MASSIVE NEUTRON STARS

EFFECTS OF EQUATION OF STATE AND MAGNETIC FIELD

ECT* Workshop: Strongly Interacting Matter in Extreme Magnetic Fields

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Work done in collaboration with Banibrata Mukhopadhyay (IISc), Fridolin Weber (San Diego State)

NEUTRON STARS (NSs)

- End stages of main sequence stars with masses $10 - 25\ M_{\odot}$.
- Typically have masses of the order of $M_{\odot}$ contained within a radius of $10\ km$.
- Extremely dense! At their cores – density several times that of the nuclear saturation density.

Artist's impression of two neutron stars merging
ESA/Hubble

MASSIVE NSs : AN INTRODUCTION

There is no limit to the mass of a NS
→ high-density nuclear matter equation of state (EOS) still remains unknown
→ no physical reason to rule out massive NSs ($\mathrm{M} > 2M_{\odot}$).

Why explore massive NSs?

- Recent pulsar observations indicate NS mass can be well above 2 $M_{\odot}$ limit
  PSR J1614-2230: $M = 1.97(+0.04)M_{\odot}$, MSP J0740+6620: $M = 2.14(+0.20)M_{\odot}$,
  PSR J0952–0607: $M = 2.35(+0.17)M_{\odot}$
- Gravitational wave (GW) observations discovering objects in the "mass gap" range (2.5 $M_{\odot}$ – 5 $M_{\odot}$)
  → some of these objects (e.g. – GW190814) could be massive NSs
- Other "massive" compact objects (Super-Chandrasekhar white dwarfs) explored in recent years
  → sets a general precedent for massive degenerate stars

OBJECTIVE

How does a NS increase its mass?

- Classically → magnetic field effects, anisotropy, rotation
- Microscopic → through EOS effects → not sufficient due to competing effects like hyperon softening

In this work, we explore the **theoretical possibility of massive NSs**
→ examining how the mass changes under different relativistic mean field models for the NS EOS
→ additionally checking how adding magnetic field and anisotropy can affect the system

Following the work of *Deb, Mukhopadhyay and Weber (2021, 2022)* who did a similar analysis but with a non-relativistic EOS (SLy4) and with white dwarfs respectively

(MODIFIED) TOV EQUATIONS

$$\frac{dm}{dr} = 4\pi r^2(\rho + \frac{B^2}{8\pi})$$

$$\frac{dp_r}{dr} = \begin{cases} \dfrac{\dfrac{-(\rho+p_r)\left(4\pi r^3\left(p_r - \frac{B^2}{8\pi}\right)+m\right)}{r(r-2m)} + \frac{2}{r}\Delta}{\left[1 - \frac{d}{d\rho}\left(\frac{B^2}{8\pi}\right)\left(\frac{d\rho}{dp_r}\right)\right]} & \text{For radially oriented (RO) fields} \\ \dfrac{\dfrac{-\left(\rho+p_r+\frac{B^2}{4\pi}\right)\left(4\pi r^3\left(p_r + \frac{B^2}{8\pi}\right)+m\right)}{r(r-2m)} + \frac{2}{r}\Delta}{\left[1 + \frac{d}{d\rho}\left(\frac{B^2}{8\pi}\right)\left(\frac{d\rho}{dp_r}\right)\right]} & \text{For transversely oriented (TO) fields} \end{cases}$$

Ansatz for anisotropy

$$\Delta = \begin{cases} \dfrac{\kappa r^2\left((\rho+p_r)\left(\rho+3p_r-\frac{B^2}{4\pi}\right)\right)}{1-\frac{2m}{r}} & \text{(RO)} \\ \dfrac{\kappa r^2\left(\left(\rho+p_r+\frac{B^2}{4\pi}\right)\left(\rho+3p_r+\frac{B^2}{2\pi}\right)\right)}{1-\frac{2m}{r}} & \text{(TO)} \end{cases}$$

Bowers & Liang (1974)
Deb, Mukhopadhyay & Weber (2021, 2022)

$-2/3 \le \kappa \le 2/3$ *(Silva et. al. (2015))*

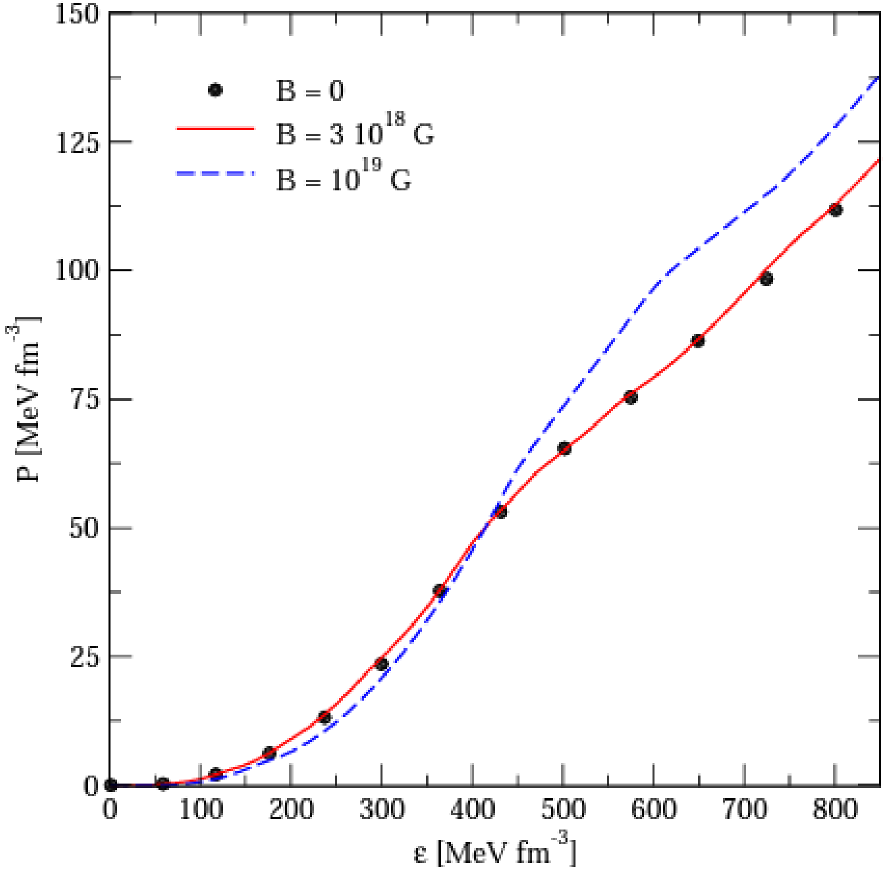
Magnetic Field profile used:

$$B(\rho) = B_s + B_0\left[1 - \exp\left\{-\eta\left(\frac{\rho}{\rho_0}\right)^{\gamma}\right\}\right]$$

Bandyopadhyay et al. (1997, 1998)

In this work we have considered central fields up to $1.32 \times 10^{18}\ G$ or less.

Beyond this value, we have to consider Landau quantization in EOS.



Energy-momentum tensor on inclusion of magnetic field

$$T^{\mu\nu} = T_m^{\mu\nu} + T_f^{\mu\nu}$$

with matter contribution

$$T_m^{\mu\nu} = \epsilon_m u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu) + MB\left(g^{\mu\nu} - u^\mu u^\nu + \frac{B^\mu B^\nu}{B^2}\right)$$

and field contribution

$$T_f^{\mu\nu} = \frac{B^2}{4\pi}\left(u^\mu u^\nu - \frac{1}{2}g^{\mu\nu}\right) - \frac{B^\mu B^\nu}{4\pi}$$

Magnetisation effects are not considered as it is negligible at strong fields.

Sinha, Mukhopadhyay, Sedrakian (2013)

CONSTRAINING THE EOS

- Constraints on NS EOS
  - Macroscopic NS properties from observations
    - ***Maximum Mass***, *Mass-Radius curves*, *Rotational Frequency*
  - Properties of Symmetric Nuclear Matter (SNM) at nuclear saturation density
    - *Binding energy per nucleon, nuclear incompressibility, isospin asymmetry energy, effective mass*

MODELLING NS MATTER

- We use an effective field theory → “Quantum Hadrodynamics”.
- Baryon-baryon interactions described in terms of meson fields:
  - **Scalar Meson** (σ): Describes attraction between baryons
  - **Vector Meson** (ω): Describes repulsion between baryons
  - **Isovector Meson** (ρ): Describes baryon-baryon interactions in isospin asymmetric systems
- Isospin symmetric nuclear matter (SNM) used as an approximation to NS matter.

