

Suppression and enhancement of the **Chiral Separation Effect** due to dynamical sea quarks in **QC₂D**

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Anomalous transport

Axial anomaly

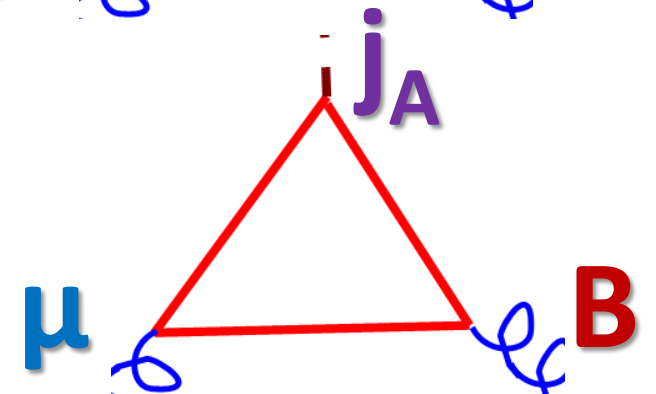
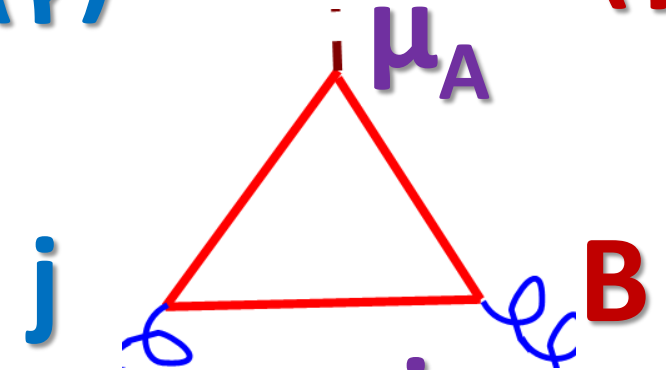
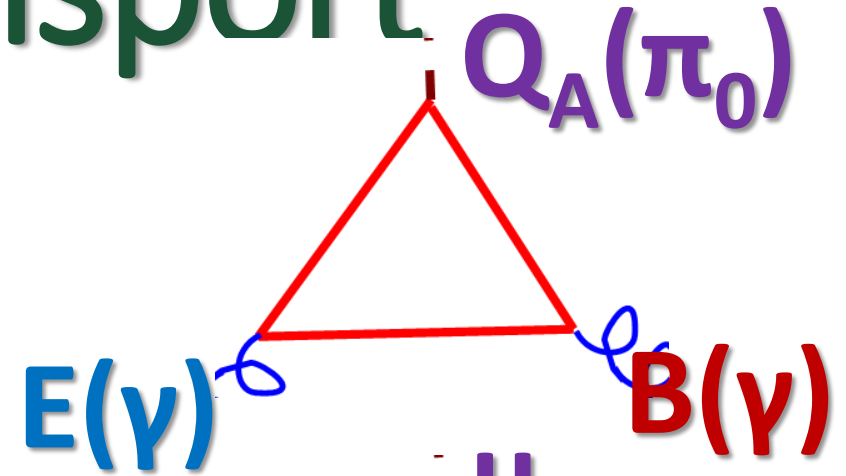
$$\partial_{\mu} j_{\mu}^A = \frac{1}{2\pi^2} \vec{E} \cdot \vec{B}$$

Chiral Magnetic Effect

$$\vec{j} = \frac{\mu_A}{2\pi^2} \vec{B}$$

Chiral Separation Effect

$$\vec{j}_A = \frac{\mu \vec{B}}{2\pi^2}$$

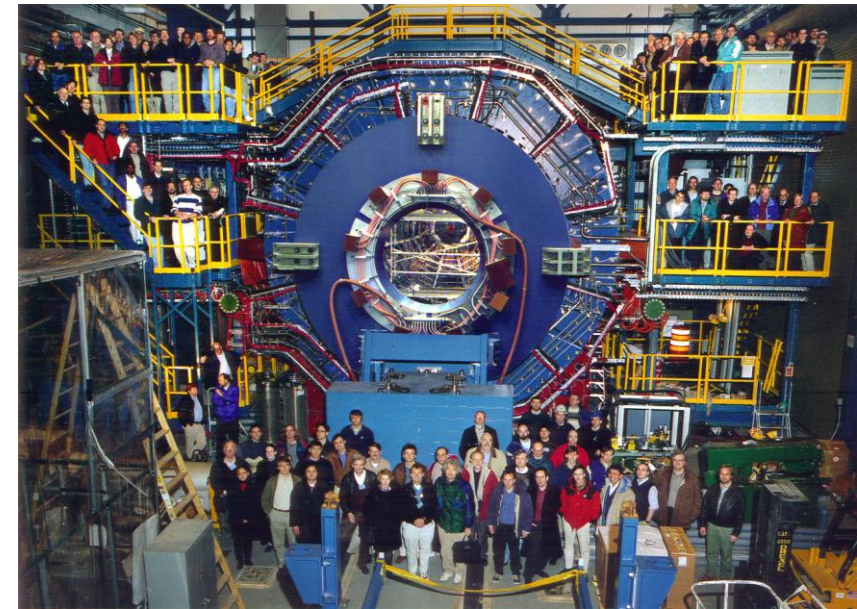


Anomalous transport and heavy ions

In order to better control the influence of signal and backgrounds, the STAR Collaboration performed a blind analysis of a large data sample of approximately 3.8 billion isobar collisions of $^{96}Zr+^{96}Zr$ and $^{96}Ru+^{96}Ru$ at $\sqrt{s_{NN}}=200$ GeV.

No CME signature that satisfies the predefined criteria has been observed in isobar collisions in this blind analysis.

[STAR Collaboration, ArXiv:2109.00131]

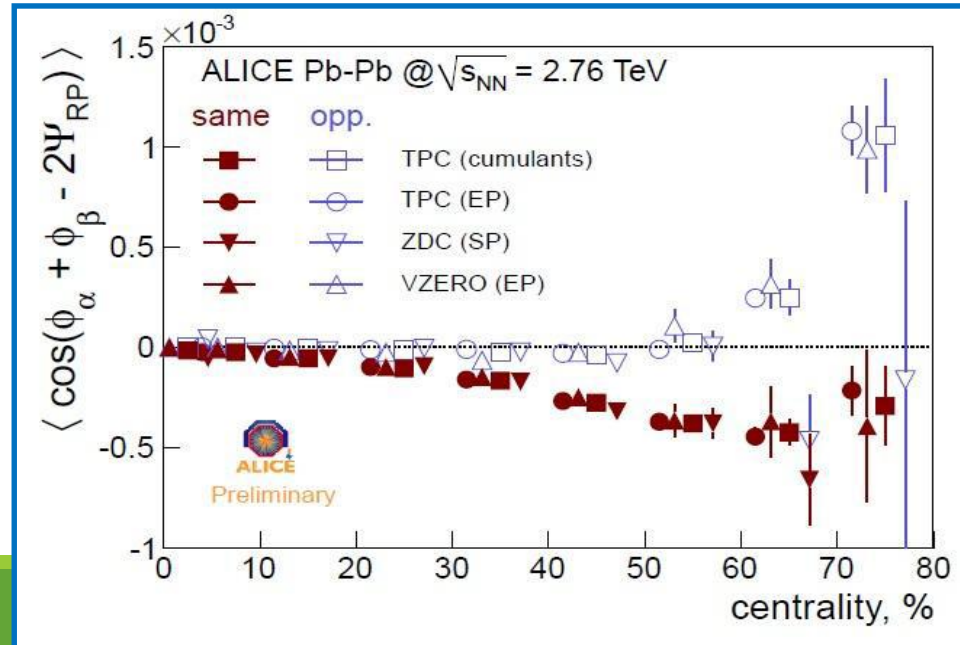
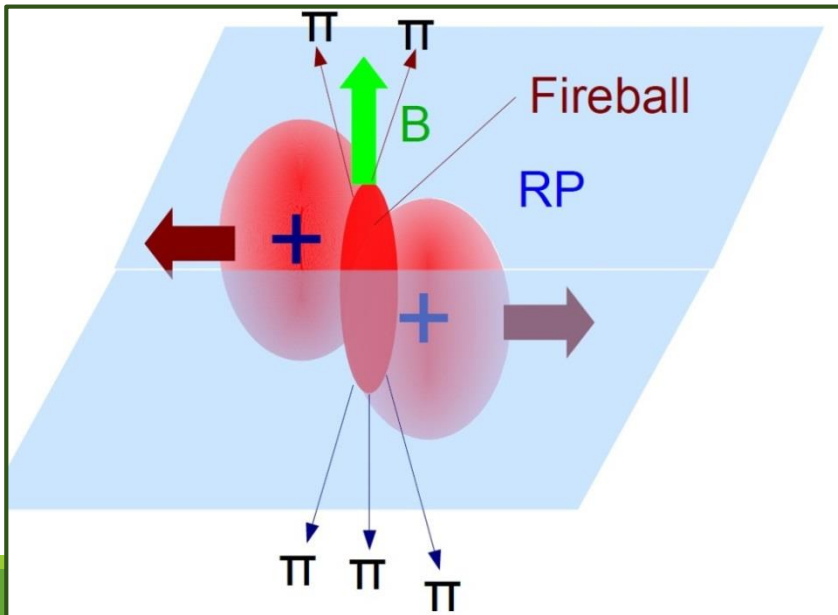


Anomalous transport and heavy ions

Ideal hydro
Viscous hydro
Anomalous hydro

Elliptic flow
Parity-odd fluctuations

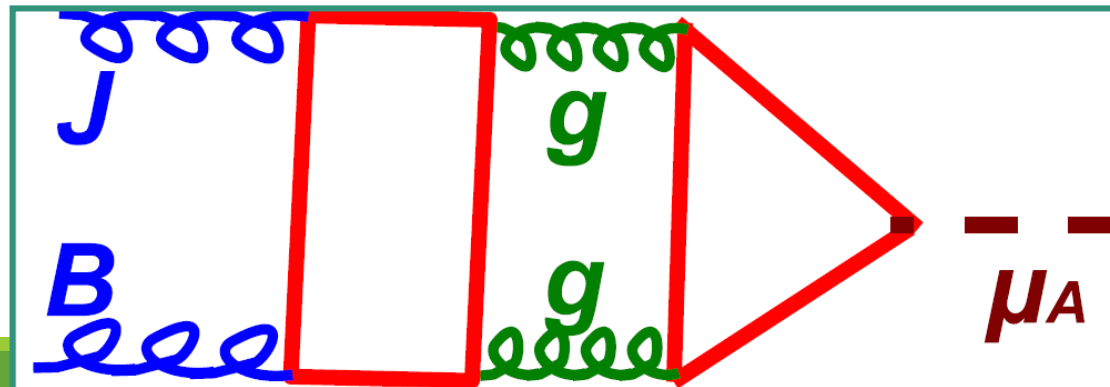
Isobar run RHIC 2018 – what's next?



