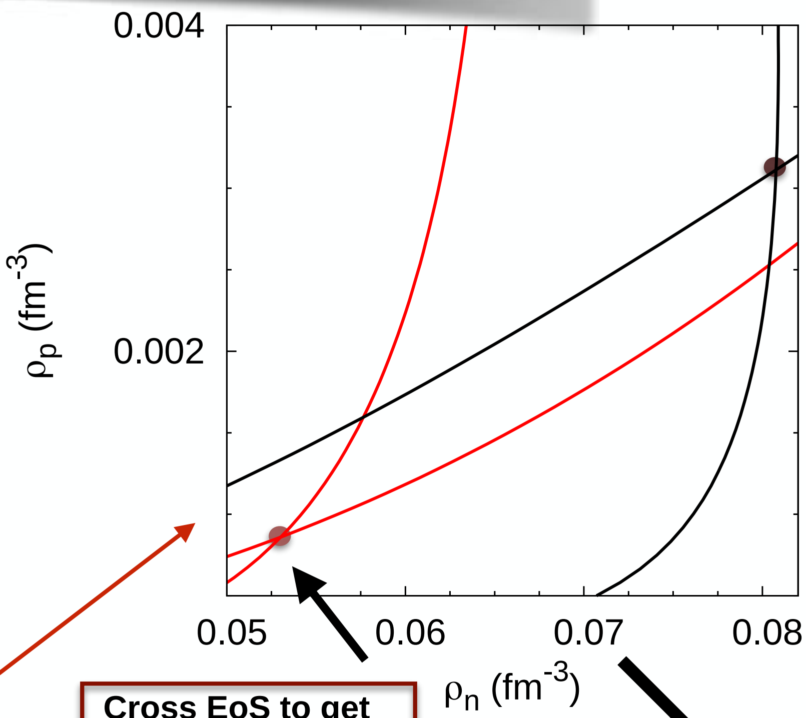
Avancini et al, PRC 82, 055807, 2010
Avancini et al, PRC 85, 059904(E), 2012

For β-eq. matter and T=0, dyn. spinodal very coincident with TF calculation

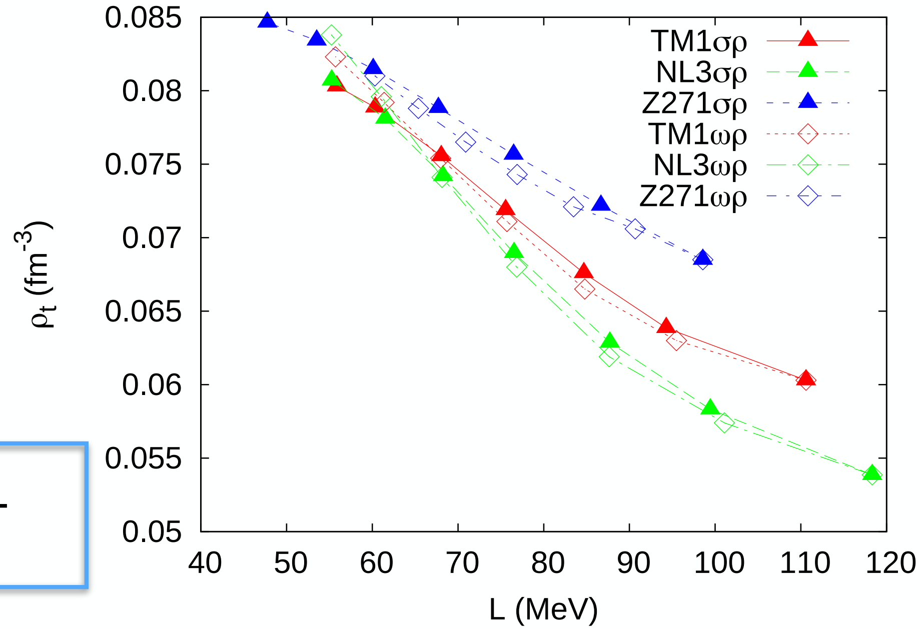
How to calculate the crust-core transition?

- dynamical spinodal:





Cross EoS to get transition density.



Instability region given by collective modes: small oscillations around equilibrium.

The boundary of the region is where the frequency of the modes is zero.

The larger L, the smaller the spinodal section.

ρ_t **decreases when L increases.**

Results: Influence of a strong external magnetic field - magnetars

Influence of a strong external magnetic field - magnetars

The strong external B makes the inner crust more complex.

The crust-core transition extends to a larger range of densities.

Note that if B=0, $\rho_1 = \rho_2$

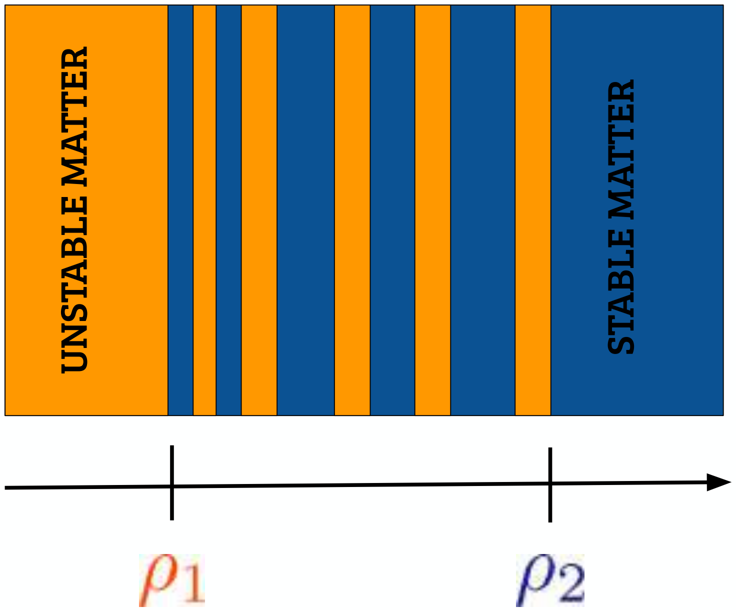


FIG. 1. (Color online) The extended crust region. The densities ρ_1 and ρ_2 define the boundaries of this region.