THE RMF LAGRANGIAN

$$\mathcal{L}_{baryons} = \sum_{b} \overline{\psi_B} \, [\gamma_\mu (i\partial^\mu - g_{\omega B}(n)\omega^\mu - \frac{1}{2} g_{\rho \mathrm{B}}(n)\tau. \ \rho^\mu) \ - (m_B - g_{\sigma \mathrm{B}}(n)\sigma]\psi_B$$

$$\mathcal{L}_{leptons} = \sum_{\lambda} \overline{\psi_\lambda} \, [i\gamma_\mu \partial^\mu - m_\lambda] \ \psi_\lambda$$

$$\mathcal{L}_{mesons} = \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2\right) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{2} m_\rho^2 \rho_\mu . \rho^\mu \ - \frac{1}{4}\rho_{\mu\nu} . \rho^{\mu\nu}$$

Necessary to introduce non-linear terms to reproduce all properties of nuclear matter at the saturation density: $\mathcal{L}_{NL\sigma}, \mathcal{L}_{NL\omega}, \mathcal{L}_{\sigma\omega\rho}$.

$$\mathcal{L}_{RMF} = \mathcal{L}_{baryons} + \mathcal{L}_{leptons} + \mathcal{L}_{mesons} + \mathcal{L}_{NL\sigma} + \mathcal{L}_{NL\omega} + \mathcal{L}_{\sigma\omega\rho}$$

SYSTEM OF NONLINEAR EQUATIONS TO BE SOLVED

$$m_\sigma^2\bar{\sigma} + b_\sigma m_N g_{\sigma N}\left[g_{\sigma N}\bar{\sigma}\right]^2 + c_\sigma g_{\sigma N}\left[g_{\sigma N}\bar{\sigma}\right]^3 + g_{\sigma\rho}g_{\sigma N}^2\bar{\sigma}g_{\rho N}^2\bar{\rho}^2 - \sum_B g_{\sigma B}n_B^S = 0\,, \qquad \text{(A.1)}$$

$$m_\omega^2\bar{\omega} + g_{\omega^4}g_{\omega N}\left[g_{\omega N}\bar{\omega}\right]^3 + g_{\omega\rho}g_{\omega N}^2\bar{\omega}g_{\rho N}^2\bar{\rho}^2 - \sum_B g_{\omega B}n_B = 0\,, \qquad \text{(A.2)}$$

$$m_\rho^2\bar{\rho} + g_{\sigma\rho}g_{\sigma N}^2\bar{\sigma}^2 g_{\rho N}^2\bar{\rho} + g_{\omega\rho}g_{\omega N}^2\bar{\omega}^2 g_{\rho N}^2\bar{\rho} - \sum_B g_{\rho B}I_{3B}n_B = 0\,, \qquad \text{(A.3)}$$

$$\sum_B n_B q_B + \sum_\lambda n_\lambda q_\lambda = 0\,, \qquad \text{(A.4)}$$

$$n - \sum_B n_B = 0\,, \qquad \text{(A.5)}$$

$$\text{(RMFL, DDRMF)} \quad \mu_B - (\mu_n - q_B\mu_e) = 0\,. \qquad \text{(A.6)}$$

Coupled non-linear equations solved to give us meson mean fields and neutron, electron Fermi momenta. Fermi momenta of rest of baryons given by imposing chemical equilibrium condition.

EXOTIC MATTER IN NS CORES

High densities at NS cores

→ Possibility of formation of **exotic particles**

HYPERONS

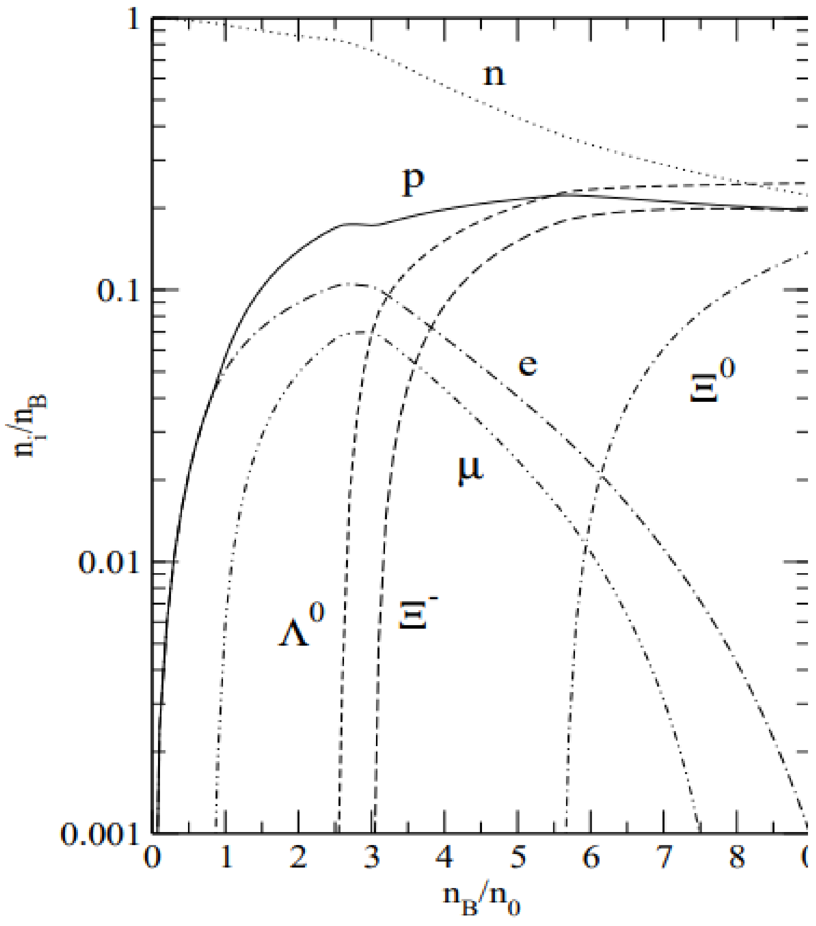
baryons that contain at least one strange quark

DELTAS

baryons consisting only of up and down quarks but with spin 3/2

Another possibility → at high densities, quarks can become deconfined → **quark stars** (not explored in current work)

Sinha, Mukhopadhyay, Sedrakian (2013)



EOS USED IN THIS WORK

GM1L
Glendenning et al., PRL 67, 2414 (1991)

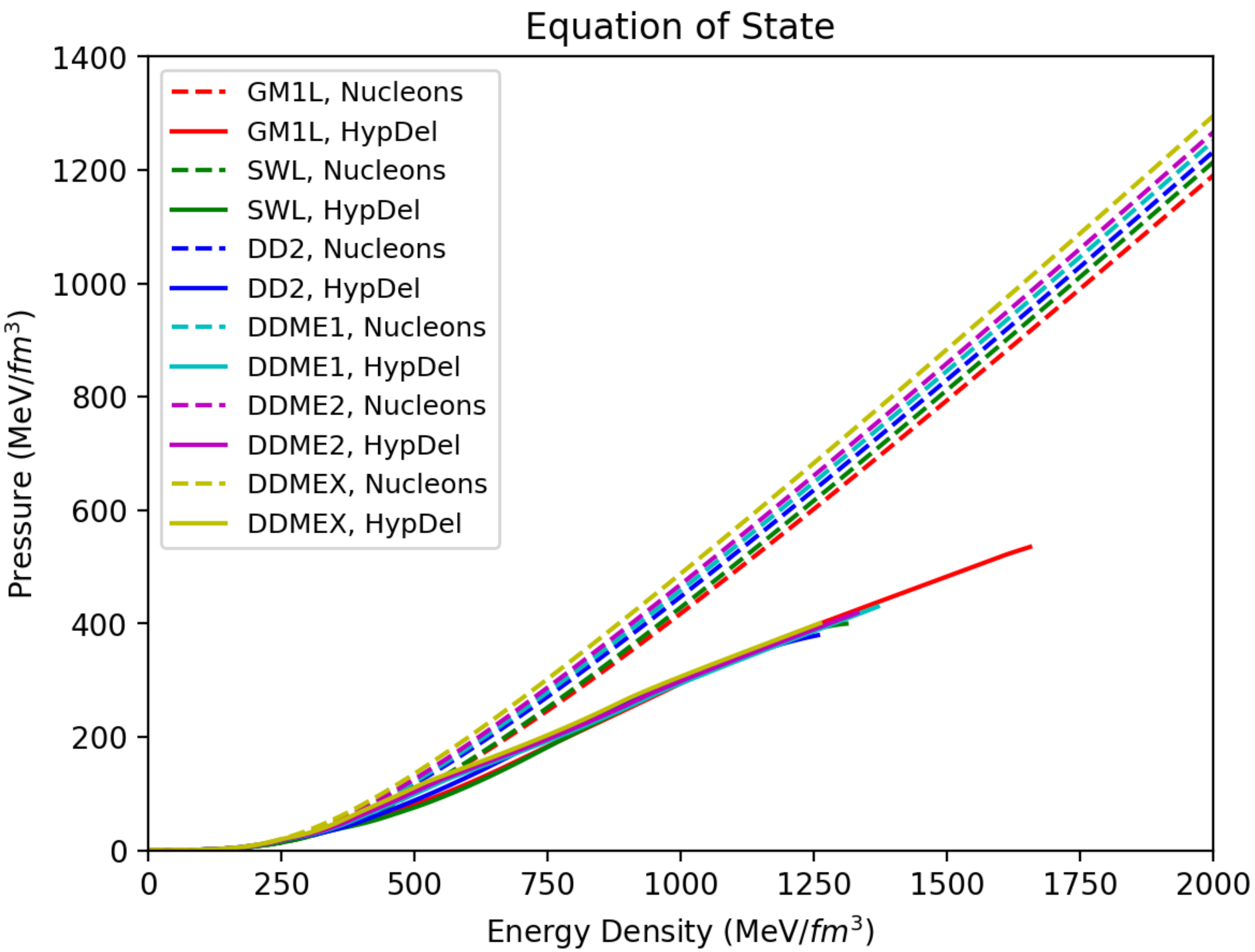
SWL
Spinella et al. (2017)

