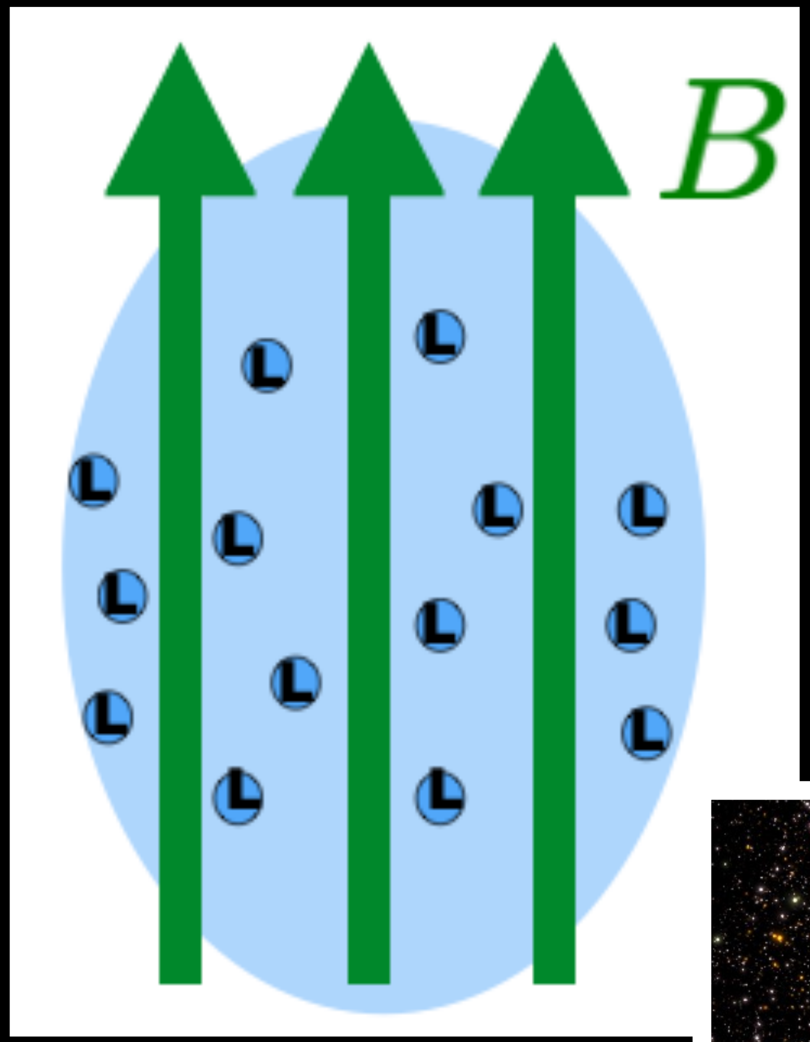
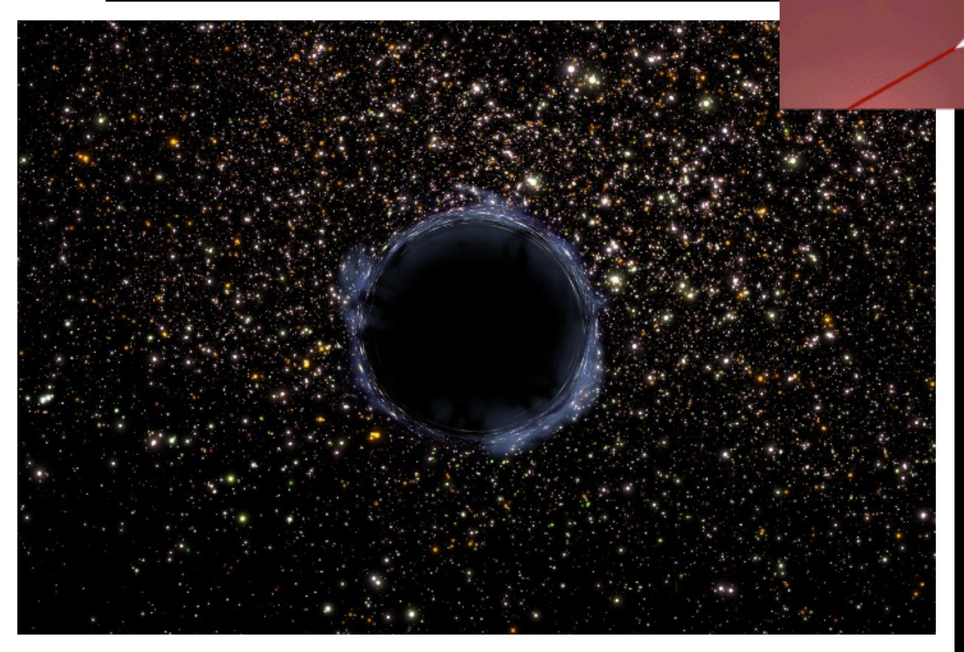
Novel hydrodynamic transport coefficients in extreme magnetic fields & the CME far from equilibrium

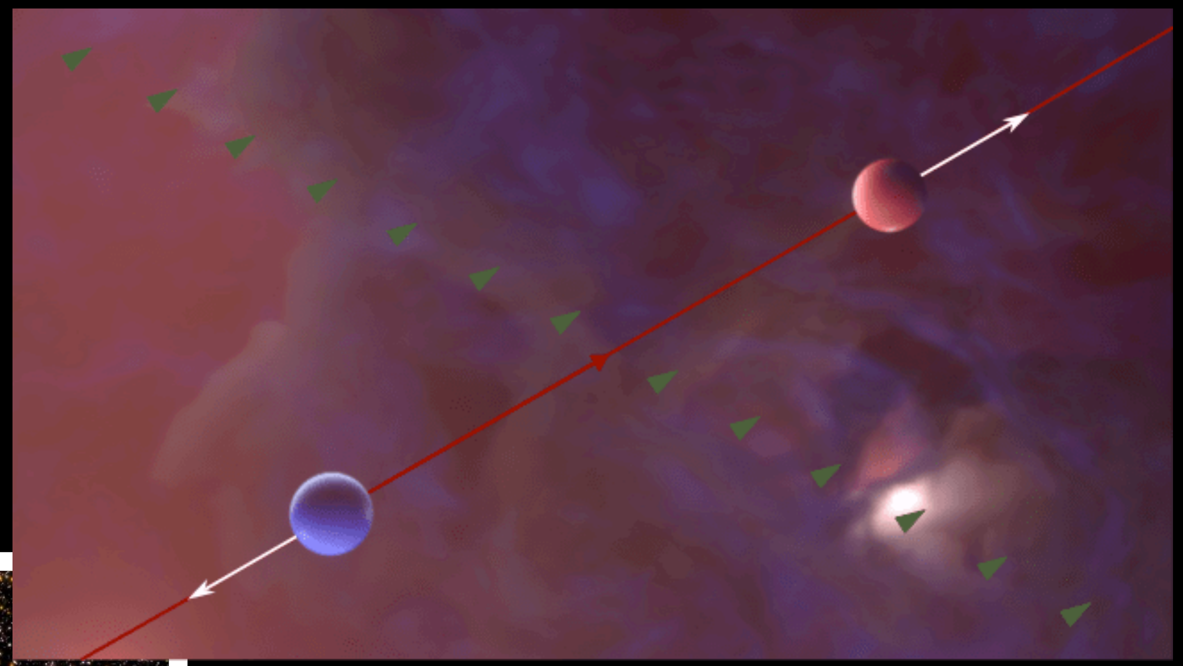
ECT* Workshop: Strongly interacting matter in extreme magnetic fields

ECT*, Trento, Italy

September 26th, 2023





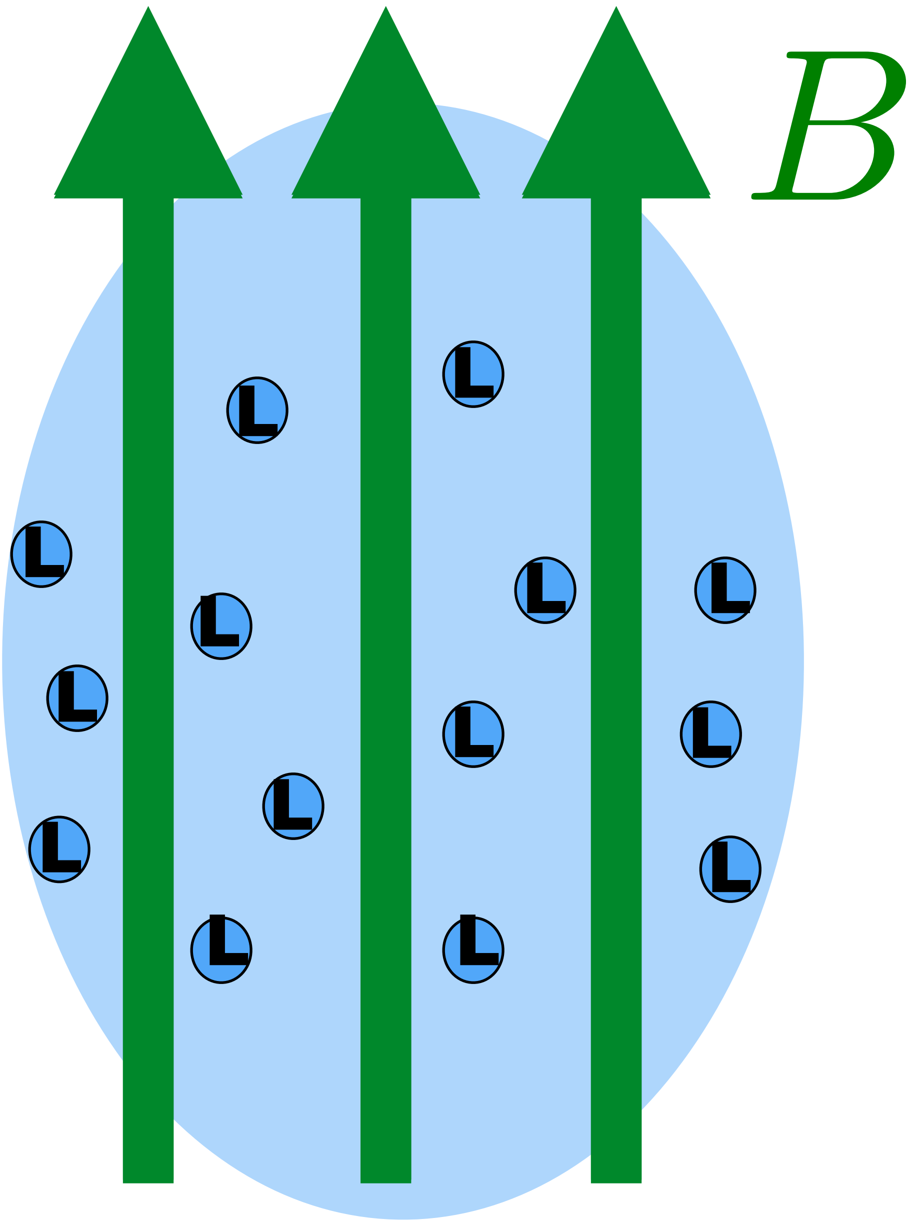


[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]
[Cartwright, Kaminski, Schenke; PRC (2022)]

Matthias Kaminski
University of Alabama

1. Novel transport coefficients in extreme magnetic fields

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]



charged (3+1)-dimensional relativistic fluid of chiral fermions in magnetic field

Preview of results: charged chiral hydrodynamics in strong magnetic fields

several novel transport effects, e.g. :

- ✦ **1 *perpendicular magnetic vorticity susceptibility***
- ✦ **1 non-dissipative *shear-induced Hall conductivity***

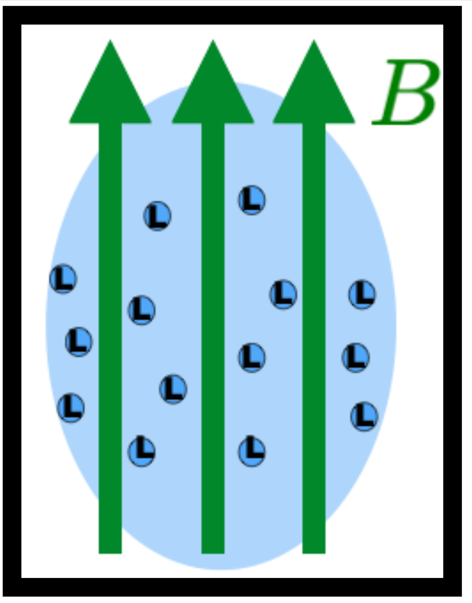
$$j_x \sim c_{10}(\partial_y v_z + \partial_z v_y)$$

- ✦ **Kubo formulae for >20 transport coefficients**

Reminder:
shear viscosity Kubo formula

$$\eta = \lim_{\omega\to 0}\frac{1}{2\omega}\int dt\, d\boldsymbol{x}\, e^{i\omega t}\,\langle[T_{xy}(x),\,T_{xy}(0)]\rangle$$

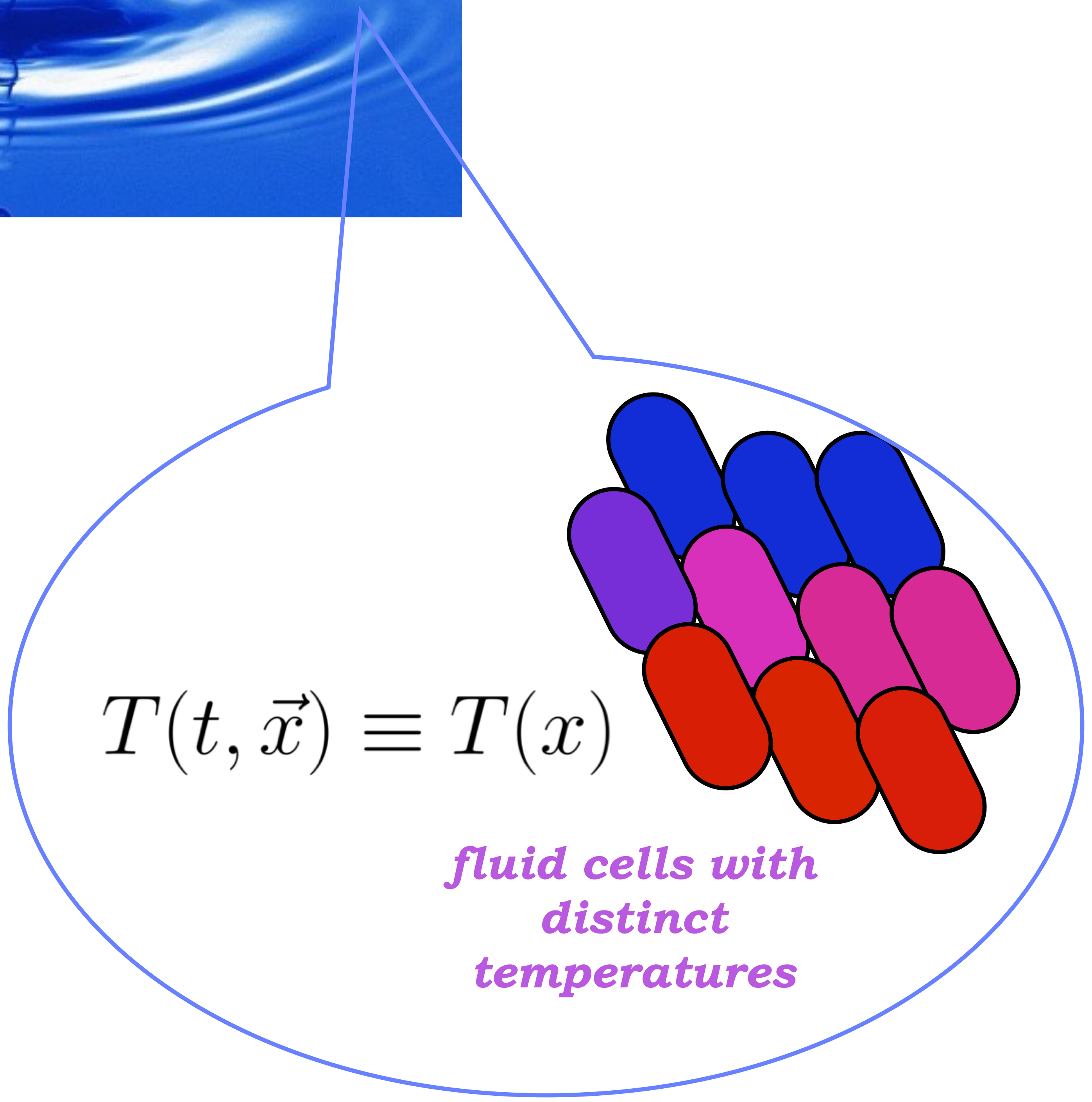
1. Chiral hydrodynamics - Concepts

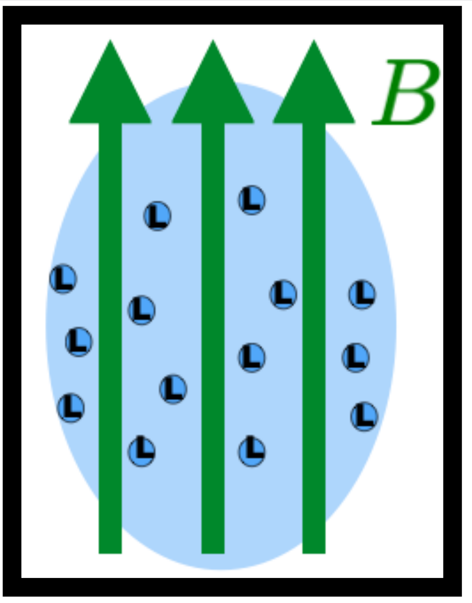


Hydrodynamics

- **effective field theory**
- expansion in gradients
- small gradients
- large temperature
- conserved quantities survive

$$\partial_t \, e^{-i\omega t} = -i\omega \, e^{-i\omega t}$$





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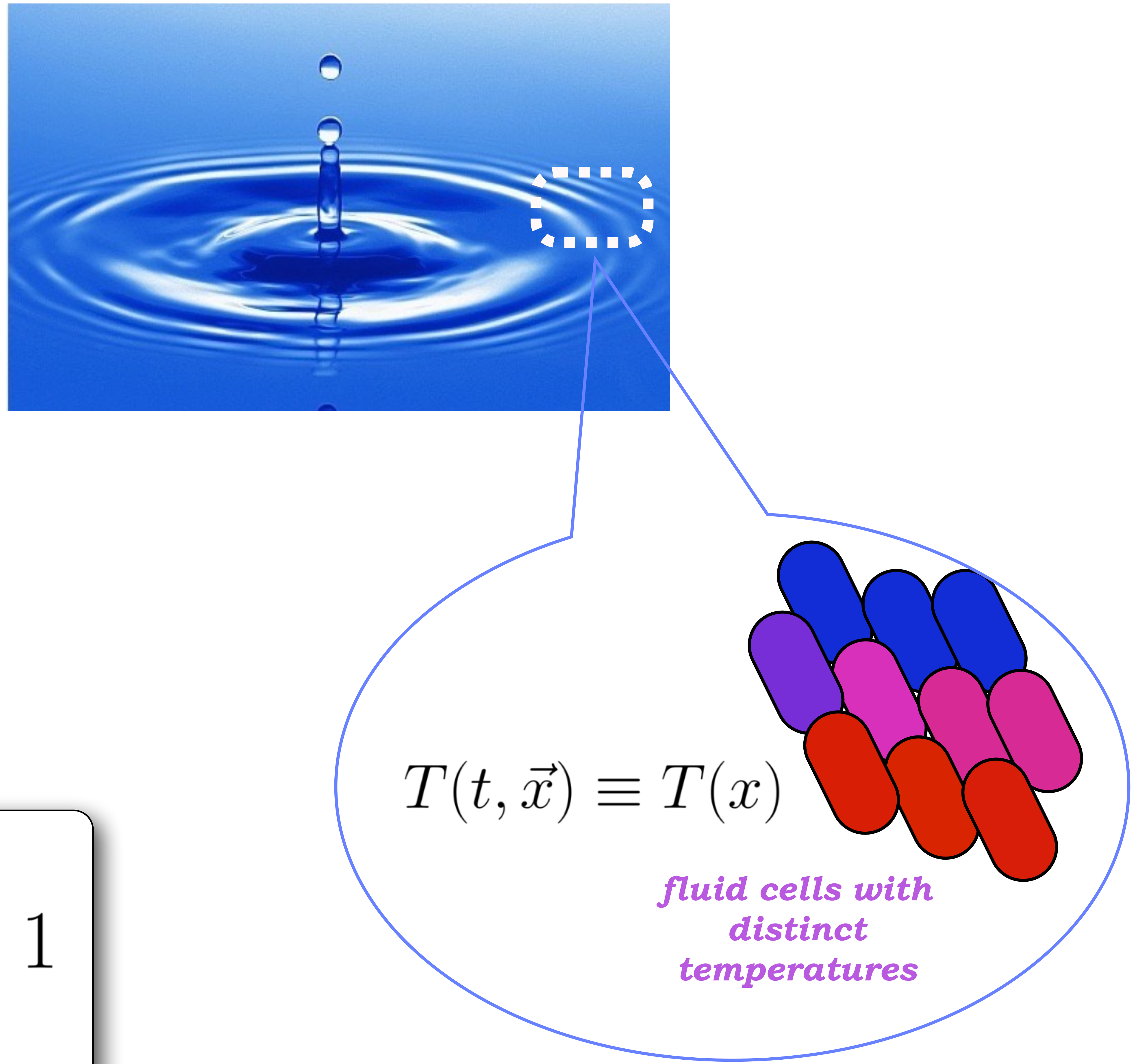
$$\partial_t\, e^{-i\omega t} = -i\omega\, e^{-i\omega t}$$

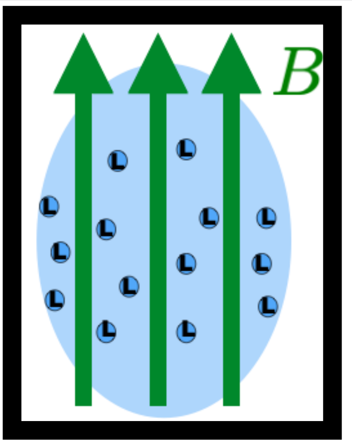
$$\frac{\omega}{T} \ll 1, \qquad \frac{|\vec{k}|}{T} \ll 1$$

In this presentation also :

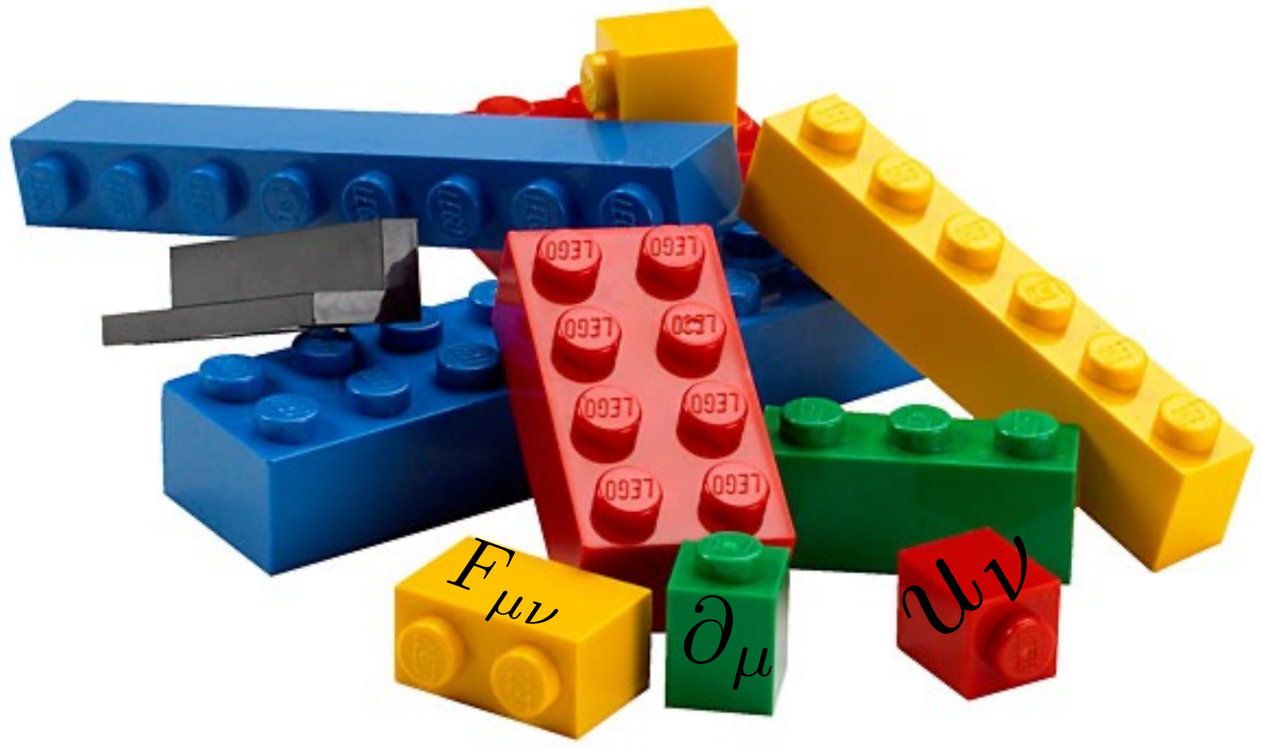
$$B \sim \mathcal{O}(1) \qquad B \ll T^2$$



1. Chiral hydrodynamics - Construction



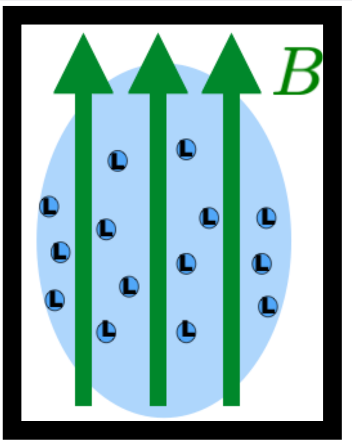
1. Constitutive equations: all (pseudo)vectors and (pseudo)tensors under Lorentz group



$$< j^\mu > = n u^\mu + \mathcal{O}(\partial) + \mathcal{O}(\partial^2) + \dots$$

Examples at $\mathcal{O}(\partial)$: $\nabla^\mu n$ *charge gradient (covariant derivative)*

vorticity $\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} u_\nu \nabla_\lambda u_\rho$



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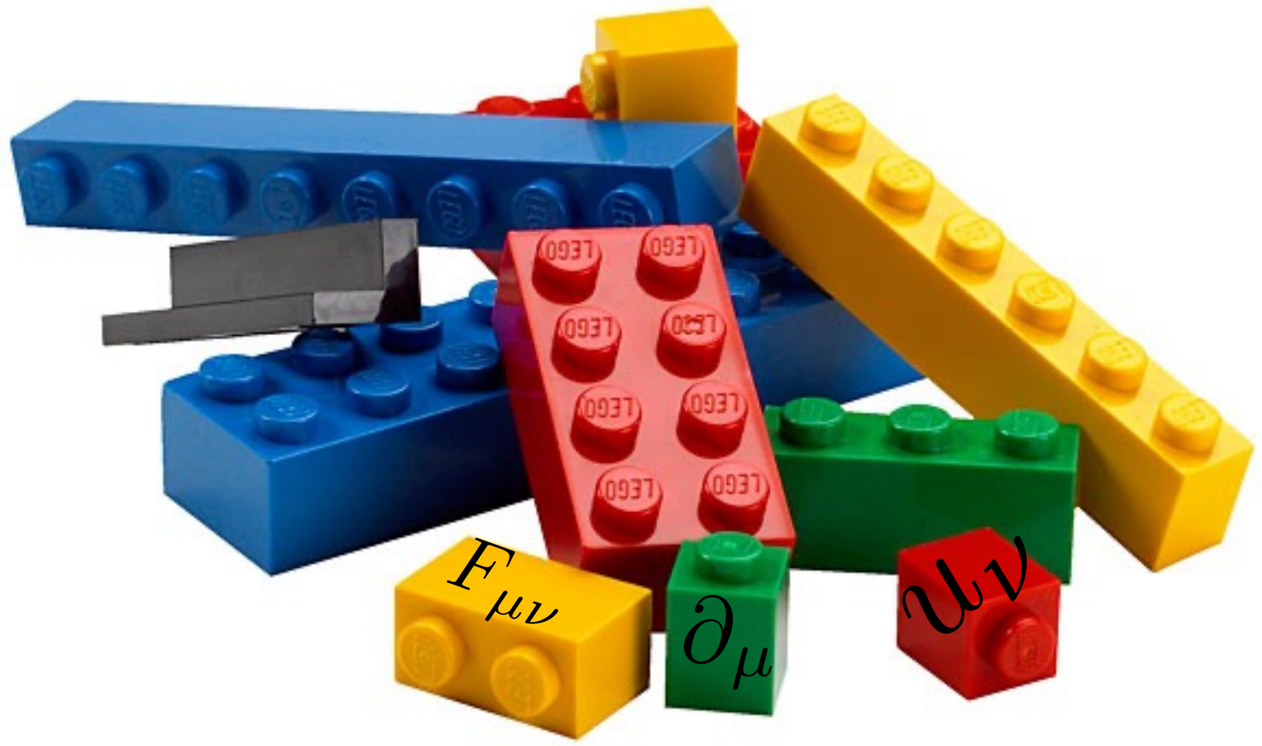

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2. Restricted by conservation equations

Example: $\nabla_\mu j^\mu_{(0)} = \nabla_\mu (n u^\mu) = 0$

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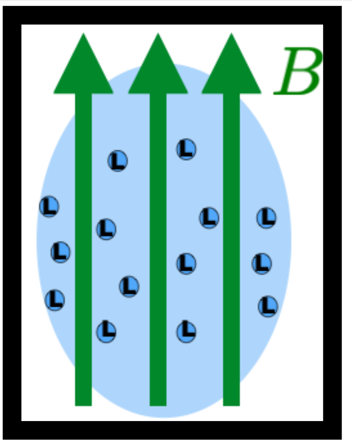

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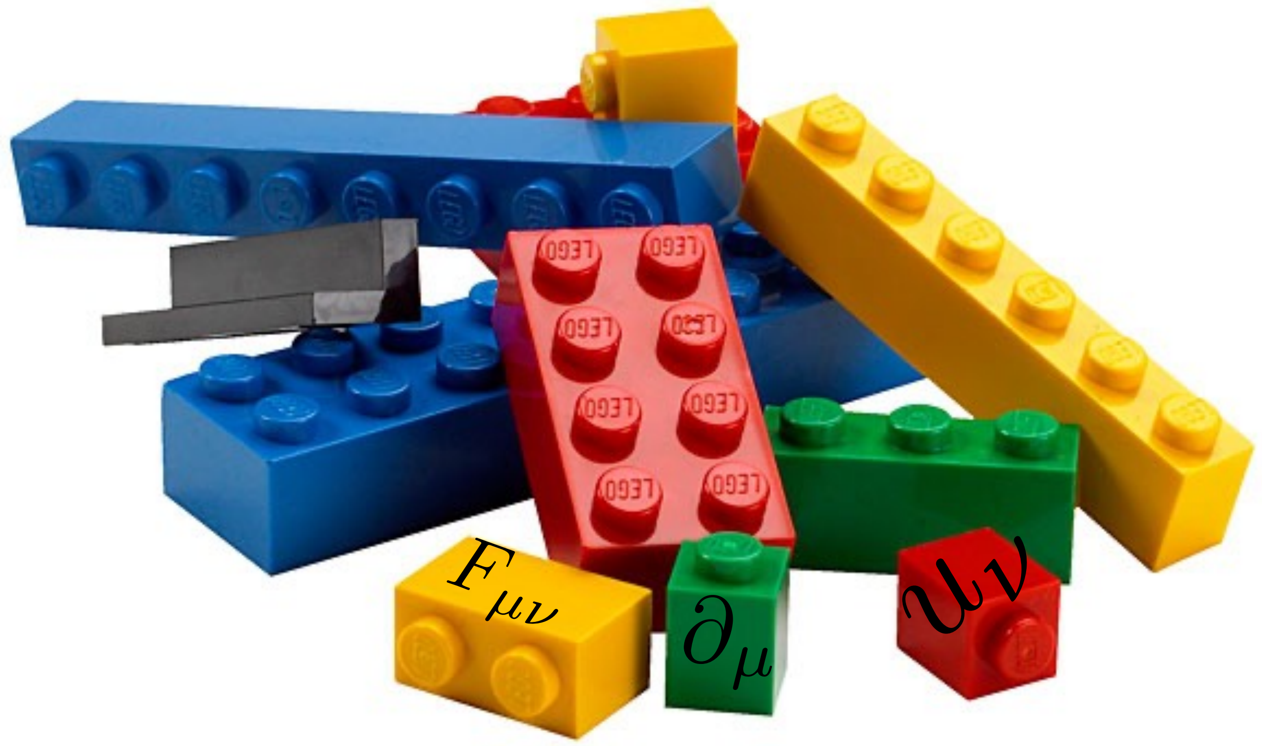

