Anomalous transport in low dimensions

ANA MIZHER*

* UNICID and IFT-UNESP, Brazil.

Strongly Interacting Matter in Extreme Magnetic Fields,
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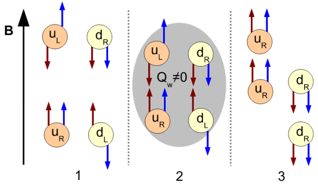
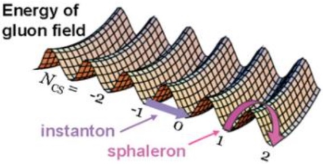
Huitzil collaboration with Cristian Villavicencio, Alfredo Raya, Adnan Bashir, Luis Albino, David Dudal, Alexandre Reily and Filipe Matusalém.

Overview

The chiral magnetic effect

- The chiral magnetic effect is an electric current that appears as a result of chiral imbalance associated to an external magnetic field.





Chiral anomaly: non-conservation of axial current

$$\partial_\mu J^5_\mu = 2\sum_f m_f \langle \bar{\psi}_f i\gamma_5 \psi_f \rangle_A - \frac{N_f g^2}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a .$$

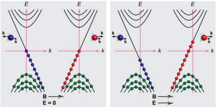
Topological invariant:

$$Q_w = \frac{g^2}{32\pi^2} \int d^4x F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a .$$

Summing over the eigenvalues of the chiral operator:

$$(N_R - N_L) = 2 N_f Q_w .$$

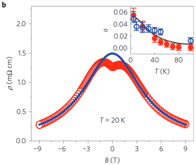
The chiral magnetic effect



[Kharzeev,Li Nuclear Physics A, 956, 107]

$$J_{CME} = \frac{e^2}{2\pi^2}\mu_5 B, \qquad \mu_5 \sim E \cdot B$$

$$J_{CME} \equiv \sigma^{ik}_{CME} E^k,\ \sigma^{zz}_{CME} \sim B^2$$



[Li et al, Nature Phys. 12, 550 (2016).]

- The magnetoresistence in $ZrTe_5$ when a magnetic field is applied parallel to an electric field is in accordance with the predictions for the CME.

After the first observation, the CME was detected in several other 3D Dirac materials.

Could it replace superconductors in certain devices? Could it perform at a higher temperature?

Overview

The chiral magnetic effect

Honeycomb lattices

Quantum electrodynamics in (2+1) dimensions

Anomalous Quantum Hall Effect

Perspectives

Final remarks

Graphene: the first material where relativistic-like quasi-particles were observed.

Vol 438|10 November 2005|**doi:10.1038/nature04233** nature

LETTERS

Two-dimensional gas of massless Dirac fermions in graphene

K. S. Novoselov[1], A. K. Geim[1], S. V. Morozov[2], D. Jiang[1], M. I. Katsnelson[3], I. V. Grigorieva[1], S. V. Dubonos[2] & A. A. Firsov[2]

Quantum electrodynamics (resulting from the merger of quantum mechanics and relativity theory) has provided a clear understanding of phenomena ranging from particle physics to cosmology and from astrophysics to quantum chemistry[1–3]. The ideas underlying quantum electrodynamics also influence the theory of condensed matter[4,5], but quantum relativistic effects are usually minute in the known experimental systems that can be described accurately

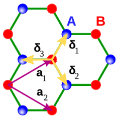
behaviour shows that substantial concentrations of electrons (holes) are induced by positive (negative) gate voltages. Away from the transition region $V_g \approx 0$, Hall coefficient $R_H = 1/ne$ varies as $1/V_g$, where n is the concentration of electrons or holes and e is the electron charge. The linear dependence $1/R_H \propto V_g$ yields $n = \alpha V_g$ with $\alpha \approx 7.3 \times 10^{10}\,\mathrm{cm}^{-2}\mathrm{V}^{-1}$, in agreement with the theoretical estimate $n/V_g \approx 7.2 \times 10^{10}\,\mathrm{cm}^{-2}\mathrm{V}^{-1}$ for the surface charge density

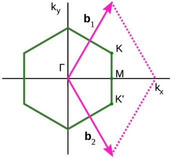
Chiral magnetic effect in graphene?

- There is no chiral anomaly in odd dimensions!
- Not possible to define a chiral operator γ_5 as a product of the all the gamma matrices that anti-commutes with all of them.

- Is it possible to construct an analogy?
- Parity anomaly!