DD2
Typel et al., PRC 81, 015803 (2010)

DDME1
Niksic et al., PRC 66, 024306 (2002)

DDME2
Lalazissis et al., PRC 71, 024312 (2005)

DDMEX
Taninah et al., PLB 800, 135065 (2020)



TIDAL DEFORMABILITY

In presence of external gravitational field (ϵ_{ij}), the star develops a quadrupole moment (Q_{ij}) such that $Q_{ij} = -\lambda\epsilon_{ij}$, where λ is the tidal deformability of the star

- λ related to dimensionless second Love number k_2 as $\lambda = \frac{2}{3}k_2R^5$
- Dimensionless tidal deformability: $\Lambda = \frac{\lambda}{M^5} = \frac{2}{3}k_2C^{-5}$, where C is the compactness (M/R)

Observational Limits

→ GW170817 : $\Lambda_{1.4} < 800$,
$\Lambda_{1.4} < 580$

Abbott et.al. (2017, 2018)
Hinderer (2007)

Computing k_2 from a given EOS

$$k_2 = (8/5)C^5(1-2C^2)[2-y_R+2C(y_R-1]$$
$$\{2C(6-3y_R+3C(5y_R-8)$$
$$+4C^3[13-11y_R+C(3y_R-2)+2C^2(1+y_R)]$$
$$+3(1-2C)^2[2-y_R+2C(y_R-1)]\log(1-2C)\}^{-1}$$

where $y_R = [rH'(r)/H(r)]$
and $H(r)$ is the solution of the differential equation

$$H''(r) + H'(r)\left[\frac{2}{r} + e^{\lambda}\left(\frac{2m(r)}{r^2} + 4\pi r(p-\rho)\right)\right]$$
$$+ H(r)\left[4\pi e^{\lambda}\left(4\rho + 8p + \frac{\rho+p}{Ac_s^2}(1+c_s^2)\right) - \frac{6e^{\lambda}}{r^2} - \nu'^2\right] = 0$$

Here, $A = \frac{dp_t}{dp} . c_s^2 = \frac{dp}{d\rho}$,

$$e^{\lambda} = \left[1 - \frac{2m(r)}{r}\right]^{-1}, \nu' = \frac{2e^{\lambda}(m+4\pi p(r)r^3)}{r^2}$$

RESULTS

EOS GENERATED

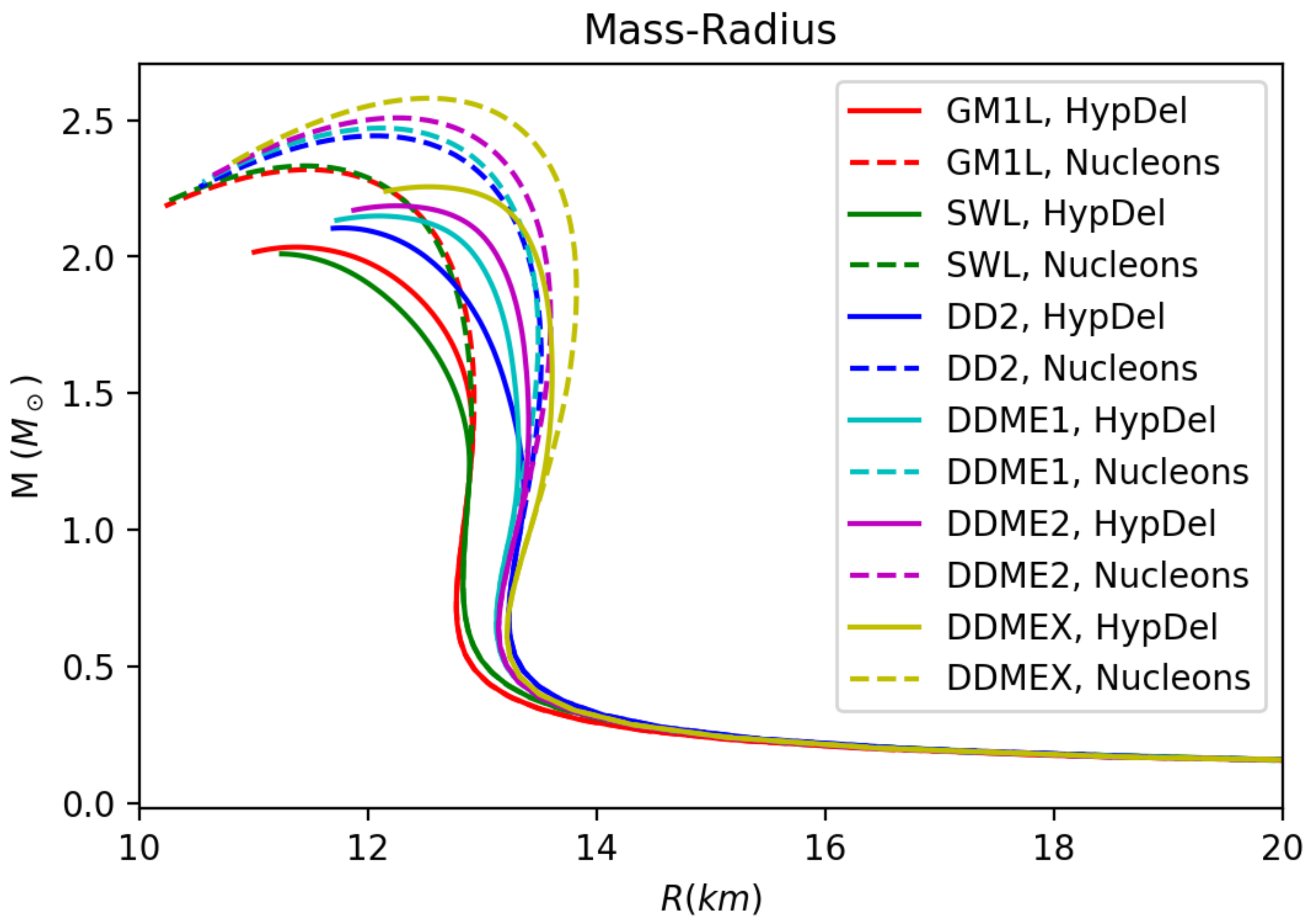
Solving mean field equations through a numerical code courtesy of Prof. Fridolin Weber (San Diego State University).

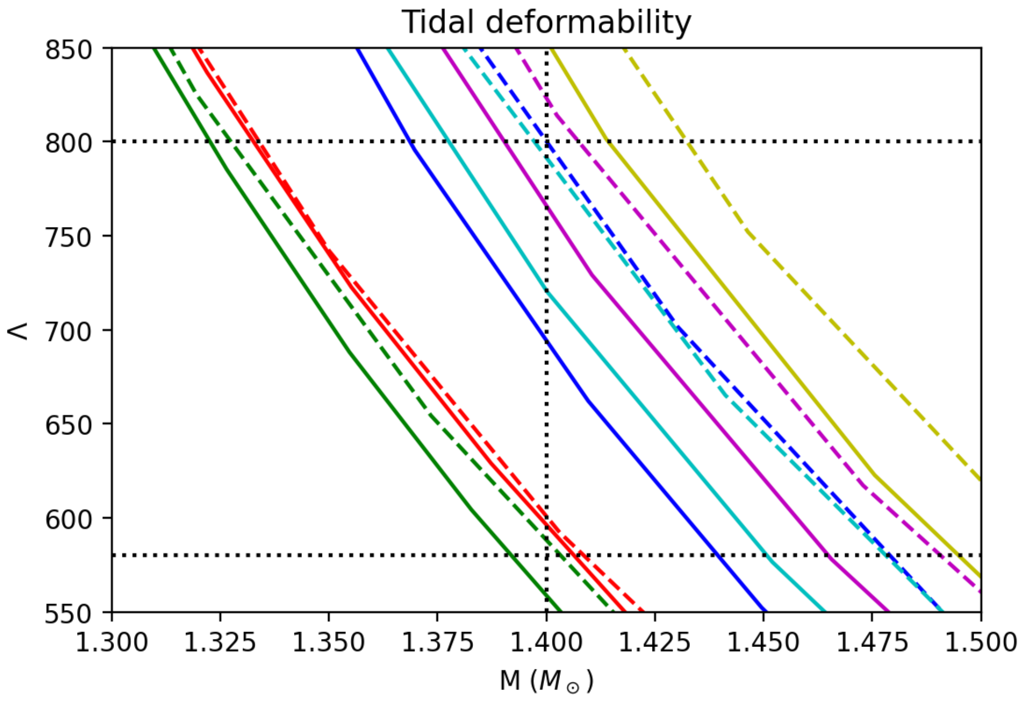
TOV SOLVER

Solving the coupled equations of stellar structure in general relativity using numerical techniques with generated EOS as input