<https://indico.bnl.gov/event/12758/>

Anomalous transport coefficients

- Input for hydrodynamic simulations of HICs
- Get unknown corrections in real QCD
- Due to broken chiral symmetry [PB'1312.1843]
- Perturbatively [Miransky 1304.4606] [Gursoy 1407.3282]
- Due to influence of heavy quark flavors [Suenaga 2012.15173]



Anomalous transport coefficients

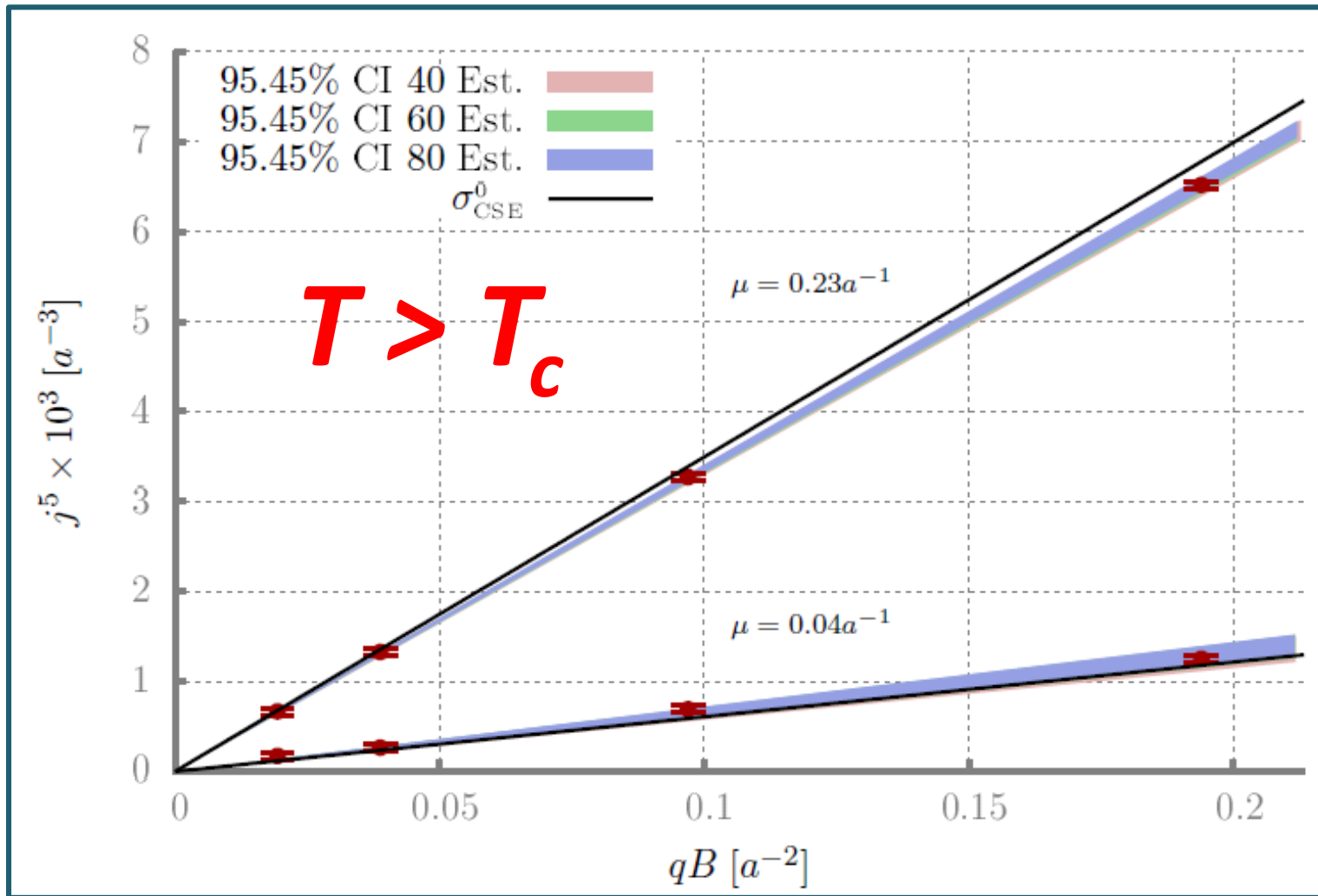
Lattice studies so far:

- [Yamamoto'1105.0385]: ~20% of Chiral Magnetic Effect
- [Braguta et al' 1401.8095]: ~5% of Chiral Vortical Effect
- So far hydro simulations with **free-fermion** transport coefficients only
- Lattice conclusions can question the **hydro interpretation** of **RHIC results**

BUT: Wilson-Dirac/Quenched overlap/non-conserved currents/energy-momentum

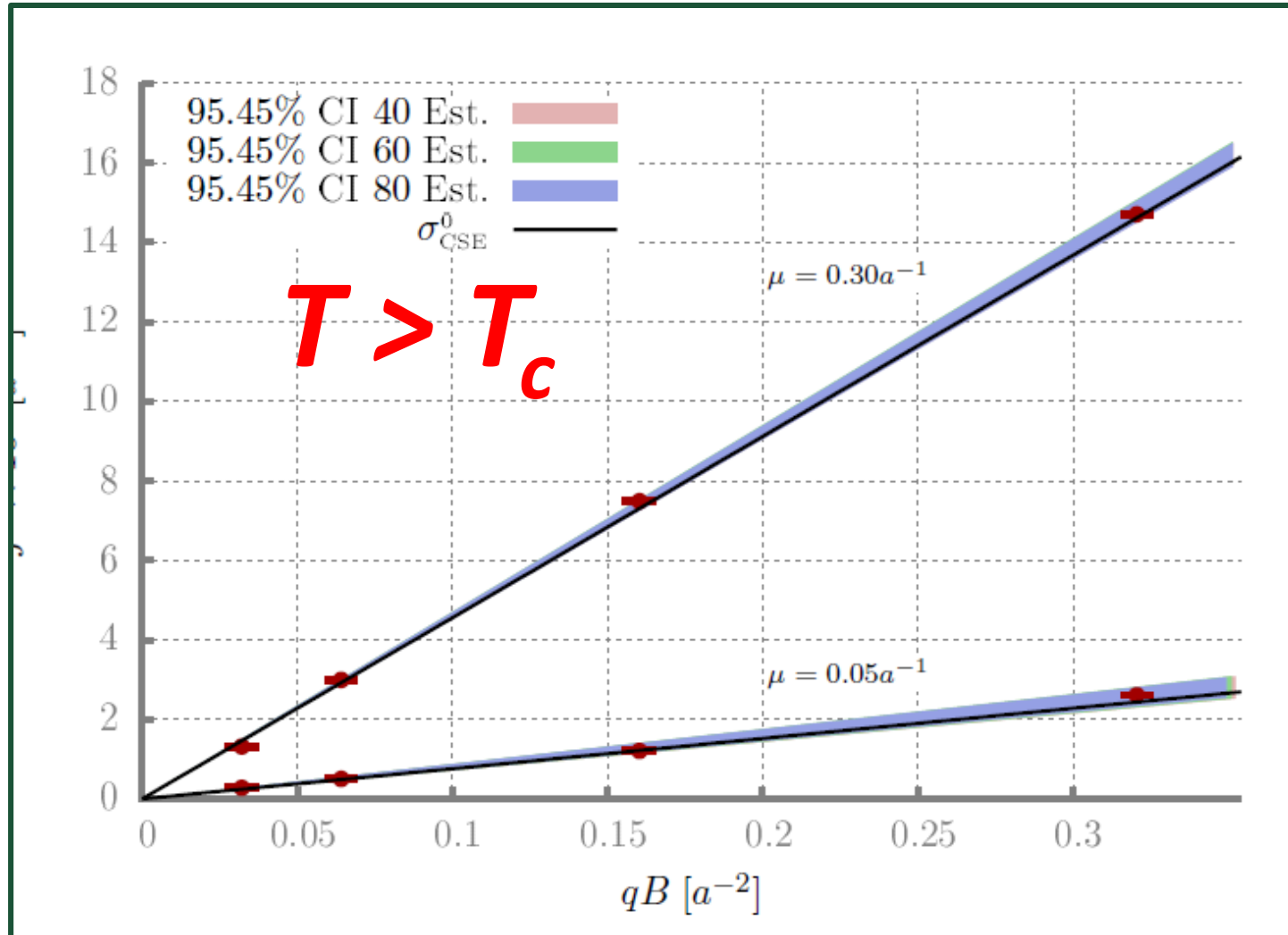
* [Brandt et al.2212.02148]: CSE suppression – proper **QCD**

Pure SU(3) gauge theory




[PB, M. Pühr,
ArXiv:
1611.07263]