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Example: $\nabla_\mu j^\mu_{(0)} = \nabla_\mu (n u^\mu) = 0$

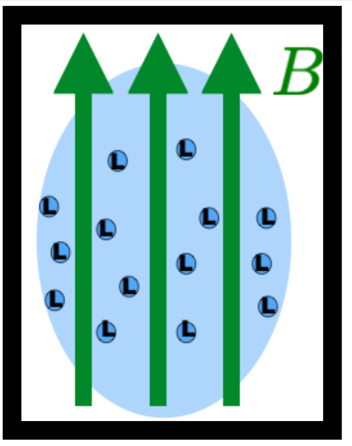
3. Further restricted by positivity of local entropy production: $\nabla_\mu J^\mu_s \geq 0$

[Landau, Lifshitz]

➡ Most general hydrodynamic 1-point functions for chiral charged fluid in strong magnetic field *[Ammon, Kaminski et al.; JHEP (2017)]*

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

1. Kubo-formula derivation example: hydrodynamic correlators in 2+1



Simple (non-chiral) example in 2+1 dims:

$$j^\mu = n u^\mu + \sigma \left[E^\mu - T \Delta^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) \right]$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$u^\mu = (1, 0, 0)$$

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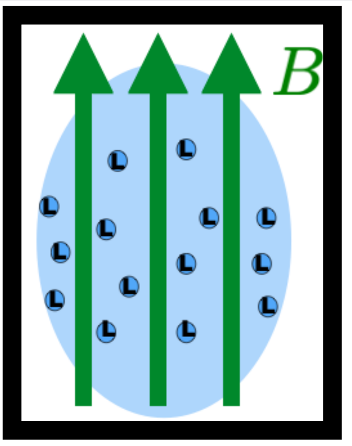

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sources $\quad A_t,\ A_x \propto e^{-i\omega t + ikx}$

fluctuations $\quad n = n(t, x, y) \propto e^{-i\omega t + ikx}$ $\quad$ (fix T and u)

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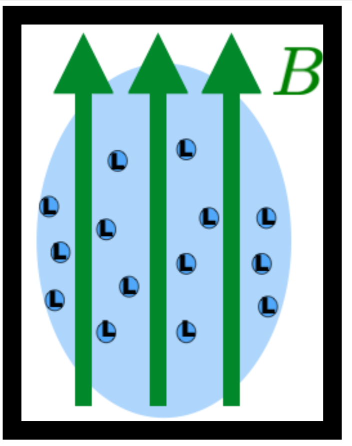

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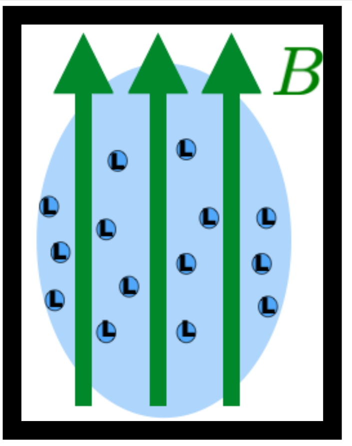

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one point functions $\quad$ (use $\nabla_\mu j^\mu = 0$)

$$\langle j^t \rangle = n(\omega, k) = \frac{ik\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle j^x \rangle = \frac{i\omega\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle j^y \rangle = 0$$

susceptibility: $\quad \chi = \frac{\partial n}{\partial \mu}$

Einstein relation: $\quad D = \frac{\sigma}{\chi}$

$\Longrightarrow$ two point functions $\quad \langle j^x j^x \rangle = \frac{\delta \langle j^x \rangle}{\delta A_x} = \frac{i\omega^2 \sigma}{\omega + iDk^2}$

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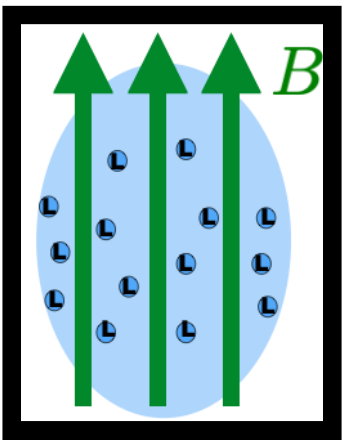

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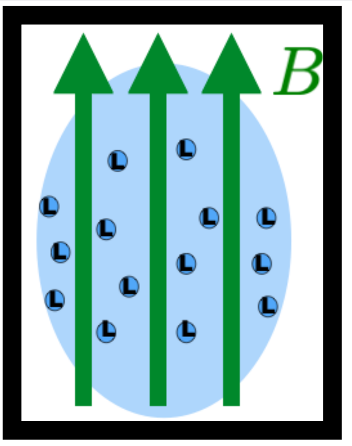
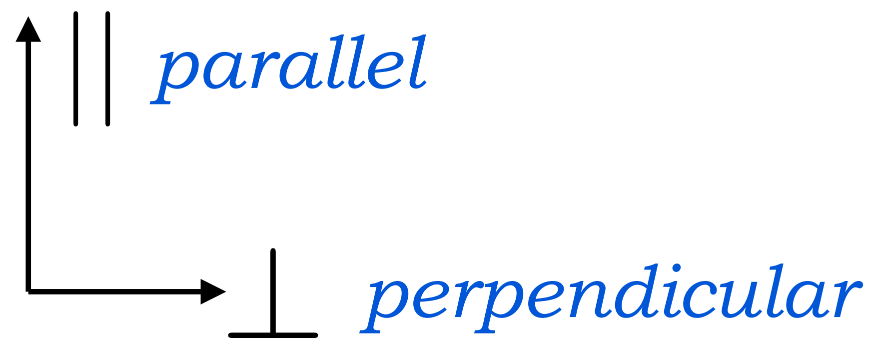
$\Longrightarrow$ **Kubo formula:** $\sigma = \lim_{\omega \to 0} \frac{1}{i\omega} \langle j^x j^x \rangle (\omega, k = 0)$

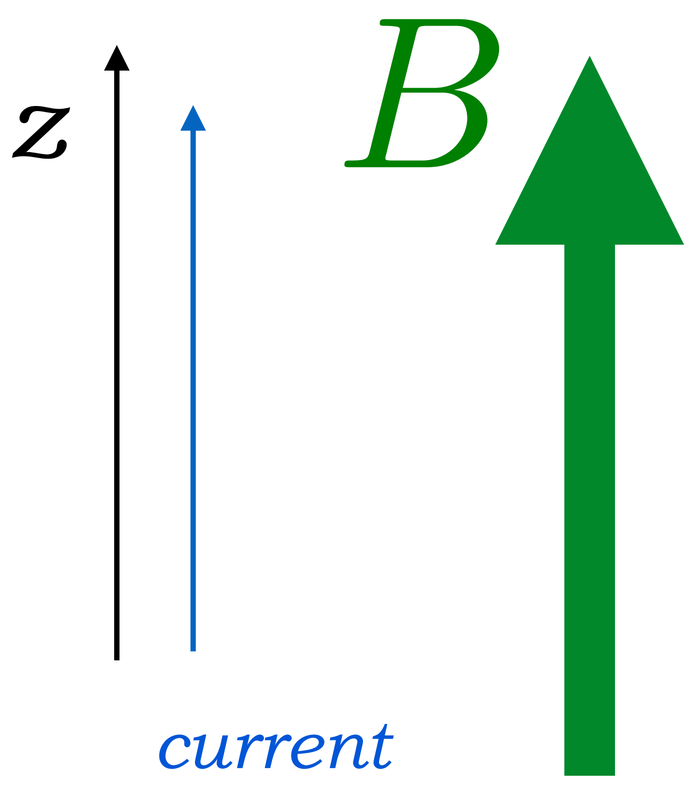
1. Chiral hydrodynamics - conductivity Kubo formulae



Parallel **conductivity**

$$\lim_{\omega\to 0} \frac{1}{\omega}\mathrm{Im}\,\langle J^z J^z\rangle(\omega, \mathbf{k}=0) = \sigma_{\|} + \cdots$$





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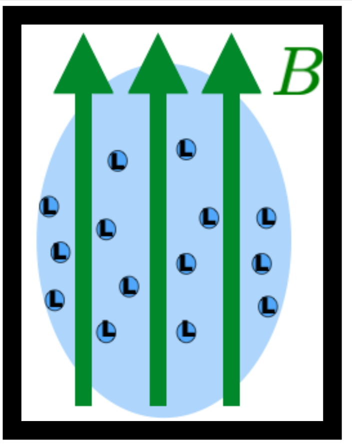

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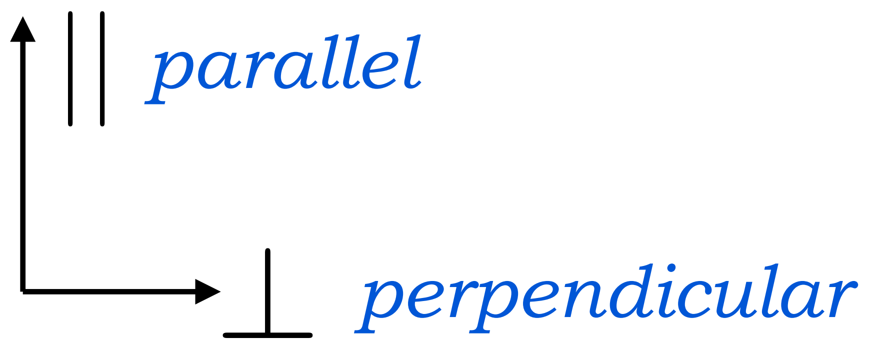
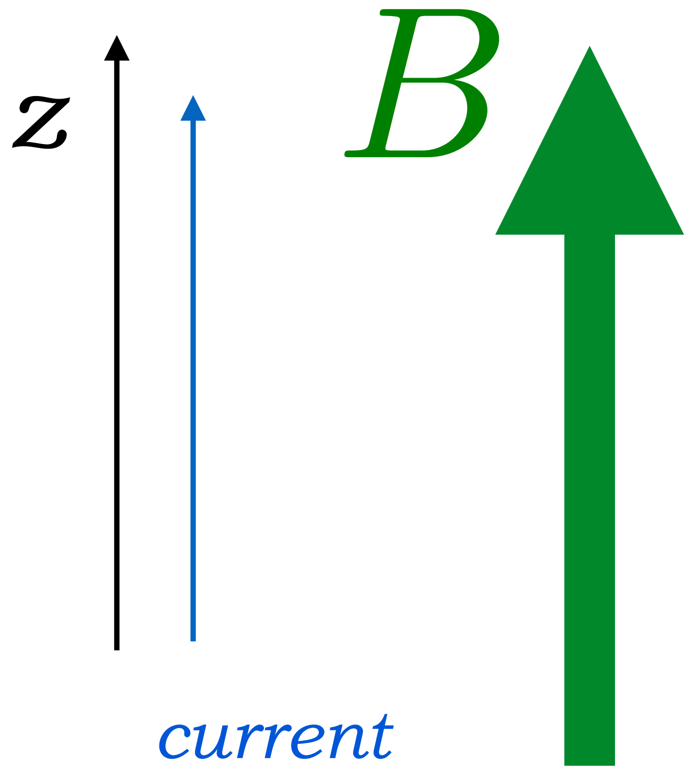
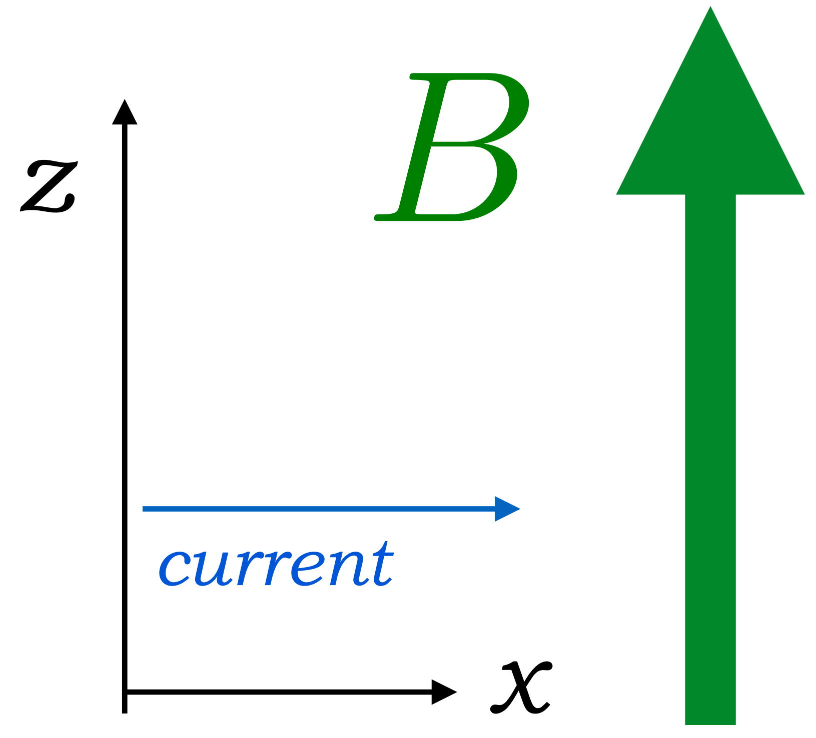




Perpendicular **resistivity**

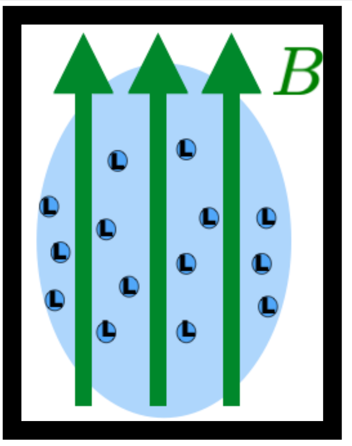
$$\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,\langle J^x J^x\rangle(\omega,\mathbf{k}{=}0) = \omega^2\rho_\perp\frac{w_0(w_0 - M_{5,\mu}B_0^2)}{B_0^4}$$



Very different parallel versus perpendicular

$$\langle J^z J^z\rangle(\omega,\mathbf{k}=0)\sim\sigma_{||}$$

$$\langle J^x J^x\rangle(\omega,\mathbf{k}=0)\sim\rho_\perp$$

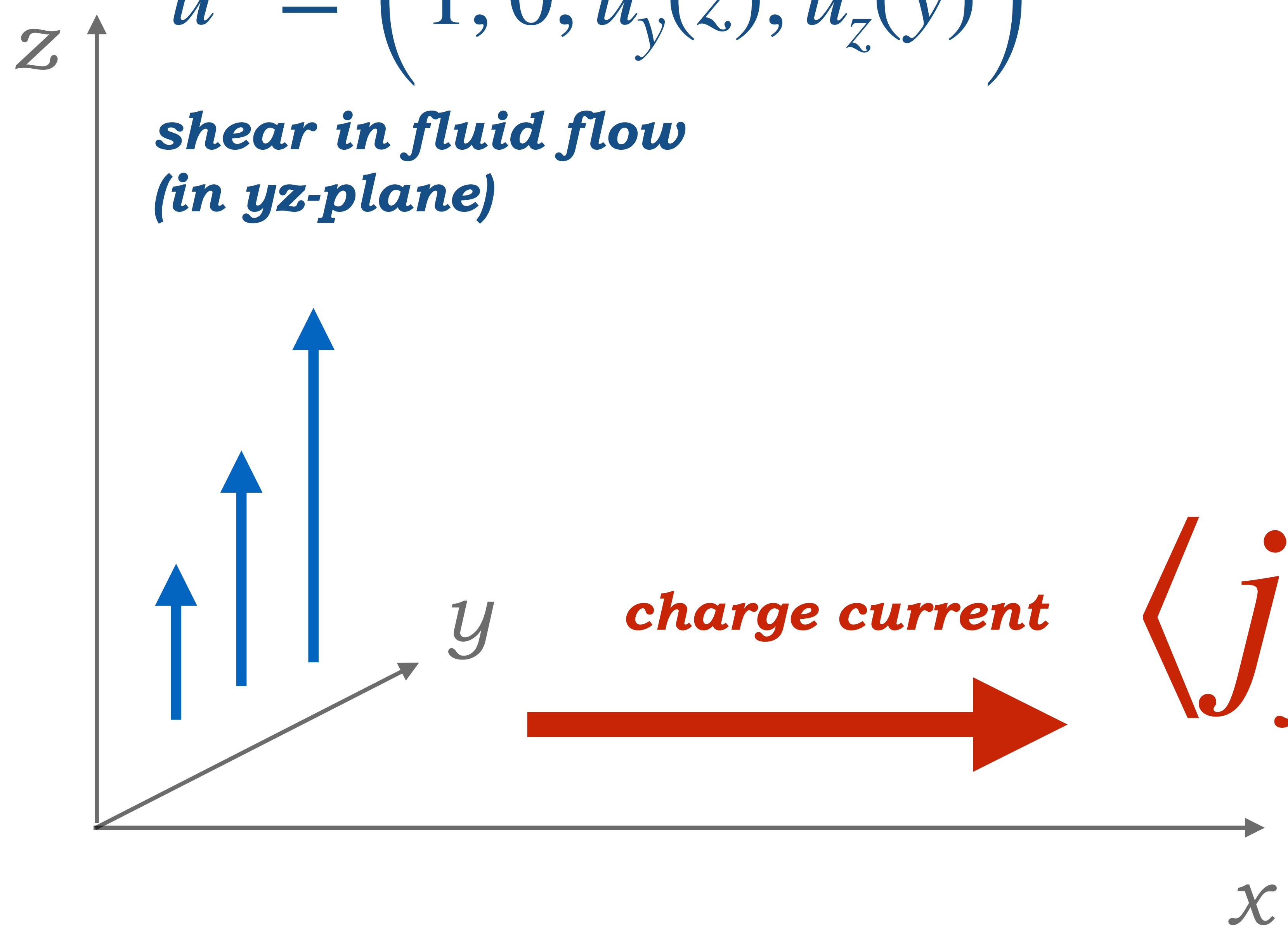


[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

Shear-induced Hall conductivity c_{10}

$$c_{10} \sim \lim_{\omega \to 0} \frac{1}{\omega} \text{Im} \langle T^{tx} T^{yz} \rangle (\omega, \vec{k} = 0)$$

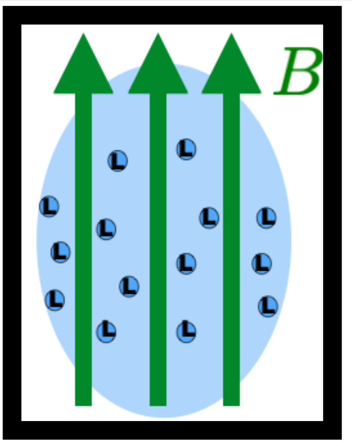
$$u^\nu = \left(1, 0, u_y(z), u_z(y)\right)$$



$$\langle j_x \rangle \sim c_{10}(\partial_y u_z + \partial_z u_y)$$

- ➡ **novel Hall response**
- ➡ **non-dissipative**
- ➡ **interplay: shear-charge**

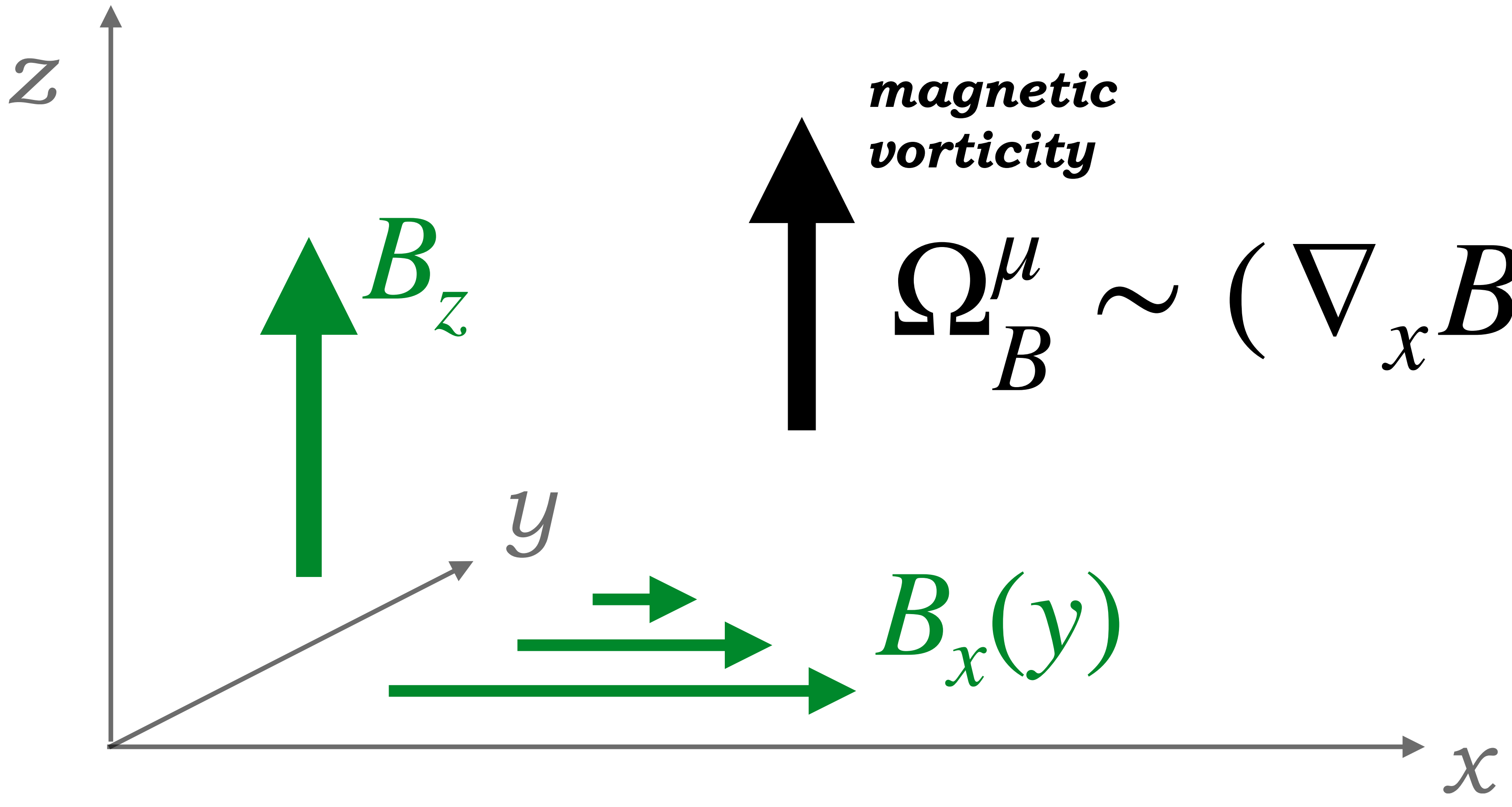
1. Chiral hydrodynamics - novel susceptibility M_2



[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

Perpendicular magnetic vorticity susceptibility M_2

$$M_2 = -\lim_{k_z \to 0} \frac{1}{2 k_z B_0^2} \mathrm{Im} \langle T^{xz} T^{yz} \rangle (\omega = 0, k_z)$$



$$\Omega_B^\mu \sim (\nabla_x B_y - \nabla_y B_x)$$

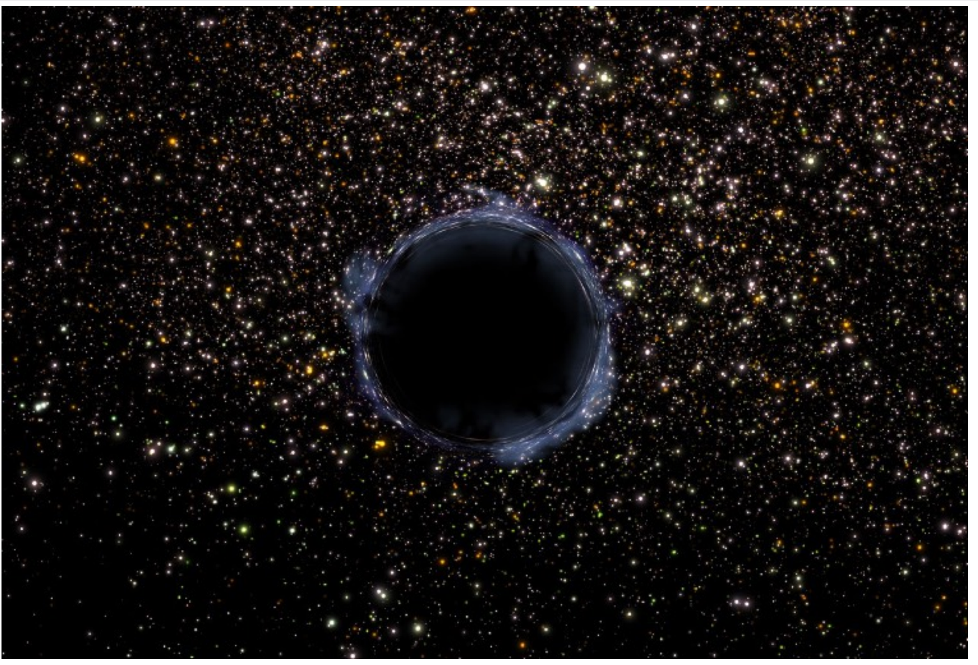
response in energy/pressure :

$$\langle T^{tt} \rangle = \mathcal{E}_{\text{eq}} \sim \mathcal{P}_{\text{eq}} \sim M_2 B \cdot \Omega_B$$

magnetic vorticity : $\Omega_B^\mu = \epsilon^{\mu\nu\rho\sigma} u_\nu \nabla_\rho B_\sigma$

➡ Can we test these Kubo formulae and constitutive relations?

2. Holographic model for chiral hydrodynamics



➡ **Construct holographic dual to charged plasma in strong *B***

➡ **Compute values for novel transport coefficients (*N*=4 SYM)**

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

cf. [Son,Surowka; PRL (2009)]
[Erdmenger,Haack,Kaminski,Yarom; JHEP (2008)]

Einstein-Maxwell-Chern-Simons action

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A\wedge F\wedge F\right]$$

5-dimensional Chern-Simons term encodes chiral anomaly

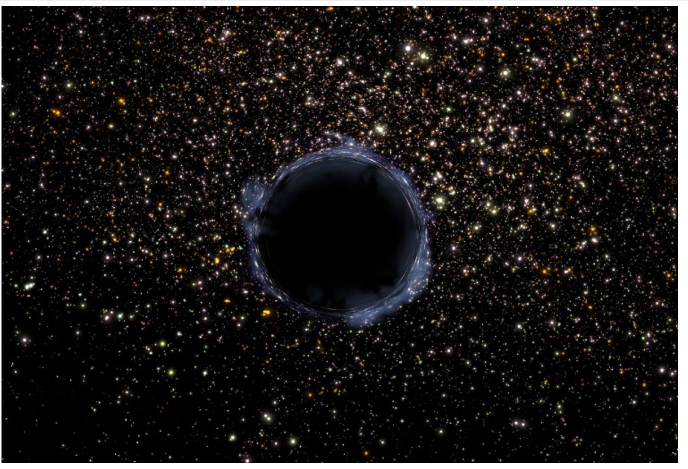
Charged magnetic black branes

[D'Hoker, Kraus; JHEP (2010)]

- **charged magnetic** analog of Reissner-Nordstrom black brane
- asymptotically AdS_5