Each EOS generates a family of stars parameterised by central density → Mass-Radius curve → Maximum mass supported by each EOS

Isotropic, non-magnetised case ($B = 0, \kappa = 0$)

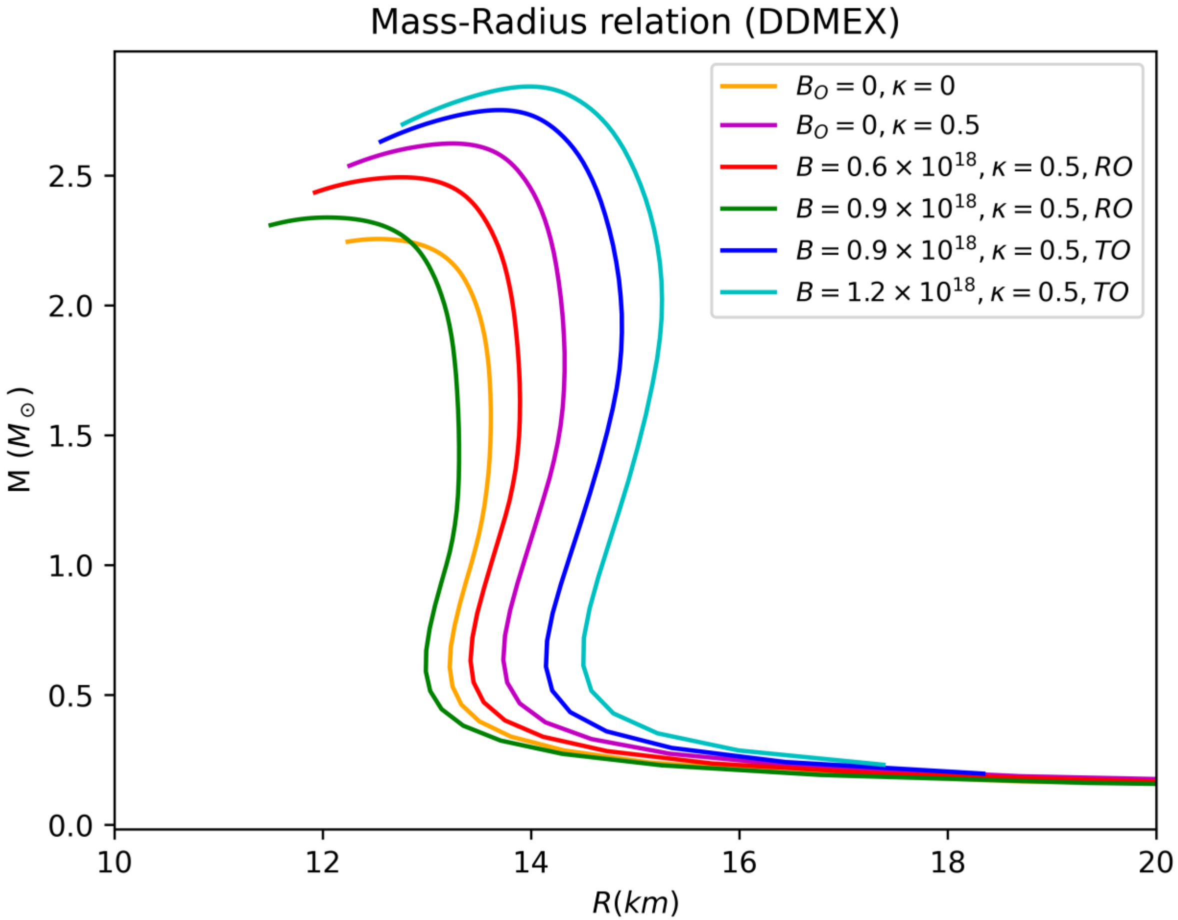




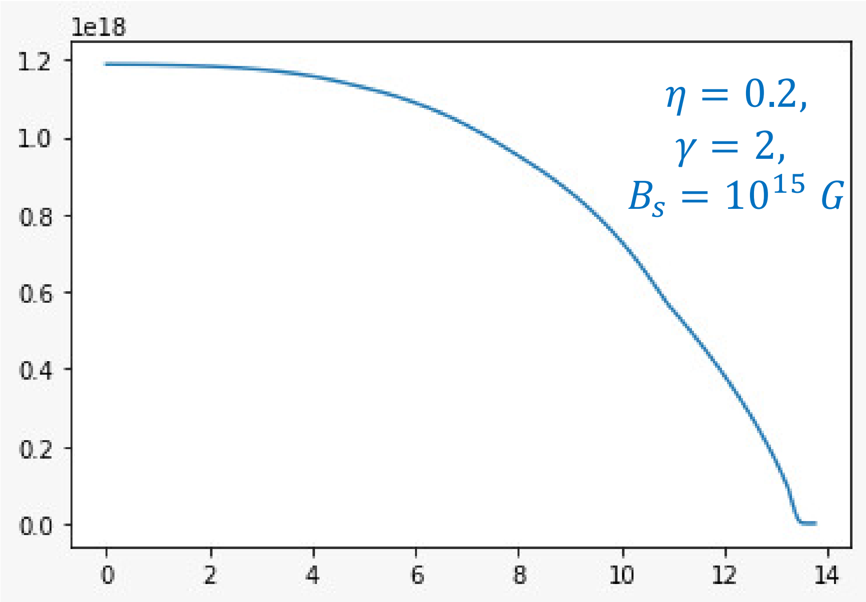
Hyperon-delta result

EOS	M_{max} ($M_\odot$)	R (km)
GM1L	2.0348	11.365
SWL	2.0098	11.246
DD2	2.1049	11.786
DDME1	2.1486	12.096
DDME2	2.1857	12.257
DDMEX	2.2555	12.532