Sengo et al, PRD 102, 063013 (2020)

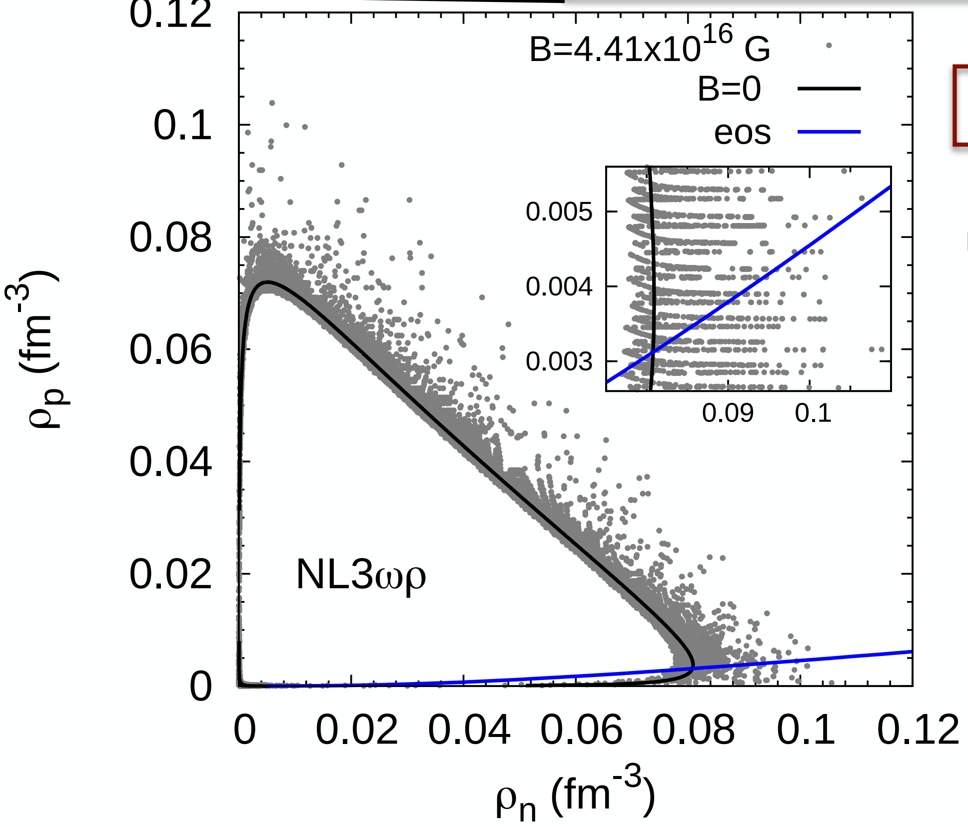
The crust-core transition - effect of strong external B - spinodal calculation

J. Fang's PhD thesis

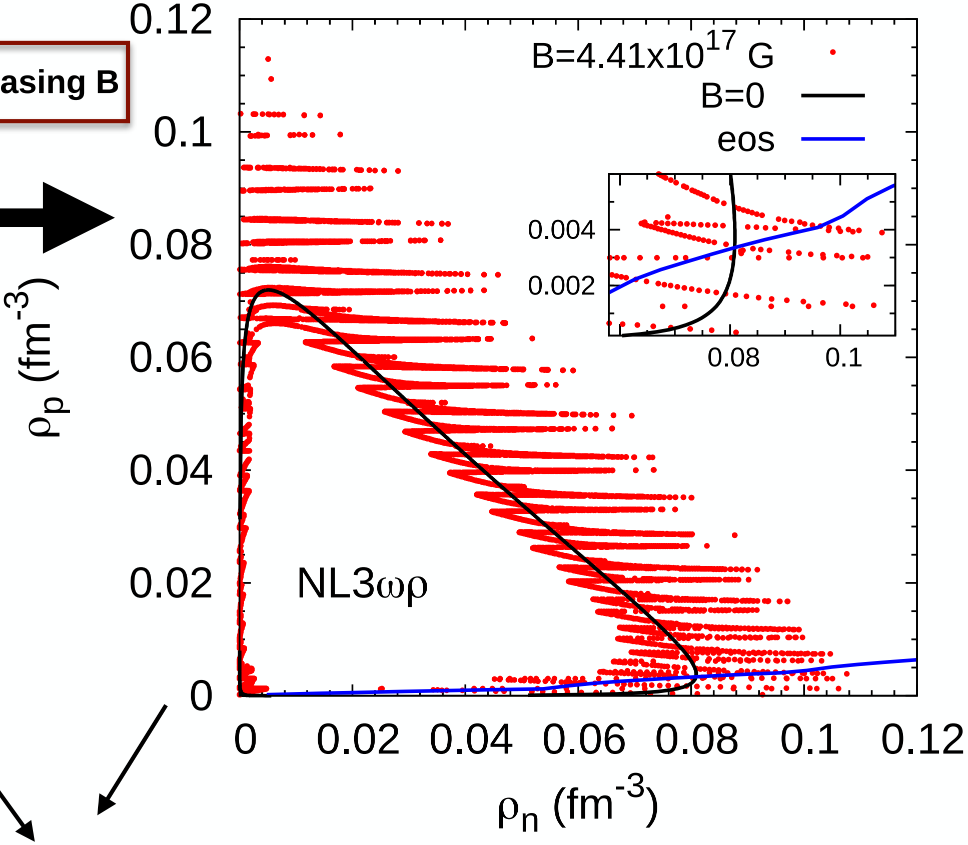
Fang et al, PRC 94, 062801(R) 2016
Fang et al, PRC 95, 045802 2017