Pure SU(3) gauge theory



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CSE with dynamical fermions

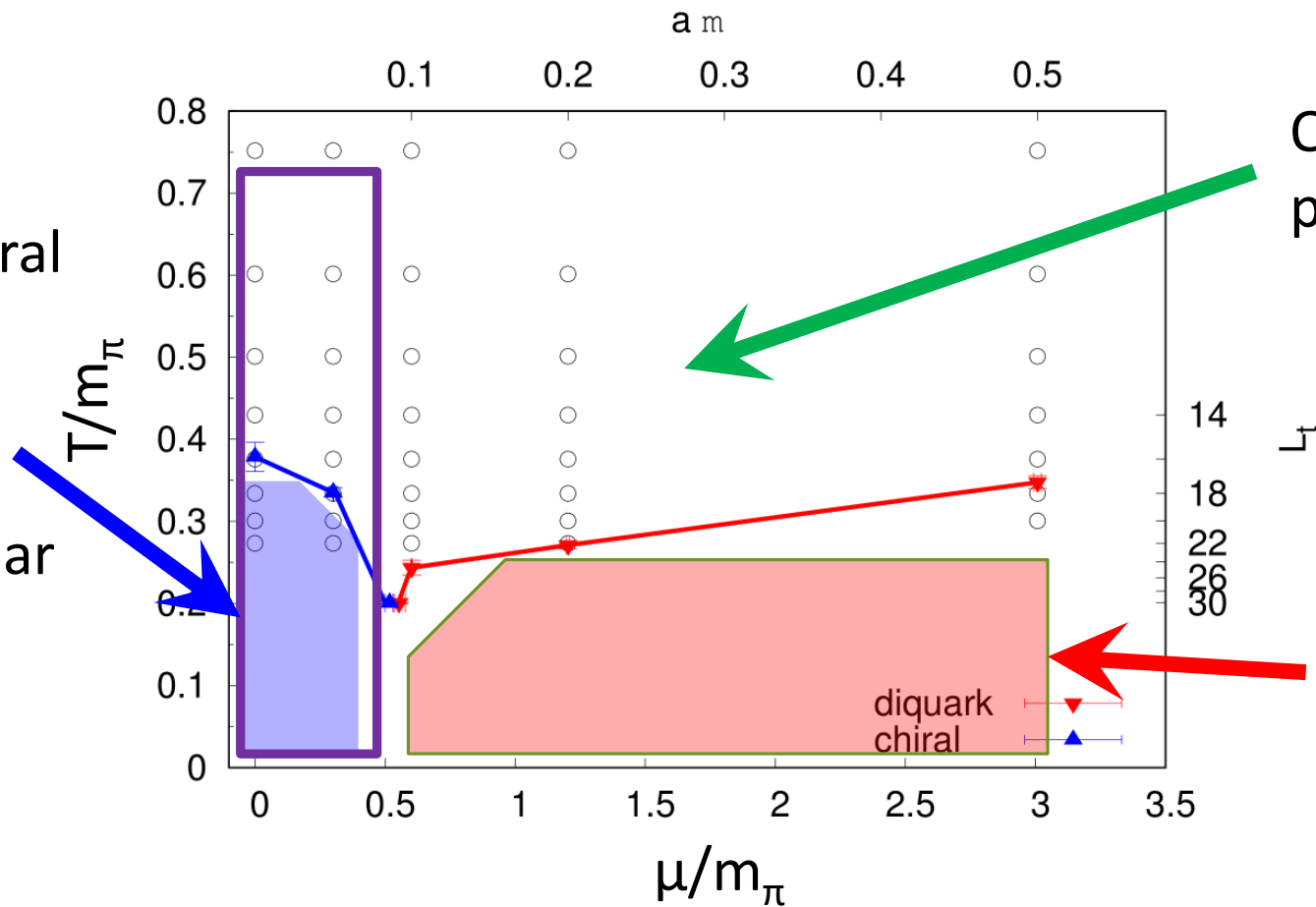
- What can be the order of magnitude of **corrections**?
- **Sign problem** in full **QCD**  use **SU(2)** gauge theory, no sign problem
- Features **confinement-deconfinement crossover** and χ SB, **QCD**-like dynamics at small $\mu < m_\pi/2$.
- **Diquark** condensation at $\mu > m_\pi/2$, absent in real **QCD**

Phase diagram of $SU(2)$ gauge theory

QCD-like

low-temperature
phase, broken chiral
symmetry,
pion excitations

Qualitatively similar
to QCD !!!



Quark-gluon plasma
phase

Diquark condensation
phase, absent in QCD

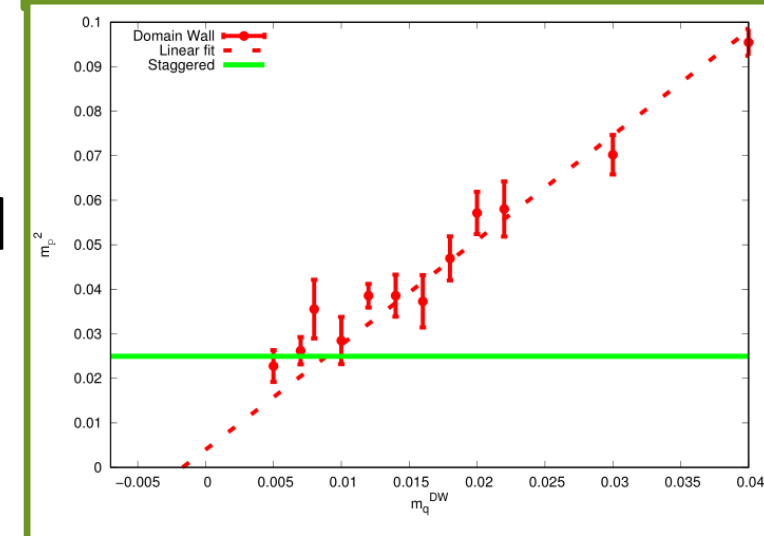
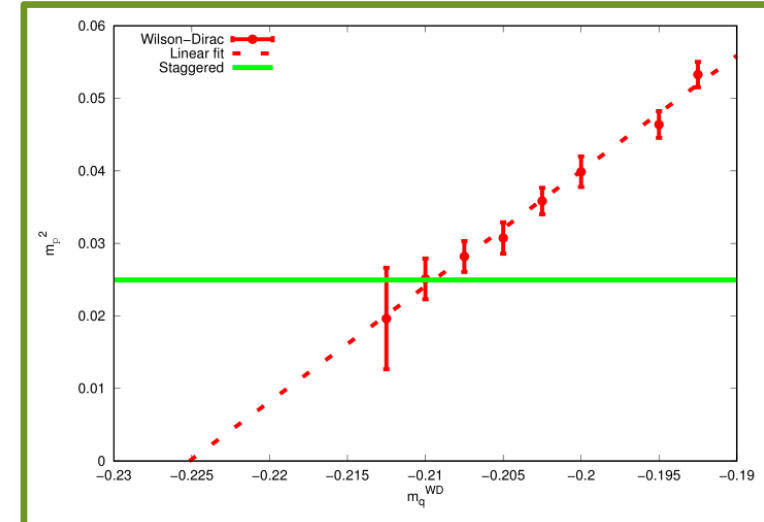
Lattice setup: sea quarks & gauge action

- $N_f=2$ light flavours with $m_u=m_d = 0.005$, pion mass $m_\pi = 0.158$
- Rooted staggered sea quarks
- Tadpole-improved gauge action
- Spatial lattice sizes $L_s=24$ and $L_s=30$
- Single gauge coupling = single lattice spacing
- Temporal lattice sizes $L_t=4 \dots 26$
- Standard Hybrid Monte Carlo
- Acceleration using **GPUs**
- Small **diquark source term** added for low temperatures to facilitate **diquark condensation**



Lattice setup: valence quarks

- **Wilson-Dirac** and **Domain-Wall** valence quarks
- **HYP-smearred gauge links** in the Dirac operator:
reduces additive mass renormalization and lattice artifacts
- Better quality of signal than for staggered quarks
- Bare mass for Wilson-Dirac/Domain-Wall quarks tuned to match the pion mass calculated with sea quarks
- **GMOR relation** works with good precision



Measuring the CSE

- Sign problem even in $SU(2)$

gauge theory at finite μ and

magnetic field

- We use linear response

approximation w.r.t.