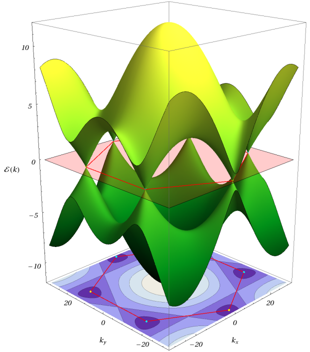
Honeycomb lattices





- Represented in terms of two triangular sublattices.
- Hexagonal reciprocal lattice.

- Tight-binding approach: nearest neighboors.
- Hopping only between sublattices.



- Linear dispersion relation: $\mathcal{H} = \bar{\psi}\, \hbar v_F \gamma \cdot \boldsymbol{k}\, \psi$.
- Dirac points: valence and conduction band touch generating no gap.

Expanding around the K and K' points:

$$H_{K'}(\vec{q}) \approx \tfrac{3at}{2} \begin{pmatrix} 0 & \alpha(q_x + iq_y) \\ \alpha^*(q_x - iq_y) & 0 \end{pmatrix}, \qquad H_K = H_{K'}^*.$$

$$H_K = -i\hbar v_f \vec{\sigma}\nabla, \qquad H' = H_K^T.$$

Considering the 4-component spinor:

$$H = \begin{pmatrix} H_K & 0 \\ 0 & H_K' \end{pmatrix}, \qquad H_K = H_{K'}^*, \qquad H = -i\hbar v \tau_0 \otimes \vec{\sigma}\nabla.$$

In the continuous limit:

$$\mathcal{L} = \sum_{\sigma=\pm} \bar{\psi}_\sigma(t,\mathbf{r}) \left[i\gamma^0(\hbar\partial_t - i\mu_\sigma) + i\hbar v_f \gamma^1 D_x + i\hbar v_f \gamma^2 D_y \right] \psi_\sigma(t,\mathbf{r}).$$

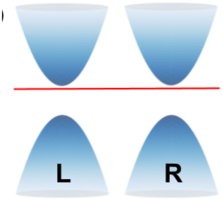
Chirality $\longleftrightarrow$ Dirac point (valley)

Overview

QED in (2+1)D

- Gaps between the conduction and valence band appear as a mass term $\hat{M}$ in the Lagrangian - interactions, deformations, substrates, doping, etc.
- $\hat{\mu}$ is a generalized chemical potential including spin interaction (Zeeman term), $\mu_\sigma = \mu - \frac{\sigma g}{2\mu_B} B$.
- QED3 fermion sector ($\hbar$=c=1):

$$\mathcal{L} = \bar{\psi}[\gamma_0(i\partial_0 + \hat{\mu}) - i(\gamma_1 D_x + \gamma_2 D_y) - \hat{M}]\psi.$$



Pseudo/Reduced QED

[Marino, Nucl. Phys. B 408, 551 (1993), Gorbar, Guysinin, Miranski, PRD 64, 105028 (2001)]

- The gauge sector is not constrained to the plane.
- Coulomb rather than logarithmic interaction.
- Reduced QED: general (3+1)D theory dimensionally reduced to a non-local effective (2+1)D theory.

$$S = \int d^D X \left(\frac{1}{4e^2} F_{ab}^2 + A_a J^a - \frac{1}{2e^2 \xi} \left(\partial_a A^a \right)^2 \right)$$

- $D = 4 \rightarrow$ Integrating over the gauge field and the third spatial dimension.
- Keeping $J^3 = 0$.
- Adding the fermion fields in (2+1)D.

$$S = \int d^3 x \left[\bar{\psi} \left(i \not{D} + m \right) \psi + \frac{1}{2} F_{\mu\nu} \frac{1}{\sqrt{-\partial^2}} F^{\mu\nu} + \frac{1}{e^2 \xi} \partial_\mu A^\mu \frac{1}{\sqrt{-\partial^2}} \partial_\nu A^\nu \right].$$

- The theory is scale invariant [**AM**, David Dudal and Pablo Pais, PRD 99, 045017 (2019)]

Overview

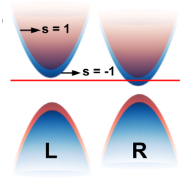
Anomalous Quantum Hall Effect

[**AJM**, A. Raya, C. Villavicencio S. Hernandez, Eur.Phy.J.C 78, 912 (2018),

[**AJM**, C. Villavicencio, D. Dudal, A. R. Rocha, F. Matusalém, Sci.Rep. 12, 5439 (2022)]

We assume different gaps for each pair of cones.

$$S = \int d^3x \left(\bar{\Psi}_+(\mu\sigma^z + m_k)\Psi_+ + \Psi_-(\mu\sigma^z + m'_k)\Psi_-\right) \qquad (1)$$



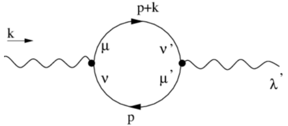
Linear response formalism: reaction of the system to external influences.

$$\delta S = \int d^4x J_\mu(x) a_\mu(x)$$

The conductivity is given by

$$\sigma_\chi = -\lim_{\omega\to 0}\frac{1}{\hbar\omega}\tilde{\Pi}_R^{xy}$$

The polarization tensor is given by the diagram



We have shown that only 1-loop contributions are non-vanishing: Coleman-Hill theorem valid for RQED

[D. Dudal, **AJM** and P. Pais, PRD (2018)].