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

2. Holographic model for chiral hydrodynamics - Results



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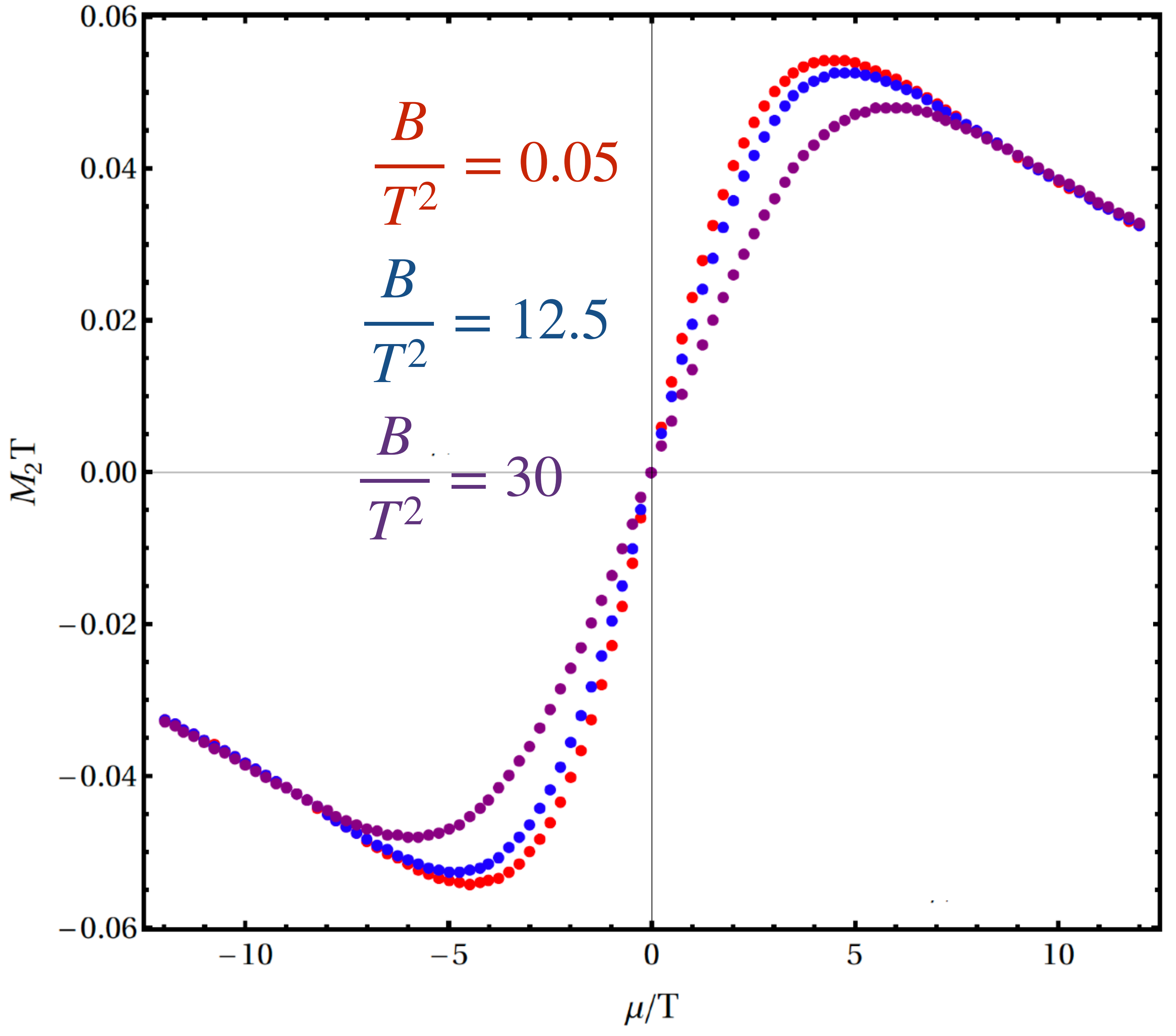
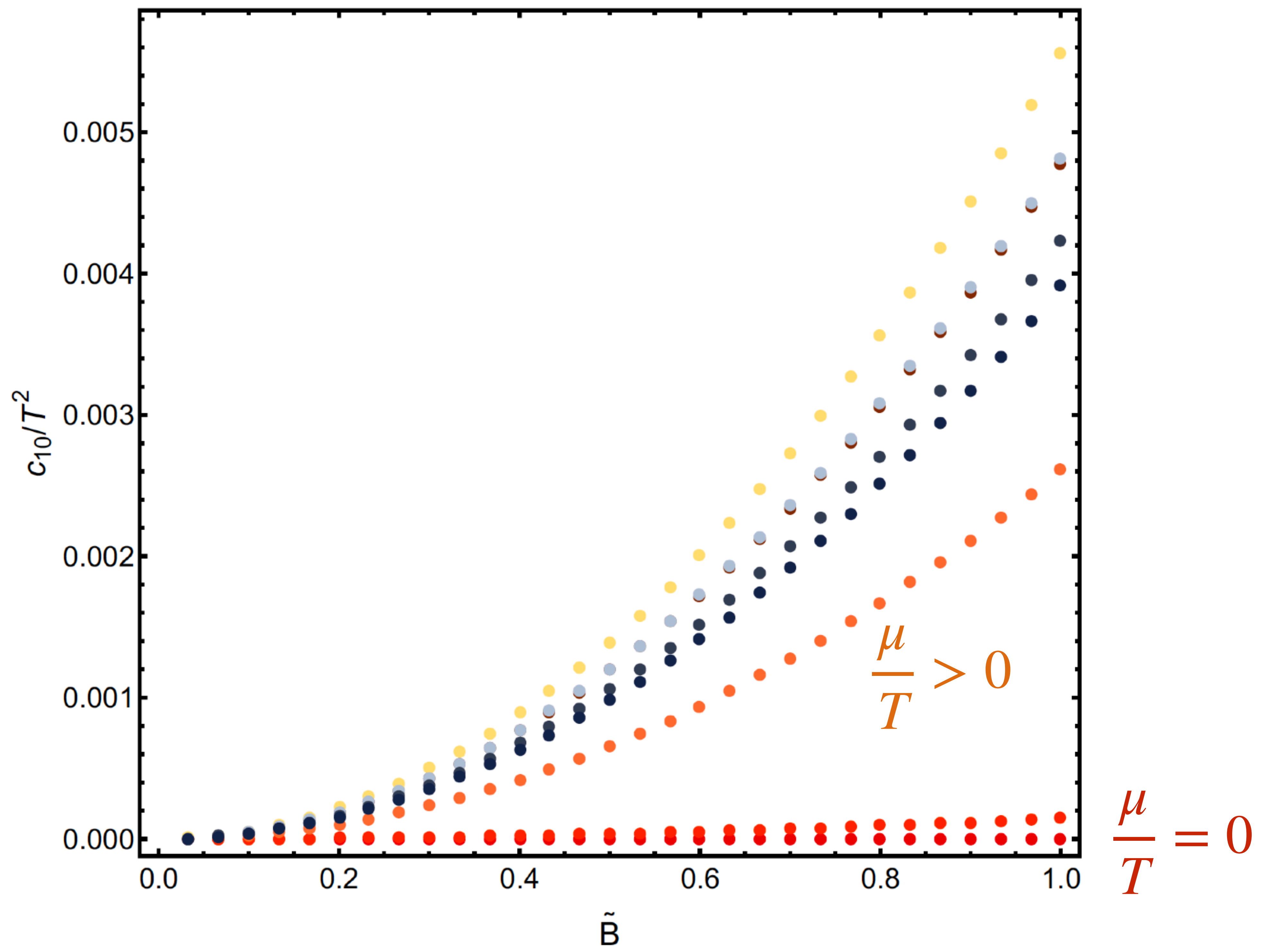


Shear-induced Hall conductivity c_{10}



➡ **not zero, finite, Onsager satisfied**
➡ ***all* Kubo formulae consistent**

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
Thermodynamic $\left(\lim_{\mathbf{k}\to 0}\lim_{\omega\to 0}\right)$, non-dissipative					
helicity 1					
M_2	perp. magnetic vorticity susceptibility	$T^{xz}T^{yz}$ (2.30)	+	-	+
M_5	magneto-vortical susceptibility	$T^{tx}T^{yz}$ (2.30,2.31)	+	-	+
ξ	chiral vortical conductivity	$J_x T_{ty}$ (2.38,2.39)	+	+	+
ξ_B	chiral magnetic conductivity	$J^x J^y$ (2.38,2.39)	+	-	+
ξ_T	chiral vortical heat conductivity	$T^{tx}T^{ty}$ (2.38,2.39)	+	-	+
helicity 0					
M_1	magneto-thermal susceptibility	$J^t T^{xx}$ (2.32)	+	+	-
M_3	magneto-acceleration susceptibility	$J^t T^{tt}$ (2.32)	+	+	-
M_4	magneto-electric susceptibility	$J^t J^t$ (2.32)	+	-	-

Non-dissipative Hydrodynamic $\left(\lim_{\omega\to 0}\lim_{\mathbf{k}\to 0}\right)$					
coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
helicity 2					
$\tilde{\eta}_\perp$	transverse Hall viscosity	$T_{xy}(T_{xx}-T_{yy})$(2.55f)	+	-	+
helicity 1					
$\boxed{c_{10}} \propto c_{17}$	shear-induced Hall cond.	$T^{tx}T^{xz}, T^{tx}T^{yz}$ (2.60,2.62a,2.62b)	+	+	+
$\tilde{\sigma}_\perp$	Hall conductivity	$J^x J^x, J^x J^y$ (2.54,2.53b,2.53c)	+	-	+

dissipative, hydrodynamic $\left(\lim_{\omega\to 0}\lim_{\mathbf{k}\to 0}\right)$					
coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
helicity 2					
$\eta_\perp$	perp. shear viscosity	$T_{xy}T_{xy}$ (2.55)	+	+	-
helicity 1					
$\eta_{\|\|}$	parallel shear viscosity	$T^{xz}T^{xz}$ (2.59a)	+	+	-
$\tilde{\eta}_{\|\|}$	parallel Hall viscosity	$T_{yz}T_{xz}$ (2.59b)	+	-	+
$\boxed{c_8} \propto c_{15}$	shear-induced conductivity	$T_{tx}T_{xz}$, $T_{tx}T_{yz}$ (2.57)	+	+	+
$\rho_\perp$	perp. resistivity	$J^x J^x$ (2.54)	+	+	-
$\tilde{\rho}_\perp$	Hall resistivity	$J^x J^y$ (2.55e)	+	+	-
$\sigma_{\|\|}$	long. conductivity	$J^z J^z$ (2.53a)	+	+	-
$\sigma_\perp$	perp. conductivity	$\rho_{ab} \equiv (\sigma^{-1})_{ab} = \rho_\perp \delta_{ab} + \tilde{\rho}_\perp \epsilon_{ab}$	+	+	-
helicity 0					
η_1	bulk viscosity	$\mathcal{O}_1\mathcal{O}_1$ (2.55c)	+	+	-
η_2	bulk viscosity	$\mathcal{O}_2\mathcal{O}_2$ (2.55d)	+	+	-
ζ_1	bulk viscosity	$T^{ij}(T^{xx}+T^{yy})$(2.55a)	+	+	-
ζ_2	bulk viscosity	$3\zeta_2 - 6\eta_1 = 2\eta_2$	+	+	-
$\boxed{c_4}$	expan.-induced long. cond.	$J_x T_{xx}$ (2.57)	+	-	-
$\boxed{c_5}$	expan.-induced long. cond.	$J_z T_{zz}$ (2.57)	+	-	-
c_3		$c_5 = -3(c_3 + c_4)$	+	-	-

cf. [Hernandez, Kovtun; JHEP (2017)]

2. Chiral hydrodynamics & holography - all coefficients

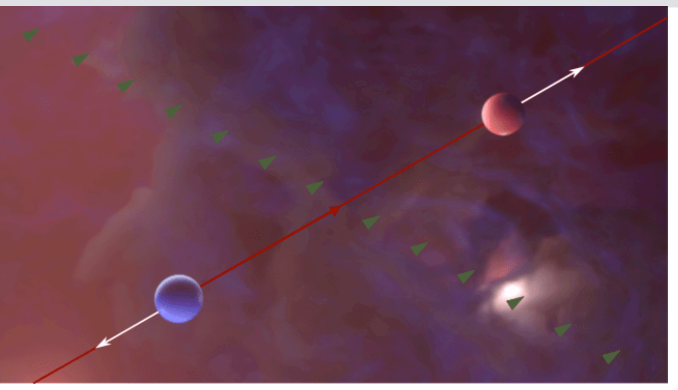
[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
Thermodynamic $\left(\lim_{\mathbf{k}\to 0}\lim_{\omega\to 0}\right)$, non-dissipative					
helicity 1					
M_2	perp. magnetic vorticity susceptibility	$T^{xz}T^{yz}$ (2.30)	+	-	+
M_5	magneto-vortical susceptibility	$T^{tx}T^{yz}$ (2.30,2.31)	+	-	+
ξ	chiral vortical conductivity	J_xT_{ty} (2.38,2.39)	+	+	+
ξ_B	chiral magnetic conductivity	J^xJ^y (2.38,2.39)	+	-	+
ξ_T	chiral vortical heat conductivity	$T^{tx}T^{ty}$ (2.38,2.39)	+	-	+
helicity 0					
M_1	magneto-thermal susceptibility	J^tT^{xx} (2.32)	+	+	-
M_3	magneto-acceleration susceptibility	J^tT^{tt} (2.32)	+	+	-
M_4	magneto-electric susceptibility	J^tJ^t (2.32)	+	-	-

coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
Non-dissipative Hydrodynamic $\left(\lim_{\omega\to 0}\lim_{\mathbf{k}\to 0}\right)$					
helicity 2					
$\tilde{\eta}_\perp$	transverse Hall viscosity	$T_{xy}(T_{xx}-T_{yy})$(2.55f)	+	-	+
helicity 1					
$\boxed{c_{10}} \propto c_{17}$	shear-induced Hall cond.	$T^{tx}T^{xz}, T^{tx}T^{yz}$ (2.60,2.62a,2.62b)	+	+	+
$\tilde{\sigma}_\perp$	Hall conductivity	J^xJ^x, J^xJ^y (2.54,2.53b,2.53c)	+	-	+

coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
dissipative, hydrodynamic $\left(\lim_{\omega\to 0}\lim_{\mathbf{k}\to 0}\right)$					
helicity 2					
$\eta_\perp$	perp. shear viscosity	$T_{xy}T_{xy}$ (2.55)	+	+	-
helicity 1					
$\eta_\parallel$	parallel shear viscosity	$T^{xz}T^{xz}$ (2.59a)	+	+	-
$\tilde{\eta}_\parallel$	parallel Hall viscosity	$T_{yz}T_{xz}$ (2.59b)	+	-	+
$\boxed{c_8} \propto c_{15}$	shear-induced conductivity	$T_{tx}T_{xz}$, $T_{tx}T_{yz}$ (2.57)	+	+	+
$\rho_\perp$	perp. resistivity	J^xJ^x (2.54)	+	+	-
$\tilde{\rho}_\perp$	Hall resistivity	J^xJ^y (2.55e)	+	+	-
$\sigma_\parallel$	long. conductivity	J^zJ^z (2.53a)	+	+	-
$\sigma_\perp$	perp. conductivity	$\rho_{ab}\equiv(\sigma^{-1})_{ab}=\rho_\perp\delta_{ab}+\tilde{\rho}_\perp\epsilon_{ab}$	+	+	-
helicity 0					
η_1	bulk viscosity	$\mathcal{O}_1\mathcal{O}_1$ (2.55c)	+	+	-
η_2	bulk viscosity	$\mathcal{O}_2\mathcal{O}_2$ (2.55d)	+	+	-
ζ_1	bulk viscosity	$T^{ij}(T^{xx}+T^{yy})$(2.55a)	+	+	-
ζ_2	bulk viscosity	$3\zeta_2-6\eta_1=2\eta_2$	+	+	-
$\boxed{c_4}$	expan.-induced long. cond.	J_xT_{xx} (2.57)	+	-	-
$\boxed{c_5}$	expan.-induced long. cond.	J_zT_{zz} (2.57)	+	-	-
c_3		$c_5=-3(c_3+c_4)$	+	-	-

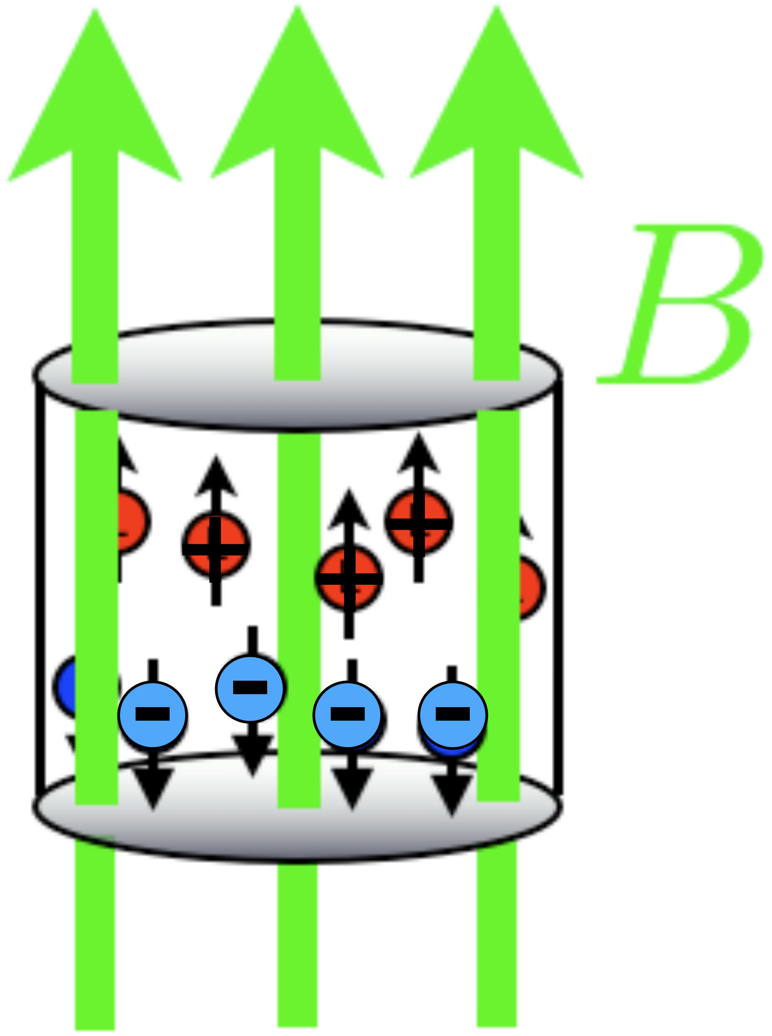
cf. [Hernandez, Kovtun; JHEP (2017)]



[DOE Highlight Article; Cartwright,Kaminski,Schenke (2023)]

The Chiral Magnetic Effect (CME) caused by chiral anomaly

[Kharzeev; PRC (2004)]
[Fukushima,Kharzeev,Warringa; PRD (2008)]
[Son,Surowka; PRL (2009)]
[Neiman,Oz; JHEP (2010)]



Electric charge current:

$$J^\mu = \xi_\chi B$$

Chiral magnetic conductivity: $\xi_\chi = C\,\mu_A$

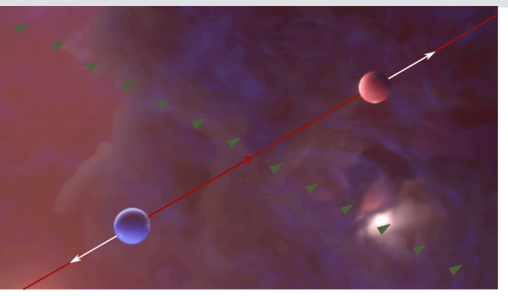
Anomalous axial current divergence: $\nabla_\mu J^\mu_A = C\, E \cdot B$

axial charges are generated in aligned E- and B-fields

Needed for observation:

- ➡**chiral anomaly**
- ➡**axial charge imbalance**
- ➡**magnetic field**
- ➡**sufficient life time**

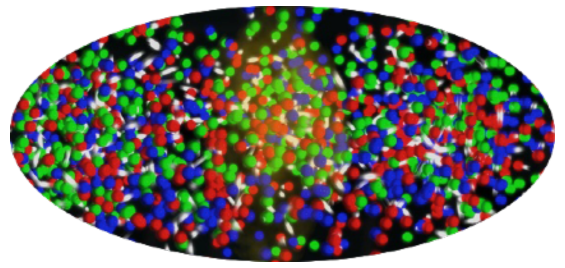
3. Far from equilibrium holography



[DOE Highlight Article; Cartwright,Kaminski, Schenke (2023)]

Thermalization in field theory:

[Janik,Peschanski; PRD (2006)]
[Chesler, Yaffe; PRL (2009)]



T=0 particle “soup”

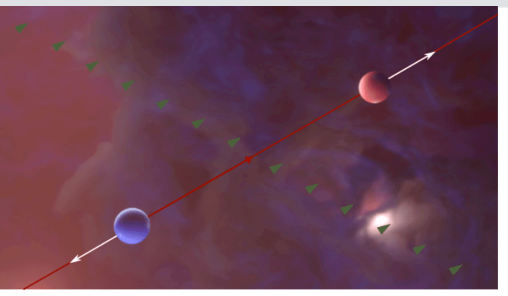
$$\langle T^{\mu\nu}\rangle(v)$$

thermal QFT

time v

nonzero T plasma

3. Far from equilibrium holography



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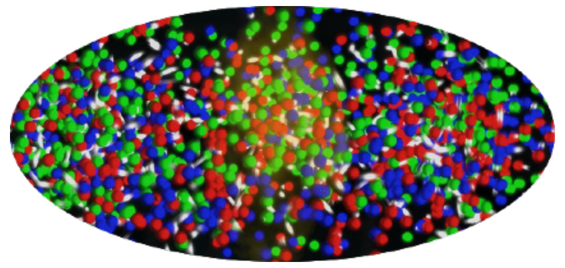

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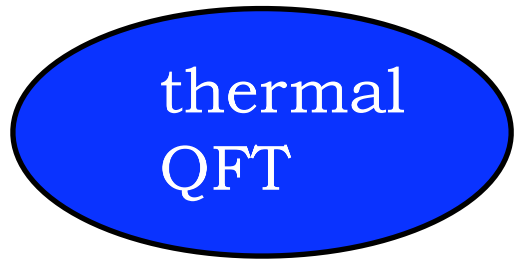
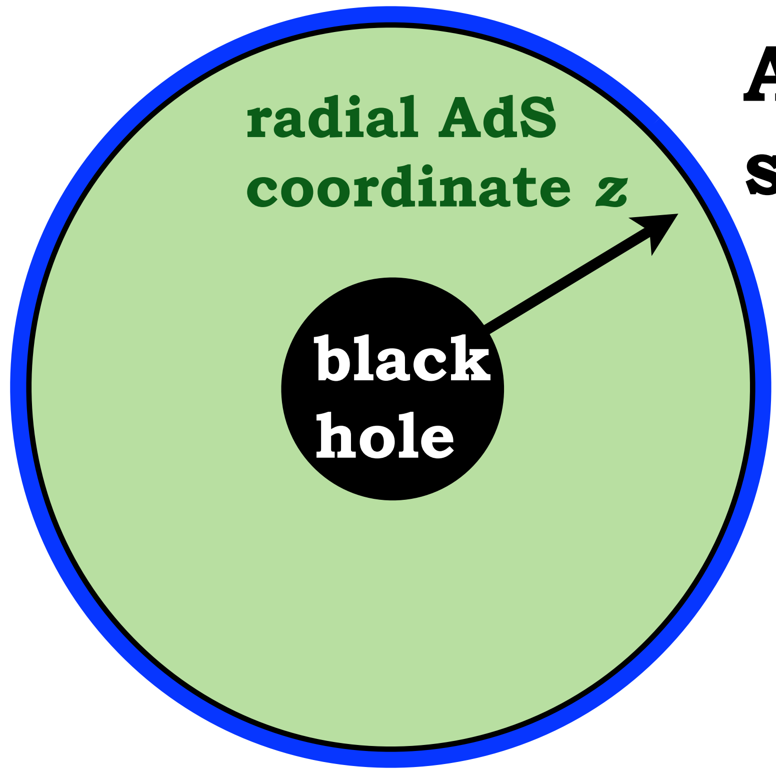


nonzero T plasma

time v

gauge/gravity correspondence

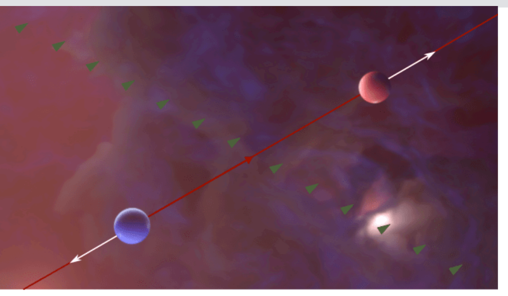
$$\langle T^{\mu\nu}\rangle(v) \longleftrightarrow g_{\mu\nu}(v)$$



Anti-de Sitter space boundary

thermal QFT "lives" here temperature T

3. Far from equilibrium holography

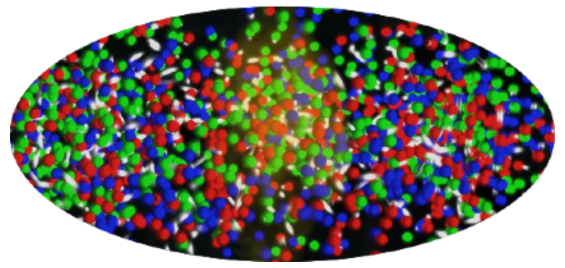


[DOE Highlight Article; Cartwright,Kaminski, Schenke (2023)]

Thermalization in field theory:

Horizon formation in gravity:

[Janik,Peschanski; PRD (2006)]
[Chesler, Yaffe; PRL (2009)]



T=0 particle "soup"

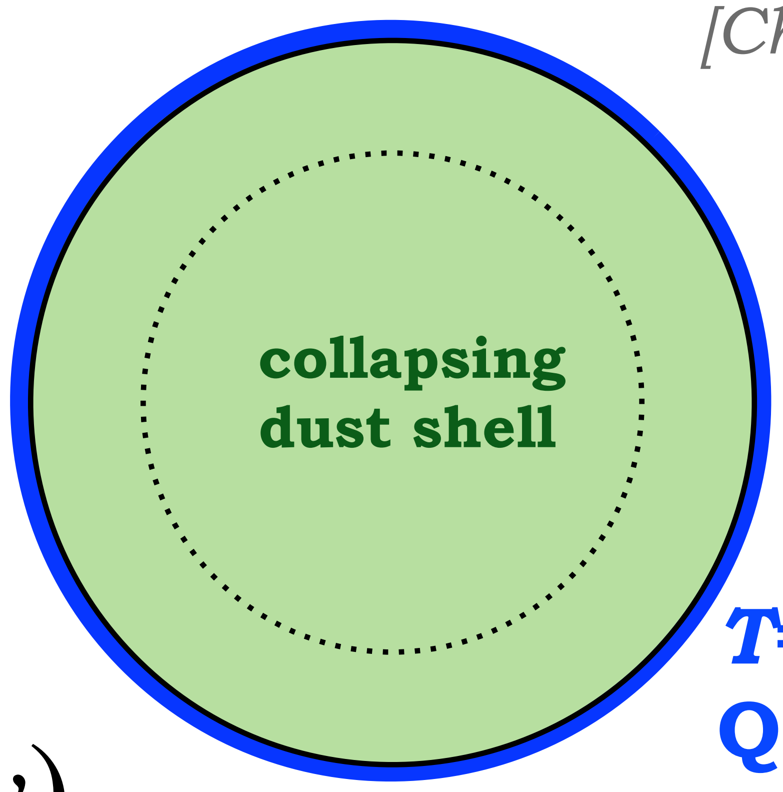
thermal QFT

nonzero T plasma

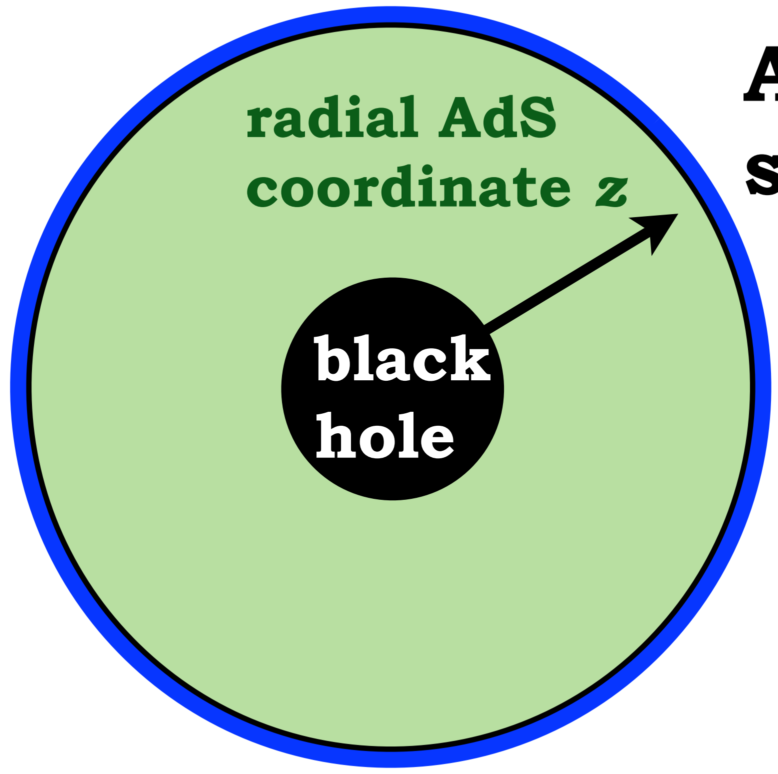
time v

gauge/gravity correspondence

$$\langle T^{\mu\nu}\rangle(v) \longleftrightarrow g_{\mu\nu}(v)$$



T=0 QFT



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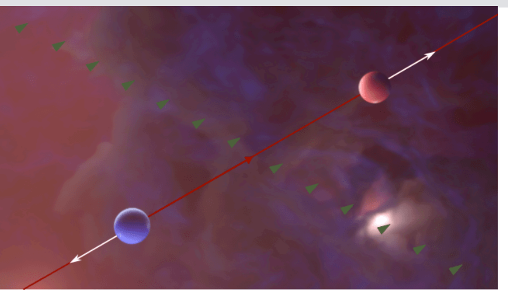

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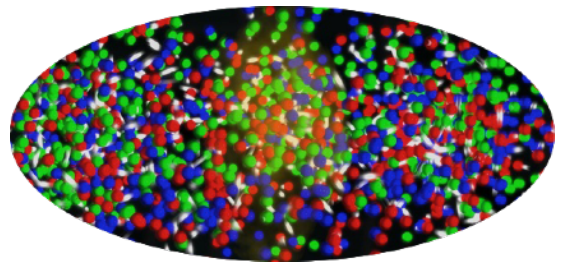
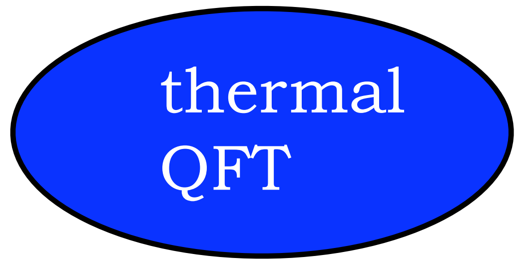


T=0 particle "soup"

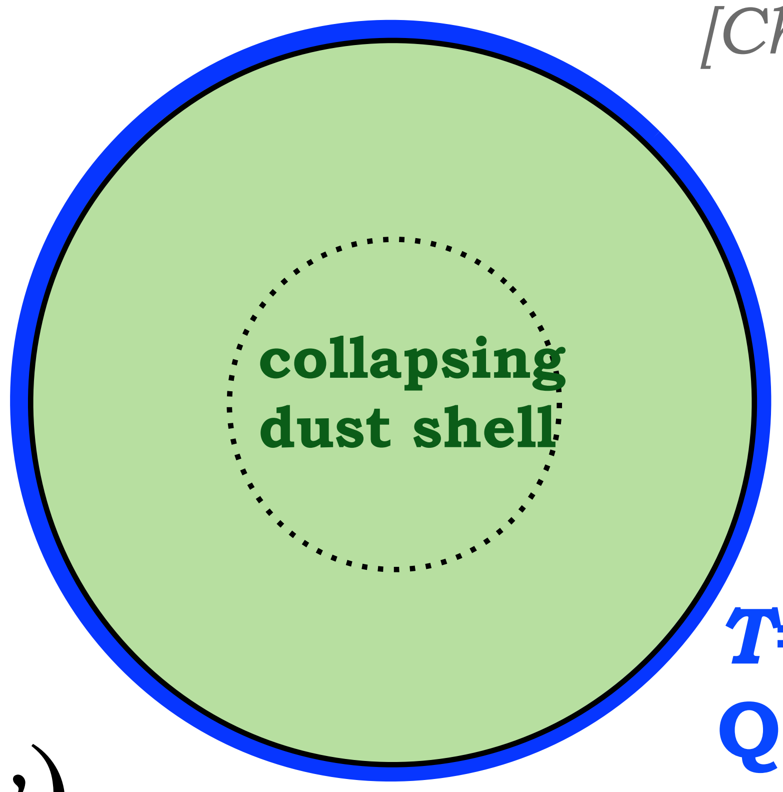
time v



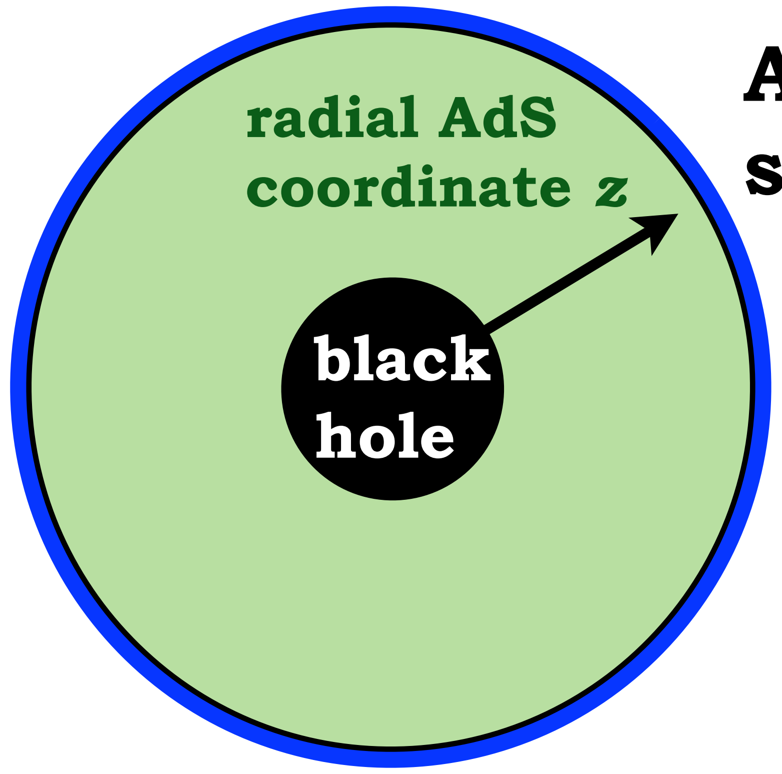
nonzero T plasma

gauge/gravity correspondence

$$\langle T^{\mu\nu}\rangle(v) \longleftrightarrow g_{\mu\nu}(v)$$



***T*=0 QFT**

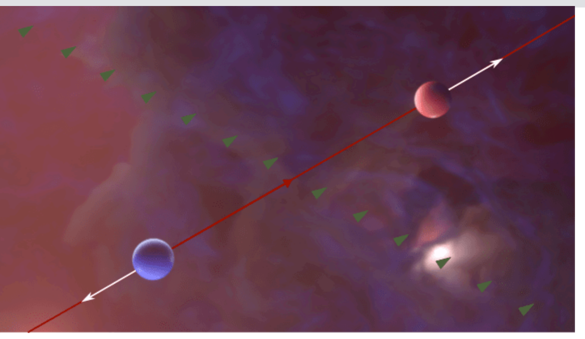


Anti-de Sitter space boundary

thermal QFT "lives" here temperature T

➡solve time-dependent Einstein equations

3. CME far from equilibrium - Bjorken-expanding plasma



[DOE Highlight Article; Cartwright,Kaminski,Schenke (2023)]