Adding magnetic field with fixed anisotropy parameter, $\kappa = 0.5$

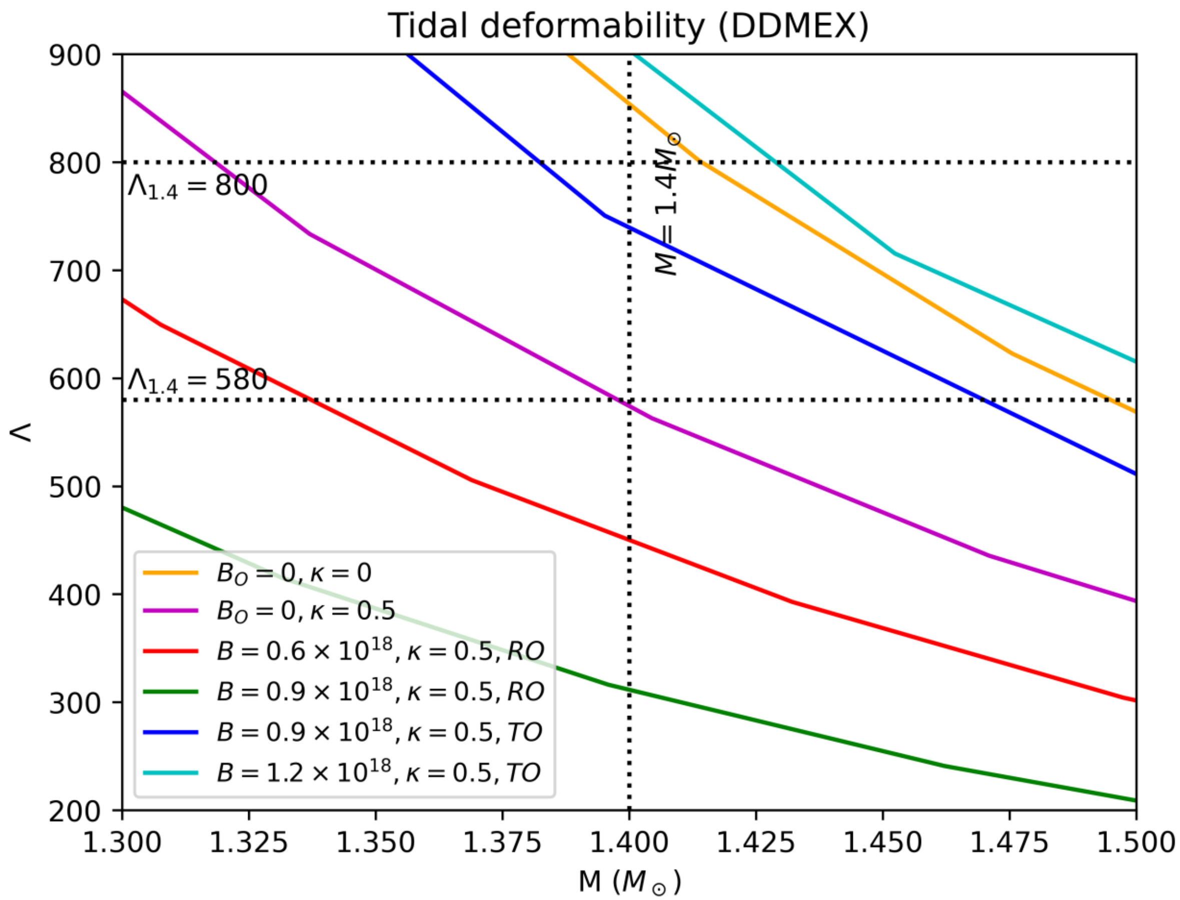


B_0 (G)	M_{max} ($M_\odot$)	R (km)
1.2×10^{18} (TO)	2.8423	13.990
0.9×10^{18} (TO)	2.7516	13.695
0 G	2.6231	13.263
0.6×10^{18} (RO)	2.4929	12.764
0.9×10^{18} (RO)	2.3383	12.046



RO – Radially oriented
TO – Transversely oriented

Adding magnetic field with fixed anisotropy parameter, $\kappa = 0.5$



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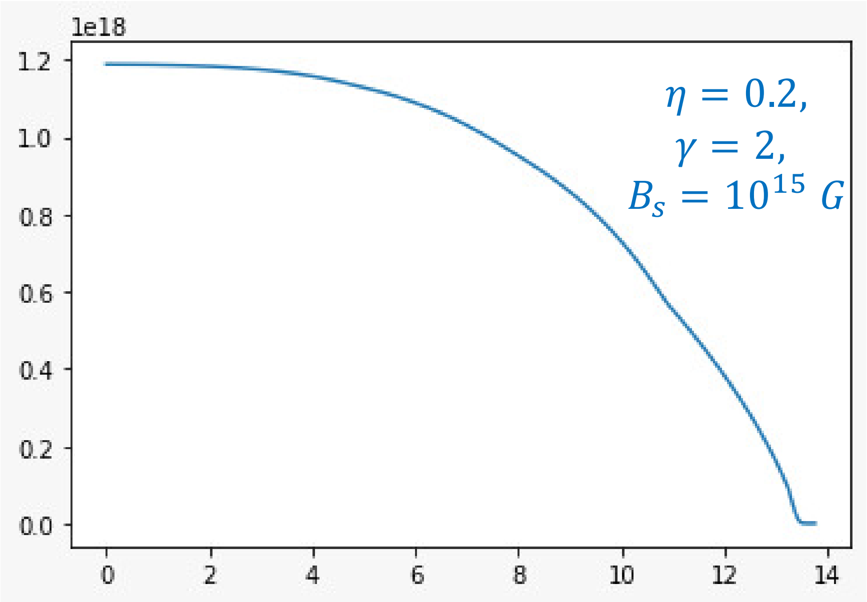
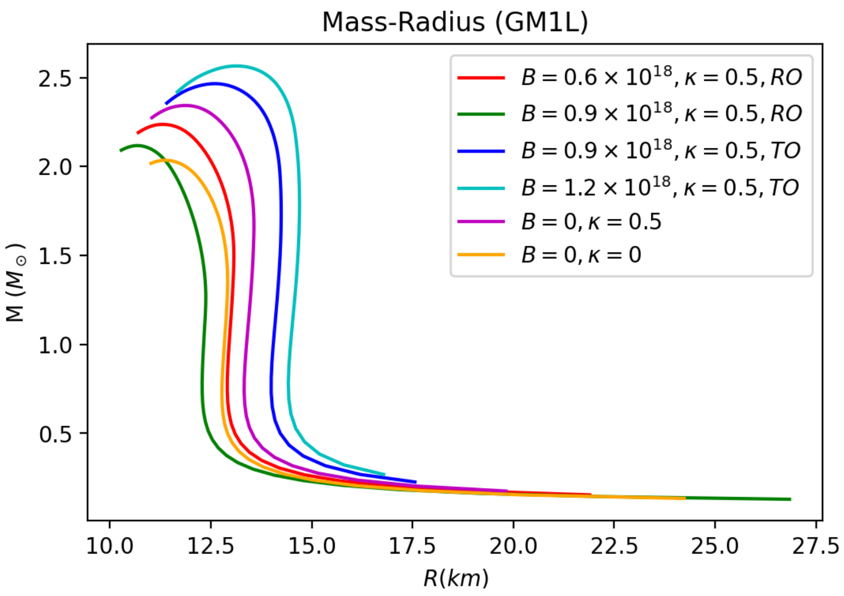
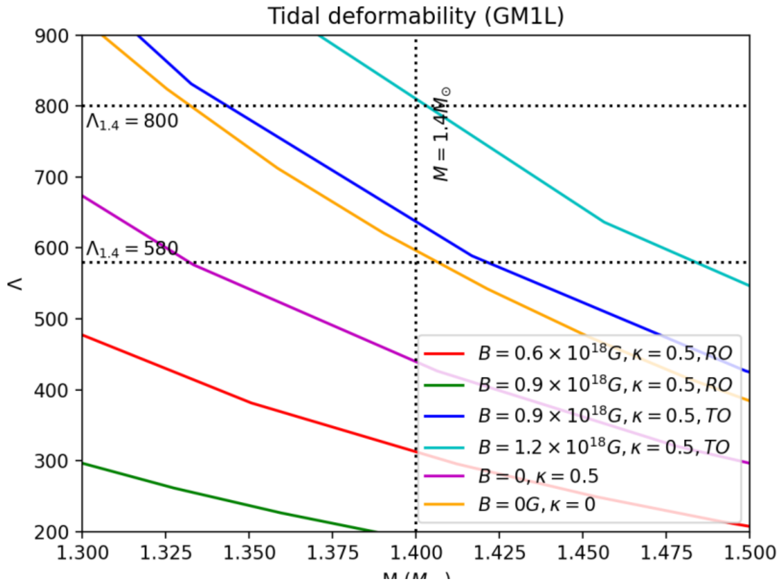
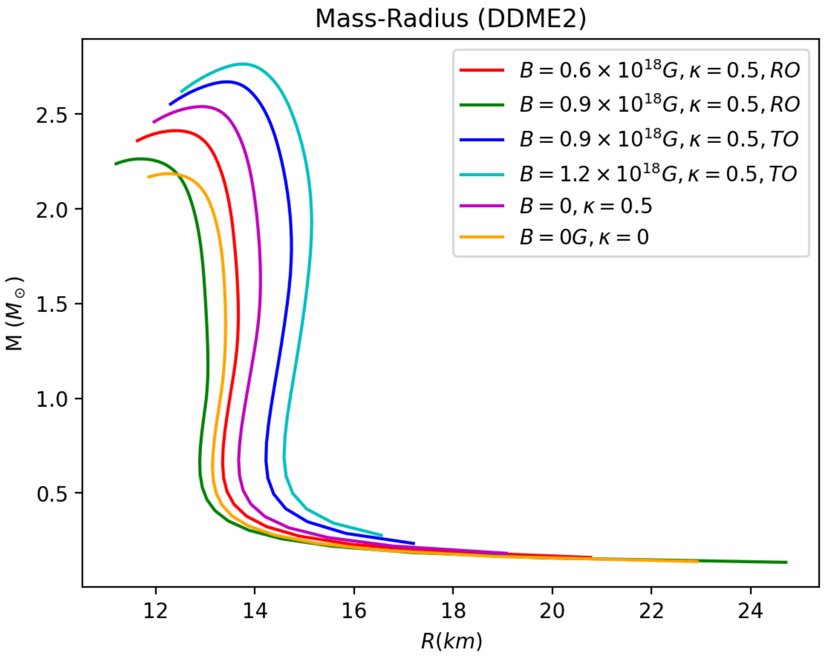
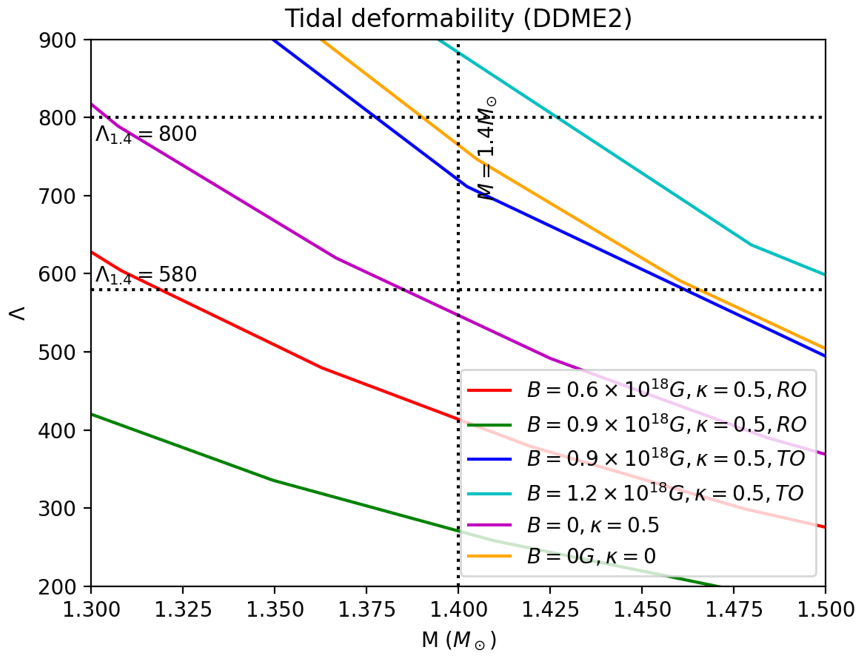