Fang et al, PRC 95, 062801(R) 2017

Pons et al, Nat. Phys 9, 431 (2013): **Fast decay of B may explain lack of pulsars with P>12s. → pasta phases?**

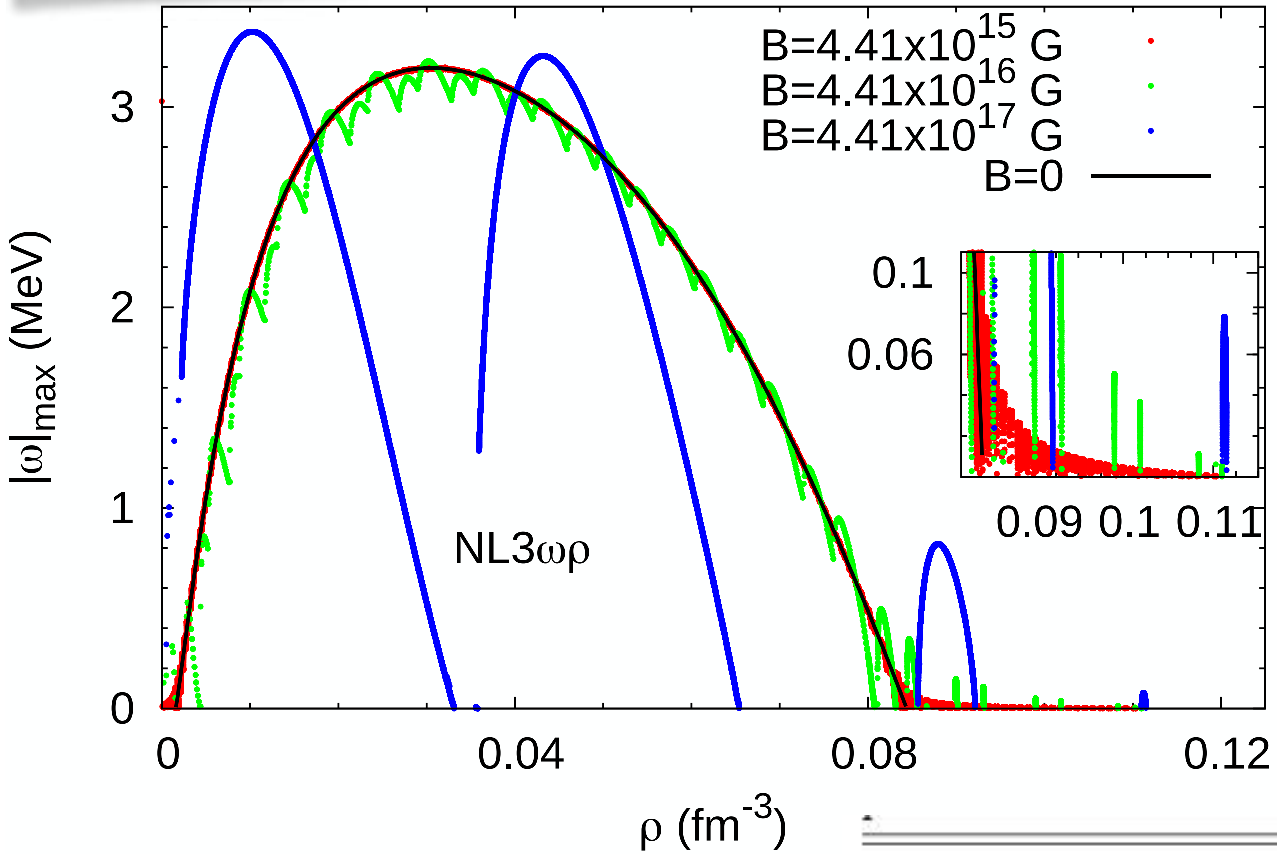


increasing B



- **The strong external B makes the inner crust more complex.**
- **New bands associated with the filling of the Landau levels.**
- **The stronger the B, the greater the spinodal section, and the smaller and wider the number of bands → decrease of the number of Landau levels when B increases.**
- **The crust-core transition extends to a range of densities.**

The crust-core transition - effect of strong external B - spinodal calculation



- Inside the region delimited by B=0 spinodal: **oscillations** around the B=0 results, the larger the B, the larger the oscillations.
- Above the B=0 crust-core transition: **alternate regions** of clustered and non-clusterized regions of matter appear.
- These regions appear when new Landau level opens.
- The smaller the B, the closer it gets to the B=0 results.