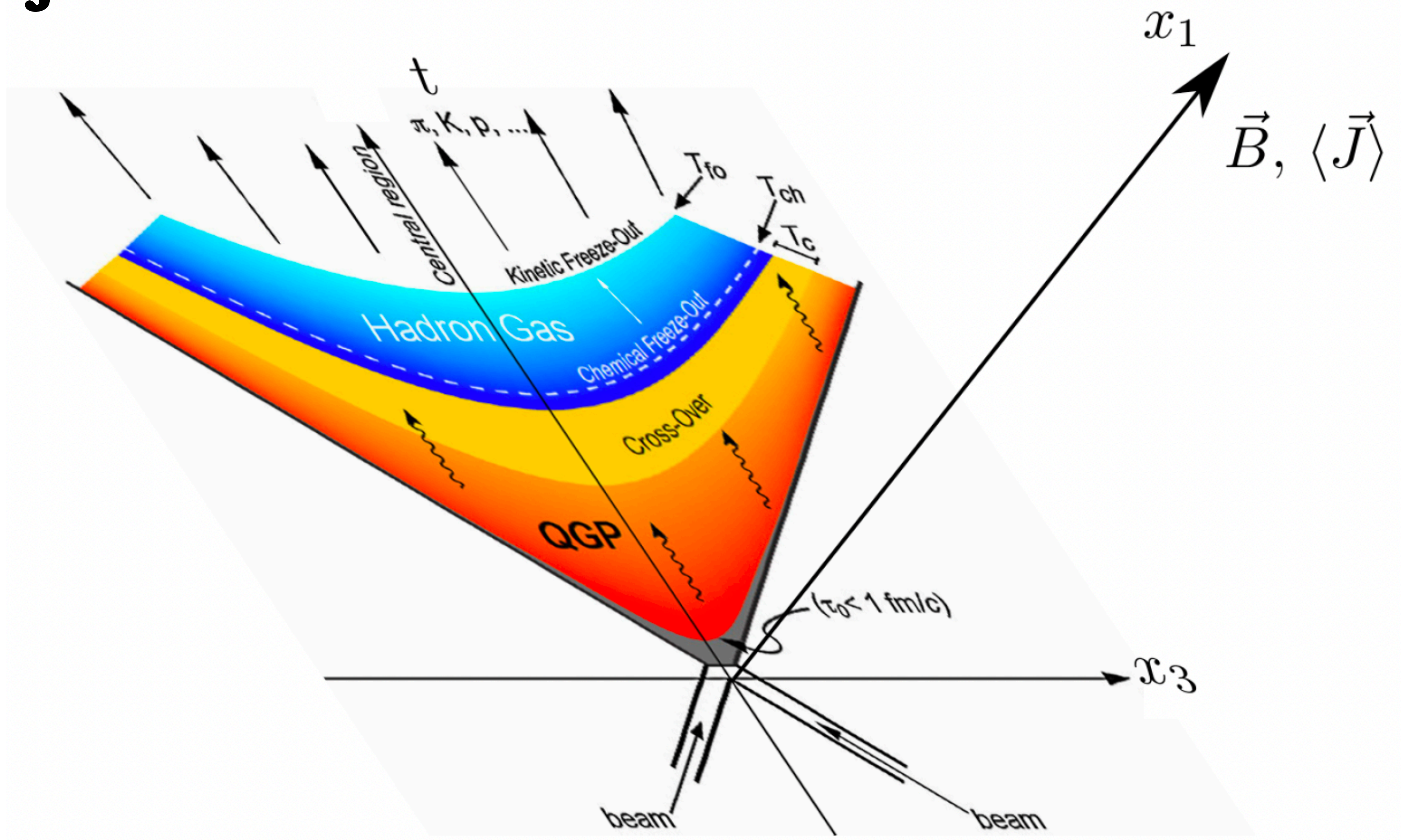
[Cartwright,Kaminski,Schenke; PRC (2022)]

Milne coordinates $(\tau, x_1, x_2, \xi; r)$

proper time $\tau = \sqrt{t^2 - x_3^2}$

rapidity $\xi = \frac{1}{2}\ln[(t+x_3)/(t-x_3)]$

Bjorken flow



taken from Casey Cartwright's talk

Metric Ansatz

AdS radial coordinate r

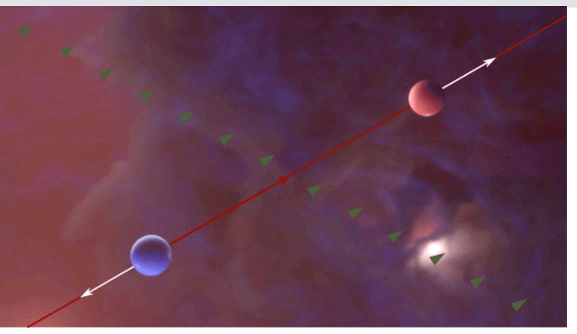
$$ds^2 = 2\mathrm{d}r\mathrm{d}v - A(v,r)\mathrm{d}v^2 + F_1(v,r)\mathrm{d}v\mathrm{d}x_1 + S(v,r)^2 e^{H_1(v,r)}\mathrm{d}x_1^2 + S(v,r)^2 e^{H_2(v,r)}\mathrm{d}x_2^2 + L^2 S(v,r)^2 e^{-H_1(v,r)-H_2(v,r)}\mathrm{d}\xi^2,$$

boundary at $r=\infty$ ***has boost invariant Milne metric:***

$$\lim_{r\to\infty}\frac{L^2}{r^2}\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \tau^2\mathrm{d}\xi^2$$

3. CME far from equilibrium - case I

[Cartwright,Kaminski,Schenke; PRC (2022)]



[DOE Highlight Article; Cartwright,Kaminski,Schenke (2023)]

Initial state:
constant B,
pressure anisotropy

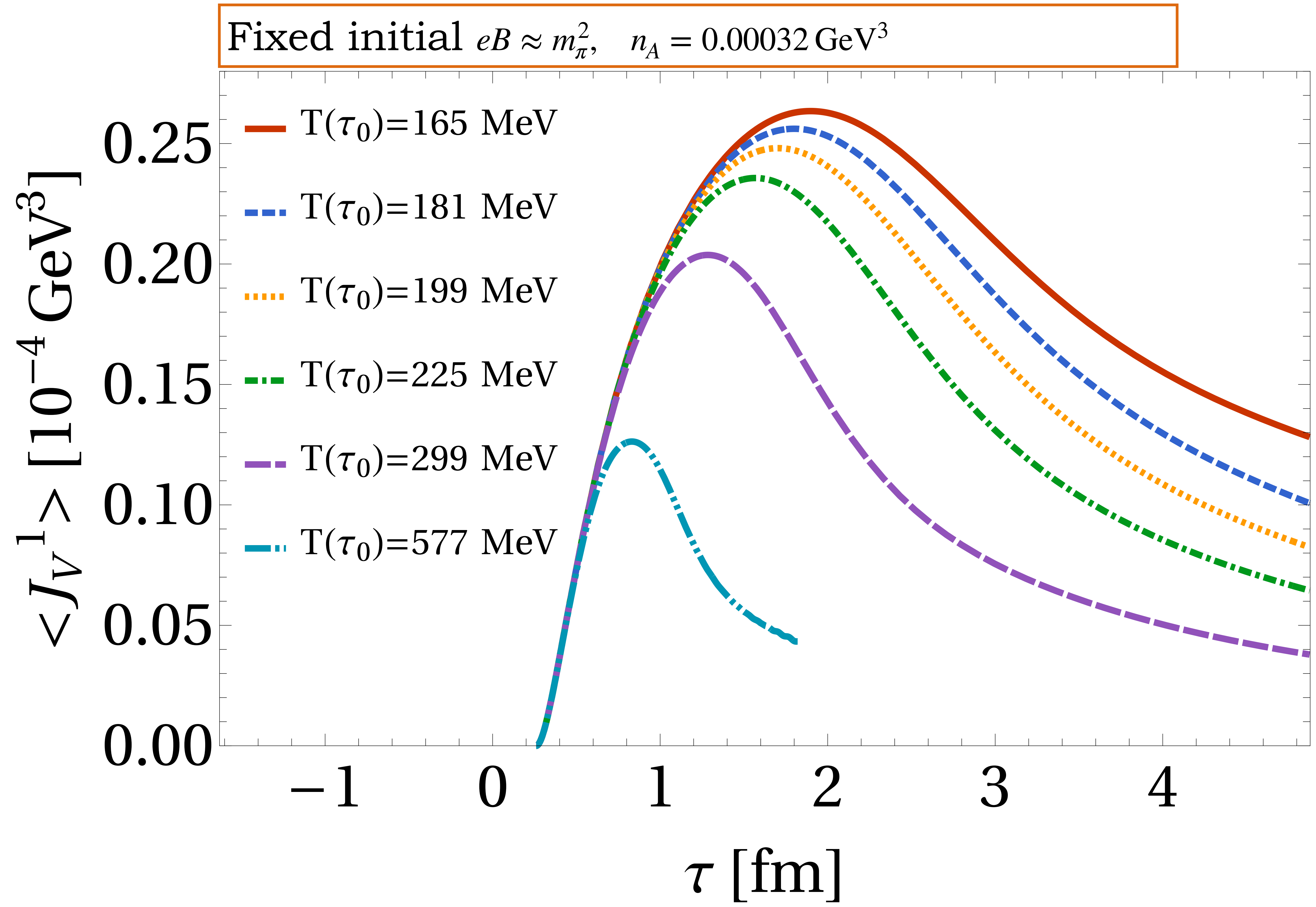
time-dependent μ_5,
plasma expanding along beam line

Matching to QCD:
SUSY value for α
L=1fm fixes κ

Near-equilibrium CME

$$J^\mu_V = \xi_\chi B \qquad \xi_\chi = C\,\mu_A$$

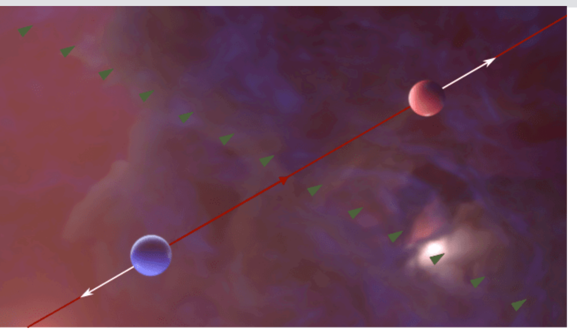
[Kharzeev; PRC (2004)]
[Fukushima,Kharzeev,Warringa; PRD (2008)]
[Son,Surowka; PRL (2009)]



➡CME-current more likely to be seen at lower energies!

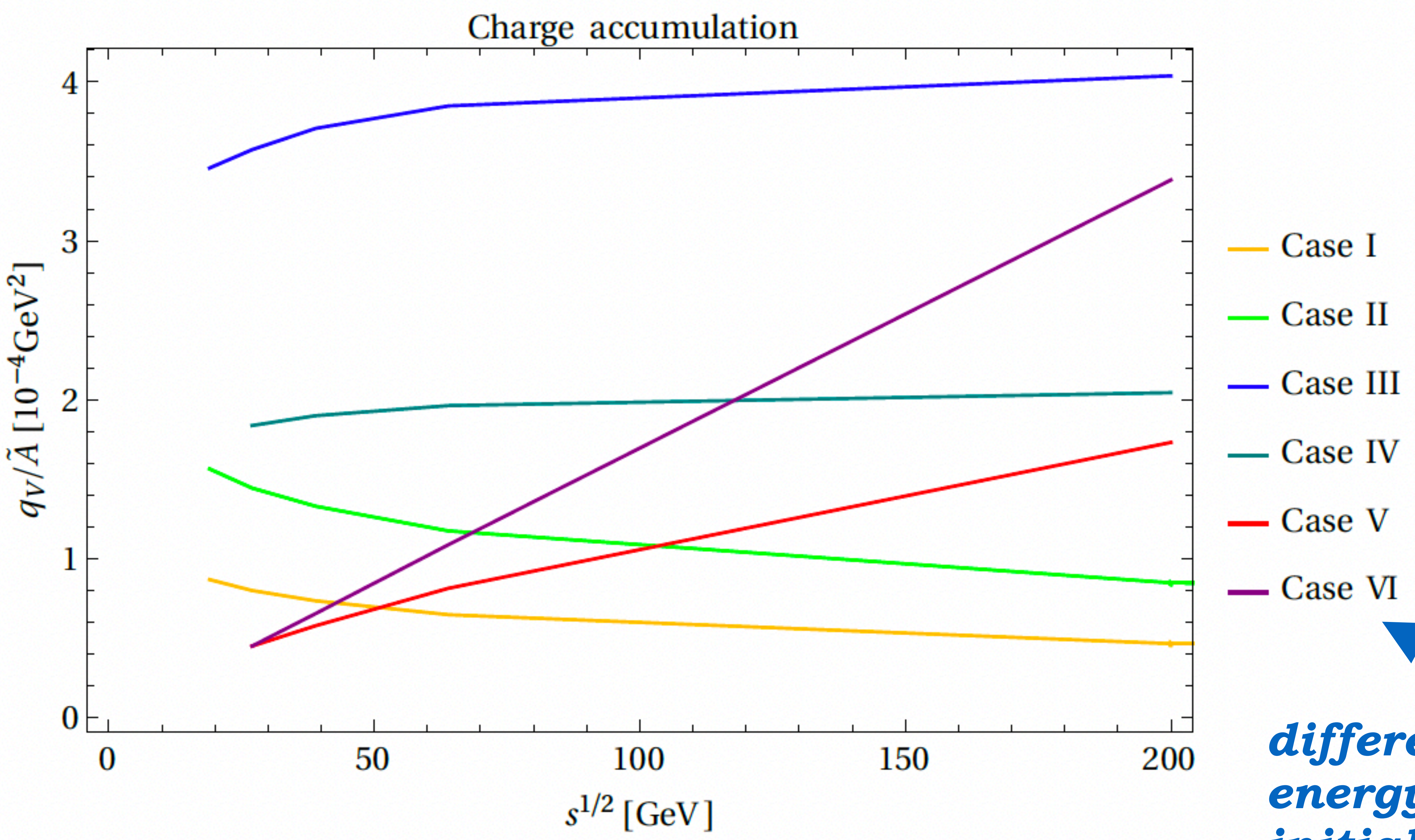
agrees with non-expanding holographic model:
[Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]

3. Chiral Magnetic Effect in Bjorken-expanding plasma



[DOE Highlight Article; Cartwright,Kaminski,Schenke (2023)]

[Cartwright,Kaminski,Schenke; PRC (2022)]



Charge accumulated in detector over time $\Delta\tau = \tau_f - \tau_i$ due to Chiral Magnetic Effect (CME)

$$q_V/\tilde{A} = \int_{\tau_i}^{\tau_f} \mathrm{d}\tau\tau \langle J^1 \rangle$$

Area: $\tilde{A} = \int \mathrm{d}x_2 \mathrm{d}\xi$

different combinations of initial energy, initial chiral imbalance, initial magnetic field

Near-equilibrium CME

$$J^\mu = \xi_\chi B \qquad \xi_\chi = C\,\mu_A$$

[Kharzeev; PRC (2004)]
[Fukushima,Kharzeev,Warringa; PRD (2008)]
[Son,Surowka; PRL (2009)]

➡Accumulated charge: CME more likely to be seen at <u>higher</u> energies!

compare: [Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]

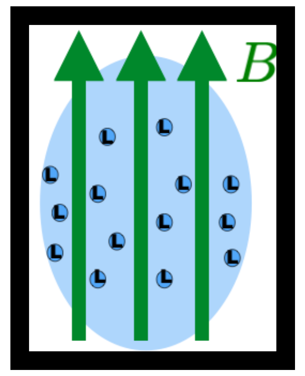
Compare to experiments: ➡ ***Talk by Fuqiang Wang***

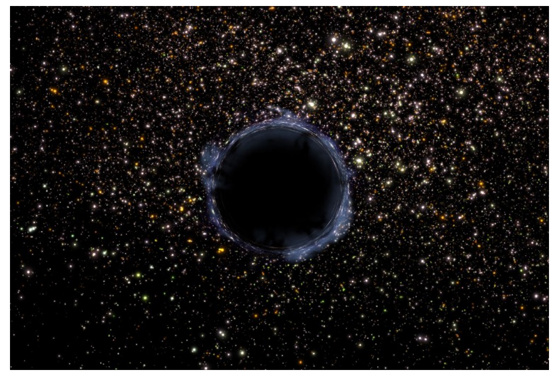
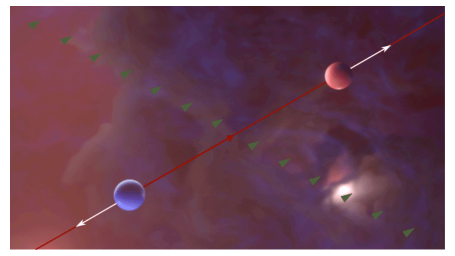
top-RHIC energy: [STAR Collaboration; (2021)]
low-energy update: [STAR Collaboration; (2022)]
high energy update: [ALICE Collaboration; (2022)]

Discussion

Summary

- Constructed most general **chiral hydrodynamics for charged chiral fluids in strong magnetic field**
- Derived **Kubo formulae** for 27 transport coefficients (8 novel)
- Confirmed Kubo formulae by computation in holographic model
- Chiral Magnetic Effect depends on initial values (axial imbalance, *B*, *T*)



Discussion

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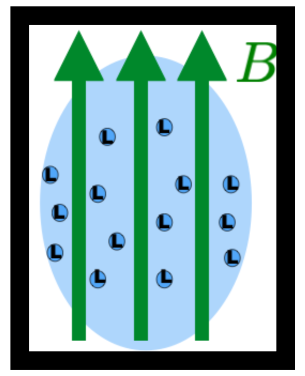
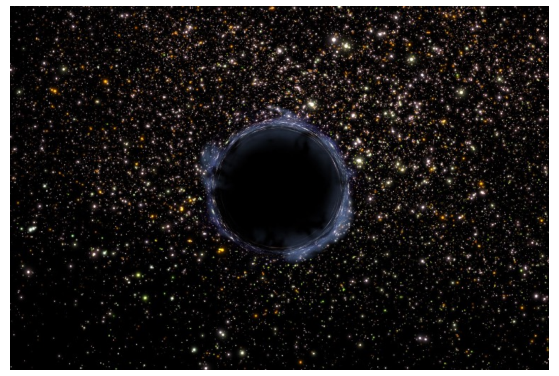
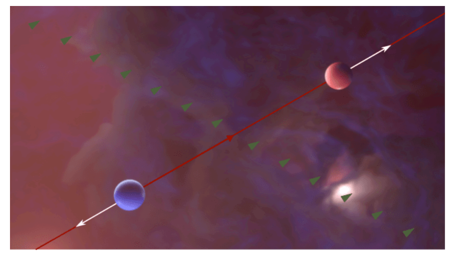
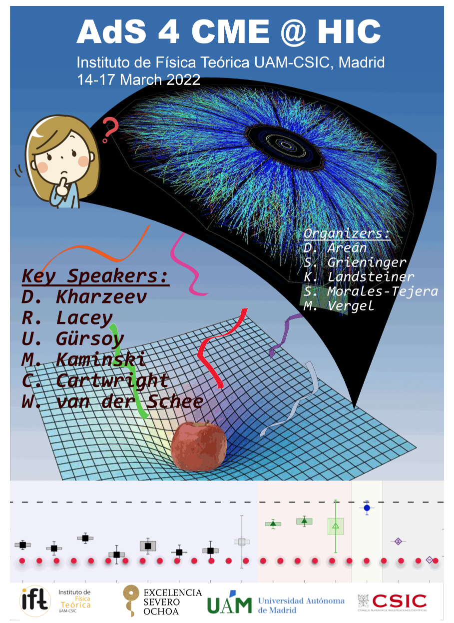


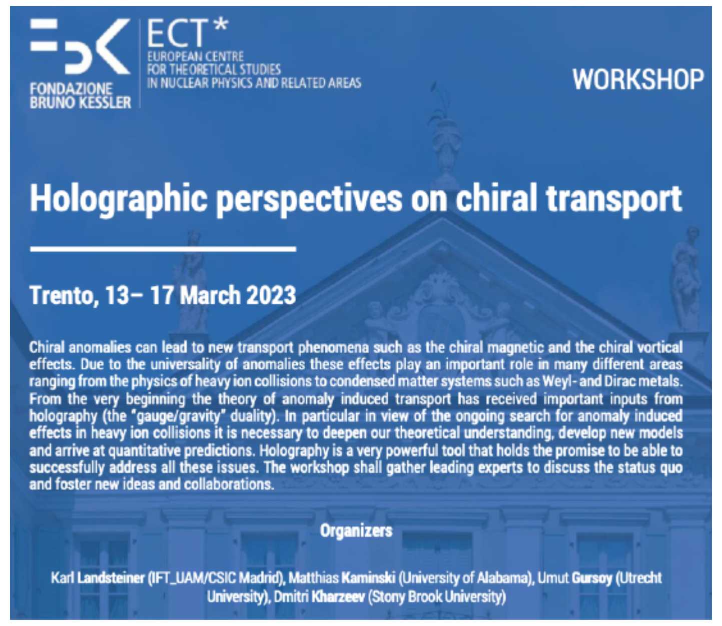




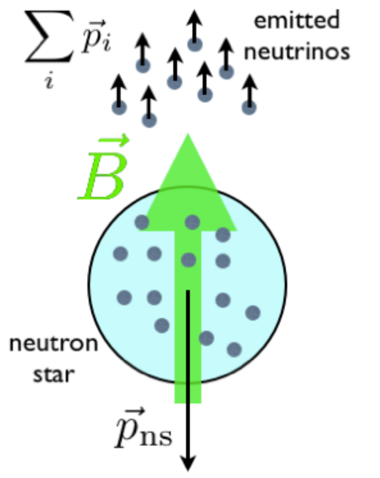
Outlook

- Can novel transport coefficients be calculated **on the lattice** or in **perturbative QCD**? → ***Talk by Gergely Marko***
- Effect on **elliptic flow**?
- Include **dynamical magnetic field and dynamically created axial imbalance** to model QGP and CME
- **Neutron star kicks** from chiral hydrodynamics *[Kaminski,Uhlemann,Bleicher,Schaffner-Bielich; Phys.Lett.B (2016)]*





[AdS4CME Collaboration]



Collaborators on these projects

Perimeter, Canada

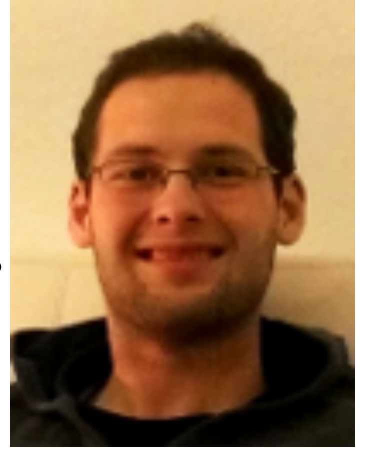
Dr. Juan Hernandez (now Brussels University)

BNL, USA

Prof. Dr. Bjoern Schenke

Friedrich-Schiller University of Jena, Germany

Prof. Dr. Martin Ammon



Dr. Julian Leiber

Dr. Sebastian Grieninger (now Stony Brook University)

University of Alabama, Tuscaloosa, USA

Dr. Jackson Wu

Dr. Roshan Koirala

Dr. Casey Cartwright

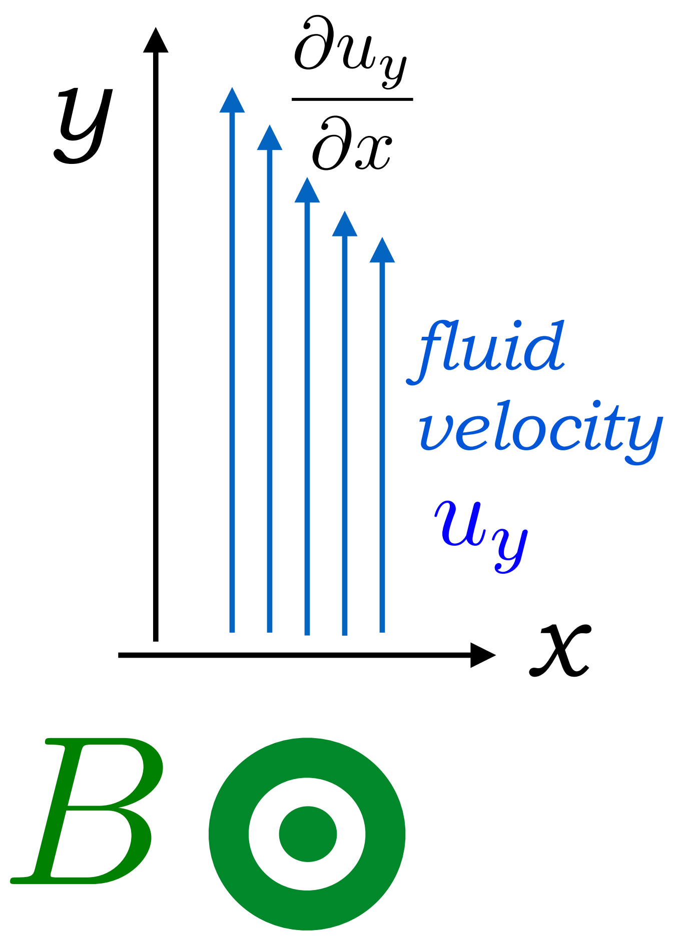
(now at Utrecht University, Netherlands)

APPENDIX

APPENDIX: Kubo formulae: two shear viscosities

Shear viscosity perpendicular

$$\frac{1}{\omega}\mathrm{Im}\, G_{T^{xy}T^{xy}}(\omega, \mathbf{k}{=}0) = \eta_\perp$$



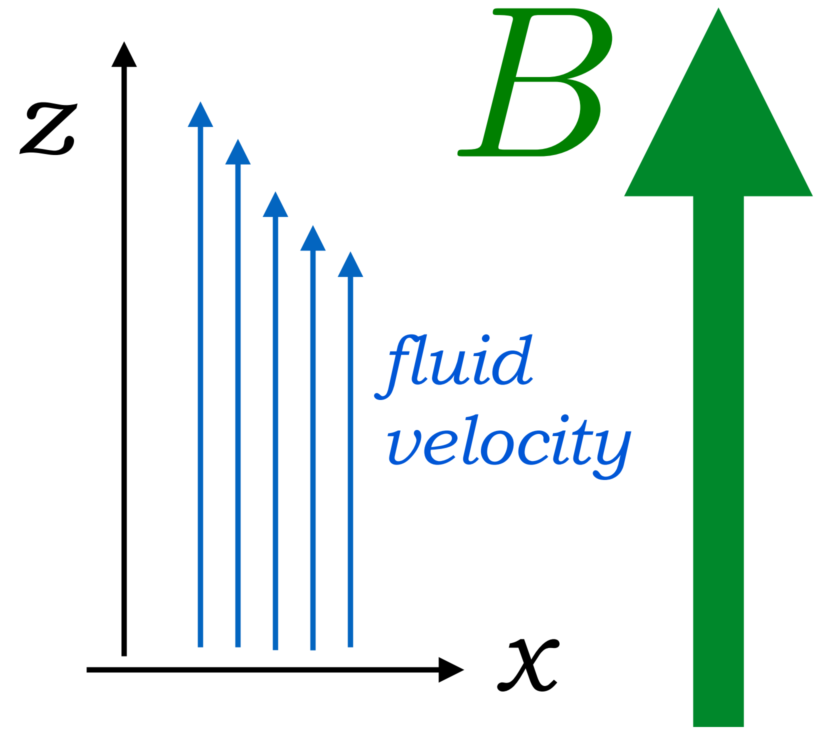
Shear viscosity parallel

$$\frac{1}{\omega}\mathrm{Im}\, G_{T^{xz}T^{xz}}(\omega, \mathbf{k}{=}0) = \eta_{||} + (\bar{c}_8 c_{15} - c_{10}\bar{c}_{17})\rho_\perp - (\bar{c}_8\bar{c}_{17} + c_{10}c_{15})\tilde{\rho}_\perp$$

perpendicular resistivity

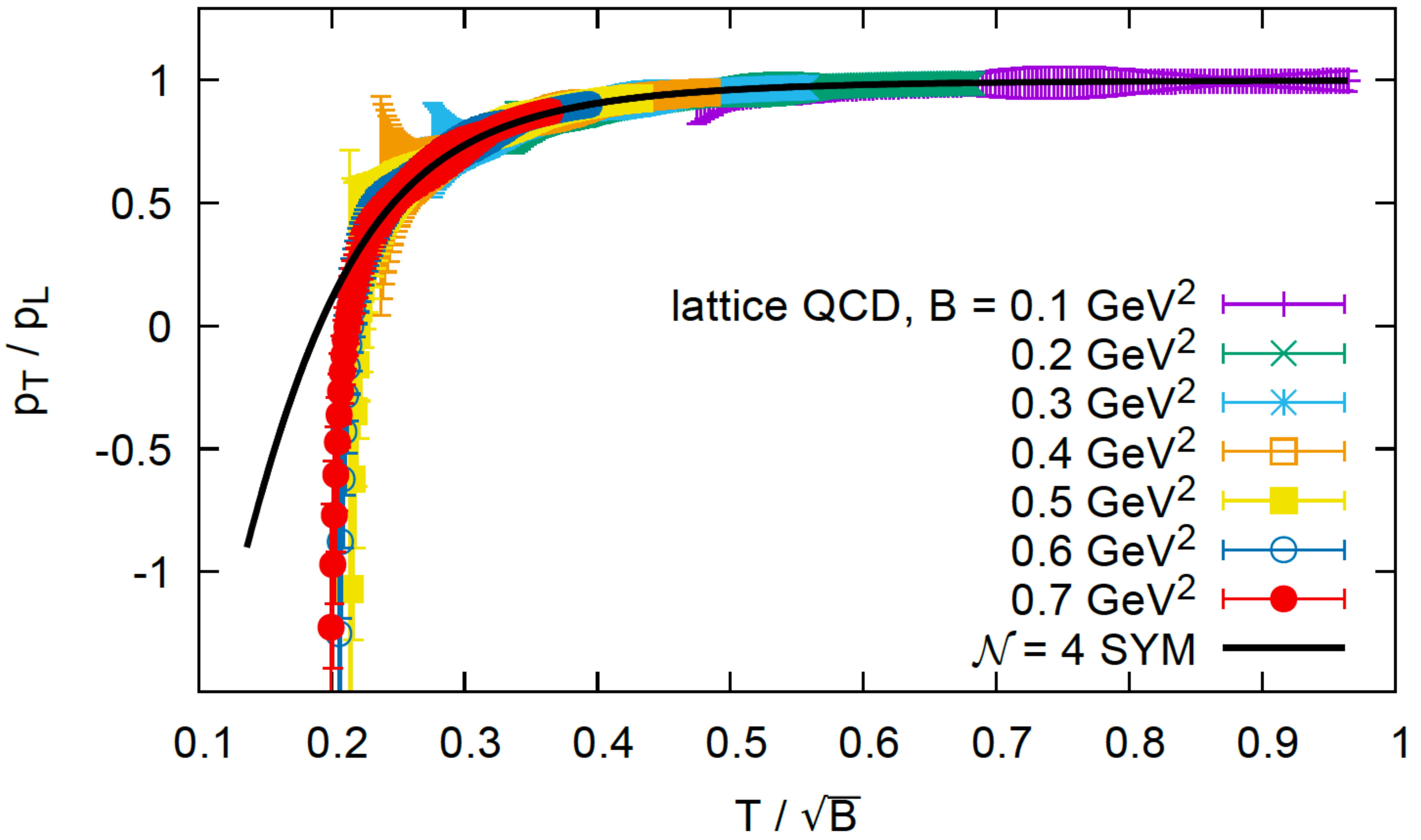
Hall resistivity

➡ **Value of shear viscosity depends on direction of magnetic field**
➡ **Can lead to creation of flow at early times**