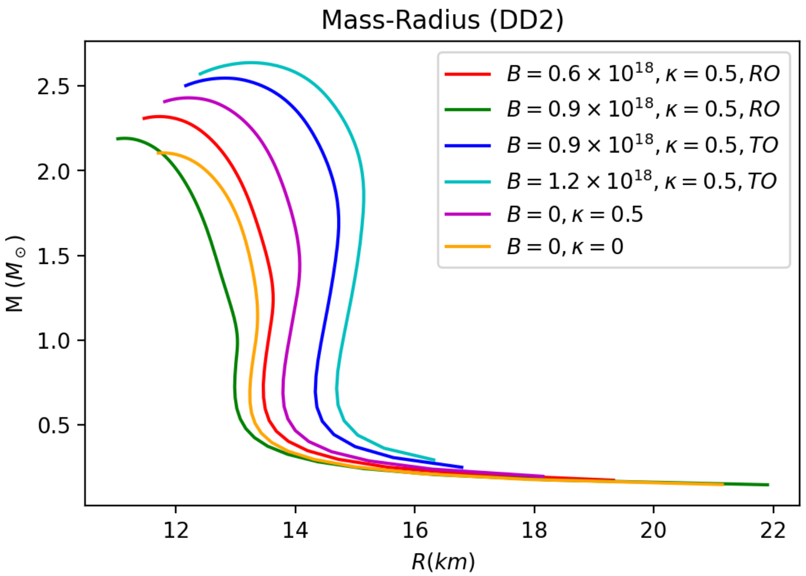
RO – Radially oriented
TO – Transversely oriented

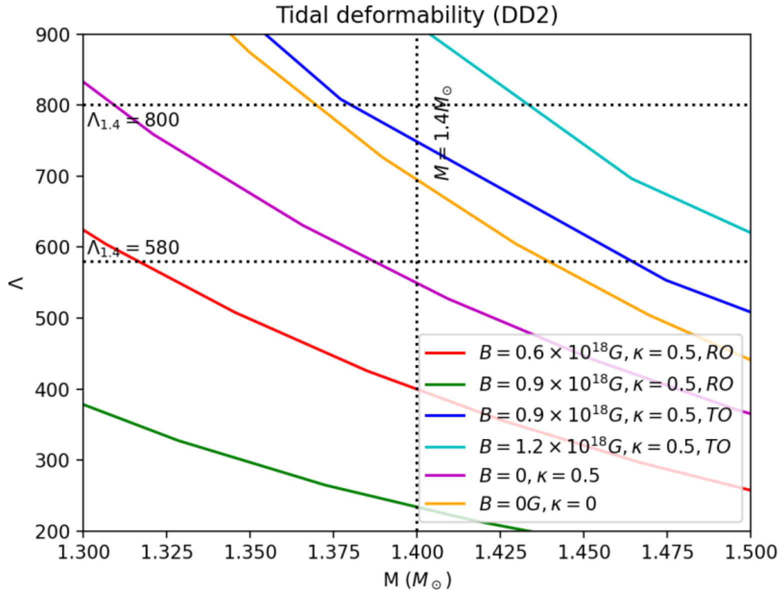

Mass-Radius (GM1L)
$B = 0.6 \times 10^{18}, \kappa = 0.5, RO$
$B = 0.9 \times 10^{18}, \kappa = 0.5, RO$
$B = 0.9 \times 10^{18}, \kappa = 0.5, TO$
$B = 1.2 \times 10^{18}, \kappa = 0.5, TO$
$B = 0, \kappa = 0.5$
$B = 0, \kappa = 0$
M ($M_\odot$)
R(km)


Tidal deformability (GM1L)
$\Lambda_{1.4} = 800$
$\Lambda_{1.4} = 580$
$M = 1.4 M_\odot$
$B = 0.6 \times 10^{18} G, \kappa = 0.5, RO$
$B = 0.9 \times 10^{18} G, \kappa = 0.5, RO$
$B = 0.9 \times 10^{18} G, \kappa = 0.5, TO$
$B = 1.2 \times 10^{18} G, \kappa = 0.5, TO$
$B = 0, \kappa = 0.5$
$B = 0G, \kappa = 0$
Λ


Mass-Radius (DDME2)
$B = 0.6 \times 10^{18} G, \kappa = 0.5, RO$
$B = 0.9 \times 10^{18} G, \kappa = 0.5, RO$
$B = 0.9 \times 10^{18} G, \kappa = 0.5, TO$
$B = 1.2 \times 10^{18} G, \kappa = 0.5, TO$
$B = 0, \kappa = 0.5$
$B = 0G, \kappa = 0$
M ($M_\odot$)
R(km)


Tidal deformability (DDME2)
$\Lambda_{1.4} = 800$
$\Lambda_{1.4} = 580$
$M = 1.4 M_\odot$
$B = 0.6 \times 10^{18} G, \kappa = 0.5, RO$
$B = 0.9 \times 10^{18} G, \kappa = 0.5, RO$
$B = 0.9 \times 10^{18} G, \kappa = 0.5, TO$
$B = 1.2 \times 10^{18} G, \kappa = 0.5, TO$
$B = 0, \kappa = 0.5$
$B = 0G, \kappa = 0$
Λ
M ($M_\odot$)


Mass-Radius (DD2)
M ($M_\odot$)
R(km)
$B = 0.6 \times 10^{18}, \kappa = 0.5, RO$
$B = 0.9 \times 10^{18}, \kappa = 0.5, RO$
$B = 0.9 \times 10^{18}, \kappa = 0.5, TO$
$B = 1.2 \times 10^{18}, \kappa = 0.5, TO$
$B = 0, \kappa = 0.5$
$B = 0, \kappa = 0$


Tidal deformability (DD2)
Λ
M ($M_\odot$)
$\Lambda_{1.4} = 800$
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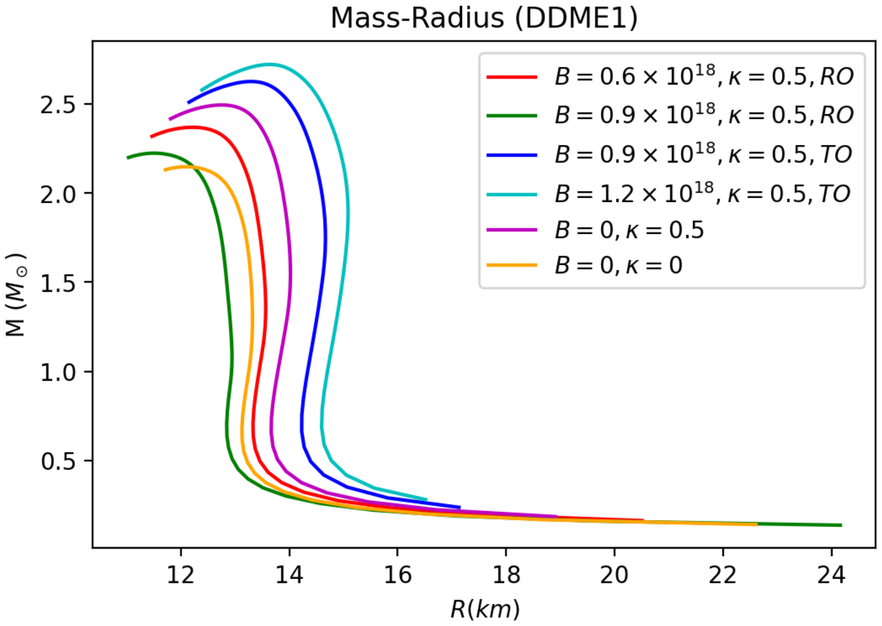
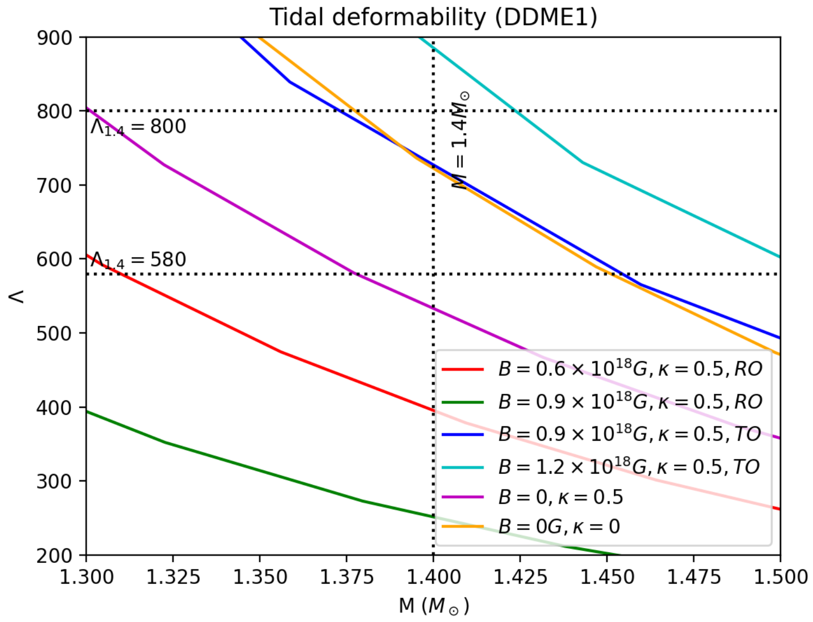
$B = 0, \kappa = 0.5$
$B = 0G, \kappa = 0$