fractional moment of inertia of the crust

- for a star of M =1.4M⊙ and R=13.734km
- $\Delta R' = R(\varrho_1) - R(\varrho_2)$
- $\Delta R_B = \Delta R - \Delta R(B=0)$

B^*	ρ_1 (fm^{-3})	ρ_2 (fm^{-3})	P_1 ($\frac{\mathrm{MeV}}{\mathrm{fm}^3}$)	P_2 ($\frac{\mathrm{MeV}}{\mathrm{fm}^3}$)	ΔR (m)	$\Delta R'$ (m)	ΔR_B (m)	$\frac{\Delta I_{cr}}{I}$
0	0.0843	0.0843	0.5196	0.5196	1368	0	0	0.0676
10^2	0.0837	0.1044	0.5119	0.8541	1551	185	182	0.0968
10^3	0.0808	0.1096	0.4758	0.9743	1609	257	240	0.1056
10^4	0.0654	0.0998	0.3274	0.8095	1503	260	134	0.0922

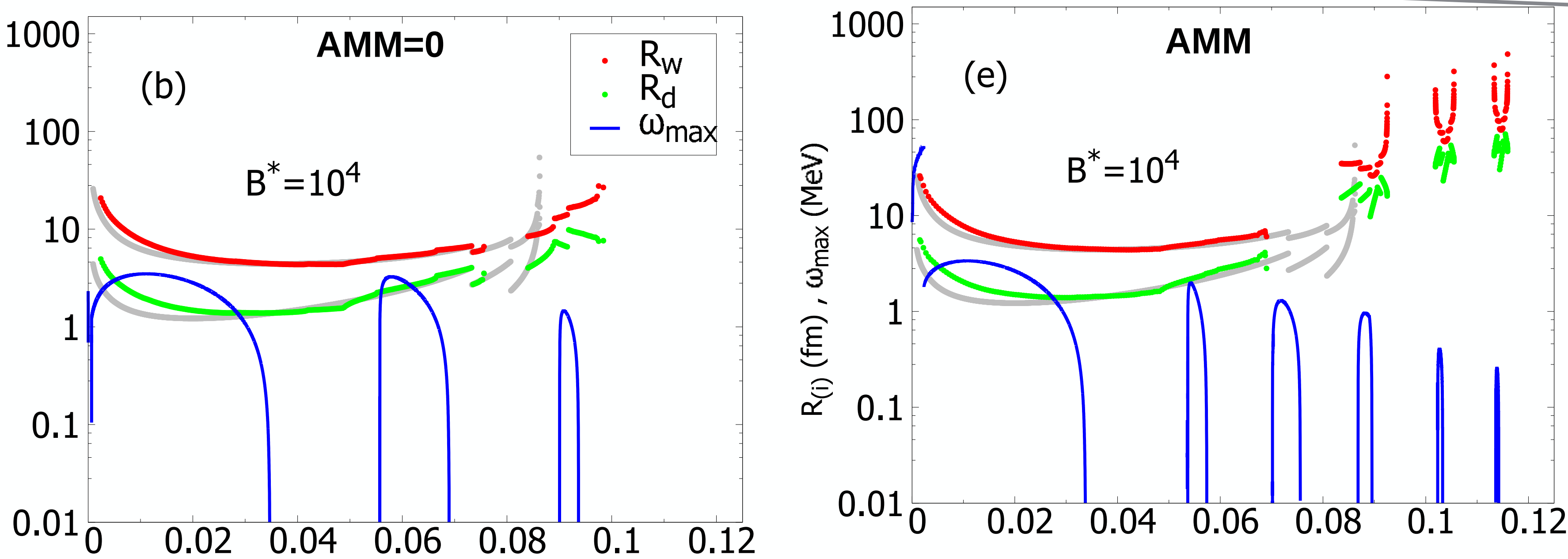
B field gives rise to larger ρ_t and P_t → higher values for ΔIcrust/I

ΔIcrust/I could be high enough for the crust to completely describe glitch mechanism

Pasta in beta-equilibrium matter - effect of strong external B - CP calculation

$B = 4.41 \times 10^{17}$G   NL3ωρ

Wang et al, PRD 105, 063004 (2022)



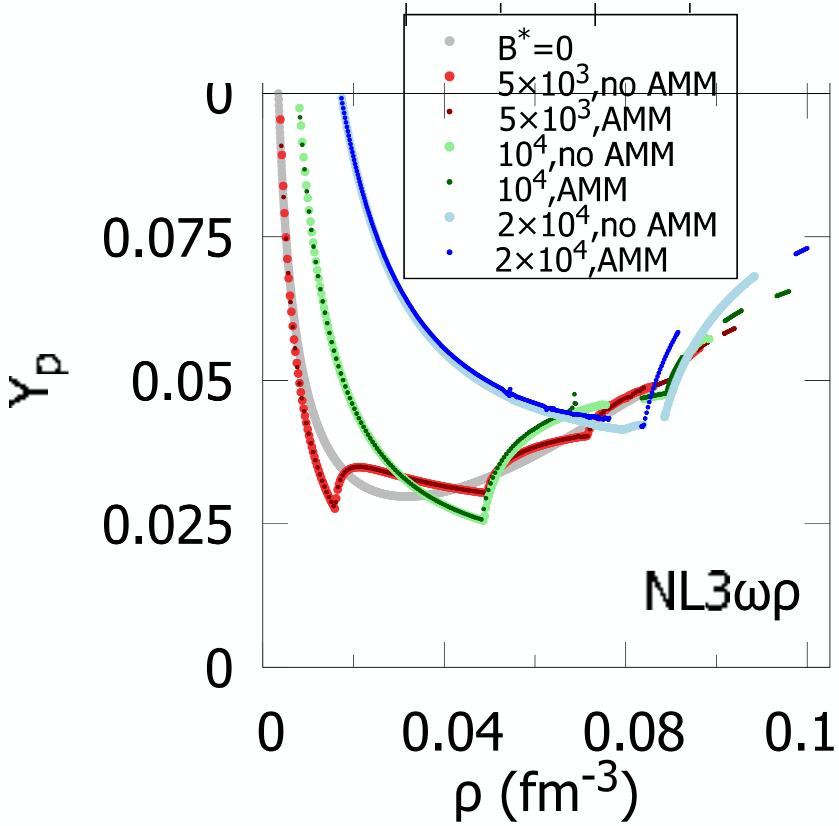
- There are several disconnected pasta regions that appear above the B=0 region. Effect caused by Landau quantisation induced by B field.
- The stronger the B, the smaller the number of regions, and the wider the density rang that they cover.
- If AMM is considered, more and narrower disconnected regions occur, because the spin polarisation degeneracy is removed.