APPENDIX: Same magneto response in LQCD and N=4 SYM with magnetic field

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; JHEP (2018)]



Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

transverse pressure: $p_{\mathrm{T}} = -\frac{L_{\mathrm{T}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{T}}}$

longitudinal pressure: $p_{\mathrm{L}} = -\frac{L_{\mathrm{L}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{L}}}$

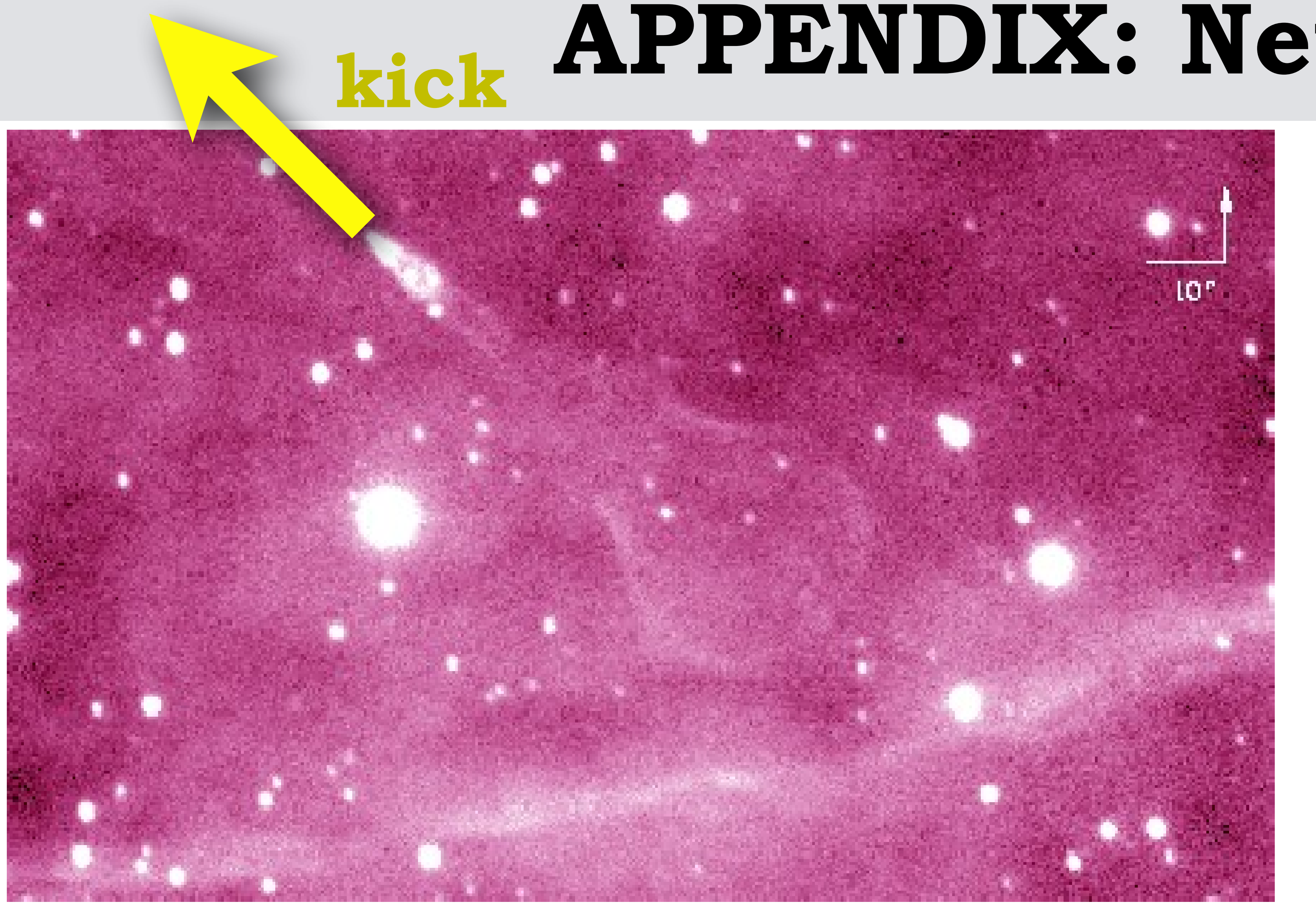
F_{QCD} ... *free energy*

L_{T} ... *transverse system size*

L_{L} ... *longitudinal system size*

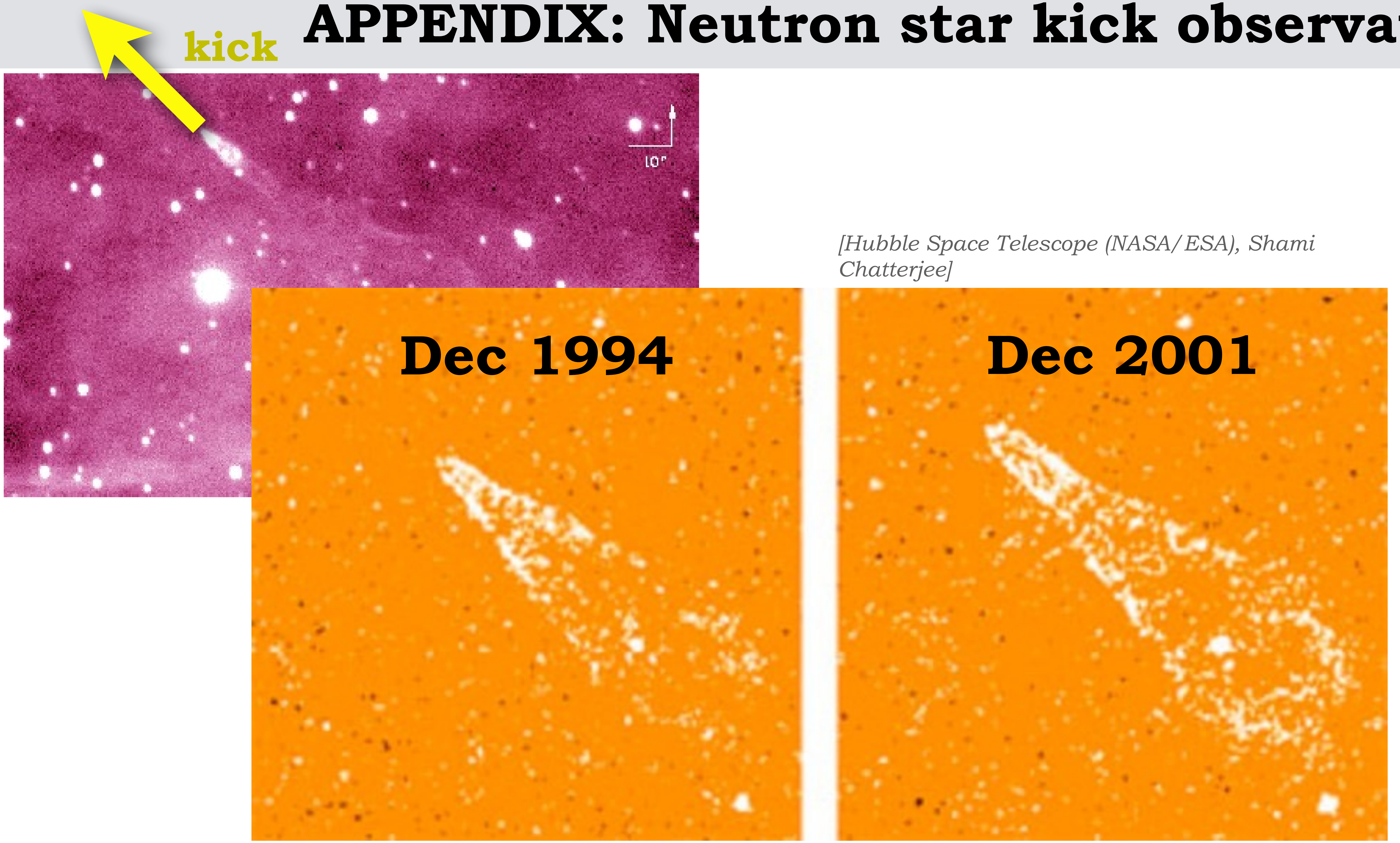
V ... *system volume*

APPENDIX: Neutron star kick observations



Neutron stars kicked out of their initial position
with velocities ~ 1000 km/s

APPENDIX: Neutron star kick observations

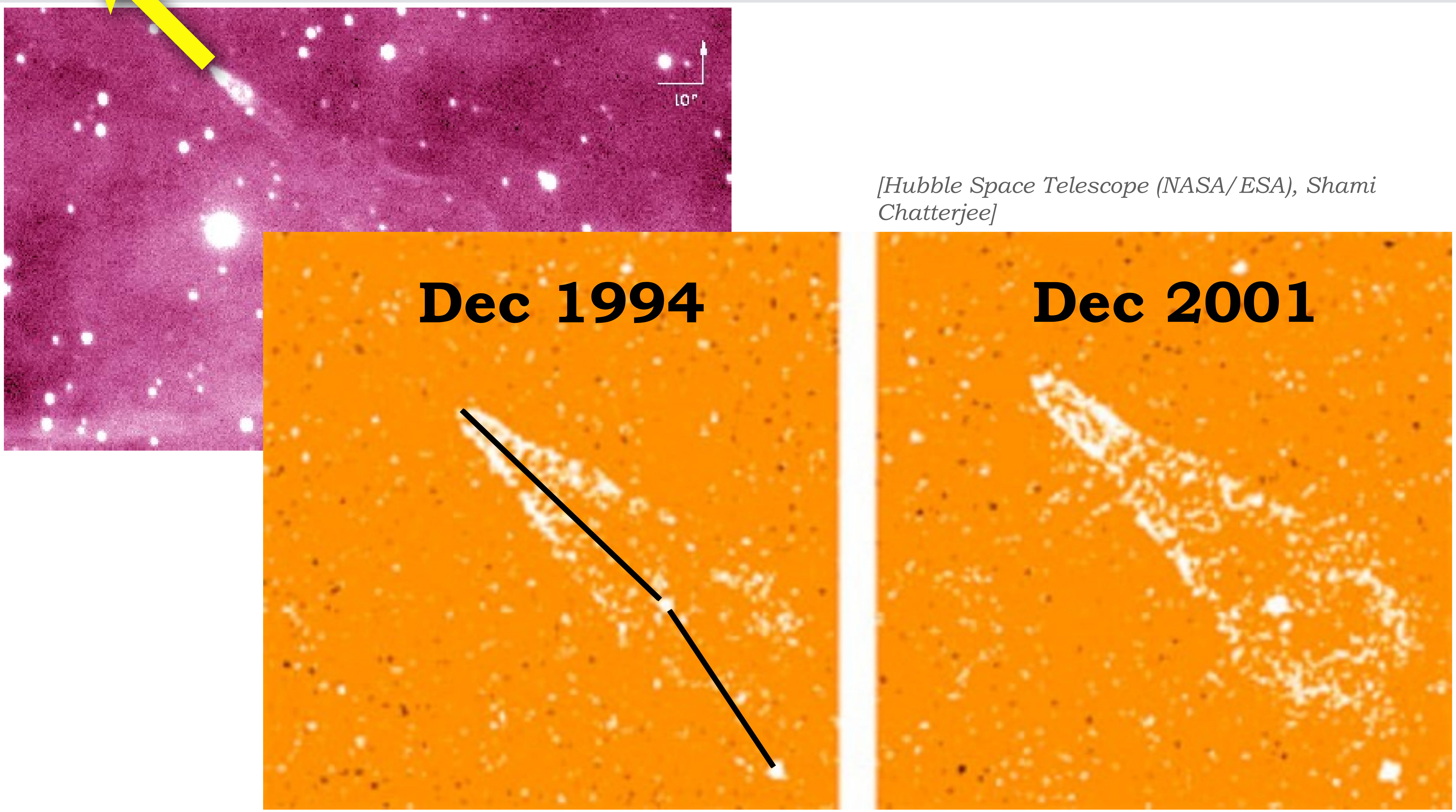


[Hubble Space Telescope (NASA/ESA), Shami Chatterjee]

Neutron stars kicked out of their initial position
with velocities ~ 1000 km/s

[Chatterjee et al.; Astrophys. J (2005)]

APPENDIX: Neutron star kick observations



[Hubble Space Telescope (NASA/ESA), Shami Chatterjee]

Neutron stars kicked out of their initial position
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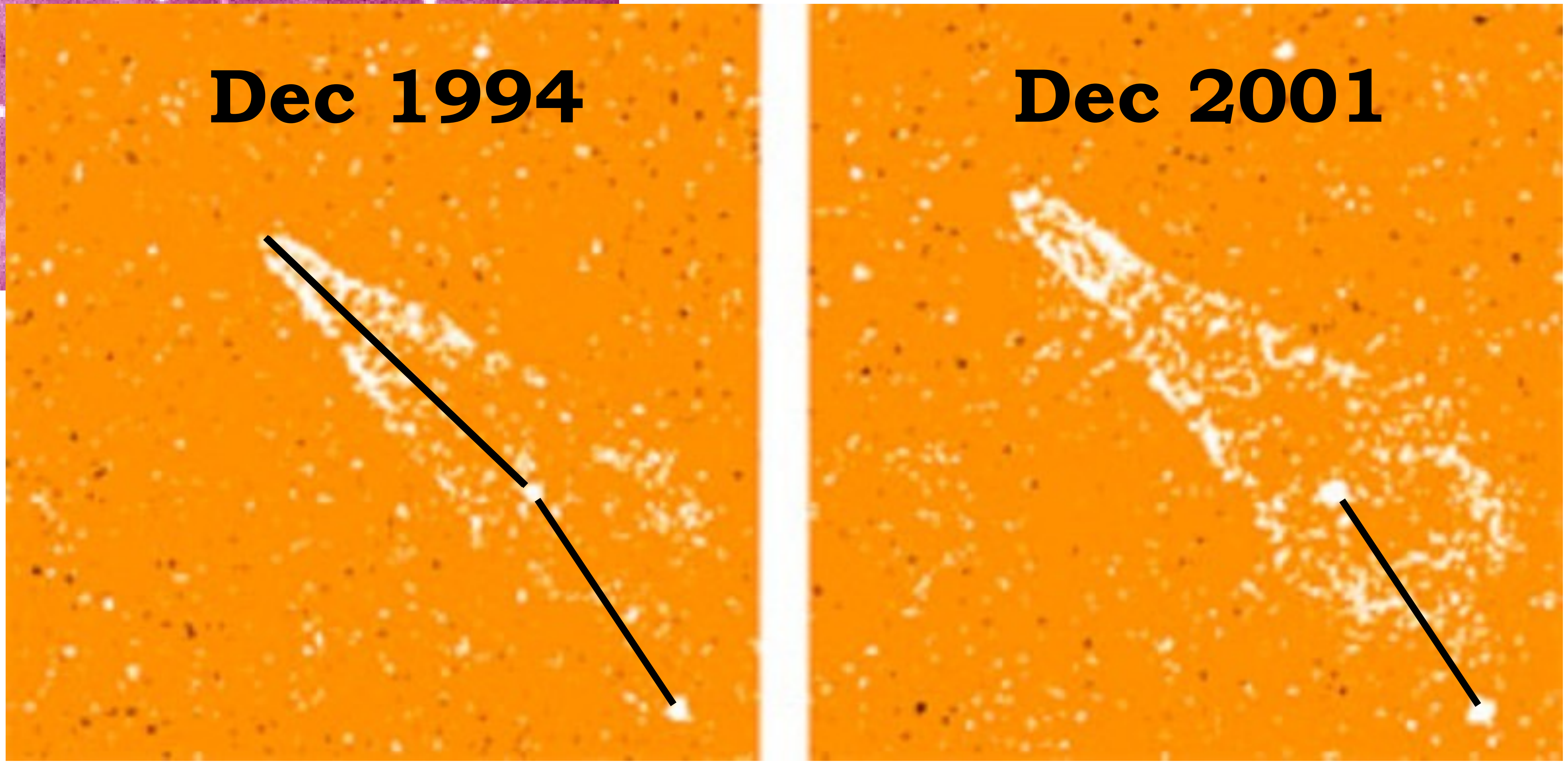
APPENDIX: Neutron star kick observations

[Hubble Space Telescope (NASA/ESA), Shami Chatterjee]



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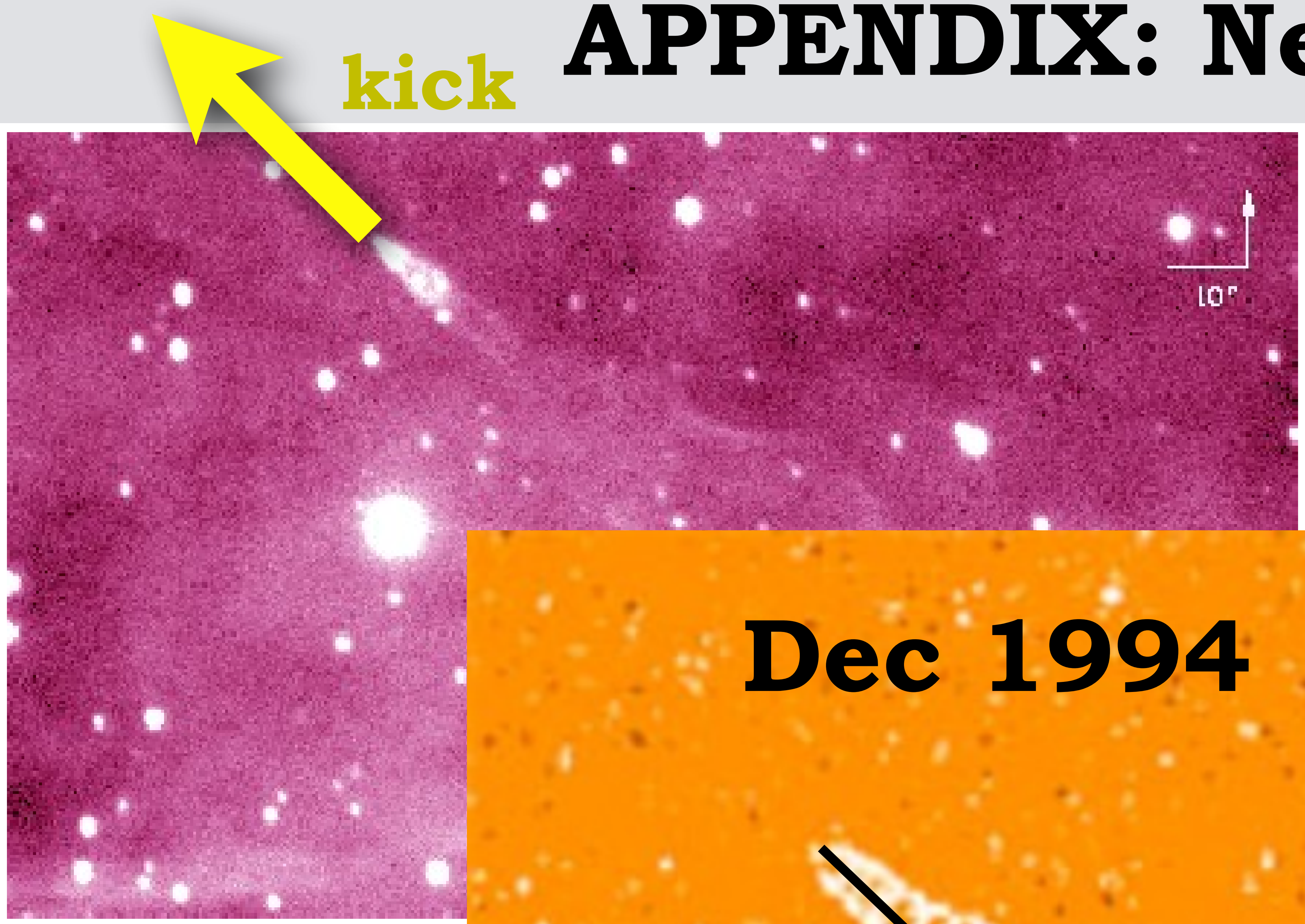
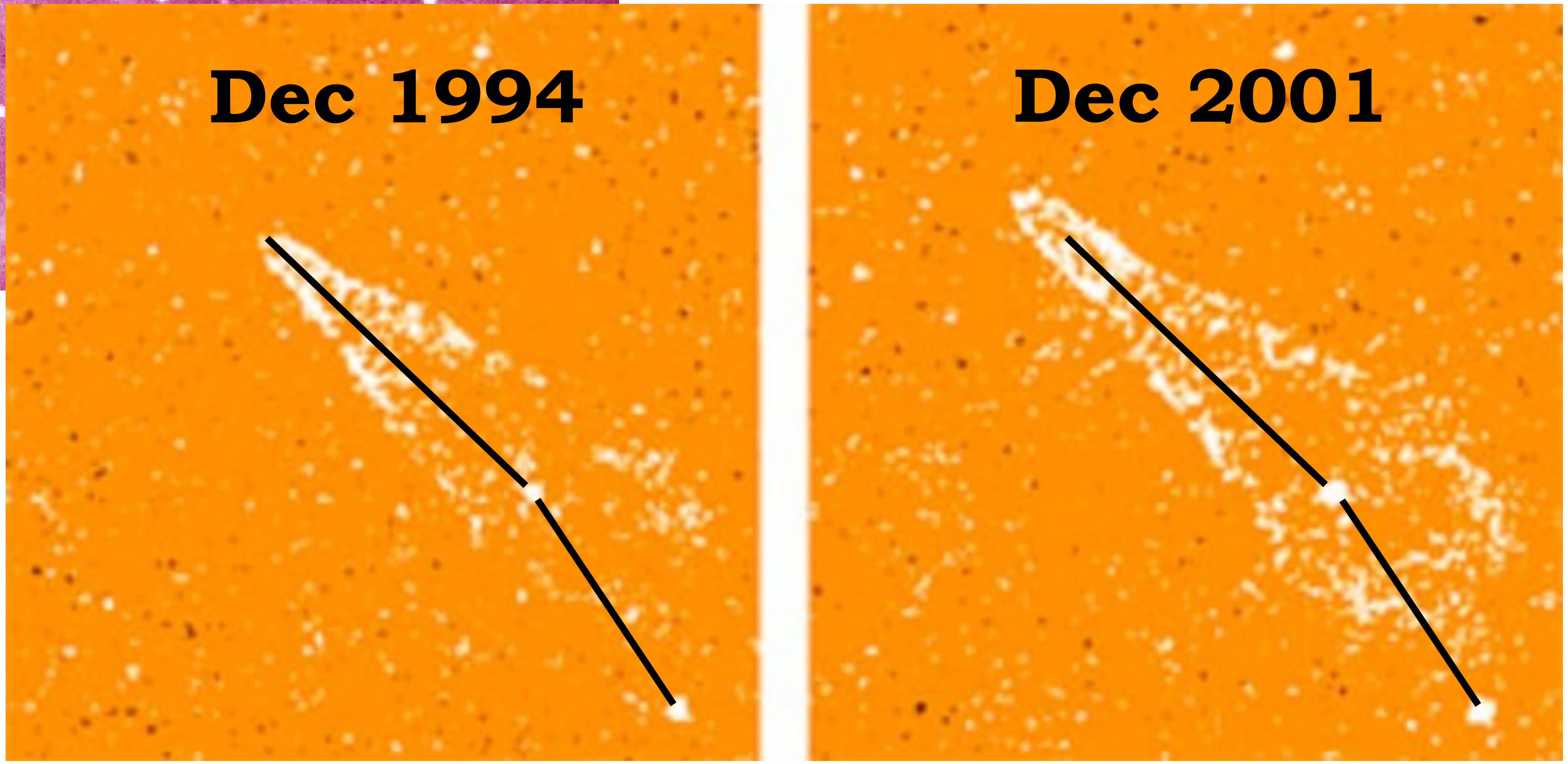
[Chatterjee et al.; Astrophys. J (2005)]

APPENDIX: Neutron star kick observations



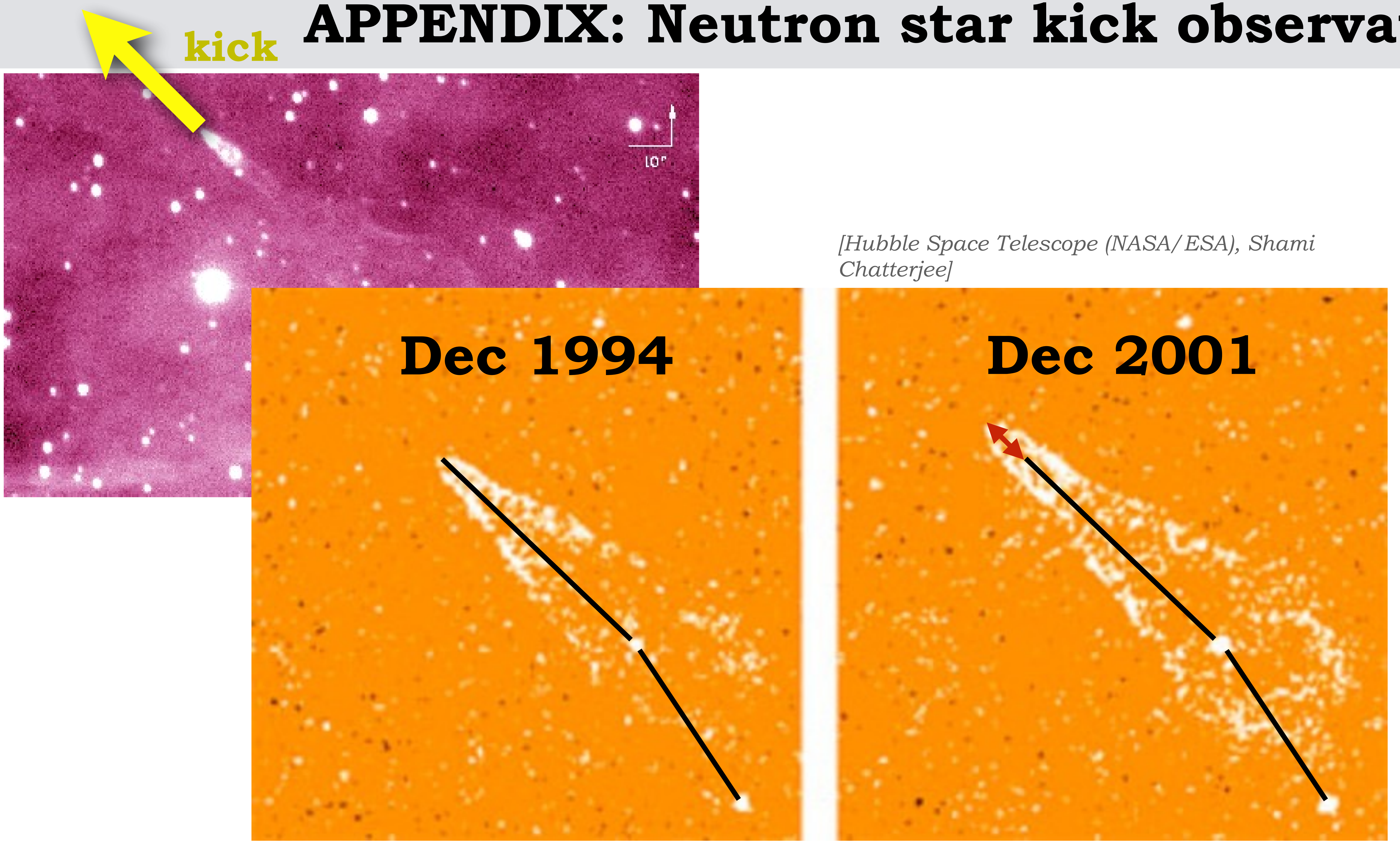
[Hubble Space Telescope (NASA/ESA), Shami Chatterjee]



Neutron stars kicked out of their initial position
with velocities ~ 1000 km/s

[Chatterjee et al.; Astrophys. J (2005)]

APPENDIX: Neutron star kick observations



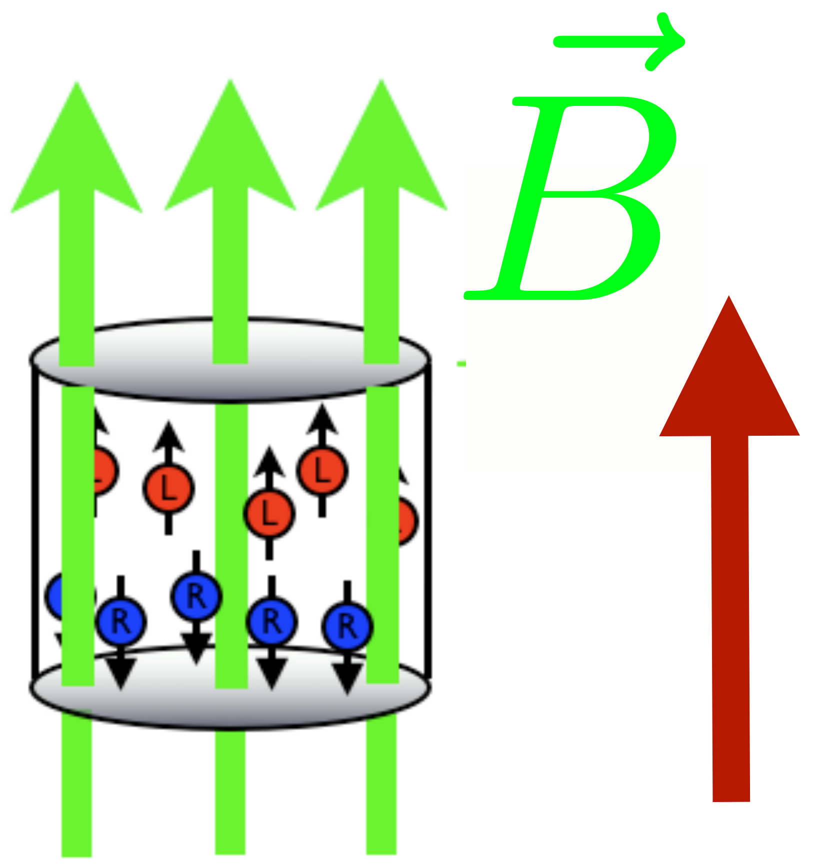
[Hubble Space Telescope (NASA/ESA), Shami Chatterjee]

Neutron stars kicked out of their initial position
with velocities ~ 1000 km/s

[Chatterjee et al.; Astrophys. J (2005)]

APPENDIX: Chiral hydrodynamics kicks neutron stars

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; Phys.Lett.B (2016)]

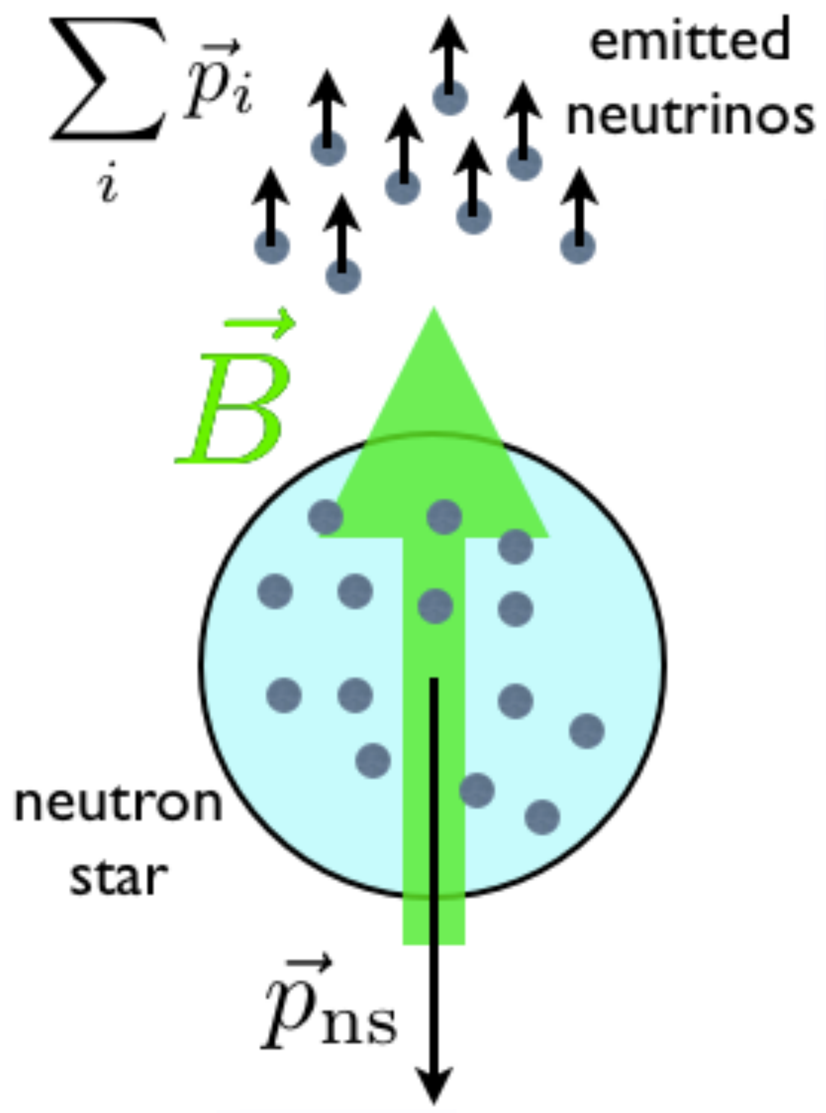


$$J^\mu = \xi_B B^\mu$$

hydrodynamics: fluids with left-handed and right-handed particles produce a **current** along magnetic field

e.g. right/left-handed electrons, neutrinos, ...