Mass-Radius (DDME1)
M ($M_\odot$)
R(km)
$B = 0.6 \times 10^{18}, \kappa = 0.5, RO$
$B = 0.9 \times 10^{18}, \kappa = 0.5, RO$
$B = 0.9 \times 10^{18}, \kappa = 0.5, TO$
$B = 1.2 \times 10^{18}, \kappa = 0.5, TO$
$B = 0, \kappa = 0.5$
$B = 0, \kappa = 0$


Tidal deformability (DDME1)
Λ
M ($M_\odot$)
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$\Lambda_{1.4} = 580$
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$B = 0.6 \times 10^{18}G, \kappa = 0.5, RO$
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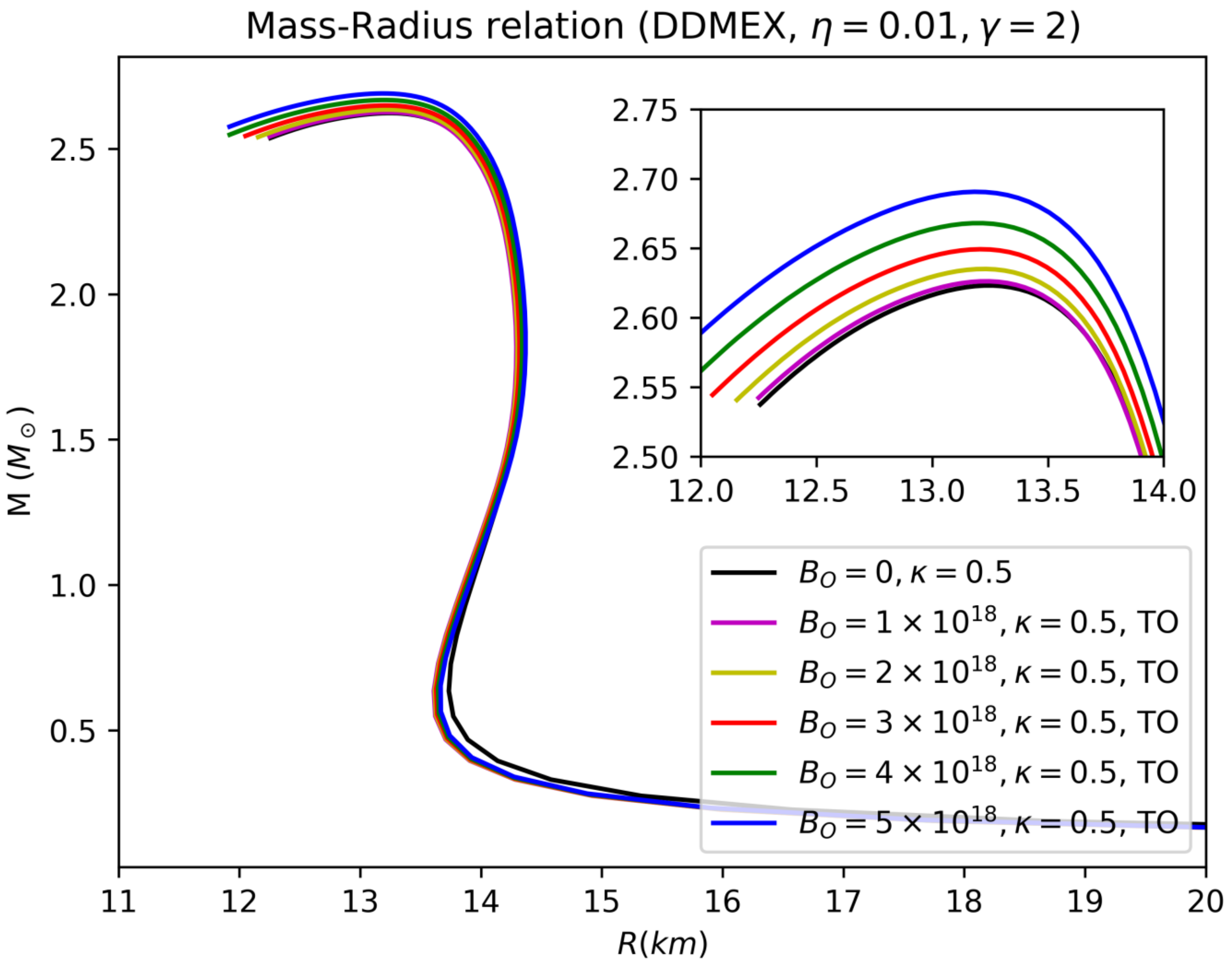
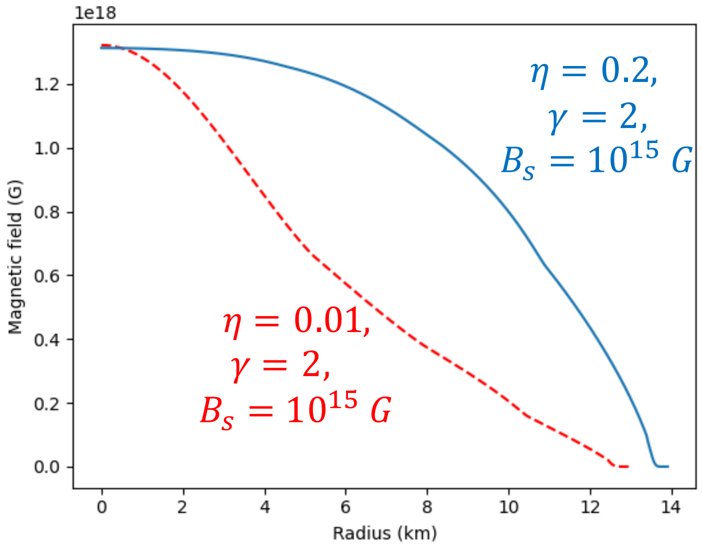
$B = 0.9 \times 10^{18}G, \kappa = 0.5, TO$
$B = 1.2 \times 10^{18}G, \kappa = 0.5, TO$
$B = 0, \kappa = 0.5$
$B = 0G, \kappa = 0$

Stability
($\kappa = 0.5$)

- Central field, B_c
- Field profile

B_c (10^{18}G)	M_{max} $(M_\odot)$	R (km)	ME/GE
1.18 (TO)	2.8423	13.990	0.192





0	2.6231	13.263	-
0.24 (TO)	2.6261	13.247	0.0023
0.49 (TO)	2.6492	13.232	0.0090
0.74 (TO)	2.6492	13.219	0.0204
0.95 (TO)	2.6680	13.207	0.0356
1.18 (TO)	2.6906	13.182	0.0558

Stability
($\kappa = 0.5$)