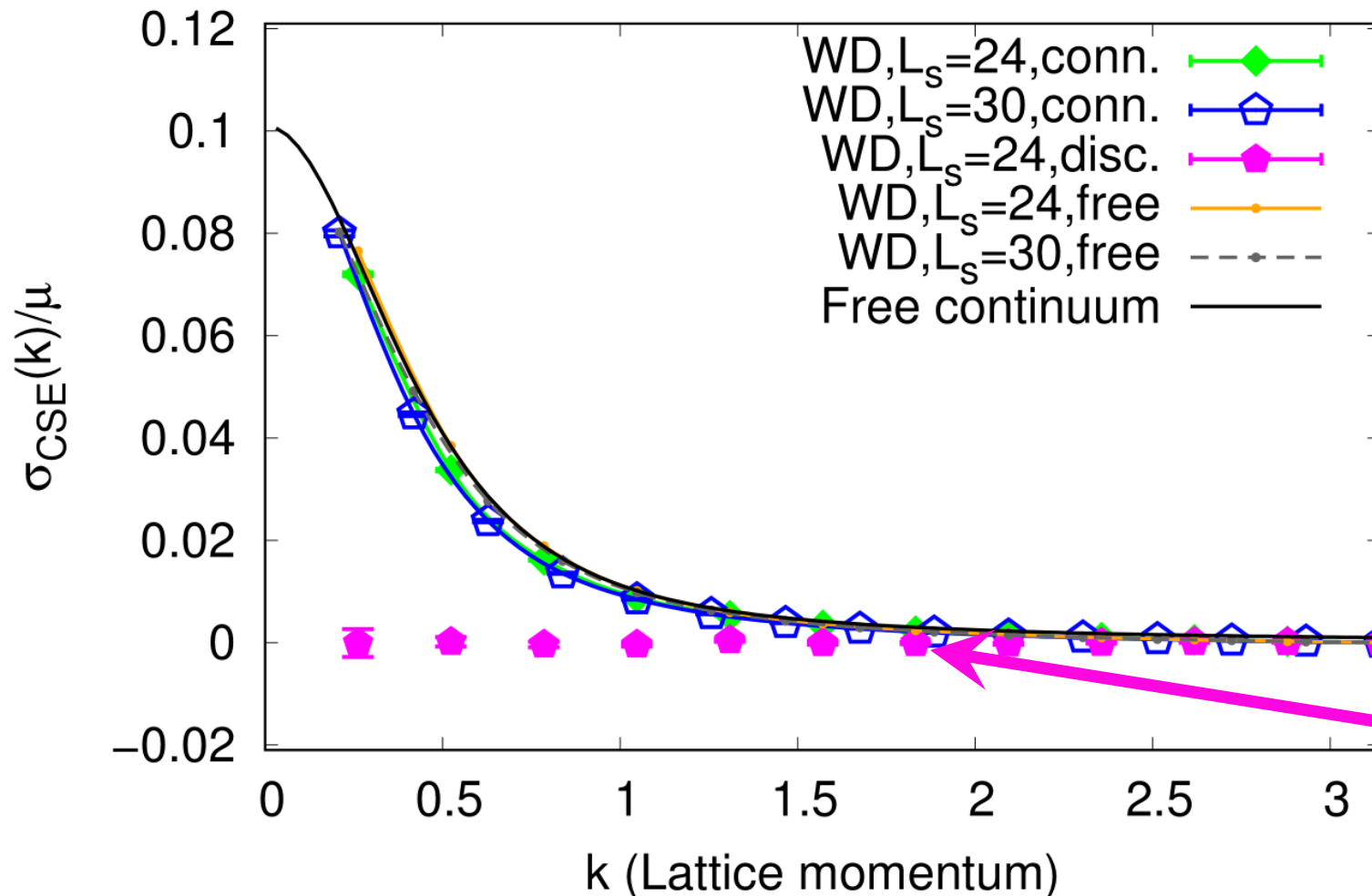
magnetic field

$$\vec{j}_A = \sigma_{CSE}(\mu, T) \vec{B}$$

$$\langle j_1^A(k_3) j_2^V(-k_3) \rangle = \sigma_{CSE} k_3$$

Numerical results

$L_t = 12, a\mu = 0.05$

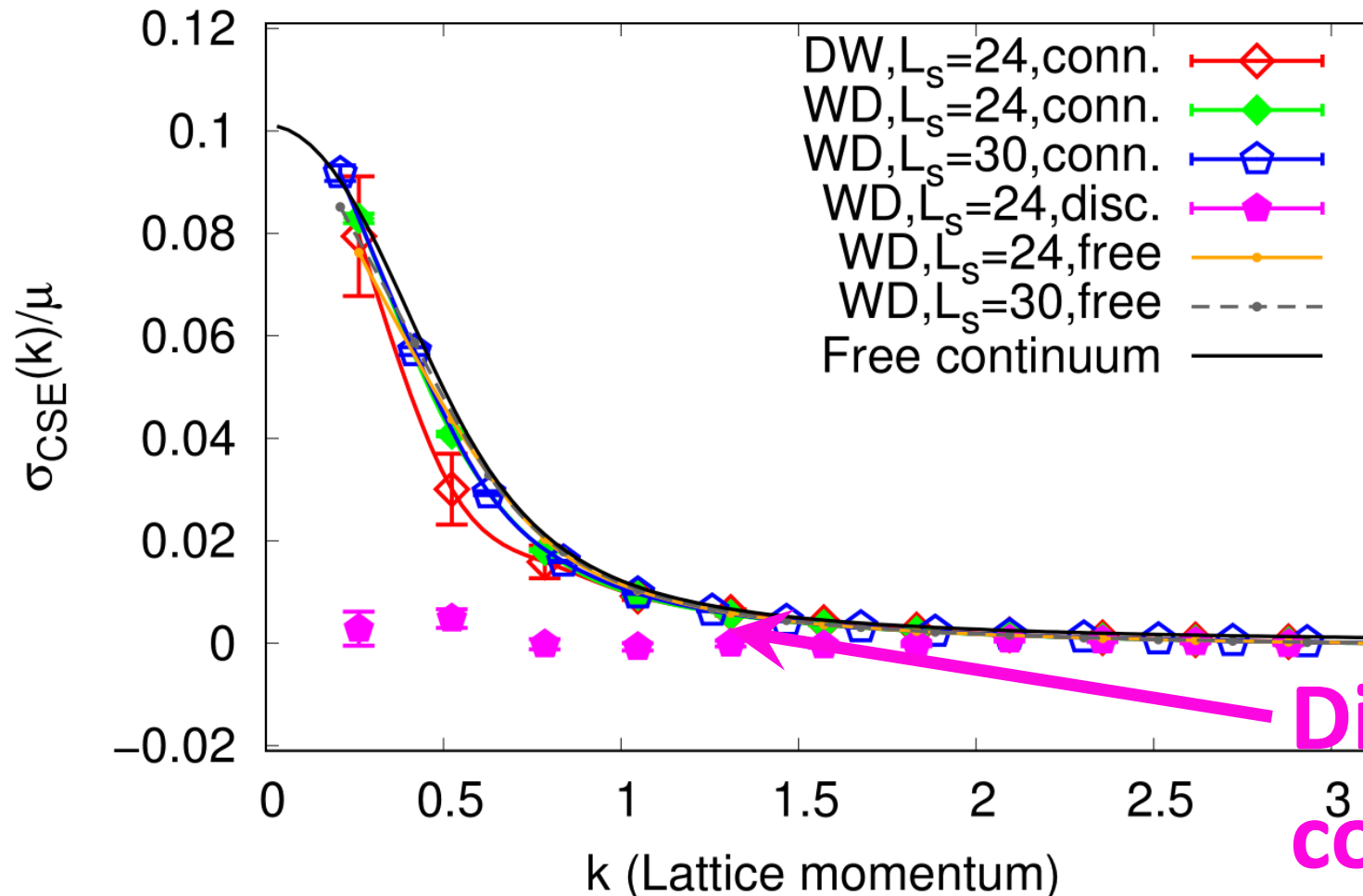


**High T,
Small μ**

**Disconnected
contribution**

Numerical results

$L_t = 16, a\mu = 0.20$

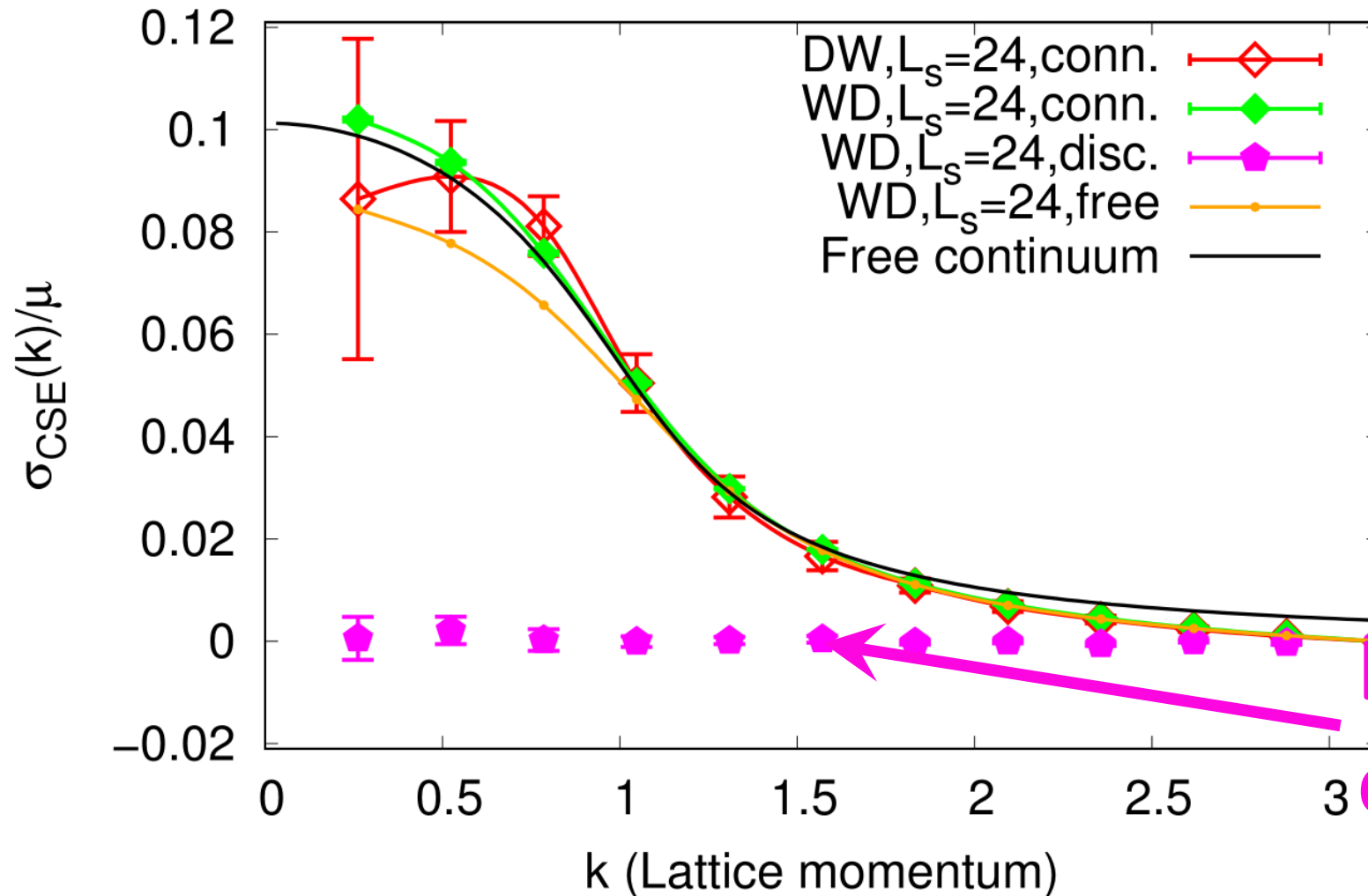


**Critical T,
Large μ**