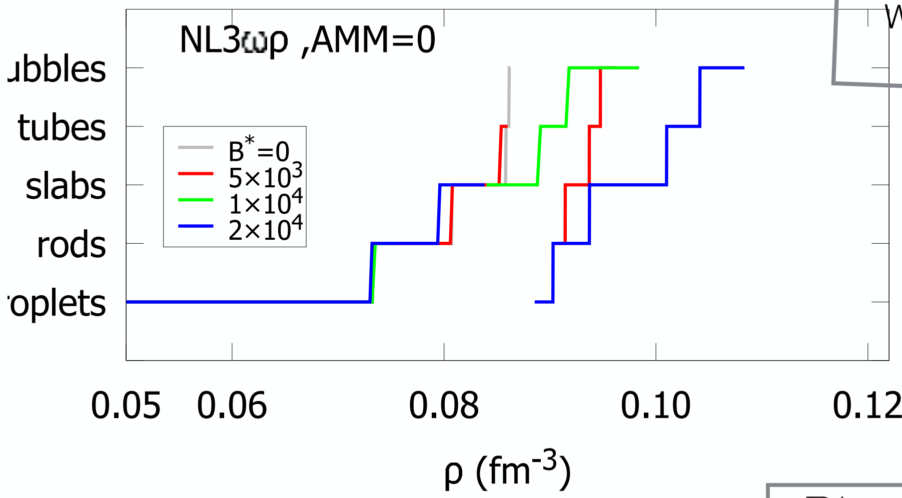
Confirmation of previous results:

PRC 94, 062801(R) 2016
PRC 95, 045802 2017

PRC 95, 062801(R) 2017

Pasta in beta-equilibrium matter - effect of strong external B - CP calculation





Wang et al, PRD 105, 063004 (2022)

$$B^* = B/B_c^e$$
$$B_c^e = 4.414 \times 10^{13}\ \mathrm{G}$$

- All geometric configurations appear in the disconnected pasta regions.
- B favors larger proton fractions, and gives rise to fluctuations due to the opening of Landau levels.

B field gives rise to larger ρ_2 and correspondent transition pressure, P2

higher values for ΔIcrust/I

Pasta in beta-equilibrium matter - effect of strong external B - CLD calculation

Scurto et al, PRC 107, 045806 (2023)

- For NL3, the crust-core transition density (orange lines) gets shifted to higher values wrt to the B=0 case (green lines).
- This extra region, that appears due to the B field, is in line with previous studies using the dynamical spinodal (light blue lines).

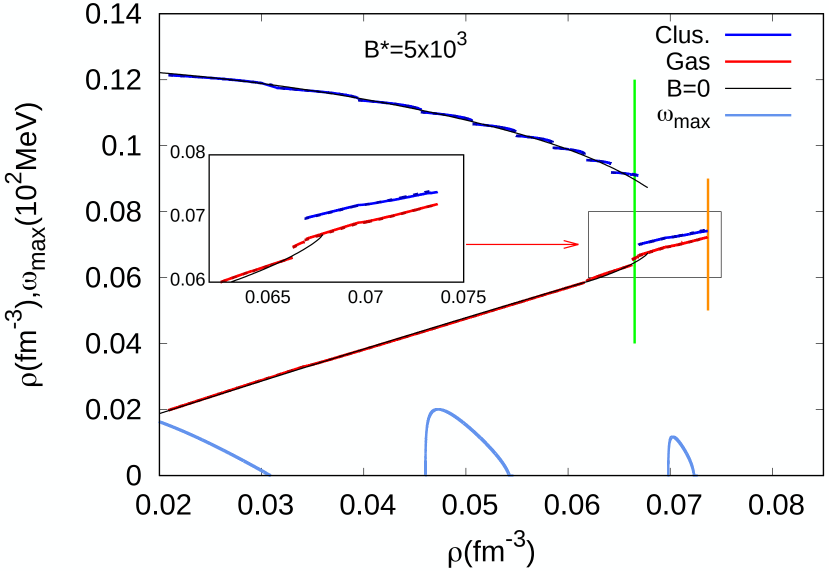


Fig.2 Baryon density of liquid (blue) and gas (red) phase for the NL3 model.