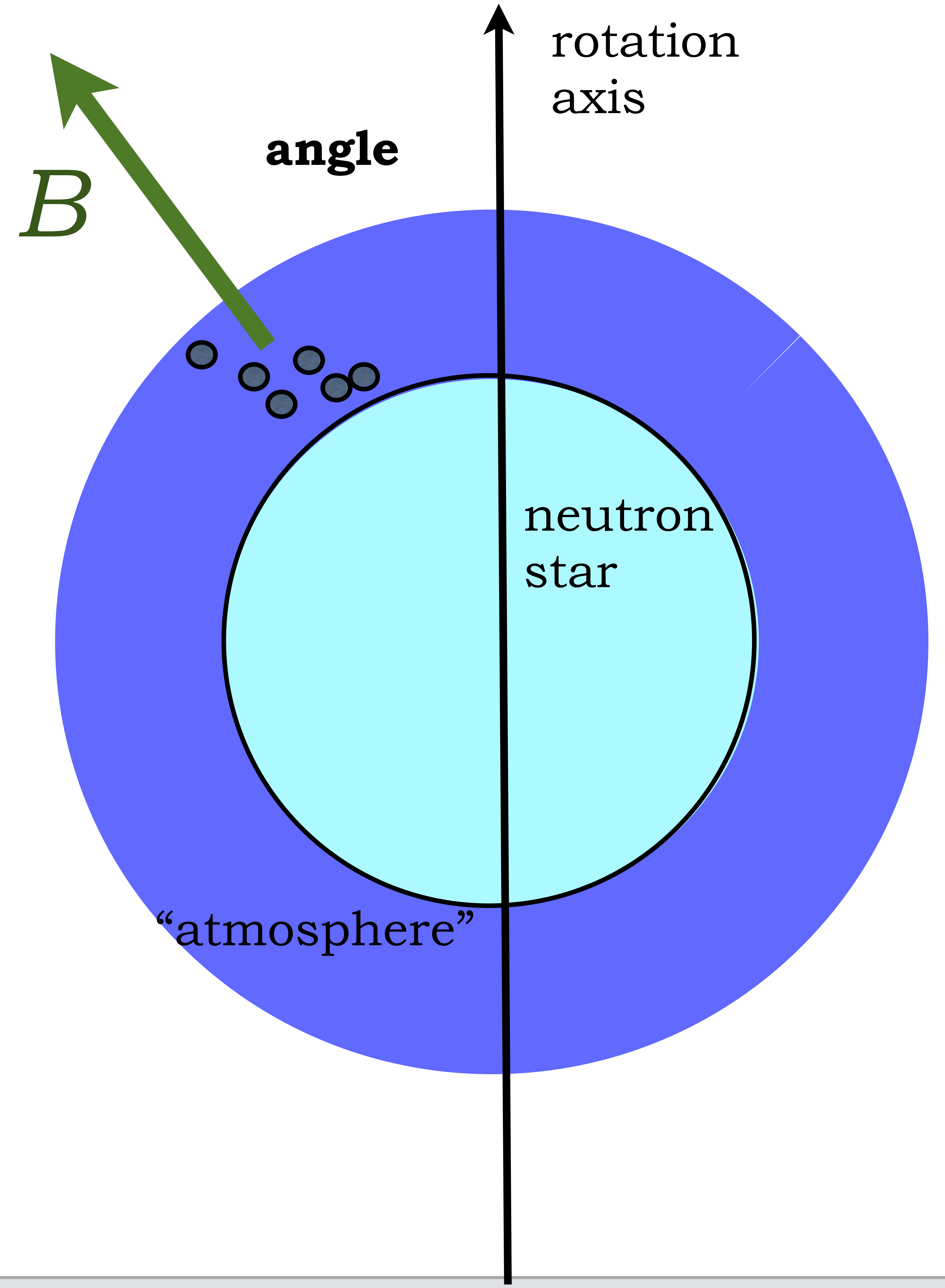
Idea:



Chiral hydrodynamics leads to neutron star kicks

APPENDIX: Observable features

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; PLB (2016)]

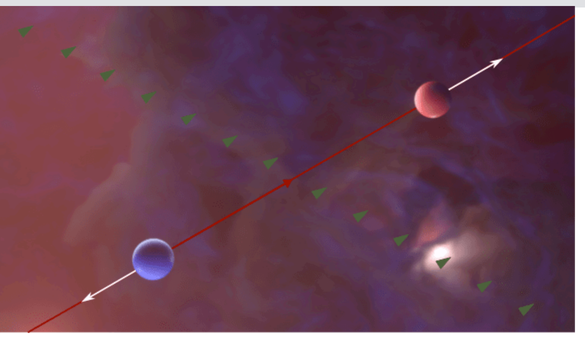


Predictions
➡Kick magnitude depends on angle between rotation axis and internal magnetic field axis
➡For fast spinning neutron stars, kick directed along rotation axis.

- check with simulation
- kick aligned with *spin*?
- compare to numerical kick mechanisms

[Scheck, Kifonidis, Janka, Muller; (2003)]
[Wongwathanarat, Janka, Muller; (2010)]
[Wongwathanarat, Janka, Muller; (2012)]
[Berdermann, Blaschke et al; (2005)]

3. CME far from equilibrium - Bjorken-expanding plasma



[DOE Highlight Article; Cartwright,Kaminski,Schenke (2023)]

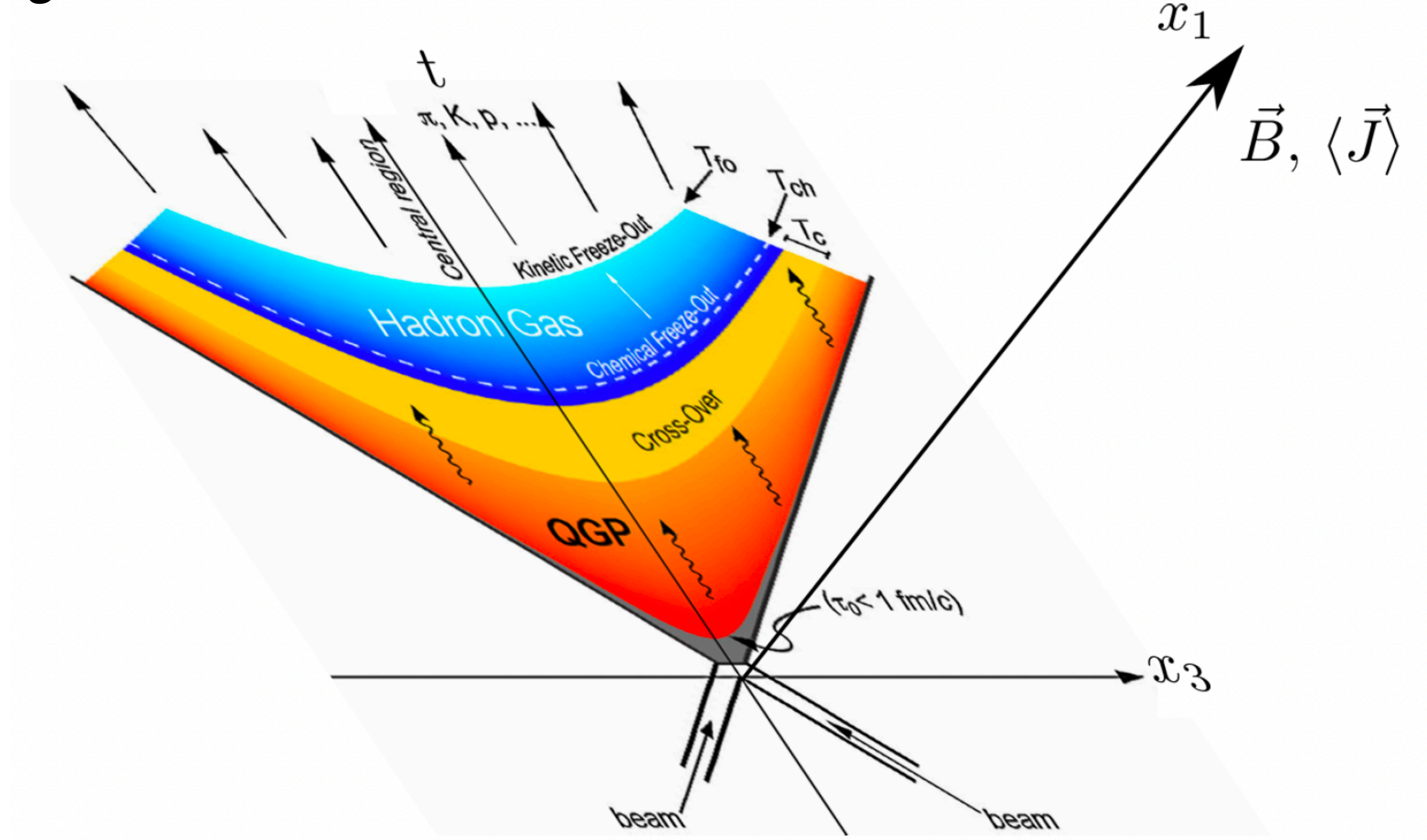
Milne coordinates $(\tau, x_1, x_2, \xi; r)$

[Cartwright,Kaminski,Schenke; PRC (2022)]

proper time $\tau = \sqrt{t^2 - x_3^2}$

rapidity $\xi = \frac{1}{2}\ln[(t+x_3)/(t-x_3)]$

Bjorken flow



taken from Casey Cartwright's talk

Metric Ansatz

AdS radial coordinate r

$$ds^2 = 2\mathrm{d}r\mathrm{d}v - A(v,r)\mathrm{d}v^2 + F_1(v,r)\mathrm{d}v\mathrm{d}x_1 + S(v,r)^2 e^{H_1(v,r)}\mathrm{d}x_1^2 + S(v,r)^2 e^{H_2(v,r)}\mathrm{d}x_2^2 + L^2 S(v,r)^2 e^{-H_1(v,r)-H_2(v,r)}\mathrm{d}\xi^2,$$

boundary at $r=\infty$ ***has boost invariant Milne metric:***

$$\lim_{r\to\infty}\frac{L^2}{r^2}\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \tau^2\mathrm{d}\xi^2$$

Definitions

$$\lim_{r\to\infty} V_a = V_a^{\text{ext}} = \frac{1}{L}(0,0,b\,\xi,0)$$

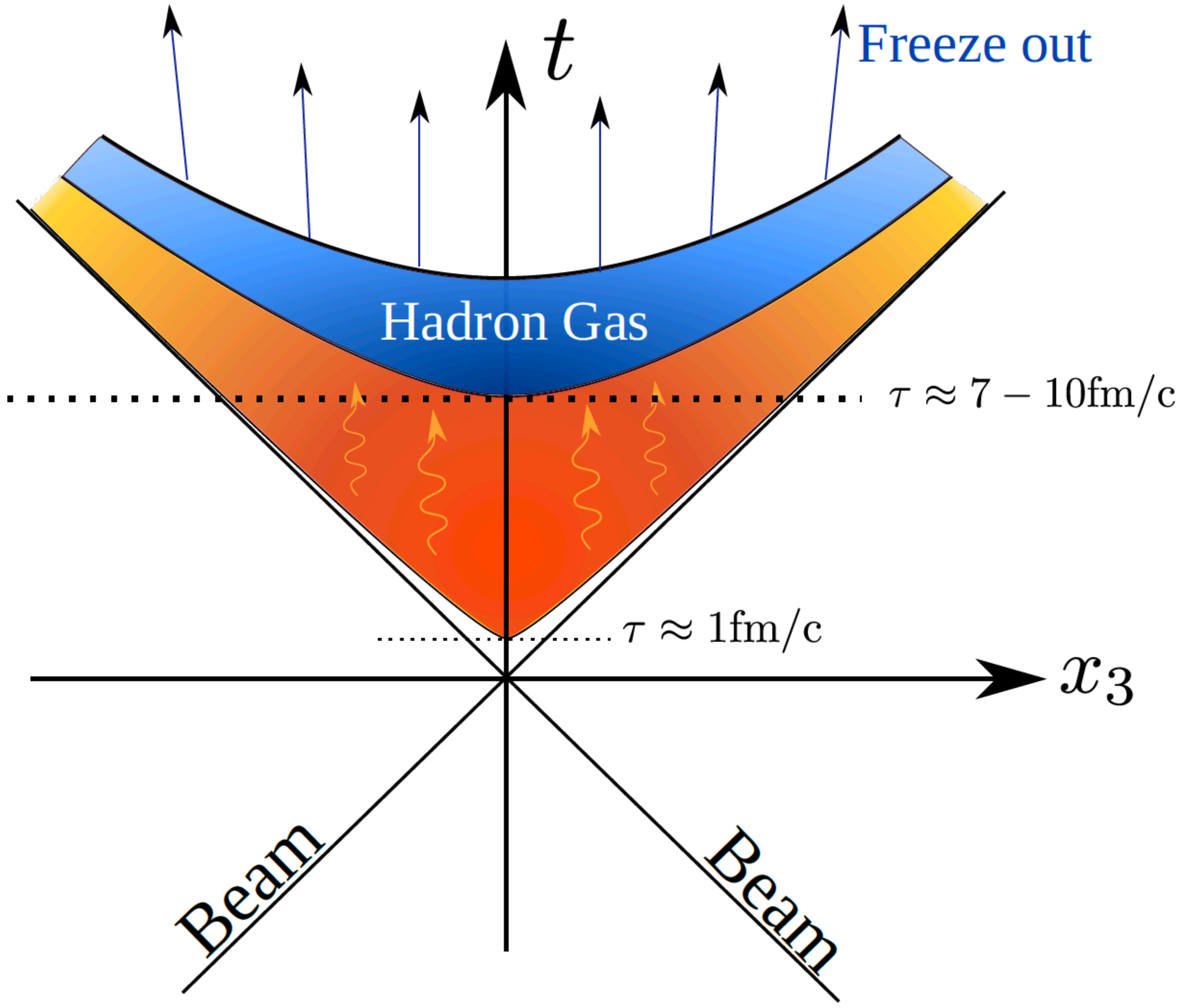
$$A_\mu = \frac{1}{L}(0,-\phi(v,r),0,0,0),$$
$$V_\mu = \frac{1}{L}(0,0,-V(v,r),b\,\xi,0)\,,$$

$$q_5/L = L^4 S(v,r)^3\phi'(v,r) + 8\alpha b V(v,r)\,,$$
$$\mathcal{E}_5 \equiv -\phi'(v,r) = \frac{q_5 L^{-1} - 8\alpha b V(v,r) L^{-4}}{S(v,r)^3}$$

APPENDIX: Bjorken - expanding plasma

[Cartwright,Kaminski,Knipfer; (2022)]

- far away from equilibrium thermodynamic quantities are not well-defined
- plasma is approximately boost invariant along the beam-line
- initially large anisotropy between that direction and the transverse plane



proper time $$\tau = \sqrt{t^2 - x_3^2}$$

Ideal hydrodynamics:

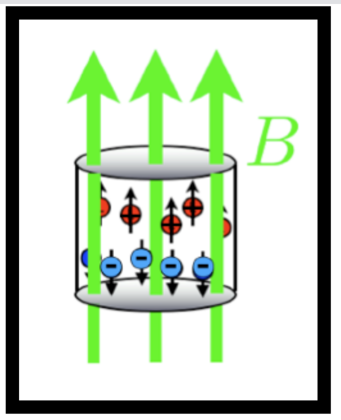
$$u_\nu \partial_\mu T^{\mu\nu} = u_\nu \partial_\mu \left((\epsilon + P) u^\mu u^\nu - P g^{\mu\nu} \right)$$
$$= \partial_\tau \epsilon + \frac{4}{3\tau}\epsilon \, , \quad \epsilon = \epsilon_0 \left(\frac{\tau_0}{\tau} \right)^{4/3} \, ,$$

$$\tilde{T}^{\mu\nu} = \begin{pmatrix} \frac{P x_3^2 + t^2 \epsilon}{t^2 - x_3^2} & 0 & 0 & \frac{t x_3 (P+\epsilon)}{t^2 - x_3^2} \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ \frac{t x_3 (P+\epsilon)}{t^2 - x_3^2} & 0 & 0 & \frac{x_3^2 (P+\epsilon)}{t^2 - x_3^2} + P \end{pmatrix}$$

Viscous hydrodynamics (second order):

$$\partial_\tau \epsilon + \frac{4\epsilon}{3\tau} = \frac{4\eta}{3\tau^2} + \frac{8\eta\tau_\pi}{9\tau^3} - \frac{8\lambda_1}{9\tau^3}$$

➡ At late times, the system is still expanding and approximately isotropic.



[Cartwright,Kaminski,Schenke; PRC (2022)]

Bjorken flow equation

$$\partial_\tau \epsilon + \frac{4}{3}\frac{\epsilon}{\tau} - \frac{4}{3}\frac{\eta}{\tau^2} = 0$$

Holographic Bjorken flow equation

$$-\frac{P_1(\tau)}{\tau} - \frac{P_2(\tau)}{\tau} - \frac{B_1(\tau)^2}{8\tau} + \partial_\tau \epsilon(\tau) + \frac{2\epsilon(\tau)}{\tau} = 0$$

Energy and pressures

$$\epsilon = \langle T_{00}\rangle = \frac{2L^3}{\kappa^2}\left(-\frac{3a_4(\tau)}{4L^4} - \frac{b^2\log(b^{1/2})}{8L^2\tau^2}\right),$$

$$P_1 = \langle T_{11}\rangle = \frac{2L^3}{\kappa^2}\left(-\frac{a_4(\tau)}{4L^4} + \frac{h_4^{(1)}(\tau)}{L^4} + \frac{b^2\log(b^{1/2})}{8L^2\tau^2} - \frac{1}{6\tau^4}\right),$$

$$P_2 = \langle T_{22}\rangle = \frac{2L^3}{\kappa^2}\left(-\frac{a_4(\tau)}{4L^4} + \frac{h_4^{(2)}(\tau)}{L^4} - \frac{b^2\log(b^{1/2})}{8L^2\tau^2} - \frac{b^2}{16L^2\tau^2} - \frac{1}{6\tau^4}\right),$$

$$\tau^2 P_\xi = \langle T_{\xi\xi}\rangle = \frac{2L^3\tau^2}{\kappa^2}\left(-\frac{a_4(\tau)}{4L^4} - \frac{h_4^{(1)}(\tau)}{L^4} - \frac{h_4^{(2)}(\tau)}{L^4} - \frac{b^2\log(b^{1/2})}{8L^2\tau^2} - \frac{b^2}{16L^2\tau^2} + \frac{1}{3\tau^4}\right)$$

$$\langle J^a_{(5)}\rangle = \frac{1}{2\kappa^2}\left(\frac{q_5 L}{\tau}, 0, 0, 0\right),$$

$$\langle J^a\rangle = \frac{1}{2\kappa^2}\left(0, 2V_2(\tau), 0, 0\right),$$

➡CME current

➡time-dependent axial charge and *B*

$$B^a = \frac{1}{2}\epsilon^{abcd}u_b F_{cd} \quad \Rightarrow \quad B^1 = \frac{b}{L\tau}$$

Recall the metric:

$$\begin{aligned} ds^2 &= 2\mathrm{d}r\mathrm{d}v - A(v,r)\mathrm{d}v^2 + F_1(v,r)\mathrm{d}v\mathrm{d}x_1 \\ &+ S(v,r)^2 e^{H_1(v,r)}\mathrm{d}x_1^2 + S(v,r)^2 e^{H_2(v,r)}\mathrm{d}x_2^2 \\ &+ L^2 S(v,r)^2 e^{-H_1(v,r)-H_2(v,r)}\mathrm{d}\xi^2, \end{aligned}$$

APPENDIX: Holographic Bjorken - expanding plasma

[Cartwright,Kaminski,Knipfer; (2022)]

Metric Ansatz :

$$ds^2 = 2\mathrm{d}r\mathrm{d}v - A(v,r)\mathrm{d}v^2 + e^{B(v,r)}S(v,r)^2(\mathrm{d}x_1^2 + \mathrm{d}x_2^2) + S(v,r)^2 e^{-2B(v,r)}\mathrm{d}\xi^2$$

$$\lim_{r\to\infty}\frac{1}{r^2}\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \tau^2\mathrm{d}\xi^2$$

Anisotropy function :

$$B = z^4 B_\mathrm{s} + \Delta_B$$

Initial conditions :

$$B_\mathrm{s}(z, v_0) = \Omega_1 \cos(\gamma_1 z) + \Omega_2 \tan(\gamma_2 z) + \Omega_3 \sin(\gamma_3 z) + \sum_{i=0}^{5} \beta_i z^i$$
$$+ \frac{\alpha}{z^4}\left[-\frac{2}{3}\ln\left(1 + \frac{z}{v_0}\right) + \frac{2z^3}{9v_0^3} - \frac{z^2}{3v_0^2} + \frac{2z}{3v_0}\right],$$

APPENDIX: Strong B thermodynamics

$$\boxed{B \sim \mathcal{O}(1)}$$

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

Strong B thermodynamics with anomaly : $\langle T^{\alpha\beta}\rangle = \epsilon\, u^\alpha u^\beta + p\Delta^{\alpha\beta} + \tau^{\alpha\beta}$

Energy momentum tensor:

equilibrium heat current

$$\langle T^{\mu\nu}\rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \underline{\xi_V^{(0)} B} \\ 0 & \underline{P_0 - \chi_{BB} B^2} & 0 & 0 \\ 0 & 0 & \underline{P_0 - \chi_{BB} B^2} & 0 \\ \underline{\xi_V^{(0)} B} & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

"magnetic pressure shift"

Axial current:

$$\langle J^\mu\rangle = \left(n_0,\, 0,\, 0,\, \underline{\xi_B^{(0)} B}\right) + \mathcal{O}(\partial)$$

equilibrium charge current

➡ new contributions to thermodynamic equilibrium observables

previous work:

[Kovtun; JHEP (2016)]

[Jensen, Loganayagam, Yarom; JHEP (2014)]

[Israel; Gen.Rel.Grav. (1978)]

APPENDIX: CPT symmetries

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

quantity	C	P	T
t	+	+	-
x^i	+	-	+
r	+	+	+
T, h_{tt}, T^{tt}	+	+	+
μ_A, A_t, J^t	+	-	+
μ_V, V_t, J^t_V	-	+	+
A_i, J^i	+	+	-
V_i, J^i_V	-	-	-
A_r	+	-	-
V_r	-	+	-
u^i, h_{ti}, T^{ti}	+	-	-
h_{ij}, T^{ij}	+	+	+
B^i	+	-	-
B^i_V	-	+	-
E^i	+	+	+
E^i_V	-	-	+
$dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma \wedge dx^\kappa$	+	-	-
$\int_i^f A \wedge F \wedge F$	+	+	+
$\int_i^f V \wedge F_V \wedge F_V$	-	-	+
u^t	+	+	+
generating functional W (axial $U(1)_A$)	+	+	+

APPENDIX: EFT calculation: chiral hydrodynamics with magnetic field

For any theory with chiral anomaly

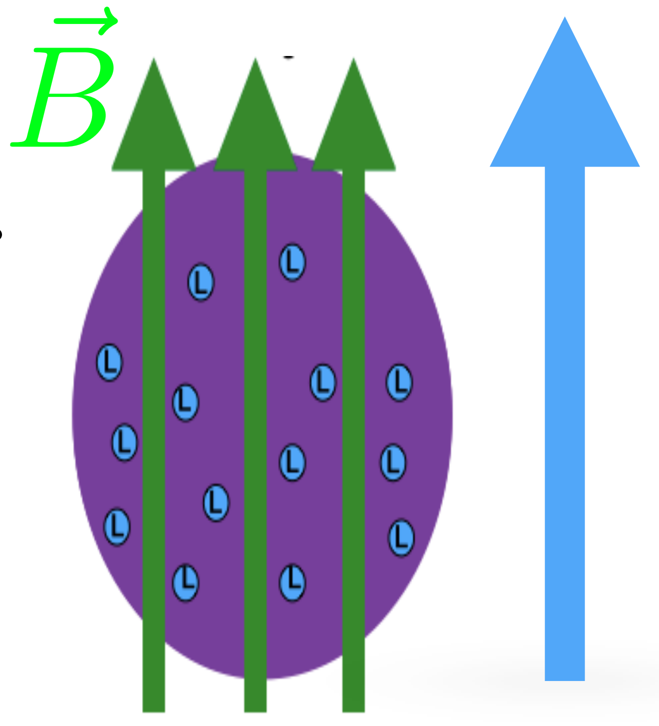
$$\partial_\mu J_A{}^\mu = C\,\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

[Son,Surowka; PRL (2009)]
[Erdmenger, Haack, Kaminski, Yarom; JHEP (2008)]
[Banerjee et al.; JHEP (2011)]

Axial current with **weak** external B field:

$$\langle J_A{}^\mu \rangle = n u^\mu + \sigma E^\mu - \sigma T \Delta^{\mu\nu} \nabla_\nu \left(\frac{\mu}{T}\right) + \xi_B B^\mu + \xi_V \Omega^\mu + \dots$$

(ideal) charge flow — *conductivity term* — *charge diffusion* — *chiral magnetic conductivity term* — *chiral vortical conductivity term*



axial current

Energy momentum tensor with weak external B field:

$$\langle T^{\mu\nu} \rangle = \underbrace{\epsilon u^\mu u^\nu + P \Delta^{\mu\nu}}_{\text{ideal fluid}} + \underline{u^\mu q^\nu + u^\nu q^\mu} + \tau^{\mu\nu}$$

heat current

measured in Weyl semi metals
e.g. [Huang et al; PRX (2015)]

neutron stars?
[Kaminski et al.; PLB (2014)]

Now calculate hydrodynamic 1- and 2-point functions and determine their poles!
[Landau, Lifshitz]
[Kadanoff; Martin]

APPENDIX: Dispersion relations: weak B hydrodynamics

Weak B hydrodynamics - poles of 2-point functions

: $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle,\ \langle T^{\mu\nu}\, J^{\alpha}\rangle,\ \langle J^{\mu}\, T^{\alpha\beta}\rangle,\ \langle J^{\mu}\, J^{\alpha}\rangle$

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

spin 1 modes under SO(2) rotations around *B*

$$\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}$$

former momentum diffusion modes

$$\mathfrak{s}_0 = s_0 / n_0$$
$$\tilde{c}_P = T_0 (\partial \mathfrak{s} / \partial T)_P$$

spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$

former charge diffusion mode

$$\omega_+ = v_+\, k - i\Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_- = v_-\, k - i\Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$

former sound modes

➡ **a chiral magnetic wave**

[Kharzeev, Yee; PRD (2011)]

$$v_0 = \frac{2B\, T_0}{\tilde{c}_P n_0} \left(\tilde{C} - 3C \mathfrak{s}_0^2 \right)$$

$$D_0 = \frac{w_0^2\, \sigma}{\tilde{c}_P n_0^3 T_0}$$

➡ **dispersion relations of hydrodynamic modes are heavily modified by anomaly and *B***

APPENDIX: EFT result III: weak B details

Weak B hydrodynamics - poles of 2-point functions:

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$ *former charge diffusion mode*

$$\omega_+ = v_+\, k - i\Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_- = v_-\, k - i\Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$

former sound modes

$$w_0 = \epsilon_0 + P_0$$
$$\mathfrak{s}_0 = s_0/n_0$$
$$\tilde{c}_P = T_0(\partial\mathfrak{s}/\partial T)_P$$
$$c_s^2 = (\partial P/\partial\epsilon)_{\mathfrak{s}}$$

damping coefficients:

$$\Gamma_\pm = \frac{3\zeta + 4\eta}{6w_0} + c_s^2\,\frac{w_0\,\sigma}{2n_0^2}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)^2 \qquad D_0 = \frac{w_0^2\,\sigma}{\tilde{c}_P n_0^3 T_0}$$

velocities:

$$v_\pm = \pm c_s - B\,\frac{c_s^2}{n_0}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)\left[3CT_0\mathfrak{s}_0 + \frac{\alpha_P T_0^2}{\tilde{c}_P}(\tilde{C} - 3C\mathfrak{s}_0^2) + \frac{1}{2}\xi_B^{(0)} - \frac{n_0}{w_0}\xi_V^{(0)}\right] + B\,\frac{1 - c_s^2}{w_0}\,\xi_V^{(0)}\,, \qquad v_0 = \frac{2B\,T_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3C\mathfrak{s}_0^2\right)$$

chiral conductivities:

$$\xi_V = -3C\mu^2 + \tilde{C}T^2, \quad \xi_B = -6C\mu, \quad \xi_3 = -2C\mu^3 + 2\tilde{C}\mu T^2$$

known from entropy current argument
[Son,Surowka; PRL (2009)]
[Neiman,Oz; JHEP (2010)]

APPENDIX: weak B hydrodynamics comparison

Spin-1 modes

No knowledge of anisotropic (B-dependent) transport coefficients
— take B=0 values of this model instead

except zero charge: [Finazzo, Critelli, Rougemont, Noronha; PRD (2016)]

weak B hydro prediction:

$$\omega = \mp \frac{Bn_0}{\epsilon_0 + P_0} - ik^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{Bn_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{iB^2 \sigma}{\epsilon_0 + P_0}$$

calculate from holography

We find agreement between hydrodynamic prediction and holographic model for small values of B, increasing deviations for larger B.