**Disconnected
contribution**

Numerical results

$L_t = 16, a\mu = 0.50$

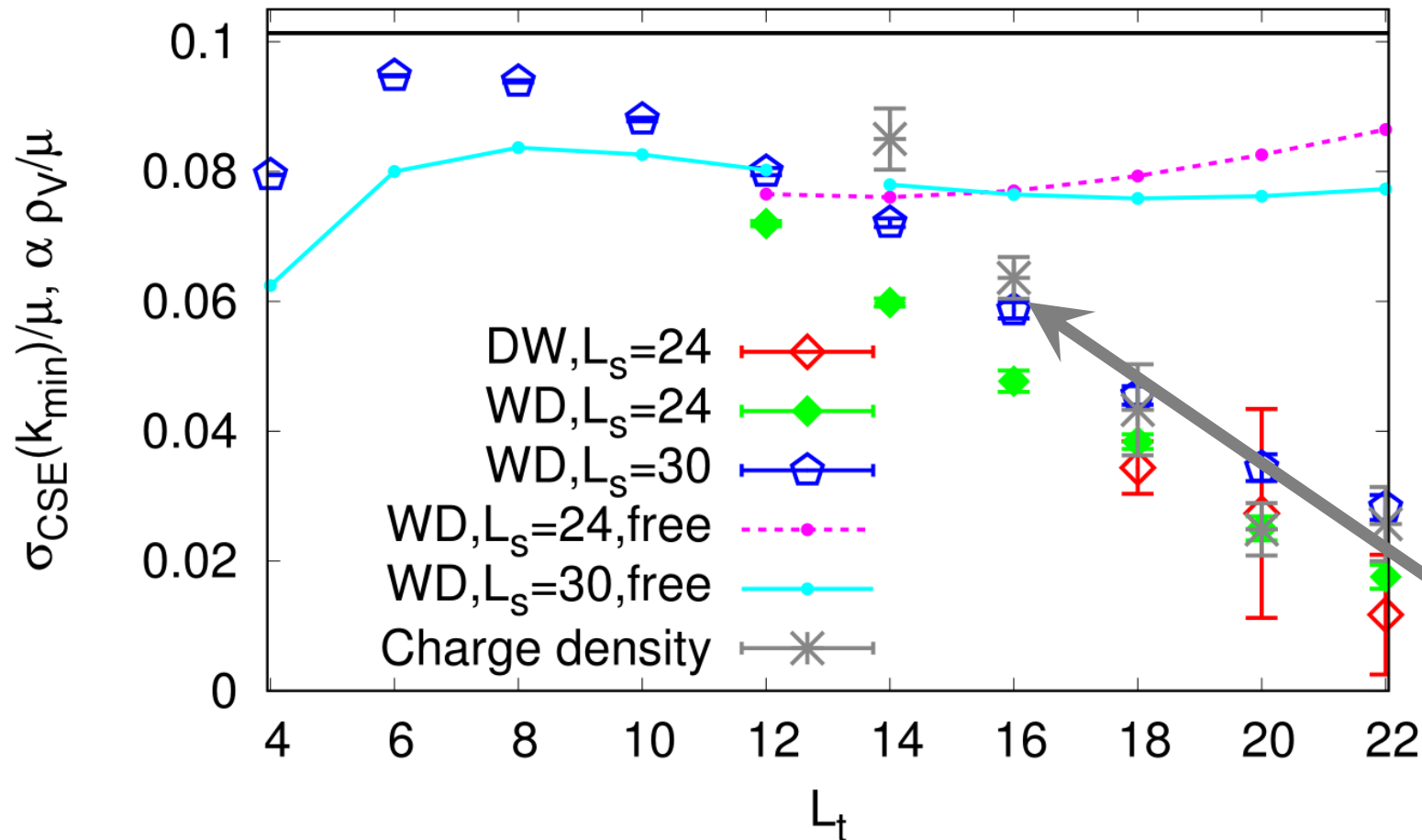


**Critical T ,
Large μ**

**Disconnected
contribution**

σ_{CSE} vs temperature, low μ

$a\mu = 0.05$



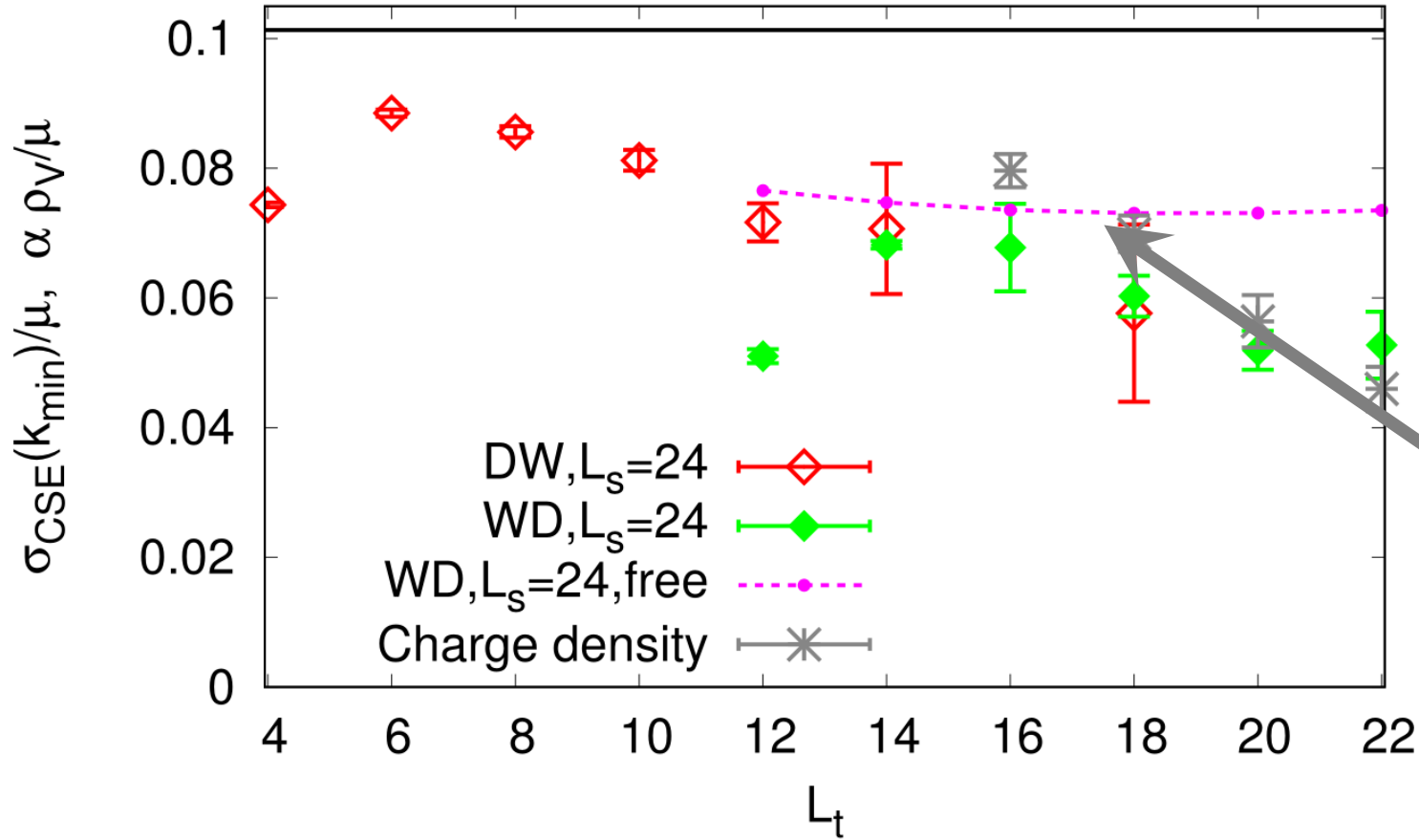
**Significant
suppression
towards low
temperatures!**

Rescaled
charge
density

Qualitative agreement with [\[2212.02148\]](#)!