- In this new region, both the proton and baryon densities of the liquid (blue) and gas (red) become very similar, in line with the previous study using the CP approximation.

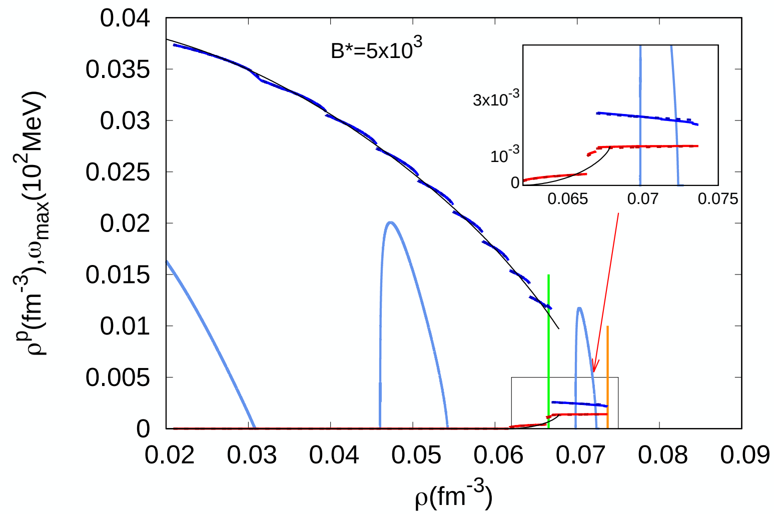
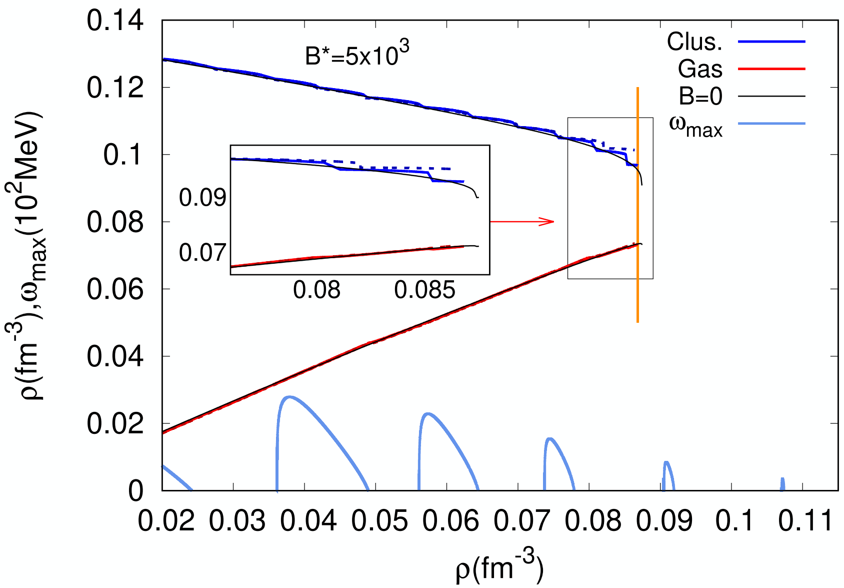


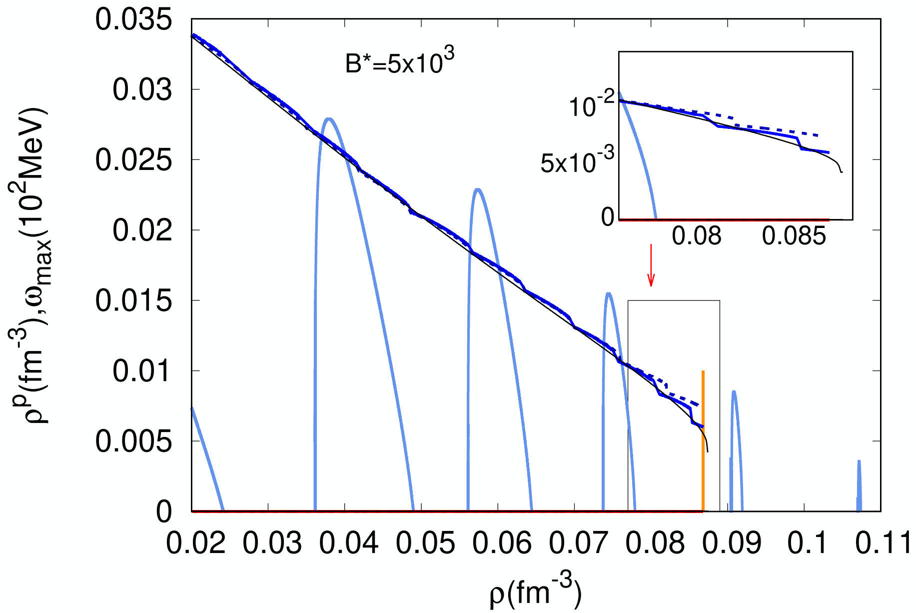
Fig.3 Proton density of liquid (blue) and gas (red) phase for the NL3 model.