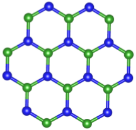
The limit can only be taken if we consider a configuration of the magnetic field that implies an electric field when the limit $\omega \to 0$ is taken.

Considering a chemical potential, we obtain for the net current

$$\sigma_\chi = \sum_s \frac{e^2}{4\pi}\left[\frac{m_{s,k}}{|m_{s,k}|}\theta(m_{s,k}^2 - \mu^2) - \frac{m_{s,k'}}{|m_{s,k'}|}\theta(m_{s,k'}^2 - \mu^2)\right]$$

Quantum Hall Effect, with fractional Chern-number. **TOPOLOGICALLY PROTECTED!**

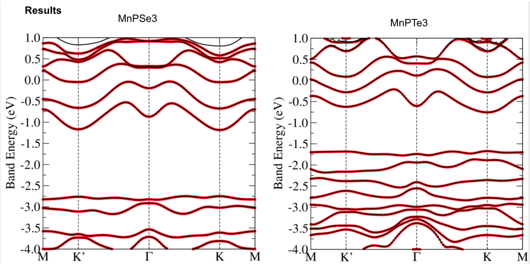
- $m_R = M_+ + M_-$, $m_L = M_+ - M_-$
- Center symmetry breaking "mass": $M_+ = m_3\gamma_3$
- This can be obtained if **sublattice symmetry is broken**.

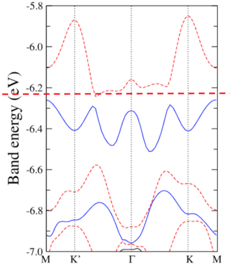


- Broken T symmetry: complex next to nearest neighbors term. In the Lagrangian: $M_- = m_3\gamma_3\gamma_5$. **Spin orbit?**
- Both mechanisms occurring simultaneously lead to different gaps in each Dirac cone.

We look for materials that can present an intrinsic effect.

Band structure of $MnPX_3$, $X = Se, Te$





Doped with 1 Cu atom in order to generate a Zeeman effect and lift spin degeneracy.

Other promising materials:

- Heterostructures: MnPSe3/CrBr3, MnPSe3/MoS2 and WS2/h-VN
- Dichalcogenides: NbSe2 and WS2

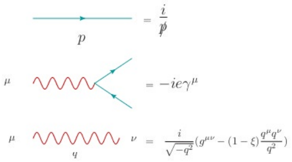
Schwinger-Dyson calculations

Gap generation from the interaction with external fields.

[J. Olivares, L. Albino, **AJM**, A. Raya, PRD 102 (2020) 9, 096023]

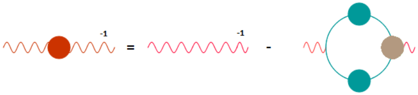
$$
\begin{aligned}
\mathcal{L}^{CS}_{RQED} &= -\frac{1}{4}F^{\mu\nu}\frac{2}{(-\Box)^{1/2}}F_{\mu\nu} + \bar{\psi}(i\gamma^{\mu}\partial_{\mu} + e\gamma^{\mu}A_{\mu})\psi \\
&+ \frac{1}{2\zeta}(\partial\cdot A)^2 + \frac{-i\theta}{4}\varepsilon^{\mu\nu\rho}A_{\mu}\partial_{\nu}A_{\rho},
\end{aligned}
$$

Feynman rules for P(R)QED



$$
\hat{\Delta}_{\mu\nu}(\vec{q}) = \frac{1}{2q}\frac{1}{(1+\theta^2)}\left(\delta_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2}\right)
$$

SDE for the inverse fermion propagator



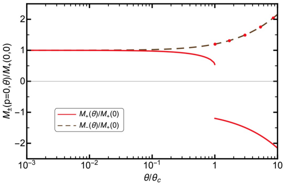
SDE for the inverse gauge boson propagator