Real part of spin-1 modes matches exactly even at large B!

APPENDIX: strong B hydrodynamics

[Hernandez, Kovtun; JHEP (2017)]

Spin-1 modes

Anisotropic transport coefficients

strong B: $\omega = \pm \frac{B_0 n_0}{w_0} - \frac{iB_0^2}{w_0}(\sigma_\perp \pm i\tilde{\sigma}) - iD_c k^2$

parity-odd

weak B: $\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - ik^2 \frac{\eta}{\epsilon_0 + P_0} + k\frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{iB^2\sigma}{\epsilon_0 + P_0}$

Agreement in form

Exact agreement in real part!

Spin-0 modes

strong B: $\omega = \pm k v_s - i\frac{\Gamma_{s,\|}}{2}k^2\,,$

$\omega = -iD_\| k^2\,,$

Anisotropic transport coefficients

$$D_\| = \frac{\sigma_\| w_0^2}{n_0^2 \chi_{11} + w_0^2 \chi_{33} - 2n_0 w_0 \chi_{13}}$$

parity-odd

weak B: $\omega_0 = v_0 k - iD_0 k^2 + \mathcal{O}(\partial^3)$

$\omega_+ = v_+ k - i\Gamma_+ k^2 + \mathcal{O}(\partial^3)$

$\omega_- = v_- k - i\Gamma_- k^2 + \mathcal{O}(\partial^3)$

Agreement in form

$$D_0 = \frac{w_0^2 \sigma}{\tilde{c}_P n_0^3 T_0} \qquad v_0 = \frac{2B T_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3C\mathfrak{s}_0^2\right)$$

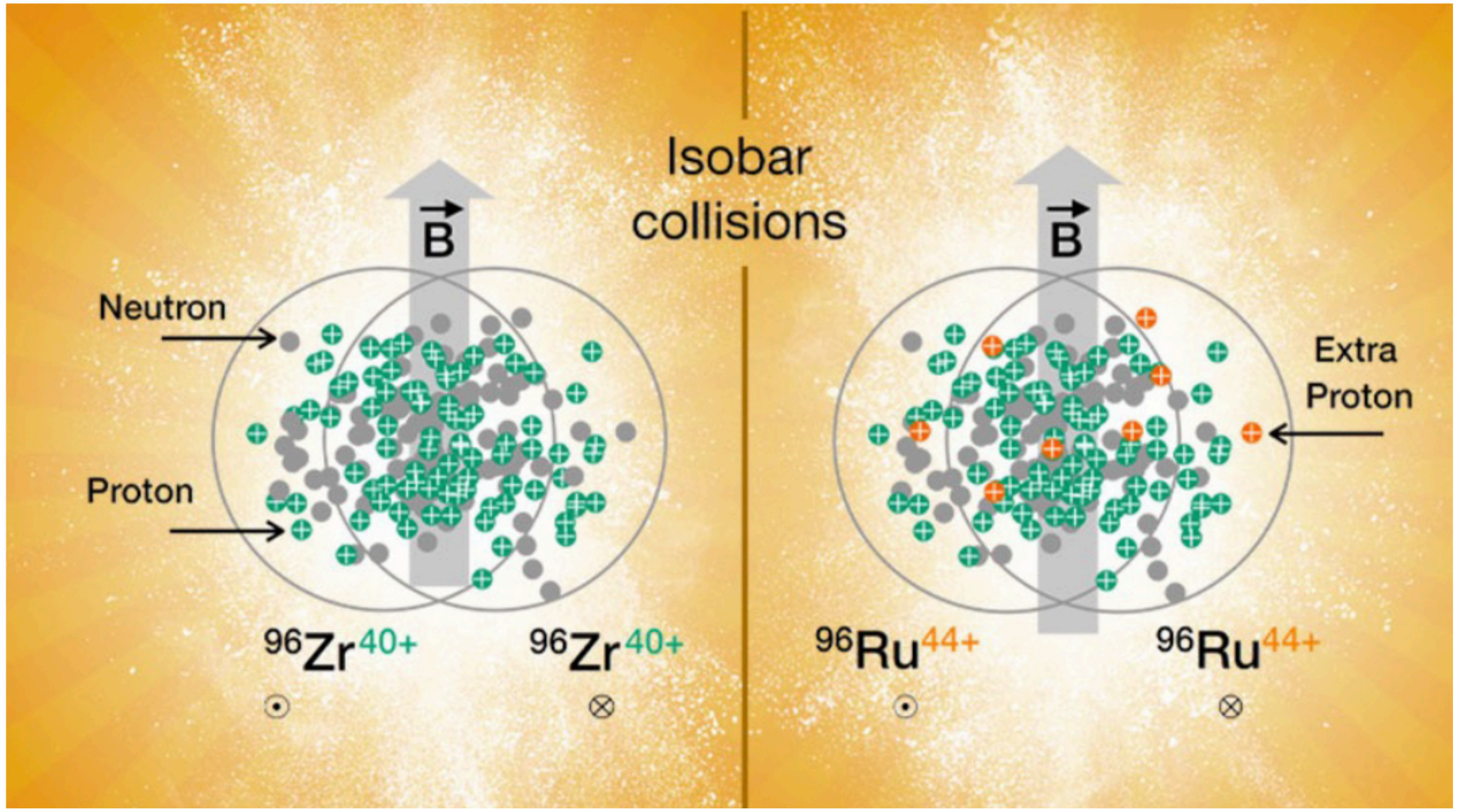
$$\tilde{c}_P = T_0(\partial\mathfrak{s}/\partial T)_P$$

APPENDIX: CME in heavy ion collisions - RHIC isobar run

Magnetic field B is large in collision experiments:

RHIC	$B \approx 10^{19} G$
LHC	$B \approx 10^{20} G$

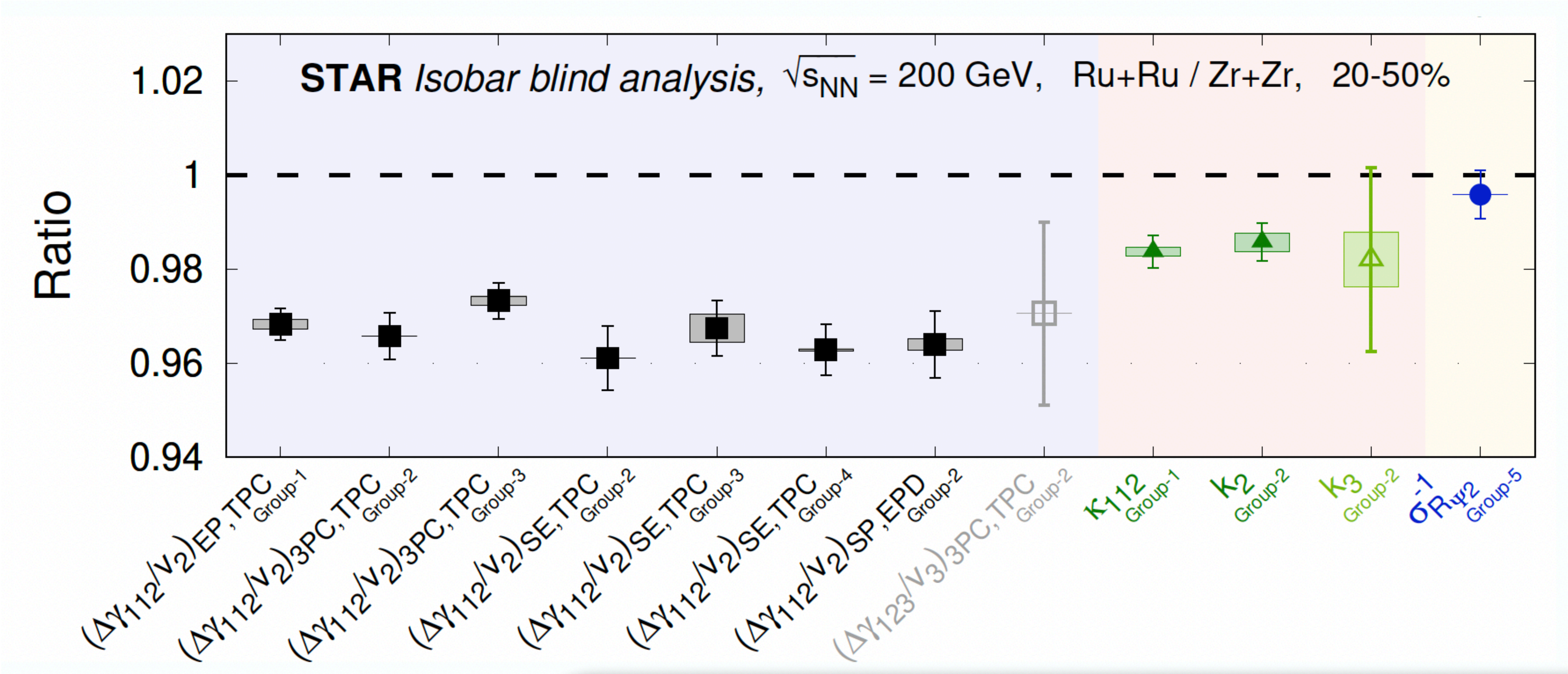
- early RHIC (2009, 2014) and LHC (2013) results hint at CME, but inconclusive; too dirty (cond-mat observed CME)
- isobar run approved at RHIC (2017)



***taken from Helen Caines' talk at 6th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions** (Nov 1-5, 2021)*

➡Larger charge creates larger magnetic field, so larger CME in Ru

➡otherwise identical (?)



If CME present, we expect:

$$\frac{\mathrm{Measure(Ru+Ru)}}{\mathrm{Measure(Zr+Zr)}} > 1$$

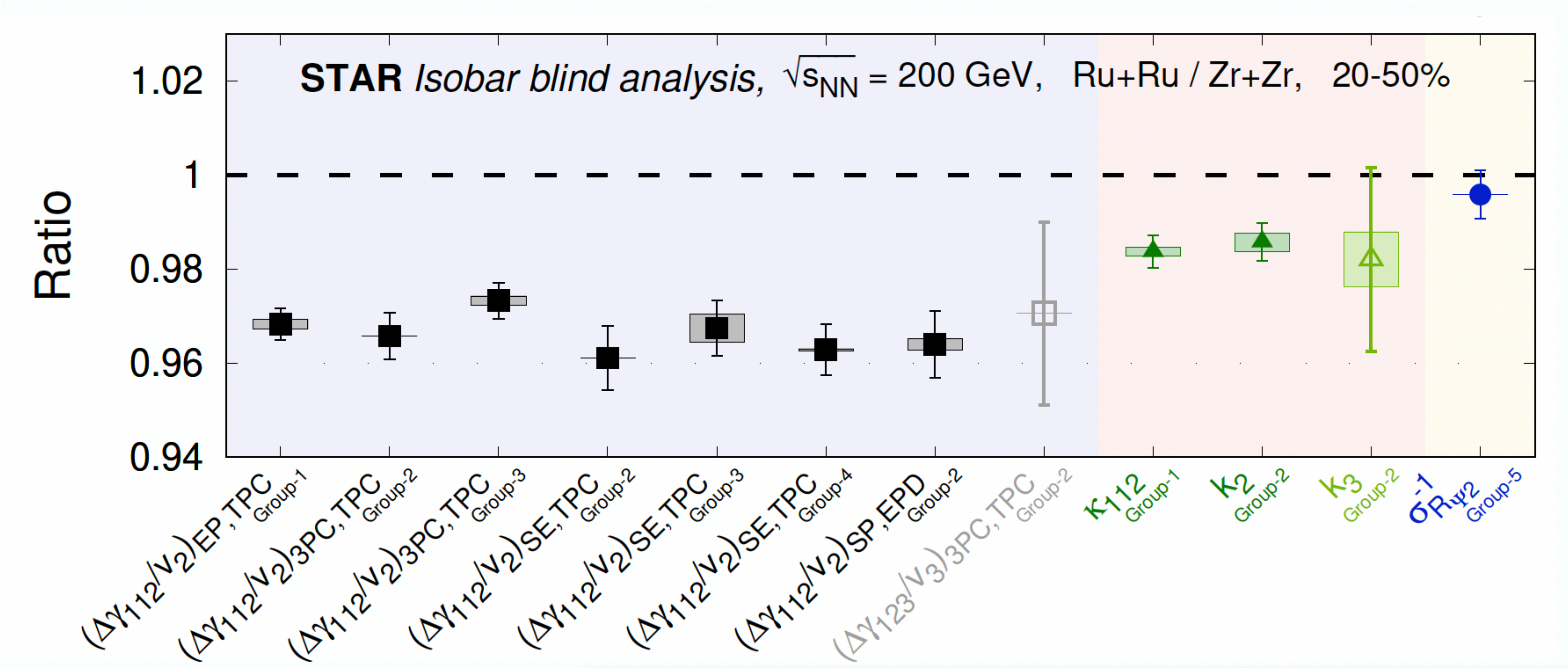
But plot shows all ratios <1 !

- ➡**No CME according to pre-blind criteria**
- ➡**Ru and Zr not as identical as expected: multiplicities and initial geometries differ**
- ➡**don't know axial charge or magnetic field**
- ➡**signal-to-background ratio unclear**
- ➡**more runs? need theoretical understanding**

APPENDIX: CME in heavy ion collisions - RHIC isobar analysis

top-RHIC energy: [STAR Collaboration; (2021)]
low-energy update: [STAR Collaboration; (2022)]

high energy update: [ALICE Collaboration; (2022)]



If CME present, we expect:

$$\frac{\text{Measure}(\text{Ru}+\text{Ru})}{\text{Measure}(\text{Zr}+\text{Zr})} > 1$$

But plot shows all ratios <1 !

➡No CME according to pre-blind criteria

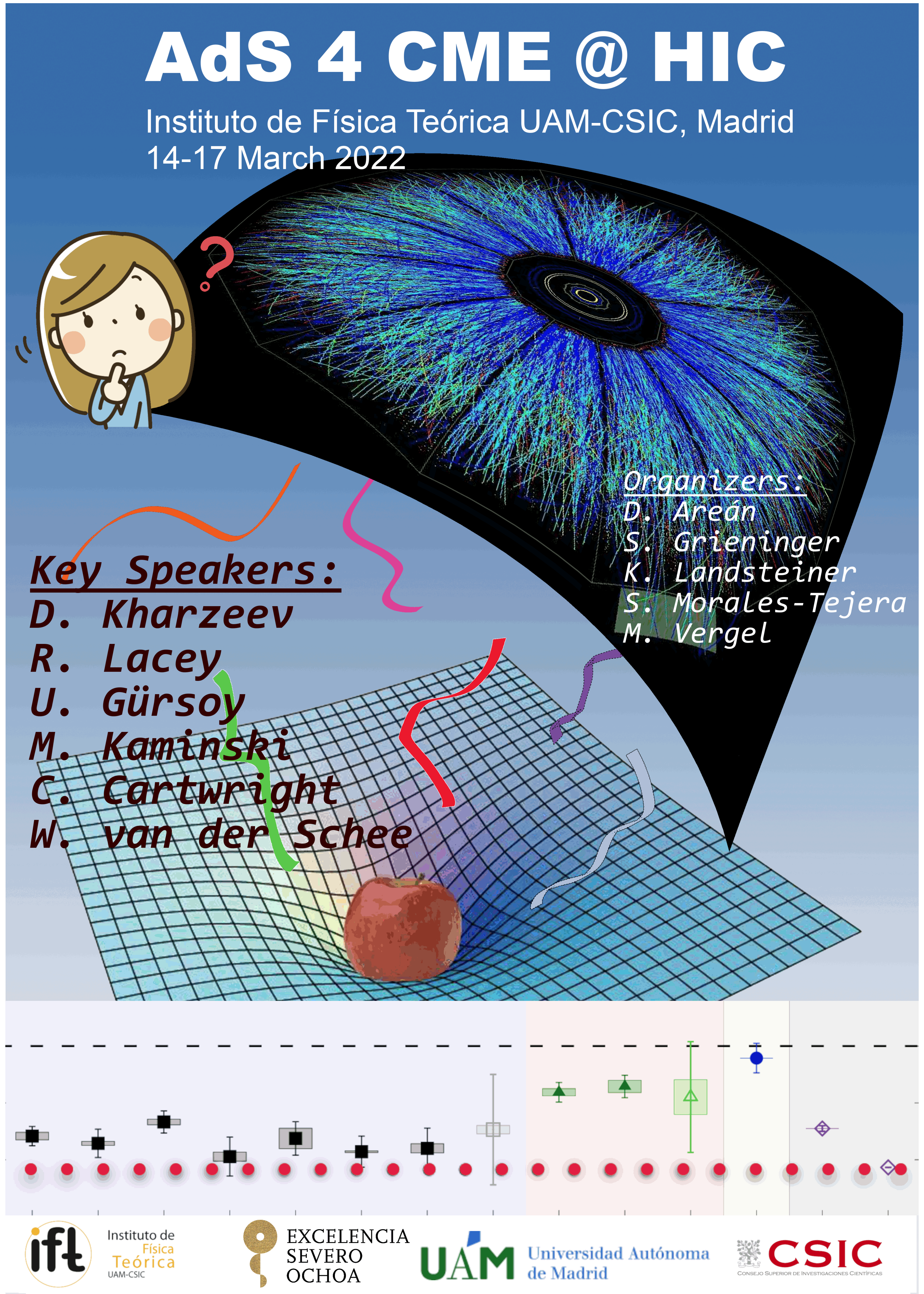
➡Ru and Zr not as identical as expected: multiplicities and initial geometries differ

➡don't know axial charge or magnetic field

➡signal-to-background ratio unclear

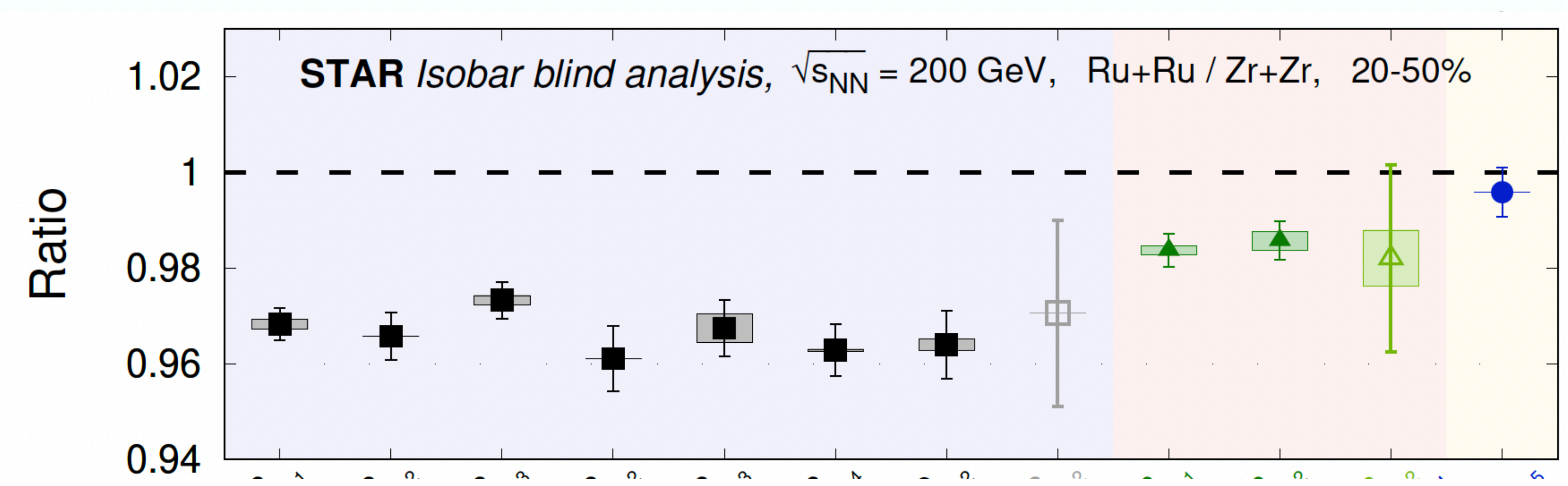
➡more runs? need theoretical understanding

APPENDIX: AdS4CME Collaboration



Participants: Dmitri Kharzeev, Karl Landsteiner, Umut Gürsoy, MK

To be invited: Wilke van der Schee, Daniel Arean, Björn Schenke, Sebastian Grieninger, Casey Cartwright, Sergio Morales Tejera, Pablo Saura Bastida, Nabil Iqbal, Nick Poovuttikul, Martin Ammon, Matti Jarvinen, Ho Ung Yee, Misha Stephanov, Jenfing Liao, Saso Grozdanov, Ruth Gregory, Arpit Das, Helen Caines, Andrea Danu, Mei Huang, Jacquelyn Noronha-Hostler, …



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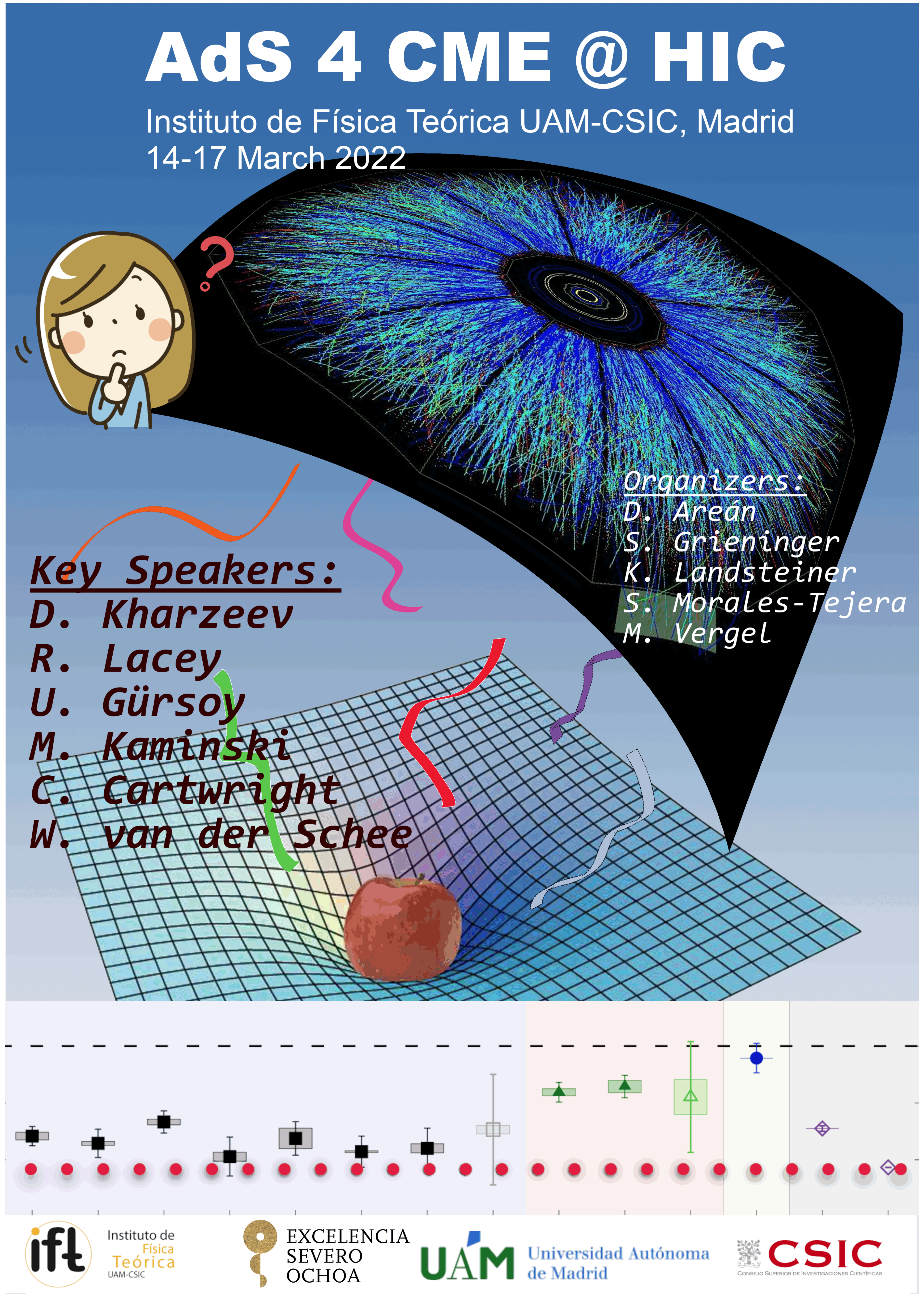

https://ads4cme.wixsite.com/ads4cme

Workshop at ECT*, Trento, Italy
March 13-17, 2023

Participants: Dmitri Kharzeev, Karl Landsteiner, Umut Gürsoy, MK

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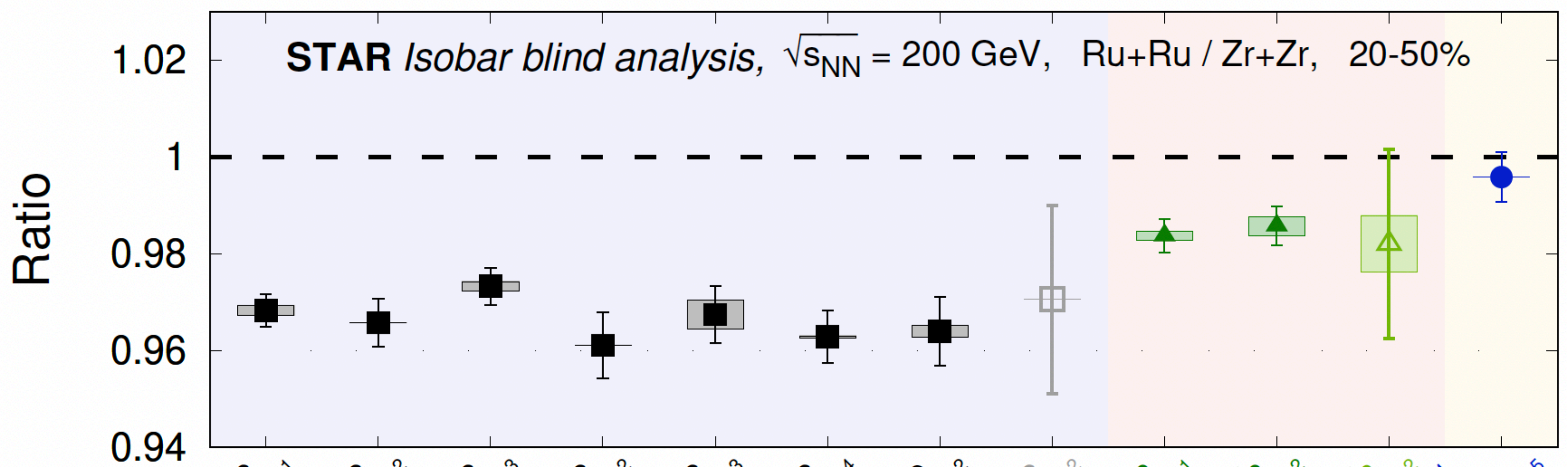
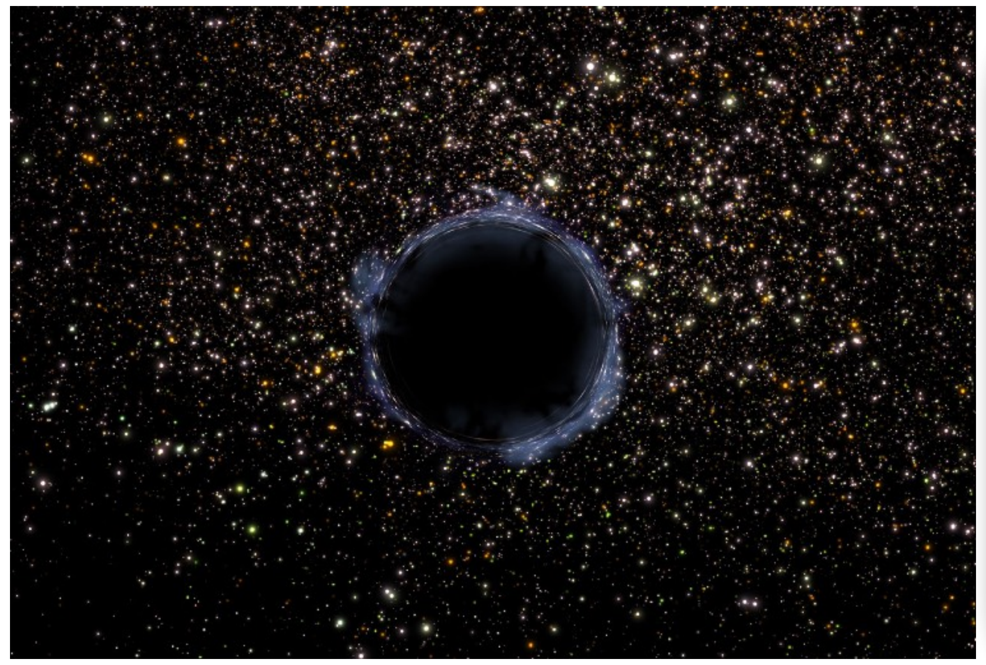


Preliminary re-analysis of isobar data suggested lower baseline, implying a CME-signal (with 1 to 5 sigma)

Holographic model with axial current only



➡ **use as holographic dual to charged state in strong *B***

➡ ***N*=4 Super-Yang-Mills coupled to external (E,B)-fields**

[Ammon, Kaminski et al.; JHEP (2017)]

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; arXiv:2012.09183]

Einstein-Maxwell-Chern-Simons action

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A\wedge F\wedge F\right]$$

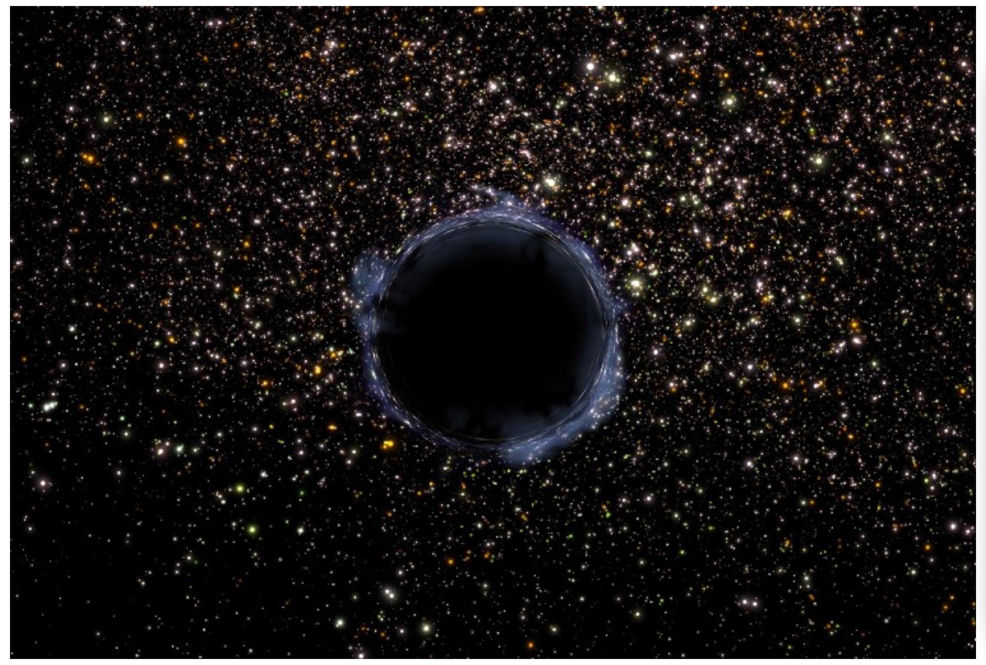
5-dimensional Einstein-Maxwell action encodes N=4 Super-Yang-Mills theory with axial U(1) gauge symmetry