Pasta in beta-equilibrium matter - effect of strong external B - CLD calculation

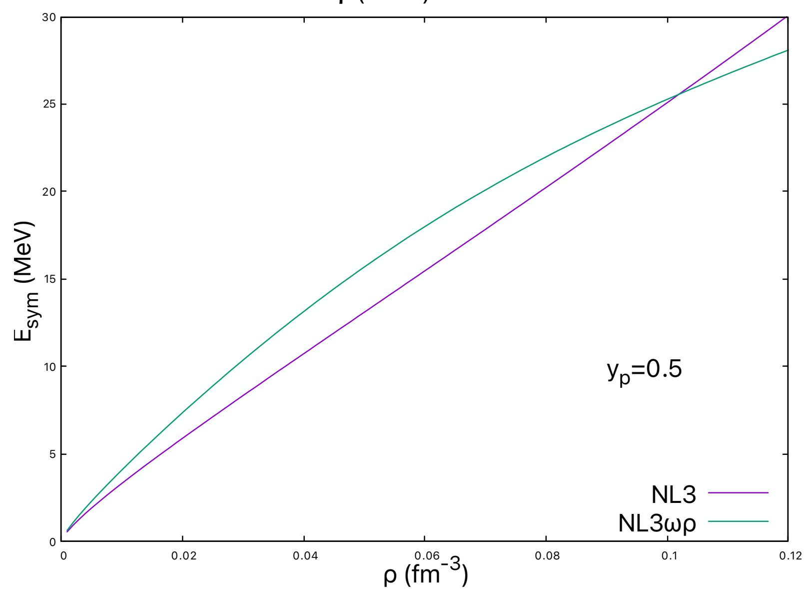
Scurto et al, PRC 107, 045806 (2023)

- However, for NL3wr, the behaviour seems to be opposite: with increasing B, the crust-core transition decreases, and this extra regions does not appear:





- This seems to be related with Esym: even though L(NL3)>L(NL3wr), for densities below 0.1. fm^{-3}, Esym(NL3wr)>Esym(NL3):



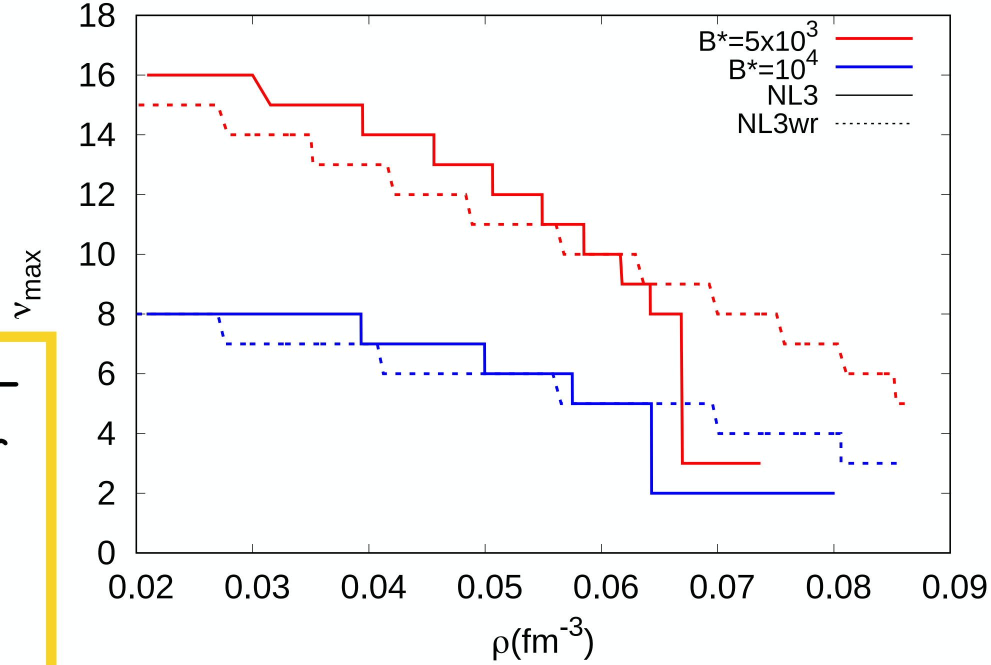
Pasta in beta-equilibrium matter - effect of strong external B - CLD calculation

Scurto et al, PRC 107, 045806 (2023)

Looking at the figure:

- For same B: LL(NL3wr)>LL(NL3)
- The higher the B, the lower the LL
- The higher the LL, the higher the rho_p—> the higher the yp

LL_max of protons in high-density phase



- NL3wr: higher Esym (below 0.1) —> larger yp—> larger LL —> smaller Bfield effect—> smaller extension of crust
- NL3: smaller Esym (below 0.1)—>smaller yp—> smaller LL —> larger B-field effect —> wider extension of crust

- The Esym behaviour favours larger proton fractions for NL3wr, and smaller B-field effects, since the number of Landau levels will be larger, favouring a smaller crust, when compared to NL3.

Pasta in beta-equilibrium matter - effect of strong external B - CLD calculation

Scurto et al, PRC 107, 045806 (2023)

			$M_1(M_\odot)$	$M_2(M_\odot)$	$R_T(km)$	$\Delta R_1(km)$	$\Delta R_2(km)$
$M_T = 1.4M_\odot$	NL3	$B=0$	0.0588	0.0	14.685	1.4270	0.0
		$B^*=5\times10^3$	0.0597	0.0258	14.908	1.6532	0.1541
		$B^*=10^4$	0.0574	0.0414	15.025	1.7427	0.3148
	NL3$\omega\rho$	B=0	0.0457	0.0	13.747	1.3665	0.0
		$B^*=5\times10^3$	0.0526	0.0	13.871	1.5431	0.0
		$B^*=10^4$	0.0526	0.0	13.991	1.6556	0.0
$M_T = 2.0M_\odot$	NL3	$B=0$	0.0394	0.0	14.777	0.8691	0.0
		$B^*=5\times10^3$	0.0400	0.0132	14.914	1.0064	0.0932
		$B^*=10^4$	0.0385	0.0288	14.989	1.0617	0.1973
	NL3$\omega\rho$	B=0	0.0326	0.0	14.079	0.8632	0.0
		$B^*=5\times10^3$	0.0384	0.0	14.161	0.9769	0.0
		$B^*=10^4$	0.0383	0.0	14.234	1.0437	0.0

- The mass and radius of the extended crust (region 2) increases with B.
- For the higher B-field, the mass of the extended region becomes comparable with M1.