Schwinger-Dyson equations

$$
\begin{aligned}
S^{-1}(p) &= S_0^{-1}(p) - \Xi(p), \\
\Delta_{\mu\nu}^{-1}(p) &= \Delta_{0\mu\nu}^{-1}(p) - \Pi_{\mu\nu}(p)
\end{aligned}
$$

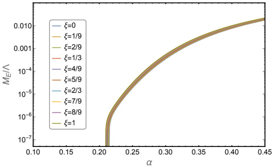
Schwinger-Dyson: coupled equations

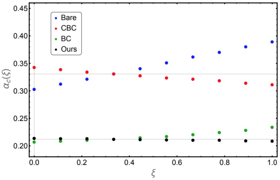
In the infrared Haldane mass is generated above a critical value of θ



- More educated truncations of SDE lower the value of the critical coupling

L. Albino, A. Bashir, **AJM**, A. Raya, PRD 106 (2022) 9, 096007





Overview

Old approach, new point of view

[**AJM**, Saul Hernandez-Ortiz, Alfredo Raya, Cristian Villavicencio, Eur.Phys.J.C 78 (2018) 11, 912]

$$\mathcal{L} = \sum_{\chi=\pm} \bar{\Psi}_\chi \left[i\partial\!\!\!/ + \mu\gamma^0 + \left(eA_3^{ext} - m_\chi \right) \right] \Psi_\chi .$$

Using the Schwinger proper time method:

$$\begin{aligned} \tilde{G}(k;\xi) \quad = \quad & \int_{-\infty}^{\infty} ds\, r_s(k^0, K_\parallel^0)\, e^{is{K_\parallel}^2 - i\left[{k_2}^2+\xi^2\right]\tan(eBs)/eB} \\ & \left\{ K\!\!\!/_\parallel \left[1+\gamma^2\gamma^3\tan(eBs) \right] + \left[k_2\gamma^2+\xi\gamma^3 \right] \sec^2(eBs) \right\}, \end{aligned}$$

The CME shows up:

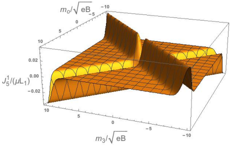


Figure: $L_2\sqrt{eB} = 0.2$

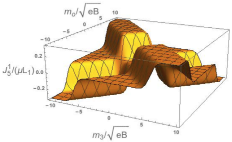


Figure: $L_2\sqrt{eB} = 8$

Pseudo-chirality: a physical quantity or an elegant theoretical modeling?

- Mecklenburg and Regan: what if graphene is not so $2+1$D? (PRL 106, 116803 (2011)).
  - Missing angular momentum: $[H, \mathbf{L}] \neq \mathbf{0}$ while the Hamiltonian has rotational symmetry around the axis perpendicular to to the graphene plane.
  - Possible to define a vector S analogous to spin collectively generated by the background lattice such that $[H, \mathbf{L}+\mathbf{S}] = \mathbf{0}$

Two possible algebras for the pseudo-spin σ:

- Rotations in 3 spatial dimensions:
  $[S_i, S_j] = i\hbar\varepsilon_{ijk} S_k, \qquad \{i,j,k\} \in \{1,2,3\}$

- 2 boosts and 1 rotation:
  $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \qquad \{\mu,\nu\} \in \{0,1,2\}$

"Thus the pseudospin Pauli matrices σ can be connected both to the generators of the Lorentz group in $2+1$ dimensions and the generators of the rotation group in $3+1$ dimensions. The choice of algebra corresponds to considering the honeycomb lattice as a strictly two dimensional structure, or as a quasi-two dimensional structure embedded in three dimensional space. Since experiments on graphene occur in three dimensional space containing inherently three dimensional objects, this second perspective is sometimes unavoidable."

"Unveiling pseudospin and angular momentum in photonic graphene"
[Nature comm. 6, 6272 (2015)].

Photonic graphene with broken sublattice symmetry exhibits vorticities that can be associated to an angular momentum of the pseudo-spin.

- Work in progress: to modify RQED to allow for a small thickness. The CME may emerge in quasi-planar materials?

Overview

Final remarks

- The chiral magnetic effect has been observed in condensed matter systems: three-dimensional Dirac/Weyl materials.
- Although the chiral magnetic effect is not allowed in two-dimensional materials, an analogue effect based on the parity anomaly is possible.
- First principle simulations are on the way: band structure, conductivity, stability.
- SDE show that the symmetry pattern necessary for the A-QHE to happen can be generated by external fields.
- We have recently started a collaboration with Michel Houssa from KULeuven-Belgium: strong background on simulations and nanomaterial facilities.
- To consider a small thickness may allow the CME to be manifested in materials that in principle are considered 2+1D

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Thank you