5-dimensional Chern-Simons term encodes chiral anomaly

Charged magnetic black branes dual to charged thermal state with *B*

[D'Hoker, Kraus; JHEP (2010)]

- charged magnetic analog of Reissner-Nordstrom black brane
- asymptotically AdS_5



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➡**axial *B***
➡**axial charge**
➡**axial current only**

Chiral effects in vector and axial currents

see e.g. [Jensen, Kovtun, Ritz; JHEP (2013)]

Vector current (e.g. QCD electromagnetic *U(1)*)

$$J^\mu_V = \cdots + \xi_V \omega^\mu + \xi_\chi \, B^\mu + \xi_{VA} B^\mu_A$$

chiral
magnetic
effect

Axial current (e.g. QCD axial *U(1)*)

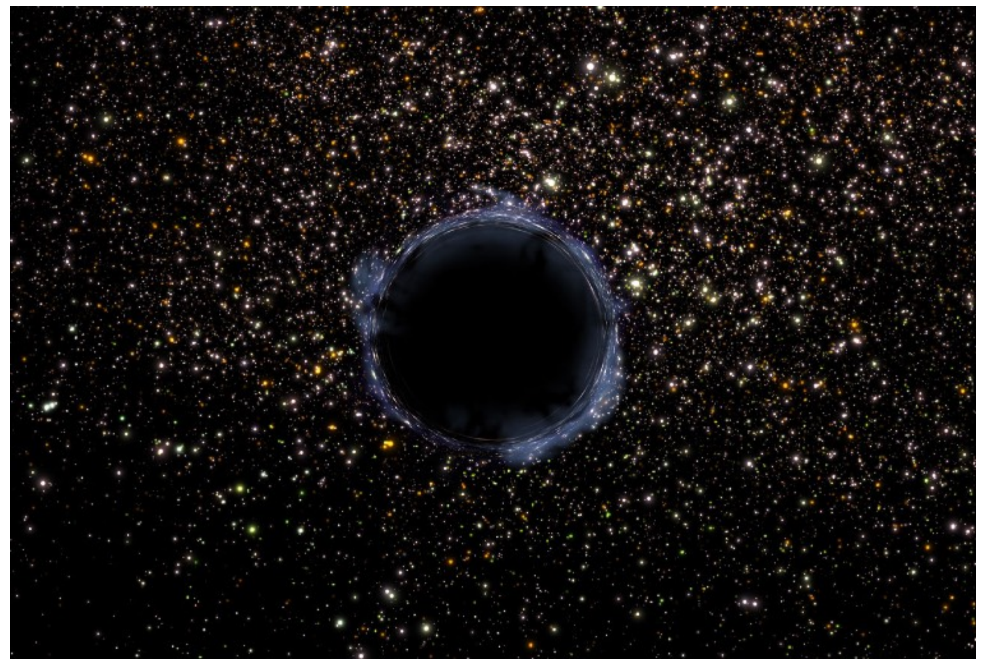
$$J^\mu_A = \cdots + \xi \omega^\mu + \xi_B B^\mu + \xi_{AA} B^\mu_A$$

chiral
vortical
effect

chiral
separation
effect

➡**phenomenology needs *both* currents**

Holographic model with two currents



Einstein-Maxwell-Chern-Simons action with two gauge fields A_μ and V_μ

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(R - 2\Lambda - \frac{L^2}{4}F_{\mu\nu}F^{\mu\nu} - \frac{L^2}{4}F^{(5)}_{\mu\nu}F^{\mu\nu}_{(5)} + \frac{\alpha}{3}\epsilon^{\mu\nu\rho\sigma\tau}A_\mu\left(3F_{\nu\rho}F_{\sigma\tau} + F^{(5)}_{\nu\rho}F^{(5)}_{\sigma\tau}\right)\right)$$

gravitational coupling κ

Einstein-Hilbert

Maxwell

"axial Maxwell"

Chern-Simons coupling α

Chern-Simons term encoding chiral anomaly

5D vector gauge field V_μ ⟷ *4D conserved vector current*

$$J^\mu_V = \cdots + \xi_V\omega^\mu + \xi_{VV}B^\mu + \xi_{VA}B^\mu_A$$

5D axial gauge field A_μ ⟷ *4D anomalous axial current*

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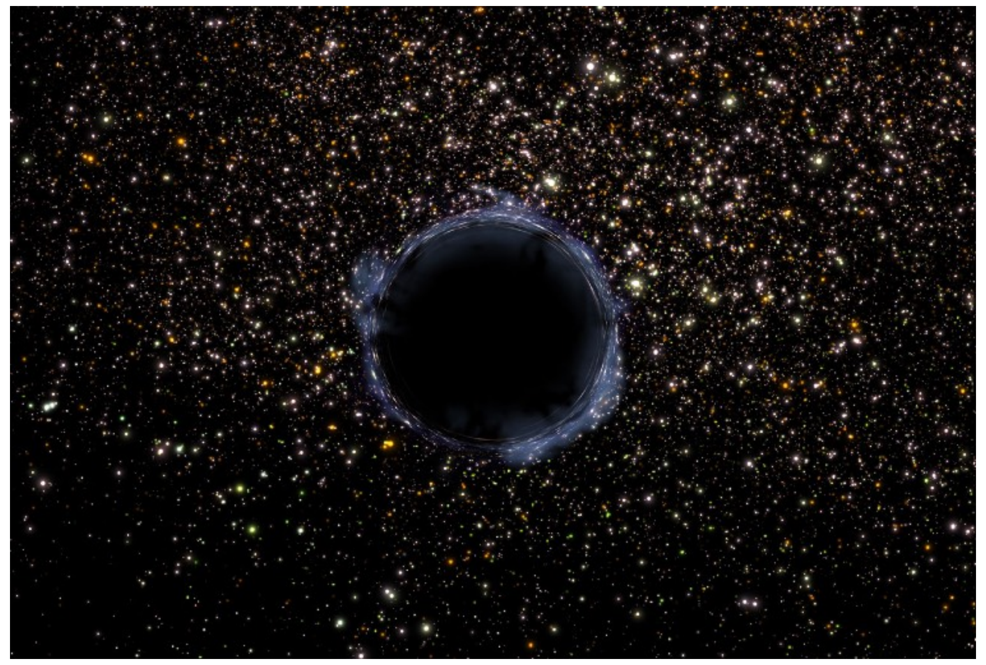

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[Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]

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gravitational coupling κ — *Einstein-Hilbert* — *Maxwell* — *"axial Maxwell"* — *Chern-Simons coupling α* — *Chern-Simons term encoding chiral anomaly*

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Isotropization (non-expanding plasma)

Initial state:

- Energy and axial charge corresponding to (T,μ_5) in final state
- Magnetic field is uniform and constant in time
- Dynamical pressure anisotropy vanishes
- CME current is absent

	"RHIC"	"LHC"
T	300MeV	1000MeV
μ_5	10 (100) MeV	10 (100) MeV
B	1 (0.1) m_π^2	15 (1.5) m_π^2

Matching couplings to QCD:

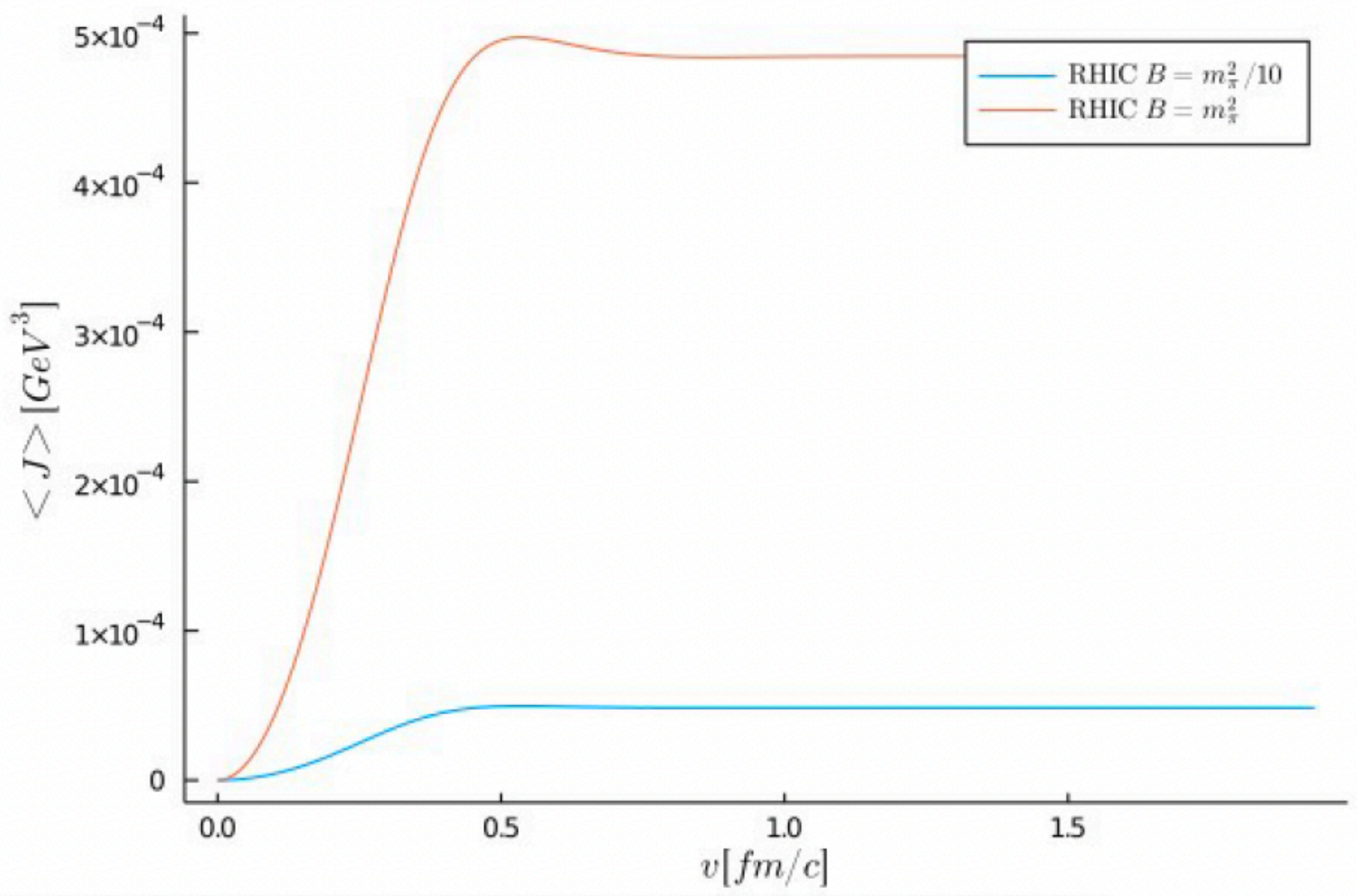
➔ Gravitational coupling: match to entropy

$$s_{BH} = \frac{4\pi^2 T^3}{2\kappa^2} \qquad s_{SB} = 4\left(\nu_b + \frac{7}{4}\nu_f\right)\frac{\pi^2 T^3}{90}$$

$$s_{BH} = \frac{3}{4} s_{SB} \qquad \Longrightarrow \quad \kappa^2 \approx 12.5$$

➔ Chern Simons coupling: match to anomaly

$$\frac{\alpha}{2\kappa^2} = A_{QCD} = \frac{1}{8\pi^2} \qquad \Longrightarrow \alpha \approx 0.316$$



➡CME more likely to be seen at RHIC than at LHC

➡lifetime of *B* crucial

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taken from Karl Landsteiner's talk at 6th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

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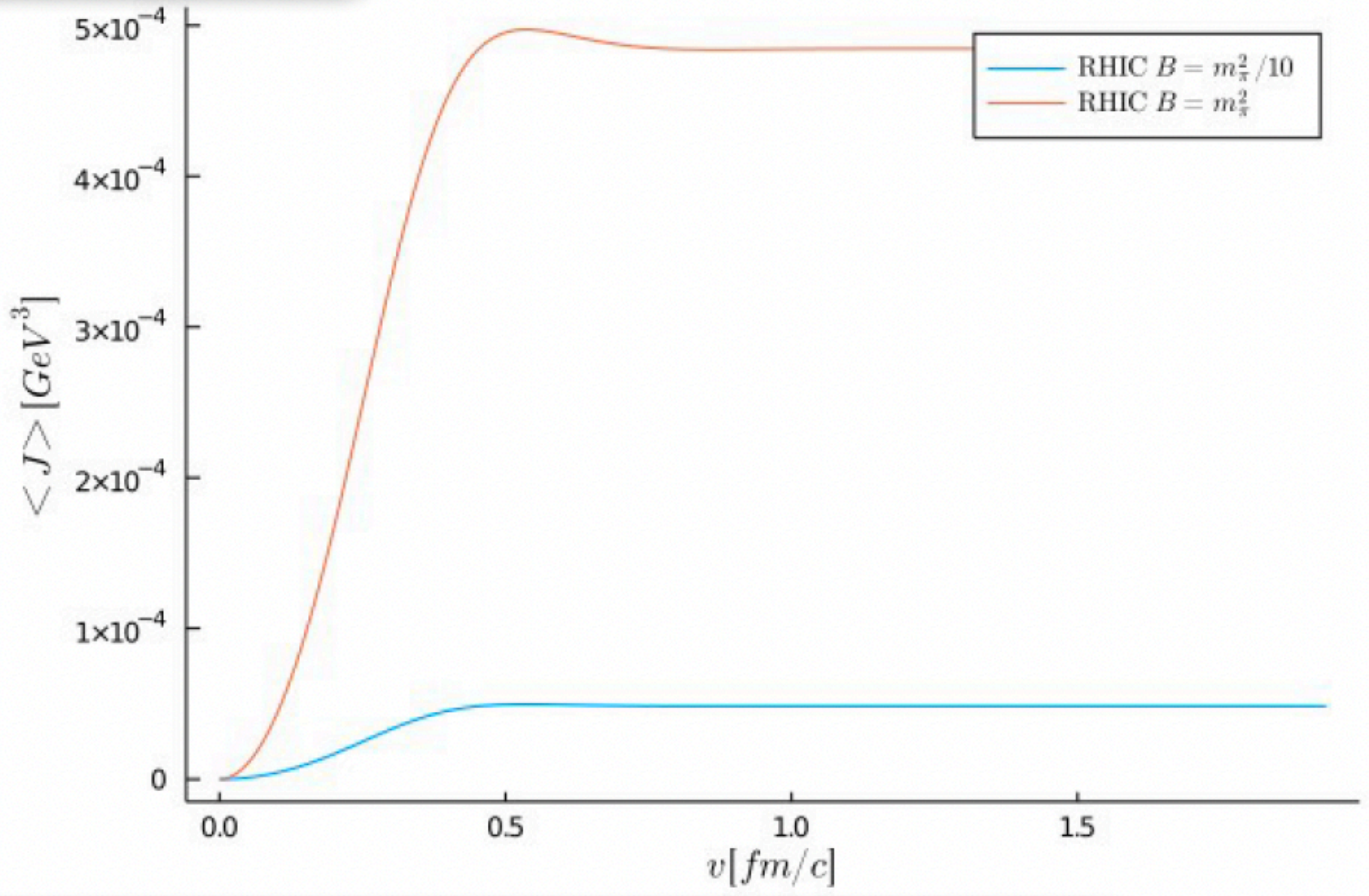
$$J^\mu = \xi_\chi B$$



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Bjorken - expanding plasma: C^3-code

[Cartwright,Kaminski,Schenke; PRC (2022)]

Holographic Model

Numerical routine

[Cartwright,Kaminski,Schenke, 2021]

1. Provide initial data $H_i(v_0,r), V(v_0,r), \epsilon(v_0), \xi(v_0)$

2. Solve line by line

$$
\begin{aligned}
0 &= zS(v,z)^2\left(H_1'(v,z)H_2'(v,z) + H_1'(v,z)^2 + H_2'(v,z)^2\right) + ze^{-H_1(v,z)}V'(v,z)^2 \\
&\quad + 6\left(2S'(v,z) + zS''(v,z)\right)S(v,z)\,, \\
0 &= L^6 b^2 e^{H_1(v,z)}S(v,z)^2 + \left(L^3 q_5 - 8\alpha b V(v,z)\right)^2 - 24L^6 z^2 S(v,z)^4 S'(v,z)\dot{S}(v,z) \\
&\quad - 12L^6 z^2 S(v,z)^5 \dot{S}'(v,z) - 24L^6 S(v,z)^6\,, \\
0 &= -64\alpha^2 b^2 e^{H_1(v,z)}V(v,z) + 8\alpha b L^3 q_5 e^{H_1(v,z)} - L^6 z^2 S(v,z)^3\left(S'(v,z)\dot{V}(v,z) + \dot{S}(v,z)V'(v,z)\right) \\
&\quad + L^6 z^2 S(v,z)^4\left(H_1'(v,z)\dot{V}(v,z) + \dot{H}_1(v,z)V'(v,z) - 2\dot{V}'(v,z)\right)\,, \\
0 &= -9z^2 S(v,z)^3\left(H_1'(v,z)\dot{S}(v,z) + \dot{H}_1(v,z)S'(v,z)\right) - 4z^2 e^{-H_1(v,z)}S(v,z)^2 V'(v,z)\dot{V}(v,z) \\
&\quad - 6z^2 \dot{H}_1{}'(v,z)S(v,z)^4 - 2b^2 e^{H_1(v,z)}\,, \\
0 &= -6z^2 \dot{H}_2{}'(v,z)S(v,z)^4 + b^2 e^{H_1(v,z)} + 2z^2 e^{-H_1(v,z)}S(v,z)^2 V'(v,z)\dot{V}(v,z) \\
&\quad - 9z^2 S(v,z)^3\left(H_2'(v,z)\dot{S}(v,z) + \dot{H}_2(v,z)S'(v,z)\right)\,, \\
0 &= 3L^4 S(v,z)^6\Big(2L^2 z^4 A''(v,z) + 4z^3 A'(v,z) - L^2 z^2 \dot{H}_1(v,z)\left(2H_1'(v,z) + H_2'(v,z)\right) \\
&\quad - L^2 z^2 H_1'(v,z)\dot{H}_2(v,z) - 2L^2 z^2 H_2'(v,z)\dot{H}_2(v,z) + 8L^2\Big) - 5b^2 L^6 e^{H_1(v,z)}S(v,z)^2 \\
&\quad + 2L^6 z^2 e^{-H_1(v,z)}S(v,z)^4\left(36e^{H_1(v,z)}S'(v,z)\dot{S}(v,z) - V'(v,z)\dot{V}(v,z)\right) \\
&\quad - 7\left(L^3 q_5 - 8\alpha b V(v,z)\right)^2\,, \\
0 &= 3z^2 A'(v,z)S(v,z)\dot{S}(v,z) + L^2 e^{-H_1(v,z)}\dot{V}(v,z)^2 + L^2 \dot{H}_1(v,z)\dot{H}_2(v,z)S(v,z)^2 \\
&\quad + L^2 \dot{H}_1(v,z)^2 S(v,z)^2 + L^2 \dot{H}_2(v,z)^2 S(v,z)^2 + 6L^2 S(v,z)\ddot{S}(v,z)\,.
\end{aligned}
$$

3. Obtain time derivative

$$\partial_v H_i(r,v) = \dot{H}_i - \frac{1}{2}A(r,v)\partial_r H_i(r,v)$$

$$\partial_v V(r,v) = \dot{V} - \frac{1}{2}A(r,v)\partial_r V(r,v)$$

4. Step forward in time

5. Provide new initial condition $H_i(v_0+n\Delta v,r), V(v_0+n\Delta v,r), \epsilon(v_0+n\Delta v), \xi(v_0+n\Delta v)$

6. Repeat steps 2-5 until final time is reached

Casey Cartwright, AdS4CME@HIC - 3/15/2022

15

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taken from Casey Cartwright's talk

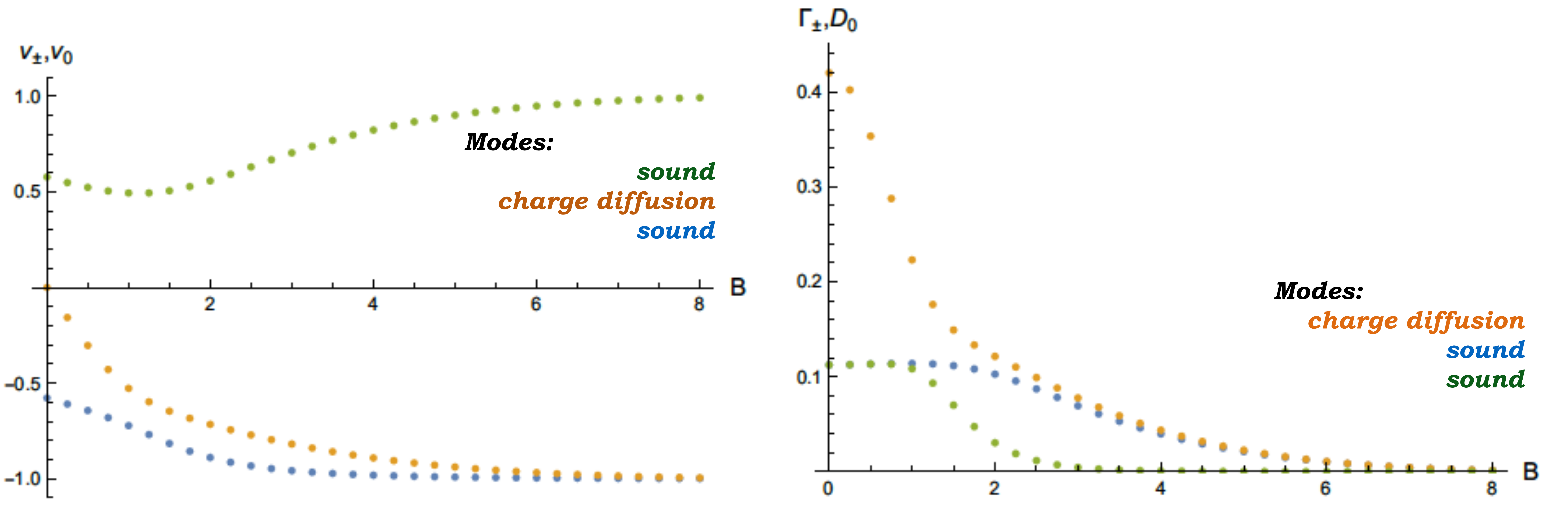
APPENDIX: Holographic result: hydrodynamic poles

[Ammon, Kaminski et al.; JHEP (2017)]

Fluctuations around charged magnetic black branes (QNMs)

- Weak *B*: **holographic results are in "agreement" with hydrodynamics.**
- Strong *B*: holographic result in agreement with thermodynamics, and numerical result indicates that **chiral waves propagate:**

(i) at the speed of light **and (ii) without attenuation**



confirming conjectures and results in probe brane approach [Kharzeev,Yee; PRD (2011)]

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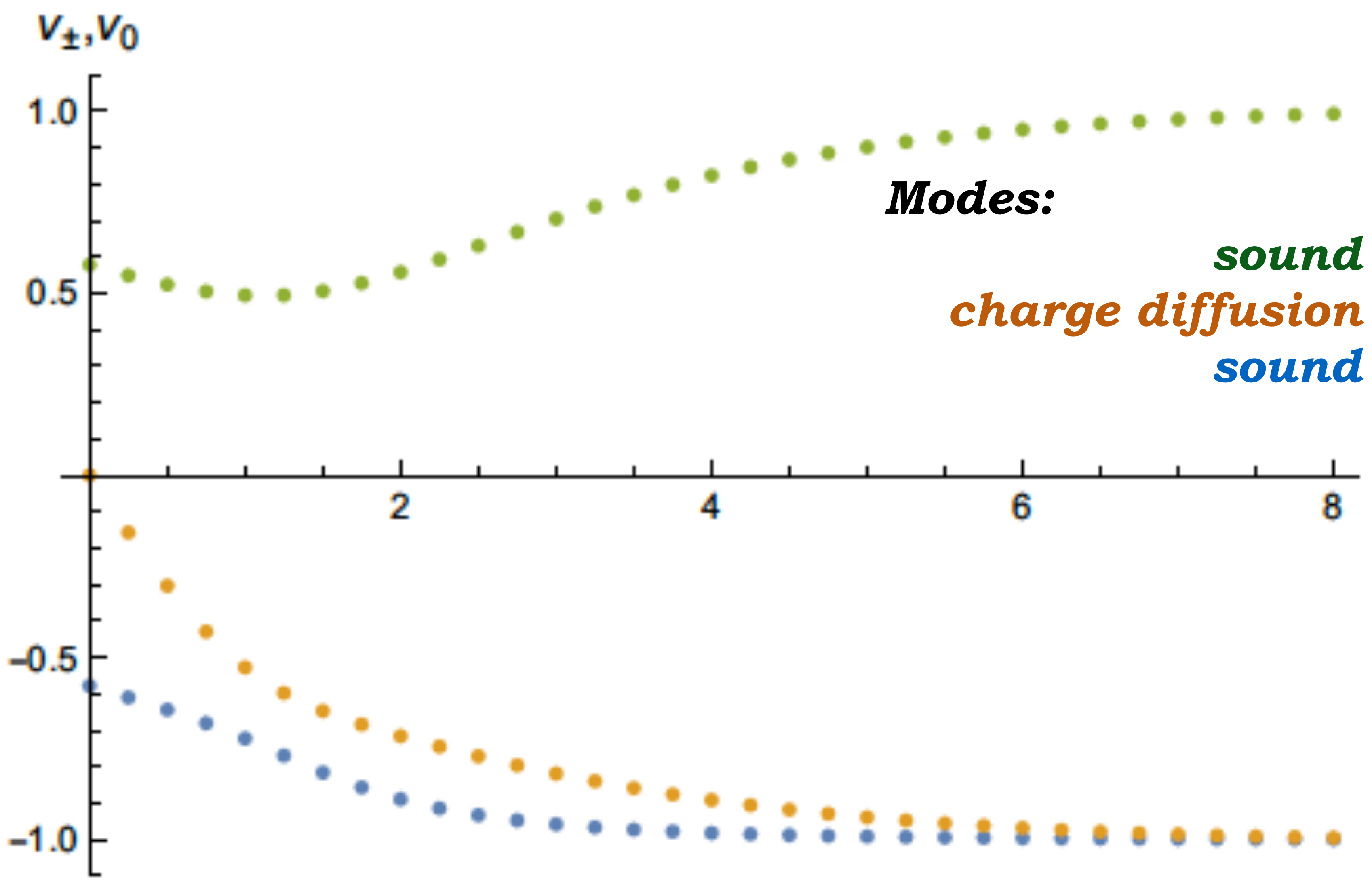

RECALL: weak B hydrodynamic poles

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$ *former charge diffusion mode*

$$\omega_+ = v_+\, k - i\Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_- = v_-\, k - i\Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$ *former sound modes*

confirming conjectures and results in probe brane approach *[Kharzeev, Yee; PRD (2011)]*

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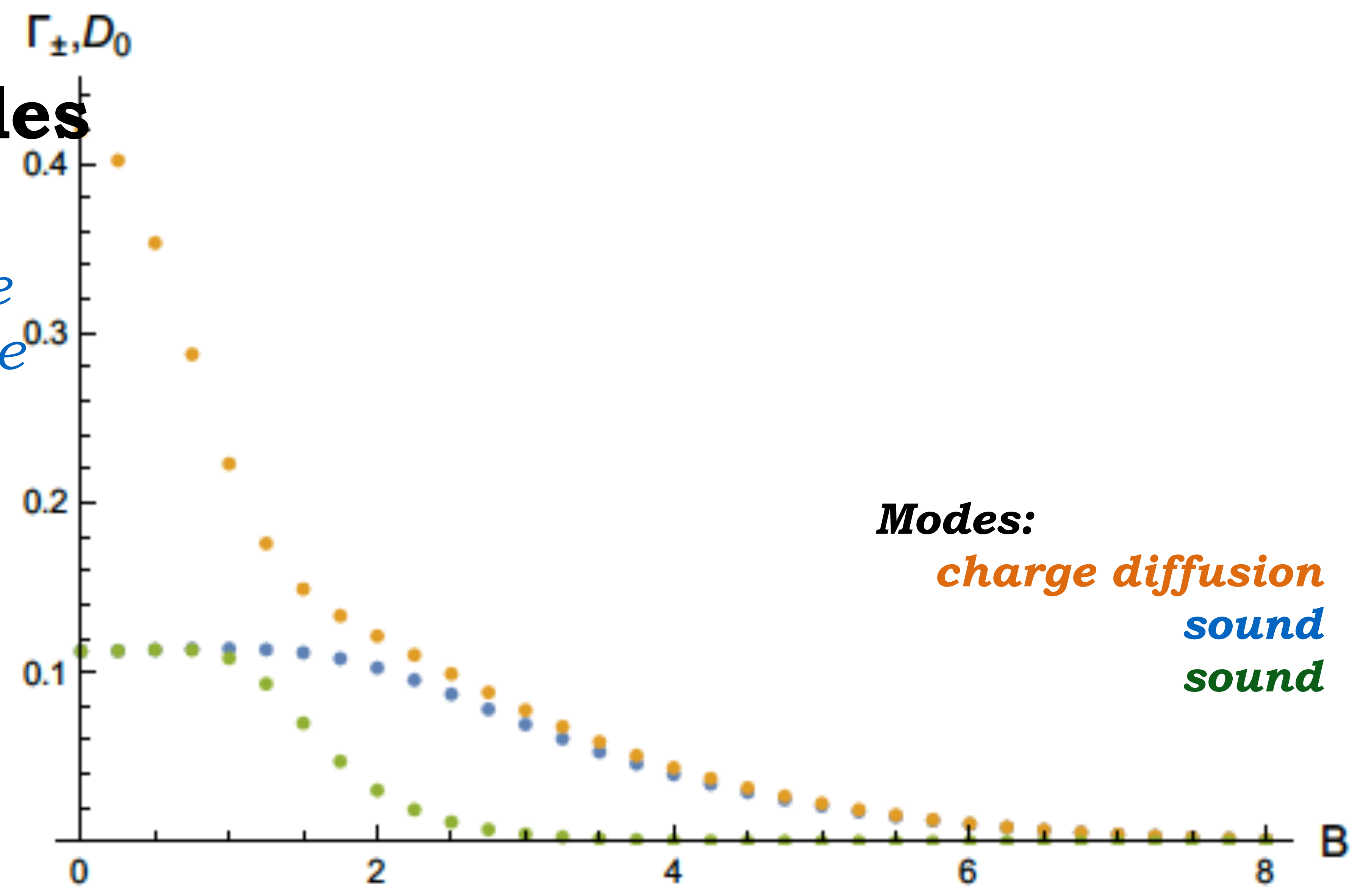
former sound modes



confirming conjectures and results in probe brane approach *[Kharzeev,Yee; PRD (2011)]*

APPENDIX: More thermodynamic transport coefficients

Magneto-thermal susceptibility M_1 **:**

$$\mathcal{E}_{\text{eq}} \sim M_1 B^\mu \partial_\mu \left(\frac{B^2}{T^4} \right)$$

Magneto-acceleration susceptibility M_3 **:**

$$\mathcal{E}_{\text{eq}} \sim \mathcal{P}_{\text{eq}} \sim M_{3,B^2}\, B{\cdot}a$$

Magneto-electric susceptibility M_4 **:**

$$\mathcal{E}_{\text{eq}} \sim M_{4,T}\, B{\cdot}E \,, \qquad \mathcal{P}_{\text{eq}} \sim M_{4,B^2}\, B{\cdot}E$$

Magneto-vortical susceptibility M_5 **:** $\mathcal{E}_{\text{eq}} \sim \mathcal{P}_{\text{eq}}$

$$\sim M_5 B{\cdot}\Omega$$