σ_{CSE} vs temperature, medium μ

$a\mu = 0.10$

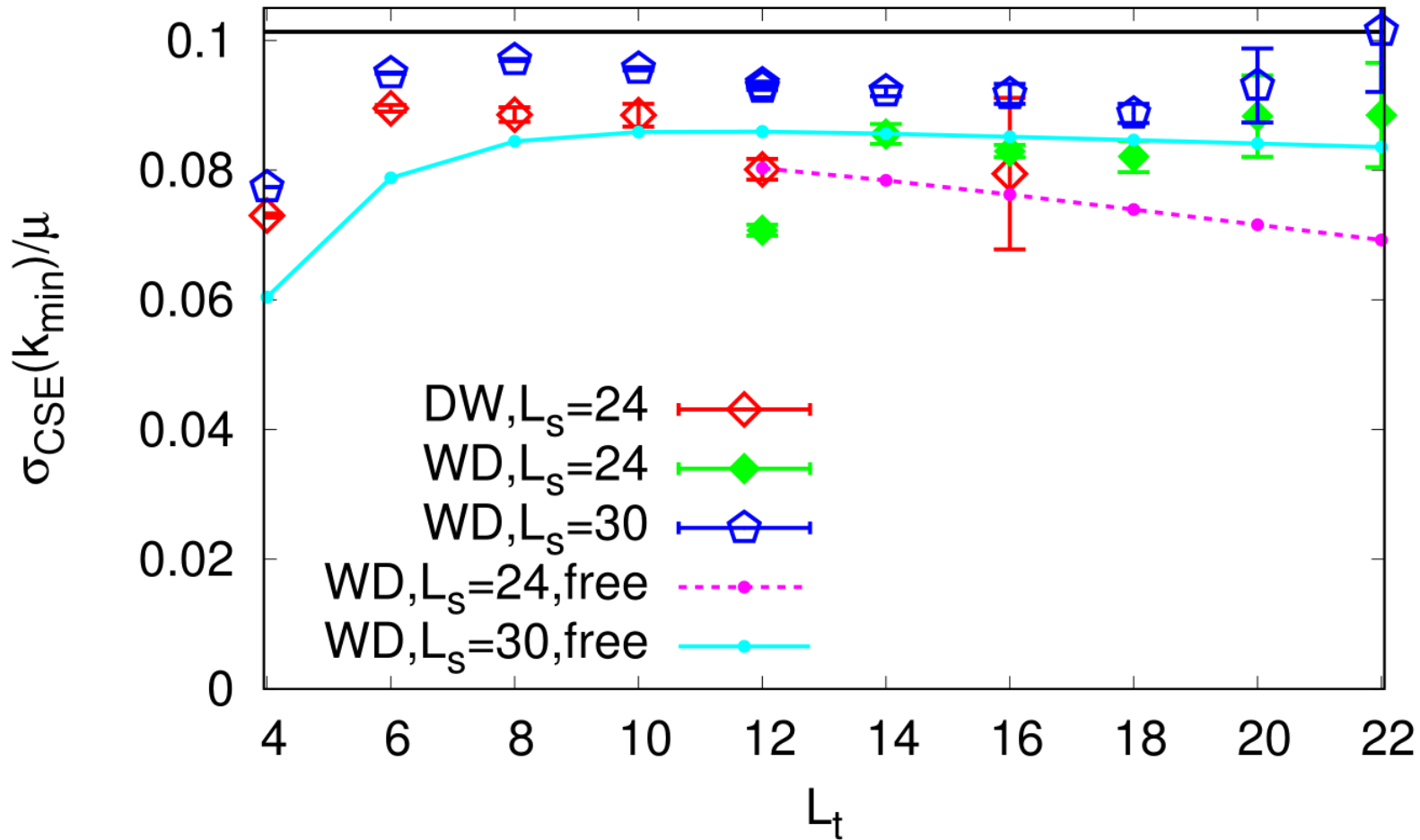


Data moving closer to the free fermion results

Rescaled charge density (same coefficient)

σ_{CSE} vs temperature, large μ

$$a\mu = 0.20$$



Data quite close to the free fermion results

Describing CSE suppression

- ChPT result for flavor-**non-singlet axial current** [Avdoshkin,Sadofyev,Zakharov' 1712.01256]:
- We work with flavor-**singlet axial current**, has different status in ChPT
- Singlet and non-singlet currents become similar at **large N_c**
- **Phenomenological formula** works well in the low- T , low- μ regime even for singlet axial current in $SU(2)$ gauge theory

$$\vec{j}_A^a = \frac{N_c \text{Tr} (Q)}{(2\pi f_\pi)^2} \rho_V^a \vec{B}$$

Disconnected contribution appears to be small!

$$\sigma_{CSE} (\mu, T) = \alpha \rho_V (\mu, T)$$

Kondo effect in non-Abelian gauge theory

- Suppression of an interesting effect feels somewhat unfortunate...
- Is there something that can **enhance the CSE?**
- Yes, **QCD** Kondo Effect [Suenaga et al., 2012.15173]

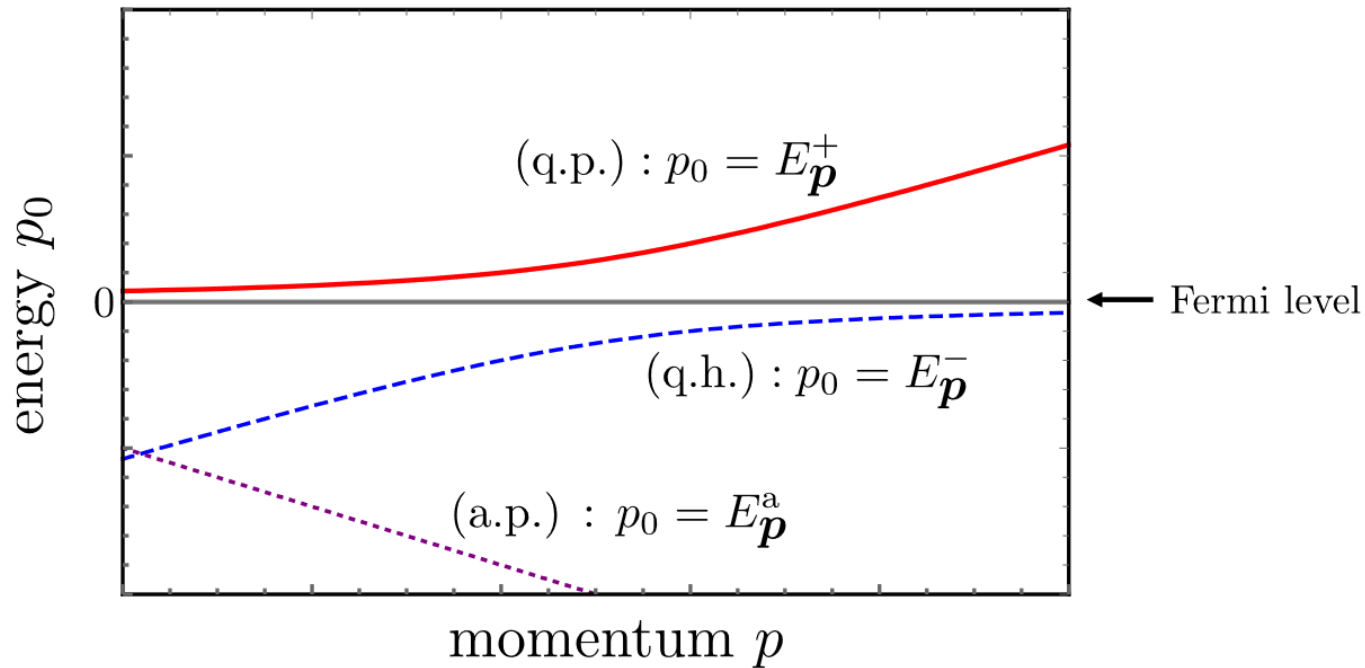
Kondo effect in non-Abelian gauge theory

- **Kondo effect**: scattering of **light fermion** near a Fermi surface off a **heavy fermion** of mass M enhanced as $\log(M)$
- Mean-field approach for **QCD** [Yasui,Suzuki,Itakura, 1604.07208]: **spontaneous emergence of Kondo condensate** $\langle \bar{Q} q \rangle$
- **Suppresses low-T, finite- μ conductivity** [Yasui,Ozaki, 1710.03434]
- **But... CSE is enhanced** [Suenaga,Araki,Suzuki,Yasui, 2012.15173]
- **We only consider CSE of light quarks**

CSE enhancement: mean-field description

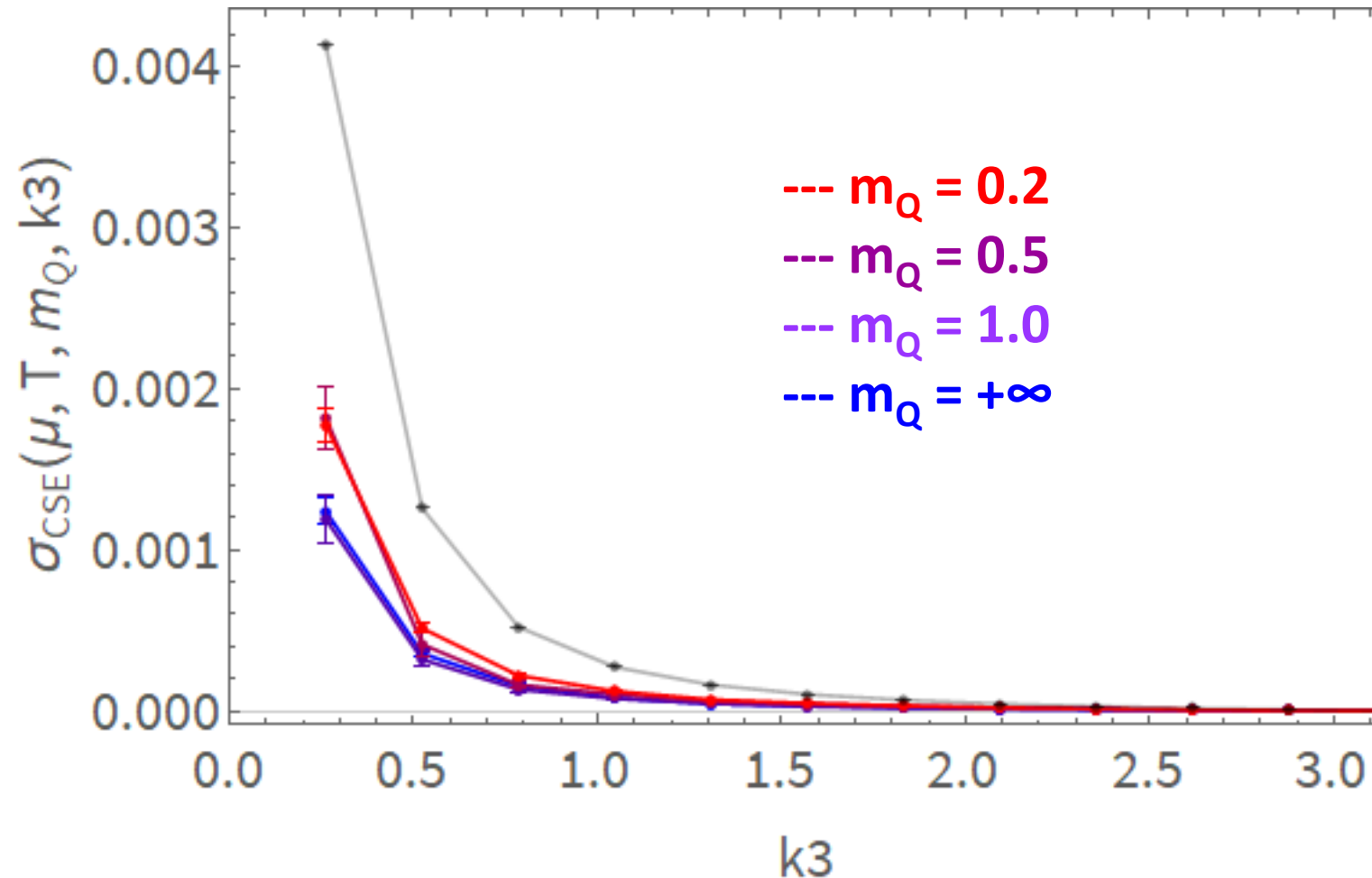
from [Suenaga et al. 2012.15173]

$$\mathcal{L} = \bar{q}\mathcal{D}q + \Delta \left(\bar{q}Q + \bar{q}\gamma_\mu Q \frac{p_\mu}{|p|} \right)$$