- Central field, B_c
- Field profile

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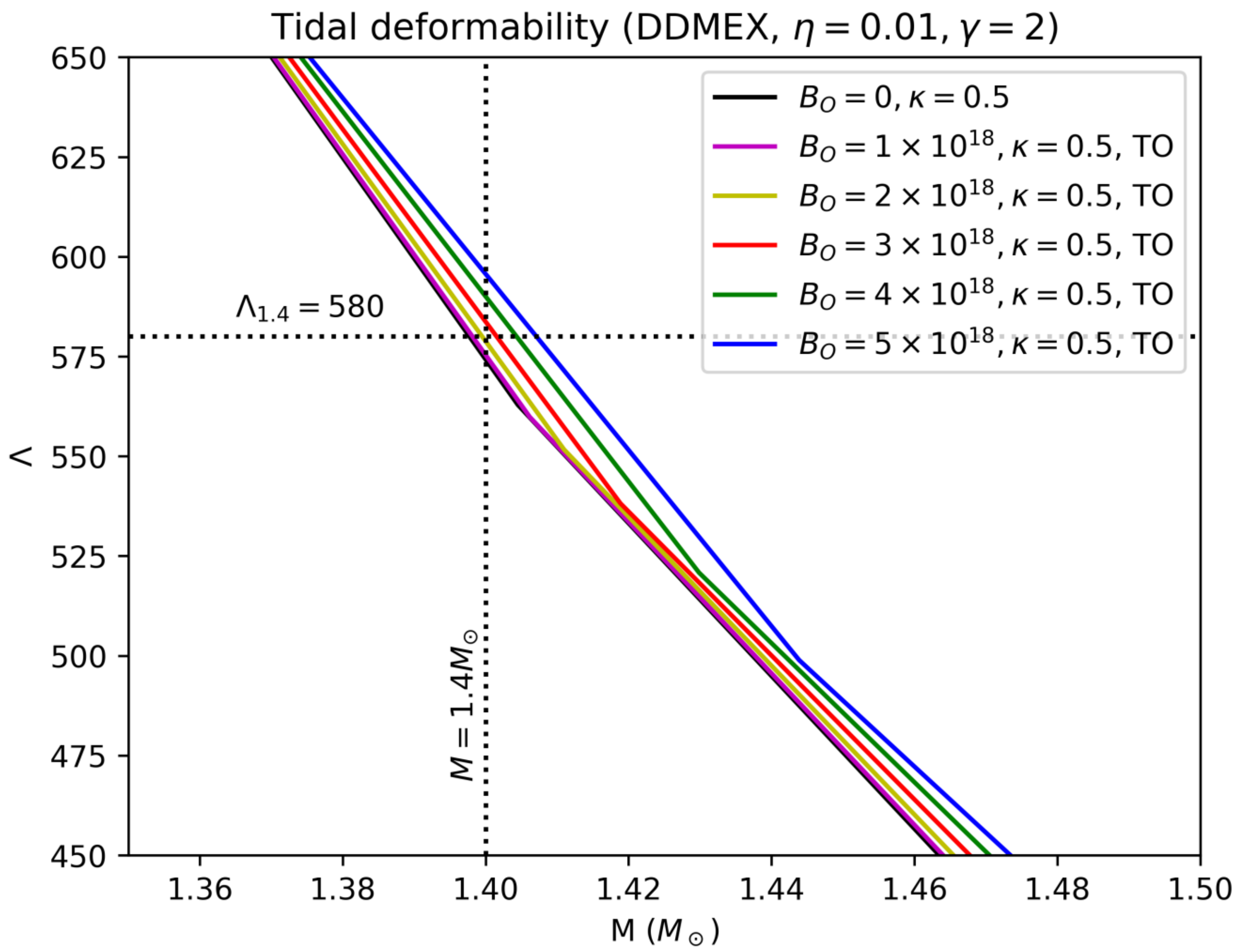

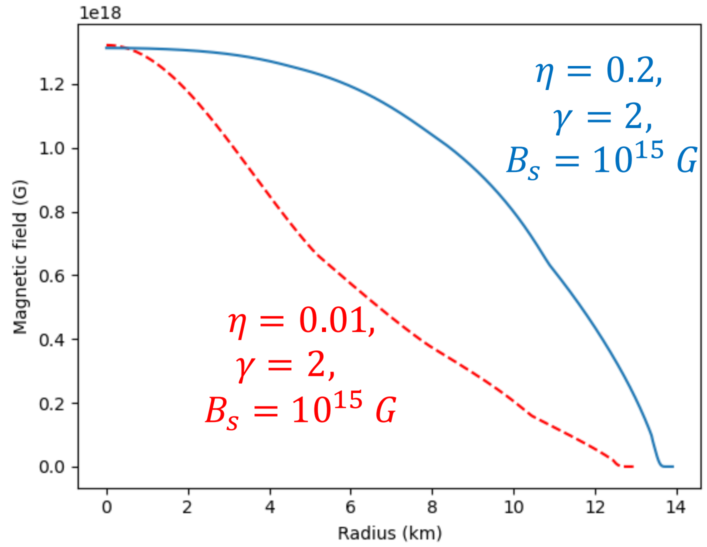

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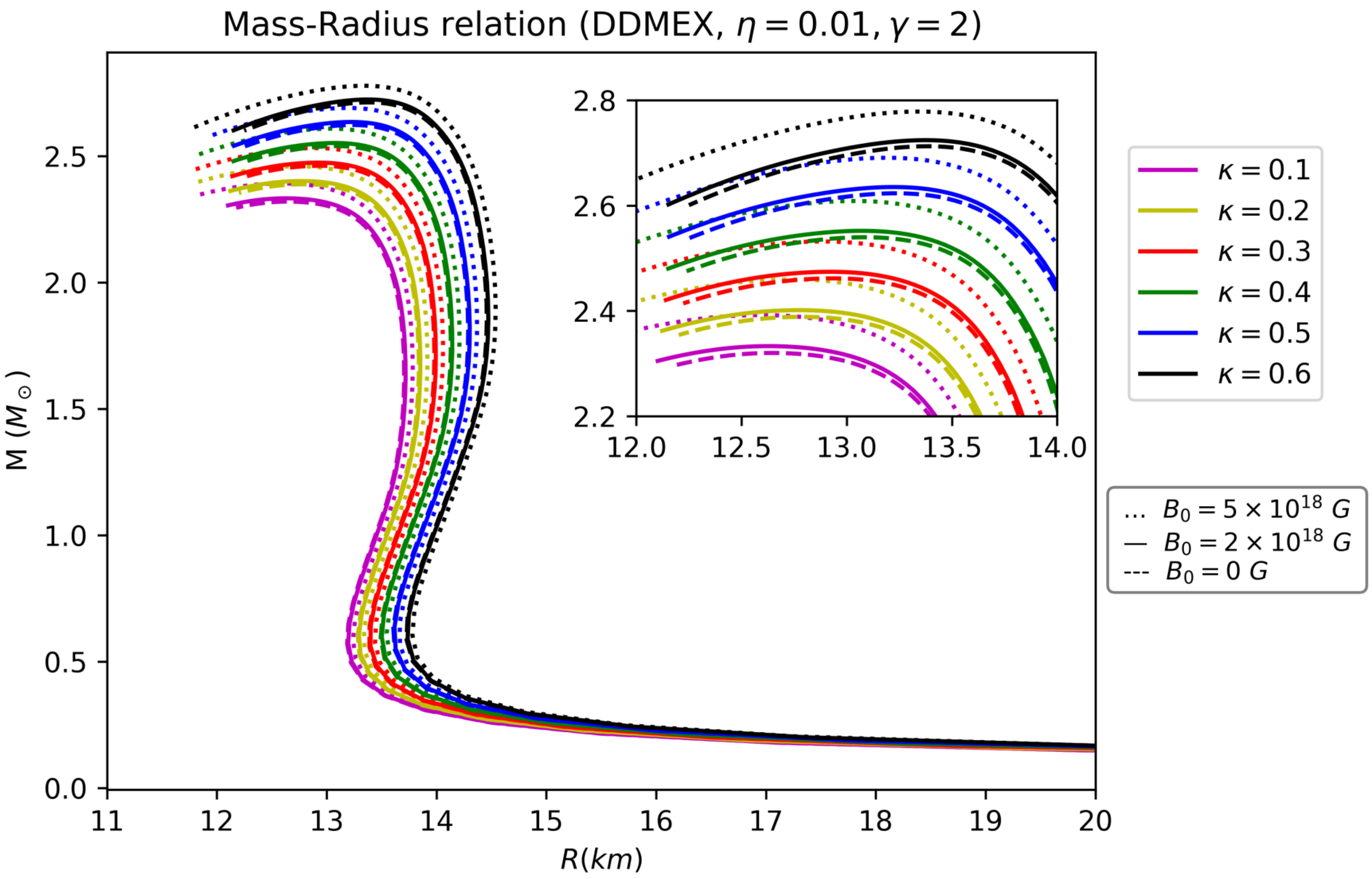
Changing anisotropy parameter, κ



SUMMARY

- We explored theoretical possibility of massive neutron stars which could potentially be candidates that fall in the "mass gap"
- Pure EOS effect not sufficient for mass gap.
- Require additional physics → magnetic field and/or anisotropy
- Field does not necessarily increase mass → RO vs. TO. Geometry determines how mass changes with respect to that without field.
- Tidal deformability further constrains mass of NS. Strong field induced very massive NS may not satisfy tidal deformability constraint.
- $2.5\ M_{\odot}$ NS is demonstrated to be still possible even with all constraints.