Pasta in beta-equilibrium matter - effect of strong external B - CLD calculation

Scurto et al, PRC 107, 045806 (2023)

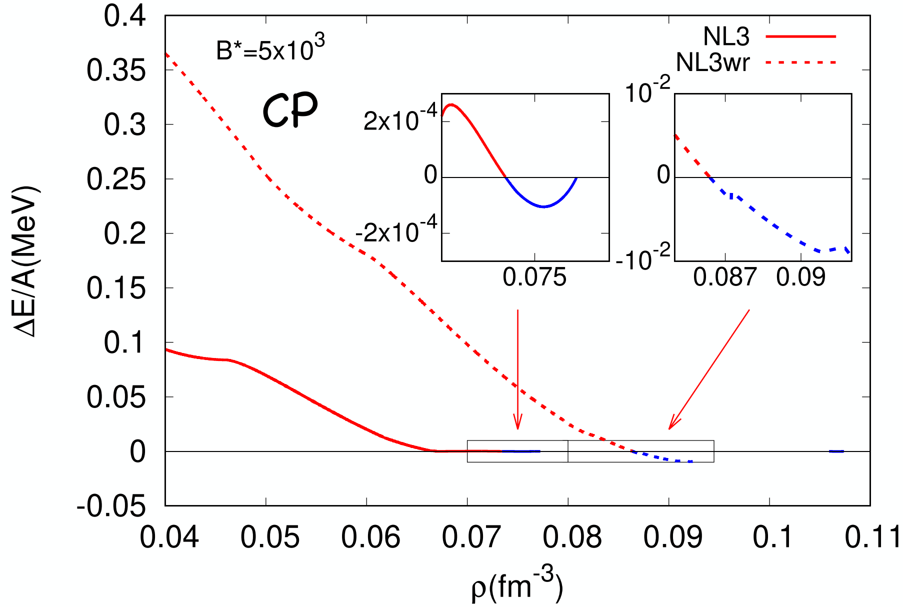
- The difference between E/A (homogenous matter) and E/A (crust):
- Red: Clusters favoured
- Blue: HM favoured
- The transition to HM occurs when this difference is zero, E/A(HM)>E/A(crust)
- The two approaches tend to give similar results, **however**

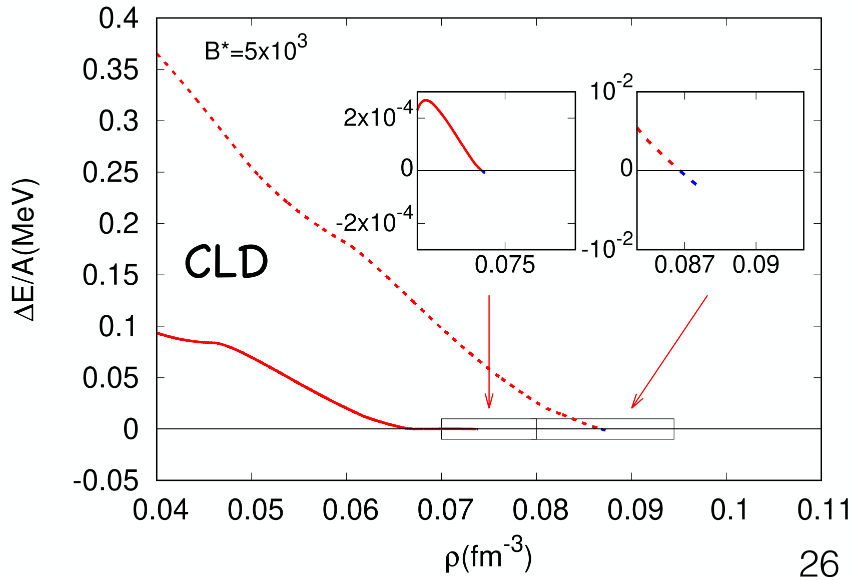
- CP and CLD are not self-consistent (surface tension is parametrised from a fit to TF), therefore pasta curve intersects HM curve.
- In a consistent calculation (eg TF) the pasta curve merges continuously with HM curve.
- Surface tension influences the transition density

Open questions:

Will the crust be even more complex, wider?

Need to know surface tension with B





Conclusions

- An extended region of clusters appears due to the presence of the magnetic field. This extra region seems to depend on the behaviour of the symmetry energy in the crustal EoS.
- The mass and radius of this extended crust seems to be comparable in size with the one of the B=0 crust.
- The surface tension is a crucial property and needs to be explored in the presence of strong magnetic fields.

- Heavy clusters are relevant and should be explicitly included in the NS EoS (and also CCSN simulations and NS mergers).
- Strong external B fields make inner crust more complex, and this may have consequences in the glitch mechanism.

Thank you!