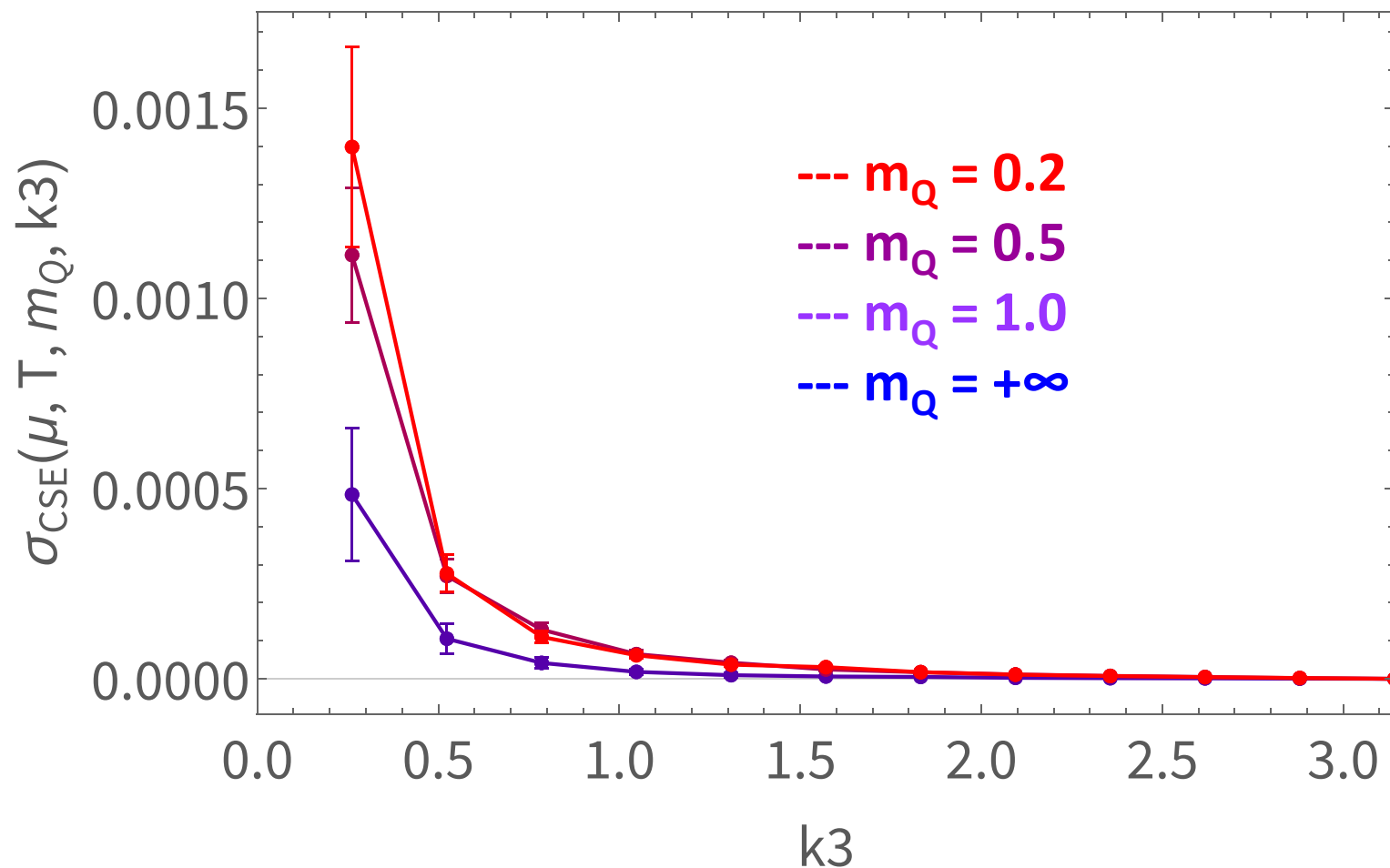
Is the increased **density of states** near the Fermi surface driving the CSE enhancement?

Numerical results for CSE in $N_f=2+1$ SU(2) LGT



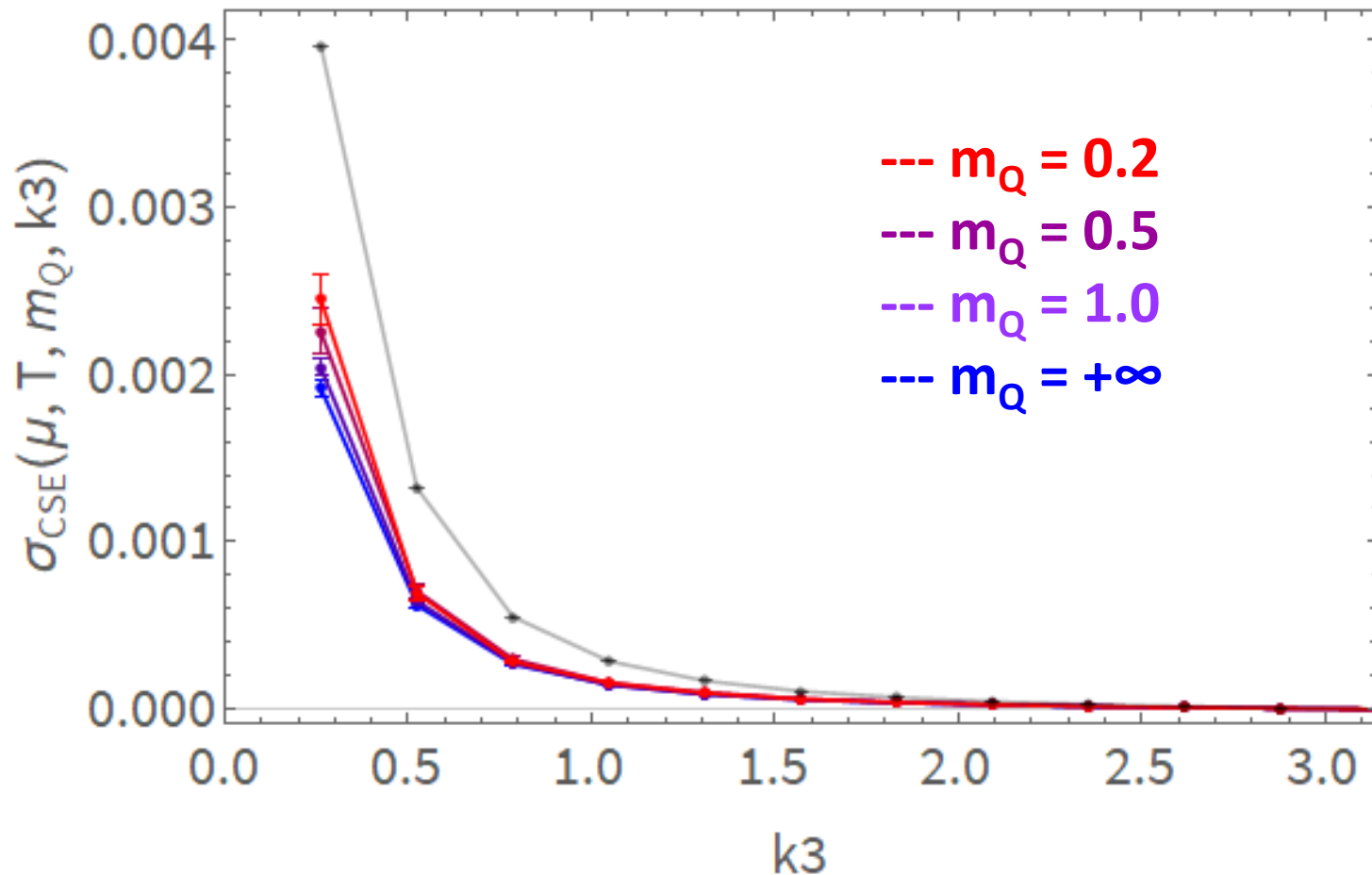
- $Lt=20$, low-temperature regime
- CSE enhanced by more than **30%** for $m_Q=0.2$ *a*

Numerical results for CSE in $N_f=2+1$ SU(2) LGT



- $Lt=24$, low-temperature regime
- CSE enhanced by a factor of 3 for $m_Q=0.2 a$

Numerical results for CSE in $N_f=2+1$ SU(2) LGT



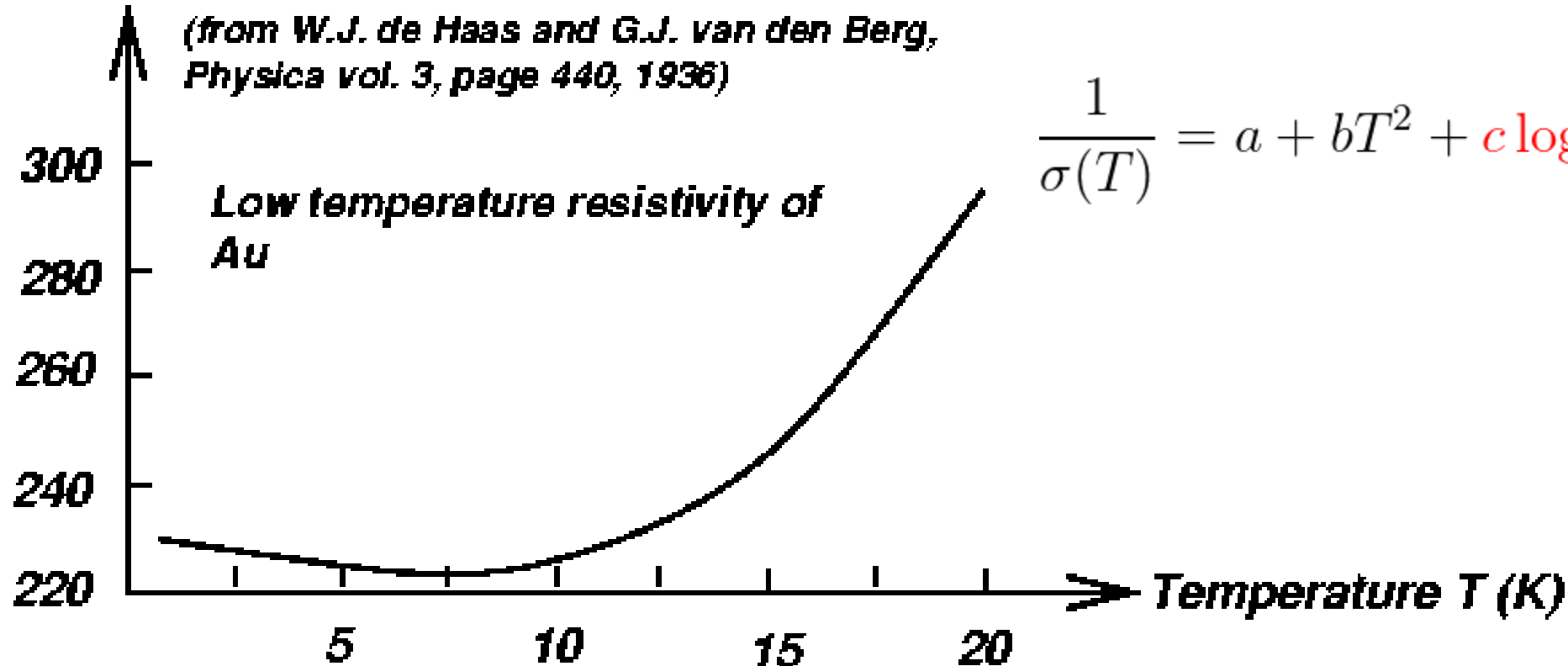
- $Lt=18$, a bit higher temperature
- Enhancement not so large
- Not a conventional Kondo, Fermi surface not well-defined

Kondo physics for electric conductivity?

Resistance/Resistance(T=0 Celsius) x 10000

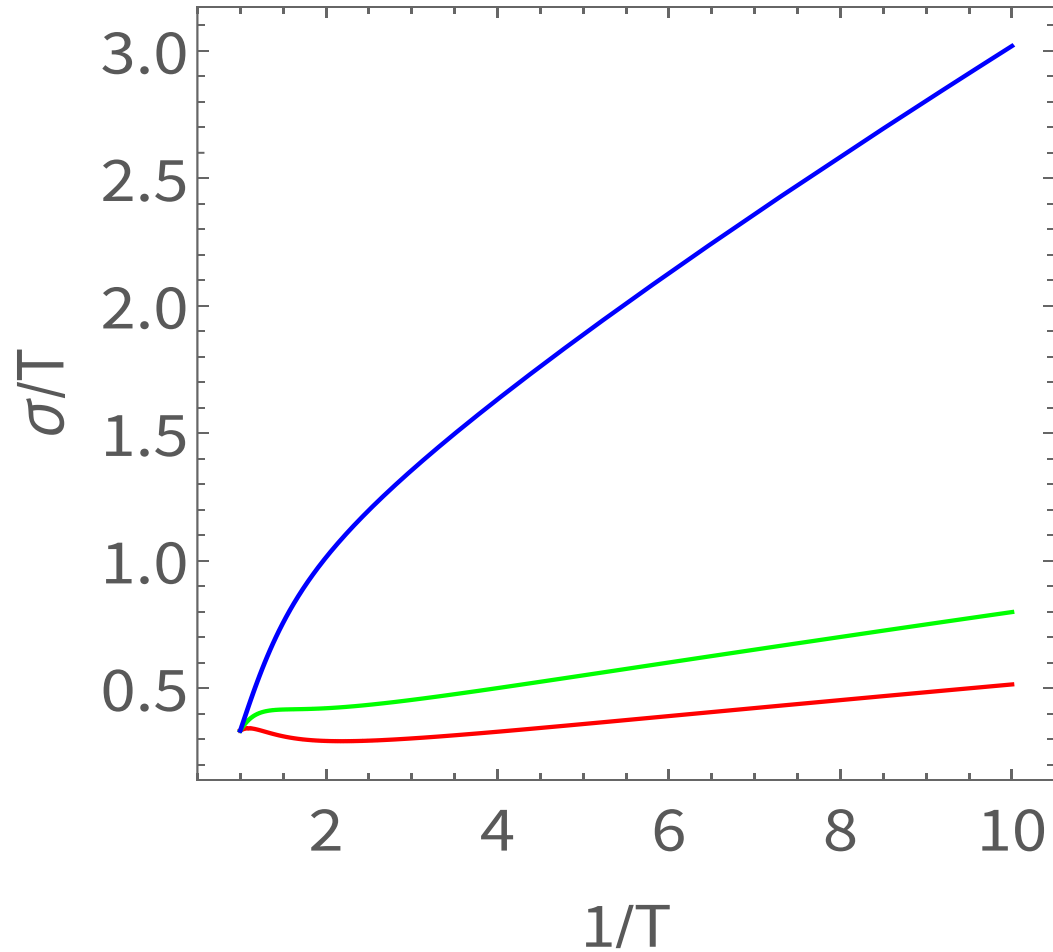
[Wikipedia article on Kondo effect]

*(from W.J. de Haas and G.J. van den Berg,
Physica vol. 3, page 440, 1936)*



$$\frac{1}{\sigma(T)} = a + bT^2 + c \log(\mu/T) + dT^5$$

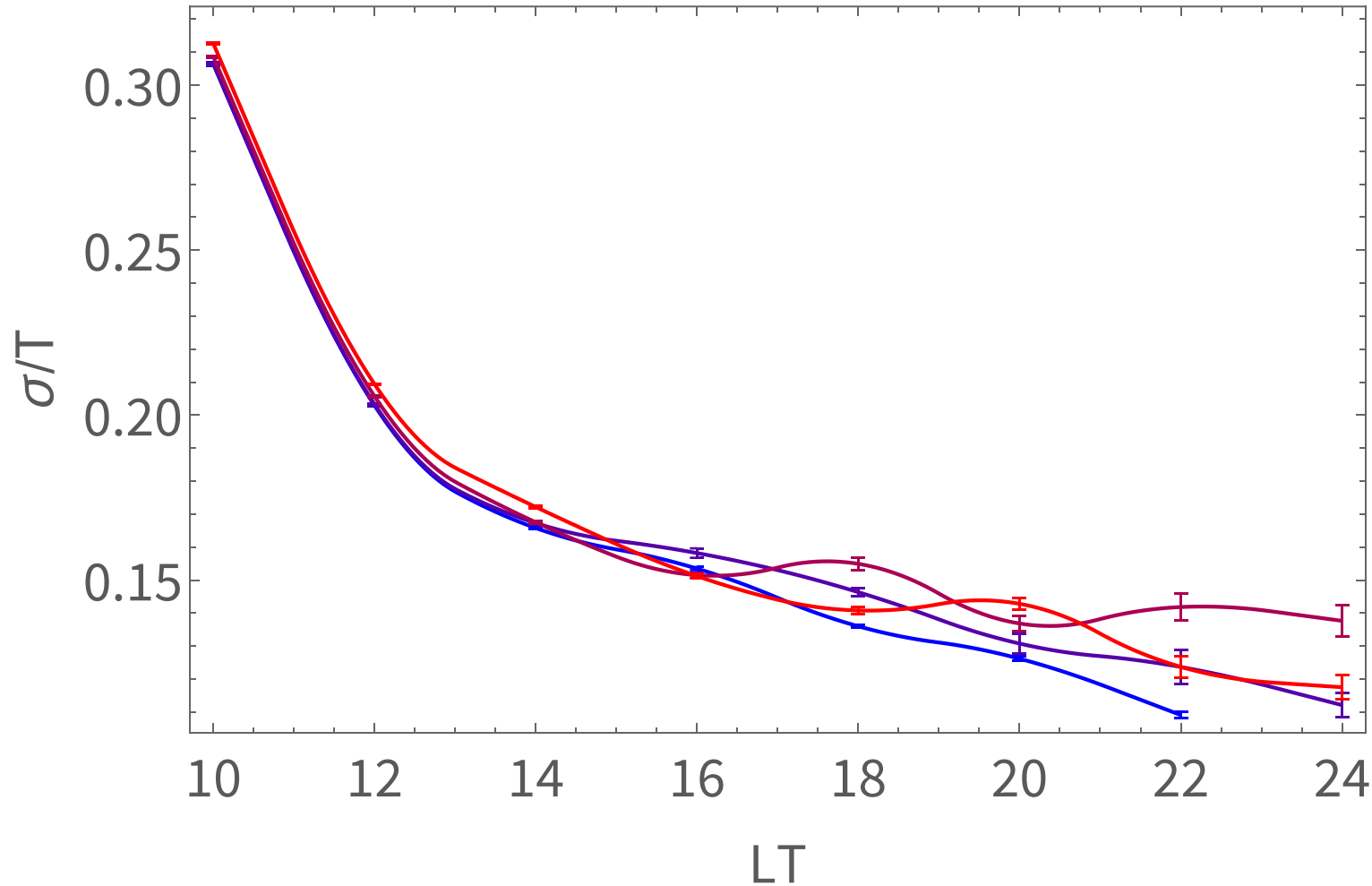
Kondo physics for electric conductivity?



- **Blue line** = no Kondo effect
- **Green line** = intermediate strength
- **Red line** = strong Kondo effect

Kondo physics for electric conductivity?

$$\mu = 0.05$$



Conductivity vs.
temperature:
lattice data

Overall dependence is
even **qualitatively**
different

No clear evidence

Conclusions

- CSE close to **free-quark result** at high temperatures and/or high densities
- Significant **suppression** at low temperatures and low densities
- σ_{CSE} approximately proportional to charge density rather than chemical potential
- Similar to **ChPT calculation** of [Avdoshkin,Sadofyev,Zakharov' 1712.01256] for axial non-flavor-singlet current, although non-singlet and singlet axial currents are physically quite different
- CSE can be **enhanced** in the presence of additional fermion flavors – signature of **Kondo effect**?
- **Measuring the Kondo condensate on the lattice?**